Introduction to neutrosophic soft topological spatial region

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Abstract. Spatial information often deals with regions which are vague or incompletely determined. Understanding vagueness, indeterminacy and imprecision are the most important in GIS. Smarandache’s neutrosophic set is a computational method to tackle problems involving incomplete, infinite and reliable data. The definition of soft sets was introduced by Molodtsov as a new mathematical method to tackle uncertainty. Maji presented the Neutrosophic Soft Set theory. This paper provides concepts of a neutrosophic soft spatial region for its possible application in GIS. The notions of neutrosophic soft α-open, neutrosophic soft pre-open, neutrosophic soft semi-open and neutrosophic soft β-open sets are introduced.

Keywords: Neutrosophic soft set; neutrosophic soft topology; neutrosophic soft connected; neutrosophic soft spatial region; GIS.

1. Introduction

Many real-life issues deal with uncertainties in economics, engineering, environment, social sciences, medical sciences, and business management. There are difficulties with classical mathematical modeling in solving the uncertainties in these data. Theories such as fuzzy set[1], rough set[2] and intuitionist fuzzy set[3] are used to prevent difficulties in dealing with uncertainty. But all of these hypotheses have some difficulties in addressing the indeterminate or contradictory data problems. Smarandache[4] described the neutrosophical set as a mathematical method for dealing with indeterminate and inaccurate problems in nature. There is a lot of use in all fields, such as IT, information systems and decision support systems.
Abdel-Basset has developed a Novel Intelligent Medical Decision Support Model based on soft computing and IoT as the use of neutrosophical sets for decision-making. In the researchers developed neutrosophic multi-criterion approach to help healthcare professionals predict illness. In a solution is proposed to Neutrosophic Linear Fractional Programming Problem (NLFP) in the case of triangular neutrosophic number costs of the objective function, capital and engineering coefficients. In the researchers suggest the method to help the patient and doctor know whether the patient is having a heart failure through neutrophic multi-criteria decision making (NMCDM).

The neutrosophical topological space theory was proposed in. Further neutrosophic topological space was studied in. Subsequently, the sets were added similar to the neutrosophic open and neutrosophic closed sets. Neutrosophic semi-open set and neutrosophic semi-closed sets have been introduced by Iswaraya et al. Imran et al. proposed neutrosophic semi-α open sets and analysed their basic properties. Arokiarani et al. studied about neutrosophic semi-open (resp. pre-open and α-open) functions and examined their relations. Rao et al. proposed neutrosophic pre-open sets.

In the researchers investigate new kind of neutrosophic continuity in neutrosophic topological spaces known as Neutrosophic αgs continuity maps and also the properties and characterization Neutrosophic αgs Irresolute Maps were examined. Anitha et al. proposed the concept of NGSR-closed sets and NGSR-open sets. NGSR continuous and NGSR-contra continuous mappings are also further studied. Dhavaseelan et al. introduced neutrosophic almost α-contra-continuous function and studied their properties. In the authors introduced neutrosophic generalized b-closed sets and Neutrosophic generalized b-continuity in Neutrosophic topological spaces.

Molodstov introduced the soft set theory as a computational method for tackling insecurity. Maji combined the concept of soft set and neutrosophic set together by introducing the current mathematical framework called neutrosophic soft set. In neutrosophic soft set was applied in making decision. Several researchers applied in various mathematical systems the concept of neutrosophic soft sets. Bera introduced neurosophic soft topological spaces. Neutrosophic spatial region as introduced by A.A.Salama. This paper explores the theory and some of its features of neutrosophic soft topological space. The notions of neutrosophic soft α-open, neutrosophic soft pre-open, neutrosophic soft semi-open and neutrosophic soft β-open sets are introduced. Furthermore, for possible application in GIS, the simple neutrosophic soft region is introduced.
2. Preliminaries

Definition 2.1. \((F, E)\) is a soft set in \(X\) where \(F : E \rightarrow \mathcal{P}(Y)\) is a mapping where \(\mathcal{P}(Y)\) is a power set of \(Y\). We express \((F, E)\) by \(\widetilde{F}\). \(\widetilde{F} = \{(e, F(e)) : e \in E\}\).

