



## Neutrosophic Soft Structures

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**Abstract:** In paper, neutrosophic soft points with the concept of one point greater than the other and their properties, generalized neutrosophic soft open set known as soft  $b$ -open set, neutrosophic soft separation axioms theoretically with support of suitable examples with respect to soft points, neutrosophic soft  $b_0$ -space engagement with generalized neutrosophic soft closed set, neutrosophic soft  $b_2$ -space engagement with generalized neutrosophic soft open set are addressed. In continuation, neutrosophic soft  $b_0$ -space behave as neutrosophic soft  $b_2$ -space with the plantation of some extra condition on soft  $b_0$ -space, neutrosophic soft  $b_3$ -space and related theorems, neutrosophic soft  $b_4$ -space, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft separation axioms, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft close sets are reflected. Secondly, long touched has been given to neutrosophic soft countability connection with bases and sub-bases, neutrosophic soft product spaces and its engagement through different generalized neutrosophic soft open set and close sets, neutrosophic soft coordinate spaces and its engagement through different generalized neutrosophic soft open set and close sets, Finally, neutrosophic soft countability and its relationship with Bolzano Weirstrass Property through engagement of compactness, neutrosophic soft strongly spaces and its related theorems, neutrosophic soft sequences and its relation with neutrosophic soft compactness, neutrosophic soft Lindelof space and related theorems are supposed to address.

**Keywords** Neutrosophic soft set (*NSS*), neutrosophic soft point, neutrosophic soft  $b$ -open set and neutrosophic  $b$ -separation axioms.

## 1. Introduction

Cagman et al. [1] defined the concept of soft topology on a soft set, and presented its related properties. The authors also discussed the foundations of the theory of soft topological spaces. Shabir and Naz [2] addressed soft topological spaces with a fixed set of parameters over an initial universe. The notions of soft open sets, soft closed sets, soft closure, soft interior points, soft point neighborhood, soft separation axioms and their basic characteristics are addressed. The authors reflected that a soft topological space gives birth to a family of crisp topological spaces that are parameterized. The authors scanned a soft topological space's subspaces, and explored open and soft closed sets of characterization w. r. t. soft open set. Finally, the authors tackled in depth the notion  $T_i$  spaces, soft normal space and soft regular spaces.

Bayramov and Gunduz [3] investigated some basic notions of STS by using soft point concept. Later on the authors addressed  $T_i$ -soft spaces and the ties between them. Finally, the authors defined soft compactness and leaked out some of its important characteristics.

Khattak et al [4] introduced the concept of soft  $\alpha$ -open soft  $\beta$ -open, soft  $\alpha$ -separations axioms and soft  $\beta$ -separation axioms in soft single point topology. The authors have addressed soft  $(\alpha, \beta)$  separation axioms with regard to ordinary points and soft points in soft topological spaces.

Zadeh [5] exposed the concept of fuzzy set. The author described that a fuzzy set is a class of objects with a continuum of grades of membership. The authors further defined the set through a membership feature, assigning membership grade to each group candidate. The notions of inclusion, union, intersection, complement, relation, convexity, etc. have been applied to such sets, different properties of these notions have been developed in the sense of fuzzy sets. In particular, it has been proven that a soft separation axiom theorem for convex fuzzy set ignored the prerequisites of mutually exclusive fuzzy sets.

Atanassov [6] developed the 'intuitionistic fuzzy set' (IFS) concept, which is an extension of the 'fuzzy set' definition. The authors explored different properties including operations and set-over relationships. Bayramov and Gunduz [7] introduced some important features of intuitive fuzzy soft topological spaces and established the intuitive soft closure and interior of an intuitive soft set. In addition, their research also addressed intuitionistic fuzzy continuous mapping and structural characteristics. Deli and Broumi [8] defined for the first a relation on neutrosophic soft sets. The new concept allows two neutrosophical soft sets to be composed. It is conceived to extract useful information by combing neutrosophical soft sets. Eventually a decision making approach is based on neutrosophic soft sets.

In a new approach, Bera and Mahapatra [9] introduced the concept of *cartesian* product and the neutrosophic soft sets in a new approach. Some properties of this principle were discussed and checked with relevant examples from real life. Smarandache [10] for the first time initiated the concept of neutrosophic set which is generalization of the intuitionistic fuzzy set (IFS), and intuitionistic set (NS). Some related examples are presented. Peculiarities between NS and IFS are underlined.

Maji [11] broadened the Smarandache analysis. The author used the idea of soft set neutrosophic set and incorporated neutrosophic soft set. On neutrosophic soft set those meaning and related operations were addressed.

Bera and Mahapatra [12] developed topology formulation on a neutrosophic soft set (NSS). This study studies the notion of neutrosophic soft interior, neutrosophic soft closure, neutrosophic soft neighborhood, neutrosophic soft boundary, normal NSS and their basic properties. Topology and topology for subspaces on the NSS are described with appropriate examples. It also developed some related properties. In addition to this, the concept of separation axioms on neutrosophic soft topological space was introduced along with investigation of several structural features.

Khattak et al. [13] for the first time leaked out the idea of neutrosophic soft b-open set, neutrosophic soft b-closed sets and their properties. Also the idea of neutrosophic soft b-neighborhood and neutrosophic soft b-separation axioms in neutrosophic soft topological structures are reflected. Later on the important results are discussed related to these newly defined concepts *with respect to soft points*. The concept of neutrosophic soft b-separation axioms of neutrosophic soft topological spaces is diffused in different results with respect to soft points. Furthermore, properties of neutrosophic soft  $bT_i$ -space ( $i = 0, 1, 2, 3, 4$ ) and some associations between them are discussed.

C.G. Aras et al. [14] leaked out *some basic* notions of *neutrosophic soft sets* and redefined some *neutrosophic soft point concept*. Later on the authors addressed some neutrosophic soft  $T_i$ -space and the relationships among them.

T. Y. Ozturk et al. [15] re-defined some operations on neutrosophic soft sets differently as defined by others authors. The authors supported and defended their approach through interesting examples. The authors further beautifully addressed different results with this new approach.

M Al-Tahan, B Davvaz [16] discussed a relationship between SVN and neutrosophic  $\mathfrak{N}$ -structures and study it. Moreover, the authors apply results to algebraic structures (hyper structures) and prove that the results on neutrosophic  $\mathfrak{N}$ -substructure (sub hyper structure) of a given algebraic structure (hyper structure) can be deduced from single valued neutrosophic algebraic structure (hyper structure) and vice versa.

Adeleke et al. [17] studied refined neutrosophic rings, Substructures of refined neutrosophic rings and their elementary properties and it is shown that every refined neutrosophic ring is a ring. Adeleke et al. [18] studied refined neutrosophic ideals and refined neutrosophic homomorphism along their elementary properties. Madeleine et al. [19] provided a connection between neutrosophic  $\mathfrak{N}$ -structures and subtraction algebras. In this regard, the authors introduced the concept of neutrosophic  $\mathfrak{N}$ -ideals in subtraction algebra. Moreover, the authors studied its properties and find out a necessary and sufficient condition for a neutrosophic  $\mathfrak{N}$ -structure to be a neutrosophic  $\mathfrak{N}$ -ideal. M. Parimala et al. [20] introduced the notion of neutrosophic  $\alpha\omega$ -closed sets and study some of the properties of neutrosophic  $\alpha\omega$ -closed sets. Further, the authors investigated neutrosophic  $\alpha\omega$ -continuity, neutrosophic  $\alpha\omega$ -irresoluteness, neutrosophic  $\alpha\omega$  connectedness and neutrosophic contra  $\alpha\omega$  continuity along with examples. Abdel-Basset et al. [21] proposed a powerful framework based on neutrosophic sets to aid with patients with cancer. Abdel-Basset et al. [22] developed a novel intelligent medical decision support model based on soft computing and IOT as the use of neutrosophical sets decision-making. Abdel-Basset et al. [23] concentrated on the evaluation of supply chain sustainability based on the two critical dimensions. The authors further added that the first is the importance of evolution metrics based on economic, environmental and social aspect, and

the second is the degree of difficulty of information gathering. The authors guaranteed that the aim of this paper increase the accuracy of the evacuation. Abdel-Basset et al. [24] suggested that this article proposed a hybrid combination between analytical hierarchical process (AHP) as an MCDM method and neutrosophic theory to successfully detect and handle the uncertainty and inconsistency challenges.

A. Mehmood et al. [26] introduced generalized neutrosophic separation axioms in neutrosophic soft topological spaces. A. Mehmood et al. [27] discussed soft  $\alpha$ -connectedness, soft  $\alpha$ -dis-connectedness and soft  $\alpha$ -compact spaces in bi-polar soft topological spaces with respect to ordinary points. For better understanding the authors provided suitable examples.

## 2. Preliminaries

In this section we now state certain useful definitions, theorems, and several existing results for neutrosophic soft sets that we require in the next sections.

**Definition 2.1** [13] *NSS on a father set  $\langle X \rangle$  is characterized as:*

$$\mathcal{A}^{\text{neutrosophic}} = \left\{ \begin{array}{l} (x, \mathbb{T}_{\mathcal{A}^{\text{ntophic}}}(x), \mathbb{I}_{\mathcal{A}^{\text{ntophic}}}(x), \mathbb{F}_{\mathcal{A}^{\text{ntophic}}}(x) : x \in \langle X \rangle) \\ \mathbb{T} : \langle X \rangle \rightarrow ]0^-, 1^+[ \\ , \mathbb{I} : \langle X \rangle \rightarrow ]0^-, 1^+[ \\ \mathbb{F} : \langle X \rangle \rightarrow ]0^-, 1^+[ \\ , \text{so that's it} \\ 0^- \leq \{ \mathbb{T} + \mathbb{I} + \mathbb{F} \} \leq 3^+ . \end{array} \right.$$

**Definition 2.2**[10] let  $\langle X \rangle$  be a father set,  $\mathfrak{A}^{\text{parameter}}$  be a set of all conditions, and  $\mathcal{L}(\langle X \rangle)$  denote the efficiency set of  $\langle X \rangle$ . A pair  $(\mathfrak{f}, \mathfrak{A}^{\text{parameter}})$  is referred to as a soft set over  $\langle X \rangle$ , where  $\mathfrak{f}$  is a map given by:  $\mathfrak{A}^{\text{parameter}} \rightarrow \mathcal{L}(\langle X \rangle)$ . For  $n \in \mathfrak{A}^{\text{parameter}}$ ,  $\mathfrak{f}(n)$  may be viewed as the set of softset elements  $(\mathfrak{f}, \mathfrak{A}^{\text{parameter}})$ , or as a set of  $n$ -estimated the soft set components, i.e.  $(\mathfrak{f}, \mathfrak{A}^{\text{parameter}}) = \{(n, \mathfrak{f}(n)) : n \in \mathfrak{A}^{\text{parameter}}, \mathfrak{f} : \mathfrak{A}^{\text{parameter}} \rightarrow \mathcal{L}(\langle X \rangle)\}$ .

**Definition 2.3** [7] Let  $\langle X \rangle$  be a father set,  $\mathfrak{A}^{\text{parameter}}$  be a set of all conditions, and  $\mathcal{L}(\langle X \rangle)$  denote the efficiency Set of  $\langle X \rangle$ . A pair  $(\mathfrak{f}, \mathfrak{A}^{\text{parameter}})$  is referred to as a soft set over  $\langle X \rangle$ , where  $\mathfrak{f}$  is a map given by:  $\mathfrak{A}^{\text{parameter}} \rightarrow \mathcal{L}(\langle X \rangle)$ . Then a (NS) set  $(\mathfrak{f}, \mathfrak{A}^{\text{parameter}})$  over  $\langle X \rangle$  is a set defined by a set of valued functions signifying a mapping  $\mathfrak{A}^{\text{parameter}} \rightarrow$

$\mathcal{L}(\langle X \rangle)$ , is referred to as the approximate (NS) function  $(\mathfrak{f}, \mathfrak{A}^{\text{parameter}})$ . In other words, the (NS) is a group of conditions of certain elements of the set  $\mathcal{L}(\langle X \rangle)$  so it can be written as a set of ordered pairs:

$(\mathfrak{f}, \mathfrak{A}^{\text{parameter}}) = \{((n, [\mathbb{T}_{\mathfrak{f}(x)}(x), \mathbb{I}_{\mathfrak{f}(x)}(x), \mathbb{F}_{\mathfrak{f}(x)}(x) : x \in \langle X \rangle])) : n \in \mathfrak{A}^{\text{parameter}}\}$ . Obviously,  $\mathbb{T}_{\mathfrak{f}(x)}(x), \mathbb{I}_{\mathfrak{f}(x)}(x), \mathbb{F}_{\mathfrak{f}(x)}(x) \in [0, 1]$  are membership of truth, membership of indeterminacy and membership of falsehood  $\mathfrak{f}(n)$ . Since the supremum of each  $\mathbb{T}, \mathbb{I}, \mathbb{F}$  is 1, the inequality that  $0^- \leq \mathbb{T}_{\mathfrak{f}(x)}(x) + \mathbb{I}_{\mathfrak{f}(x)}(x) + \mathbb{F}_{\mathfrak{f}(x)}(x) \leq 3^+$  is obvious.

**Definition 2.4** [5] let  $(\tilde{f}, \mathfrak{A}^{parameter})$  be a (NSS) over the father set  $\langle \mathcal{X} \rangle$ . The complement of  $(\tilde{f}, \mathfrak{A}^{parameter})$  is signified  $(\tilde{f}, \mathfrak{A}^{parameter})^c$  and is defined as follows:

$$(\tilde{f}, \mathfrak{A}^{parameter})^c = \{((n, [x, \mathbb{T}_{\tilde{f}(x)}(x), 1 - \mathbb{I}_{\tilde{f}(x)}(x), \mathbb{F}_{\tilde{f}(x)}(x) : x \in \langle \mathcal{X} \rangle])) : n \in \mathfrak{A}^{parameter}\}$$

It's clear that

$$((\tilde{f}, \mathfrak{A}^{parameter})^c)^c = (\tilde{f}, \mathfrak{A}^{parameter}).$$

**Definition 2.5** [9] Let  $(\tilde{f}, n)$  and  $(\tilde{\rho}, n)$  two (NSS) over a father  $\langle \mathcal{X} \rangle$ .  $(\tilde{f}, n)$  is supposed to be NSSS of  $(\tilde{\rho}, n)$  if  $\mathbb{T}_{\tilde{f}(x)}(x) \leq \mathbb{T}_{\tilde{\rho}(x)}(x), \mathbb{I}_{\tilde{f}(x)}(x) \leq \mathbb{I}_{\tilde{\rho}(x)}(x), \mathbb{F}_{\tilde{f}(x)}(x) \geq \mathbb{F}_{\tilde{\rho}(x)}(x), \forall n \in \mathfrak{A}^{parameter} \& \forall x \in \langle \mathcal{X} \rangle$ . It is signifies as  $(\tilde{f}, n) \subseteq (\tilde{\rho}, n)$ .  $(\tilde{f}, n)$  is said to be (NS) equal to  $(\tilde{\rho}, n)$  if  $(\tilde{f}, n)$  is (NSSS) of  $(\tilde{\rho}, n)$  and  $(\tilde{\rho}, n)$  is NSSS of  $(\tilde{f}, n)$ . It is symbolized as  $(\tilde{f}, n) = (\tilde{\rho}, n)$ .

