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Interval Valued Intuitionistic Neutrosophic Soft Set and its Application on Diagnosing Psychiatric Disorder by Using Similarity Measure

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Abstract. The primary focus of this manuscript comprises three sections. Initially, we discuss the notion of an interval-valued intuitionistic neutrosophic soft set. We impose an intuitionistic condition between the membership grades of truth and falsity such that their supremum sum does not exceed unity. Similarly, for indeterminacy, the membership grade is in interval from the closed interval $[0, 1]$. Hence in this case, the supremum sum of membership grades of truth, indeterminacy, and falsity does not exceed two. We present the notion of necessity, possibility, concentration, and dilation operators and establish some of its properties. Second, we define the similarity measure between two interval-valued intuitionistic neutrosophic soft sets. Also, we discuss its superiority by comparing it with existing methods. Finally, we develop an algorithm and illustrate with an example of diagnosing psychiatric disorders. Even though the similarity measure plays a vital role in diagnosing psychiatric disorders, existing methods deal hardly in diagnosing psychiatric disorders. By nature, most of the psychiatric disorder behaviors are ambivalence. Hence, it is vital to capture the membership grades by using interval-valued intuitionistic neutrosophic soft set. In this manuscript, we provide a solution in diagnosing psychiatric disorders, and the proposed similarity measure is valuable and compatible in diagnosing psychiatric disorders in any neutrosophic environment.

Keywords: neutrosophic set, intuitionistic neutrosophic set; similarity measures; decision making.

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1. Introduction

Zadeh [35] coined the notion of a fuzzy set (FS) to the world. In FS theory, the membership grade of each element in a set is specified by a real number from the closed interval of $[0, 1]$. Later, Atanassov [3] defined the notion of an intuitionistic FS (IFS) as an extension of FS. In IFS theory, the elements are assumed to posses both membership and non-membership grades with the condition that their sum does not exceed unity. Also, Atanassov [5] established some properties of IFS. Both FS and IFS theories have significant roles in handling decision-making problems. But in today's decision-making scenarios, the primary focus of the decision-makers (DMs) is to select the best option under different precise or imprecise criteria. DMs may fall short of adequate level of knowledge of the problem in cognitive terms and therefore have difficulty in selecting the right object. This difficulty is overcome by the use of the notion of neutrosophic set (NS), which is characterized by the grades of truth, indeterminacy and falsity membership for each element of the set. Smarandache [27] presented the concept of NS. DMs applied this concept widely to show the importance of truth, indeterminacy, and falsity information on which humans handle the decisions. Wang et al. [30] defined the concept of single-valued NS (SVNS) with the restricted conditions for the membership grades to facilitate the real-life applications and to overcome the constraints faced in neutrosophic theory. We cite some recent developments of IFS, NS theories, and also on similarity measures (SMs) below. Beg and Tabasam [7] introduced the concept of comparative linguistic expression based on hesitant IFSs. Anita et al. [2] developed an application to solve multi-criteria decision-making (MCDM) problems using interval-valued IFS of root type. Jianming et al. [22] defined the concepts of weighted aggregation operators in neutrosophic cubic sets (NCSs) and provided applications in MCDM. Majid Khan et al. [24] presented the notions of algebraic and Einstein operators on NCSs and developed an MCDM application based on these operators. Hashim et al. [20] introduced SMs in neutrosophic bipolar FS with a purpose to build a children's hospital with the help of the HOPE foundation. Chinnadurai et al. [15] presented the concept of unique ranking by using the parameters in a neutrosophic environment. Chinnadurai and Bobin [16] used prospect theory in real-life applications to solve MCDM problems. Saranya et al. [25] introduced an application for finding the similarity value of any two NSs in the neutrosophic environment by using programming language. Broumi and Smarandache [9] developed SMs using Hausdorff distance. Liu et al. [23] introduced the concept of SMs using Euclidean distance. Chahhtterjee et al. [13] presented various concepts of SMs in neutrosophic environment. Shahzadi et al. [26] diagnosed the medical symptoms using SVNSs. Hamidi and Borumand [19] developed the concept of neutrosophic graphs to analyze the sensor networks.

Smarandache [28] studied the concept of soul in psychology by using neutrosophic theory. Christianto and Smarandache [17] reviewed the concept of cultural psychology as one of the seven applications using neutrosophic logic. Chicaiza et al. [14] studied the concept of emotional intelligence of the students using neutrosophic psychology. Abdel-Basset et al. [1] used cosine SMs in bipolar and interval-valued bipolar SVNS to diagnose bipolar disorder behaviors. The domination of NS and SVNS in psychology is clear from the above-cited works. Hence, in this research, we enlighten the concept of neutrosophic theory in the field of psychology. In general, the psychotherapist considers that each person holds a mental structure called the self, which acts as an origin of personality. Winnicott [33] introduced the concept of true self and false self. He applied the thought of true-self to sense out the individual's aliveness or conceiving real. True-self is always be in part or hidden completely. The notion of false-self acts as a defense mechanism to protect the true-self by hibernating it. He explained that the behavior of a person in society is a gentle and self-conscious attitude because of the defense mechanism of false-self. In brief, we can establish the separation of true and false self on a continuum between the normal and the pathological behaviors. There always exists a doubtful amount of unconscious substance that pertains to the whole of the self. The above statements explain the need for an intuitionistic neutrosophic set (INS) to deal with this new concept. We cite the literature review of INS and its role in decision making below. Bhowmik and Pal [8] presented the concept of INS and studied its properties. Broumi and Smarandache [10] defined the concept of intuitionistic neutrosophic soft set (INSS) and established some of its properties. Both INS and INSS have a significant role in handling decision-making problems. They defined the restricted conditions as i) the minimum of membership grades between truth and indeterminacy to be less than or equal to 0.5, ii) the minimum of membership grades between truth and falsity to be less than or equal to 0.5, and iii) the minimum of membership grades between falsity and indeterminacy to be less than or equal to 0.5, such that the sum of membership grades of truth, indeterminacy, and falsity cannot exceed two. Let us consider an example $\mathcal{N} = \langle 0.4, 0.7, 0.6 \rangle$. Now according to INS definition, we have $\min\{0.4, 0.6\} < 0.5$, $\min\{0.4, 0.7\} < 0.5$ and $\min\{0.6, 0.7\} \nless 0.5$ but satisfies the condition $0 < 0.4 + 0.7 + 0.6 < 2$. It is evident that the given example is not an INS. However, the DM may have a situation where the membership grades of falsity and indeterminacy are greater than 0.5. Similarly, let us consider another example $\mathcal{N} = \langle 0.7, 0.8, 0.3 \rangle$. According to INS definition, we have $\min\{0.7, 0.3\} < 0.5$, $\min\{0.8, 0.3\} < 0.5$ and $\min\{0.7, 0.8\} \nless 0.5$ but satisfies the condition $0 < 0.7+0.8+0.3 < 2$. It is clear that the given example is not an INS. However, the DM may have a situation where the membership grades of truth and falsity are greater than 0.5. Therefore DM may have some constraints while handling these information in INS environment. Now, when the membership grades are in interval as well the membership grades of true and false are a continuum and the membership grade of indeterminacy is independent, it becomes a challenge to input the grades during decision-making with the help of INS and INSS. Hence, it is very clear that there is a need for a new INS with simplified conditions. So, we introduce an interval-valued INS (IVINS) and interval-valued INSS (IVINSS) to effectively handle the decision-making problems.

One of the purpose of this study is to bring out the importance of IVINSS when experts provide membership grades in truth, indeterminacy, and falsity in a restricted environment. In recent years, human beings face many decisions-making problems in multiple fields and analyzing the psychiatric disorder of the subject is one of them. Similarly, SM plays a significant factor in diagnosing psychiatric disorders, but hardly no existing methods deal with it. Therefore, it is necessary to provide a working model for determining the same. DMs look for many novel extensions of the NS to compete with other working models. So, we propose a new extension of NS and a model for diagnosing the psychiatric disorders using SM, which provides an advantage for the DMs to make well-defined decisions.

The manuscript enfolds the following sections. Section 2 provides a glimpse of existing definitions. Section 3 introduces the concept of IVINSS and some basic definitions related to IVINSS. Section 4 and 5 deal with the necessity, possibility, and two new operators ($\pm$ and $\mp$) with their properties. Section 6 explains the concept of $\mathcal{N}_{\epsilon}$, $\mathcal{N}_{\epsilon,\rho}$ and $\mathcal{I}_{\epsilon,\rho}$ operators on IVINSS. Section 7 provides insight on concentration and dilation operators on SINSS. Section 8 highlights the concept of SM with a new definition and also with a comparison study to show the importance of the proposed method. Section 9 wraps up with a conclusion.

2. **Preliminaries**

We discuss some essential definitions required for this study. Let us consider the following notations throughout this manuscript unless otherwise specified . Let $\mathcal{V}$ represent universe and $v \in \mathcal{V}$, $\mathcal{Q}$ be a set of parameters, $\mathcal{S} \subseteq \mathcal{Q}$, C[0,1] denotes the set of all closed sub intervals of [0,1] and $\mathcal{I}^N$ be the set of all IVINS over $\mathcal{V}$.

Definition 2.1. [6] An interval-valued IFS (IVIFS) is a set of the form $\mathcal{F} = \{\langle v, \alpha_{\mathcal{F}}(v), \gamma_{\mathcal{F}}(v)\rangle\}$, where $\alpha_{\mathcal{F}}(v) : \mathcal{V} \to C[0,1]$ and $\gamma_{\mathcal{F}}(v) : \mathcal{V} \to C[0,1]$ are the interval-valued membership and non-membership grades respectively. The lower and upper ends of $\alpha_{\mathcal{F}}(v)$ and $\gamma_{\mathcal{F}}(v)$ are denoted by $\underline{\alpha}_{\mathcal{F}}(v)$, $\overline{\alpha}_{\mathcal{F}}(v)$ and $\underline{\gamma}_{\mathcal{F}}(v)$, $\overline{\gamma}_{\mathcal{F}}(v)$, where $0 \leq \overline{\alpha}_{\mathcal{F}}(v)+\overline{\gamma}_{\mathcal{F}}(v) \leq 1$ and $\underline{\alpha}_{\mathcal{F}}(v), \underline{\gamma}_{\mathcal{F}}(v) \geq 0$.

Definition 2.2. [31] An interval-valued NS (IVNS) is a set of the form $\mathcal{N} = \{v, \langle \alpha_{\mathcal{N}}(v), \beta_{\mathcal{N}}(v), \gamma_{\mathcal{N}}(v)\rangle\}$, where $\alpha_{\mathcal{N}}(v) : \mathcal{V} \to C[0,1]$, $\beta_{\mathcal{N}}(v) : \mathcal{V} \to C[0,1]$ and $\gamma_{\mathcal{N}}(v) : \mathcal{V} \to C[0,1]$ are the interval-valued membership of truth, indeterminacy and falsity respectively. The lower and upper limits of $\alpha_{\mathcal{N}}(v)$, $\beta_{\mathcal{N}}(v)$ and $\gamma_{\mathcal{N}}(v)$ are denoted by $\underline{\alpha}_{\mathcal{N}}(v)$, $\overline{\alpha}_{\mathcal{N}}(v)$, $\underline{\beta}_{\mathcal{N}}(v)$, $\overline{\beta}_{\mathcal{N}}(v)$, and $\underline{\gamma}_{\mathcal{N}}(v)$, $\overline{\gamma}_{\mathcal{N}}(v)$, where $0 \leq \overline{\alpha}_{\mathcal{N}}(v) + \overline{\beta}_{\mathcal{N}}(v) + \overline{\gamma}_{\mathcal{N}}(v) \leq 3$.

3. **Interval-valued intuitionistic neutrosophic soft set**

We present the notion of IVINSS and investigate some of its properties. We generalize the operations and properties on IVINSS by the concepts discussed in [3] and [21].

Definition 3.1. An IVINS in $\mathcal{V}$ is a set of the form $\mathcal{I} = \{\langle v, \alpha_{\mathcal{I}}(v), \beta_{\mathcal{I}}(v), \gamma_{\mathcal{I}}(v)\rangle\}$, where $\alpha_{\mathcal{I}}(v) : \mathcal{V} \to C[0,1]$, $\beta_{\mathcal{I}}(v) : \mathcal{V} \to C[0,1]$ and $\gamma_{\mathcal{I}}(v) : \mathcal{V} \to C[0,1]$. $\alpha_{\mathcal{I}}(v)$, $\beta_{\mathcal{I}}(v)$ and $\gamma_{\mathcal{I}}(v)$ are closed sub intervals of [0,1], representing the membership grades of truth, indeterminacy and falsity of the element $v \in \mathcal{V}$. The lower and upper ends of $\alpha_{\mathcal{I}}(v)$, $\beta_{\mathcal{I}}(v)$ and $\gamma_{\mathcal{I}}(v)$ are denoted, respectively by $\underline{\alpha}_{\mathcal{I}}(v)$, $\overline{\alpha}_{\mathcal{I}}(v)$, $\underline{\beta}_{\mathcal{I}}(v)$, $\overline{\beta}_{\mathcal{I}}(v)$, and $\underline{\gamma}_{\mathcal{I}}(v)$, $\overline{\gamma}_{\mathcal{I}}(v)$, where $0 \leq \overline{\alpha}_{\mathcal{I}}(v) + \overline{\gamma}_{\mathcal{I}}(v) \leq 1$ and $\underline{\alpha}_{\mathcal{I}}(v), \underline{\beta}_{\mathcal{I}}(v), \underline{\gamma}_{\mathcal{I}}(v) \geq 0$, $0 \leq \overline{\alpha}_{\mathcal{I}}(v) + \overline{\beta}_{\mathcal{I}}(v) + \overline{\gamma}_{\mathcal{I}}(v) \leq 2$, $\forall\ v \in \mathcal{V}$.

Example 3.2. Let $\mathcal{V} = \{v_1, v_2, v_3\}$ be a non-empty set. Then an IVINS on $\mathcal{V}$ can be represented as,

$$\mathcal{I} = \Big\{ \langle v_1, [0.3, 0.4], [0.7, 0.8], [0.1, 0.2] \rangle, \langle v_2, [0.4, 0.5], [0.8, 0.9], [0.2, 0.3] \rangle, \langle v_3, [0.6, 0.7], [0.2, 0.3], [0.2, 0.3] \rangle \Big\}.$$

Definition 3.3. A pair $(\Omega, \mathcal{S})$ is called IVINSS over $\mathcal{V}$, where $\Omega : \mathcal{S} \rightarrow \mathcal{I}^N$. Thus for any parameter $q \in \mathcal{S}$, $\Omega(q)$ is an IVINSS.

Example 3.4. Let $\mathcal{V} = \{v_1, v_2, v_3\}$ represent clients with cognitive disorders and $\mathcal{S} = \{q_1, q_2, q_3\}$ represent symptoms which stand for inability of motor coordination (IMC), loss of memory (LM) and identity confusion (IC) respectively. An IVINSS $(\Omega, \mathcal{S})$ is a collection of subsets of $\mathcal{V}$, given by a psychiatrist based on the description in Table 1.

TABLE 1. Shows client with cognitive disorders in IVINSS $(\Omega, \mathcal{S})$ form.

$\mathcal{V}$	IMC(q_1)	LM(q_2)	IC(q_3)
v_1	$\langle [0.2, 0.4], [0.4, 0.5], [0.4, 0.5] \rangle$	$\langle [0.3, 0.4], [0.5, 0.6], [0.3, 0.5] \rangle$	$\langle [0.2, 0.3], [0.5, 0.8], [0.6, 0.7] \rangle$
v_2	$\langle [0.4, 0.6], [0.3, 0.5], [0.1, 0.2] \rangle$	$\langle [0.7, 0.8], [0.2, 0.5], [0.1, 0.2] \rangle$	$\langle [0.6, 0.7], [0.7, 0.8], [0.1, 0.2] \rangle$
v_3	$\langle [0.6, 0.7], [0.2, 0.7], [0.1, 0.2] \rangle$	$\langle [0.1, 0.3], [0.6, 0.7], [0.5, 0.6] \rangle$	$\langle [0.2, 0.3], [0.7, 0.8], [0.4, 0.5] \rangle$

Definition 3.5. Let $(\Omega_1, \mathcal{S}_1)$ and $(\Omega_2, \mathcal{S}_2)$ be two IVINSS over $\mathcal{V}$. Then,
(i) $(\Omega_1, \mathcal{S}_1)$ OR $(\Omega_2, \mathcal{S}_2)$ is an IVINSS represented as $(\Omega_1, \mathcal{S}_1) \vee (\Omega_2, \mathcal{S}_2) = (\Omega_{\vee}, \mathcal{S}_1 \times \mathcal{S}_2)$, where $\Omega_{\vee}(q_1, q_2) = \Omega_1(q_1) \cup \Omega_2(q_2)$, $\forall\ (q_1, q_2) \in \mathcal{S}_1 \times \mathcal{S}_2$.

$$\begin{aligned}\Omega_{\vee}(q_1, q_2) = \Big\langle & [\vee(\underline{\alpha}_{\Omega_1(q_1)}(v), \underline{\alpha}_{\Omega_2(q_2)}(v)), \vee(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v))], \\ & [\vee(\underline{\beta}_{\Omega_1(q_1)}(v), \underline{\beta}_{\Omega_2(q_2)}(v)), \vee(\overline{\beta}_{\Omega_1(q_1)}(v), \overline{\beta}_{\Omega_2(q_2)}(v))], \\ & [\wedge(\underline{\gamma}_{\Omega_1(q_1)}(v), \underline{\gamma}_{\Omega_2(q_2)}(v)), \wedge(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v))], \Big\rangle, \forall\ (q_1, q_2) \in \mathcal{S}_1 \times \mathcal{S}_2.\end{aligned}$$

(ii) $(\Omega_1, \mathcal{S}_1)$ AND $(\Omega_2, \mathcal{S}_2)$ is an IVINSS represented as $(\Omega_1, \mathcal{S}_1) \wedge (\Omega_2, \mathcal{S}_2) = (\Omega_{\wedge}, \mathcal{S}_1 \times \mathcal{S}_2)$, where $\Omega_{\wedge}(q_1, q_2) = \Omega_1(q_1) \cap \Omega_2(q_2)$, $\forall\ (q_1, q_2) \in \mathcal{S}_1 \times \mathcal{S}_2$.

$$\begin{aligned}\Omega_{\wedge}(q_1, q_2) = \Big\langle & [\wedge(\underline{\alpha}_{\Omega_1(q_1)}(v), \underline{\alpha}_{\Omega_2(q_2)}(v)), \wedge(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v))], \\ & [\wedge(\underline{\beta}_{\Omega_1(q_1)}(v), \underline{\beta}_{\Omega_2(q_2)}(v)), \wedge(\overline{\beta}_{\Omega_1(q_1)}(v), \overline{\beta}_{\Omega_2(q_2)}(v))], \\ & [\vee(\underline{\gamma}_{\Omega_1(q_1)}(v), \underline{\gamma}_{\Omega_2(q_2)}(v)), \vee(\overline{\gamma}_{\Omega_1(q_1)}(v), \overline{\gamma}_{\Omega_2(q_2)}(v))] \Big\rangle, \forall\ (q_1, q_2) \in \mathcal{S}_1 \times \mathcal{S}_2.\end{aligned}$$