Definition 2.2. \((A)\). A neutrosophic set (NS) \(A\) on \(Y\) is defined as: \(A = \{<y, T_A(y), I_A(y), F_A(y)> : y \in Y\}\) where \(T, I, F : Y \rightarrow \mathbb{R}^{+, 0, -}\) and \(0 \leq T_A(y) + I_A(y) + F_A(y) \leq 3\)

Definition 2.3. Let \(Y\) be an set and \(E\) be parameter set. Let \(\mathcal{P}(Y)\) denotes the set of all neutrosophic soft set (NSS) of \(Y\). Then \((F, E)\) is called a NSS over \(Y\) where \(F : E \rightarrow \mathcal{P}(Y)\) is a mapping. We express the NSS \((F, E)\) by \(\widetilde{F}_N\).

That is, \(\widetilde{F}_N = \{(e, < y, T_{\widetilde{F}_N(e)}(y), I_{\widetilde{F}_N(e)}(y), F_{\widetilde{F}_N(e)}(y) > : y \in Y\} e \in E\}

Definition 2.4. The complement of the NSS \(\widetilde{F}_N\) is denoted by \((\widetilde{F}_N)^c\) and is defined by \(\widetilde{F}_N^c = \{(e, < y, F_{\widetilde{F}_N(e)}(y), I_{\widetilde{F}_N(e)}(y), T_{\widetilde{F}_N(e)}(y) > : y \in Y\} e \in E\}

Definition 2.5. For any two NSS \(\widetilde{F}_N\) and \(\widetilde{G}_N\) over \(Y\), \(\widetilde{F}_N\) is a neutrosophic soft subset of \(\widetilde{G}_N\) if \(T_{\widetilde{F}_N(e)}(y) \leq T_{\widetilde{G}_N(e)}(y)\); \(I_{\widetilde{F}_N(e)}(y) \leq I_{\widetilde{G}_N(e)}(y)\); \(F_{\widetilde{F}_N(e)}(y) \geq F_{\widetilde{G}_N(e)}(y)\); for all \(e \in E\) and \(y \in Y\).

Definition 2.6. A NSS \(\widetilde{F}_N\) over \(Y\) is said to be null NSS if \(T_{\widetilde{F}_N(e)}(y) = 0\); \(I_{\widetilde{F}_N(e)}(y) = 0\); \(F_{\widetilde{F}_N(e)}(y) = 1\); for all \(e \in E\) and \(y \in Y\). It is denoted by \(\widetilde{\Phi}_N\).

Definition 2.7. A NSS \(\widetilde{F}_N\) over \(Y\) is said to be absolute NSS if \(T_{\widetilde{F}_N(e)}(y) = 1\); \(I_{\widetilde{F}_N(e)}(y) = 1\); \(F_{\widetilde{F}_N(e)}(y) = 0\); for all \(e \in E\) and \(y \in Y\). It is denoted by \(\widetilde{Y}_N\).

Definition 2.8. The union of two NSS \(\widetilde{F}_N\) and \(\widetilde{G}_N\) is denoted by \(\widetilde{F}_N \cup \widetilde{G}_N\) and is defined by \(\widetilde{H}_N = \widetilde{F}_N \cup \widetilde{G}_N\), where the truth-membership, indeterminacy-membership and falsity membership of \(\widetilde{H}_N\) are as follows:

\[
T_{\widetilde{H}_N(e)}(y) = \begin{cases} T_{\widetilde{F}_N(e)}(y) & \text{if } e \in A - B \\ T_{\widetilde{G}_N(e)}(y) & \text{if } e \in B - A \\ \max\{T_{\widetilde{F}_N(e)}(y), T_{\widetilde{G}_N(e)}(y)\} & \text{if } e \in A \cap B \end{cases}
\]

\[
I_{\widetilde{H}_N(e)}(y) = \begin{cases} I_{\widetilde{F}_N(e)}(y) & \text{if } e \in A - B \\ I_{\widetilde{G}_N(e)}(y) & \text{if } e \in B - A \\ \frac{I_{\widetilde{F}_N(e)}(y) + I_{\widetilde{G}_N(e)}(y)}{2} & \text{if } e \in A \cap B \end{cases}
\]
If 

\[ F_{h_{N}(e)}(y) = \begin{cases} 
F_{F_{N}(e)}(y) & \text{if } e \in A - B \\
F_{G_{N}(e)}(y) & \text{if } e \in B - A \\
\min\{F_{F_{N}(e)}(y), F_{G_{N}(e)}(y)\} & \text{if } e \in A \cap B 
\end{cases} \]

