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Decision Making Application Based on Neutrosophic Parameterized Hypersoft Set Theory

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Abstract. Hypersoft set is the generalization of soft set as it converts single attribute function to multi-attribute function. The core purpose of this study is to make the existing literature regarding neutrosophic parameterized soft set in line with the need of multi-attribute function. We first conceptualize the neutrosophic parameterized hypersoft set along with some of its elementary properties and operations. Then we propose decision making based algorithm with the help of this theory. Moreover, an illustrative example is presented which depicts its validity for successful application to the problems involving vagueness and uncertainties.

Keywords: Neutrosophic Set; Hypersoft Set; Neutrosophic Parameterized Soft Set; Neutrosophic Parameterized Hypersoft set.

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1. Introduction

Fuzzy sets theory (FST) [1] and intuitionistic fuzzy set theory (IFST) [2] are considered apt mathematical modes to tackle many intricate problems involving various uncertainties, in different mathematical disciplines. The former one emphasizes on the degree of true belongingness of a certain object from the initial sample space whereas the later one accentuates on degree of true membership and degree of non-membership with condition of their dependency on each other. These theories depict some kind of inadequacy regarding the provision of due status to degree of indeterminacy. Such impediment is addressed with the introduction of neutrosophic set theory (NST) [3, 4] which not only considers the due status of degree of indeterminacy but also waives off the condition of dependency. This theory is more flexible and appropriate to deal with uncertainty and vagueness. NST has attracted the keen concentration of many researchers [5–19] to further utilization in statistics, topological spaces as well as in the development of certain neutrosophic-like blended structures with other existing models for useful applications in decision making.

FST, IFST and NST have some kind of complexities which restrain them to solve problem involving uncertainty professionally. The reason for these hurdles is, possibly, the inadequacy of the parametrization tool. It demands a mathematical tool free of all such impediments to tackle such issues. This scantiness is resolved with the development of soft set theory [20] which is a new parameterized family of subsets of the universe of discourse. The researchers [21–30] studied and investigated some elementary properties, operations, laws and hybrids of SST with applications in decision making. The gluing concept of NST and SST, is studied in [31] to make the NST adequate with parameterized tool. In many real life situations, distinct attributes are further partitioned in disjoint attribute-valued sets but existing SST is insufficient for dealing with such kind of attribute-valued sets. Hypersoft set theory (HST) [32] is developed to make the SST in line with attribute-valued sets to tackle real life scenarios. Certain elementary properties, aggregation operations, laws, relations and functions of HST, are investigated by [33–35] for proper understanding and further utilization in different fields. The applications of HST in decision making is studied by [36–39] and the intermingling study of HST with complex sets, convex and concave sets is studied by [40, 41]. The core aim of this study is to develop a novel theory of embedding structure of parameterized neutrosophic set and hypersoft set with the extension of concept investigated in [42,43]. A decision-making based algorithm is proposed to solve a real life problem relating to the purchase of most suitable and appropriate product with the help of some essential operations of this presented theory. The rest of the paper is systemized as:

Section 2	Some essential definitions and terminologies are recalled.
Section 3	Theory of neutrosophic parameterized hypersoft set is developed with suitable examples.
Section 4	Neutrosophic decision system is constructed with proposed decision making algorithm and application.
Section 5	Paper is summarized with future directions.

2. Preliminaries

Here some basic terms are recalled from existing literature to support the proposed work. Throughout the paper, $\mathbb{X}$, $\mathbb{P}(\mathbb{X})$ and $\mathbb{I}$ will denote the universe of discourse, power set of $\mathbb{X}$ and closed unit interval respectively.

Definition 2.1. [1]
A *fuzzy set* $\mathcal{X}$ defined as $\mathcal{X} = \{(\epsilon, \zeta_{\mathcal{X}}(\epsilon))|\epsilon \in \mathbb{X}\}$ such that $\zeta_{\mathcal{X}} : \mathbb{X} \to \mathbb{I}$ where $\zeta_{\mathcal{X}}(\epsilon)$ denotes the belonging value of $\epsilon \in \mathcal{X}$.

Definition 2.2. [2]

An *intuitionistic fuzzy set* $\mathcal{Y}$ defined as $\mathcal{Y} = \{(\beta, < \zeta_{\mathcal{Y}}(\beta), \xi_{\mathcal{Y}}(\beta) >) | \beta \in \mathbb{X}\}$ such that $\zeta_{\mathcal{Y}} : \mathbb{X} \rightarrow \mathbb{I}$ and $\xi_{\mathcal{Y}} : \mathbb{X} \rightarrow \mathbb{I}$, where $\zeta_{\mathcal{Y}}(\beta)$ and $\xi_{\mathcal{Y}}(\beta)$ denote the belonging value and not-belonging value of $\beta \in \mathcal{Y}$ with condition of $0 \leq \zeta_{\mathcal{Y}}(\beta) + \xi_{\mathcal{Y}}(\beta) \leq 1$.

Definition 2.3. [3]

A *neutrosophic set* $\mathcal{Z}$ defined as $\mathcal{Z} = \{(\gamma, < \mathcal{A}_{\mathcal{Z}}(\gamma), \mathcal{B}_{\mathcal{Z}}(\gamma), \mathcal{C}_{\mathcal{Z}}(\gamma) >) | \gamma \in \mathbb{X}\}$ such that $\mathcal{A}_{\mathcal{Z}}(\gamma), \mathcal{B}_{\mathcal{Z}}(\gamma), \mathcal{C}_{\mathcal{Z}}(\gamma) : \mathbb{X} \rightarrow (^{-}0, 1^{+})$, where $\mathcal{A}_{\mathcal{Z}}(\gamma), \mathcal{B}_{\mathcal{Z}}(\gamma)$ and $\mathcal{C}_{\mathcal{Z}}(\gamma)$ denote the degrees of membership, indeterminacy and non-membership of $\gamma \in \mathcal{Z}$ with condition of $^{-}0 \leq \mathcal{A}_{\mathcal{Z}}(\gamma) + \mathcal{B}_{\mathcal{Z}}(\gamma) + \mathcal{C}_{\mathcal{Z}}(\gamma) \leq 3^{+}$.