### 3. Neutrosophic Soft Points and Their Characteristics

**Definition 3.1** Let  $(\tilde{f}_1, n) \& (\tilde{f}_2, n)$  be two (NSSS) over a father set  $\langle \mathcal{X} \rangle$  s.t.  $(\tilde{f}_1, n) \neq (\tilde{f}_2, n)$ . Then their union is signified as  $(\tilde{f}_1, n) \sqcup (\tilde{f}_2, n) = (\tilde{f}_3, n)$  & is defined as  $(\tilde{f}_3, n) = \left\{ \begin{matrix} (n, x, \mathbb{T}_{\tilde{f}_3(x)}(x), \mathbb{I}_{\tilde{f}_3(x)}(x), \mathbb{F}_{\tilde{f}_3(x)}(x) : x) \\ : n \in \mathfrak{A}^{parameter} \end{matrix} \right.$

Where, 
$$\begin{cases} \mathbb{T}_{\tilde{f}_3(x)}(x) = \max[\mathbb{T}_{\tilde{f}_1(x)}(x), \mathbb{T}_{\tilde{f}_2(x)}(x), \\ \mathbb{I}_{\tilde{f}_3(x)}(x) = \max[\mathbb{I}_{\tilde{f}_1(x)}(x), \mathbb{I}_{\tilde{f}_2(x)}(x), \\ \mathbb{F}_{\tilde{f}_3(x)}(x) = \min[\mathbb{F}_{\tilde{f}_1(x)}(x), \mathbb{F}_{\tilde{f}_2(x)}(x) \end{cases}$$

**Definition 3.2** Let  $(\tilde{f}_1, n) \& (\tilde{f}_2, n)$  be two (NSSS) over the father set  $\langle \mathcal{X} \rangle$  s.t.  $(\tilde{f}_1, n) \neq (\tilde{f}_2, n)$ . Then their intersection is signified as  $(\tilde{f}_1, n) \tilde{\cap} (\tilde{f}_2, n) = (\tilde{f}_3, n)$  & is defined as follows  $(\tilde{f}_3, n) = \{((n, [x, \mathbb{T}_{\tilde{f}_3(x)}(x), \mathbb{I}_{\tilde{f}_3(x)}(x), \mathbb{F}_{\tilde{f}_3(x)}(x) : \langle \mathcal{X} \rangle])) : n \in \mathfrak{A}^{parameter}\}$  where  $\mathbb{T}_{\tilde{f}_3(x)}(x) = \min[\mathbb{T}_{\tilde{f}_1(x)}(x), \mathbb{T}_{\tilde{f}_2(x)}(x), \mathbb{I}_{\tilde{f}_3(x)}(x) = \min[\mathbb{I}_{\tilde{f}_1(x)}(x), \mathbb{I}_{\tilde{f}_2(x)}(x), \mathbb{F}_{\tilde{f}_3(x)}(x) = \max[\mathbb{F}_{\tilde{f}_1(x)}(x), \mathbb{F}_{\tilde{f}_2(x)}(x)$

**Definition 3.3** NSSet  $(\tilde{f}, n)$  be a (NSS) over the father set  $\langle \mathcal{X} \rangle$  is said to be a null neutrosophic soft set If  $\mathbb{T}_{\tilde{f}(x)}(x) = 0, \mathbb{I}_{\tilde{f}(x)}(x) = 0, \mathbb{F}_{\tilde{f}(x)}(x) = 1; , \forall e \in n \& \forall x \in \langle \mathcal{X} \rangle$ . it is signifies as  $0_{(\langle \mathcal{X} \rangle, n)}$ .

**Definition 3.4** NSS  $(\tilde{f}, n)$  over the father set  $\langle \mathcal{X} \rangle$  It is said to be an absolute neutrosophical softness if  $\mathbb{T}_{\tilde{f}(x)}(x) = 1, \mathbb{I}_{\tilde{f}(x)}(x) = 1, \mathbb{F}_{\tilde{f}(x)}(x) = 0; , \forall e \in n \& \forall x \in \langle \mathcal{X} \rangle$ .

It is signifies as  $1_{(\langle \mathcal{X} \rangle, n)}$ . clearly,  $0_{(\langle \mathcal{X} \rangle, n)}^c = 1_{(\langle \mathcal{X} \rangle, n)} \& 1_{(\langle \mathcal{X} \rangle, n)}^c = 0_{(\langle \mathcal{X} \rangle, n)}$ .

**Definition 3.5** Let  $NSS(\langle \widetilde{X} \rangle, \mathfrak{A}^{parameter})$  be the family of all NS soft sets over the father set  $\langle \widetilde{X} \rangle$  and  $\tau \subset NSS(\langle \widetilde{X} \rangle, \mathfrak{A}^{parameter})$ . then  $\tau$  is said to be a NS soft topology on  $\langle \widetilde{X} \rangle$  if:

- (1).  $0_{(\langle \widetilde{X} \rangle, n)}, 1_{(\langle \widetilde{X} \rangle, n)} \in \tau$ ,
- (2). The union of any number of NS soft sets in  $\tau$  belongs to  $\tau$ ,
- (3). The intersection of a finite number of (NS) soft sets in  $\tau$  belongs to  $\tau$ . Then  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is said to be a (NSTS) over  $\langle \widetilde{X} \rangle$ . Each member of  $\tau$  is said to be a NS soft open set.

**Definition 3.6** Let  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a NSTS over  $\langle \widetilde{X} \rangle$  &  $(\mathfrak{F}, \mathfrak{A}^{parameter})$  be a NS set over  $\langle \widetilde{X} \rangle$ . Then  $(\mathfrak{F}, \mathfrak{A}^{parameter})$  is supposed to be a NS closed set iff its complement is a NS open set.

**Definition 3.7** Let NS be the family of all NS over father set  $\langle \widetilde{X} \rangle$  and  $x \in \langle \widetilde{X} \rangle$  the NS  $x_{(a,b,c)}$  is supposed to be a N point, for  $0 < a, b, c \leq 1$  and is defined as follows:  $x_{(a,b,c)}^{(\psi)} = \{(a, b, c) \text{ provided } \psi = x\}$ . It is obvious that every (NS) is actually the union of its N points  $\{(0,0,1) \text{ provided } \psi \neq x\}$ .

**Example 3.8** Suppose that  $\langle \widetilde{X} \rangle = \{x_1, x_2\}$  then N set  $A = \{(x_1, 0.1, 0.3, 0.5), (x_2, 0.5, 0.4, 0.7)\}$  is the union of N points  $x_{1(0.1,0.3,0.5)}$  &  $x_{2(0.5,0.4,0.7)}$ . Now we define the concept of NS points for NS sets.

**Definition 3.9** Let  $NSS(\langle \widetilde{X} \rangle)$  be the family of all N soft sets over the father set  $\langle \widetilde{X} \rangle$ . Then  $NSS(x_{(a,b,c)})^e$  is called a NS point, for every  $x \in \langle \widetilde{X} \rangle, 0 < \{a, b, c\} \leq 1, e \propto \mathfrak{A}^{parameter}$ , and is defined as follows:

$$x_{(a,b,c)}^{e(\psi)} = \begin{cases} (a, b, c) \text{ provided } e = e \wedge \psi = x \\ (a, b, c) \text{ provided } e \neq e \wedge \psi \neq x \end{cases}$$

**Definition 3.10** Suppose that the father set  $\langle \widetilde{X} \rangle$  is assumed to be  $\langle \widetilde{X} \rangle = \{x_1, x_2\}$  & the set of conditions by  $\mathfrak{A}^{parameter} = \{e_1, e_2\}$ . Let us consider  $NSS(\mathfrak{F}, \mathfrak{A}^{parameter})$  over the father set  $\langle \widetilde{X} \rangle$  as follows:  $(\mathfrak{F}, \mathfrak{A}^{parameter}) = \{e_1 = \{(x_1, 0.3, 0.7, 0.6), (x_2, 0.4, 0.3, 0.8)\}, e_2 = \{(x_1, 0.4, 0.6, 0.8), (x_2, 0.3, 0.7, 0.2)\}\}$ . It is clear that

$$(\mathfrak{F}, \mathfrak{A}^{parameter}) \text{ is the union of its NS points } \begin{cases} x^{e_1}_{1(0.3,0.7,0.6)}, \\ x^{e_2}_{1(0.4,0.6,0.8)}, \\ x^{e_1}_{2(0.4,0.3,0.8)} \\ x^{e_2}_{2(0.3,0.7,0.2)}. \end{cases}$$

$$\text{Where, } \begin{cases} x^{e_1}_{1(0.3,0.7,0.6)} = \left[ \begin{matrix} e_1 = \{(x_1, 0.3, 0.7, 0.6), (x_2, 0, 0, 1)\} \\ e_2 = \{(x_1, 0, 0, 1), (x_2, 0, 0, 1)\} \end{matrix} \right] \\ x^{e_2}_{1(0.4,0.6,0.8)} = \left[ \begin{matrix} e_1 = \{(x_1, 0, 0, 1), (x_2, 0, 0, 1)\} \\ e_2 = \{(x_1, 0.4, 0.6, 0.8), (x_2, 0, 0, 1)\} \end{matrix} \right] \\ x^{e_1}_{2(0.3,0.7,0.6)} = \left[ \begin{matrix} e_1 = \{(x_1, 0, 0, 1), (x_2, 0.4, 0.3, 0.8)\} \\ e_2 = \{(x_1, 0, 0, 1), (x_2, 0, 0, 1)\} \end{matrix} \right] \\ x^{e_2}_{2(0.3,0.7,0.2)} = \left[ \begin{matrix} e_1 = \{(x_1, 0, 0, 1), (x_2, 0, 0, 1)\} \\ e_2 = \{(x_1, 0, 0, 1), (0.3, 0.7, 0.2)\} \end{matrix} \right] \end{cases}$$

**Definition 3.11** Let  $(\tilde{f}, \mathfrak{A}^{parameter})$  be a NSS over the father set  $\langle \tilde{X} \rangle$ . we say that  $x^e_{(a,b,c)} \in (\tilde{f}, \mathfrak{A}^{parameter})$  read as belonging to the NSS  $(\tilde{f}, \mathfrak{A}^{parameter})$  whenever  $a \leq T_{\tilde{f}(x)}(x), b \leq I_{\tilde{f}(x)}(x), c \geq F_{\tilde{f}(x)}(x)$ .

**Definition 3.12** Let  $x^e_{(a,b,c)}$  and  $x^{e'}_{(a',b',c')}$  be two NSpoints. For the NS points Over father set  $\langle \tilde{X} \rangle$ , we say that the NSpoints are distinct points  $x^e_{(a,b,c)} \cap x^{e'}_{(a',b',c')} = 0_{(\langle \tilde{X} \rangle, \mathfrak{A}^{parameter})}$ . It is clear that  $x^e_{(a,b,c)}$  and  $x^{e'}_{(a',b',c')}$  are distinct NS points if and only if  $x > y$  or  $x < y$  or  $e' > e$  or  $e' < e$ .

#### 4. Neutrosophic Soft b-Separation Axioms

In this phase we define generalized neutrosophic soft separation axioms.

**Definition 4.1** Let  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a NSTS over  $\langle \tilde{X} \rangle$  &  $(\tilde{f}, \mathfrak{A}^{parameter})$  be a neutrosophic soft set over  $\langle \tilde{X} \rangle$ . Then  $(\tilde{f}, \mathfrak{A}^{parameter})$  is supposed to be a NS b-open if  $(\tilde{f}, \mathfrak{A}^{parameter}) \subseteq cl(int((\tilde{f}, \mathfrak{A}^{parameter}))) \cup int(cl((\tilde{f}, \mathfrak{A}^{parameter})))$  and NS b-close if  $(\tilde{f}, \mathfrak{A}^{parameter}) \supseteq int(cl((\tilde{f}, \mathfrak{A}^{parameter}))) \cap cl(int((\tilde{f}, \mathfrak{A}^{parameter})))$ .

**Definition 4.2** Let  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a NSTS over  $\langle \tilde{X} \rangle$ , and  $x^e_{(a,b,c)} > y^{e'}_{(a',b',c')}$  or  $x^e_{(a,b,c)} < y^{e'}_{(a',b',c')}$  are NSPoints. If there exist NSb open sets  $(\tilde{f}, \mathfrak{A}^{parameter})$  &  $(\tilde{g}, \mathfrak{A}^{parameter})$  such that  $x^e_{(a,b,c)} \in (\tilde{f}, \mathfrak{A}^{parameter}), x^e_{(a,b,c)} \cap (\tilde{g}, \mathfrak{A}^{parameter}) = 0_{(\langle \tilde{X} \rangle, \mathfrak{A}^{parameter})}$  or  $y^{e'}_{(a',b',c')} \in (\tilde{g}, \mathfrak{A}^{parameter}), y^{e'}_{(a',b',c')} \cap (\tilde{f}, \mathfrak{A}^{parameter}) = 0_{(\langle \tilde{X} \rangle, \mathfrak{A}^{parameter})}$ , Then  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is called a NSb<sub>0</sub>

**Definition 4.3** Let  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a NSTS over  $\langle \tilde{X} \rangle$ , and  $x^e_{(a,b,c)} > y^{e'}_{(a',b',c')}$  or  $x^e_{(a,b,c)} < y^{e'}_{(a',b',c')}$  are NS points. If there exist NSb open sets  $(\tilde{f}, \mathfrak{A}^{parameter})$  &  $(\tilde{g}, \mathfrak{A}^{parameter})$  :

$$x^e_{(a,b,c)} \in (\tilde{f}, \mathfrak{A}^{parameter}), x^e_{(a,b,c)} \cap (\tilde{g}, \mathfrak{A}^{parameter}) = 0_{(\langle \tilde{X} \rangle, \mathfrak{A}^{parameter})} \text{ OR } y^{e'}_{(a',b',c')} \in (\tilde{g}, \mathfrak{A}^{parameter}), y^{e'}_{(a',b',c')} \cap (\tilde{f}, \mathfrak{A}^{parameter}) = 0_{(\langle \tilde{X} \rangle, \mathfrak{A}^{parameter})}$$

Then  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is called a NSb<sub>1</sub>.

**Definition 4.4** Let  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a NSTS over  $\langle \tilde{X} \rangle$ , and  $x^e_{(a,b,c)} > y^{e'}_{(a',b',c')}$

$x^e_{(a,b,c)} < y^e_{(a',b',c')}$  are NSpoints. if  $\exists$  NSb open sets  $(\tilde{f}, \mathfrak{A}^{parameter})$  &  $(\tilde{g}, \mathfrak{A}^{parameter})$  s. t.

$x^e_{(a,b,c)} \in (\tilde{f}, \mathfrak{A}^{parameter})$  &  $y^e_{(a',b',c')} \in (\tilde{g}, \mathfrak{A}^{parameter})$  &  $(\tilde{f}, \mathfrak{A}^{parameter}) \cap (\tilde{g}, \mathfrak{A}^{parameter}) =$

$0_{(\tilde{X}, \mathfrak{A}^{parameter})}$ , Then  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  is called a NSb<sub>2</sub> space.

**Example 4.5** Suppose that the father set  $\tilde{X}$  is assumed to be

$\tilde{X} = \{x_1, x_2\}$  & the set of conditions by  $\mathfrak{A}^{parameter} = \{e_1, e_2\}$ . Let us consider NS set  $(\tilde{f}, \mathfrak{A}^{parameter})$  over the father set  $\tilde{X}$  &  $x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{1(0.2,0.5,0.6)}, x^{e_1}_{2(0.3,0.3,0.5)}$  &  $x^{e_1}_{2(0.4,0.4,0.4)}$  be NS points. Then the family  $\tau = \{0_{(\tilde{X}, \mathfrak{A}^{parameter})}, 1_{(\tilde{X}, \mathfrak{A}^{parameter})}, (\tilde{f}_1, \mathfrak{A}^{parameter}), (\tilde{f}_2, \mathfrak{A}^{parameter}),$

$(\tilde{f}_3, \mathfrak{A}^{parameter}), (\tilde{f}_4, \mathfrak{A}^{parameter}), (\tilde{f}_5, \mathfrak{A}^{parameter}), (\tilde{f}_6, \mathfrak{A}^{parameter}), (\tilde{f}_7, \mathfrak{A}^{parameter}), (\tilde{f}_8, \mathfrak{A}^{parameter})\}$ ,

where  $(\tilde{f}_1, \mathfrak{A}^{parameter}) = x^{e_1}_{1(0.1,0.4,0.7)}, (\tilde{f}_2, \mathfrak{A}^{parameter}) = x^{e_2}_{1(0.2,0.5,0.6)}, (\tilde{f}_3, \mathfrak{A}^{parameter}) =$

$x^{e_1}_{2(0.3,0.3,0.5)}, (\tilde{f}_4, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup (\tilde{f}_2, \mathfrak{A}^{parameter}), (\tilde{f}_5, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup$

$(\tilde{f}_3, \mathfrak{A}^{parameter}), (\tilde{f}_6, \mathfrak{A}^{parameter}) = (\tilde{f}_2, \mathfrak{A}^{parameter}) \cup (\tilde{f}_3, \mathfrak{A}^{parameter}), (\tilde{f}_7, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup$

$(\tilde{f}_2, \mathfrak{A}^{parameter}) \cup (\tilde{f}_3, \mathfrak{A}^{parameter}),$

$(\tilde{f}_8, \mathfrak{A}^{parameter}) = \{x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{1(0.2,0.5,0.6)}, x^{e_1}_{2(0.3,0.3,0.5)}, x^{e_2}_{2(0.4,0.4,0.4)}\}$  is a NSTS over the father set

$\tilde{X}$ . Thus  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a NSTS over the father set  $\tilde{X}$ . Also  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  is NSb<sub>0</sub>

structure but it is not NSb<sub>1</sub> because for NSpoints  $x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{2(0.4,0.4,0.4)}, (\tilde{X}, \tau, \mathfrak{A}^{parameter})$  not NSb<sub>1</sub>.