Definition 3.6. Let $(\Omega_1, \mathcal{S}_1)$ and $(\Omega_2, \mathcal{S}_2)$ be two IVINSS over $\mathcal{V}$. Then,

(i) $(\Omega_1, \mathcal{S}_1)$ union $(\Omega_2, \mathcal{S}_2)$ is an IVINSS represented as $(\Omega_1, \mathcal{S}_1) \Cup (\Omega_2, \mathcal{S}_2) = (\Omega_{\Cup}, \mathcal{S}_{\Cup})$, where $\mathcal{S}_{\Cup} = \mathcal{S}_1 \cup \mathcal{S}_2$ and $\forall\ q \in \mathcal{S}_{\Cup}$,

$$\Omega_{\Cup}(q) = \begin{cases} \left\{\left\langle v, (\alpha_{\Omega_1(q)}(v), \beta_{\Omega_1(q)}(v), \gamma_{\Omega_1(q)}(v))\right\rangle; & \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\right\}, \\ \left\{\left\langle v, (\alpha_{\Omega_2(q)}(v), \beta_{\Omega_2(q)}(v), \gamma_{\Omega_2(q)}(v))\right\rangle; & \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\right\}, \\ \left\{\left\langle v, [\vee(\underline{\alpha}_{\Omega_1(q)}(v), \underline{\alpha}_{\Omega_2(q)}(v)), \vee(\overline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v))], \right.\right. & \\ \quad [\vee(\underline{\beta}_{\Omega_1(q)}(v), \underline{\beta}_{\Omega_2(q)}(v)), \vee(\overline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_2(q)}(v))], & \\ \quad \left.[\wedge(\underline{\gamma}_{\Omega_1(q)}(v), \underline{\gamma}_{\Omega_2(q)}(v)), \wedge(\overline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v))]\right\rangle; & \left.\text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\right\}. \end{cases}$$

(ii) $(\Omega_1, \mathcal{S}_1)$ intersection $(\Omega_2, \mathcal{S}_2)$ is an IVINSS represented as $(\Omega_1, \mathcal{S}_1) \Cap (\Omega_2, \mathcal{S}_2) = (\Omega_{\Cap}, \mathcal{S}_{\Cap})$, where $\mathcal{S}_{\Cap} = \mathcal{S}_1 \cup \mathcal{S}_2$ and $\forall\ q \in \mathcal{S}_{\Cap}$,

$$\Omega_{\Cap}(q) = \begin{cases} \left\{\left\langle v, (\alpha_{\Omega_1(q)}(v), \beta_{\Omega_1(q)}(v), \gamma_{\Omega_1(q)}(v))\right\rangle; & \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\right\}, \\ \left\{\left\langle v, (\alpha_{\Omega_2(q)}(v), \beta_{\Omega_2(q)}(v), \gamma_{\Omega_2(q)}(v))\right\rangle; & \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\right\}, \\ \left\{\left\langle v, [\wedge(\underline{\alpha}_{\Omega_1(q)}(v), \underline{\alpha}_{\Omega_2(q)}(v)), \wedge(\overline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v))], \right.\right. & \\ \quad [\wedge(\underline{\beta}_{\Omega_1(q)}(v), \underline{\beta}_{\Omega_2(q)}(v)), \wedge(\overline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_2(q)}(v))], & \\ \quad \left.[\vee(\underline{\gamma}_{\Omega_1(q)}(v), \underline{\gamma}_{\Omega_2(q)}(v)), \vee(\overline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v))]\right\rangle; & \left.\text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\right\}. \end{cases}$$

Definition 3.7. The complement of an IVINSS $(\Omega, \mathcal{S})$ is represented as,

$$(\Omega, \mathcal{S})^c = \left\{\left\langle v, \gamma_{\Omega(q)}(v), [(1 - \overline{\beta}_{\Omega(q)}(v)), (1 - \underline{\beta}_{\Omega(q)}(v))], \alpha_{\Omega(q)}(v)\right\rangle; \text{and } q \in \mathcal{S}\right\}.$$

Theorem 3.8. *Let $(\Omega_1, \mathcal{S}_1)$ and $(\Omega_2, \mathcal{S}_2)$ be two IVINSS over $\mathcal{V}$. Then,*
(i) $\big((\Omega_1, \mathcal{S}_1) \vee (\Omega_2, \mathcal{S}_2)\big)^c = (\Omega_1, \mathcal{S}_1)^c \wedge (\Omega_2, \mathcal{S}_2)^c$;
(ii) $\big((\Omega_1, \mathcal{S}_1) \wedge (\Omega_2, \mathcal{S}_2)\big)^c = (\Omega_1, \mathcal{S}_1)^c \vee (\Omega_2, \mathcal{S}_2)^c$.

Proof. We give the proof of (i), and proof of (ii) is analogous.
(i) $(\Omega_1, \mathcal{S}_1) \vee (\Omega_2, \mathcal{S}_2) = (\Omega_\vee, \mathcal{S}_1 \times \mathcal{S}_2)$.
$\big((\Omega_1, \mathcal{S}_1) \vee (\Omega_2, \mathcal{S}_2)\big)^c = (\Omega_\vee, \mathcal{S}_1 \times \mathcal{S}_2)^c$.

$$\begin{aligned} \Omega^c_\vee(q_1, q_2) = \big\langle &[\wedge(\underline{\gamma}_{\Omega_1(q_1)}(v), \underline{\gamma}_{\Omega_2(q_2)}(v)), \wedge(\overline{\gamma}_{\Omega_1(q_1)}(v), \overline{\gamma}_{\Omega_2(q_2)}(v))], \\ &[\wedge((1 - \overline{\beta}_{\Omega_1(q_1)}(v)), (1 - \overline{\beta}_{\Omega_2(q_2)}(v))), \wedge((1 - \underline{\beta}_{\Omega_1(q_1)}(v)), (1 - \underline{\beta}_{\Omega_2(q_2)}(v)))], \\ &[\vee(\underline{\alpha}_{\Omega_1(q_1)}(v), \underline{\alpha}_{\Omega_2(q_2)}(v)), \vee(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v))]\big\rangle, \forall\ (q_1, q_2) \in \mathcal{S}_1 \times \mathcal{S}_2. \end{aligned}$$

and $(\Omega_1, \mathcal{S}_1)^c \wedge (\Omega_2, \mathcal{S}_2)^c = (\Omega_\wedge, \mathcal{S}_1 \times \mathcal{S}_2)$.

$$\begin{aligned} \Omega_\wedge(q_1, q_2) = \big\langle &[\wedge(\underline{\gamma}_{\Omega_1(q_1)}(v), \underline{\gamma}_{\Omega_2(q_2)}(v)), \wedge(\overline{\gamma}_{\Omega_1(q_1)}(v), \overline{\gamma}_{\Omega_2(q_2)}(v))], \\ &[\wedge((1 - \overline{\beta}_{\Omega_1(q_1)}(v)), (1 - \overline{\beta}_{\Omega_2(q_2)}(v))), \wedge((1 - \underline{\beta}_{\Omega_1(q_1)}(v)), (1 - \underline{\beta}_{\Omega_2(q_2)}(v)))], \\ &[\vee(\underline{\alpha}_{\Omega_1(q_1)}(v), \underline{\alpha}_{\Omega_2(q_2)}(v)), \vee(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v))]\big\rangle, \forall\ (q_1, q_2) \in \mathcal{S}_1 \times \mathcal{S}_2. \end{aligned}$$

Thus $\left((\Omega_1,\mathcal{S}_1)\vee(\Omega_2,\mathcal{S}_2)\right)^c=(\Omega_1,\mathcal{S}_1)^c\wedge(\Omega_2,\mathcal{S}_2)^c$. □

Definition 3.9. Let $\mathcal{S}_1,\mathcal{S}_2\subseteq\mathcal{Q}$. $(\Omega_1,\mathcal{S}_1)$ is an interval-valued intuitionistic neutrosophic soft subset (IVINSSS) of $(\Omega_2,\mathcal{S}_2)$ represented as $(\Omega_1,\mathcal{S}_1)\Subset(\Omega_2,\mathcal{S}_2)$ if and only if (iff)
(i) $\mathcal{S}_1\subseteq\mathcal{S}_2$;
(ii)$\Omega_1(q)$ is an IVINSSS of $\Omega_2(q)$ that is for all $q\in\mathcal{S}_1$,
$\underline{\alpha}_{\Omega_1(q)}(v)\le\underline{\alpha}_{\Omega_2(q)}(v)$, $\overline{\alpha}_{\Omega_1(q)}(v)\le\overline{\alpha}_{\Omega_2(q)}(v)$; $\underline{\beta}_{\Omega_1(q)}(v)\le\underline{\beta}_{\Omega_2(q)}(v)$,
$\overline{\beta}_{\Omega_1(q)}(v)\le\overline{\beta}_{\Omega_2(q)}(v)$, $\underline{\gamma}_{\Omega_1(q)}(v)\ge\underline{\gamma}_{\Omega_2(q)}(v)$ and $\overline{\gamma}_{\Omega_1(q)}(v)\ge\overline{\gamma}_{\Omega_2(q)}(v)$.
Also, $(\Omega_2,\mathcal{S}_2)$ is called an interval-valued intuitionistic neutrosophic soft superset of $(\Omega_1,\mathcal{S}_1)$ and represented as $(\Omega_2,\mathcal{S}_2)\Supset(\Omega_1,\mathcal{S}_1)$.

Definition 3.10. If $(\Omega_1,\mathcal{S}_1)$ and $(\Omega_2,\mathcal{S}_2)$ are two IVINSS, then $(\Omega_1,\mathcal{S}_1)=(\Omega_2,\mathcal{S}_2)$ iff $(\Omega_1,\mathcal{S}_1)\Subset(\Omega_2,\mathcal{S}_2)$ and $(\Omega_2,\mathcal{S}_2)\Subset(\Omega_1,\mathcal{S}_1)$.

4. Necessity ($\oplus$) and possibility ($\ominus$) operators on IVINSS

We provide the definition of $\oplus$ and $\ominus$ operators on IVINSS and its properties. We generalize these operations and some properties on IVINSS using the concepts discussed in [3] and [21].

Definition 4.1. If $(\Omega,\mathcal{S})$ is an IVINSS over $\mathcal{V}$ and $\Omega:\mathcal{S}\to\mathcal{I}^N$, then,
(i) the necessity operator ($\oplus$) is represented as,

$$\oplus(\Omega,\mathcal{S})=\left\{\left\langle v,\alpha_{\oplus\Omega(q)}(v),\beta_{\oplus\Omega(q)}(v),\gamma_{\oplus\Omega(q)}(v)\right\rangle;\ q\in\mathcal{S}\right\}.$$

Here, $\alpha_{\oplus\Omega(q)}(v)=[\underline{\alpha}_{\Omega(q)}(v),\overline{\alpha}_{\Omega(q)}(v)]$, $\beta_{\oplus\Omega(q)}(v)=[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)]$ and
$\gamma_{\oplus\Omega(q)}(v)=[(1-\overline{\alpha}_{\Omega(q)}(v)),(1-\underline{\alpha}_{\Omega(q)}(v))]$, are the membership grades of truth, indeterminacy and falsity for the object v on the parameter q.
(ii) the possibility operator ($\ominus$) is represented as,

$$\ominus(\Omega,\mathcal{S})=\left\{\left\langle v,\alpha_{\ominus\Omega(q)}(v),\beta_{\ominus\Omega(q)}(v),\gamma_{\ominus\Psi(q)}(v)\right\rangle;\ q\in\mathcal{S}\right\}.$$

Here, $\alpha_{\ominus\Omega(q)}(v)=[(1-\overline{\gamma}_{\Omega(q)}(v)),(1-\underline{\gamma}_{\Omega(q)}(v))]$, $\beta_{\ominus\Omega(q)}(v)=[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)]$ and
$\gamma_{\ominus\Omega(q)}(v)=[\underline{\gamma}_{\Omega(q)}(v),\overline{\gamma}_{\Omega(q)}(v)]$, are the membership grades of truth, indeterminacy and falsity for the object v on the parameter q.

Example 4.2. (i) The IVINSS $\oplus(\Omega,\mathcal{S})$ for Example 3.4 is shown in Table 2.

TABLE 2. Shows client with cognitive disorders using $\oplus$ operator.

$\mathcal{V}$	IMC(q_1)	LM(q_2)	IC(q_3)
v_1	$\langle[0.2,0.4],[0.4,0.5],[0.6,0.8]\rangle$	$\langle[0.3,0.4],[0.5,0.6],[0.6,0.7]\rangle$	$\langle[0.2,0.3],[0.5,0.8],[0.7,0.8]\rangle$
v_2	$\langle[0.4,0.6],[0.3,0.5],[0.4,0.6]\rangle$	$\langle[0.7,0.8],[0.2,0.5],[0.2,0.3]\rangle$	$\langle[0.6,0.7],[0.7,0.8],[0.3,0.4]\rangle$
v_3	$\langle[0.6,0.7],[0.2,0.7],[0.3,0.4]\rangle$	$\langle[0.1,0.3],[0.6,0.7],[0.7,0.9]\rangle$	$\langle[0.2,0.3],[0.7,0.8],[0.7,0.8]\rangle$

Table 3. Shows client with cognitive disorders using $\ominus$ operator.

$\mathcal{V}$	IMC(q_1)	LM(q_2)	IC(q_3)
v_1	$\langle[0.5,0.6],[0.4,0.5],[0.4,0.5]\rangle$	$\langle[0.5,0.7],[0.5,0.6],[0.3,0.5]\rangle$	$\langle[0.3,0.4],[0.5,0.8],[0.6,0.7]\rangle$
v_2	$\langle[0.8,0.9],[0.3,0.5],[0.1,0.2]\rangle$	$\langle[0.8,0.9],[0.2,0.5],[0.1,0.2]\rangle$	$\langle[0.8,0.9],[0.7,0.8],[0.1,0.2]\rangle$
v_3	$\langle[0.8,0.9],[0.2,0.7],[0.1,0.2]\rangle$	$\langle[0.4,0.5],[0.6,0.7],[0.5,0.6]\rangle$	$\langle[0.5,0.6],[0.7,0.8],[0.4,0.5]\rangle$

(ii) The IVINSS $\ominus(\Omega,\mathcal{S})$ for Example 3.4 is shown in Table 3.

Theorem 4.3. *Let $(\Omega_1,\mathcal{S}_1)$ and $(\Omega_2,\mathcal{S}_2)$ be two IVINSS over $\mathcal{V}$. Then,*
(i) $\oplus((\Omega_1,\mathcal{S}_1)\Cup(\Omega_2,\mathcal{S}_2)) = \oplus(\Omega_1,\mathcal{S}_1)\Cup\oplus(\Omega_2,\mathcal{S}_2)$;
(ii) $\oplus((\Omega_1,\mathcal{S}_1)\Cap(\Omega_2,\mathcal{S}_2)) = \oplus(\Omega_1,\mathcal{S}_1)\Cap\oplus(\Omega_2,\mathcal{S}_2)$;
(iii) $\oplus\oplus(\Omega_1,\mathcal{S}_1)= \oplus(\Omega_1,\mathcal{S}_1)$.

Proof. We present the proof of (i), and proof of (ii) is analogous.
(i) Let $(\Omega_1,\mathcal{S}_1)\Cup(\Omega_2,\mathcal{S}_2) = (\Omega_{\Cup},\mathcal{S}_{\Cup})$, where $\mathcal{S}_{\Cup} = \mathcal{S}_1\cup\mathcal{S}_2$, $\forall\ q\in\mathcal{S}_{\Cup}$.
Consider,

$$\oplus\Psi_{\Cup}(q)$$

$$= \begin{cases} \{\langle v,[\underline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_1(q)}(v)],[\underline{\beta}_{\Omega_1(q)}(v),\overline{\beta}_{\Omega_1(q)}(v)], \\ \qquad [(1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\underline{\alpha}_{\Omega_1(q)}(v))]\rangle; & \text{if } q\in\mathcal{S}_1-\mathcal{S}_2\}, \\ \{\langle v,[\underline{\alpha}_{\Omega_2(q)}(v),\overline{\alpha}_{\Omega_2(q)}(v)],[\underline{\beta}_{\Omega_2(q)}(v),\overline{\beta}_{\Omega_2(q)}(v)], \\ \qquad [(1-\overline{\alpha}_{\Omega_2(q)}(v)),(1-\underline{\alpha}_{\Omega_2(q)}(v))]\rangle; & \text{if } q\in\mathcal{S}_2-\mathcal{S}_1\}, \\ \{\langle v,\vee(\alpha_{\Omega_1(q)}(v),\alpha_{\Omega_2(q)}(v)),\vee(\beta_{\Omega_1(q)}(v),\beta_{\Omega_2(q)}(v)), \\ \quad [(1-\vee(\overline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_2(q)}(v))),(1-\vee(\underline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_2(q)}(v)))]\rangle; & \text{if } q\in\mathcal{S}_1\cap\mathcal{S}_2\}. \end{cases}$$

$$= \begin{cases} \{\langle v,[\underline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_1(q)}(v)],[\underline{\beta}_{\Omega_1(q)}(v),\overline{\beta}_{\Omega_1(q)}(v)], \\ \qquad [(1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\underline{\alpha}_{\Omega_1(q)}(v))]\rangle; & \text{if } q\in\mathcal{S}_1-\mathcal{S}_2\}, \\ \{\langle v,[\underline{\alpha}_{\Omega_2(q)}(v),\overline{\alpha}_{\Omega_2(q)}(v)],[\underline{\beta}_{\Omega_2(q)}(v),\overline{\beta}_{\Omega_2(q)}(v)], \\ \qquad [(1-\overline{\alpha}_{\Omega_2(q)}(v)),(1-\underline{\alpha}_{\Omega_2(q)}(v))]\rangle; & \text{if } q\in\mathcal{S}_2-\mathcal{S}_1\}, \\ \{\langle v,\vee(\alpha_{\Omega_1(q)}(v),\alpha_{\Omega_2(q)}(v)),\vee(\beta_{\Omega_1(q)}(v),\beta_{\Omega_2(q)}(v)), \\ \quad [\wedge((1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\overline{\alpha}_{\Omega_2(q)}(v))),\wedge((1-\underline{\alpha}_{\Omega_1(q)}(v)),(1-\overline{\alpha}_{\Omega_2(q)}(v)))]\rangle; \\ & \text{if } q\in\mathcal{S}_1\cap\mathcal{S}_2\}. \end{cases}$$

We know that,

$$\oplus(\Omega_1,\mathcal{S}_1) = \{\langle v,[\underline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_1(q)}(v)],[\underline{\beta}_{\Omega_1(q)}(v),\overline{\beta}_{\Omega_1(q)}(v)], [(1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\underline{\alpha}_{\Omega_1(q)}(v))]\rangle; q\in\mathcal{S}_1\},$$

$$\oplus(\Omega_2,\mathcal{S}_2) = \{\langle v,[\underline{\alpha}_{\Omega_2(q)}(v),\overline{\alpha}_{\Omega_2(q)}(v)],[\underline{\beta}_{\Omega_2(q)}(v),\overline{\beta}_{\Omega_2(q)}(v)], [(1-\overline{\alpha}_{\Omega_2(q)}(v)),(1-\underline{\alpha}_{\Omega_2(q)}(v))]\rangle; q\in\mathcal{S}_2\},$$

Let $\oplus(\Omega_1,\mathcal{S}_1)\Cup\oplus(\Omega_2,\mathcal{S}_2)=(\Omega_{\oplus\Cup},\mathcal{S}_{\oplus\Cup})$, where $\mathcal{S}_{\oplus\Cup}=\mathcal{S}_1\cup\mathcal{S}_2$.