Definition 2.4. [20]

A pair (ζ_S, Λ) is called a *soft set* over $\mathbb{X}$, where $\zeta_S : \Lambda \rightarrow \mathbb{P}(\mathbb{X})$ and Λ be a subset of a set of attributes E.

For more detail on soft set, see [21–30].

Definition 2.5. [32]

The pair (Ψ, G) is called a hypersoft set over $\mathbb{X}$, where G is the cartesian product of n disjoint sets $G_1, G_2, G_3, ..., G_n$ having attribute values of n distinct attributes $g_1, g_2, g_3, ..., g_n$ respectively and $\Psi : G \rightarrow \mathbb{P}(\mathbb{X})$.

For more definitions and operations of hypersoft set, see [33–35].

3. Neutrosophic Parameterized Hypersoft Set (*nphs*-set)with Application

In this section, neutrosophic parameterized hypersoft set is conceptualized and some of its fundamentals are discussed.

Definition 3.1. Let $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3, ..., \mathcal{A}_n\}$ be a collection of disjoint attribute-valued sets corresponding to n distinct attributes $\alpha_1, \alpha_2, \alpha_3, ..., \alpha_n$ respectively. A NP-hypersoft set (*nphs*-set) $\Psi_{\mathcal{N}}$ over $\mathbb{X}$ is defined as

$$\Psi_{\mathcal{N}} = \{(< P_{\mathcal{N}}(g), Q_{\mathcal{N}}(g), R_{\mathcal{N}}(g) > /g, \psi_{\mathcal{N}}(g)) : g \in \mathbb{G}\}$$

where

(i) $\mathbb{G} = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3 \times .... \times \mathcal{A}_n$

(ii) $\mathcal{N}$ is a neutrosophic set over $\mathbb{G}$ with $P_{\mathcal{N}}, Q_{\mathcal{N}}, R_{\mathcal{N}} : \mathbb{G} \rightarrow \mathbb{I}$ as membership function, indeterminacy function and nonmembership function of *nphs*-set.

(iii) $\psi_{\mathcal{N}} : \mathbb{G} \rightarrow \mathbb{P}(\mathbb{X})$ is called approximate function of *nphs*-set.

Note that collection of all *nphs*-sets is represented by $\Omega_{NPHS}(\mathbb{X})$.

Definition 3.2. Let $\Psi_{\mathcal{N}} \in \Omega_{NPHS}(\mathbb{X})$. If $\psi_{\mathcal{N}}(g) = \phi, P_{\mathcal{N}}(g) = 0, Q_{\mathcal{N}}(g) = 1, R_{\mathcal{N}}(g) = 1$ for all $g \in \mathbb{G}$, then $\Psi_{\mathcal{N}}$ is called $\mathcal{N}$-empty *nphs*-set, denoted by $\Psi_{\Phi_{\mathcal{N}}}$. If $\mathcal{N} = \phi$, then $\mathcal{N}$-empty *nphs*-set is called an empty *nphs*-set, denoted by Ψ_{Φ}.

Definition 3.3. Let $\Psi_{\mathcal{N}} \in \Omega_{NPHS}(\mathbb{X})$. If $\psi_{\mathcal{N}}(g) = \mathbb{X}, P_{\mathcal{N}}(g) = 1, Q_{\mathcal{N}}(g) = 0, R_{\mathcal{N}}(g) = 0$ for all $g \in \mathbb{G}$, then $\Psi_{\mathcal{N}}$ is called $\mathcal{N}$-universal *nphs*-set, denoted by $\Psi_{\tilde{\mathcal{N}}}$. If $\mathcal{N} = \mathbb{G}$, then the $\mathcal{N}$-universal *nphs*-set is called universal *nphs*-set, denoted by $\Psi_{\tilde{\mathbb{G}}}$.

Example 3.4. Consider $\mathbb{X} = \{u_1, u_2, u_3, u_4, u_5\}$ and $\mathcal{A} = \{\mathcal{A}_1, \mathcal{A}_2, \mathcal{A}_3\}$ with $\mathcal{A}_1 = \{a_{11}, a_{12}\}$, $\mathcal{A}_2 = \{a_{21}, a_{22}\}, \mathcal{A}_3 = \{a_{31}\}$, then
$G = \mathcal{A}_1 \times \mathcal{A}_2 \times \mathcal{A}_3$
$G = \{(a_{11}, a_{21}, a_{31}), (a_{11}, a_{22}, a_{31}), (a_{12}, a_{21}, a_{31}), (a_{12}, a_{22}, a_{31})\} = \{g_1, g_2, g_3, g_4\}$.
Case 1.
If $\mathcal{N}_1 = \{< 0.2, 0.3, 0.4 > /g_2, < 0, 1, 1 > /g_3, < 1, 0, 0 > /g_4\}$ and
$\psi_{\mathcal{N}_1}(g_2) = \{u_2, u_4\}$, $\psi_{\mathcal{N}_1}(g_3) = \phi$, and $\psi_{\mathcal{N}_1}(g_4) = \mathbb{X}$, then
$\Psi_{\mathcal{N}_1} = \{(< 0.2, 0.3, 0.4 > /g_2, \{u_2, u_4\}), (< 0, 1, 1 > /g_3, \phi), (< 1, 0, 0 > /g_4, \mathbb{X})\}$.
Case 2.
If $\mathcal{N}_2 = \{< 0, 1, 1 > /g_2, < 0, 1, 1 > /g_3\}, \psi_{\mathcal{N}_2}(g_2) = \phi$ and $\psi_{\mathcal{N}_2}(g_3) = \phi$, then $\Psi_{\mathcal{N}_2} = \Psi_{\Phi_{\mathcal{N}_2}}$.
Case 3.
If $\mathcal{N}_3 = \phi$ corresponding to all elements of $\mathbb{G}$, then $\Psi_{\mathcal{N}_3} = \Psi_{\Phi}$.
Case 4.
If $\mathcal{N}_4 = \{< 1, 0, 0 > /g_1, < 1, 0, 0 > /g_2\}, \psi_{\mathcal{N}_4}(g_1) = \mathbb{X}$, and $\psi_{\mathcal{N}_4}(g_2) = \mathbb{X}$, then $\Psi_{\mathcal{N}_4} = \Psi_{\tilde{\mathcal{N}_4}}$.
Case 5.
If $\mathcal{N}_5 = \mathbb{X}$ with respect to all elements of $\mathbb{G}$, then $\Psi_{\mathcal{N}_5} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 3.5. Let $\Psi_{\mathcal{N}_1}$, $\Psi_{\mathcal{N}_2} \in \Omega_{NPHS}(\mathbb{X})$ then $\Psi_{\mathcal{N}_1}$ is an *nphs*-subset of $\Psi_{\mathcal{N}_2}$, denoted by $\Psi_{\mathcal{N}_1} \tilde{\subseteq} \Psi_{\mathcal{N}_2}$ if
$P_{\mathcal{N}_1}(g) \le P_{\mathcal{N}_2}(g), Q_{\mathcal{N}_1}(g) \ge Q_{\mathcal{N}_2}(g), R_{\mathcal{N}_1}(g) \ge R_{\mathcal{N}_2}(g)$ and $\psi_{\mathcal{N}_1}(g) \subseteq \psi_{\mathcal{N}_2}(g)$ for all $g \in \mathbb{G}$.