**Example 4.6** Suppose that the father set  $\tilde{X}$  is assumed to be

$\tilde{X} = \{x_1, x_2\}$  & the set of conditions by  $\mathfrak{A}^{parameter} = \{e_1, e_2\}$ . Let us consider NS set  $(\tilde{f}, \mathfrak{A}^{parameter})$  over the father

set  $\tilde{X}$  &  $x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{1(0.2,0.5,0.6)}, x^{e_1}_{2(0.3,0.3,0.5)}$  &  $x^{e_1}_{2(0.4,0.4,0.4)}$  be NSpoints. Then the family  $\tau =$

$\{0_{(\tilde{X}, \mathfrak{A}^{parameter})}, 1_{(\tilde{X}, \mathfrak{A}^{parameter})}, (\tilde{f}_1, \mathfrak{A}^{parameter}), (\tilde{f}_2, \mathfrak{A}^{parameter}), (\tilde{f}_3, \mathfrak{A}^{parameter}), (\tilde{f}_4, \mathfrak{A}^{parameter}), (\tilde{f}_5, \mathfrak{A}^{parameter}), (\tilde{f}_6, \mathfrak{A}^{parameter})$

$, (\tilde{f}_7, \mathfrak{A}^{parameter}), (\tilde{f}_8, \mathfrak{A}^{parameter}), \dots \dots \dots (\tilde{f}_{15}, \mathfrak{A}^{parameter})\}$ , where  $(\tilde{f}_1, \mathfrak{A}^{parameter}) = x^{e_1}_{1(0.1,0.4,0.7)}, (\tilde{f}_2, \mathfrak{A}^{parameter}) =$

$x^{e_2}_{1(0.2,0.5,0.6)}, (\tilde{f}_3, \mathfrak{A}^{parameter}) = x^{e_1}_{2(0.3,0.3,0.5)}, (\tilde{f}_4, \mathfrak{A}^{parameter}) = x^{e_2}_{2(0.4,0.4,0.4)}, (\tilde{f}_5, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup$

$(\tilde{f}_2, \mathfrak{A}^{parameter}), (\tilde{f}_6, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup (\tilde{f}_3, \mathfrak{A}^{parameter}), (\tilde{f}_7, \mathfrak{A}^{parameter}) = (\tilde{f}_2, \mathfrak{A}^{parameter}) \cup$

$(\tilde{f}_4, \mathfrak{A}^{parameter}), (\tilde{f}_8, \mathfrak{A}^{parameter}) = (\tilde{f}_2, \mathfrak{A}^{parameter}) \cup (\tilde{f}_3, \mathfrak{A}^{parameter}), (\tilde{f}_9, \mathfrak{A}^{parameter}) = (\tilde{f}_2, \mathfrak{A}^{parameter}) \cup$

$(\tilde{f}_4, \mathfrak{A}^{parameter}), (\tilde{f}_{10}, \mathfrak{A}^{parameter}) = (\tilde{f}_3, \mathfrak{A}^{parameter}) \cup (\tilde{f}_4, \mathfrak{A}^{parameter}), (\tilde{f}_{11}, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup$

$(\tilde{f}_2, \mathfrak{A}^{parameter}) \cup (\tilde{f}_3, \mathfrak{A}^{parameter}), (\tilde{f}_{12}, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup (\tilde{f}_2, \mathfrak{A}^{parameter}) \cup$



$(\tilde{f}_4, \mathfrak{A}^{parameter}), (\tilde{f}_{13}, \mathfrak{A}^{parameter}) = (\tilde{f}_2, \mathfrak{A}^{parameter}) \cup (\tilde{f}_3, \mathfrak{A}^{parameter}) \cup (\tilde{f}_4, \mathfrak{A}^{parameter}), (\tilde{f}_{14}, \mathfrak{A}^{parameter}) = (\tilde{f}_1, \mathfrak{A}^{parameter}) \cup (\tilde{f}_3, \mathfrak{A}^{parameter}) \cup (\tilde{f}_4, \mathfrak{A}^{parameter}) \cup (\tilde{f}_{15}, \mathfrak{A}^{parameter}) = \{x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{1(0.2,0.5,0.6)}, x^{e_1}_{2(0.3,0.3,0.5)}, x^{e_2}_{2(0.4,0.4,0.4)}\}$  is a NSTS over the father set  $(\tilde{X})$ . Thus  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a (NSTS) over the father set  $(\tilde{X})$ . Also  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  is  $NSb_1$  structure.  $x^{e_1}_{1(0.1,0.4,0.7)}, x^{e_2}_{2(0.4,0.4,0.4)}, (\tilde{X}, \tau, \mathfrak{A}^{parameter})$  not  $NSb_2$ .

**Theorem 4.7** Let  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a (NSTS) over the father set  $(\tilde{X})$ . Then  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a  $NSb_1$  structure iff each NSpoint is a NS b-closed set.

**Proof.** Let  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a NSTS over the father set  $(\tilde{X})$ .  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  be an arbitrary NSpoint. We establish  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  is a N soft b-open set. Let  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$ . Then either  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) > (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) < (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) >> (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) << (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$ .

This means that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  &  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  are mutually exclusive NSpoints.

Thus  $x > y$  or  $x < y$  or  $e' > e$  or  $e' < e$  or  $x >> y$  or  $x << y$  or  $e' >> e$  or  $e' << e$ . Since  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a  $NSb_1$  structure,  $\exists$  a NSb-open set  $(\tilde{\mathcal{G}}, \mathfrak{A}^{parameter})$  so that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in (\tilde{\mathcal{G}}, \mathfrak{A}^{parameter})$  &  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (\tilde{\mathcal{G}}, \mathfrak{A}^{parameter}) = 0_{((\tilde{X}), \mathfrak{A}^{parameter})}$ . Since,  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (\tilde{\mathcal{G}}, \mathfrak{A}^{parameter}) = 0_{((\tilde{X}), \mathfrak{A}^{parameter})}$ . So  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in (\tilde{\mathcal{G}}, \mathfrak{A}^{parameter}) \subset (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$ . Thus  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  is a NSb-open set, i.e.,  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  is a NSb-closed set. Suppose that each NSpoint  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  is a NSb-closed set. Then  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})^c$  is a NSb-open set. Let  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) = 0_{((\tilde{X}), \mathfrak{A}^{parameter})}$ . Thus  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})^c$  &  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})^c = 0_{((\tilde{X}), \mathfrak{A}^{parameter})}$ . So  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a  $NSb-b_1$  space.

**Theorem 4.8** Let  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  be a (NSTS) over the father set  $(\tilde{X})$ . Then  $(\tilde{X}, \tau, \mathfrak{A}^{parameter})$  is  $NSb_2$  space iff for distinct NS points  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  &  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$ , there exists a NSb-open set  $(\tilde{f}, \mathfrak{A}^{parameter})$  containing  $\exists$  but not  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  s. t.  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin \overline{(\tilde{f}, \mathfrak{A}^{parameter})}$ .

**Proof** Let  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) > (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  be two NSpoints in NSbT<sub>2</sub> space. Then  $\exists$  disjoint NSopen sets  $(\tilde{f}, \mathfrak{A}^{parameter}) \& (\tilde{g}, \mathfrak{A}^{parameter})$  s.t.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\tilde{f}, \mathfrak{A}^{parameter}) \& (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in (\tilde{g}, \mathfrak{A}^{parameter})$ . Since  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) = 0_{(\tilde{x}, \mathfrak{A}^{parameter})} \& (\tilde{f}, \mathfrak{A}^{parameter}) \cap (\tilde{g}, \mathfrak{A}^{parameter}) = 0_{(\tilde{x}, \mathfrak{A}^{parameter})} \cdot (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin (\tilde{f}, \mathfrak{A}^{parameter}) \Rightarrow (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin \overline{(\tilde{f}, \mathfrak{A}^{parameter})}$ . Next suppose that,  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) > (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$ ,  $\exists$  a NSb open set  $(\tilde{f}, \mathfrak{A}^{parameter})$  containing  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  but not  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  s.t.  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin \overline{(\tilde{f}, \mathfrak{A}^{parameter})}^c$  that is  $(\tilde{f}, \mathfrak{A}^{parameter}) \& \overline{(\tilde{f}, \mathfrak{A}^{parameter})}^c$  are mutually exclusive NSbopen sets supposing  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \& (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  in turn.

**Theorem 4.9** Let  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a NSTS over the father set  $\langle \tilde{X} \rangle$ . Then  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is NS b-T<sub>1</sub> space if every NS point  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\tilde{f}, \mathfrak{A}^{parameter}) \in (\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$ . If there exists a NSb open set  $(\tilde{g}, \mathfrak{A}^{parameter})$  s.t.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\tilde{g}, \mathfrak{A}^{parameter}) \subset \overline{(\tilde{g}, \mathfrak{A}^{parameter})} \subset (\tilde{f}, \mathfrak{A}^{parameter})$ , Then  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  a NSb<sub>2</sub> space.

**Proof.** Suppose  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) = 0_{(\tilde{x}, \mathfrak{A}^{parameter})}$ . Since  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is NSb<sub>1</sub> space.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \& (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  are NS b close sets in  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$ . Then  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in ((y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}))^c \in (\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$ . Thus  $\exists$  a NS b open set  $(\tilde{g}, \mathfrak{A}^{parameter}) \in (\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  s.t.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\tilde{g}, \mathfrak{A}^{parameter}) \subset \overline{(\tilde{g}, \mathfrak{A}^{parameter})} \subset ((y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}))^c$ . So we have  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in (\tilde{g}, \mathfrak{A}^{parameter}) \& (\tilde{g}, \mathfrak{A}^{parameter}) \cap ((\tilde{g}, \mathfrak{A}^{parameter}))^c = 0_{(\tilde{x}, \mathfrak{A}^{parameter})}$ , i. e.  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is a NS soft NSb<sub>2</sub> space.

### 5. Characterization of other NS b-Separation Axioms

**Definition 5.1.** Let  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a NSTS over the father set  $\langle \tilde{X} \rangle$ .  $(\tilde{f}, \mathfrak{A}^{parameter})$  be a NSb closed set and  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (\tilde{f}, \mathfrak{A}^{parameter}) = 0_{(\tilde{x}, \mathfrak{A}^{parameter})}$ . If  $\exists$  NS b-open sets  $(\tilde{g}_1, \mathfrak{A}^{parameter}) \& (\tilde{g}_2, \mathfrak{A}^{parameter})$  s.t.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\tilde{g}_1, \mathfrak{A}^{parameter})$ ,  $(\tilde{f}, \mathfrak{A}^{parameter}) \subset (\tilde{g}_2, \mathfrak{A}^{parameter}) \& (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (\tilde{g}_1, \mathfrak{A}^{parameter}) = 0_{(\tilde{x}, \mathfrak{A}^{parameter})}$ , then  $(\langle \tilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is called

a NS b-regular space.  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is said to be *NSb<sub>3</sub>space*, if is both a NSregular and *NSb<sub>1</sub>space*.

**Theorem 5.2.** Let  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a *NSTSover* the father set  $\langle \widetilde{X} \rangle$ .  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is soft b-*T<sub>3</sub> space* iff for every  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{f}, \mathfrak{A}^{parameter}), t.e, (\check{g}, \mathfrak{A}^{parameter}) \in (\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  s.t.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{g}, \mathfrak{A}^{parameter}) \subset \overline{(\check{g}, \mathfrak{A}^{parameter})} \subset (\check{f}, \mathfrak{A}^{parameter})$ .

**Proof.** Let  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is *NSb<sub>3</sub>space*

$\&(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{f}, \mathfrak{A}^{parameter}) \in (\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$ . Since  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is *NSNST<sub>3</sub> space* for the NS point  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \&$  b-closed set  $(\check{f}, \mathfrak{A}^{parameter})^c, \exists (\check{g}_1, \mathfrak{A}^{parameter}) \& (\check{g}_2, \mathfrak{A}^{parameter})$  s.t.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{g}_1, \mathfrak{A}^{parameter}), (\check{f}, \mathfrak{A}^{parameter})^c \subset (\check{g}_2, \mathfrak{A}^{parameter}) \& (\check{g}_1, \mathfrak{A}^{parameter}) \cap (\check{g}_2, \mathfrak{A}^{parameter}) = 0_{(\langle \widetilde{X} \rangle, \mathfrak{A}^{parameter})}$ . Then we have  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{g}_1, \mathfrak{A}^{parameter}) \subset (\check{g}_2, \mathfrak{A}^{parameter})^c \subset (\check{f}, \mathfrak{A}^{parameter})$ . Since  $(\check{g}_2, \mathfrak{A}^{parameter})^c$  NSb closed set  $\overline{(\check{g}_1, \mathfrak{A}^{parameter})} \subset (\check{g}_2, \mathfrak{A}^{parameter})^c$ .

Conversely, let  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \cap (\check{H}, \mathfrak{A}^{parameter}) = 0_{(\langle \widetilde{X} \rangle, \mathfrak{A}^{parameter})} \& (\check{H}, \mathfrak{A}^{parameter})$  be a NSb closed set.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{H}, \mathfrak{A}^{parameter})^c$  & from the condition of the theorem, we have  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{g}, \mathfrak{A}^{parameter}) \subset \overline{(\check{g}, \mathfrak{A}^{parameter})} \subset (\check{H}, \mathfrak{A}^{parameter})^c$ . Thus  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\check{g}, \mathfrak{A}^{parameter}), (\check{H}, \mathfrak{A}^{parameter}) \subset \overline{(\check{g}, \mathfrak{A}^{parameter})} \& (\check{g}, \mathfrak{A}^{parameter}) \cap \overline{(\check{g}, \mathfrak{A}^{parameter})}^c = 0_{(\langle \widetilde{X} \rangle, \mathfrak{A}^{parameter})}$ . So  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is *NSb<sub>3</sub>space*.

**Definition 5.3.** Let  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a *NSTSover* the father set  $\langle \widetilde{X} \rangle$ . This space is a *NSnormal space*, if for every pair of disjoint NSb closed sets  $(\check{f}_1, \mathfrak{A}^{parameter}) \& (\check{f}_2, \mathfrak{A}^{parameter}), \exists$  disjoint NSb open sets  $(\check{g}_1, \mathfrak{A}^{parameter}) \& (\check{g}_2, \mathfrak{A}^{parameter})$  s.t.  $(\check{f}_1, \mathfrak{A}^{parameter}) \subset (\check{g}_1, \mathfrak{A}^{parameter}) \& (\check{f}_2, \mathfrak{A}^{parameter}) \subset (\check{g}_2, \mathfrak{A}^{parameter})$ .  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  is said to be a *NSb<sub>4</sub>space* if it is both a *NSnormal* and *NSb<sub>1</sub>space*.

**Theorem 5.4.** Let  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a *NSTSover* the father set  $\langle \widetilde{X} \rangle$ . This space is a *NS bT<sub>4</sub> space*  $\Leftrightarrow$ , for each NS b closed set  $(\check{f}, \mathfrak{A}^{parameter})$  and NSb open set  $(\check{g}, \mathfrak{A}^{parameter})$  with  $(\check{f}, \mathfrak{A}^{parameter}) \subset (\check{g}, \mathfrak{A}^{parameter}), \exists$  a NSb open set  $(\check{D}, \mathfrak{A}^{parameter})$  s.t.  $(\check{f}, \mathfrak{A}^{parameter}) \subset (\check{D}, \mathfrak{A}^{parameter}) \subset \overline{(\check{D}, \mathfrak{A}^{parameter})} \subset (\check{g}, \mathfrak{A}^{parameter})$ .