**Definition 2.9.** The intersection of two NSS \( F_{N} \) and \( G_{N} \) is denoted by \( F_{N} \cap G_{N} \) and is defined by \( \bar{H}_{N} = F_{N} \cap G_{N} \), where the truth-membership, indeterminacy-membership and falsity membership of \( \bar{H}_{N} \) are as follows

\[ T_{\bar{H}_{N}(e)}(y) = \min\{T_{F_{N}(e)}(y), T_{G_{N}(e)}(y)\}, \]

\[ I_{\bar{H}_{N}(e)}(y) = \frac{I_{F_{N}(e)}(y) + I_{G_{N}(e)}(y)}{2}, \]

\[ F_{\bar{H}_{N}(e)}(y) = \max\{F_{F_{N}(e)}(y), F_{G_{N}(e)}(y)\} \]

3. Neutrosophic soft topological space

**Definition 3.1.** Let \( NSS(Y, E) \) be the family of all NSS over \( Y \) and \( \tau_{N} \subset NSS(Y, E) \). Then \( \tau_{N} \) is called neutrosophic soft topology(NST) on \( (Y, E) \) if the following conditions are satisfied:

(i) \( \tilde{\Phi}_{N}, \tilde{Y}_{N} \subset \tau_{N} \)

(ii) \( \tau_{N} \) is closed under arbitrary union.

(iii) \( \tau_{N} \) is closed under finite intersection.

Then the triplet \( (Y, \tau_{N}, E) \) is called neutrosophic soft topological space(NSTS). The members of \( \tau_{N} \) are called neutrosophic soft open sets in \( (Y, \tau_{N}, E) \). A NSS \( F_{N} \) in \( NSS(Y, E) \) is soft closed in \( (Y, \tau_{N}, E) \) if its complement \( (F_{N})^{e} \) is neutrosophic soft open set in \( (Y, \tau_{N}, E) \).

The neutrosophic soft closure of \( F_{N} \) is the NSS, \( Nscl(F_{N}) = \cap\{G_{N} : G_{N} \text{ is neutrosophic soft closed and } F_{N} \subseteq G_{N}\} \).

The neutrosophic soft interior of \( F_{N} \) is the NSS, \( Nsint(F_{N}) = \cup\{O_{N} : O_{N} \text{ is neutrosophic soft closed and } O_{N} \subseteq F_{N}\} \).

It is easy to see that \( F_{N} \) is neutrosophic soft open if and only if \( F_{N} = Nsint(F_{N}) \) and neutrosophic soft closed if and only if \( \tilde{F}_{N} = Nscl(F_{N}) \).

**Theorem 3.2.** Let \( (Y, \tau_{N}, E) \) be a NSTS over \( (Y, E) \) and \( F_{N} \) and \( G_{N} \in NSS(Y, E) \) then

(i) \( Nsint(F_{N}) \subset F_{N} \) and \( Nsint(F_{N}) \) is the largest open set.

(ii) \( \tilde{F}_{N} \subset F_{N} \) implies \( Nsint(F_{N}) \subset Nsint(\tilde{F}_{N}) \)

(iii) \( Nsint(F_{N}) \) is an neutrosophic soft open set. That is \( Nsint(F_{N}) \in \tau_{N} \)

(iv) \( F_{N} \) is neutrosophic soft open iff \( Nsint(F_{N}) = F_{N} \)

(v) \( Nsint(Nsint(F_{N})) = Nsint(\tilde{F}_{N}) \)

(vi) \( Nsint(\tilde{F}_{N}) = \tilde{F}_{N} \) and \( Nsint(\tilde{Y}_{N}) = \tilde{Y}_{N} \)

(vii) \( Nsint(F_{N} \cap G_{N}) = Nsint(F_{N}) \cap Nsint(G_{N}) \)