Proposition 3.6. *Let* $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2}, \Psi_{\mathcal{N}_3} \in \Omega_{NPHS}(\mathbb{X})$ *then*

(1) $\Psi_{\mathcal{N}_1} \tilde{\subseteq} \Psi_{\tilde{\mathbb{G}}}$.
(2) $\Psi_{\Phi} \tilde{\subseteq} \Psi_{\mathcal{N}_1}$.
(3) $\Psi_{\mathcal{N}_1} \tilde{\subseteq} \Psi_{\mathcal{N}_1}$.
(4) *if* $\Psi_{\mathcal{N}_1} \tilde{\subseteq} \Psi_{\mathcal{N}_2}$ *and* $\Psi_{\mathcal{N}_2} \tilde{\subseteq} \Psi_{\mathcal{N}_3}$ *then* $\Psi_{\mathcal{N}_1} \tilde{\subseteq} \Psi_{\mathcal{N}_3}$.

Definition 3.7. Let $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2} \in \Omega_{NPHS}(\mathbb{X})$ then, $\Psi_{\mathcal{N}_1}$ and $\Psi_{\mathcal{N}_2}$ are *nphs*-equal, represented as $\Psi_{\mathcal{N}_1} = \Psi_{\mathcal{N}_2}$, if and only if $P_{\mathcal{N}_1}(g) = P_{\mathcal{N}_2}(g), Q_{\mathcal{N}_1}(g) = Q_{\mathcal{N}_2}(g), R_{\mathcal{N}_1}(g) = R_{\mathcal{N}_2}(g)$ and $\psi_{\mathcal{N}_1}(g) = \psi_{\mathcal{N}_2}(g)$ for all $g \in \mathbb{G}$.

Proposition 3.8. *Let* $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2}, \Psi_{\mathcal{N}_3} \in \Omega_{NPHS}(\mathbb{X})$ *then,*

(1) *if* $\Psi_{\mathcal{N}_1} = \Psi_{\mathcal{N}_2}$ *and* $\Psi_{\mathcal{N}_2} = \Psi_{\mathcal{N}_3}$ *then* $\Psi_{\mathcal{N}_1} = \Psi_{\mathcal{N}_3}$.

(2) *if* $\Psi_{\mathcal{N}_1} \tilde{\subseteq} \Psi_{\mathcal{N}_2}$ *and* $\Psi_{\mathcal{N}_2} \tilde{\subseteq} \Psi_{\mathcal{N}_1} \Leftrightarrow \Psi_{\mathcal{N}_1} = \Psi_{\mathcal{N}_2}$.

Definition 3.9. Let $\Psi_{\mathcal{N}} \in \Omega_{NPHS}(\mathbb{X})$ then, complement of $\Psi_{\mathcal{N}}$(i.e. $\Psi_{\mathcal{N}}^{\tilde{c}}$) is an *nphs*-set given as $P_{\mathcal{N}}^{\tilde{c}}(g) = 1 - P_{\mathcal{N}}(g), Q_{\mathcal{N}}^{\tilde{c}}(g) = 1 - Q_{\mathcal{N}}(g), R_{\mathcal{N}}^{\tilde{c}}(g) = 1 - R_{\mathcal{N}}(g)$ and $\psi_{\mathcal{N}}^{\tilde{c}}(g) = \mathbb{X} \setminus \psi_{\mathcal{N}}(g)$

Proposition 3.10. *Let* $\Psi_{\mathcal{N}} \in \Omega_{NPHS}(\mathbb{X})$ *then,*

(1) $(\Psi_{\mathcal{N}}^{\tilde{c}})^{\tilde{c}} = \Psi_{\mathcal{N}}$.

(2) $\Psi_{\phi}^{\tilde{c}} = \Psi_{\tilde{\mathbb{G}}}$.

Definition 3.11. Let $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2} \in \Omega_{NPHS}(\mathbb{X})$ then, union of $\Psi_{\mathcal{N}_1}$ and $\Psi_{\mathcal{N}_2}$, denoted by $\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\mathcal{N}_2}$, is defined by

(i) $P_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g) = max\{P_{\mathcal{N}_1}(x), P_{\mathcal{N}_2}(g)\}$,

(ii) $Q_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g) = min\{Q_{\mathcal{N}_1}(x), Q_{\mathcal{N}_2}(g)\}$,

(iii) $R_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g) = min\{R_{\mathcal{N}_1}(x), R_{\mathcal{N}_2}(g)\}$,

(iv) $\psi_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g) = \psi_{\mathcal{N}_1}(g) \cup \psi_{\mathcal{N}_2}(g)$, for all $g \in \mathbb{G}$.

Proposition 3.12. *Let* $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2}, \Psi_{\mathcal{N}_3} \in \Omega_{NPHS}(\mathbb{X})$ *then,*

(1) $\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\mathcal{N}_1} = \Psi_{\mathcal{N}_1}$,

(2) $\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\Phi} = \Psi_{\mathcal{N}_1}$,

(3) $\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\tilde{\mathbb{G}}} = \Psi_{\tilde{\mathbb{G}}}$,

(4) $\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\mathcal{N}_2} = \Psi_{\mathcal{N}_2} \tilde{\cup} \Psi_{\mathcal{N}_1}$,

(5) $(\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\mathcal{N}_2}) \tilde{\cup} \Psi_{\mathcal{N}_3} = \Psi_{\mathcal{N}_1} \tilde{\cup} (\Psi_{\mathcal{N}_2} \tilde{\cup} \Psi_{\mathcal{N}_3})$.