**Proof.** Let  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a *NS b<sub>4</sub> over* the father set  $\langle \widetilde{X} \rangle$ . & let  $(\check{f}, \mathfrak{A}^{parameter}) \subset (\check{g}, \mathfrak{A}^{parameter})$ . Then  $(\check{g}, \mathfrak{A}^{parameter})^c$  is a NSb closed set and  $(\check{f}, \mathfrak{A}^{parameter}) \cap (\check{g}, \mathfrak{A}^{parameter}) = 0_{(\langle \widetilde{X} \rangle, \mathfrak{A}^{parameter})}$ . Since  $(\langle \widetilde{X} \rangle, \tau, \mathfrak{A}^{parameter})$  be a *NS b<sub>4</sub> space*,  $\exists$  NS b-open sets  $(\check{D}_1, \mathfrak{A}^{parameter}) \& (\check{D}_2, \mathfrak{A}^{parameter})$  s.t.  $(\check{f}, \mathfrak{A}^{parameter}) \subset (\check{D}_1, \mathfrak{A}^{parameter}), (\check{g}, \mathfrak{A}^{parameter})^c \subset (\check{D}_2, \mathfrak{A}^{parameter}) \& (\check{D}_1, \mathfrak{A}^{parameter}) \cap (\check{D}_2, \mathfrak{A}^{parameter}) = 0_{(\langle \widetilde{X} \rangle, \mathfrak{A}^{parameter})}$ . Thus  $(\check{f}, \mathfrak{A}^{parameter}) \subset (\check{D}_1, \mathfrak{A}^{parameter}) \subset (\check{D}_2, \mathfrak{A}^{parameter})^c \subset (\check{g}, \mathfrak{A}^{parameter}), (\check{D}_2, \mathfrak{A}^{parameter})^c$  is a NS b closed set and  $\overline{(\check{D}_1, \mathfrak{A}^{parameter})} \subset (\check{D}_2, \mathfrak{A}^{parameter})^c$ . So  $(\check{f}, \mathfrak{A}^{parameter}) \subset (\check{D}_1, \mathfrak{A}^{parameter}) \subset \overline{(\check{D}_1, \mathfrak{A}^{parameter})} \subset (\check{g}, \mathfrak{A}^{parameter})$ .

Conversely, let  $(\tilde{f}_1, \mathfrak{A}^{parameter}) \& (\tilde{f}_2, \mathfrak{A}^{parameter})$  be two disjoint NSb closed sets. Then  $(\tilde{f}_1, \mathfrak{A}^{parameter}) \subset (\tilde{f}_2, \mathfrak{A}^{parameter})^c$ . From the condition of theorem, there exists a NS b open set  $(\tilde{\mathcal{D}}, \mathfrak{A}^{parameter})$  s. t.  $(\tilde{f}_1, \mathfrak{A}^{parameter}) \subset (\tilde{\mathcal{D}}, \mathfrak{A}^{parameter}) \subset \overline{(\tilde{\mathcal{D}}, \mathfrak{A}^{parameter})} \subset (\tilde{f}_2, \mathfrak{A}^{parameter})^c$ . Thus  $(\tilde{\mathcal{D}}, \mathfrak{A}^{parameter}) \& (\overline{(\tilde{\mathcal{D}}, \mathfrak{A}^{parameter})})^c$  are NSb open sets and  $(\tilde{f}_1, \mathfrak{A}^{parameter}) \subset (\tilde{f}_2, \mathfrak{A}^{parameter})^c$ ,  $(\tilde{f}_2, \mathfrak{A}^{parameter}) \subset \overline{(\tilde{\mathcal{D}}, \mathfrak{A}^{parameter})} \& (\overline{(\tilde{\mathcal{D}}, \mathfrak{A}^{parameter})})^c = 0_{(\tilde{\mathcal{X}}, \mathfrak{A}^{parameter})}$ .

$(\tilde{\mathcal{X}}, \tau, \mathfrak{A}^{parameter})$  be a NS  $b_4$  space

### 6. Monotonous behavior of NS b-Separation Axioms

**Theorem 6.1.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be NSST such that it is NSb Hausdorff space and  $\langle Y^{crip}, \mathfrak{B}, \partial \rangle$  be any NSST.

Let  $\langle \mathfrak{f}, \partial \rangle: \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \rightarrow \langle Y^{crip}, \mathfrak{B}, \partial \rangle$  be a soft fuction such that it is soft monotone and continuous. Then  $\langle Y^{crip}, \mathfrak{B}, \partial \rangle$  is also of characteristics of NSb Hausdorffness.

**Proof:** Suppose  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1, (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2 \in \mathcal{X}^{crip}$  such that either  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 > (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2$  or  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 < (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2$ . Since  $\langle \mathfrak{f}, \partial \rangle$  is soft monotone. Let us suppose the monotonically increasing case. So,  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 > \mathfrak{f}(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2$  or  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 < \mathfrak{f}(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2$  implies that  $\mathfrak{f}(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 > \mathfrak{f}(x_2)$  or  $\mathfrak{f}(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 < \mathfrak{f}(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2$  respectively. Suppose

$(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1, (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2 \in Y^{crip}$  such that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 > (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 < (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$ . So,  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 > (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 < (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$  respectively such that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) = \mathfrak{f}(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1, (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2 = \mathfrak{f}(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2$ . Since,  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is

NSb Hausdorff space so there exists mutually disjoint NS b-open sets  $\langle \mathfrak{k}_1, \partial \rangle$  and  $\langle \mathfrak{k}_2, \partial \rangle \in \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \Rightarrow \mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle)$  and  $\mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) \in \langle Y^{crip}, \mathfrak{B}, \partial \rangle$ . We claim that  $\mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle) \tilde{\cap} \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) = 0_{(\tilde{\mathcal{X}}, \widetilde{\mathfrak{A}^{parameter}})}$ . Otherwise

$\mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle) \tilde{\cap} \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) \neq 0_{(\tilde{\mathcal{X}}, \widetilde{\mathfrak{A}^{parameter}})}$ . Suppose  $\exists (t^{e''}_{(a'',b'',c'')}, \mathfrak{A}^{parameter})_1 \in \mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle) \tilde{\cap} \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) \Rightarrow$

$(t^{e''}_{(a'',b'',c'')}, \mathfrak{A}^{parameter})_1 \in \mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle) \& (t^{e''}_{(a'',b'',c'')}, \mathfrak{A}^{parameter})_1 \in \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle),$

$(t^{e''}_{(a'',b'',c'')}, \mathfrak{A}^{parameter})_1 \in \mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle), \mathfrak{f}$  is soft one – one  $\Rightarrow \exists (t^{e''}_{(a'',b'',c'')}, \mathfrak{A}^{parameter})_2 \in \langle \mathfrak{k}_1, \partial \rangle$  s.t.

$(\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_1 = \mathfrak{f}((\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_2), (\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_1 \in \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) \Rightarrow$   
 $\exists (\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_3 \in \langle \mathfrak{k}_2, \partial \rangle$  s.t.  $(\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_1 = \mathfrak{f}((\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_3) \Rightarrow$   
 $\mathfrak{f}((\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_2) = \mathfrak{f}((\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_3)$  Since,  $\mathfrak{f}$  is soft one – one  $\Rightarrow$   
 $(\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_2 = (\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_3 \Rightarrow (\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_2 \in$   
 $\mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle), (\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_2 \in \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) \Rightarrow (\mathfrak{t}^{e//}_{(a//,b//,c//)}, \mathfrak{A}parameter)_2 \in \mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle) \tilde{\cap} \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle)$ . This  
 is contradiction because  $\langle \mathfrak{k}_1, \partial \rangle \tilde{\cap} \langle \mathfrak{k}_2, \partial \rangle = 0_{(\tilde{\mathfrak{X}}, \mathfrak{A}parameter)}$ . so,  $\mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle) \tilde{\cap} \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) =$

$$0_{(\tilde{\mathfrak{X}}, \mathfrak{A}parameter)}. \text{ Finally, } \begin{cases} (x^e_{(a,b,c)}, \mathfrak{A}parameter)_1 > (x^e_{(a,b,c)}, \mathfrak{A}parameter)_2 \text{ or} \\ (x^e_{(a,b,c)}, \mathfrak{A}parameter)_1 < (x^e_{(a,b,c)}, \mathfrak{A}parameter)_2 \Rightarrow \\ (x^e_{(a,b,c)}, \mathfrak{A}parameter)_1 \neq \mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_2) \quad \mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_1) > \\ \text{or } \mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_1) < \mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_2) \\ \Rightarrow \mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_1) \neq \mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_2) \end{cases}$$

Given a pair of points

$(\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_1, (\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_2 \in Y^{crip} \ni (\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_1 \neq$   
 $(\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_2$  We are able to find out mutually exclusive NSb open sets  $\mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle), \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle) \in$   
 $\langle Y^{crip}, \mathfrak{F}, \partial \rangle$  s.t.  $(\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_1 \in \mathfrak{f}(\langle \mathfrak{k}_1, \partial \rangle), (\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_2 \in \mathfrak{f}(\langle \mathfrak{k}_2, \partial \rangle)$ . this proves that  
 $\langle Y^{crip}, \mathfrak{F}, \partial \rangle$  is NSb Hausdorff space.

**Theorem 6.2.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  be NSST and  $\langle Y^{crip}, \mathfrak{F}, \partial \rangle$  be an-other NSST which satisfies one more condition of NSb Hausdorffness. Let  $\langle \mathfrak{f}, \partial \rangle : \langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle \rightarrow \langle Y^{crip}, \mathfrak{F}, \partial \rangle$  be a soft fuction s.t. it is soft monotone and continuous. Then  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  is also of characteristics of NSb Hausdorffness.

**Proof:** Suppose  $(x^e_{(a,b,c)}, \mathfrak{A}parameter)_1, (x^e_{(a,b,c)}, \mathfrak{A}parameter)_2 \in \mathcal{X}^{crip}$  such that either  $(x^e_{(a,b,c)}, \mathfrak{A}parameter)_1 >$   
 $(x^e_{(a,b,c)}, \mathfrak{A}parameter)_2$  or  $(x^e_{(a,b,c)}, \mathfrak{A}parameter)_1 < (x^e_{(a,b,c)}, \mathfrak{A}parameter)_2$  Let us suppose the NS monotonically  
 increasing case. So,  $(x^e_{(a,b,c)}, \mathfrak{A}parameter)_1 > (x^e_{(a,b,c)}, \mathfrak{A}parameter)_2$  or  $(x^e_{(a,b,c)}, \mathfrak{A}parameter)_1 <$   
 $(x^e_{(a,b,c)}, \mathfrak{A}parameter)_2$  implies that  $\mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_1) >$   
 $\mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_2)$  or  $\mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_1) <$   
 $\mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}parameter)_2)$  respectively. Suppose  $(\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_1, (\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_2 \in Y^{crip}$   
 such that  $(\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_1 > (\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_2$  or  $(\mathfrak{y}^{e/}_{(a',b',c')}, \mathfrak{A}parameter)_1 <$

$(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$ . So,  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 > (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 <$   
 $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$  respectively such that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 =$   
 $\mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1), (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2 = \mathfrak{f}((x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2)$  s. t.  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 =$   
 $\mathfrak{f}^{-1}(y_1)$  and  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2 =$   
 $\mathfrak{f}^{-1}((y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2)$ . since  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1, (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2 \in Y^{crip}$  but  $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$   
 is NSb Hausdorff space. So according to definition  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 >$   
 $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 < (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2$ . There definitely exists NS b-  
 open sets  $\langle \mathfrak{k}_1, \partial \rangle$  and  $\langle \mathfrak{k}_2, \partial \rangle \in \langle Y^{crip}, \mathfrak{I}, \partial \rangle$  such that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 \in \langle \mathfrak{k}_1, \partial \rangle$  and  
 $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2 \in \langle \mathfrak{k}_2, \partial \rangle$  and these two NS b-open sets are guaranteedly mutually exclusive because  
 the possibility of one rules out the possibility of other. Since  $\mathfrak{f}^{-1}(\langle \mathfrak{k}_1, \partial \rangle)$  and  $\mathfrak{f}^{-1}(\langle \mathfrak{k}_2, \partial \rangle)$  are NS open in  
 $\langle X^{crip}, \mathfrak{I}, \partial \rangle$ . Now,  $\mathfrak{f}^{-1}(\langle \mathfrak{k}_1, \partial \rangle) \tilde{\cap} \mathfrak{f}^{-1}(\langle \mathfrak{k}_2, \partial \rangle) = \mathfrak{f}^{-1}(\langle \mathfrak{k}_1, \partial \rangle \tilde{\cap} \langle \mathfrak{k}_2, \partial \rangle) = \mathfrak{f}^{-1}(\tilde{\emptyset}) =$   
 $0_{(\tilde{X}, \tilde{\partial}^{parameter})}$  and  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1 \in \langle \mathfrak{k}_1, \partial \rangle \Rightarrow \mathfrak{f}^{-1}((y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_1) \in \mathfrak{f}^{-1}(\langle \mathfrak{k}_1, \partial \rangle) \Rightarrow$   
 $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 \in (\langle \mathfrak{k}_1, \partial \rangle), (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2 \in \langle \mathfrak{k}_2, \partial \rangle \Rightarrow \mathfrak{f}^{-1}((y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})_2) \in$   
 $\mathfrak{f}^{-1}(\langle \mathfrak{k}_2, \partial \rangle)$  implies that  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2 \in (\langle \mathfrak{k}_2, \partial \rangle)$ . We see that it has been shown for every pair of distinct  
 points of  $X^{crip}$ , there suppose disjoint NS b-open sets namely,  $\mathfrak{f}^{-1}(\langle \mathfrak{k}_1, \partial \rangle)$  and  $\mathfrak{f}^{-1}(\langle \mathfrak{k}_2, \partial \rangle)$  belong to  
 $\langle X^{crip}, \mathfrak{I}, \partial \rangle$  such that  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_1 \in \mathfrak{f}^{-1}(\langle \mathfrak{k}_1, \partial \rangle)$  and  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_2 \in \mathfrak{f}^{-1}(\langle \mathfrak{k}_2, \partial \rangle)$ .  
 Accordingly, NSST is NS b Hausdorff space.

**Theorem 6.3.** Let  $\langle X^{crip}, \mathfrak{I}, \partial \rangle$  be NSST and  $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$  be an-other NSST. Let  $\mathfrak{f}, \partial: \langle X^{crip}, \mathfrak{I}, \partial \rangle \rightarrow \langle Y^{crip}, \mathfrak{I}, \partial \rangle$   
 be a soft mapping such that it is continuous mapping. Let  $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$  is NSb Hausdorff space Then it is  
 guaranteed that  $\left\{ \left( (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}), (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \right) : \mathfrak{f} \left( (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \right) = \right.$   
 $\left. \mathfrak{f} \left( (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \right) \right\}$  is a NSb closed sub-set of  $\langle X^{crip}, \mathfrak{I}, \partial \rangle \times \langle X^{crip}, \mathfrak{I}, \partial \rangle$ .