For $q\in\mathcal{S}_{\oplus\Cup}$,

$\Omega_{\oplus\Cup}(q)$

$$=\begin{cases}\big\{\big\langle v,[\underline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_1(q)}(v)],[\underline{\beta}_{\Omega_1(q)}(v),\overline{\beta}_{\Omega_1(q)}(v)],\\ \qquad [(1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\underline{\alpha}_{\Omega_1(q)}(v))]\big\rangle; \quad \text{if } q\in\mathcal{S}_1-\mathcal{S}_2\big\},\\ \big\{\big\langle v,[\underline{\alpha}_{\Omega_2(q)}(v),\overline{\alpha}_{\Omega_2(q)}(v)],[\underline{\beta}_{\Omega_2(q)}(v),\overline{\beta}_{\Omega_2(q)}(v)],\\ \qquad [(1-\overline{\alpha}_{\Omega_2(q)}(v)),(1-\underline{\alpha}_{\Omega_2(q)}(v))]\big\rangle; \quad \text{if } q\in\mathcal{S}_2-\mathcal{S}_1\big\},\\ \big\{\big\langle v,\vee(\alpha_{\Omega_1(q)}(v),\alpha_{\Omega_2(q)}(v)),\vee(\beta_{\Omega_1(q)}(v),\beta_{\Omega_2(q)}(v)),\\ \quad [\wedge((1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\overline{\alpha}_{\Omega_2(q)}(v))),\wedge((1-\underline{\alpha}_{\Omega_1(q)}(v)),(1-\overline{\alpha}_{\Omega_2(q)}(v)))]\big\rangle;\\ \qquad \text{if } q\in\mathcal{S}_1\cap\mathcal{S}_2\big\}.\end{cases}$$

Thus $\oplus((\Omega_1,\mathcal{S}_1)\Cup(\Omega_2,\mathcal{S}_2))=\oplus(\Omega_1,\mathcal{S}_1)\Cup\oplus(\Omega_2,\mathcal{S}_2)$.

(iii) $\oplus\oplus(\Omega_1,\mathcal{S}_1)$

$$=\oplus\big\{\big\langle v,[\underline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_1(q)}(v)],[\underline{\beta}_{\Omega_1(q)}(v),\overline{\beta}_{\Omega_1(q)}(v)],[(1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\underline{\alpha}_{\Omega_1(q)}(v))]\big\rangle;q\in\mathcal{S}_1\big\}$$
$$=\big\{\big\langle v,[\underline{\alpha}_{\Omega_1(q)}(v),\overline{\alpha}_{\Omega_1(q)}(v)],[\underline{\beta}_{\Omega_1(q)}(v),\overline{\beta}_{\Omega_1(q)}(v)],[(1-\overline{\alpha}_{\Omega_1(q)}(v)),(1-\underline{\alpha}_{\Omega_1(q)}(v))]\big\rangle;q\in\mathcal{S}_1\big\}$$
$$=\oplus(\Omega_1,\mathcal{S}_1).$$

□

Theorem 4.4. *Let $(\Omega_1,\mathcal{S}_1)$ and $(\Omega_2,\mathcal{S}_2)$ be two IVINSS over $\mathcal{V}$. Then,*
(i) $\ominus((\Omega_1,\mathcal{S}_1)\Cup(\Omega_2,\mathcal{S}_2))=\ominus(\Omega_1,\mathcal{S}_1)\Cup\ominus(\Omega_2,\mathcal{S}_2)$;
(ii) $\ominus((\Omega_1,\mathcal{S}_1)\Cap(\Omega_2,\mathcal{S}_2))=\ominus(\Omega_1,\mathcal{S}_1)\Cap\ominus(\Omega_2,\mathcal{S}_2)$;
(iii) $\ominus\ominus(\Omega_1,\mathcal{S}_1)=\ominus(\Omega_1,\mathcal{S}_1)$.

Proof. We give the proof of (i), and proof of (ii) is analogous.

(i) Let $(\Omega_1,\mathcal{S}_1)\Cup(\Omega_2,\mathcal{S}_2)=(\Omega_{\Cup},\mathcal{S}_{\Cup})$, where $\mathcal{S}_{\Cup}=\mathcal{S}_1\cup\mathcal{S}_2$, $\forall\ q\in\mathcal{S}_{\Cup}$.

Consider, $\ominus\Psi_{\Cup}(q)$

$$=\begin{cases}\big\{\big\langle v,[(1-\overline{\gamma}_{\Omega_1(q)}(v)),(1-\underline{\gamma}_{\Omega_1(q)}(v))],[\underline{\beta}_{\Omega_1(q)}(v),\overline{\beta}_{\Omega_1(q)}(v)],\\ \qquad [\underline{\gamma}_{\Omega_1(q)}(v),\overline{\gamma}_{\Omega_1(q)}(v)]\big\rangle; \quad \text{if } q\in\mathcal{S}_1-\mathcal{S}_2\big\},\\ \big\{\big\langle v,[(1-\overline{\gamma}_{\Omega_2(q)}(v)),(1-\underline{\gamma}_{\Omega_2(q)}(v))],[\overline{\beta}_{\Omega_2(q)}(v),\overline{\beta}_{\Omega_2(q)}(v)]\\ \qquad [\overline{\gamma}_{\Omega_2(q)}(v),\overline{\gamma}_{\Omega_2(q)}(v)]\big\rangle; \quad \text{if } q\in\mathcal{S}_2-\mathcal{S}_1\big\},\\ \big\{\big\langle v,[(1-\wedge(\overline{\gamma}_{\Omega_1(q)}(v),\overline{\gamma}_{\Omega_2(q)}(v))),(1-\wedge(\underline{\gamma}_{\Omega_1(q)}(v),\overline{\gamma}_{\Omega_2(q)}(v)))],\\ \qquad \wedge(\beta_{\Omega_1(q)}(v),\beta_{\Omega_2(q)}(v)),\wedge(\gamma_{\Omega_1(q)}(v),\gamma_{\Omega_2(q)}(v))\big\rangle; \quad \text{if } q\in\mathcal{S}_1\cap\mathcal{S}_2\big\}.\end{cases}$$

$$= \begin{cases} \{\langle v, [(1-\overline{\gamma}_{\Omega_1(q)}(v)), (1-\underline{\gamma}_{\Omega_1(q)}(v))], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], \\ \qquad [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\}, \\ \{\langle v, [(1-\overline{\gamma}_{\Omega_2(q)}(v)), (1-\underline{\gamma}_{\Omega_2(q)}(v))], [\overline{\beta}_{\Omega_2(q)}(v), \overline{\beta}_{\Omega_2(q)}(v)] \\ \qquad [\overline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\}, \\ \{\langle v, [\vee((1-\overline{\gamma}_{\Omega_1(q)}(v)), (1-\overline{\gamma}_{\Omega_2(q)}(v))), \vee((1-\underline{\gamma}_{\Omega_1(q)}(v)), (1-\overline{\gamma}_{\Omega_2(q)}(v)))], \\ \qquad \vee(\beta_{\Omega_1(q)}(v), \beta_{\Omega_2(q)}(v)), \wedge(\gamma_{\Omega_1(q)}(v), \gamma_{\Omega_2(q)}(v))\rangle; \quad \text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\}. \end{cases}$$

We know that,

$$\ominus(\Omega_1, \mathcal{S}_1) = \{\langle v, [(1-\overline{\gamma}_{\Omega_1(q)}(v)), (1-\underline{\gamma}_{\Omega_1(q)}(v))], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\rangle; q \in \mathcal{S}_1\},$$

$$\ominus(\Omega_2, \mathcal{S}_2) = \{\langle v, [(1-\overline{\gamma}_{\Omega_2(q)}(v)), (1-\underline{\gamma}_{\Omega_2(q)}(v))], [\underline{\beta}_{\Omega_2(q)}(v), \overline{\beta}_{\Omega_2(q)}(v)], [\underline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega(q)}(v)]\rangle; q \in \mathcal{S}_2\},$$

Let $\ominus(\Omega_1, \mathcal{S}_1) \Cup \ominus(\Omega_2, \mathcal{S}_2) = (\Omega_{\ominus\Cup}, \mathcal{S}_{\ominus\Cup})$, where $\mathcal{S}_{\ominus\Cup} = \mathcal{S}_1 \cup \mathcal{S}_2$.
For $q \in \mathcal{S}_{\ominus\Cup}$,
$\Omega_{\ominus\Cup}(q)$

$$= \begin{cases} \{\langle v, [(1-\overline{\gamma}_{\Omega_1(q)}(v)), (1-\underline{\gamma}_{\Omega_1(q)}(v))], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], \\ \qquad [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\}, \\ \{\langle v, [(1-\overline{\gamma}_{\Omega_2(q)}(v)), (1-\underline{\gamma}_{\Omega_2(q)}(v))], [\overline{\beta}_{\Omega_2(q)}(v), \overline{\beta}_{\Omega_2(q)}(v)] \\ \qquad [\overline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\}, \\ \{\langle v, [\vee((1-\overline{\gamma}_{\Omega_1(q)}(v)), (1-\overline{\gamma}_{\Omega_2(q)}(v))), \vee((1-\underline{\gamma}_{\Omega_1(q)}(v)), (1-\overline{\gamma}_{\Omega_2(q)}(v)))], \\ \qquad \vee(\beta_{\Omega_1(q)}(v), \beta_{\Omega_2(q)}(v)), \wedge(\gamma_{\Omega_1(q)}(v), \gamma_{\Omega_2(q)}(v))\rangle; \quad \text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\}. \end{cases}$$

Thus $\ominus((\Omega_1, \mathcal{S}_1) \Cup (\Omega_2, \mathcal{S}_2)) = \ominus(\Omega_1, \mathcal{S}_1) \Cup \ominus(\Omega_2, \mathcal{S}_2)$.
(iii) $\ominus\ominus(\Omega_1, \mathcal{S}_1)$

$$= \ominus\{\langle v, [(1-\overline{\gamma}_{\Omega_1(q)}(v)), (1-\underline{\gamma}_{\Omega_1(q)}(v))], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\rangle; q \in \mathcal{S}_1\}$$
$$= \{\langle v, [(1-\overline{\gamma}_{\Omega_1(q)}(v)), (1-\underline{\gamma}_{\Omega_1(q)}(v))], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\rangle; q \in \mathcal{S}_1\}$$
$$= \ominus(\Omega_1, \mathcal{S}_1).$$

□

Theorem 4.5. *Let $(\Omega, \mathcal{S})$ be an IVINSS over $\mathcal{V}$. Then,*
(i) $\ominus\oplus(\Omega, \mathcal{S}) = \oplus(\Omega, \mathcal{S})$;
(ii) $\oplus\ominus(\Omega, \mathcal{S}) = \ominus(\Omega, \mathcal{S})$.

Proof. (i)$\ominus\oplus(\Omega,\mathcal{S})$

$$= \{\langle v, [(1-(1-\underline{\alpha}_{\Omega(q)}(v)), (1-(1-\overline{\alpha}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [(1-\overline{\alpha}_{\Omega(q)}(v)), (1-\underline{\alpha}_{\Omega(q)}(v))]\rangle; q \in \mathcal{S}\}$$

$$= \{\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [(1-\overline{\alpha}_{\Omega(q)}(v)), (1-\underline{\alpha}_{\Omega(q)}(v))]\rangle; q \in \mathcal{S}\}$$

$$= \oplus(\Omega,\mathcal{S}).$$

(ii) $\oplus\ominus(\Omega,\mathcal{S})$

$$= \{\langle v, [(1-\overline{\gamma}_{\Omega(q)}(v)), (1-\underline{\gamma}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [(1-(1-\underline{\gamma}_{\Omega(q)}(v)), (1-(1-\overline{\gamma}_{\Omega(q)}(v))]\rangle; q \in \mathcal{S}\}$$

$$= \{\langle v, [(1-\overline{\gamma}_{\Omega(q)}(v)), (1-\underline{\gamma}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)]\rangle; q \in \mathcal{S}\}$$

$$= \ominus(\Omega,\mathcal{S}).$$

□

Theorem 4.6. *Let $(\Omega_1,\mathcal{S}_1)$ and $(\Omega_2,\mathcal{S}_2)$ be two IVINSS over $\mathcal{V}$. Then,*
(i) $\oplus((\Omega_1,\mathcal{S}_1)\wedge(\Omega_2,\mathcal{S}_2)) = \oplus(\Omega_1,\mathcal{S}_1)\wedge\oplus(\Omega_2,\mathcal{S}_2)$;
(ii) $\oplus((\Omega_1,\mathcal{S}_1)\vee(\Omega_2,\mathcal{S}_2)) = \oplus(\Omega_1,\mathcal{S}_1)\vee\oplus(\Omega_2,\mathcal{S}_2)$;
(iii) $\ominus((\Omega_1,\mathcal{S}_1)\wedge(\Omega_2,\mathcal{S}_2)) = \ominus(\Omega_1,\mathcal{S}_1)\wedge\oplus(\Omega_2,\mathcal{S}_2)$;
(iv) $\ominus((\Omega_1,\mathcal{S}_1)\vee(\Omega_2,\mathcal{S}_2)) = \ominus(\Omega_1,\mathcal{S}_1)\vee\oplus(\Omega_2,\mathcal{S}_2)$.

Proof. We present the proofs of (i) and (iii), and proofs of (ii) and (iv) are analogous.
(i) $\oplus((\Omega_1,\mathcal{S}_1)\wedge(\Omega_2,\mathcal{S}_2))$

$$= \{\langle v, [\wedge(\underline{\alpha}_{\Omega_1(q_1)}(v), \underline{\alpha}_{\Omega_2(q_2)}(v)), \wedge(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v))], [\wedge(\underline{\beta}_{\Omega_1(q_1)}(v), \underline{\beta}_{\Omega_2(q_2)}(v)), \wedge(\overline{\beta}_{\Omega_1(q_1)}(v), \overline{\beta}_{\Omega_2(q_2)}(v))], [(1-\wedge(\underline{\alpha}_{\Omega_1(q_1)}(v), \underline{\alpha}_{\Omega_2(q_2)}(v))), (1-\wedge(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v)))], \forall(q_1,q_2)\in\mathcal{S}_1\times\mathcal{S}_2\}.$$

$$= \{\langle v, [\wedge(\underline{\alpha}_{\Omega_1(q_1)}(v), \underline{\alpha}_{\Omega_2(q_2)}(v)), \wedge(\overline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_2(q_2)}(v))], [\wedge(\underline{\beta}_{\Omega_1(q_1)}(v), \underline{\beta}_{\Omega_2(q_2)}(v)), \wedge(\overline{\beta}_{\Omega_1(q_1)}(v), \overline{\beta}_{\Omega_2(q_2)}(v))], [\vee((1-\overline{\alpha}_{\Omega_1(q_1)}(v)), (1-\overline{\alpha}_{\Omega_2(q_2)}(v))), \vee((1-\underline{\alpha}_{\Omega_1(q_1)}(v)), (1-\underline{\alpha}_{\Omega_2(q_2)}(v)))], \forall(q_1,q_2)\in\mathcal{S}_1\times\mathcal{S}_2\}.$$

Also,

$$\oplus(\Omega_1,\mathcal{S}_1) = \{\langle v, [\underline{\alpha}_{\Omega_1(q_1)}(v), \overline{\alpha}_{\Omega_1(q_1)}(v)], [\underline{\beta}_{\Omega_1(q_1)}(v), \overline{\beta}_{\Omega_1(q_1)}(v)], [(1-\overline{\alpha}_{\Omega_1(q_1)}(v)), (1-\underline{\alpha}_{\Omega_1(q_1)}(v))]\rangle;\ q_1\in\mathcal{S}_1\},$$

$$\oplus(\Omega_2,\mathcal{S}_2)=\big\{\big\langle v,[\underline{\alpha}_{\Omega_2(q_2)}(v),\overline{\alpha}_{\Omega_2(q_2)}(v)],[\underline{\beta}_{\Omega_2(q_2)}(v),\overline{\beta}_{\Omega_2(q_2)}(v)],$$
$$[(1-\overline{\alpha}_{\Omega_2(q_2)}(v)),(1-\underline{\alpha}_{\Omega_2(q_2)}(v))]\big\rangle;\ q_2\in\mathcal{S}_2\big\}.$$

Therefore, we have

$\oplus(\Omega_1,\mathcal{S}_1)\wedge\oplus(\Omega_2,\mathcal{S}_2)$

$$=\big\{\big\langle v,[\wedge(\underline{\alpha}_{\Omega_1(q_1)}(v),\underline{\alpha}_{\Omega_2(q_2)}(v)),\wedge(\overline{\alpha}_{\Omega_1(q_1)}(v),\overline{\alpha}_{\Omega_2(q_2)}(v))],$$
$$[\wedge(\underline{\beta}_{\Omega_1(q_1)}(v),\underline{\beta}_{\Omega_2(q_2)}(v)),\wedge(\overline{\beta}_{\Omega_1(q_1)}(v),\overline{\beta}_{\Omega_2(q_2)}(v))],[\vee((1-\overline{\alpha}_{\Omega_1(q_1)}(v)),(1-\overline{\alpha}_{\Omega_2(q_2)}(v))),$$
$$\vee((1-\underline{\alpha}_{\Omega_1(q_1)}(v)),(1-\underline{\alpha}_{\Omega_2(q_2)}(v)))],\forall(q_1,q_2)\in\mathcal{S}_1\times\mathcal{S}_2\big\}.$$
$$=\ \oplus\left((\Omega_1,\mathcal{S}_1)\wedge(\Omega_2,\mathcal{S}_2)\right).$$

(iii) $\ominus\left((\Omega_1,\mathcal{S}_1)\wedge(\Omega_2,\mathcal{S}_2)\right)$

$$=\big\{\big\langle v,[(1-\vee(\overline{\gamma}_{\Omega_1(q_1)}(v),\overline{\gamma}_{\Omega_2(q_2)}(v)),(1-\vee(\underline{\gamma}_{\Omega_1(q_1)}(v),\underline{\alpha}_{\Omega_2(q_2)}(v))],$$
$$[\vee(\underline{\beta}_{\Omega_1(q_1)}(v),\underline{\beta}_{\Omega_2(q_2)}(v)),\vee(\overline{\beta}_{\Omega_1(q_1)}(v),\overline{\beta}_{\Omega_2(q_2)}(v))],$$
$$[\vee(\underline{\gamma}_{\Omega_1(q_1)}(v)),\underline{\gamma}_{\Omega_2(q_2)}(v)),\vee(\overline{\gamma}_{\Omega_1(q_1)}(v),\overline{\gamma}_{\Omega_2(q_2)}(v))],\forall(q_1,q_2)\in\mathcal{S}_1\times\mathcal{S}_2\big\}.$$
$$=\big\{\big\langle v,[\wedge((1-\overline{\gamma}_{\Omega_1(q_1)}(v),(1-\overline{\gamma}_{\Omega_2(q_2)}(v))),\wedge((1-\underline{\gamma}_{\Omega_1(q_1)}(v)),(1-\underline{\gamma}_{\Omega_2(q_2)}(v)))],$$
$$[\vee(\underline{\beta}_{\Omega_1(q_1)}(v),\underline{\beta}_{\Omega_2(q_2)}(v)),\vee(\overline{\beta}_{\Omega_1(q_1)}(v),\overline{\beta}_{\Omega_2(q_2)}(v))],$$
$$[\vee(\underline{\gamma}_{\Omega_1(q_1)}(v),\underline{\gamma}_{\Omega_2(q_2)}(v)),\vee(\overline{\gamma}_{\Omega_1(q_1)}(v),\overline{\gamma}_{\Omega_2(q_2)}(v))],\forall(q_1,q_2)\in\mathcal{S}_1\times\mathcal{S}_2\big\}.$$