Definition 3.13. Let $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2} \in \Omega_{NPHS}(\mathbb{X})$ then intersection of $\Psi_{\mathcal{N}_1}$ and $\Psi_{\mathcal{N}_2}$, denoted by $\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\mathcal{N}_2}$, is an *nphs*-set defined by

(i) $P_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g) = min\{P_{\mathcal{N}_1}(x), P_{\mathcal{N}_2}(g)\}$,

(ii) $Q_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g) = max\{Q_{\mathcal{N}_1}(x), Q_{\mathcal{N}_2}(g)\}$,

(iii) $R_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g) = max\{R_{\mathcal{N}_1}(x), R_{\mathcal{N}_2}(g)\}$,

(iv) $\psi_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g) = \psi_{\mathcal{N}_1}(g) \cap \psi_{\mathcal{N}_2}(g)$, for all $g \in \mathbb{G}$.

Proposition 3.14. *Let* $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2}, \Psi_{\mathcal{N}_3} \in \Omega_{NPHS}(\mathbb{X})$ *then*

(1) $\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\mathcal{N}_1} = \Psi_{\mathcal{N}_1}$.

(2) $\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\Phi} = \Psi_{\Phi}$.

(3) $\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\tilde{\mathbb{G}}} = \Psi_{\tilde{\mathcal{N}_1}}$.

(4) $\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\mathcal{N}_2} = \Psi_{\mathcal{N}_2} \tilde{\cap} \Psi_{\mathcal{N}_1}$.

(5) $(\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\mathcal{N}_2}) \tilde{\cap} \Psi_{\Psi_{\mathcal{N}_3}} = \Psi_{\mathcal{N}_1} \tilde{\cap} (\Psi_{\mathcal{N}_2} \tilde{\cap} \Psi_{\Psi_{\mathcal{N}_3}})$.

Remark 3.15. Let $\Psi_{\mathcal{N}} \in \Omega_{NPHS}(\mathbb{X})$. If $\Psi_{\mathcal{N}} \neq \Psi_{\tilde{\mathbb{G}}}$, then $\Psi_{\mathcal{N}} \tilde{\cup} \Psi_{\mathcal{N}}^{\tilde{c}} \neq \Psi_{\tilde{\mathbb{G}}}$ and $\Psi_{\mathcal{N}} \tilde{\cap} \Psi_{\mathcal{N}}^{\tilde{c}} \neq \Psi_{\Phi}$

Proposition 3.16. *Let* $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2} \in \Omega_{NPHS}(\mathbb{X})$ *D'Morgans laws are valid*

(1) $(\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\mathcal{N}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{N}_1} \tilde{\cap} \Psi^{\tilde{c}}_{\mathcal{N}_2}$.
(2) $(\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\mathcal{N}_2})^{\tilde{c}} = \Psi^{\tilde{c}}_{\mathcal{N}_1} \tilde{\cup} \Psi^{\tilde{c}}_{\mathcal{N}_2}$.

Proof. For all $g \in \mathbb{G}$,
(1). Since $(P_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2})^{\tilde{c}}(g) = 1 - P_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g)$
$= 1 - max\{P_{\mathcal{N}_1}(g), P_{\mathcal{N}_2}(g)\}$
$= min\{1 - P_{\mathcal{N}_1}(g), 1 - P_{\mathcal{N}_2}(g)\}$
$= min\{P^{\tilde{c}}_{\mathcal{N}_1}(g), P^{\tilde{c}}_{\mathcal{N}_2}(g)\}$
$= P^{\tilde{c}}_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g)$
also
$(Q_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2})^{\tilde{c}}(g) = 1 - Q_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g)$
$= 1 - min\{Q_{\mathcal{N}_1}(g), Q_{\mathcal{N}_2}(g)\}$
$= max\{1 - Q_{\mathcal{N}_1}(g), 1 - Q_{\mathcal{N}_2}(g)\}$
$= max\{Q^{\tilde{c}}_{\mathcal{N}_1}(g), Q^{\tilde{c}}_{\mathcal{N}_2}(g)\}$
$= Q^{\tilde{c}}_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g)$
and
$(R_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2})^{\tilde{c}}(g) = 1 - R_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g)$
$= 1 - min\{R_{\mathcal{N}_1}(g), R_{\mathcal{N}_2}(g)\}$
$= max\{1 - R_{\mathcal{N}_1}(g), 1 - R_{\mathcal{N}_2}(g)\}$
$= max\{R^{\tilde{c}}_{\mathcal{N}_1}(g), R^{\tilde{c}}_{\mathcal{N}_2}(g)\}$
$= R^{\tilde{c}}_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g)$
and
$(\psi_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2})^{\tilde{c}}(g) = \mathbb{X} \setminus \psi_{\mathcal{N}_1 \tilde{\cup} \mathcal{N}_2}(g)$
$= \mathbb{X} \setminus (\psi_{\mathcal{N}_1}(g) \cup \psi_{\mathcal{N}_2}(g))$
$= (\mathbb{X} \setminus \psi_{\mathcal{N}_1}(g)) \cap (\mathbb{X} \setminus \psi_{\mathcal{N}_2}(g))$
$= \psi^{\tilde{c}}_{\mathcal{N}_1}(g) \tilde{\cap} \psi^{\tilde{c}}_{\mathcal{N}_2}(g)$
$= \psi^{\tilde{c}}_{\mathcal{N}_1 \tilde{\cap} \mathcal{N}_2}(g)$.
similarly (2) can be proved easily. □

Proposition 3.17. *Let* $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2}, \Psi_{\mathcal{N}_3} \in \Omega_{NPHS}(\mathbb{X})$ *then*

(1) $\Psi_{\mathcal{N}_1} \tilde{\cup} (\Psi_{\mathcal{N}_2} \tilde{\cap} \Psi_{\mathcal{N}_3}) = (\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\mathcal{N}_2}) \tilde{\cap} (\Psi_{\mathcal{N}_1} \tilde{\cup} \Psi_{\mathcal{N}_3})$.
(2) $\Psi_{\mathcal{N}_1} \tilde{\cap} (\Psi_{\mathcal{N}_2} \tilde{\cup} \Psi_{\mathcal{N}_3}) = (\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\mathcal{N}_2}) \tilde{\cup} (\Psi_{\mathcal{N}_1} \tilde{\cap} \Psi_{\mathcal{N}_3})$.

