Extension of HyperGraph to n-SuperHyperGraph and to Plithogenic n-SuperHyperGraph, and Extension of HyperAlgebra to n-ary (Classical-/Neutro-/Anti-)HyperAlgebra

Florentin Smarandache 1,*

1 University of New Mexico, 705 Gurley Ave., Gallup Campus, New Mexico 87301, United States

* Correspondence: smarand@unm.edu

Abstract: We recall and improve our 2019 concepts of n-Power Set of a Set, n-SuperHyperGraph, Plithogenic n-SuperHyperGraph, and n-ary HyperAlgebra, n-ary NeutroHyperAlgebra, n-ary AntiHyperAlgebra respectively, and we present several properties and examples connected with the real world.

Keywords: n-Power Set of a Set, n-SuperHyperGraph (n-SHG), n-SHG-vertex, n-SHG-edge, Plithogenic n-SuperHyperGraph, n-ary HyperOperation, n-ary HyperAxiom, n-ary HyperAlgebra, n-ary NeutroHyperOperation, n-ary NeutroHyperAxiom, n-ary NeutroHyperAlgebra, n-ary AntiHyperOperation, n-ary AntiHyperAxiom, n-ary AntiHyperAlgebra

1. Introduction

In this paper, with respect to the classical HyperGraph (that contains HyperEdges), we add the SuperVertices (a group of vertices put all together form a SuperVertex), in order to form a SuperHyperGraph (SHG). Therefore, each SHG-vertex and each SHG-edge belong to \( P(V) \), where \( V \) is the set of vertices, and \( P(V) \) means the power set of \( V \).

Further on, since in our world we encounter complex and sophisticated groups of individuals and complex and sophisticated connections between them, we extend the SuperHyperGraph to n-SuperHyperGraph, by extending \( P(V) \) to \( P^n(V) \) that is the n-power set of the set \( V \) (see below).

Therefore, the n-SuperHyperGraph, through its n-SHG-vertices and n-SHG-edges that belong to \( P^n(V) \), can the best (so far) to model our complex and sophisticated reality.

In the second part of the paper, we extend the classical HyperAlgebra to n-ary HyperAlgebra and its alternatives n-ary NeutroHyperAlgebra and n-ary AntiHyperAlgebra.

2. n-Power Set of a Set

Let \( U \) be a universe of discourse, and a subset \( V \subseteq U \). Let \( n \geq 1 \) be an integer.

Let \( P(V) \) be the Power Set of the Set \( V \) (i.e. all subsets of \( V \), including the empty set \( \emptyset \) and the whole set \( V \)). This is the classical definition of power set.

For example, if \( V = \{a, b\} \), then \( P(V) = \{\emptyset, a, b, \{a, b\}\} \).

But we have extended the power set to n-Power Set of a Set [1].
For $n = 1$, one has the notation (identity): $P^1(V) = P(V)$.

For $n = 2$, the 2-Power Set of the Set $V$ is defined as follows:

$P^2(V) = P(P(V))$.

In our previous example, we get:

$P^3(V) = P(P(P(V)) = P(\{ \phi, a, b, [a, b] \}) = \{ \phi, \phi, a, \phi, b, \phi, b, [a, b], [a, b], [a, b] \}; \{ \phi, a, b, [a, b] \}$.

**Definition of n-Power Set of a Set**

In general, the $n$-Power Set of a Set $V$ is defined as follows:

$P^n(V) = P(P^{n-1}(V))$, for integer $n \geq 1$.

3. **Definition of SuperHyperGraph (SHG)**

A SuperHyperGraph (SHG) [1] is an ordered pair $SHG = (G \subseteq P(V), E \subseteq P(V))$, where

(i) $V = \{V_1, V_2, \ldots, V_m\}$ is a finite set of $m \geq 0$ vertices, or an infinite set.

(ii) $P(V)$ is the power set of $V$ (all subset of $V$). Therefore, an **SHG-vertex** may be a single (classical) vertex, or a super-vertex (a subset of many vertices) that represents a group (organization), or even an indeterminate-vertex (unclear, unknown vertex); $\phi$ represents the null-vertex (vertex that has no element).

(iii) $E = \{E_1, E_2, \ldots, E_n\}$, for $m \geq 1$, is a family of subsets of $V$, and each $E_i$ is an SHG-edge, $E_i \in P(V)$. An **SHG-edge** may be a (classical) edge, or a super-edge (edge between super-vertices) that represents connections between two groups (organizations), or hyper-super-edge (edge that means there is no connection between the given vertices).