**Proof:** Given that  $\langle X^{crip}, \mathfrak{I}, \partial \rangle$  be NSST and  $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$  be an-other NSST. Let  $\mathfrak{f}, \partial: \langle X^{crip}, \mathfrak{I}, \partial \rangle \rightarrow \langle Y^{crip}, \mathfrak{I}, \partial \rangle$   
 be a soft mapping such that it is continuous mapping,  $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$  is NSb Hausdorff space Then we  
 will prove that  $\left\{ \left( (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}), (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \right) : \mathfrak{f} \left( (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \right) = \right.$

$\mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$  is a NSb closed sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \times \langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$ . Equavilintly, we will prove that  $\left\{\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right): \mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right\}^c$  is NS b-open sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \times \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Let  $\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right) \in \left\{\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)\right\}$  with  $\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right) > \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right): \mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) > \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$  or  $\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right) \in \left\{\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)\right\}$  with  $\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right) < \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right): \mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) < \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$ . Then,  $\mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) > \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$  or  $< \mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) < \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$  accordingly. Since,  $\langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$  is NSb Hausdorff space. Definitely,  $\mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right), \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$  are points of  $\langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$ , there exists NS b-open sets  $\langle \mathcal{G}, \partial \rangle, \langle \mathcal{H}, \partial \rangle \in \langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$  such that  $\mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) \in \langle \mathcal{G}, \partial \rangle$  &  $\mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) \in \langle \mathcal{H}, \partial \rangle$  provided  $\langle \mathcal{G}, \partial \rangle \tilde{\cap} \langle \mathcal{H}, \partial \rangle = 0_{(\widetilde{\mathcal{X}}, \widetilde{\mathfrak{A}^{parameter}})}_Y$ . Since,  $\langle \mathfrak{f}, \partial \rangle$  is soft continuous,  $\mathfrak{f}^{-1}(\langle \mathcal{G}, \partial \rangle)$  &  $\mathfrak{f}^{-1}(\langle \mathcal{H}, \partial \rangle)$  are NS b-open sets in  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  supposing  $\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)$  and  $\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)$  respectively and so  $\mathfrak{f}^{-1}(\langle \mathcal{G}, \partial \rangle) \times \mathfrak{f}^{-1}(\langle \mathcal{H}, \partial \rangle)$  is basic NS b-open set in  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \times \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  containing  $\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$ . Since  $\langle \mathcal{G}, \partial \rangle \tilde{\cap} \langle \mathcal{H}, \partial \rangle = 0_{(\widetilde{\mathcal{X}}, \widetilde{\mathfrak{A}^{parameter}})}_Y$ , it is clear by the definition of  $\left\{\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right): \mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)\right\}$  that  $\left\{\mathfrak{f}^{-1}(\langle \mathcal{G}, \partial \rangle) \& \mathfrak{f}^{-1}(\langle \mathcal{H}, \partial \rangle)\right\} \tilde{\cap} \left\{\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right): \mathfrak{f}(x) = \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)\right\} = 0_{(\widetilde{\mathcal{X}}, \widetilde{\mathfrak{A}^{parameter}})}$ , that is  $\mathfrak{f}^{-1}(\langle \mathcal{G}, \partial \rangle) \times \mathfrak{f}^{-1}(\langle \mathcal{H}, \partial \rangle) \in \left\{\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right): \mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)\right\}^c$ . Hence,  $\left\{\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right), \left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right): \mathfrak{f}\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \mathfrak{f}\left(\left(\mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)\right\}$

$\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right)^c$  implies that  $\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) = \mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \right\}$  is NS b-closed.

### 7. Mixed NS b-Separation Axioms

**Theorem 7.1.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be (NSSTS) and  $\langle \mathcal{Y}^{crip}, \mathfrak{B}, \partial \rangle$  be an-other (NSSTS). Let  $\langle \mathfrak{f}, \partial \rangle : \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \rightarrow \langle \mathcal{Y}^{crip}, \mathfrak{B}, \partial \rangle$  be NSb open mapping such that it is onto. If the soft set  $\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) = \mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \right\}$  is NS b-closed in  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \times \langle \mathcal{Y}^{crip}, \mathfrak{B}, \partial \rangle$ , then  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  will behave as NSb Hausdorff space.

**Proof:** Suppose  $\mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right), \mathfrak{f}(\mathfrak{y})$  be two points of  $\mathcal{Y}^{crip}$  such that either

$$\mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) > \mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \text{ or } < \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) <$$

$\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right)$ . Then  $\left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \notin$

$\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) >$

$\left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) >$

$\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \}$  or  $\left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \notin$

$\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) <$

$\left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) < \mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \}$ , that is

$\left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \in$

$\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) >$

$\left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) >$

$\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \}$  or  $\left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \in$

$\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) <$

$\left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{A}^{parameter} \right) \right) <$



$\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right)^c$ . Since,  $\left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \in$   
 $\left\{ \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter}, \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) > \right.$   
 $\left. \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) \right) > \right.$   
 $\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right)^c$  or  $\left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \in$   
 $\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) < \right.$   
 $\left. \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) \right) < \mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \right\}^c$  is soft in  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \times$   
 $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$ , then  $\exists NS b - \text{open sets}$   
 $\langle \mathcal{G}, \partial \rangle$  and  $\langle \mathfrak{k}, \partial \rangle$  in  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  s.t.  $\left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \in$   
 $\langle \mathcal{G}, \partial \rangle \times \langle \mathfrak{k}, \partial \rangle \in \left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) > \right.$   
 $\left. \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) \right) > \right.$   
 $\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right)^c$  or  $\left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \in \langle \mathcal{G}, \partial \rangle \times \langle \mathfrak{k}, \partial \rangle \in$   
 $\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) < \right.$   
 $\left. \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) \right) < \mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \right\}^c$ . Then, since  $\mathfrak{f}$  is NS b-open,  
 $\mathfrak{f}(\langle \mathcal{G}, \partial \rangle)$  and  $\mathfrak{f}(\langle \mathfrak{k}, \partial \rangle)$  are NS b-open sets in  $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$  containing  $\mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) \right)$  and  
 $\mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right)$  respectively, and  $\mathfrak{f}(\langle \mathcal{G}, \partial \rangle) \tilde{\cap} \mathfrak{f}(\langle \mathfrak{k}, \partial \rangle) = 0_{(\overline{\langle \mathcal{X} \rangle}, \widetilde{\mathfrak{I}^{parameter}})}$  otherwise  $\mathfrak{f}(\langle \mathcal{G}, \partial \rangle) \times$   
 $\mathfrak{f}(\langle \mathfrak{k}, \partial \rangle) \tilde{\cap} \left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) > \right.$   
 $\left. \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) \right) > \right.$   
 $\mathfrak{f}(\mathfrak{y})$  or  $\left( \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \right) \notin$   
 $\left\{ \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right), \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \text{ with } \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) < \right.$   
 $\left. \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) : \mathfrak{f} \left( \left( x^e_{(a,b,c)}, \mathfrak{I}^{parameter} \right) \right) < \mathfrak{f} \left( \left( \mathfrak{y}^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter} \right) \right) \right\} = 0_{(\overline{\langle \mathcal{X} \rangle}, \widetilde{\mathfrak{I}^{parameter}})}$ . It follows that  
 $\langle Y^{crip}, \mathfrak{I}, \partial \rangle$  is NSb Hausdorff space.

**Theorem 7.2.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  be a NS second countable space then it is guaranteed that every family of non-empty dis-joint NS b-open subsets of a NS second countable space  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  is NSb countable.

**Proof:** Given that  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  be a NS second countable space.

Then,  $\exists$  a NS countable base  $\mathfrak{B} = \langle \mathcal{B}^1, \mathcal{B}^2, \mathcal{B}^3, \mathcal{B}^4, \dots, \mathcal{B}^n : n \in \mathbb{N} \rangle$  for  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$ . Let  $\langle \mathcal{C}, \partial \rangle$  be a family of non-vacuous mutually exclusive NS b-open sub-sets of  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$ . Then, for each  $\langle \mathcal{f}, \partial \rangle$  of in  $\langle \mathcal{C}, \partial \rangle \exists$  a soft  $\mathcal{B}^n \in \mathfrak{B}$  in such a way that  $\mathcal{B}^n \subseteq \langle \mathcal{f}, \partial \rangle$ . Let us attach with  $\langle \mathcal{f}, \partial \rangle$ , the smallest positive interger  $n$  such that  $\mathcal{B}^n \subseteq \langle \mathcal{f}, \partial \rangle$ . Since the candidates of  $\langle \mathcal{C}, \partial \rangle$  are mutully exclusive because of this behavior distinct candidates will be associated with distinct positive integers. Now, if we put the elements of  $\langle \mathcal{C}, \partial \rangle$  in order so that the order is increasing relative to the positive integers associated with them, we obtain a sequence of of candidates of  $\langle \mathcal{C}, \partial \rangle$ . This verifies that  $\langle \mathcal{C}, \partial \rangle$  is NS countable.

**Theorem 7.3.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  be a NS second countable space and let  $\langle \mathcal{f}, \partial \rangle$  be NS uncountable suset of  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$ . Then, at least one point of  $\langle \mathcal{f}, \partial \rangle$  is a soft limit point of  $\langle \mathcal{f}, \partial \rangle$ .

**Proof:** Let  $\mathfrak{B} = \langle \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \dots, \mathcal{B}_n : n \in \mathbb{N} \rangle$  for  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$ . Let, if possible, no point of  $\langle \mathcal{f}, \partial \rangle$  be a soft limitpoint of  $\langle \mathcal{f}, \partial \rangle$ . Then, for each  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \in \langle \mathcal{f}, \partial \rangle \exists$  NSb – open set

$\langle \rho, \partial \rangle_{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})}$  such that  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \in \langle \rho, \partial \rangle_{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})}$  and

$\langle \rho, \partial \rangle_{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})} \tilde{\cap} \langle \mathcal{f}, \partial \rangle = \{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})\}$ . Since  $\mathfrak{B}$  is soft base  $\exists \mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \in \mathfrak{B}$  such

that  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \in \mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \subseteq \langle \rho, \partial \rangle_{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})}$ . Therefore,  $\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \tilde{\cap} \langle \mathcal{f}, \partial \rangle \subseteq$

$\langle \rho, \partial \rangle_{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})} \tilde{\cap} \langle \mathcal{f}, \partial \rangle = \{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})\}$ . More –

over, if  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1$  and  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2$  be any two NS points such that  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1 \neq$

$(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2$  which means either  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1 \succ (x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2$  or

$(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1 \prec (x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2$  then  $\exists \mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1$  and  $\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2$  in  $\mathfrak{B}$  such that

$\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1 \tilde{\cap} \langle \mathcal{f}, \partial \rangle = \{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1\}$  and  $\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2 \tilde{\cap} \langle \mathcal{f}, \partial \rangle = \{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2\}$ .

Now,  $(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1 \neq (x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2$  which guarantees that  $\{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1\} \neq$

$\{(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2\}$  which  $\Rightarrow$

$\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1 \tilde{\cap} \langle \mathcal{f}, \partial \rangle \neq \mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2 \tilde{\cap} \langle \mathcal{f}, \partial \rangle$  which implies  $\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_1 \neq$

$\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}})_2$ . Thus,  $\exists$  a one to one soft correspondence of  $\langle \mathcal{f}, \partial \rangle$  on to

$\{\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) : (x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \in \langle \mathcal{f}, \partial \rangle\}$ . Now,  $\langle \mathcal{f}, \partial \rangle$  being NS uncountable, it follows that

$\{\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) : (x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \in \langle \mathcal{f}, \partial \rangle\}$  is NS uncountable. But, this is purely a contradiction, since

$\{\mathcal{B}_n(x^e_{(a,b,c), \mathfrak{I}^{parameter}}) : (x^e_{(a,b,c), \mathfrak{I}^{parameter}}) \in \langle \mathcal{f}, \partial \rangle\}$  benig a NS sub-family of the NS countable collection  $\mathfrak{B}$ . This

contradiction is taking birth that on point of  $\langle f, \partial \rangle$  is a soft limit point of  $\langle f, \partial \rangle$ , so at least one point of  $\langle f, \partial \rangle$  is a soft limit point of  $\langle f, \partial \rangle$ .

**Theorem 7.4.** Let  $\langle X^{crisp}, \mathfrak{A}, \partial \rangle$  NSSTS such that it is NS countably compact then this space has the characteristics of Bolzano Weierstrass Property (BWP).

**Proof:** Let  $\langle X^{crisp}, \mathfrak{A}, \partial \rangle$  be a NS countably compact space and suppose, if possible, that it negate the Bolzano Weierstrass Property (BWP). Then there must exist an infinite NS set  $\langle f, \partial \rangle$  supposing no soft limit point. Further suppose  $\langle \rho, \partial \rangle$  be soft countably infinite soft sub-set  $\langle f, \partial \rangle$  that is  $\langle \rho, \partial \rangle \subseteq \langle f, \partial \rangle$ . Then this guarantees  $\langle \rho, \partial \rangle$  has no soft limit point. It follows that  $\langle \rho, \partial \rangle$  is NSb closed set. Also for each  $(x^e_{(a,b,c), \mathfrak{A}^{parameter}})_n \in \langle \rho, \partial \rangle$ ,  $(x^e_{(a,b,c), \mathfrak{A}^{parameter}})_n$  is not a soft limit point of  $\langle \rho, \partial \rangle$ . Hence there exists NS b-open set  $\langle G_n, \partial \rangle$ , such that  $(x^e_{(a,b,c), \mathfrak{A}^{parameter}})_n \in \langle G_n, \partial \rangle$  and  $\langle G_n, \partial \rangle \cap \langle \rho, \partial \rangle = \{(x^e_{(a,b,c), \mathfrak{A}^{parameter}})_n\}$ . The collection  $\{\langle G_n, \partial \rangle : n \in N\} \cap \langle \rho, \partial \rangle^c$  is countable NS b-open cover of  $\langle X^{crisp}, \mathfrak{A}, \partial \rangle$ . This soft cover has no finite sub-cover. For this we exhaust a single  $\langle G_n, \partial \rangle$ , it would not be a soft cover of  $\langle X^{crisp}, \mathfrak{A}, \partial \rangle$  since then  $\{(x^e_{(a,b,c), \mathfrak{A}^{parameter}})_n\}$  would be covered. Result in  $\langle X^{crisp}, \mathfrak{A}, \partial \rangle$  is not NS countably compact. But this contradicts the given. Hence, we are compelled to accept  $\langle X^{crisp}, \mathfrak{A}, \partial \rangle$  must have Bolzano Weierstrass Property.

**Theorem 7.5.** Let  $(X^{crisp^1}, \mathfrak{F}, \mathcal{A})$  and  $(X^{crisp^2}, \mathfrak{T}, \mathcal{A})$  be two NSSTS and suppose  $\langle f, \partial \rangle$  be a NS continuous function such that  $\langle f, \partial \rangle : (X^{crisp^1}, \mathfrak{F}, \mathcal{A}) \rightarrow (X^{crisp^2}, \mathfrak{T}, \mathcal{A})$  is NS continuous function and let  $\langle L, \partial \rangle \subseteq (X^{crisp^1}, \mathfrak{F}, \mathcal{A})$  supposes the B.V.P. then safely  $\langle f(L, \partial) \rangle$  has the B.V.P.

**Proof:** Suppose  $\langle L, \partial \rangle$  be an infinite NS sub-set of  $\langle f, \partial \rangle$ , so that  $\langle L, \partial \rangle$  contains an enumerable NS set  $\langle (x^e_{(a,b,c), \mathfrak{A}^{parameter}})_n : n \in N \rangle$  then there exists enumerable NS set  $\langle (y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_n : n \in N \rangle \subseteq \langle L, \partial \rangle$  s.t.  $\langle f((y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_n) \rangle = (x^e_{(a,b,c), \mathfrak{A}^{parameter}})_n \cdot \langle L, \partial \rangle$  has B.V.P.  $\Rightarrow$  every infinite soft subset of  $\langle L, \partial \rangle$  supposes soft accumulation point belonging to  $\langle L, \partial \rangle$ .  $\Rightarrow \langle (y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_n : n \in N \rangle$  has soft neutrosophic limit point, say,  $(y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_0 \in \langle L, \partial \rangle$ .  $\Rightarrow$  the limit of soft sequence  $\langle (y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_n : n \in N \rangle$  is  $(y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_0 \in \langle L, \partial \rangle \Rightarrow (y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_n \rightarrow (y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_0 \in \langle L, \partial \rangle$ .  $\langle f \rangle$  is soft continuous  $\Rightarrow$  it is soft continuous. Furthermore  $(y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_n \rightarrow (y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_0 \in \langle L, \partial \rangle \Rightarrow \langle f((y^{e'}_{(a',b',c'), \mathfrak{A}^{parameter}})_n) \rangle \rightarrow$

$\tilde{f}(\langle \langle \mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \rangle_0 \rangle) \in \tilde{f}(\langle \mathcal{L}, \partial \rangle) \Rightarrow (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_n \rightarrow \tilde{f}(\langle \mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \rangle_0) \in \tilde{f}(\langle \mathcal{L}, \partial \rangle) \Rightarrow$  limit of a soft sequence  $\langle (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_n : n \in N \rangle$  is  $\tilde{f}(\langle \mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \rangle_0) \in \tilde{f}(\langle \mathcal{L}, \partial \rangle) \Rightarrow$  limit of a soft sequence  $\langle (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_n : n \in N \rangle$  is  $\tilde{f}(\langle \mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \rangle_0) \in \langle \tilde{f}, \partial \rangle(\langle \mathcal{L}, \partial \rangle)$ . Finally we have shown that there exists infinite soft subset  $\langle (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_n : n \in N \rangle$  of  $\tilde{f}(\langle \mathcal{L}, \partial \rangle)$  containing a limit point  $\tilde{f}(\langle \mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter} \rangle_0) \in \tilde{f}(\langle \mathcal{L}, \partial \rangle)$ . This guarantees that  $\tilde{f}(\langle \mathcal{L}, \partial \rangle)$  has *B.V.P.*

**Theorem 7.6.** Let  $\langle \mathcal{X}^{crisp}, \mathfrak{A}, \partial \rangle$  be a NSSTS so that (i)  $\langle \mathcal{F}, \partial \rangle$  is NSb compact soft sub-set of  $\langle \mathcal{X}^{crisp}, \mathfrak{A}, \partial \rangle$  and  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  be a crisp point in  $\mathcal{X}^{crisp}$  such that  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  can be strongly separated from every point  $(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  in  $\langle \mathcal{F}, \partial \rangle$ , then it is guaranteed that  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  and  $\langle \mathcal{F}, \partial \rangle$  can also be soft strongly separated in  $\langle \mathcal{X}^{crisp}, \mathfrak{A}, \partial \rangle$ . (ii) if  $\langle \mathcal{F}, \partial \rangle$  and  $\langle \mathcal{F}, \partial \rangle$  are two NSb compact soft sub-sets of  $\langle \mathcal{X}^{crisp}, \mathfrak{A}, \partial \rangle$  such that every point  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  in  $\langle \mathcal{F}, \partial \rangle$  can be strongly separated from every point  $(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  in  $\langle \mathcal{F}, \partial \rangle$ , then it is guaranteed that  $\langle \mathcal{F}, \partial \rangle$  and  $\langle \mathcal{F}, \partial \rangle$  can be strongly separated in  $\langle \mathcal{X}^{crisp}, \mathfrak{A}, \partial \rangle$ .