Also,

$$\ominus(\Omega_1,\mathcal{S}_1)=\big\{\big\langle v,[(1-\overline{\gamma}_{\Omega_1(q_1)}(v)),(1-\underline{\gamma}_{\Omega_1(q_1)}(v))],$$
$$[\underline{\beta}_{\Omega_1(q_1)}(v),\overline{\beta}_{\Omega_1(q_1)}(v)],[\underline{\gamma}_{\Omega_1(q_1)}(v),\overline{\gamma}_{\Omega_1(q_1)}(v)]\big\rangle;\ q_1\in\mathcal{S}_1\big\},$$

$$\ominus(\Omega_2,\mathcal{S}_2)=\big\{\big\langle v,[(1-\overline{\gamma}_{\Omega_2(q_2)}(v)),(1-\underline{\gamma}_{\Omega_2(q_2)}(v))],$$
$$[\underline{\beta}_{\Omega_2(q_2)}(v),\overline{\beta}_{\Omega_2(q_2)}(v)],[\underline{\gamma}_{\Omega_2(q_2)}(v),\overline{\gamma}_{\Omega_2(q_2)}(v)]\big\rangle;\ q_2\in\mathcal{S}_2\big\}.$$

Therefore, we have

$\ominus(\Omega_1,\mathcal{S}_1)\wedge\ominus(\Omega_2,\mathcal{S}_2)$

$$=\big\{\big\langle v,[\wedge((1-\overline{\gamma}_{\Omega_1(q_1)}(v)),(1-\overline{\gamma}_{\Omega_2(q_2)}(v)),\wedge((1-\underline{\gamma}_{\Omega_1(q_1)}(v)),(1-\underline{\gamma}_{\Omega_2(q_2)}(v))],$$
$$[\vee(\underline{\beta}_{\Omega_1(q_1)}(v),\underline{\beta}_{\Omega_2(q_2)}(v)),\vee(\overline{\beta}_{\Omega_1(q_1)}(v),\overline{\beta}_{\Omega_2(q_2)}(v))],$$
$$[\vee(\underline{\gamma}_{\Omega_1(q_1)}(v)),\underline{\gamma}_{\Omega_2(q_2)}(v)),\vee(\overline{\gamma}_{\Omega_1(q_1)}(v),\overline{\gamma}_{\Omega_2(q_2)}(v)))],\forall(q_1,q_2)\in\mathcal{S}_1\times\mathcal{S}_2\big\}.$$
$$=\ \ominus\left((\Omega_1,\mathcal{S}_1)\wedge(\Omega_2,\mathcal{S}_2)\right).$$

□

5. $\pm$ and $\mp$ operators on IVINSS

We provide the definition of two new operators ($\pm$ and $\mp$) on IVINSS and discuss some of their properties. We generalize these operations and properties on IVINSS using the concepts given in [5].

Definition 5.1. Let $(\Omega_1, \mathcal{S}_1)$ and $(\Omega_2, \mathcal{S}_2)$ be two IVINSS over $\mathcal{V}$. Then,
(i) the operator $\pm$ is represented as $(\Omega_1, \mathcal{S}_1) \pm (\Omega_2, \mathcal{S}_2) = (\Omega_\pm, \mathcal{S}_\pm)$, where $\mathcal{S}_\pm = \mathcal{S}_1 \cup \mathcal{S}_2$. $\forall$ $q \in \mathcal{S}_\pm$,

$$\Omega_\pm(q) = \begin{cases} \left\{\left\langle v, [\underline{\alpha}_{\Omega_1(q)}(v) + \overline{\alpha}_{\Omega_1(q)}(v)], [\underline{\beta}_{\Omega_1(q)}(v) + \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v) + \overline{\gamma}_{\Omega_1(q)}(v)]\right\rangle; \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\right\}, \\ \left\{\left\langle v, [\underline{\alpha}_{\Omega_2(q)}(v) + \overline{\alpha}_{\Omega_2(q)}(v)], [\underline{\beta}_{\Omega_2(q)}(v) + \overline{\beta}_{\Omega_2(q)}(v)], [\underline{\gamma}_{\Omega_2(q)}(v) + \overline{\gamma}_{\Omega_2(q)}(v)]\right\rangle; \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\right\}, \\ \Bigg\{\Bigg\langle v, \left[\frac{\underline{\alpha}_{\Omega_1(q)}(v)+\underline{\alpha}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\alpha}_{\Omega_1(q)}(v)+\overline{\alpha}_{\Omega_2(q)}(v)}{2}\right], \left[\frac{\underline{\beta}_{\Omega_1(q)}(v)+\underline{\beta}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\beta}_{\Omega_1(q)}(v)+\overline{\beta}_{\Omega_2(q)}(v)}{2}\right], \\ \qquad \left[\frac{\underline{\gamma}_{\Omega_1(q)}(v)+\underline{\gamma}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\gamma}_{\Omega_1(q)}(v)+\overline{\gamma}_{\Omega_2(q)}(v)}{2}\right]\Bigg\rangle; \text{ if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\Bigg\}. \end{cases}$$

(ii) the operator $\mp$ is represented as $(\Omega_1, \mathcal{S}_1) \mp (\Omega_2, \mathcal{S}_2) = (\Omega_\mp, \mathcal{S}_\mp)$, where $\mathcal{S}_\mp = \mathcal{S}_1 \cup \mathcal{S}_2$. $\forall$ $q \in \mathcal{S}_\mp$,

$$\Omega_\mp(q) = \begin{cases} \left\{\left\langle v, [\underline{\alpha}_{\Omega_1(q)}(v) + \overline{\alpha}_{\Omega_1(q)}(v)], [\underline{\beta}_{\Omega_1(q)}(v) + \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v) + \overline{\gamma}_{\Omega_1(q)}(v)]\right\rangle; \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\right\}, \\ \left\{\left\langle v, [\underline{\alpha}_{\Omega_2(q)}(v) + \overline{\alpha}_{\Omega_2(q)}(v)], [\underline{\beta}_{\Omega_2(q)}(v) + \overline{\beta}_{\Omega_2(q)}(v)], [\underline{\gamma}_{\Omega_2(q)}(v) + \overline{\gamma}_{\Omega_2(q)}(v)]\right\rangle; \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\right\}, \\ \Bigg\{\Bigg\langle v, \left[\frac{2\underline{\alpha}_{\Omega_1(q)}(v).\underline{\alpha}_{\Omega_2(q)}(v)}{\underline{\alpha}_{\Omega_1(q)}(v)+\underline{\alpha}_{\Omega_2(q)}(v)}, \frac{2\overline{\alpha}_{\Omega_1(q)}(v).\overline{\alpha}_{\Omega_2(q)}(v)}{\overline{\alpha}_{\Omega_1(q)}(v)+\overline{\alpha}_{\Omega_2(q)}(v)}\right], \left[\frac{\underline{\beta}_{\Omega_1(q)}(v)+\underline{\beta}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\beta}_{\Omega_1(q)}(v)+\overline{\beta}_{\Omega_2(q)}(v)}{2}\right], \\ \qquad \left[\frac{2\underline{\gamma}_{\Omega_1(q)}(v).\underline{\gamma}_{\Omega_2(q)}(v)}{\underline{\gamma}_{\Omega_1(q)}(v)+\underline{\gamma}_{\Omega_2(q)}(v)}, \frac{2\overline{\gamma}_{\Omega_1(q)}(v).\overline{\gamma}_{\Omega_2(q)}(v)}{\overline{\gamma}_{\Omega_1(q)}(v)+\overline{\gamma}_{\Omega_2(q)}(v)}\right]\Bigg\rangle; \text{ if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\Bigg\}. \end{cases}$$

Example 5.2. Consider that a psychiatrist has conducted two counseling sessions for the clients. Assume the psychiatrist has given the values in the IVINSS form for the first session $(\Omega_1, \mathcal{S}_1)$, as in Table 1 and for the second session $(\Omega_2, \mathcal{S}_2)$ in Table 4. Now we calculate the combined results of the two sessions using $(\Omega_1, \mathcal{S}_1) \pm (\Omega_2, \mathcal{S}_2)$, $(\Omega_1, \mathcal{S}_1) \mp (\Omega_2, \mathcal{S}_2)$ and present the results in Table 5 and 6 respectively.

TABLE 4. Shows client with cognitive disorders in IVINSS $(\Omega_2, \mathcal{S}_2)$ form.

$\mathcal{V}$	IMC(q_1)	LM(q_2)	IC(q_3)
v_1	$\langle[0.1, 0.3], [0.6, 0.7], [0.2, 0.3]\rangle$	$\langle[0.2, 0.3], [0.7, 0.8], [0.4, 0.6]\rangle$	$\langle[0.3, 0.4], [0.70.9], [0.4, 0.5]\rangle$
v_2	$\langle[0.3, 0.5], [0.5, 0.8], [0.2, 0.4]\rangle$	$\langle[0.6, 0.7], [0.5, 0.6], [0.2, 0.3]\rangle$	$\langle[0.5, 0.6], [0.30.5], [0.2, 0.3]\rangle$
v_3	$\langle[0.5, 0.6], [0.6, 0.9], [0.3, 0.4]\rangle$	$\langle[0.2, 0.4], [0.9, 1.0], [0.3, 0.4]\rangle$	$\langle[0.1, 0.2], [0.20.4], [0.3, 0.4]\rangle$

(i) The IVINSS $(\Omega_1, \mathcal{S}_1) \pm (\Omega_2, \mathcal{S}_2)$ is shown in Table 5.

(ii) The IVINSS $(\Omega_1, \mathcal{S}_1) \mp (\Omega_2, \mathcal{S}_2)$ is given in Table 6.

Table 5. Representation of clients with cognitive disorders in IVINSS $(\Omega_1, \mathcal{S}_1) \pm (\Omega_2, \mathcal{S}_2)$ form.

$\mathcal{V}$	IMC(q_1)	LM(q_2)	IC(q_3)
v_1	$\langle[0.15, 0.35], [0.50, 0.60], [0.30, 0.40]\rangle$	$\langle[0.25, 0.35], [0.60, 0.70], [0.35, 0.55]\rangle$	$\langle[0.25, 0.35], [0.60, 0.85], [0.50, 0.60]\rangle$
v_2	$\langle[0.35, 0.55], [0.40, 0.65], [0.15, 0.30]\rangle$	$\langle[0.65, 0.75], [0.35, 0.55], [0.15, 0.25]\rangle$	$\langle[0.55, 0.65], [0.50, 0.65], [0.15, 0.25]\rangle$
v_3	$\langle[0.55, 0.65], [0.40, 0.80], [0.20, 0.30]\rangle$	$\langle[0.15, 0.35], [0.75, 0.85], [0.40, 0.50]\rangle$	$\langle[0.15, 0.25], [0.45, 0.60], [0.35, 0.45]\rangle$

Table 6. Shows clients with cognitive disorders in IVINSS $(\Omega_1, \mathcal{S}_1) \mp (\Omega_2, \mathcal{S}_2)$ form.

$\mathcal{U}$	IMC(q_1)	LM(q_2)	IC(q_3)
v_1	$\langle[0.13, 0.34], [0.50, 0.60], [0.26, 0.37]\rangle$	$\langle[0.24, 0.34], [0.60, 0.70], [0.34, 0.54]\rangle$	$\langle[0.24, 0.34], [0.60, 0.85], [0.48, 0.58]\rangle$
v_2	$\langle[0.34, 0.54], [0.40, 0.65], [0.13, 0.26]\rangle$	$\langle[0.64, 0.74], [0.35, 0.55], [0.13, 0.24]\rangle$	$\langle[0.54, 0.64], [0.50, 0.65], [0.13, 0.24]\rangle$
v_3	$\langle[0.54, 0.64], [0.40, 0.80], [0.15, 0.26]\rangle$	$\langle[0.13, 0.34], [0.75, 0.85], [0.37, 0.48]\rangle$	$\langle[0.13, 0.24], [0.45, 0.60], [0.34, 0.44]\rangle$

Proposition 5.3. *Let* $(\Omega_1, \mathcal{S}_1)$ *and* $(\Omega_2, \mathcal{S}_2)$ *be non-empty over* $\mathcal{V}$. *Then,*
(i) $(\Omega_1, \mathcal{S}_1) \pm (\Omega_2, \mathcal{S}_2) = (\Omega_2, \mathcal{S}_2) \pm (\Omega_1, \mathcal{S}_1)$;
(ii) $[(\Omega_1, \mathcal{S}_1)^c \pm (\Omega_2, \mathcal{S}_2)^c]^c = (\Omega_1, \mathcal{S}_1) \pm (\Omega_1, \mathcal{S}_1)$.

Proof. (i) Proof straightforward.
(ii) Let

$$(\Omega_1, \mathcal{S}_1) = \{\langle v, [\underline{\alpha}_{\Omega_1(q)}(v) + \overline{\alpha}_{\Omega_1(q)}(v)], [\underline{\beta}_{\Omega_1(q)}(v) + \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v) + \overline{\gamma}_{\Omega_1(q)}(v)]\rangle; q \in \mathcal{S}_1\},$$

and

$$(\Omega_2, \mathcal{S}_2) = \{\langle v, [\underline{\alpha}_{\Omega_2(q)}(v) + \overline{\alpha}_{\Omega_2(q)}(v)], [\underline{\beta}_{\Omega_2(q)}(v) + \overline{\beta}_{\Omega_2(q)}(v)], [\underline{\gamma}_{\Omega_2(q)}(v) + \overline{\gamma}_{\Omega_2(q)}(v)]\rangle; q \in \mathcal{S}_2\}$$

be two IVINSS.
Then, $[(\Omega_1, \mathcal{S}_1)^c \pm (\Omega_2, \mathcal{S}_2)^c]$

$$= \begin{cases} \{\langle v, [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)], [(1 - \overline{\beta}_{\Omega_1(q)}(v)), (1 - \underline{\beta}_{\Omega_1(q)}(v))], \\ \qquad [\underline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_1(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\}, \\ \{\langle v, [\underline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)], [(1 - \overline{\beta}_{\Omega_2(q)}(v)), (1 - \underline{\beta}_{\Omega_2(q)}(v))], \\ \qquad [\underline{\alpha}_{\Omega_2(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\}, \\ \left\{\left\langle v, \left[\frac{\underline{\gamma}_{\Omega_1(q)}(v)+\underline{\gamma}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\gamma}_{\Omega_1(q)}(v)+\overline{\gamma}_{\Omega_2(q)}(v)}{2}\right], \left[\frac{(1-\underline{\beta}_{\Omega_1(q)}(v))+(1-\underline{\beta}_{\Omega_2(q)}(v))}{2}, \right.\right.\right. \\ \qquad \left.\left.\left.\frac{(1-\overline{\beta}_{\Omega_1(q)}(v))+(1-\overline{\beta}_{\Omega_2(q)}(v)}{2}\right], \left[\frac{\underline{\alpha}_{\Omega_1(q)}(v)+\underline{\alpha}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\alpha}_{\Omega_1(q)}(v)+\overline{\alpha}_{\Omega_2(q)}(v)}{2}\right]\right\rangle; \text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\right\}. \end{cases}$$

Now consider, $[(\Omega_1,\mathcal{S}_1)^c \pm (\Omega_2,\mathcal{S}_2)^c]^c$

$$= \begin{cases} \big\{\big\langle v, [\underline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_1(q)}(v)], [(1-\overline{\beta}_{\Omega_1(q)}(v)), (1-\underline{\beta}_{\Omega_1(q)}(v))], \\ \qquad [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\big\}, \\ \big\{\big\langle v, [\underline{\alpha}_{\Omega_2(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v)], [(1-\overline{\beta}_{\Omega_2(q)}(v)), (1-\underline{\beta}_{\Omega_2(q)}(v))], \\ \qquad [\underline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\big\}, \\ \Big\{\Big\langle v, \Big[\frac{\underline{\alpha}_{\Omega_1(q)}(v)+\underline{\alpha}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\alpha}_{\Omega_1(q)}(v)+\overline{\alpha}_{\Omega_2(q)}(v)}{2}\Big]\Big[\frac{(1-\underline{\beta}_{\Omega_1(q)}(v))+(1-\underline{\beta}_{\Omega_2(q)}(v))}{2}, \\ \frac{(1-\overline{\beta}_{\Omega_1(q)}(v))+(1-\overline{\beta}_{\Omega_2(q)}(v)}{2}\Big], \Big[\frac{\underline{\gamma}_{\Omega_1(q)}(v)+\underline{\gamma}_{\Omega_2(q)}(v}{2}, \frac{\overline{\gamma}_{\Omega_1(q)}(v)+\overline{\gamma}_{\Omega_2(q)}(v)}{2}\Big]\Big\rangle; \quad \text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\Big\}. \end{cases}$$

Hence $[(\Omega_1,\mathcal{S}_1)^c \pm (\Omega_2,\mathcal{S}_2)^c]^c = (\Omega_1,\mathcal{S}_1) \pm (\Omega_2,\mathcal{S}_2)$. □

Proposition 5.4. *Let $(\Omega_1,\mathcal{S}_1)$ and $(\Omega_2,\mathcal{S}_2)$ be non-empty over $\mathcal{V}$. Then,*
(i) $(\Omega_1,\mathcal{S}_1) \mp (\Omega_2,\mathcal{S}_2) = (\Omega_2,\mathcal{S}_2) \mp (\Omega_1,\mathcal{S}_1)$;
(ii) $[(\Omega_1,\mathcal{S}_1)^c \mp (\Omega_2,\mathcal{S}_2)^c]^c = (\Omega_1,\mathcal{S}_1) \mp (\Omega_1,\mathcal{S}_1)$.