Proof. For all $g \in \mathbb{G}$,
(1). Since $P_{\mathcal{N}_1 \tilde{\cup} (\mathcal{N}_2 \tilde{\cap} \mathcal{N}_3)}(g) = max\{P_{\mathcal{N}_1}(g), P_{\mathcal{N}_2 \tilde{\cap} \mathcal{N}_3}(g)\}$
$= max\{P_{\mathcal{N}_1}(g), min\{P_{\mathcal{N}_2}(g), P_{\mathcal{N}_3}(g)\}\}$

$= min\{max\{P_{\mathcal{N}_1}(g), P_{\mathcal{N}_2}(g)\}, max\{P_{\mathcal{N}_1}(g), P_{\mathcal{N}_3}(g)\}\}$
$= min\{P_{\mathcal{N}_1\tilde{\cup}\mathcal{N}_2}(g), P_{\mathcal{N}_1\tilde{\cup}\mathcal{N}_3}(g)\}$
$= P_{(\mathcal{N}_1\tilde{\cup}\mathcal{N}_2)\tilde{\cap}(\mathcal{N}_1\tilde{\cup}\mathcal{N}_3)}(g)$
and
$Q_{\mathcal{N}_1\tilde{\cup}(\mathcal{N}_2\tilde{\cap}\mathcal{N}_3)}(g) = min\{Q_{\mathcal{N}_1}(g), Q_{\mathcal{N}_2\tilde{\cap}\mathcal{N}_3}(g)\}$
$= min\{Q_{\mathcal{N}_1}(g), max\{Q_{\mathcal{N}_2}(g), Q_{\mathcal{N}_3}(g)\}\}$
$= max\{min\{Q_{\mathcal{N}_1}(g), Q_{\mathcal{N}_2}(g)\}, min\{Q_{\mathcal{N}_1}(g), Q_{\mathcal{N}_3}(g)\}\}$
$= max\{Q_{\mathcal{N}_1\tilde{\cup}\mathcal{N}_2}(g), Q_{\mathcal{N}_1\tilde{\cup}\mathcal{N}_3}(g)\}$
$= Q_{(\mathcal{N}_1\tilde{\cup}\mathcal{N}_2)\tilde{\cap}(\mathcal{N}_1\tilde{\cup}\mathcal{N}_3)}(g)$
and
$R_{\mathcal{N}_1\tilde{\cup}(\mathcal{N}_2\tilde{\cap}\mathcal{N}_3)}(g) = min\{R_{\mathcal{N}_1}(g), R_{\mathcal{N}_2\tilde{\cap}\mathcal{N}_3}(g)\}$
$= min\{R_{\mathcal{N}_1}(g), max\{R_{\mathcal{N}_2}(g), R_{\mathcal{N}_3}(g)\}\}$
$= max\{min\{R_{\mathcal{N}_1}(g), R_{\mathcal{N}_2}(g)\}, min\{R_{\mathcal{N}_1}(g), R_{\mathcal{N}_3}(g)\}\}$
$= max\{R_{\mathcal{N}_1\tilde{\cup}\mathcal{N}_2}(g), R_{\mathcal{N}_1\tilde{\cup}\mathcal{N}_3}(g)\}$
$= R_{(\mathcal{N}_1\tilde{\cup}\mathcal{N}_2)\tilde{\cap}(\mathcal{N}_1\tilde{\cup}\mathcal{N}_3)}(g)$
and
$\psi_{\mathcal{N}_1\tilde{\cup}(\mathcal{N}_2\tilde{\cap}\mathcal{N}_3)}(g) = \psi_{\mathcal{N}_1}(g) \cup \psi_{\mathcal{N}_2\tilde{\cap}\mathcal{N}_3}(g)$
$= \psi_{\mathcal{N}_1}(g) \cup (\psi_{\mathcal{N}_2}(g) \cap \psi_{\mathcal{N}_3}(g))$
$= (\psi_{\mathcal{N}_1}(g) \cup \psi_{\mathcal{N}_2}(g)) \cap (\psi_{\mathcal{N}_1}(g) \cup \psi_{\mathcal{N}_3}(g))$
$= \psi_{\mathcal{N}_1\tilde{\cup}\mathcal{N}_2}(g) \cap \psi_{\mathcal{N}_1\tilde{U}\mathcal{N}_3}(g)$
$= \psi_{(\mathcal{N}_1\tilde{\cup}\mathcal{N}_2)\tilde{\cap}(\mathcal{N}_1\tilde{\cup}\mathcal{N}_3)}(g)$
In the same way, (2) can be proved. □

Definition 3.18. Let $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2} \in \Omega_{NPHS}(\mathbb{X})$ then OR-operation of $\Psi_{\mathcal{N}_1}$ and $\Psi_{\mathcal{N}_2}$, denoted by $\Psi_{\mathcal{N}_1} \tilde{\oplus} \Psi_{\mathcal{N}_2}$, is an *nphs*-set defined by

(i) $P_{\mathcal{N}_1\tilde{\oplus}\mathcal{N}_2}(g_1, g_2) = max\{P_{\mathcal{N}_1}(g_1), P_{\mathcal{N}_2}(g_2)\}$,
(ii) $Q_{\mathcal{N}_1\tilde{\oplus}\mathcal{N}_2}(g_1, g_2) = min\{Q_{\mathcal{N}_1}(g_1), Q_{\mathcal{N}_2}(g_2)\}$,
(iii) $R_{\mathcal{N}_1\tilde{\oplus}\mathcal{N}_2}(g_1, g_2) = min\{R_{\mathcal{N}_1}(g_1), R_{\mathcal{N}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{N}_1\tilde{\oplus}\mathcal{N}_2}(g_1, g_2) = \psi_{\mathcal{N}_1}(g_1) \cup \psi_{\mathcal{N}_2}(g_2)$, for all $(g_1, g_2) \in \mathcal{N}_1 \times \mathcal{N}_2$.