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(viii) $Nsint(\tilde{F}_N) \cup Nsint(\tilde{G}_N) \subset Nsint(\tilde{F}_N \cup \tilde{G}_N)$

**Theorem 3.3.** Let $(Y, \tilde{\tau}_N, E)$ be a NSTS $(Y, E)$ and $\tilde{F}_N$ and $\tilde{G}_N \in NSS(Y, E)$ then

(i) $\tilde{F}_N \subset Nsc(\tilde{F}_N)$ and $Nsc(\tilde{F}_N)$ is the smallest closed set
(ii) $\tilde{F}_N \subset \tilde{F}_N$ implies $Nsc(\tilde{F}_N) \subset Nsc(\tilde{F}_N)$
(iii) $Nsc(\tilde{F}_N)$ is neutrosophic soft closed set. That is $Nsc(\tilde{F}_N) \in (\tilde{\tau}_N)^c$
(iv) $\tilde{F}_N$ is neutrosophic soft closed iff $Nsc(\tilde{F}_N) = \tilde{F}_N$
(v) $Nsc(Nsc(\tilde{F}_N)) = Nsc(\tilde{F}_N)$
(vi) $Nsc(\tilde{\Phi}_N) = \tilde{\Phi}_N$ and $Nsc(\tilde{Y}_N) = \tilde{Y}_N$
(vii) $Nsc(\tilde{F}_N \cup \tilde{G}_N) = Nsc(\tilde{F}_N) \cup Nsc(\tilde{G}_N)$
(viii) $Nsc(\tilde{F}_N) \cap Nsc(\tilde{G}_N) \subset Nsc(\tilde{F}_N \cap \tilde{G}_N)$

4. Neutrosophic soft nearly open sets

**Definition 4.1.** Let $(Y, \tilde{\tau}_N, E)$ be a NSTS and $\tilde{F}_N$ be a neutrosophic soft open set in $(Y, E)$, then $\tilde{F}_N$ is called

(i) Neutrosophic soft $\alpha$-open iff $\tilde{F}_N \subseteq Nsint(Nsc(\tilde{F}_N)))$
(ii) Neutrosophic soft pre-open iff $\tilde{F}_N \subseteq Nsint(Nsc(\tilde{F}_N))$
(iii) Neutrosophic soft semi-open iff $\tilde{F}_N \subseteq Nscl(Nscl(\tilde{F}_N))$
(iv) Neutrosophic soft $\beta$-open iff $\tilde{F}_N \subseteq Nscl(Nscl(Nscl(\tilde{F}_N))))$
(v) Neutrosophic soft regular-open iff $\tilde{F}_N = Nsint(Nscl(\tilde{F}_N))$

**Definition 4.2.** Let $(Y, \tilde{\tau}_N, E)$ be a NSTS and $\tilde{F}_N \in NSS(Y, E)$, then $\tilde{F}_N$ is called

(i) Neutrosophic soft $\alpha$-closed iff $Nsc(Nscl(Nscl(\tilde{F}_N)))) \subseteq \tilde{F}_N$
(ii) Neutrosophic soft pre-closed iff $Nsc(Nscl(\tilde{F}_N)) \subseteq \tilde{F}_N$
(iii) Neutrosophic soft semi-closed iff $Nscl(Nscl(\tilde{F}_N)) \subseteq \tilde{F}_N$
(iv) Neutrosophic soft $\beta$-closed iff $Nsint(Nscl(Nscl(\tilde{F}_N)))) \subseteq \tilde{F}_N$
(v) Neutrosophic soft regular-closed iff $\tilde{F}_N = Nsc(Nscl(\tilde{F}_N))$

5. Neutrosophic soft region

Topological relationships have played a significant role during space search, analysis and reasoning through Geographical information systems (GIS) and Geospatial databases. The topological relations between smooth, unstable and fuzzy spatial regions have been developed on the basis of the nine-intersection model. In the past couple of decades a lot of emphasis has been given to the topological relationship research issue, particularly between uncertain spatial regions. Nevertheless, formal representation and calculation of topological links between unknown regions remains an open issue and needs further investigation. We discuss further Evanzalin, Jude and Sivaranjani, Introduction to neutrosophic soft topological spatial region
definitions and proposals for a neutrosophic soft topological region, which provide an theoretical framework for the modeling of neutrosophic soft topology relations among uncertain regions.