Proof. (i) Consider, $(\Omega_1,\mathcal{S}_1) \mp (\Omega_2,\mathcal{S}_2)$

$$= \begin{cases} \big\{\big\langle v, [\underline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_1(q)}(v)], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\big\}, \\ \big\{\big\langle v, [\underline{\alpha}_{\Omega_2(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v)], [\underline{\beta}_{\Omega_2(q)}(v), \overline{\beta}_{\Omega_2(q)}(v)], [\underline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\big\}, \\ \{\Big\langle v, \Big[\frac{2\underline{\alpha}_{\Omega_1(q)}(v).\underline{\alpha}_{\Omega_2(q)}(v)}{\underline{\alpha}_{\Omega_1(q)}(v)+\underline{\alpha}_{\Omega_2(q)}(v)}, \frac{2\overline{\alpha}_{\Omega_1(q)}(v).\overline{\alpha}_{\Omega_2(q)}(v)}{\overline{\alpha}_{\Omega_1(q)}(v)+\overline{\alpha}_{\Omega_2(q)}(v)}\Big], \Big[\frac{\underline{\beta}_{\Omega_1(q)}(v)+\underline{\beta}_{\Omega_2(q)}(v)}{2}, \frac{\overline{\beta}_{\Omega_1(q)}(v)+\overline{\beta}_{\Omega_2(q)}(v)}{2}\Big], \\ \qquad \Big[\frac{2\underline{\gamma}_{\Omega_1(q)}(v).\underline{\gamma}_{\Omega_2(q)}(v)}{\underline{\gamma}_{\Omega_1(q)}(v)+\underline{\gamma}_{\Omega_2(q)}(v)}, \frac{2\overline{\gamma}_{\Omega_1(q)}(v).\overline{\gamma}_{\Omega_2(q)}(v)}{\overline{\gamma}_{\Omega_1(q)}(v)+\overline{\gamma}_{\Omega_2(q)}(v)}\Big]\Big\rangle; \text{ if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\}. \end{cases}$$

$$= \begin{cases} \big\{\big\langle v, [\underline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_1(q)}(v)], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\big\}, \\ \big\{\big\langle v, [\underline{\alpha}_{\Omega_2(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v)], [\underline{\beta}_{\Omega_2(q)}(v), \overline{\beta}_{\Omega_2(q)}(v)], [\underline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\big\}, \\ \{\Big\langle v, \Big[\frac{2\underline{\alpha}_{\Omega_2(q)}(v).\underline{\alpha}_{\Omega_1(q)}(v)}{\underline{\alpha}_{\Omega_2(q)}(v)+\underline{\alpha}_{\Omega_1(q)}(v)}, \frac{2\overline{\alpha}_{\Omega_2(q)}(v).\overline{\alpha}_{\Omega_1(q)}(v)}{\overline{\alpha}_{\Omega_2(q)}(v)+\overline{\alpha}_{\Omega_1(q)}(v)}\Big], \Big[\frac{\underline{\beta}_{\Omega_2(q)}(v)+\underline{\beta}_{\Omega_1(q)}(v)}{2}, \frac{\overline{\beta}_{\Omega_2(q)}(v)+\overline{\beta}_{\Omega_1(q)}(v)}{2}\Big], \\ \qquad \Big[\frac{2\underline{\gamma}_{\Omega_2(q)}(v).\underline{\gamma}_{\Omega_1(q)}(v)}{\underline{\gamma}_{\Omega_2(q)}(v)+\underline{\gamma}_{\Omega_1(q)}(v)}, \frac{2\overline{\gamma}_{\Omega_2(q)}(v).\overline{\gamma}_{\Omega_1(q)}(v)}{\overline{\gamma}_{\Omega_2(q)}(v)+\overline{\gamma}_{\Omega_1(q)}(v)}\Big]\Big\rangle; \quad \text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\}. \end{cases}$$

Hence $(\Omega_1,\mathcal{S}_1) \mp (\Omega_2,\mathcal{S}_2) = (\Omega_2,\mathcal{S}_2) \mp (\Omega_1,\mathcal{S}_1)$.

(ii) Consider, $(\Omega_1,\mathcal{S}_1)^c \mp (\Omega_2,\mathcal{S}_2)^c$

$$= \begin{cases} \big\{\big\langle v, [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)], [(1-\overline{\beta}_{\Omega_1(q)}(v)), (1-\underline{\beta}_{\Omega_1(q)}(v))], \\ \qquad [\underline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_1(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\big\}, \\ \big\{\big\langle v, [\underline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)], [(1-\overline{\beta}_{\Omega_2(q)}(v)), (1-\underline{\beta}_{\Omega_2(q)}(v))], \\ \qquad [\underline{\alpha}_{\Omega_2(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v)]\big\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\big\}, \\ \Big\{\Big\langle v, \Big[\frac{2\underline{\gamma}_{\Omega_1(q)}(v).\underline{\gamma}_{\Omega_2(q)}(v)}{\underline{\gamma}_{\Omega_1(q)}(v)+\underline{\gamma}_{\Omega_2(q)}(v)}, \frac{2\overline{\gamma}_{\Omega_1(q)}(v).\overline{\gamma}_{\Omega_2(q)}(v)}{\overline{\gamma}_{\Omega_1(q)}(v)+\overline{\gamma}_{\Omega_2(q)}(v)}\Big], \Big[\frac{(1-\underline{\beta}_{\Omega_1(q)}(v))+(1-\underline{\beta}_{\Omega_2(q)}(v))}{2}, \\ \frac{(1-\overline{\beta}_{\Omega_1(q)}(v))+(1-\overline{\beta}_{\Omega_2(q)}(v)}{2}\Big], \Big[\frac{2\underline{\alpha}_{\Omega_1(q)}(v).\underline{\alpha}_{\Omega_2(q)}(v)}{\underline{\alpha}_{\Omega_1(q)}(v)+\underline{\alpha}_{\Omega_2(q)}(v)}, \frac{2\overline{\alpha}_{\Omega_1(q)}(v).\overline{\alpha}_{\Omega_2(q)}(v)}{\overline{\alpha}_{\Omega_1(q)}(v)+\overline{\alpha}_{\Omega_2(q)}(v)}\Big]\Big\rangle; \text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\Big\}. \end{cases}$$

Then, $[(\Omega_1,\mathcal{S}_1)^c \mp (\Omega_2,\mathcal{S}_2)^c]^c$

$$= \begin{cases} \{\langle v, [\underline{\alpha}_{\Omega_1(q)}(v), \overline{\alpha}_{\Omega_1(q)}(v)], [\underline{\beta}_{\Omega_1(q)}(v), \overline{\beta}_{\Omega_1(q)}(v)], \\ \qquad [\underline{\gamma}_{\Omega_1(q)}(v), \overline{\gamma}_{\Omega_1(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_1 - \mathcal{S}_2\}, \\ \{\langle v, [\underline{\alpha}_{\Omega_2(q)}(v), \overline{\alpha}_{\Omega_2(q)}(v)], [\underline{\beta}_{\Omega_2(q)}(v), \overline{\beta}_{\Omega_2(q)}(v)], \\ \qquad [\underline{\gamma}_{\Omega_2(q)}(v), \overline{\gamma}_{\Omega_2(q)}(v)]\rangle; \quad \text{if } q \in \mathcal{S}_2 - \mathcal{S}_1\}, \\ \left\{\left\langle v, \left[\frac{2\underline{\alpha}_{\Omega_1(q)}(v).\underline{\alpha}_{\Omega_2(q)}(v)}{\underline{\alpha}_{\Omega_1(q)}(v)+\underline{\alpha}_{\Omega_2(q)}(v)}, \frac{2\overline{\alpha}_{\Omega_1(q)}(v).\overline{\alpha}_{\Omega_2(q)}(v)}{\overline{\alpha}_{\Omega_1(q)}(v)+\overline{\alpha}_{\Omega_2(q)}(v)}\right], \left[\frac{(1-\underline{\beta}_{\Omega_1(q)}(v))+(1-\underline{\beta}_{\Omega_2(q)}(v))}{2},\right.\right.\right. \\ \quad \left.\left.\left.\frac{(1-\overline{\beta}_{\Omega_1(q)}(v))+(1-\overline{\beta}_{\Omega_2(q)}(v)}{2}\right], \left[\frac{2\underline{\gamma}_{\Omega_1(q)}(v).\underline{\gamma}_{\Omega_2(q)}(v)}{\underline{\gamma}_{\Omega_1(q)}(v)+\underline{\gamma}_{\Omega_2(q)}(v)}, \frac{2\overline{\gamma}_{\Omega_1(q)}(v).\overline{\gamma}_{\Omega_2(q)}(v)}{\overline{\gamma}_{\Omega_1(q)}(v)+\overline{\gamma}_{\Omega_2(q)}(v)}\right]\right\rangle; \quad \text{if } q \in \mathcal{S}_1 \cap \mathcal{S}_2\right\}. \end{cases}$$

Hence $[(\Omega_1,\mathcal{S}_1)^c \mp (\Omega_2,\mathcal{S}_2)^c]^c = (\Omega_1,\mathcal{S}_1) \mp (\Omega_2,\mathcal{S}_2)$. □

6. $\mathcal{N}_\epsilon$, $\mathcal{N}_{\epsilon,\rho}$ and $\mathcal{I}_{\epsilon,\rho}$ operators on IVINSS

In this section, we define the operators $\mathcal{N}_\epsilon$, $\mathcal{N}_{\epsilon,\rho}$ and $\mathcal{I}_{\epsilon,\rho}$ on IVINSS and discuss some of their properties in detail. We generalize these operations and properties on IVINSS by the concepts discussed in [4].

Definition 6.1. Let $\epsilon \in [0,1]$. Then the operator $\mathcal{N}_\epsilon(\Omega,\mathcal{S})$ is represented as,
$\mathcal{N}_\epsilon(\Omega,\mathcal{S})$

$$\begin{aligned} &= \{\langle v, [\underline{\alpha}_{\Omega(q)}(v) + \epsilon(1-\underline{\alpha}_{\Omega(q)}(v) - \underline{\gamma}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \epsilon(1-\overline{\alpha}_{\Omega(q)}(v) - \overline{\gamma}_{\Omega(q)}(v))], \\ &\quad [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\overline{\gamma}_{\Omega(q)}(v) + (1-\epsilon)(1-\overline{\alpha}_{\Omega(q)}(v) - \overline{\gamma}_{\Omega(q)}(v)), \\ &\quad \underline{\gamma}_{\Omega(q)}(v) + (1-\epsilon)(1-\underline{\alpha}_{\Omega(q)}(v) - \underline{\gamma}_{\Omega(q)}(v))]\rangle; q \in \mathcal{S}\}. \\ &= \{\langle v, [\underline{\alpha}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \epsilon(\overline{\pi}_{\Omega(q)}(v))], \\ &\quad [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\overline{\gamma}_{\Omega(q)}(v) + (1-\epsilon)(\overline{\pi}_{\Omega(q)}(v)), \\ &\quad \underline{\gamma}_{\Omega(q)}(v) + (1-\epsilon)(\underline{\pi}_{\Omega(q)}(v))]\rangle; q \in \mathcal{S}\}, \\ &\quad \text{where } \underline{\pi}_{\Omega(q)}(v) = (1-\underline{\alpha}_{\Omega(q)}(v) - \underline{\gamma}_{\Omega(q)}(v)) \text{and } \overline{\pi}_{\Omega(q)}(v) = (1-\overline{\alpha}_{\Omega(q)}(v) - \overline{\gamma}_{\Omega(q)}(v)). \end{aligned}$$

Proposition 6.2. *Let $\epsilon, \rho \in [0,1]$ and $\epsilon \leq \rho$. Then for every IVINSS $(\Omega,\mathcal{S})$ the following hold:*
(i) $\mathcal{N}_\epsilon(\Omega,\mathcal{S}) \Subset \mathcal{N}_\rho(\Omega,\mathcal{S})$;
(ii) $\mathcal{N}_0(\Omega,\mathcal{S}) = \oplus(\Omega,\mathcal{S})$;
(iii) $\mathcal{N}_1(\Omega,\mathcal{S}) = \ominus(\Omega,\mathcal{S})$.

Proof. (i) $\mathcal{N}_\epsilon(\Omega,\mathcal{S})$

$$= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v)+\epsilon(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v)+\epsilon(\overline{\pi}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\overline{\gamma}_{\Omega(q)}(v)+(1-\epsilon)(\overline{\pi}_{\Omega(q)}(v)), \underline{\gamma}_{\Omega(q)}(v)+(1-\epsilon)(\underline{\pi}_{\Omega(q)}(v))]\big\rangle; q\in\mathcal{S}\big\}, \text{and}$$

$\mathcal{N}_\rho(\Omega,\mathcal{S})$

$$= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v)+\rho(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v)+\rho(\overline{\pi}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\overline{\gamma}_{\Omega(q)}(v)+(1-\rho)(\overline{\pi}_{\Omega(q)}(v)), \underline{\gamma}_{\Omega(q)}(v)+(1-\rho)(\underline{\pi}_{\Omega(q)}(v))]\big\rangle; q\in\mathcal{S}\big\}.$$

Since $\epsilon \le \rho$, we have

$$(\overline{\alpha}_{\Omega(q)}(v)+\epsilon(\overline{\pi}_{\Omega(q)}(v)) \le (\overline{\alpha}_{\Omega(q)}(v)+\rho(\overline{\pi}_{\Omega(q)}(v)).$$

Also, $(1-\epsilon)\ge(1-\rho)$, we have

$$(\overline{\gamma}_{\Omega(q)}(v)+(1-\epsilon)(\overline{\pi}_{\Omega(q)}(v)) \ge (\overline{\gamma}_{\Omega(q)}(v)+(1-\rho)(\overline{\pi}_{\Omega(q)}(v)).$$

Hence $\mathcal{N}_\epsilon(\Omega,\mathcal{S}) \Subset \mathcal{N}_\rho(\Omega,\mathcal{S})$.
(ii) Consider, $\epsilon = 0$
$\mathcal{N}_0(\Omega,\mathcal{S})$

$$\begin{aligned}
&= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v)+0, \overline{\alpha}_{\Omega(q)}(v)+0], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [(\underline{\gamma}_{\Omega(q)}(v)+1.(\underline{\pi}_{\Omega(q)}(v))), \\
&\quad (\overline{\gamma}_{\Omega(q)}(v)+1.(\overline{\pi}_{\Omega(q)}(v)))]\big\rangle; q\in\mathcal{S}\big\} \\
&= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [(1-\overline{\alpha}_{\Omega(q)}(v)), (1-\underline{\alpha}_{\Omega(q)}(v))]\big\rangle; q\in\mathcal{S}\big\} \\
&= \oplus(\Omega,\mathcal{S}).
\end{aligned}$$

Hence $\mathcal{N}_0(\Omega,\mathcal{S}) = \oplus(\Omega,\mathcal{S})$.
(iii) Consider, $\epsilon = 1$
$\mathcal{N}_1(\Omega,\mathcal{S})$

$$\begin{aligned}
&= \big\{\big\langle v, [(\underline{\alpha}_{\Omega(q)}(v)+\underline{\pi}_{\Omega(q)}(v)), (\overline{\alpha}_{\Omega(q)}(v)+\overline{\pi}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], \\
&\quad [\underline{\gamma}_{\Omega(q)}(v)+0, \overline{\gamma}_{\Omega(q)}(v)+0]\big\rangle; q\in\mathcal{S}\big\} \\
&= \big\{\big\langle v, [(1-\overline{\gamma}_{\Omega(q)}(v)), (1-\underline{\gamma}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)]\big\rangle; q\in\mathcal{S}\big\} \\
&= \oplus(\Omega,\mathcal{S}).
\end{aligned}$$

Hence $\mathcal{N}_1(\Omega,\mathcal{S}) = \ominus(\Omega,\mathcal{S})$. □

Remark 6.3. The operator $\mathcal{N}_\epsilon$ is an extension of $\oplus$ and $\ominus$ operators.

Definition 6.4. Let $\epsilon, \rho \in [0,1]$ and $\epsilon + \rho \leq 1$. Then the operator $\mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S})$ is represented as,

$\mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S})$

$$= \{\langle v, [\underline{\alpha}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \epsilon(\overline{\pi}_{\Omega(q)}(v))],$$
$$[\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\gamma}_{\Omega(q)}(v) + \rho(\underline{\pi}_{\Omega(q)}(v)), \overline{\gamma}_{\Omega(q)}(v) + \rho(\overline{\pi}_{\Omega(q)}(v)]\rangle; q \in \mathcal{S}\},$$

where $\underline{\pi}_{\Omega(q)}(v) = (1 - \underline{\alpha}_{\Omega(q)}(v) - \underline{\gamma}_{\Omega(q)}(v))$and $\overline{\pi}_{\Omega(q)}(v) = (1 - \overline{\alpha}_{\Omega(q)}(v) - \overline{\gamma}_{\Omega(q)}(v))$.

Theorem 6.5. *Let $\epsilon, \rho, \sigma \in [0,1]$ and $\epsilon + \rho \leq 1$. Then for every IVINSS $(\Omega, \mathcal{S})$ the following hold:*

(i) $\mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S})$ is an IVINSS;

(ii) If $0 \leq \sigma \leq \epsilon$ then $\mathcal{N}_{\sigma,\rho}(\Omega, \mathcal{S}) \Subset \mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S})$;

(iii) If $0 \leq \sigma \leq \rho$ then $\mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S}) \Subset \mathcal{N}_{\epsilon,\sigma}(\Omega, \mathcal{S})$;

(iv) $\mathcal{N}_{\epsilon}(\Omega, \mathcal{S}) = \mathcal{N}_{\epsilon,(1-\epsilon)}(\Omega, \mathcal{S})$;

(v) $\oplus(\Omega, \mathcal{S}) = \mathcal{N}_{0,1}(\Omega, \mathcal{S})$;

(vi) $\ominus(\Omega, \mathcal{S}) = \mathcal{N}_{1,0}(\Omega, \mathcal{S})$;

(vii) $(\mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S})^c)^c = (\mathcal{N}_{\rho,\epsilon}(\Omega, \mathcal{S}))$.

Proof. (i) Consider,

$$\begin{aligned}
\frac{\overline{\alpha}_{\Omega(q)}(v) + \epsilon(\overline{\pi}_{\Omega(q)}(v))}{2} + \overline{\beta}_{\Omega(q)}(v) + \frac{\overline{\gamma}_{\Omega(q)}(v) + \rho(\overline{\pi}_{\Omega(q)}(v))}{2} \\
&= \frac{\overline{\alpha}_{\Omega(q)}(v) + \overline{\gamma}_{\Omega(q)}(v)}{2} + \overline{\beta}_{\Omega(q)}(v) + (\epsilon + \rho)\frac{(\overline{\pi}_{\Omega(q)}(v))}{2} \\
&\leq \frac{\overline{\alpha}_{\Omega(q)}(v) + \overline{\gamma}_{\Omega(q)}(v)}{2} + \overline{\beta}_{\Omega(q)}(v) + \frac{(1 - \overline{\alpha}_{\Omega(q)}(v) - \overline{\gamma}_{\Omega(q)}(v)}{2} \\
&\leq \frac{1}{2} + 1 < 2, \qquad \text{since } \epsilon + \rho \leq 1 \text{ and } \overline{\beta}_{\Omega(q)}(v) \leq 1
\end{aligned}$$

Hence $\mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S})$ is an IVINSS.

(ii) Consider,

$$\mathcal{N}_{\sigma,\rho}(\Omega, \mathcal{S}) = \{\langle v, [\underline{\alpha}_{\Omega(q)}(v) + \sigma(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \sigma(\overline{\pi}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],$$
$$[\underline{\gamma}_{\Omega(q)}(v) + \rho(\underline{\pi}_{\Omega(q)}(v)), \overline{\gamma}_{\Omega(q)}(v) + \rho(\overline{\pi}_{\Omega(q)}(v)]\rangle; q \in \mathcal{S}\}$$

$$\mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S}) = \{\langle v, [\underline{\alpha}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \epsilon(\overline{\pi}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],$$
$$[\underline{\gamma}_{\Omega(q)}(v) + \rho(\underline{\pi}_{\Omega(q)}(v)), \overline{\gamma}_{\Omega(q)}(v) + \rho(\overline{\pi}_{\Omega(q)}(v)]\rangle; q \in \mathcal{S}\}$$

Now, $\underline{\alpha}_{\Omega(q)}(v) + \sigma(\underline{\pi}_{\Omega(q)}(v)) \leq \underline{\alpha}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v))$, since $\sigma \leq \epsilon$

Similarly, $\overline{\alpha}_{\Omega(q)}(v) + \sigma(\underline{\pi}_{\Omega(q)}(v)) \leq \overline{\alpha}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v))$.