Definition 3.19. Let $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2} \in \Omega_{NPHS}(\mathbb{X})$ then AND-operation of $\Psi_{\mathcal{N}_1}$ and $\Psi_{\mathcal{N}_2}$, denoted by $\Psi_{\mathcal{N}_1} \tilde{\otimes} \Psi_{\mathcal{N}_2}$, is an *nphs*-set defined by

(i) $P_{\mathcal{N}_1\tilde{\otimes}\mathcal{N}_2}(g_1, g_2) = min\{P_{\mathcal{N}_1}(g_1), P_{\mathcal{N}_2}(g_2)\}$,
(ii) $Q_{\mathcal{N}_1\tilde{\otimes}\mathcal{N}_2}(g_1, g_2) = max\{Q_{\mathcal{N}_1}(g_1), Q_{\mathcal{N}_2}(g_2)\}$,
(iii) $R_{\mathcal{N}_1\tilde{\otimes}\mathcal{N}_2}(g_1, g_2) = max\{R_{\mathcal{N}_1}(g_1), R_{\mathcal{N}_2}(g_2)\}$,
(iv) $\psi_{\mathcal{N}_1\tilde{\otimes}\mathcal{N}_2}(g_1, g_2) = \psi_{\mathcal{N}_1}(g_1) \cap \psi_{\mathcal{N}_2}(g_2)$, for all $(g_1, g_2) \in \mathcal{N}_1 \times \mathcal{N}_2$.

Proposition 3.20. *Let* $\Psi_{\mathcal{N}_1}, \Psi_{\mathcal{N}_2}, \Psi_{\mathcal{N}_3} \in \Omega_{NPHS}(\mathbb{X})$ *then*

(1) $\Psi_{\mathcal{N}_1} \tilde{\otimes} \Psi_{\Phi} = \Psi_{\Phi}$.

(2) $(\Psi_{\mathcal{N}_1} \tilde{\otimes} \Psi_{\mathcal{N}_2}) \tilde{\otimes} \Psi_{\mathcal{N}_3} = \Psi_{\mathcal{N}_1} \tilde{\otimes} (\Psi_{\mathcal{N}_2} \tilde{\otimes} \Psi_{\mathcal{N}_3})$.

(3) $(\Psi_{\mathcal{N}_1} \tilde{\oplus} \Psi_{\mathcal{N}_2}) \tilde{\oplus} \Psi_{\mathcal{N}_3} = \Psi_{\mathcal{N}_1} \tilde{\oplus} (\Psi_{\mathcal{N}_2} \tilde{\oplus} \Psi_{\mathcal{N}_3})$.

4. Neutrosophic Decision Set of *nphs*-set

Having motivation from decision making methods stated in [42–50], here an algorithm is presented with the help of characterization of neutrosophic decision set on *nphs*-set which based on decision making technique and is explained with example.

Definition 4.1. Let $\Psi_{\mathcal{N}} \in \Omega_{NPHS}(\mathbb{X})$ then a neutrosophic decision set of $\Psi_{\mathcal{N}}$ (i.e. $\Psi^D_{\mathcal{N}}$) is represented as

$$\Psi^D_{\mathcal{N}} = \{< \mathcal{T}^D_{\mathcal{N}}(u), \mathcal{I}^D_{\mathcal{N}}(u), \mathcal{F}^D_{\mathcal{N}}(u) > /u : u \in \mathbb{X}\}$$

where $\mathcal{T}^D_{\mathcal{N}}, \mathcal{I}^D_{\mathcal{N}}, \mathcal{F}^D_{\mathcal{N}} : \mathbb{X} \to \mathbb{I}$ and

$$\mathcal{T}^D_{\mathcal{N}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{N})} \mathcal{T}_{\mathcal{N}}(v) \Gamma_{\psi_{\mathcal{N}(v)}}(u)$$

$$\mathcal{I}^D_{\mathcal{N}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{N})} \mathcal{I}_{\mathcal{N}}(v) \Gamma_{\psi_{\mathcal{N}(v)}}(u)$$

$$\mathcal{F}^D_{\mathcal{N}}(u) = \frac{1}{|\mathbb{X}|} \sum_{v \in S(\mathcal{N})} \mathcal{F}_{\mathcal{N}}(v) \Gamma_{\psi_{\mathcal{N}(v)}}(u)$$

where $| \bullet |$ denotes set cardinality with

$$\Gamma_{\psi_{\mathcal{N}(v)}}(u) = \begin{cases} 1 & ; \quad u \in \Gamma_{\psi_{\mathcal{N}}}(v) \\ 0 & ; \quad u \notin \Gamma_{\psi_{\mathcal{N}}}(v) \end{cases}$$

Definition 4.2. If $\Psi_{\mathcal{N}} \in \Omega_{NPHS}(\mathbb{X})$ with neutrosophic decision set $\Psi^D_{\mathcal{N}}$ then reduced fuzzy set of $\Psi^D_{\mathcal{N}}$ is a fuzzy set represented as

$$\mathbb{R}(\Psi^D_{\mathcal{N}}) = \left\{ \zeta_{\Psi^D_{\mathcal{N}}}(u)/u : u \in \mathbb{X} \right\}$$

where $\zeta_{\Psi^D_{\mathcal{N}}} : \mathbb{X} \to \mathbb{I}$ with $\zeta_{\Psi^D_{\mathcal{N}}}(u) = \mathcal{T}^D_{\mathcal{N}}(u) + \mathcal{I}^D_{\mathcal{N}}(u) - \mathcal{F}^D_{\mathcal{N}}(u)$

4.1. *Proposed Algorithm*

Once $\Psi^D_{\mathcal{N}}$ has been established, it may be indispensable to select the best single substitute from the options. Therefore, decision can be set up with the help of following algorithm.