4. **Characterization of the SuperHyperGraph**

Therefore, a SuperHyperGraph (SHG) may have any of the below:

- SingleVertices ($V_i$), as in classical graphs, such as: $V_1$, $V_2$, etc.;
- SuperVertices (or SubsetVertices) ($SV_i$), belonging to $P(V)$, for example: $SV_{1,1} = V_1V_3$, $SV_{2,5} = V_2V_5$, etc. that we introduce now for the first time. A super-vertex may represent a group (organization, team, club, city, country, etc.) of many individuals;

The comma between indexes distinguishes the single vertices assembled together into a single SuperVertex. For example $SV_{1,2,3}$ means the single vertex $S_{12}$ and single vertex $S_{13}$ are put together to form a super-vertex. But $SV_{1,2,3}$ means the single vertices $S_1$ and $S_2$ are put together; while $SV_{1,2,3}$ means $S_1$, $S_2$, $S_3$ as single vertices are put together as a super-vertex. In no comma in between indexes, i.e. $SV_{1,2,3}$ means just a single vertex $V_{123}$, whose index is 123, or $SV_{1,2,3} = V_{123}$.

- IndeterminateVertices (i.e. unclear, unknown vertices); we denote them as: $IV_1$, $IV_2$, etc. that we introduce now for the first time;

- NullVertex (i.e. vertex that has no elements, let’s for example assume an abandoned house, whose all occupants left), denoted by $\phi V$.
- **SingleEdges**, as in classical graphs, i.e. edges connecting only two single-vertices, for example: $E_{1,5} = \{V_1, V_5\}$, $E_{2,3} = \{V_2, V_3\}$, etc.;

- **HyperEdges**, i.e. edges connecting three or more single-vertices, for example $HE_{1,4,6} = \{V_1, V_4, V_6\}$, $HE_{2,4,5,7,8,9} = \{V_2, V_4, V_5, V_7, V_8, V_9\}$, etc. as in hypergraphs;

- **SuperEdges** (or **SubsetEdges**), i.e. edges connecting only two SHG-vertices (and at least one vertex is SuperVertex), for example $SE_{13,6,45,79} = \{SV_{13,6}, SV_{45,79}\}$ connecting two SuperVertices, $SE_{9,2,345} = \{SV_9, SV_{2,345}\}$ connecting one SingleVertex $V_9$ with one SuperVertex, $SV_{2,345}$, etc. that we introduce now for the first time;

- **HyperSuperEdges** (or **HyperSubsetEdges**), i.e. edges connecting three or more vertices (and at least one vertex is SuperVertex), for example $HSE_{3,45,236} = \{V_3, V_{45}, V_{236}\}$, $HSE_{1234,456789,567,5679} = \{SV_{1234}, SV_{456789}, SV_{567}, SV_{5679}\}$, etc. that we introduce now for the first time;

- **MultiEdges**, i.e. two or more edges connecting the same (single-/super-/indeterminate-) vertices; each vertex is characterized by many attribute values, thus with respect to each attribute value there is an edge, the more attribute values the more edges (= multiedge) between the same vertices;

- **IndeterminateEdges** (i.e. unclear, unknown edges; either we do not know their value, or we do not know what vertices they might connect): $IE_1, IE_2$, etc. that we introduce now for the first time;

- **NullEdge** (i.e. edge that represents no connection between some given vertices; for example two people that have no connections between them whatsoever): denoted by $\phi E$.

5. **Definition of the n-SuperHyperGraph (n-SHG)**

A **n-SuperHyperGraph** $(n\text{-SHG})$ [1] is an ordered pair $n\text{-SHG} = (G_n \subseteq P^n(V), E_n \subseteq P^n(V))$, where $P^n(V)$ is the $n$-power set of the set $V$, for integer $n \geq 1$.

6. **Examples of 2-SuperHyperGraph, SuperVertex, IndeterminateVertex, SingleEdge, Indeterminate Edge, HyperEdge, SuperEdge, MultiEdge, 2-SuperHyperEdge**
Let $V_1$ and $V_2$ be two single-vertices, characterized by the attributes $a_1 = \text{size}$, whose attribute values are \{short, medium, long\}, and $a_2 = \text{color}$, whose attribute values are \{red, yellow\}. Thus we have the attributes values \{(Size{short, medium, long}, Color{red, yellow})\}, whence: $V_1(a_1{s_1, m_1, l_1}, a_2{r_1, y_1})$, where $s_1$ is the degree of short, $m_1$ degree of medium, $l_1$ degree of long, while $r_1$ is the degree of red and $y_1$ is the degree of yellow of the vertex $V_1$.

And similarly $V_2(a_1{s_2, m_2, l_2}, a_2{r_2, y_2})$.

The degrees may be fuzzy, neutrosophic etc.

Example of fuzzy degree:

$V_1(a_1{0.8, 0.2, 0.1}, a_2{0.3, 0.5})$.

Example of neutrosophic degree:

$V_1(a_1{(0.7,0.3,0.0), (0.4,0.2,0.1),(0.3,0.1,0.1)}), a_2{(0.5,0.1,0.3), (0.0,0.2,0.7)})$.

Examples of the SVG-edges connecting single vertices $V_1$ and $V_2$ are below:

$\{V_1, V_2\} = E_{12}$
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and Intuitionistic Fuzzy-$n$-SHG-edge($a(t_1, f_1), a(t_2, f_2), \ldots, a(t_n, f_n)$);
Neutrosophic-$n$-SHG-vertex($a(t_1, i_1, f_1), a(t_2, i_2, f_2), \ldots, a(t_n, i_n, f_n)$)
and Neutrosophic-$n$-SHG-edge($a(t_1, i_1, f_1), a(t_2, i_2, f_2), \ldots, a(t_n, i_n, f_n)$);

etc.