Hence $\mathcal{N}_{\sigma,\rho}(\Omega, \mathcal{S}) \Subset \mathcal{N}_{\epsilon,\rho}(\Omega, \mathcal{S})$.

(iii) Similar to proof (ii).

(iv) Consider,

$$\begin{aligned}\mathcal{N}_{\epsilon,1-\epsilon}(\Omega,\mathcal{S}) = \big\{\big\langle v, [&\underline{\alpha}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \epsilon(\overline{\pi}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\ &[\underline{\gamma}_{\Omega(q)}(v) + (1-\epsilon)(\underline{\pi}_{\Omega(q)}(v)), \overline{\gamma}_{\Omega(q)}(v) + (1-\epsilon)(\overline{\pi}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\}\\ =&\mathcal{N}_{\epsilon}(\Omega,\mathcal{S}).\end{aligned}$$

Hence $\mathcal{N}_{\epsilon}(\Omega,\mathcal{S}) = \mathcal{N}_{\epsilon,(1-\epsilon)}(\Omega,\mathcal{S})$.

(v) Let $\epsilon = 0$ and $\rho = 1$,

$$\begin{aligned}\mathcal{N}_{0,1}(\Omega,\mathcal{S}) &= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\gamma}_{\Omega(q)}(v) + \underline{\pi}_{\Omega(q)}(v),\\ &\quad \overline{\gamma}_{\Omega(q)}(v) + \overline{\pi}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\}\\ &=\big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [(1-\overline{\alpha}_{\Omega(q)}(v)),\\ &\quad (1-\underline{\alpha}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\}\\ &= \oplus\,(\Omega,\mathcal{S}).\end{aligned}$$

Hence $\oplus(\Omega,\mathcal{S}) = \mathcal{N}_{0,1}(\Omega,\mathcal{S})$.

(vi)Let $\alpha = 1$ and $\beta = 0$,

$$\begin{aligned}\mathcal{N}_{1,0}(\Omega,\mathcal{S}) &= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v) + \underline{\pi}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v) + \overline{\pi}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\ &\quad [\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\}\\ &=\big\{\big\langle v, [(1-\overline{\gamma}_{\Omega(q)}(v)), (1-\underline{\gamma}_{\Omega(q)}(v))], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\gamma}_{\Omega(q)}(v),\\ &\quad \overline{\gamma}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\}\\ &= \ominus\,(\Omega,\mathcal{S}).\end{aligned}$$

Hence $\ominus(\Omega,\mathcal{S}) = \mathcal{N}_{1,0}(\Omega,\mathcal{S})$.

(vii) Consider,

$$\begin{aligned}\mathcal{N}_{\epsilon,\rho}(\Omega,\mathcal{S})^c = \big\{\big\langle v, [&\underline{\gamma}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v)), \overline{\gamma}_{\Omega(q)}(v) + \epsilon(\overline{\pi}_{\Omega(q)}(v))],\\ &[\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\alpha}_{\Omega(q)}(v) + \rho(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \rho(\overline{\pi}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\}\\ (\mathcal{N}_{\epsilon,\rho}(\Omega,\mathcal{S})^c)^c = \big\{\big\langle v, [&\underline{\alpha}_{\Omega(q)}(v) + \rho(\underline{\pi}_{\Omega(q)}(v)), \overline{\alpha}_{\Omega(q)}(v) + \rho(\overline{\pi}_{\Omega(q)}(v)],\\ &[\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\gamma}_{\Omega(q)}(v) + \epsilon(\underline{\pi}_{\Omega(q)}(v)), \overline{\gamma}_{\Omega(q)}(v) + \epsilon(\overline{\pi}_{\Omega(q)}(v))]\big\rangle; q \in \mathcal{S}\big\}\\ =&(\mathcal{N}_{\rho,\epsilon}(\Omega,\mathcal{S})).\end{aligned}$$

Hence $(\mathcal{N}_{\epsilon,\rho}(\Omega,\mathcal{S})^c)^c = (\mathcal{N}_{\rho,\epsilon}(\Omega,\mathcal{S}))$. □

Remark 6.6. If $\epsilon + \rho = 1$, then $\mathcal{N}_{\epsilon,\rho}(\Omega,\mathcal{S}) = \mathcal{N}_{\epsilon}(\Omega,\mathcal{S})$.

Definition 6.7. Let $\epsilon, \rho \in [0,1]$ and $\epsilon + \rho \leq 1$. Then the operator $\mathcal{I}_{\epsilon,\rho}(\Omega,\mathcal{S})$ is represented as,

$$\mathcal{I}_{\epsilon,\rho}(\Omega,\mathcal{S}) = \big\{\big\langle v, [\epsilon.\underline{\alpha}_{\Omega(q)}(v), \epsilon.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\rho.\underline{\gamma}_{\Omega(q)}(v), \rho.\overline{\gamma}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\}$$

Theorem 6.8. *Let $\epsilon, \rho, \gamma \in [0,1]$. Then for every IVINSS $(\Omega, \mathcal{S})$ the following hold:*
(i) $\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S})$ is an IVINSS;
(ii) If $\epsilon \leq \sigma$ then $\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S}) \Subset \mathcal{I}_{\sigma,\rho}(\Omega, \mathcal{S})$;
(iii) If $\rho \leq \sigma$ then $\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S}) \supset \mathcal{I}_{\epsilon,\sigma}(\Omega, \mathcal{S})$;
(iv) If $\delta \in [0,1]$ then $\mathcal{I}_{\epsilon,\rho}(\mathcal{I}_{\sigma,\delta}(\Omega, \mathcal{S})) = \mathcal{I}_{\epsilon\sigma,\rho\delta}(\Omega, \mathcal{S}) = \mathcal{I}_{\sigma,\delta}(\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S}))$;
(v)$(\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S})^c)^c = (\mathcal{I}_{\rho,\epsilon}(\Omega, \mathcal{S}))$.

Proof. (i) Proof straightforward.
(ii) Consider,

$$\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S}) = \Big\{\Big\langle v, [\epsilon.\underline{\alpha}_{\Omega(q)}(v), \epsilon.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\rho.\underline{\gamma}_{\Omega(q)}(v), \rho.\overline{\gamma}_{\Omega(q)}(v)]\Big\rangle; q \in \mathcal{S}\Big\},$$
$$\mathcal{I}_{\sigma,\rho}(\Omega, \mathcal{S}) = \Big\{\Big\langle v, [\sigma.\underline{\alpha}_{\Omega(q)}(v), \sigma.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\rho.\underline{\gamma}_{\Omega(q)}(v), \rho.\overline{\gamma}_{\Omega(q)}(v)]\Big\rangle; q \in \mathcal{S}\Big\}.$$

Since $\epsilon \leq \sigma$, $\epsilon.\underline{\alpha}_{\Omega(q)}(v) \leq \sigma.\underline{\alpha}_{\Omega(q)}(v)$ and $\epsilon.\overline{\alpha}_{\Omega(q)}(v) \leq \sigma.\overline{\alpha}_{\Omega(q)}(v)$.
Hence $\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S}) \Subset \mathcal{I}_{\sigma,\rho}(\Omega, \mathcal{S})$.
(iii) Consider,

$$\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S}) = \Big\{\Big\langle v, [\epsilon.\underline{\alpha}_{\Omega(q)}(v), \epsilon.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\rho.\underline{\gamma}_{\Omega(q)}(v), \rho.\overline{\gamma}_{\Omega(q)}(v)]\Big\rangle; q \in \mathcal{S}\Big\},$$
$$\mathcal{I}_{\epsilon,\sigma}(\Omega, \mathcal{S}) = \Big\{\Big\langle v, [\epsilon.\underline{\alpha}_{\Omega(q)}(v), \epsilon.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\sigma.\underline{\gamma}_{\Omega(q)}(v), \sigma.\overline{\gamma}_{\Omega(q)}(v)]\Big\rangle; q \in \mathcal{S}\Big\}.$$

Since, $\rho \leq \sigma$, $\rho.\underline{\gamma}_{\Omega(q)}(v) \leq \sigma.\underline{\gamma}_{\Omega(q)}(v)$ and $\rho.\overline{\gamma}_{\Omega(q)}(v) \leq \sigma.\overline{\gamma}_{\Omega(q)}(v)$.
Hence $\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S}) \supset \mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S})$.
(iv) Consider,

$$\begin{aligned}
\mathcal{I}_{\epsilon,\rho}(\mathcal{I}_{\sigma,\delta}(\Omega, \mathcal{S})) = &\Big\{\Big\langle v, [\epsilon\sigma.\underline{\alpha}_{\Omega(q)}(v), \epsilon\sigma.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\
&[\rho\delta.\underline{\gamma}_{\Omega(q)}(v), \rho\delta.\overline{\gamma}_{\Omega(q)}(v)]\Big\rangle; q \in \mathcal{S}\Big\},\\
=&\mathcal{I}_{\epsilon\sigma,\rho\delta}(\Omega, \mathcal{S}).\\
\mathcal{I}_{\sigma,\delta}(\mathcal{I}_{\epsilon,\rho}(\Omega, \mathcal{S})) = &\Big\{\Big\langle v, [\sigma\epsilon.\underline{\alpha}_{\Omega(q)}(v), \sigma\epsilon.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\
&[\delta\rho.\underline{\gamma}_{\Omega(q)}(v), \delta\rho.\overline{\gamma}_{\Omega(q)}(v)]\Big\rangle; q \in \mathcal{S}\Big\},\\
= &\Big\{\Big\langle v, [\epsilon\sigma.\underline{\alpha}_{\Omega(q)}(v), \epsilon\sigma.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\
&[\rho\delta.\underline{\gamma}_{\Omega(q)}(v), \rho\delta.\overline{\gamma}_{\Omega(q)}(v)]\Big\rangle; q \in \mathcal{S}\Big\},\\
=&\mathcal{I}_{\epsilon\sigma,\rho\delta}(\Omega, \mathcal{S})).
\end{aligned}$$

Hence $\mathcal{I}_{\epsilon,\rho}(\mathcal{I}_{\sigma,\delta}(\Omega,\mathcal{S})) = \mathcal{I}_{\epsilon\sigma,\rho\delta}(\Omega,\mathcal{S}) = \mathcal{I}_{\sigma,\delta}(\mathcal{I}_{\epsilon,\rho}(\Omega,\mathcal{S}))$.

(v) Consider,

$$\begin{aligned}
(\Omega,\mathcal{S})^c &= \big\{\big\langle v, [\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\},\\
\mathcal{I}_{\epsilon,\rho}(\Omega,\mathcal{S})^c &= \big\{\big\langle v, [\epsilon.\underline{\gamma}_{\Omega(q)}(v), \epsilon.\overline{\gamma}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\rho.\underline{\alpha}_{\Omega(q)}(v), \rho.\overline{\alpha}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\},\\
(\mathcal{I}_{\epsilon,\rho}(\Omega,\mathcal{S})^c)^c &= \big\{\big\langle v, [\rho.\underline{\alpha}_{\Omega(q)}(v), \rho.\overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [\epsilon.\underline{\gamma}_{\Omega(q)}(v), \epsilon.\overline{\gamma}_{\Omega(q)}(v)]\big\rangle; q \in \mathcal{S}\big\},\\
&= \mathcal{I}_{\rho,\epsilon}(\Omega,\mathcal{S}).
\end{aligned}$$

Hence $(\mathcal{I}_{\epsilon,\rho}(\Omega,\mathcal{S})^c)^c = \mathcal{I}_{\rho,\epsilon}(\Omega,\mathcal{S})$. □

7. Concentration $(\mathcal{CO})$ and dilation $(\mathcal{DO})$ operators on IVINSS

We provide the definition of $(\mathcal{CO})$ and $(\mathcal{DO})$ on IVINSS and discuss their properties in detail. We generalize these operations and properties on IVINSS by the concepts discussed in [32], [18] and [2].

Definition 7.1. Let $(\Omega,\mathcal{S})$ be an IVINSS over $\mathcal{V}$. Then,

(i) the $\mathcal{CO}$ of $(\Omega,\mathcal{S})$ is represented as,

$$\begin{aligned}
\mathcal{C}(\Omega,\mathcal{S}) = \big\{\big\langle v, &[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\
&[1-(1-\underline{\gamma}_{\Omega(q)}(v))^2, 1-(1-\overline{\gamma}_{\Omega(q)}(v))^2]\big\rangle; q \in \mathcal{S}\big\};
\end{aligned}$$

(ii) the $\mathcal{DO}$ of $(\Omega,\mathcal{S})$ is represented as,

$$\begin{aligned}
\mathcal{D}(\Omega,\mathcal{S}) = \big\{\big\langle v, &[(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}, (\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\
&[1-(1-\underline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}}, 1-(1-\overline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}}]\big\rangle; q \in \mathcal{S}\big\};
\end{aligned}$$

Proposition 7.2. *Let $\mathcal{V}$ denote a non-empty set and $(\Omega,\mathcal{S})$ be an IVINSS over $\mathcal{V}$.*

(i) If $[\underline{\pi}_{\Omega(q)}(v), \overline{\pi}_{\Omega(q)}(v)]$ = [0,0], then $[\underline{\pi}_{\mathcal{C}\Omega(q)}(v), \overline{\pi}_{\mathcal{C}\Omega(q)}(v)]$ = [0,0] iff $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)]$ = [0,0] or $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)]$ = [1,1] ;

(ii) $\oplus[\mathcal{C}(\Omega,\mathcal{S})] = \mathcal{C}[\oplus(\Omega,\mathcal{S})]$ iff $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)]$ = [0,0] or $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)]$ = [1,1] ;

(iii) $\ominus[\mathcal{C}(\Omega,\mathcal{S})] = \mathcal{C}[\ominus(\Omega,\mathcal{S})]$ iff $[\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)]$ = [0,0] or $[\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)]$ = [1,1] .

Proof. (i) If $[\underline{\pi}_{\Omega(q)}(v), \overline{\pi}_{\Omega(q)}(v)] = [0,0]$

$\Rightarrow 1 - \underline{\alpha}_{\Omega(q)}(v) - \underline{\gamma}_{\Omega(q)}(v) = 0$ and $1 - \overline{\alpha}_{\Omega(q)}(v) - \overline{\gamma}_{\Omega(q)}(v) = 0$

$\Rightarrow \underline{\alpha}_{\Omega(q)}(v) + \underline{\gamma}_{\Omega(q)}(v) = 1$ and $\overline{\alpha}_{\Omega(q)}(v) + \overline{\gamma}_{\Omega(q)}(v) = 1$

$$\begin{aligned}\mathcal{C}(\Omega,\mathcal{S}) = \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\ [1-(1-\underline{\gamma}_{\Omega(q)}(v))^2, 1-(1-\overline{\gamma}_{\Omega(q)}(v))^2]\big\rangle; q\in\mathcal{S}\big\}.\\ =\big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)],\\ [1-(\underline{\alpha}_{\Omega(q)}(v))^2, 1-(\overline{\alpha}_{\Omega(q)}(v))^2]\big\rangle; q\in\mathcal{S}\big\}.\end{aligned}$$

If $\underline{\pi}_{\mathcal{C}\Omega(q)}(v) = 0 \Leftrightarrow 1-\underline{\alpha}_{\Omega(q)}(v) - (1-(\underline{\alpha}_{\Omega(q)}(v))^2) = 0.$

Then $\underline{\alpha}_{\Omega(q)}(v)(\underline{\alpha}_{\Omega(q)}(v)-1) = 0 \Leftrightarrow \underline{\alpha}_{\Omega(q)}(v) = 0$ or $\underline{\alpha}_{\Omega(q)}(v) = 1.$

Similarly, $\overline{\alpha}_{\Omega(q)}(v) = 0$ or $\overline{\alpha}_{\Omega(q)}(v) = 1.$

(ii) We know that, $\oplus[\mathcal{C}(\Omega,\mathcal{S})]$

$$= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [(1-\overline{\alpha}_{\Omega(q)}(v)), (1-\underline{\alpha}_{\Omega(q)}(v))]\big\rangle; q\in\mathcal{S}\big\} \quad (1)$$

Also,

$\mathcal{C}[\oplus(\Omega,\mathcal{S})]$

$$\begin{aligned}&= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [1-(1-(1-\underline{\alpha}_{\Omega(q)}(v)))^2,\\ &\quad 1-(1-(1-\overline{\alpha}_{\Omega(q)}(v)))^2]\big\rangle; q\in\mathcal{S}\big\} \qquad (2)\\ &= \big\{\big\langle v, [\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [1-(\underline{\alpha}_{\Omega(q)}(v))^2, 1-(\overline{\alpha}_{\Omega(q)}(v))^2]\big\rangle; q\in\mathcal{S}\big\}.\end{aligned}$$

From (1) and (2), we conclude that

$$\begin{aligned}\oplus[\mathcal{C}(\Omega,\mathcal{S})] = \mathcal{C}[\oplus(\Omega,\mathcal{S})] &\Leftrightarrow 1-\underline{\alpha}_{\Omega(q)}(v) = 1-(\underline{\alpha}_{\Omega(q)}(v))^2\\ &\Leftrightarrow \underline{\alpha}_{\Omega(q)}(v)(\underline{\alpha}_{\Omega(q)}(v)-1) = 0\\ &\Leftrightarrow \underline{\alpha}_{\Omega(q)}(v) = 0 \text{ or } \underline{\alpha}_{\Omega(q)}(v) = 1.\end{aligned}$$

Similarly, $\overline{\alpha}_{\Omega(q)}(v) = 0$ or $\overline{\alpha}_{\Omega(q)}(v) = 1$.

(iii) Proof is similar to (ii). □

Proposition 7.3. *Let $\mathcal{V}$ denote a non-empty set and $(\Omega,\mathcal{S})$ be an IVINSS over $\mathcal{V}$.*

(i) If $[\underline{\pi}_{\Omega(q)}(v), \overline{\pi}_{\Omega(q)}(v)] =$ [0,0], then $[\underline{\pi}_{\mathcal{D}\Omega(q)}(v), \overline{\pi}_{\mathcal{C}\Omega(q)}(v)] =$ [0,0] iff $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)] =$ [0,0] or $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)] =$ [1,1] ;

(ii) $\oplus[\mathcal{D}(\Omega,\mathcal{S})] = \mathcal{D}[\oplus(\Omega,\mathcal{S})]$ iff $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)] =$ [0,0] or $[\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v)] =$ [1,1] ;

(iii) $\ominus[\mathcal{D}(\Omega,\mathcal{S})] = \mathcal{D}[\ominus(\Omega,\mathcal{S})]$ iff $[\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)] =$ [0,0] or $[\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v)] =$ [1,1] .