Step 1 Determine $\mathcal{N} = \{< \mathcal{T}_{\mathcal{N}}(g), \mathcal{I}_{\mathcal{N}}(g), \mathcal{F}_{\mathcal{N}}(g) > /g : \mathcal{T}_{\mathcal{N}}(g), \mathcal{I}_{\mathcal{N}}(g), \mathcal{F}_{\mathcal{N}}(g) \in \mathbb{I}, g \in \mathbb{G}\}$,

Step 2 Find $\psi_{\mathcal{N}}(g)$

Step 3 Construct $\Psi_{\mathcal{N}}$ over $\mathbb{X}$,

Step 4 Compute $\Psi^D_{\mathcal{N}}$,

TABLE 1. Degrees of Membership $\mathcal{T}_{\mathcal{N}}(g_i)$

$\mathcal{T}_{\mathcal{N}}(g_i)$	Degree	$\mathcal{T}_{\mathcal{N}}(g_i)$	Degree
$\mathcal{T}_{\mathcal{N}}(g_1)$	0.1	$\mathcal{T}_{\mathcal{N}}(g_9)$	0.9
$\mathcal{T}_{\mathcal{N}}(g_2)$	0.2	$\mathcal{T}_{\mathcal{N}}(g_{10})$	0.16
$\mathcal{T}_{\mathcal{N}}(g_3)$	0.3	$\mathcal{T}_{\mathcal{N}}(g_{11})$	0.25
$\mathcal{T}_{\mathcal{N}}(g_4)$	0.4	$\mathcal{T}_{\mathcal{N}}(g_{12})$	0.45
$\mathcal{T}_{\mathcal{N}}(g_5)$	0.5	$\mathcal{T}_{\mathcal{N}}(g_{13})$	0.35
$\mathcal{T}_{\mathcal{N}}(g_6)$	0.6	$\mathcal{T}_{\mathcal{N}}(g_{14})$	0.75
$\mathcal{T}_{\mathcal{N}}(g_7)$	0.7	$\mathcal{T}_{\mathcal{N}}(g_{15})$	0.65
$\mathcal{T}_{\mathcal{N}}(g_8)$	0.8	$\mathcal{T}_{\mathcal{N}}(g_{16})$	0.85

Step 5 Choose the maximum of $\zeta_{\Psi^D_{\mathcal{N}}}(u)$.

Example 4.3. Suppose that Mr. James Peter wants to buy a mobile from a mobile market. There are eight kinds of mobiles (options) which form the set of discourse $\mathbb{X} = \{m_1, m_2, m_3, m_4, m_5, m_6, m_7, m_8\}$. The best selection may be evaluated by observing the attributes i.e. $a_1 =$ Company, $a_2 =$ Camera Resolution, $a_3 =$ Size, $a_4 =$ RAM, and $a_5 =$ Battery power. The attribute-valued sets corresponding to these attributes are:
$B_1 = \{b_{11}, b_{12}\}$
$B_2 = \{b_{21}, b_{22}\}$
$B_3 = \{b_{31}, b_{32}\}$
$B_4 = \{b_{41}, b_{42}\}$
$B_5 = \{b_{51}\}$
then $\mathbb{G} = B_1 \times B_2 \times B_3 \times B_4 \times B_5$
$\mathbb{G} = \{g_1, g_2, g_3, g_4, ..., g_{16}\}$ where each $g_i, i = 1, 2, ..., 16$, is a 5-tuples element.
Step 1 :
From tables 1, 2, 3 we can construct $\mathcal{N}$ as

$$\mathcal{N} = \left\{ \begin{array}{l} <0.1, 0.2, 0.3> /g_1, <0.2, 0.3, 0.4> /g_2, <0.3, 0.4, 0.5> /g_3, <0.4, 0.5, 0.6> /g_4, \\ <0.5, 0.6, 0.7> /g_5, <0.6, 0.7, 0.8> /g_6, <0.7, 0.8, 0.9> /g_7, <0.8, 0.9, 0.1> /g_8, \\ <0.9, 0.1, 0.2> /g_9, <0.16, 0.27, 0.37> /g_{10}, <0.25, 0.35, 0.45> /g_{11}, <0.45, 0.55, 0.65> /g_{12}, \\ <0.35, 0.45, 0.55> /g_{13}, <0.75, 0.85, 0.95> /g_{14}, <0.65, 0.75, 0.85> /g_{15}, <0.85, 0.95, 0.96> /g_{16} \end{array} \right\}$$

Step 2 :
Table 4 presents $\psi_{\mathcal{N}}(g_i)$ corresponding to each element of $\mathbb{G}$.
Step 3 :

TABLE 2. Degrees of Indeterminacy $\mathcal{I}_{\mathcal{N}}(g_i)$

$\mathcal{I}_{\mathcal{N}}(g_i)$	Degree	$\mathcal{I}_{\mathcal{N}}(g_i)$	Degree
$\mathcal{I}_{\mathcal{N}}(g_1)$	0.2	$\mathcal{I}_{\mathcal{N}}(g_9)$	0.1
$\mathcal{I}_{\mathcal{N}}(g_2)$	0.3	$\mathcal{I}_{\mathcal{N}}(g_{10})$	0.27
$\mathcal{I}_{\mathcal{N}}(g_3)$	0.4	$\mathcal{I}_{\mathcal{N}}(g_{11})$	0.35
$\mathcal{I}_{\mathcal{N}}(g_4)$	0.5	$\mathcal{I}_{\mathcal{N}}(g_{12})$	0.55
$\mathcal{I}_{\mathcal{N}}(g_5)$	0.6	$\mathcal{I}_{\mathcal{N}}(g_{13})$	0.45
$\mathcal{I}_{\mathcal{N}}(g_6)$	0.7	$\mathcal{I}_{\mathcal{N}}(g_{14})$	0.85
$\mathcal{I}_{\mathcal{N}}(g_7)$	0.8	$\mathcal{I}_{\mathcal{N}}(g_{15})$	0.75
$\mathcal{I}_{\mathcal{N}}(g_8)$	0.9	$\mathcal{I}_{\mathcal{N}}(g_{16})$	0.95

TABLE 3. Degrees of Non-Membership $\mathcal{F}_{\mathcal{N}}(g_i)$

$\mathcal{F}_{\mathcal{N}}(g_i)$	Degree	$\mathcal{F}_{\mathcal{N}}(g_i)$	Degree
$\mathcal{F}_{\mathcal{N}}(g_1)$	0.3	$\mathcal{F}_{\mathcal{N}}(g_9)$	0.2
$\mathcal{F}_{\mathcal{N}}(g_2)$	0.4	$\mathcal{F}_{\mathcal{N}}(g_{10})$	0.37
$\mathcal{F}_{\mathcal{N}}(g_3)$	0.5	$\mathcal{F}_{\mathcal{N}}(g_{11})$	0.45
$\mathcal{F}_{\mathcal{N}}(g_4)$	0.6	$\mathcal{F}_{\mathcal{N}}(g_{12})$	0.65
$\mathcal{F}_{\mathcal{N}}(g_5)$	0.7	$\mathcal{F}_{\mathcal{N}}(g_{13})$	0.55
$\mathcal{F}_{\mathcal{N}}(g_6)$	0.8	$\mathcal{F}_{\mathcal{N}}(g_{14})$	0.95
$\mathcal{F}_{\mathcal{N}}(g_7)$	0.9	$\mathcal{F}_{\mathcal{N}}(g_{15})$	0.85
$\mathcal{F}_{\mathcal{N}}(g_8)$	0.1	$\mathcal{F}_{\mathcal{N}}(g_{16})$	0.96