Whence we get:

8. The Plithogenic ( Crip / Fuzzy / Intuitionistic Fuzzy / Picture Fuzzy / Spherical Fuzzy / etc. / Neutrosophic / Refined Neutrosophic ) $n$-SuperHyperGraph.

9. Introduction to $n$-ary HyperAlgebra
Let $U$ be a universe of discourse, a nonempty set $S \subset U$. Let $P(S)$ be the power set of $S$ (i.e. all subsets of $S$, including the empty set $\emptyset$ and the whole set $S$), and an integer $n \geq 1$.

We formed [2] the following neutrosophic triplets, which are defined in below sections:
$(n$-ary HyperOperation, $n$-ary NeutroHyperOperation, $n$-ary AntiHyperOperation),
$(n$-ary HyperAxiom, $n$-ary NeutroHyperAxiom, $n$-ary AntiHyperAxiom), and
$(n$-ary HyperAlgebra, $n$-ary NeutroHyperAlgebra, $n$-ary AntiHyperAlgebra).

10. $n$-ary HyperOperation ($n$-ary HyperLaw)
A $n$-ary HyperOperation ($n$-ary HyperLaw) $*_{n}$ is defined as:

$*_{n} : S^n \rightarrow P(S)$, and

$\forall a_1, a_2, \ldots, a_n \in S$ one has $*_{n}(a_1, a_2, \ldots, a_n) \in P(S)$.

The $n$-ary HyperOperation ($n$-ary HyperLaw) is well-defined.

11. $n$-ary HyperAxiom
A $n$-ary HyperAxiom is an axiom defined of $S$, with respect the above $n$-ary operation $*_{n}$, that is true for all $n$-plets of $S^n$.

12. $n$-ary HyperAlgebra
A $n$-ary HyperAlgebra $(S, *_{n})$, is the $S$ endowed with the above $n$-ary well-defined HyperOperation $*_{n}$.

13. Types of $n$-ary HyperAlgebras
Adding one or more $n$-ary HyperAxioms to $S$ we get different types of $n$-ary HyperAlgebras.

14. $n$-ary NeutroHyperOperation ($n$-ary NeutroHyperLaw)
A $n$-ary NeutroHyperOperation is a $n$-ary HyperOperation $*_{n}$ that is well-defined for some $n$-plets of $S^n$ [i.e. $\exists (a_1, a_2, \ldots, a_n) \in S^n, *_{n}(a_1, a_2, \ldots, a_n) \in P(S)$],

and indeterminate [i.e. $\exists (b_1, b_2, \ldots, b_n) \in S^n, *_{n}(b_1, b_2, \ldots, b_n) = \text{indeterminate}$]
or outer-defined [i.e. \( \exists (c_1, c_2, \ldots, c_n) \in S^n, *_n (c_1, c_2, \ldots, c_n) \notin P(S) \)] (or both), on other \( n \)-plets of \( S^n \).

15. n-ary NeutroHyperAxiom

A \( n \)-ary NeutroHyperAxiom is an \( n \)-ary HyperAxiom defined of \( S \), with respect the above \( n \)-ary operation \( *_n \) that is true for some \( n \)-plets of \( S^n \), and indeterminate or false (or both) for other \( n \)-plets of \( S^n \).

16. n-ary NeutroHyperAlgebra is an \( n \)-ary HyperAlgebra that has some \( n \)-ary NeutroHyperOperations or some \( n \)-ary NeutroHyperAxioms.

17. n-ary AntiHyperOperation (n-ary AntiHyperLaw)

A \( n \)-ary AntiHyperOperation is a \( n \)-ary HyperOperation \( *_n \) that is outer-defined for all \( n \)-plets of \( S^n \) [i.e. \( \forall (s_1, s_2, \ldots, s_n) \in S^n, *_n (s_1, s_2, \ldots, s_n) \notin P(S) \)].

18. n-ary AntiHyperAxiom

A \( n \)-ary AntiHyperAxiom is an \( n \)-ary HyperAxiom defined of \( S \), with respect the above \( n \)-ary operation \( *_n \) that is false for all \( n \)-plets of \( S^n \).

19. n-ary AntiHyperAlgebra is an \( n \)-ary HyperAlgebra that has some \( n \)-ary AntiHyperOperations or some \( n \)-ary AntiHyperAxioms.

20. Conclusion

We have recalled our 2019 concepts of \( n \)-Power Set of a Set, \( n \)-SuperHyperGraph and Plithogenic \( n \)-SuperHyperGraph [1], afterwards the \( n \)-ary HyperAlgebra together with its alternatives \( n \)-ary NeutroHyperAlgebra and \( n \)-ary AntiHyperAlgebra [2], and we presented several properties, explanations, and examples inspired from the real world.

References


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