Proof. (i) If $[\underline{\pi}_{\Omega(q)}(v), \overline{\pi}_{\Omega(q)}(v)] =$ [0,0]

$\Rightarrow 1-\underline{\alpha}_{\Omega(q)}(v) - \underline{\gamma}_{\Omega(q)}(v) = 0$ and $1-\overline{\alpha}_{\Omega(q)}(v) - \overline{\gamma}_{\Omega(q)}(v) = 0$

$\Rightarrow \underline{\alpha}_{\Omega(q)}(v)+\underline{\gamma}_{\Omega(q)}(v)=1$ and $\overline{\alpha}_{\Omega(q)}(v)+\overline{\gamma}_{\Omega(q)}(v)=1$

$$\begin{aligned}\mathcal{D}(\Omega,\mathcal{S}) = \big\{\big\langle v, &[(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}},(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}],[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)],\\ &[1-(1-\underline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}},1-(1-\overline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}}]\big\rangle; q\in\mathcal{S}\big\}\\ =\big\{\big\langle v, &[(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}},(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}],[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)],\\ &[1-(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{4}},1-(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{4}}]\big\rangle; q\in\mathcal{S}\big\}.\end{aligned}$$

If $\underline{\pi}_{\mathcal{D}\Omega(q)}(v)=0 \Leftrightarrow 1-(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}-1+(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{4}}=0 \Leftrightarrow (\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{4}}=(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}$.
Then $\underline{\alpha}_{\Omega(q)}(v)(\underline{\alpha}_{\Omega(q)}(v)-1)=0 \Leftrightarrow \underline{\alpha}_{\Omega(q)}(v)=0$ or $\underline{\alpha}_{\Omega(q)}(v)=1$.
Similarly, $\overline{\alpha}_{\Omega(q)}(v)=0$ or $\overline{\alpha}_{\Omega(q)}(v)=1$.

(ii) We know that,

$$\begin{aligned}\oplus[\mathcal{D}(\Omega,\mathcal{S})] = \big\{\big\langle v, &[(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}},(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}],[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)],\\ &[1-(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}},1-(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}]\big\rangle; q\in\mathcal{S}\big\}.\end{aligned} \tag{3}$$

Also,

$$\begin{aligned}\mathcal{D}[\oplus(\Omega,\mathcal{S})] = \big\{\big\langle v, &[(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}},(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}],[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)],\\ &[1-(1-(1-\underline{\alpha}_{\Omega(q)}(v)))^{\frac{1}{4}},1-(1-(1-\overline{\alpha}_{\Omega(q)}(v)))^{\frac{1}{4}}\big\rangle; q\in\mathcal{S}\big\}\\ =\big\{\big\langle v, &[(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}},(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}],[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)],\\ &[1-(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{4}},1-(\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{4}}]\big\rangle; q\in\mathcal{S}\big\}.\end{aligned} \tag{4}$$

From (3) and (4), we conclude that

$$\begin{aligned}\oplus[\mathcal{D}(\Omega,\mathcal{S})]=\mathcal{D}[\oplus(\Omega,\mathcal{S})] &\Leftrightarrow 1-(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}=1-(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{4}}.\\ &\Leftrightarrow \underline{\alpha}_{\Omega(q)}(v)(1-\underline{\alpha}_{\Omega(q)}(v))=0.\\ &\Leftrightarrow \underline{\alpha}_{\Omega(q)}(v)=0 \text{ or } \underline{\alpha}_{\Omega(q)}(v)=1.\end{aligned}$$

Similarly, $\overline{\alpha}_{\Omega(q)}(v)=0$ or $\overline{\alpha}_{\Omega(q)}(v)=1$.
(iii) Proof is similar to (ii). □

Proposition 7.4. *For any IVINSS* $(\Omega,\mathcal{S})$*,* $\mathcal{C}(\Omega,\mathcal{S})\Subset(\Omega,\mathcal{S})\Subset\mathcal{D}(\Omega,\mathcal{S})$.

Proof. Consider,

$$(\Omega,\mathcal{S})=\left\{\left\langle v,[\underline{\alpha}_{\Omega(q)}(v),\overline{\alpha}_{\Omega(q)}(v)],[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)],[\underline{\gamma}_{\Omega(q)}(v),\overline{\gamma}_{\Omega(q)}(v)]\right\rangle;\ q\in\mathcal{S}\right\}.$$

$$\begin{aligned}\mathcal{C}(\Omega,\mathcal{S}) = \big\{\big\langle v, &[\underline{\alpha}_{\Omega(q)}(v),\overline{\alpha}_{\Omega(q)}(v)],[\underline{\beta}_{\Omega(q)}(v),\overline{\beta}_{\Omega(q)}(v)],[1-(1-\underline{\gamma}_{\Omega(q)}(v))^2,\\ &1-(1-\overline{\gamma}_{\Omega(q)}(v))^2]\big\rangle; q\in\mathcal{S}\big\};\end{aligned}$$

Since, $\underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v) \in [0,1], (1-(1-\underline{\gamma}_{\Omega(q)}(v))^2) \geq \underline{\gamma}_{\Omega(q)}(v)$and $(1-(1-\overline{\gamma}_{\Omega(q)}(v))^2) \geq \overline{\gamma}_{\Omega(q)}(v)$.

$$\text{Hence } \mathcal{C}(\Omega, \mathcal{S}) \Subset (\Omega, \mathcal{S}). \tag{5}$$

$$\mathcal{D}(\Omega, \mathcal{S}) = \Big\{\big\langle v, [(\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}, (\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}], [\underline{\beta}_{\Omega(q)}(v), \overline{\beta}_{\Omega(q)}(v)], [1-(1-\underline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}}, 1-(1-\overline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}}]\big\rangle; q \in \mathcal{S}\Big\}.$$

Since, $\underline{\alpha}_{\Omega(q)}(v), \overline{\alpha}_{\Omega(q)}(v), \underline{\gamma}_{\Omega(q)}(v), \overline{\gamma}_{\Omega(q)}(v) \in [0,1]$,
$\underline{\alpha}_{\Omega(q)}(v) \leq (\underline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}$, $\overline{\alpha}_{\Omega(q)}(v) \leq (\overline{\alpha}_{\Omega(q)}(v))^{\frac{1}{2}}$, $\underline{\gamma}_{\Omega(q)}(v) \geq (1-(1-\underline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}})$ and $\overline{\gamma}_{\Omega(q)}(v) \geq (1-(1-\overline{\gamma}_{\Omega(q)}(v))^{\frac{1}{4}})$.

$$\text{Hence } (\Omega, \mathcal{S}) \Subset \mathcal{D}(\Omega, \mathcal{S}). \tag{6}$$

From (5) and (6), we get $\mathcal{C}(\Omega, \mathcal{S}) \Subset (\Omega, \mathcal{S}) \Subset \mathcal{D}(\Omega, \mathcal{S})$. □

8. Similarity measures between IVINSS

We provide a new similarity measure (SM) between IVINSS and explain its use with an application. We illustrate the working model with an algorithm and examples. Also, we bring out the importance of the proposed SM by comparing with existing SMs.

Definition 8.1. Let $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ represent the universe and $\mathcal{S} = \{q_1, q_2, ...q_m\}$ represent the parameters. Then the SM between IVINSSs $(\Omega_1, \mathcal{S})$ and $(\Omega_2, \mathcal{S})$ is represented as,

$$S_M \left\langle (\Omega_1, \mathcal{S}), (\Omega_2, \mathcal{S}) \right\rangle$$
$$= 1 - \frac{1}{4m} \sum_{i=1}^{m} \sum_{j=1}^{n} \Bigg[\frac{|\underline{\alpha}_{\Omega_1(q_i)}(v_j) - \underline{\alpha}_{\Omega_2(q_i)}(v_j)|}{2 + \underline{\alpha}_{\Omega_1(q_i)}(v_j) + \underline{\alpha}_{\Omega_2(q_i)}(v_j)} + \frac{|\overline{\alpha}_{\Omega_1(q_i)}(v_j) - \overline{\alpha}_{\Omega_2(q_i)}(v_j)|}{2 + \overline{\alpha}_{\Omega_1(q_i)}(v_j) + \overline{\alpha}_{\Omega_2(q_i)}(v_j)} + \frac{|\underline{\beta}_{\Omega_1(q_i)}(v_j) - \underline{\beta}_{\Omega_2(q_i)}(v_j)|}{2 + \underline{\beta}_{\Omega_1(q_i)}(v_j) + \underline{\beta}_{\Omega_2(q_i)}(v_j)} +$$
$$\frac{|\overline{\beta}_{\Omega_1(q_i)}(v_j) - \overline{\beta}_{\Omega_2(q_i)}(v_j)|}{2 + \overline{\beta}_{\Omega_1(q_i)}(v_j) + \overline{\beta}_{\Omega_2(q_i)}(v_j)} + \frac{|\underline{\gamma}_{\Omega_1(q_i)}(v_j) - \underline{\gamma}_{\Omega_2(q_i)}(v_j)|}{2 + \underline{\gamma}_{\Omega_1(q_i)}(v_j) + \underline{\gamma}_{\Omega_2(q_i)}(v_j)} + \frac{|\overline{\gamma}_{\Omega_1(q_i)}(v_j) - \overline{\gamma}_{\Omega_2(q_i)}(v_j)|}{2 + \overline{\gamma}_{\Omega_1(q_i)}(v_j) + \overline{\gamma}_{\Omega_2(q_i)}(v_j)} +$$
$$\left| \frac{(\underline{\alpha}_{\Omega_1(q_i)}(v_j) - \underline{\gamma}_{\Omega_1(q_i)}(v_j))}{2} - \frac{(\underline{\alpha}_{\Omega_2(q_i)}(v_j) - \underline{\gamma}_{\Omega_2(q_i)}(v_j))}{2} \right| +$$
$$\left| \frac{(\overline{\alpha}_{\Omega_1(q_i)}(v_j) - \overline{\gamma}_{\Omega_1(q_i)}(v_j))}{2} - \frac{(\overline{\alpha}_{\Omega_2(q_i)}(v_j) - \overline{\gamma}_{\Omega_2(q_i)}(v_j))}{2} \right| \Bigg].$$

8.1. *Comparison analysis with existing SMs*

In this section, we analyze some existing SMs on IVINSS. DMs apply SM to identify the most similar pattern between the precise and imprecise values. The framework of existing measures are given below.

(i) $\mathcal{S}_Y(\Omega_1,\Omega_2)$ [11]

$$= 1-\frac{1}{n}\sum_{i=1}^{n} w_j\Big[|\underline{\alpha}_{\Omega_1(q_i)}(v_j)-\underline{\alpha}_{\Omega_2(q_i)}(v_j)|+|\overline{\alpha}_{\Omega_1(q_i)}(v_j)-\overline{\alpha}_{\Omega_2(q_i)}(v_j)|+|\underline{\beta}_{\Omega_1(q_i)}(v_j)-\underline{\beta}_{\Omega_2(q_i)}(v_j)|$$
$$+|\overline{\beta}_{\Omega_1(q_i)}(v_j)-\overline{\beta}_{\Omega_2(q_i)}(v_j)|+|\underline{\gamma}_{\Omega_1(q_i)}(v_j)-\underline{\gamma}_{\Omega_2(q_i)}(v_j)|+|\overline{\gamma}_{\Omega_1(q_i)}(v_j)-\overline{\gamma}_{\Omega_2(q_i)}(v_j)|\Big].$$

(ii) $\mathcal{S}_C(\Omega_1,\Omega_2)$ [11]

$$= 1-\frac{1}{n}\sum_{i=1}^{n}\frac{(\tilde{\alpha}_{\Omega_1(q_i)}(v_j))(\tilde{\alpha}_{\Omega_2(q_i)}(v_j))+(\tilde{\beta}_{\Omega_1(q_i)}(v_j))(\tilde{\beta}_{\Omega_2(q_i)}(v_j))+(\tilde{\gamma}_{\Omega_1(q_i)}(v_j))(\tilde{\gamma}_{\Omega_1(q_i)}(v_j))}{(\sqrt{(\tilde{\alpha}_{\Omega_1(q_i)}(v_j))^2+(\tilde{\beta}_{\Omega_1(q_i)}(v_j))^2+(\tilde{\gamma}_{\Omega_1(q_i)}(v_j))^2})(\sqrt{(\tilde{\alpha}_{\Omega_2(q_i)}(v_j))^2+(\tilde{\beta}_{\Omega_2(q_i)}(v_j))^2+(\tilde{\gamma}_{\Omega_2(q_i)}(v_j))^2})},$$

where $\tilde{\alpha}_{\Omega_1(q_i)}(v_j)=\underline{\alpha}_{\Omega_1(q_i)}(v_j)+\overline{\alpha}_{\Omega_1(q_i)}(v_j), \tilde{\alpha}_{\Omega_2(q_i)}(v_j)=\underline{\alpha}_{\Omega_2(q_i)}(v_j)+\overline{\alpha}_{\Omega_2(q_i)}(v_j)$,
$\tilde{\beta}_{\Omega_1(q_i)}(v_j)=\underline{\beta}_{\Omega_1(q_i)}(v_j)+\overline{\beta}_{\Omega_1(q_i)}(v_j), \tilde{\beta}_{\Omega_2(q_i)}(v_j)=\underline{\beta}_{\Omega_2(q_i)}(v_j)+\overline{\beta}_{\Omega_2(q_i)}(v_j)$,
$\tilde{\gamma}_{\Omega_1(q_i)}(v_j)=\underline{\gamma}_{\Omega_1(q_i)}(v_j)+\overline{\gamma}_{\Omega_1(q_i)}(v_j)$ and $\tilde{\gamma}_{\Omega_2(q_i)}(v_j)=\underline{\gamma}_{\Omega_2(q_i)}(v_j)+\overline{\gamma}_{\Omega_2(q_i)}(v_j)$.

(iii) $\mathcal{S}_T(\Omega_1,\Omega_2)$ [12]

$$=\frac{\sum_{i=1}^{n}\Big(\begin{array}{l}min(\underline{\alpha}_{\Omega_1(q_i)}(v_j),\underline{\alpha}_{\Omega_2(q_i)}(v_j))+min(\overline{\alpha}_{\Omega_1(q_i)}(v_j),\overline{\alpha}_{\Omega_2(q_i)}(v_j))+min(\underline{\beta}_{\Omega_1(q_i)}(v_j),\underline{\beta}_{\Omega_2(q_i)}(v_j))\\ +min(\overline{\beta}_{\Omega_1(q_i)}(v_j),\overline{\beta}_{\Omega_2(q_i)}(v_j))+min(\underline{\gamma}_{\Omega_1(q_i)}(v_j),\underline{\gamma}_{\Omega_2(q_i)}(v_j))+min(\overline{\gamma}_{\Omega_1(q_i)}(v_j),\overline{\gamma}_{\Omega_2(q_i)}(v_j))\end{array}\Big)}{\sum_{i=1}^{n}\Big(\begin{array}{l}max(\underline{\alpha}_{\Omega_1(q_i)}(v_j),\underline{\alpha}_{\Omega_2(q_i)}(v_j))+max(\overline{\alpha}_{\Omega_1(q_i)}(v_j),\overline{\alpha}_{\Omega_2(q_i)}(v_j))+max(\underline{\beta}_{\Omega_1(q_i)}(v_j),\underline{\beta}_{\Omega_2(q_i)}(v_j))\\ +max(\overline{\beta}_{\Omega_1(q_i)}(v_j),\overline{\beta}_{\Omega_2(q_i)}(v_j))+max(\underline{\gamma}_{\Omega_1(q_i)}(v_j),\underline{\gamma}_{\Omega_2(q_i)}(v_j))+max(\overline{\gamma}_{\Omega_1(q_i)}(v_j),\overline{\gamma}_{\Omega_2(q_i)}(v_j))\end{array}\Big)},$$

(iv) $\mathcal{S}_H(\Omega_1,\Omega_2)$ [12]

$$=\frac{1}{6}\sum_{i=1}^{n} w_j\Big[|\underline{\alpha}_{\Omega_1(q_i)}(v_j)-\underline{\alpha}_{\Omega_2(q_i)}(v_j)|+|\overline{\alpha}_{\Omega_1(q_i)}(v_j)-\overline{\alpha}_{\Omega_2(q_i)}(v_j)|+|\underline{\beta}_{\Omega_1(q_i)}(v_j)-\underline{\beta}_{\Omega_2(q_i)}(v_j)|$$
$$+|\overline{\beta}_{\Omega_1(q_i)}(v_j)-\overline{\beta}_{\Omega_2(q_i)}(v_j)|+|\underline{\gamma}_{\Omega_1(q_i)}(v_j)-\underline{\gamma}_{\Omega_2(q_i)}(v_j)|+|\overline{\gamma}_{\Omega_1(q_i)}(v_j)-\overline{\gamma}_{\Omega_2(q_i)}(v_j)|\Big].$$

(v) $\mathcal{S}_E(\Omega_1,\Omega_2)$ [12]

$$=\Big(\sum_{i=1}^{n} w_j\Big[|\underline{\alpha}_{\Omega_1(q_i)}(v_j)-\underline{\alpha}_{\Omega_2(q_i)}(v_j)|^2+|\overline{\alpha}_{\Omega_1(q_i)}(v_j)-\overline{\alpha}_{\Omega_2(q_i)}(v_j)|^2+|\underline{\beta}_{\Omega_1(q_i)}(v_j)-\underline{\beta}_{\Omega_2(q_i)}(v_j)|^2$$
$$+|\overline{\beta}_{\Omega_1(q_i)}(v_j)-\overline{\beta}_{\Omega_2(q_i)}(v_j)|^2+|\underline{\gamma}_{\Omega_1(q_i)}(v_j)-\underline{\gamma}_{\Omega_2(q_i)}(v_j)|^2+|\overline{\gamma}_{\Omega_1(q_i)}(v_j)-\overline{\gamma}_{\Omega_2(q_i)}(v_j)|^2\Big]\Big)^{\frac{1}{2}}.$$

(vi) $\mathcal{S}_{C_1}(\Omega_1,\Omega_2)$ [34]

$$=\frac{1}{n}\sum_{i=1}^{n}\text{Cos}\Big[\frac{\pi}{12}\Big(|\underline{\alpha}_{\Omega_1(q_i)}(v_j)-\underline{\alpha}_{\Omega_2(q_i)}(v_j)|+|\overline{\alpha}_{\Omega_1(q_i)}(v_j)-\overline{\alpha}_{\Omega_2(q_i)}(v_j)|+|\underline{\beta}_{\Omega_1(q_i)}(v_j)-\underline{\beta}_{\Omega_2(q_i)}(v_j)|$$
$$+|\overline{\beta}_{\Omega_1(q_i)}(v_j)-\overline{\beta}_{\Omega_2(q_i)}(v_j)|+|\underline{\gamma}_{\Omega_1(q_i)}(v_j)-\underline{\gamma}_{\Omega_2(q_i)}(v_j)|+|\overline{\gamma}_{\Omega_1(q_i)}(v_j)-\overline{\gamma}_{\Omega_2(q_i)}(v_j)|\Big)\Big].$$

(vii) $\mathcal{S}_{C_2}(\Omega_1,\Omega_2)$ [34]

$$=\frac{1}{n}\sum_{i=1}^{n}\text{Cos}\Big[\frac{\pi}{4}\Big(|\underline{\alpha}_{\Omega_1(q_i)}(v_j)-\underline{\alpha}_{\Omega_2(q_i)}(v_j)|\vee|\underline{\beta}_{\Omega_1(q_i)}(v_j)-\underline{\beta}_{\Omega_2(q_i)}(v_j)|\vee|\underline{\gamma}_{\Omega_1(q_i)}(v_j)-\underline{\gamma}_{\Omega_2(q_i)}(v_j)|$$
$$+|\overline{\alpha}_{\Omega_1(q_i)}(v_j)-\overline{\alpha}_{\Omega_2(q_i)}(v_j)|\vee|\overline{\beta}_{\Omega_1(q_i)}(v_j)-\overline{\beta}_{\Omega_2(q_i)}(v_j)|\vee|\overline{\gamma}_{\Omega_1(q_i)}(v_j)-\overline{\gamma}_{\Omega_2(q_i)}(v_j)|\Big)\Big].$$

Example 8.2. Consider the following values, as in Table 7. Table 8 shows the superiority of the proposed SM than the existing SMs. It illustrates that the proposed SM can identify similar patterns (refer third column) even when the existing SMs have some limitations (refer first column). For computation purpose, let us consider w_j=1 for $\mathcal{S}_Y(\Omega_1,\Omega_2)$ and $\mathcal{S}_H(\Omega_1,\Omega_2)$.