**Proof i)** Let  $\langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}((x^e_{(a,b,c)}, \mathfrak{A}^{parameter}))$  and  $\langle \mathcal{L}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}((\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}))$  separate strongly the point  $x$  from a point  $(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in \langle \mathcal{F}, \partial \rangle$ . As  $(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  runs over  $\langle \mathcal{F}, \partial \rangle$ , the corresponding NS sets  $\langle \mathcal{L}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}((\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}))$  form NS b-open covering of  $\langle \mathcal{F}, \partial \rangle$ , for which there exists a finite soft sub-covering,  $\langle \mathcal{L}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}((\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}))_1, \dots, \langle \mathcal{L}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}((\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}))_n$ , say, since  $\langle \mathcal{F}, \partial \rangle$  is NSb compact. Let

$$\langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})} \left( \langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}_1, \langle \mathcal{U}, \partial \rangle_{y_n} \right) \langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}_1, \dots, \langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}_n \langle \mathcal{U}, \partial \rangle_{y_4} \left( \langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}_1, \dots, \langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}_n \right) \langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}_5 \left( (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \right)$$

be the corresponding NSb open sets supposing the point  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$ .

$$\text{Let } \langle \mathcal{U}, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \right) = \langle \mathcal{U}, \partial \rangle_{(\mathcal{Y}^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}_1 \left( (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \right) \tilde{\cap}, \dots, \dots$$

$$\begin{aligned} & \tilde{N} \langle U, \partial \rangle_{\left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_1} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \right) \text{ and } \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \langle \mathcal{F}, \partial \rangle \right) \\ & = \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_1 \right) \tilde{U} \end{aligned}$$

$$\langle V, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_2 \right) \tilde{U} \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \left( \left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_3 \right) \tilde{U} \dots$$

$\tilde{U} \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_n \right)$ . Then  $\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \in$

$\langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \right)$  and  $\langle \mathcal{F}, \partial \rangle \in \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \langle \mathcal{F}, \partial \rangle \right)$ . Also,

since  $\langle U, \partial \rangle_{\left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_i} \left( \left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right) \right) \tilde{N} \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_i \right) =$

$0_{\langle \tilde{\mathcal{X}} \rangle, \mathfrak{A}parameter}$ , and  $\langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \right) \in \langle U, \partial \rangle_{\left( \mathcal{Y}^{e'}_{(a',b',c'), \mathfrak{A}parameter} \right)_i} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \right)$ , for  $i =$

$1, 2, 3, 4, \dots, n$ , it follows that  $\langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \right) \tilde{N} \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \langle \mathcal{F}, \partial \rangle \right) = 0_{\langle \tilde{\mathcal{X}} \rangle, \mathfrak{A}parameter}$ .

Thus  $\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)$  and  $\langle \mathcal{F}, \partial \rangle$  are separated strongly by the pair of disjoint NSb open sets

$\langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \right)$  and  $\langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)} \left( \langle \mathcal{F}, \partial \rangle \right)$  in  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ .

(ii) Suppose  $\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)$  runs over  $\langle \wp, \partial \rangle$ , then corresponding soft sets  $\langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right) \right)$  generate soft covering of  $\langle \wp, \partial \rangle$ , for which there exists a finite soft sub-covering

$$\left\{ \begin{aligned} & \langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_1 \right), \\ & \langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_2 \right), \\ & \dots, \langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_m \right) \end{aligned} \right\}, \text{ say, for } \langle \wp, \partial \rangle \text{ (since } \langle \wp, \partial \rangle \text{ is soft NSb compact). Let}$$

$$\langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_1} \left( \langle \mathcal{F}, \partial \rangle \right), \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_2} \left( \langle \mathcal{F}, \partial \rangle \right), \dots$$

$\langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_m} \left( \langle \mathcal{F}, \partial \rangle \right)$  be the corresponding NSb open sets containing  $\langle \mathcal{F}, \partial \rangle$ . Then  $\langle U, \partial \rangle_{\langle \wp, \partial \rangle} =$

$$\left\{ \begin{aligned} & \langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_1 \right) \tilde{U} \\ & \langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_2 \right) \tilde{U}, \dots \\ & \tilde{U} \langle U, \partial \rangle_{\langle \mathcal{F}, \partial \rangle} \left( \left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_m \right) \end{aligned} \right\} \text{ and } \langle \mathcal{L}, \partial \rangle_{\langle \wp, \partial \rangle} = \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_1} \left( \langle \mathcal{F}, \partial \rangle \right)$$

$$\tilde{N} \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_2} \left( \langle \mathcal{F}, \partial \rangle \right) \tilde{N}, \dots$$

$\tilde{N} \langle \mathcal{L}, \partial \rangle_{\left( x^e_{(a,b,c), \mathfrak{A}parameter} \right)_m} \left( \langle \mathcal{F}, \partial \rangle \right)$  are two disjoint NS open sets, (as in (i)), which separate  $\langle \wp, \partial \rangle$  and  $\langle \mathcal{F}, \partial \rangle$  strongly.

**Theorem 7.7.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be a NSSTS and  $\langle \mathfrak{f}^{crip}, \mathfrak{A}, \partial \rangle$  be NS sub-space of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . The necessary and sufficient condition for  $\langle \mathfrak{f}^{crip}, \partial \rangle$  to be NS compact relative to  $\langle \mathfrak{f}^{crip}, \mathfrak{A}, \partial \rangle$  is that  $\langle \mathfrak{f}^{crip}, \partial \rangle$  is NS compact relative to  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ .

**Proof:** First we prove that  $\langle \mathfrak{f}^{crip}, \partial \rangle$  relative to  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . Let  $\{ \langle \mathfrak{f}, \partial \rangle_i : i \in I \}$  that is  $\{ \langle \mathfrak{f}, \partial \rangle_1, \langle \mathfrak{f}, \partial \rangle_2, \langle \mathfrak{f}, \partial \rangle_3, \langle \mathfrak{f}, \partial \rangle_4, \dots \}$  be  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  - NSb open cover of  $\langle \mathfrak{f}^{crip}, \partial \rangle$ , then  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \bigcup_i \langle \mathfrak{f}, \partial \rangle_i$ .  $\langle \mathfrak{f}, \partial \rangle_i \in$

$\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \Rightarrow \exists \langle \mathcal{G}, \partial \rangle_i \in \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  s.t.  $\langle \mathfrak{f}, \partial \rangle_i = \langle \mathcal{G}, \partial \rangle_i \tilde{\cap} \langle \mathfrak{f}^{crip}, \partial \rangle \in \langle \mathcal{G}, \partial \rangle_i \Rightarrow \exists \langle \mathcal{G}, \partial \rangle_i \in$

$\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  s.t.  $\langle \mathfrak{f}, \partial \rangle_i \in \langle \mathcal{G}, \partial \rangle_i \Rightarrow \widetilde{\cup}_i \langle \mathfrak{f}, \partial \rangle_i \in \widetilde{\cup}_i$  but  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \langle \mathfrak{f}, \partial \rangle_i$ . So that  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \widetilde{\cup}_i \langle \mathfrak{f}, \partial \rangle_i$ . This guarantees

that  $\{\langle \mathcal{G}, \partial \rangle_i : i \in I\}$  is a  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - NSb$  open cover of  $\langle \mathfrak{f}^{crip}, \partial \rangle$  which is known to be  $NSb$  compact relative

$\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  and hence the soft cover  $\{\langle \mathcal{G}, \partial \rangle_i : i(x_{(a,b,c)}^e, \mathfrak{d}^{parameter})I\}$  must be freezable to a finite soft sub cover,

say,

$\{\langle \mathcal{G}, \partial \rangle_{ir} : r = 1, 2, 3, 4, \dots, n\}$ , Then  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \bigcup_{r=1}^n \widetilde{\langle \mathcal{G}, \partial \rangle}_{ir} \Rightarrow$

$$\langle \mathfrak{f}^{crip}, \partial \rangle \tilde{\cap} \langle \mathfrak{f}^{crip}, \partial \rangle \in \langle \mathfrak{f}^{crip}, \partial \rangle \tilde{\cap} \left[ \bigcup_{r=1}^n \widetilde{\langle \mathcal{G}, \partial \rangle}_{ir} \right]$$

$= \bigcup_{r=1}^n (\langle \mathfrak{f}^{crip}, \partial \rangle \tilde{\cap} \langle \mathcal{G}, \partial \rangle_{ir}) = \bigcup_{r=1}^n \widetilde{\langle \mathfrak{f}, \partial \rangle}_{ir}$  or  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \bigcup_{r=1}^n \widetilde{\langle \mathfrak{f}, \partial \rangle}_{ir} \Rightarrow \{\langle \mathfrak{f}, \partial \rangle_{ir} : 1 \leq r \leq n\}$  is a  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - NS$

open cover of  $\langle \mathfrak{f}^{crip}, \partial \rangle$ . Stepping from an arbitrary  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle -$ open cover of  $\langle \mathfrak{f}^{crip}, \partial \rangle$ , we are able to show that

the  $NS$  cover is freezable to a finite soft subcover  $\{\langle \mathfrak{f}, \partial \rangle_{ir} : 1 \leq r \leq n\}$  of  $\langle \mathfrak{f}^{crip}, \partial \rangle$ , meaning there by  $\langle \mathfrak{f}^{crip}, \partial \rangle$  is

$\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - NSbcompact$ . The condition is sufficient: Suppose  $\langle \mathfrak{f}^{crip}, \mathfrak{S}, \partial \rangle$  be soft sub-space of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$

and also  $\langle \mathfrak{f}^{crip}, \partial \rangle$  is  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - NSb compact$ . We have to prove that  $\langle \mathfrak{f}^{crip}, \partial \rangle$  is  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - NScompact$ .

Let  $\{\langle \mathfrak{f}, \partial \rangle_1, \langle \mathfrak{f}, \partial \rangle_2, \langle \mathfrak{f}, \partial \rangle_3, \langle \mathfrak{f}, \partial \rangle_4, \dots\}$  be soft  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - NS$  b-open cover of  $\langle \mathfrak{f}^{crip}, \partial \rangle$ , so that  $\langle \mathfrak{f}^{crip}, \partial \rangle \in$

$\widetilde{\cup}_i \langle \mathcal{G}, \partial \rangle_i$  from which  $\langle \mathfrak{f}^{crip}, \partial \rangle \tilde{\cap} \langle \mathfrak{f}^{crip}, \partial \rangle \in \langle \mathfrak{f}^{crip}, \partial \rangle \tilde{\cap} (\widetilde{\cup}_i \langle \mathcal{G}, \partial \rangle_i)$  or,  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \widetilde{\cup}_i (\langle \mathfrak{f}^{crip}, \partial \rangle \tilde{\cap} \langle \mathcal{G}, \partial \rangle_i)$ . On

taking  $\langle \mathfrak{f}, \partial \rangle_i = \langle \mathcal{G}, \partial \rangle_i \tilde{\cap} \langle \mathfrak{f}^{crip}, \partial \rangle$ , we get  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \widetilde{\cup}_i \langle \mathfrak{f}, \partial \rangle_i, \langle \mathcal{G}, \partial \rangle_i \in \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \Rightarrow \langle \mathfrak{f}, \partial \rangle_i =$

$\langle \mathcal{G}, \partial \rangle_i \tilde{\cap} \langle \mathfrak{f}^{crip}, \partial \rangle \in \langle \mathfrak{f}^{crip}, \mathfrak{S}, \partial \rangle \dots (1)$ . Now from (1) it is clear that  $\{\langle \mathfrak{f}, \partial \rangle_1, \langle \mathfrak{f}, \partial \rangle_2, \langle \mathfrak{f}, \partial \rangle_3, \langle \mathfrak{f}, \partial \rangle_4, \dots\}$  is  $\langle \mathfrak{f}^{crip}, \mathfrak{S}, \partial \rangle -$

$NSopen$  soft cover of  $\langle \mathfrak{f}^{crip}, \partial \rangle$  which is known to be  $\langle \mathfrak{f}^{crip}, \mathfrak{S}, \partial \rangle - NS b compact$  hence this sof cover must be

reducible to a finite soft sub-cover. say,  $\{\langle \mathfrak{f}, \partial \rangle_{ir} : 1 \leq r \leq n\}$ . This  $\Rightarrow \langle \mathfrak{f}^{crip}, \partial \rangle \in \bigcup_{r=1}^n \widetilde{\langle \mathfrak{f}, \partial \rangle}_{ir} =$

$\bigcup_{r=1}^n (\langle \mathcal{G}, \partial \rangle_{ir}) \tilde{\cap} \langle \mathfrak{f}^{crip}, \partial \rangle \in \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ , or

$$\langle \mathfrak{f}^{crip}, \partial \rangle \in \left( \bigcup_{r=1}^n (\langle \mathcal{G}, \partial \rangle_{ir}) \tilde{\cap} \langle \mathfrak{f}^{crip}, \partial \rangle \right) \in \bigcup_{r=1}^n \widetilde{\langle \mathcal{G}, \partial \rangle}_{ir},$$

or  $\langle \mathfrak{f}^{crip}, \partial \rangle \in \bigcup_{r=1}^n \widetilde{\langle \mathcal{G}, \partial \rangle}_{ir}$ . This proves that  $\{\langle \mathcal{G}, \partial \rangle_{ir} : 1 \leq r \leq n\}$  is a finite soft sub-cover of the soft cover  $\langle \mathcal{G}, \partial \rangle_i$ .

Starting from an arbitrary  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - NS$  b-open soft cover of  $\langle \mathfrak{f}^{crip}, \partial \rangle$ , we are able to show that this soft

neutrosophic b- open cover is freezable to a finite soft sub-cover, showing there by  $\langle \mathfrak{f}^{crip}, \partial \rangle$  is  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle -$

$Nbcompact$ .

**Theorem 7.8.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  NSSTS and let  $\langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n \rangle$  be a NS sequence in  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  such that it converges to a point  $(x^e_{(a,b,c), \mathfrak{A}parameter})_0$  then the soft set  $\langle \mathcal{G}, \partial \rangle$  consisting of the points  $(x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0}$  and  $(x^e_{(a,b,c), \mathfrak{A}parameter})_n$  ( $n = 1, 2, 3, \dots$ ) is soft NSb compact.