**Definition 5.1.** Let \((Y, \tilde{\tau}_N, E)\) be a NSTS over \((Y, E)\) and \(\tilde{F}_N \in NSS(Y, E)\). Then neutrosophic soft boundary of \(\tilde{F}_N\) is defined by \(\partial \tilde{F}_N = Nscl(\tilde{F}_N) \cap Nscl((\tilde{F}_N)^c)\)

**Definition 5.2.** Let \((Y, \tilde{\tau}_N, E)\) be a NSTS over \((Y, E)\). Then the neutrosophic soft exterior of \(\tilde{F}_N \in NSS(Y, E)\) is denoted by \((\tilde{F}_N)_o\) and is defined by \((\tilde{F}_N)_o = Nsint((\tilde{F}_N)^c)\)

**Theorem 5.3.** Let \(\tilde{F}_N\) and \(\tilde{G}_N\) be two NSS over \((Y, E)\). Then

(i) \((\tilde{F}_N)_o = Nsint((\tilde{F}_N)^c)\)

(ii) \((\tilde{F}_N \cup \tilde{G}_N)_o = (\tilde{F}_N)_o \cap (\tilde{G}_N)_o\)

(iii) \((\tilde{F}_N)_o \cup (\tilde{G}_N)_o \subset (\tilde{F}_N \cap \tilde{G}_N)_o\)

**Theorem 5.4.** Let \((Y, \tilde{\tau}_N, E)\) be a NSTS over \((Y, E)\) and \(\tilde{F}_N, \tilde{G}_N \in NSS(Y, E)\). Then

(i) \((\partial \tilde{F}_N)^c = Nsint(\tilde{F}_N) \cup Nsint((\tilde{F}_N)^c)\)

(ii) \(Nscl(\tilde{F}_N) = Nsint(\tilde{F}_N) \cup \partial \tilde{F}_N\)

(iii) \(\partial \tilde{F}_N = Nscl(\tilde{F}_N) \cap Nscl((\tilde{F}_N)^c)\)

(iv) \(\partial \tilde{F}_N \cap Nsint(\tilde{F}_N) = \tilde{\Phi}_N\)

(v) \(\partial(\partial(\partial(\tilde{F}_N))) = \partial(\partial(\tilde{F}_N))\)

**Definition 5.5.** Let \((Y, \tilde{\tau}_N, E)\) be a NSTS over \((Y, E)\). Then a pair of non-empty neutrosophic soft open sets \(\tilde{F}_N, \tilde{G}_N\) is called a neutrosophic soft separation of \((Y, \tilde{\tau}_N, E)\) if \(\tilde{Y}_N = \tilde{F}_N \cup \tilde{G}_N\) and \(\tilde{F}_N \cap \tilde{G}_N = \tilde{\Phi}_N\)

**Definition 5.6.** A NSTS \((Y, \tilde{\tau}_N, E)\) is said to be neutrosophic soft connected if there does not exist a neutrosophic soft separation of \((Y, \tilde{\tau}_N, E)\). Otherwise \((Y, \tilde{\tau}_N, E)\) is said to be neutrosophic soft disconnected.

Now we shall describe a model for basic spatial neutrosophic soft region based on neutrosophic soft connectedness.

**Definition 5.7.** Let \((Y, \tilde{\tau}_N, E)\) be a NSTS. A spatial neutrosophic soft region in \((Y, E)\) is a non empty neutrosophic soft subset \(\tilde{F}_N\) such that

(i) \(Nsint(\tilde{F}_N)\) is neutrosophic soft connected.

(ii) \(\tilde{F}_N = Nscl(Nsint(\tilde{F}_N))\)

6. **Conclusion**

The neutrosophic soft 4-intersection model can be implemented as an application to GIS for neutrosophic soft topological relationships between neutrosophic soft regions with sharp Evanzalin, Jude and Sivaranjani, Introduction to neutrosophic soft topological spatial region
neutrosophical soft boundaries and for neutrosophic soft regions with broad neutrosophical soft boundaries. These models can be used to formulate spatial database consistency constraints and can also be used in information systems such as mobile robots and route navigation systems.

References


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