Table 7. Shows precise and imprecise values.

Precise value	Imprecise values
$\Omega = \langle[0.20, 0.30], [0.50, 0.60], [0.30, 0.50]\rangle$	$\Omega_1 = \langle[0.60, 0.70], [0.50, 0.60], [0.10, 0.20]\rangle$,
	$\Omega_2 = \langle[0.50, 0.60], [0.40, 0.50], [0.10, 0.20]\rangle$.
$\Omega = \langle[0.60, 0.70], [0.70, 0.80], [0.20, 0.30]\rangle$	$\Omega_1 = \langle[0.50, 0.60], [0.30, 0.60], [0.10, 0.20]\rangle$,
	$\Omega_2 = \langle[0.60, 0.70], [0.40, 0.50], [0.20, 0.30]\rangle$.
$\Omega = \langle[0.40, 0.50], [0.80, 0.90], [0.30, 0.40]\rangle$	$\Omega_1 = \langle[0.50, 0.60], [0.50, 0.60], [0.20, 0.30]\rangle$,
	$\Omega_2 = \langle[0.30, 0.40], [0.40, 0.50], [0.30, [0.40]\rangle$.
$\Omega = \langle[0.69, 0.75], [0.75, 0.85], [0.15, 0.25]\rangle$	$\Omega_1 = \langle[0.55, 0.65], [0.55, 0.66], [0.21, 0.29]\rangle$,
	$\Omega_2 = \langle[0.58, 0.69], [0.57, 0.68], [0.22, 0.29]\rangle$.
$\Omega = \langle[0.69, 0.75], [0.75, 0.85], [0.15, 0.25]\rangle$	$\Omega_1 = \langle[0.55, 0.65], [0.54, 0.66], [0.21, 0.29]\rangle$,
	$\Omega_2 = \langle[0.58, 0.69], [0.46, 0.68], [0.22, 0.29]\rangle$.
$\Omega = \langle[0.20, 0.30], [0.50, 0.60], [0.30, 0.50]\rangle$	$\Omega_1 = \langle[0.40, 0.50], [0.40, 0.50], [0.10, 0.20]\rangle$,
	$\Omega_2 = \langle[0.50, 0.60], [0.50, 0.60], [0.20, 0.30]\rangle$.

Table 8. Analysis of existing SMs.

Existing SMs	Proposed SMs	Similar pattern
$\mathcal{S}_Y(\Omega, \Omega_1) = \mathcal{S}_Y(\Omega, \Omega_2) = 0.7833$,	$\mathcal{S}_M(\Omega, \Omega_1) = 0.7198$, $\mathcal{S}_M(\Omega, \Omega_2) = 0.7435$	
$\mathcal{S}_H(\Omega, \Omega_1) = \mathcal{S}_H(\Omega, \Omega_2) = 0.2166$,	$\mathcal{S}_M(\Omega, \Omega_2) > \mathcal{S}_M(\Omega, \Omega_1) \Rightarrow \Omega_2$	Ω_2
$\mathcal{S}_{C_1}(\Omega, \Omega_1) = \mathcal{S}_{C_1}(\Omega, \Omega_2) = 0.9426$.		
$\mathcal{S}_{C_2}(\Omega, \Omega_1) = \mathcal{S}_{C_2}(\Omega, \Omega_2) = 0.9877$	$\mathcal{S}_M(\Omega, \Omega_1) = 0.9154$, $\mathcal{S}_M(\Omega, \Omega_2) = 0.9530$	
	$\mathcal{S}_M(\Omega, \Omega_2) > \mathcal{S}_M(\Omega, \Omega_1) \Rightarrow \Omega_2$	Ω_2
$\mathcal{S}_Y(\Omega, \Omega_1) = \mathcal{S}_Y(\Omega, \Omega_2) = 0.8333$,	$\mathcal{S}_M(\Omega, \Omega_1) = 0.8698$, $\mathcal{S}_M(\Omega, \Omega_2) = 0.8964$	
$\mathcal{S}_H(\Omega, \Omega_1) = \mathcal{S}_H(\Omega, \Omega_2) = 0.1666$,	$\mathcal{S}_M(\Omega, \Omega_2) > \mathcal{S}_M(\Omega, \Omega_1) \Rightarrow \Omega_2$	Ω_2
$\mathcal{S}_{C_1}(\Omega, \Omega_1) = \mathcal{S}_{C_1}(\Omega, \Omega_2) = 0.9659$.		
$\mathcal{S}_C(\Omega, \Omega_1) = \mathcal{S}_C(\Omega, \Omega_2) = 0.9937$	$\mathcal{S}_M(\Omega, \Omega_1) = 0.9003$, $\mathcal{S}_M(\Omega, \Omega_2) = 0.8702$	
	$\mathcal{S}_M(\Omega, \Omega_1) > \mathcal{S}_M(\Omega, \Omega_2) \Rightarrow \Omega_1$	Ω_1
$\mathcal{S}_T(\Omega, \Omega_1) = \mathcal{S}_T(\Omega, \Omega_2) = 0.800$	$\mathcal{S}_M(\Omega, \Omega_1) = 0.8995$, $\mathcal{S}_M(\Omega, \Omega_2) = 0.8611$	
	$\mathcal{S}_M(\Omega, \Omega_1) > \mathcal{S}_M(\Omega, \Omega_2) \Rightarrow \Omega_1$	Ω_1
$\mathcal{S}_E(\Omega, \Omega_1) = \mathcal{S}_E(\Omega, \Omega_2) = 0.1957$	$\mathcal{S}_M(\Omega, \Omega_1) = 0.7851$, $\mathcal{S}_M(\Omega, \Omega_2) = 0.8060$	
	$\mathcal{S}_M(\Omega, \Omega_2) > \mathcal{S}_M(\Omega, \Omega_1) \Rightarrow \Omega_2$	Ω_2

Theorem 8.3. *Let* $(\Omega_1, \mathcal{S})$ *and* $(\Omega_2, \mathcal{S})$ *be two IVINSS over* $\mathcal{V}$*. Then,*
(i) $0 \leq S_M((\Omega_1, \mathcal{S}), (\Omega_2, \mathcal{S})) \leq 1$*;*
(ii) $S_M((\Omega_1, \mathcal{S}), (\Omega_2, \mathcal{S})) = S_M((\Omega_2, \mathcal{S}), (\Omega_1, \mathcal{S}))$*;*
(iii) $S_M((\Omega_1, \mathcal{S}), (\Omega_2, \mathcal{S})) = 1$ *iff* $(\Omega_1, \mathcal{S}) = (\Omega_2, \mathcal{S})$.

Proof. Proof straightforward □

8.2. *Diagnosing psychiatric disorder for people with COVID-19*

In this section, we present an application on diagnosing psychiatric disorder for people with COVID-19 using IVINSS. Let us consider the SM between two IVINSS over different universes with the same set of parameters. We use this to analyze the psychiatric disorder problem. We have proposed an algorithm and illustrated the technique with a suitable example.

8.3. *Description of the problem*

Let $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ represent universe and $\mathcal{S} = \{q_1, q_2, ..., q_m\}$ represent the parameters. Also, let the precise values $(\Omega, \mathcal{S})$ describe the elements of the universe in IVINSS form given by the psychiatrist for each stage. Let the psychiatrist define the norms to identify the levels (low or moderate or high) associated with psychiatric disorder as in Table 10. Let $(\Omega_i, \mathcal{S})$, $(i = 1, 2, ..., t)$ denote the imprecise values. Each $(\Omega_i, \mathcal{S})$ is in IVINSS form representing the alternatives based on the observations on the subject by the psychiatrist made in relation to each element of the universe and for each element of the parameter set. Now the problem is to identify the level associated with $(\Omega_i, \mathcal{S})$ to the precise information $(\Omega, \mathcal{S})$.

8.4. *A new method to diagnose psychiatric disorder*

Let's assume that $(\Omega, \mathcal{S})$ and $(\Omega_i, \mathcal{S})$ represent the precise and imprecise values, respectively in IVINSS form. By using Definition 7.1, the psychiatrist identifies the SM value associated with $(\Omega_i, \mathcal{S})$ $(i = 1, 2, ..., t)$ to the precise information $(\Omega, \mathcal{S})$. Now, the psychiatrist compares the obtained SM value with the norms (Table 10) and interprets on the level of psychiatric disorder for each subject.

8.5. *Algorithm for diagnosing psychiatric disorder*

An algorithm is given below for diagnosing psychiatric disorder based on SM between IVINSS.

Step 1: Construct the precise values $(\Omega, \mathcal{S})$ and the norms based on the evaluation of psychiatrist for diagnosing psychiatric disorder.

Step 2: Construct the imprecise values $(\Omega_i, \mathcal{S})$, $(i = 1, 2, ..., t)$ by observing the behavior of the subjects.

Step 3: Compute the SM between $(\Omega, \mathcal{S})$ and $(\Omega_i, \mathcal{S})$.

Step 4: Compare the calculated SM value between $(\Omega, \mathcal{S})$ and $(\Omega_i, \mathcal{S})$ with the norms.

Step 5: Identify the level associated with each subject to diagnose the psychiatric disorder.

Example 8.4. Let $\mathcal{V} = \{o_1, o_2, o_3\}$ represent the sessions conducted by a psychiatrist. Let C_1, C_2 and C_3 represent the subjects and $\mathcal{S} = \{q_1, q_2, q_3, q_4, q_5\}$ represent the parameters, $q_1 =$

feeling sad or low, q_2 = confused thinking, q_3 = extreme mood changes, q_4 = excessive fears or worries and q_5 = sleeping problems. The psychiatrist has to diagnose the psychiatric disorder based on the norms associated with each subject.

Step 1. Construct the precise values $(\Omega, \mathcal{S})$ as in Table 9 and the norms as in Table 10 based on the evaluation of psychiatrist for diagnosing psychiatric disorder.

TABLE 9. Representation of precise values $(\Omega, \mathcal{S})$ in IVINSS form for each session.

$\mathcal{V}$	o_1	o_2	o_3
q_1	$\langle[0.6,0.7],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[0.7,0.8],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[0.6,0.7],[0.8,0.9],[0.1,0.2]\rangle$
q_2	$\langle[0.5,0.6],[0.8,0.9],[0.2,0.3]\rangle$	$\langle[0.7,0.8],[0.8,0.9],[0.1,0.2]\rangle$	$\langle[0.5,0.6],[0.9,1.0],[0.2,0.3]\rangle$
q_3	$\langle[0.4,0.5],[0.7,0.8],[0.3,0.4]\rangle$	$\langle[0.2,0.3],[0.9,1.0],[0.4,0.5]\rangle$	$\langle[0.4,0.5],[0.8,0.9],[0.4,0.5]\rangle$
q_4	$\langle[0.3,0.4],[0.6,0.7],[0.4,0.5]\rangle$	$\langle[0.4,0.5],[0.7,0.8],[0.3,0.4]\rangle$	$\langle[0.6,0.7],[0.8,0.9],[0.1,0.2]\rangle$
q_5	$\langle[0.2,0.3],[0.5,0.6],[0.4,0.5]\rangle$	$\langle[0.5,0.6],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[0.4,0.5],[0.9,1.0],[0.1,0.3]\rangle$

TABLE 10. Norms for NPD.

Range of SM values	Levels of psychiatric disorder
$0.00 \leq S_M \langle(\Omega,\mathcal{S}),(\Omega_i,\mathcal{S})\rangle < 0.40$	Low
$0.40 \leq S_M \langle(\Omega,\mathcal{S}),(\Omega_i,\mathcal{S})\rangle < 0.75$	Moderate
$0.75 \leq S_M \langle(\Omega,\mathcal{S}),(\Omega_i,\mathcal{S})\rangle \leq 1.00$	High

Step 2. Now construct the imprecise values $(\Omega_i, \mathcal{S})$, $(i = 1, 2, ..., t)$ by observing the behavior of the subjects C_1, C_2 and C_3 respectively, as in Table 11, 12 and 13.

TABLE 11. Representation of imprecise values $(\Omega_1, \mathcal{S})$ for the first subject in SINSS form for each session.

$\mathcal{V}$	o_1	o_2	o_3
q_1	$\langle[0.8,0.9],[0.7,0.8],[0.0,0.1]\rangle$	$\langle[0.2,0.3],[0.4,0.5],[0.1,0.2]\rangle$	$\langle[0.1,0.2],[0.7,0.8],[0.1,0.2]\rangle$
q_2	$\langle[0.7,0.8],[0.6,0.7],[0.1,0.2]\rangle$	$\langle[0.1,0.2],[0.3,0.4],[0.2,0.3]\rangle$	$\langle[0.2,0.3],[0.8,0.9],[0.1,0.2]\rangle$
q_3	$\langle[0.6,0.7],[0.8,0.9],[0.2,0.3]\rangle$	$\langle[0.3,0.4],[0.5,0.6],[0.3,0.4]\rangle$	$\langle[0.3,0.4],[0.9,1.0],[0.4,0.5]\rangle$
q_4	$\langle[0.6,0.7],[0.7,0.8],[0.1,0.2]\rangle$	$\langle[0.2,0.4],[0.2,0.3],[0.4,0.5]\rangle$	$\langle[0.2,0.3],[0.9,1.0],[0.2,0.3]\rangle$
q_5	$\langle[0.6,0.7],[0.9,1.0],[0.2,0.3]\rangle$	$\langle[0.1,0.3],[0.4,0.5],[0.1,0.2]\rangle$	$\langle[0.1,0.3],[0.8,0.9],[0.3,0.4]\rangle$

Step 3. By using Definition 7.1, calculate the $S_M \langle(\Omega,\mathcal{S}),(\Omega_i,\mathcal{S})\rangle$.

The values are as below:

$S_M \langle(\Omega,\mathcal{S}),(\Omega_1,\mathcal{S})\rangle = 0.396$, $S_M \langle(\Omega,\mathcal{S}),(\Omega_2,\mathcal{S})\rangle = 0.663$, $S_M \langle(\Omega,\mathcal{S}),(\Omega_3,\mathcal{S})\rangle = 0.772$.

Step 4. Now compare the calculated values of $S_M \langle(\Omega,\mathcal{S}),(\Omega_i,\mathcal{S})\rangle$ with Table 10.

The level of psychiatric disorder for the first subject shows low, for the second average and the third high.

Step 5. We can conclude from the above observation that the psychiatrist to start the next set of treatment sessions for the subjects C_2 and C_3 to lower the level of psychiatric disorder.

Table 12. Representation of imprecise values $(\Omega_2, \mathcal{S})$ for the second subject in SINSS form for each session.

$\mathcal{V}$	o_1	o_2	o_3
q_1	$\langle[0.2, 0.3], [0.7, 0.8], [0.2, 0.3]\rangle$	$\langle[0.7, 0.8], [0.8, 0.9], [0.1, 0.2]\rangle$	$\langle[0.7, 0.8], [0.7, 0.8], [0.1, 0.2]\rangle$
q_2	$\langle[0.4, 0.5], [0.8, 0.9], [0.1, 0.2]\rangle$	$\langle[0.5, 0.6], [0.9, 1.0], [0.2, 0.3]\rangle$	$\langle[0.4, 0.5], [0.8, 0.9], [0.1, 0.2]\rangle$
q_3	$\langle[0.4, 0.5], [0.9, 1.0], [0.4, 0.5]\rangle$	$\langle[0.4, 0.5], [0.8, 0.9], [0.3, 0.4]\rangle$	$\langle[0.3, 0.4], [0.9, 1.0], [0.4, 0.5]\rangle$
q_4	$\langle[0.6, 0.7], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[0.3, 0.4], [0.9, 1.0], [0.4, 0.5]\rangle$	$\langle[0.6, 0.7], [0.9, 1.0], [0.2, 0.3]\rangle$
q_5	$\langle[0.5, 0.6], [0.7, 0.8], [0.2, 0.3]\rangle$	$\langle[0.2, 0.3], [0.8, 0.9], [0.1, 0.2]\rangle$	$\langle[0.5, 0.6], [0.8, 0.9], [0.3, 0.4]\rangle$

Table 13. Representation of imprecise values $(\Omega_3, \mathcal{S})$ for the third subject in SINSS form for each session.

$\mathcal{V}$	o_1	o_2	o_3
q_1	$\langle[0.7, 0.8], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[0.7, 0.8], [0.8, 0.9], [0.1, 0.2]\rangle$	$\langle[0.7, 0.8], [0.8, 0.9], [0.1, 0.2]\rangle$
q_2	$\langle[0.4, 0.5], [0.8, 0.9], [0.1, 0.2]\rangle$	$\langle[0.6, 0.7], [0.9, 1.0], [0.1, 0.2]\rangle$	$\langle[0.5, 0.6], [0.7, 0.8], [0.2, 0.3]\rangle$
q_3	$\langle[0.2, 0.3], [0.6, 0.7], [0.2, 0.3]\rangle$	$\langle[0.1, 0.3], [0.8, 0.9], [0.2, 0.3]\rangle$	$\langle[0.1, 0.2], [0.7, 0.8], [0.1, 0.3]\rangle$
q_4	$\langle[0.4, 0.5], [0.7, 0.8], [0.1, 0.2]\rangle$	$\langle[0.4, 0.5], [0.9, 1.0], [0.1, 0.2]\rangle$	$\langle[0.6, 0.7], [0.9, 1.0], [0.1, 0.2]\rangle$
q_5	$\langle[0.1, 0.2], [0.8, 0.9], [0.2, 0.3]\rangle$	$\langle[0.3, 0.4], [0.8, 0.9], [0.1, 0.2]\rangle$	$\langle[0.5, 0.6], [0.8, 0.9], [0.2, 0.3]\rangle$

9. **Conclusion**

In this manuscript, we outline the notions of IVINS, IVINSS, and establish some of their properties. Also, we show the effectiveness of the proposed SM by comparing it with existing SMs. In today's complicated psychiatric disorder behaviors, SM plays a significant role in diagnosing the same. So, we propose a diagnosing method based on the SM for diagnosing psychiatric disorder with IVINSSs. In this method, we predict the psychiatric behavior of the subjects represented in the IVINSS form. We can apply this concept to other hybrid sets for diagnosing psychiatric disorders. Our future study would be the applications of neutrosophics in sociology [29].

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