**Proof:** Given  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  NSSTS and let  $\langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n \rangle$  be a NS sequence in  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  such that it converges to a point  $(x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0}$  that is  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n \rightarrow (x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0} \in \mathcal{X}^{crip}$ . Let

$$\langle \mathcal{G}, \partial \rangle = \langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_1, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_2, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_3, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_4, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_5, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_7, \dots \rangle$$

covering of  $\langle \mathcal{G}, \partial \rangle$  so that  $\langle \mathcal{G}, \partial \rangle \in \tilde{\cup} \{ \langle \mathfrak{S}, \partial \rangle_\alpha : \alpha \in \Delta \}, (x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0} \in \langle \mathcal{G}, \partial \rangle \Rightarrow \exists \alpha_0 \in \Delta$  s. t.  $(x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0} \in \langle \mathfrak{S}, \partial \rangle_{\alpha_0}$ . According to the definition of soft convergence,  $(x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0} \in \langle \mathfrak{S}, \partial \rangle_{\alpha_0} \in \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \Rightarrow \exists n_0 \in N$  s. t.  $n \geq n_0$  and  $(x^e_{(a,b,c), \mathfrak{A}parameter})_n \in \langle \mathfrak{S}, \partial \rangle_{\alpha_0}$ . Evidently,  $\langle \mathfrak{S}, \partial \rangle_{\alpha_0}$  contains the points  $(x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0}$ ,

$$(x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0+1}, (x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0+2}, (x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0+3}, (x^e_{(a,b,c), \mathfrak{A}parameter}), \dots$$

$(x^e_{(a,b,c), \mathfrak{A}parameter})_{n_0+n}, \dots$ . Look carefully at the points and train them in a way as,

$$(x^e_{(a,b,c), \mathfrak{A}parameter})_1, (x^e_{(a,b,c), \mathfrak{A}parameter})_2, (x^e_{(a,b,c), \mathfrak{A}parameter})_3, (x^e_{(a,b,c), \mathfrak{A}parameter})_4, \dots$$

$(x^e_{(a,b,c), \mathfrak{A}parameter}), \dots$  generating a finite soft set. Let  $1 \leq n_0-1$ . Then  $(x^e_{(a,b,c), \mathfrak{A}parameter})_i \in \langle \mathcal{G}, \partial \rangle$ . For this  $i$ ,  $(x^e_{(a,b,c), \mathfrak{A}parameter})_i \in \langle \mathcal{G}, \partial \rangle$ . Hence  $\exists \alpha_i \in \Delta$  s. t.  $(x^e_{(a,b,c), \mathfrak{A}parameter})_i \in \langle \mathfrak{S}, \partial \rangle_{\alpha_i}$ . Evidently  $\langle \mathcal{G}, \partial \rangle \in \bigcup_{r=0}^{n_0-1} \langle \mathfrak{S}, \partial \rangle_{\alpha_i}$ . This shows that  $\{ \langle \mathfrak{S}, \partial \rangle_{\alpha_i} : 0 \leq n_0-1 \}$  is NS b-open cover of  $\langle \mathcal{G}, \partial \rangle$ . Thus an arbitrary soft neutrosophic open cover  $\{ \langle \mathfrak{S}, \partial \rangle_\alpha : \alpha \in \Delta \}$  of  $\langle \mathcal{G}, \partial \rangle$  is reducible to a finite NS sub-cover  $\{ \langle \mathfrak{S}, \partial \rangle_{\alpha_i} : i = 0, 1, 2, 3, \dots, n_0-1 \}$ , it follows that  $\langle \mathcal{G}, \partial \rangle$  is soft NSb compact.

**Theorem 7.9.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be a NSSTS such that it is soft countably compact. Then every NSb closed sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NSb countably compact

**Proof:** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  NSSTS such that it is NSb countably compact and suppose  $\langle \mathcal{F}, \partial \rangle$  be a NSb closed sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . Let  $\mathcal{C} = \{ \langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \langle \mathcal{G}_5, \partial \rangle, \langle \mathcal{G}_6, \partial \rangle, \dots, \dots, \langle \mathcal{G}_n, \partial \rangle, \dots \}$  That is  $\{ \langle \mathcal{G}_n, \partial \rangle : n \in N \}$  be any countably NS b-open covering of  $\langle \mathcal{F}, \partial \rangle$ . Then,  $\langle \mathcal{F}, \partial \rangle \in \tilde{\cup} \langle \mathcal{G}_n, \partial \rangle$ . This qualifying us to write  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle = \langle \mathcal{F}, \partial \rangle \tilde{\cup} \langle \mathcal{F}, \partial \rangle^c \in \tilde{\cup} ( \langle \mathcal{G}_n, \partial \rangle ) \tilde{\cup} \langle \mathcal{F}, \partial \rangle^c$ . This guarantee that the collection  $\{ \langle \mathcal{G}_n, \partial \rangle : n \in N \}$  is a NS countable b-open

covering of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . But  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  being soft countably *NSb* compact and  $\langle \mathfrak{f}, \partial \rangle^c$  obviously absorbing no piece of  $\langle \mathfrak{f}, \partial \rangle$ . It follows that there exists finite soft number of indices  $n_1, n_1, n_1, n_1, \dots, n_k$  such that

$\langle \mathfrak{f}, \partial \rangle \in \cup_{i=1}^k \langle \mathcal{G}_{ni}, \partial \rangle$ . This shows that  $\{ \langle \mathcal{G}_{ni}, \partial \rangle : i = 1, 2, 3, 4, \dots, k \}$  is a finite soft neutrosophic sub-covering of  $\mathcal{C}$ .

**Theorem 7.10.** If  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  *NSSTS* such that it has the characteristics of soft neutrosophic sequentially compactness. Then  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  is safely *NSb* countably compact.

**Proof:** Let  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  *NSSTS* and let  $\langle \rho, \partial \rangle$  be finite soft sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Let

$(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_1, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_2, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_3,$   
 $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_4, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_5,$   
 $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_6, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_7, \dots$  be a soft sequence of soft points of

$\langle \rho, \partial \rangle$ . Then,  $\langle \rho, \partial \rangle$  being finite, at least one of the elements in  $\langle \rho, \partial \rangle$  say  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0$  must be duplicated

an in- finite number of times in the *NS* sequence.

Hence,  $\langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0, \dots \rangle$  is soft sub-sequence of  $\langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0, \dots \rangle$

$\langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n \rangle$  such that it is soft constant sequence and repeatedly constructed by single soft number  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0$  and we know that a soft constant sequence converges on its self. So it converges to  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_0$  which belongs to  $\langle \rho, \partial \rangle$ . Hence,  $\langle \rho, \partial \rangle$  is soft sequentially *NSb* compact.

**Theorem 7.11.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  *NSSTS* and  $\langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$  be another *NSSTS*. Let  $\langle \mathfrak{f}, \partial \rangle$  be a soft continuous mapping of a soft neutrosophic sequentially compact *NSb* space  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  into  $\langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$ . Then,  $\langle \mathfrak{f}, \partial \rangle(\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle)$  is *NSb* sequentially compact.

**Proof:** Given  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  *NSSTS* and  $\langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$  be another *NSSTS*. Let  $\langle \mathfrak{f}, \partial \rangle$  be a soft continuous mapping of a *NSb* sequentially compact space  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  into  $\langle \mathcal{Y}^{crip}, \mathfrak{I}, \partial \rangle$ . Then we have to prove  $\langle \mathfrak{f}, \partial \rangle(\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle)$  *NSb*

sequentially. For this we proceed as. Let  $\langle (y^{e'}_{(a',b',c'), \overline{\mathfrak{A}}parameter})_1, (y^{e'}_{(a',b',c'), \overline{\mathfrak{A}}parameter})_2,$   
 $(y^{e'}_{(a',b',c'), \overline{\mathfrak{A}}parameter})_5, (y^{e'}_{(a',b',c'), \overline{\mathfrak{A}}parameter})_6,$   
 $(y^{e'}_{(a',b',c'), \overline{\mathfrak{A}}parameter})_7, \dots, (y^{e'}_{(a',b',c'), \overline{\mathfrak{A}}parameter})_n, \dots \rangle$  be a soft

sequence of *NS* points in  $\langle \mathfrak{f}, \partial \rangle(\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle)$ , Then for each  $n \in$

$\mathbb{N} \exists \langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_1, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_2,$   
 $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_4, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_5, \dots \rangle$  such that  
 $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_7, \dots, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n, \dots \in \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$



$$\langle \mathcal{F}, \partial \rangle \left( \left\langle \begin{matrix} (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_1, (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_2, \\ (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_3, \\ (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_7, \dots (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_n, \dots \end{matrix} \right\rangle \right) =$$

$$\left\langle \begin{matrix} (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_1, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_2, \\ (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_3, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_4, \\ (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_6, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_7, \\ \dots (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_n, \dots \end{matrix} \right\rangle$$

Thus we obtain a soft sequence

$$\left\langle \begin{matrix} (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_1, (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_2, \\ (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_3, (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_4, \\ (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_6, (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_7, \\ (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_7, \dots (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_n, \dots \end{matrix} \right\rangle$$

in  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . But  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  being soft sequentially NS

compact, there is a NS sub-sequence  $\langle (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_{n_i} \rangle$  of

$\langle (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_n \rangle$  such that  $\langle (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_{n_i} \rangle \rightarrow (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter}) \in \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . So, by NS

continuity of  $\langle \mathcal{F}, \partial \rangle$ ,  $\langle (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_{n_i} \rangle \rightarrow (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter}) \rightarrow \langle \mathcal{F}, \partial \rangle (\langle (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_{n_i} \rangle) \rightarrow$

$\langle \mathcal{F}, \partial \rangle (\langle (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_n \rangle) \in \langle \mathcal{F}, \partial \rangle (\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle)$ . Thus,  $\langle \mathcal{F}, \partial \rangle (\langle (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_{n_i} \rangle)$  is a soft sub-sequence

$$\left\langle \begin{matrix} (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_1, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_2, \\ (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_4, (x^e_{(a,b,c)}, \widetilde{\mathcal{A}parameter})_4, \\ (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_5, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_6, \\ (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_7, \dots (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_n, \dots \end{matrix} \right\rangle$$

of  $\langle (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_1, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_2, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_3, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_4, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_5, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_6, (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_7, \dots (y^{e'}_{(a',b',c')}, \widetilde{\mathcal{A}parameter})_n, \dots \rangle$  converges to  $\langle \mathcal{F}, \partial \rangle (\tilde{x})$  in  $\langle \mathcal{F}, \partial \rangle (\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle)$ .

Hence,  $\langle \mathcal{F}, \partial \rangle (\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle)$  is NSb sequentially compact.

**Theorem 7.12.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be NSSTS such that is NS sequentially compact then it is guaranteed that it must be NSb countably compact.

**Proof:** Suppose  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NS sequentially compact. If  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is finite, then nothing to prove in this case because it is then automatically NSb countably compact. Suppose  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be in-infinite. We prove the contrapositive of the statement given in the theorem. Let  $\{\langle \mathcal{G}_i, \partial \rangle : i \in N\}$  that is  $\{\langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \dots : i \in N\}$  be a NS b-open cover of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  which has no finite soft

sub-cover. Now, we generate a soft sequence  $\langle (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_1, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_2, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_3, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_4, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_5, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_6, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_7, \dots \rangle$  which

may be soft monotone. That is soft monotonically non-increasing or soft monotonically strictly increasing or soft monotonically non-decreasing or soft monotonically strictly decreasing. Whatever the case may be we proceed as follows. Let  $n_1$  be the smallest positive integer such that  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_1}, \partial \rangle \neq 0_{(\overline{\mathcal{X}}, \overline{\mathfrak{A}}parameter)}$ . Choose  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_1 \in \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_1}, \partial \rangle$ . Now let  $n_2$  be the least positive integer greater than  $n_1$  such that  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_2}, \partial \rangle \neq 0_{(\overline{\mathcal{X}}, \overline{\mathfrak{A}}parameter)}$ . Choose  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_2 \in (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_2}, \partial \rangle) \setminus (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_1}, \partial \rangle)$ . It is important to be noted that such a point  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_2$  always exists, for otherwise  $\langle \mathcal{G}_{n_1}, \partial \rangle$  will be a soft cover of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Choose  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_3 \in (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_3}, \partial \rangle) \setminus (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_2}, \partial \rangle)$ . It is important to be noted that such a point  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_3$  always exists, for otherwise  $\mathcal{G}_{n_2}$  will be a soft cover of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Choose  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_4 \in (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \mathcal{G}_{n_4}) \setminus (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_3}, \partial \rangle)$ . Choose  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_5 \in (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_5}, \partial \rangle) \setminus (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_4}, \partial \rangle)$ . ..... Choose  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n \in (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_n}, \partial \rangle) \setminus (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_{n-1}}, \partial \rangle)$ . we get the soft

sequence  $\left( \begin{matrix} (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_1, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_2, \\ (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_3, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_4, \\ (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_5, (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_6, \dots, \\ (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n, \dots \end{matrix} \right)$  having the characteristics that, for each

$i \in N$ .  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_i \in (\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{n_i}, \partial \rangle)$ ,  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_i$  does not belong to  $\bigcup \left\{ \left( \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_k, \partial \rangle \right) : k = 1, 2, 3, \dots, n-1 \right\}$ ,  $m \geq n_i - 1$ . It can be seen that

$\left( (x^e_{(a,b,c), \overline{\mathfrak{A}}parameter})_n \right)$  supposes no soft convergent sub-sequence in  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . For let  $(x^e_{(a,b,c), \overline{\mathfrak{A}}parameter}) \in \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Then there exists  $\langle \mathcal{G}_{m_0}, \partial \rangle \in V \{ \langle \mathcal{G}_n, \partial \rangle : n \in N \}$  such

that  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter}) \in \langle \mathcal{G}_{m_0}, \partial \rangle$ . Since  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle \tilde{\cap} \langle \mathcal{G}_{m_0}, \partial \rangle \neq 0_{(\overline{\mathfrak{A}parameter})}$ , there exists  $k_0 \in \mathbb{N}$  such that  $\langle \mathcal{G}_{n_{k_0}}, \partial \rangle = \langle \mathcal{G}_{m_0}, \partial \rangle$ . But by the choice of the soft sequence  $\left( \begin{matrix} (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_1, (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_2, \\ (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_3, (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_4, \\ (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_5, (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_6, \dots, \\ (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_n, \dots \end{matrix} \right)$  we have  $i > k_0$  this implies that  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})$  does not  $\langle \mathcal{G}_{m_0}, \partial \rangle$ . Since  $\langle \mathcal{G}_{m_0}, \partial \rangle$  is NS b-open set containing  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})$ , Since  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})$  was arbitrary,  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is not NS sequentially compact, which is a plain contradiction. Hence  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NSb countably compact.

**Theorem 7.13.** Every NS co-finite NSTS  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NSb separable.

**Proof:** case1: If  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NS countable, then clearly  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is a NSb countable dense soft sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  and therefore, in this case,  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NSb separable.

Case2: suppose  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NS uncountable. Then, there exists an infinite NSb countable soft sub-set  $\langle \mathfrak{f}, \partial \rangle$  of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . Now,  $\overline{\langle \mathfrak{f}, \partial \rangle}$  is the smallest NSb closed superset of  $\langle \mathfrak{f}, \partial \rangle$  and in the soft co-finite space  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ , the only NSb closed sub-sets of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  are  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  and finite soft sets. Results in  $\overline{\langle \mathfrak{f}, \partial \rangle} = \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . Thus,  $\langle \mathfrak{f}, \partial \rangle$  is a NSb countable dense soft sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . This signifies that  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is soft separable. Hence, every NSb co-finite NSSTS.

**Theorem 7.14.** If  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is NSSTS such that it is NS second countable then it has the characteristics NSseparability.