With the help of step 1 and step 2, we can construct $\Psi_{\mathcal{N}}$ as

$$\Psi_{\mathcal{N}} = \left\{ \begin{array}{l} (<0.1,0.2,0.3>/g_1,\{m_1,m_2\}),(<0.2,0.3,0.4>/g_2,\{m_1,m_2,m_3\}), \\ (<0.3,0.4,0.5>/g_3,\{m_2,m_3,m_4\}),(<0.4,0.5,0.6>/g_4,\{m_4,m_5,m_6\}), \\ (<0.5,0.6,0.7>/g_5,\{m_6,m_7,m_8\}),(<0.6,0.7,0.8>/g_6,\{m_2,m_3,m_4\}), \\ (<0.7,0.8,0.9>/g_7,\{m_1,m_3,m_5\}),(<0.8,0.9,0.1>/g_8,\{m_2,m_3,m_7\}), \\ (<0.9,0.1,0.2>/g_9,\{m_2,m_7,m_8\}),(<0.16,0.27,0.37>/g_{10},\{m_6,m_7,m_8\}), \\ (<0.25,0.35,0.45>/g_{11},\{m_2,m_4,m_6\}),(<0.45,0.55,0.65>/g_{12},\{m_2,m_3,m_6\}), \\ (<0.35,0.45,0.55>/g_{13},\{m_3,m_5,m_7\}),(<0.75,0.85,0.95>/g_{14},\{m_1,m_3,m_5\}), \\ (<0.65,0.75,0.85>/g_{15},\{m_5,m_7,m_8\}),(<0.85,0.95,0.96>/g_{16},\{m_4,m_5,m_6\}) \end{array} \right\}$$

Step 4 :

From tables 5 to 8 , we can construct $\mathbb{R}(\Psi_{\mathcal{N}}^{D})$ as

$$\mathbb{R}(\Psi_{\mathcal{N}}^{D}) = \left\{ \begin{array}{l} 0.1688/m_1, 0.4625/m_2, 0.5313/m_3, 0.2488/m_4, \\ 0.3988/m_5, 0.2625/m_6, 0.4575/m_7, 0.2263/m_8 \end{array} \right\}$$

Step 5 :

Since maximum of $\zeta_{\Psi_{\mathcal{N}}^{D}}(m_i)$ is 0.5313 so the mobile m_3 is selected.

Table 4. Approximate functions $\psi_{\mathcal{N}}(g_i)$

g_i	$\psi_{\mathcal{N}}(g_i)$	g_i	$\psi_{\mathcal{N}}(g_i)$
g_1	$\{m_1, m_2\}$	g_9	$\{m_2, m_7, m_8\}$
g_2	$\{m_1, m_2, m_3\}$	g_{10}	$\{m_6, m_7, m_8\}$
g_3	$\{m_2, m_3, m_4\}$	g_{11}	$\{m_2, m_4, m_6\}$
g_4	$\{m_4, m_5, m_6\}$	g_{12}	$\{m_2, m_3, m_6\}$
g_5	$\{m_6, m_7, m_8\}$	g_{13}	$\{m_3, m_5, m_7\}$
g_6	$\{m_2, m_3, m_4\}$	g_{14}	$\{m_1, m_3, m_5\}$
g_7	$\{m_1, m_3, m_5\}$	g_{15}	$\{m_5, m_7, m_8\}$
g_8	$\{m_2, m_3, m_7\}$	g_{16}	$\{m_4, m_5, m_6\}$

Table 5. Membership values $\mathcal{T}^D_{\mathcal{N}}(m_i)$

m_i	$\mathcal{T}^D_{\mathcal{N}}(m_i)$	m_i	$\mathcal{T}^D_{\mathcal{N}}(m_i)$
m_1	0.2188	m_5	0.4625
m_2	0.4500	m_6	0.3263
m_3	0.5188	m_7	0.4200
m_4	0.3000	m_8	0.2763

Table 6. Indeterminacy values $\mathcal{I}^D_{\mathcal{N}}(m_i)$

m_i	$\mathcal{I}^D_{\mathcal{N}}(m_i)$	m_i	$\mathcal{I}^D_{\mathcal{N}}(m_i)$
m_1	0.2688	m_5	0.5375
m_2	0.4375	m_6	0.4025
m_3	0.6188	m_7	0.3838
m_4	0.3625	m_8	0.2150

Table 7. Non-Membership values $\mathcal{F}^D_{\mathcal{N}}(m_i)$

m_i	$\mathcal{F}^D_{\mathcal{N}}(m_i)$	m_i	$\mathcal{F}^D_{\mathcal{N}}(m_i)$
m_1	0.3188	m_5	0.6013
m_2	0.4250	m_6	0.4663
m_3	0.6063	m_7	0.3463
m_4	0.4138	m_8	0.2650

5. Conclusion

In this study, neutrosophic parameterized hypersoft set is conceptualized along with some of elementary properties and theoretic operations. A novel algorithm is proposed for decision making and is validated with the help of an illustrative example for appropriate purchasing

TABLE 8. Reduced Fuzzy membership $\zeta_{\Psi^{D}_{\mathcal{N}}}(m_i)$

m_i	$\zeta_{\Psi^{D}_{\mathcal{N}}}(m_i)$	m_i	$\zeta_{\Psi^{D}_{\mathcal{N}}}(m_i)$
m_1	0.1688	m_5	0.3988
m_2	0.4625	m_6	0.2625
m_3	0.5313	m_7	0.4575
m_4	0.2488	m_8	0.2263

of mobile from mobile market. Future work may include the extension of this work for other neutrosophic-like environments and the implementation for solving more real life problems in decision making.

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