**Proof:** Suppose  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be NSsecond countable space.

Let  $\mathfrak{B} = \langle \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \dots, \mathcal{B}_n : n \in \mathbb{N} \rangle$  be a NSb countable base for  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . Choose  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_n \in \mathcal{B}_n$  for each  $n$ . Then, the set  $\langle \mathfrak{f}, \partial \rangle = \{n : (x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_n \in \mathcal{B}_n\}$  is NS countable. Only remaining to prove that  $\langle \mathfrak{f}, \partial \rangle$  is soft dense in  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . suppose  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter}) \in \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  and let  $\langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})}$  be NS b open set absorbing  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})$ . Then,  $\mathfrak{B}$  being a NSbase, there exists a NS b open set  $\mathcal{B}_{n_0}$  in  $\mathfrak{B}$  such that  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter}) \in \mathcal{B}_{n_0} \subseteq \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})}$ . But, by our choice of  $\langle \mathfrak{f}, \partial \rangle$ , the soft set  $\mathcal{B}_{n_0}$  contains a point  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})_{n_0}$  of  $\langle \mathfrak{f}, \partial \rangle$  that is every NSb open set containing  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})$  contain at least one point of  $\langle \mathfrak{f}, \partial \rangle$ . So,  $(x^e_{(a,b,c)}, \overline{\mathfrak{A}parameter})$  is soft adherent point of

$\langle \mathcal{F}, \partial \rangle$ . Thus, every point of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  is soft adherent point of  $\langle \mathcal{F}, \partial \rangle$ . that is  $\overline{\langle \mathcal{F}, \partial \rangle} = \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . It follows, therefore, that  $\langle \mathcal{F}, \partial \rangle$  is soft countable dense soft sub-set of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Hence,  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  is NSb seperable.

**Theorem 7.15.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  be a second soft neutrosophic countable NS space is NSLindelof space.

**Proof:** Let  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  be a second NS countable topological space and let  $\mathfrak{B} = \langle \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \dots, \mathcal{B}_n : n \in \mathbb{N} \rangle$  soft base for  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Let  $\mathcal{C} = \{ \langle \mathcal{G}_i, \partial \rangle : i \in \mathbb{N} \}$  that is  $\{ \langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \dots : i \in \mathbb{N} \}$  be any NSb open cover of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Then for each  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  there exists a NSb open set

$\langle \mathcal{G}, \partial \rangle_{\alpha_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}}$  and  $\mathfrak{B}$  being a NSbbases, corresponding to each such NSb open set there exists a NSb open set  $\langle \mathcal{G}, \partial \rangle_{n_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}}$  in  $\mathfrak{B}$  such that  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{B}_{n_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}} \subseteq \langle \mathcal{G}, \partial \rangle_{n_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}}$ . Therefore,  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle = \bigcup \left\{ \mathcal{B}_{n_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}} : (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{X} \right\} \subseteq \bigcup \left\{ \langle \mathcal{G}, \partial \rangle_{n_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}} : (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{X} \right\}$ . Now,  $\left\{ \mathcal{B}_{n_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}} : (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{X} \right\}$  being s soft sub-family of  $\mathcal{C}$  covering  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ . Thus, every NSb open covering of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  is reducible to a soft sub-covering Hence,  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  is a NSb Lindelof space

**Theorem 7.16.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  be a NS Lindelof space, then this space need not always be second NScountable.

**Proof:** Let  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  be a NS co-finite topological space provided  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  is NS uncountable. Now, Let  $\mathcal{C} = \{ \langle \mathcal{G}_i, \partial \rangle : i \in \mathbb{N} \}$  that is  $\{ \langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \dots : i \in \mathbb{N} \}$  be any NSb open cover of  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$ .  $\langle \mathcal{G}, \partial \rangle_{\alpha_0}$  be an arbitrary member of  $\{ \langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \dots : i \in \mathbb{N} \}$ . Then,

$$(\langle \mathcal{G}, \partial \rangle_{\alpha_0})^c \text{ is finite. Let, } (\langle \mathcal{G}, \partial \rangle_{\alpha_0})^c = \left( (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})_1, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})_2, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})_3, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})_4, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})_5, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})_6, \dots, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})_n, \dots \right). \text{ Now,}$$

$\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle = \langle \mathcal{G}, \partial \rangle_{\alpha_0} \cup (\langle \mathcal{G}, \partial \rangle_{\alpha_0})^c$ , Where,  $(\langle \mathcal{G}, \partial \rangle_{\alpha_0})^c$  absorbs  $(n)$  points of  $(\langle \mathcal{G}, \partial \rangle_{\alpha_0})^c$  are NSb covered by at the most  $(n)$  sets in  $\{ \langle \mathcal{G}_i, \partial \rangle : i \in \mathbb{N} \}$  that is  $\{ \langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \dots : i \in \mathbb{N} \}$  and so  $\langle \mathcal{X}^{crip}, \mathfrak{I}, \partial \rangle$  is covered by the most  $(n + 1)$  sets in

$\{\langle \mathcal{G}_i, \partial \rangle : i \in N\}$  that is  $\{\langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \dots : i \in N\}$ . Thus,  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  is NSb compact space and therefore, a NS Lindelof space. Now, if possible, let there be a NS countable base  $\mathfrak{B}$  for  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$ . Let  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{X}$ . Then,  $\tilde{\cap} \{ \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \in \langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle \text{ and } (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} = \{ (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \}$ , for, if  $(y^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter}) \neq (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})$ , then either  $(y^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter}) > (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})$  or  $(y^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter}) < (x^e_{(a,b,c)}, \mathfrak{I}^{parameter})$ , then  $\mathcal{X} \setminus \{ (y^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter}) \}$  is clearly NSb open sets containing  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})$  but not  $(y^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter})$  and therefore any  $(y^{e'}_{(a',b',c')}, \mathfrak{I}^{parameter})$ , different from  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})$ . More-over,  $\mathfrak{B} = \langle \mathcal{B}_1, \mathcal{B}_2, \mathcal{B}_3, \mathcal{B}_4, \dots, \mathcal{B}_n : n \in \mathbb{N} \rangle$  being soft base, for each  $\langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \exists \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \in \mathfrak{B}$  such that  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \subseteq \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})}$ . Obviously,  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \tilde{\cap} \{ \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} : \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \in \mathfrak{B}, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \subseteq \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \} \subseteq \tilde{\cap} \{ \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} : \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \in \langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \} = \{ (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \}$  or  $(x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \tilde{\cup} \{ (\mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})})^c : \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \in \mathfrak{B}, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \subseteq \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \} = \mathcal{X} \setminus \{ (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \}$ . But, this is false, since  $\tilde{\cup} \{ (\mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})})^c : \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \in \mathfrak{B}, (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \in \mathcal{B}_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \subseteq \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{I}^{parameter})} \}$  being a NS countable union of finite soft sets is NS countable while  $\mathcal{X}^{crip} \setminus \{ (x^e_{(a,b,c)}, \mathfrak{I}^{parameter}) \}$  is soft un-countable. So, there does not exist a NS countable soft base for  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$ , that is,  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  is not NS countable. Thus,  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  is a NS Lindelof space which is not second soft count-able.

**Theorem 7.17.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$  be a NS Lindelof space and  $\langle Y^{crip}, \mathfrak{T}_Y, \partial \rangle$  be soft sub-space of  $\langle \mathcal{X}^{crip}, \mathfrak{T}, \partial \rangle$ , then guaranteedly,  $\langle Y^{crip}, \mathfrak{T}_Y, \partial \rangle$  is NS Lindelof space.

**Proof:** Given  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$  be a NSLindel space and  $\langle Y^{crip}, \mathfrak{A}_Y, \partial \rangle$  be soft sub-space of  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$ . Let  $C = \{\langle \mathcal{H}, \partial \rangle_\alpha\}$  be any  $\langle Y^{crip}, \mathfrak{A}_Y, \partial \rangle$  NSb open covering  $Y^{crip}$ . Then,  $Y^{crip} = \tilde{U} \langle \mathcal{H}, \partial \rangle_\alpha$ . Also,  $\langle \mathcal{H}, \partial \rangle_\alpha = Y \tilde{\cap} \langle \mathcal{G}, \partial \rangle_\alpha$  where  $\langle \mathcal{G}, \partial \rangle_\alpha \in \langle X^{crip}, \mathfrak{A}, \partial \rangle$ . Therefore  $Y = \tilde{U}_\alpha (Y \tilde{\cap} \langle \mathcal{G}, \partial \rangle_\alpha) \in \tilde{U}_\alpha \langle \mathcal{G}, \partial \rangle_\alpha$ . So  $\langle X^{crip}, \mathfrak{A}, \partial \rangle = Y \tilde{U} \tilde{Y}^c \in (\langle \mathcal{G}, \partial \rangle_\alpha) \tilde{U} \tilde{Y}^c$ . Thus,  $\mathbb{C}^* = \{\langle \mathcal{G}, \partial \rangle_\alpha, Y^c\}$  is soft b open covering of the NS Lindel space of  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$ . Since,  $Y^c$  covers no part of  $Y$ , so there exists a NScountable number of  $\langle \mathcal{G}, \partial \rangle_{\alpha_i}$ s in  $\mathbb{C}^*$  such that  $Y \in \tilde{U} \{\langle \mathcal{G}, \partial \rangle_{\alpha_i} : i \in \Lambda \in N\}$  or  $Y = \tilde{U} \{Y \tilde{\cap} \langle \mathcal{G}, \partial \rangle_{\alpha_i} : i \in \Lambda \in N\}$ . Therefore,  $Y = \tilde{U} \{\langle \mathcal{H}, \partial \rangle_{\alpha_i} : i \in \Lambda \in N\}$ . This shows that  $\mathbb{C}^*$  is reducible to a NScountable subcovering. Hence,  $\langle Y^{crip}, \mathfrak{A}_Y, \partial \rangle$  is also a NSb Lindelof space.

**Theorem 7.18.** Let  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$  be a NSb<sub>1</sub> space and  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}), (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in X^{crip}$  such that  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) > (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  or  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) < (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$ . If  $\mathfrak{B}_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}$  is a NSb local base at  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$ , then there exists at least one member of  $\mathfrak{B}_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}$  which does not supposes  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$ .

**Proof:** Since  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$  be a NSb<sub>1</sub> space and  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) > (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  or  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) < (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$ ,  $\exists$  NSb open sets  $\langle \mathcal{G}, \partial \rangle$  and  $\langle \mathcal{H}, \partial \rangle$  such that  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \langle \mathcal{G}, \partial \rangle$  but  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin \langle \mathcal{G}, \partial \rangle$  and  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in \langle \mathcal{H}, \partial \rangle$  but  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \notin \langle \mathcal{H}, \partial \rangle$ . Since,  $\mathfrak{B}_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}$  is NS local base at  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  there exists  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in B \in \langle \mathcal{G}, \partial \rangle$ . Since  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin \langle \mathcal{G}, \partial \rangle$  and  $B \in \langle \mathcal{G}, \partial \rangle$ , so  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin B$ . Thus,  $B \in \mathfrak{B}_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}$  such that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \notin B$ .

**Theorem 7.19.** Let  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$  be a NSSTS such that it is NSof b<sub>1</sub> space in which every in-finite soft subset has a soft limit point. Then,  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$  is definitely NSbcompact.

**Proof:** Let  $C$  NS open covering of  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$ . Then  $\langle X^{crip}, \mathfrak{A}, \partial \rangle$  being NS Lindelof space,  $C$  is reducible to a NS countable sub-covering, say  $C^* = \{\langle \mathcal{G}_n, \partial \rangle : n \in \Lambda \in N\}$  that is  $\{\langle \mathcal{G}_1, \partial \rangle, \langle \mathcal{G}_2, \partial \rangle, \langle \mathcal{G}_3, \partial \rangle, \langle \mathcal{G}_4, \partial \rangle, \dots : n \in \Lambda \in N\}$ . If possible, let  $C^*$  is not reducible to a finite soft subcovering. Then, for any positive integer  $k$ , the soft  $(\cup_{i=1}^k \langle \mathcal{G}_n, \partial \rangle)^c$  is NSb open proper subset of

$\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  and therefore, its complement,  $F_k = (\cup_{i=1}^k \langle \mathcal{G}_n, \partial \rangle)^c$  is non-empty NS closed subset of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ . Now, taking  $k = 1, 2, 3, \dots$  we obtain a nested soft sequence  $\langle F_k \rangle$  of soft neutrosophic closed subsets of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  such that  $(F_1, \partial) \ni (F_2, \partial) \ni (F_3, \partial) \ni (F_4, \partial) \ni (F_5, \partial) \ni (F_6, \partial) \ni \dots \ni (F_n, \partial) \ni \dots$ . Let  $A = \left\{ (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_k : (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})_k \in F_k \right\}$ , then, the soft set  $A$  is obviously an infinite soft set. So, by the given hypothesis,  $A$  has a soft limit point, suppose  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$ . But  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  being NSb<sub>1</sub> space, so every NSb open set containing  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  must therefore contain an infinite number of points of  $A$ . Result in  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  is a soft limit point of each  $F_k$  that is  $(F_1, \partial), (F_2, \partial), (F_3, \partial), (F_4, \partial), (F_5, \partial), \dots$ . But each of  $(F_1, \partial), (F_2, \partial), (F_3, \partial), (F_4, \partial), (F_5, \partial), \dots$  is soft b-closed,  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (F_1, \partial), (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (F_2, \partial), (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (F_3, \partial), (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (F_4, \partial), (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (F_5, \partial), (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (F_6, \partial), \dots$ . This contradicts the fact that  $\mathcal{C}^*$  as a soft covering of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  and hence  $\mathcal{C}$  is reducible to a finite soft sub covering.

**Theorem 7.20.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  be NSregular Lindelop space then it is safely NSbnormal.

**Proof:**  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  NSbregular Lindelop space and let  $\langle \Psi_1, \partial \rangle, \langle \Psi_2, \partial \rangle$  be two NSb closed sub-sets of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  such that these are mutually exclusive. Then, every NSb closed sub-space of a NSLindelop space is soft Lindelop space. It is then guaranteed that  $\langle \Psi_1, \partial \rangle, \langle \Psi_2, \partial \rangle$  are NS Lindelop spaces. Now, by the NSb regularity of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ , corresponding to NSb closed set  $\langle \Psi_1, \partial \rangle$  and every  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \langle \Psi_2, \partial \rangle \ni$  soft NSb open set  $\langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}$  such that  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \langle \mathcal{G}, \partial \rangle \in \overline{\langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})}} \in (\langle \Psi_1, \partial \rangle)^c$ . More-over, the soft family  $\left\{ \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})} : (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \langle \Psi_2, \partial \rangle \right\}$  is clearly NSb open covering of the Lindelop space NS set  $\langle \Psi_2, \partial \rangle$ . So, it must suppose NS countable sub-covering  $\left\{ \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})} : i \in N \right\}$ .

Again, by the NSb regularity of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ , corresponding to the NSb closed set  $\langle \Psi_2, \partial \rangle$  and every  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \langle \Psi_1, \partial \rangle \ni$  NSb open set  $\langle \Psi_3, \partial \rangle_{(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}$  such that  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in \langle \Psi_3, \partial \rangle_{(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})} \in \overline{\langle \Psi_3, \partial \rangle_{(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})}} \in (\langle \Psi_2, \partial \rangle)^c$ .

Clearly,  $\left\{ \langle \Psi_3, \partial \rangle_{(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})} : (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in \langle \Psi_1, \partial \rangle \right\}$  is NS open covering of the

Lindelof space  $\langle \Psi_1, \partial \rangle$  and therefore, it is frezzable to a *NSb* countable sub-covering  $\langle \Psi_3, \partial \rangle_{\left( \overline{y^{e'}_{(a',b',c')}} \right)^{\mathfrak{A}parameter}} : i \in N$ . Let  $\langle \mathcal{M}, \partial \rangle_n = \langle \Psi_1, \partial \rangle_{\left( \overline{y^{e'}_{(a',b',c')}} \right)^{\mathfrak{A}parameter}} - \bigcup_{i \in n} \left\{ \overline{\langle \Psi_3, \partial \rangle_{\left( \overline{y^{e'}_{(a',b',c')}} \right)^{\mathfrak{A}parameter}}} : i \leq n \right\} = \langle \Psi_1, \partial \rangle_{\left( \overline{y^{e'}_{(a',b',c')}} \right)^{\mathfrak{A}parameter}} \cap [ \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - \bigcup \left\{ \overline{\langle \Psi_3, \partial \rangle_{\left( \overline{y^{e'}_{(a',b',c')}} \right)^{\mathfrak{A}parameter}}} : i \leq n \right\} ]$  and  $\langle \omega, \partial \rangle_n = \langle \mathcal{G}, \partial \rangle_{x_n} - \bigcup_{i \in n} \left\{ \overline{\langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)})^{\mathfrak{A}parameter}}} : i \leq n \right\} = \langle \mathcal{G}, \partial \rangle_{(x^e_{(a,b,c)})^{\mathfrak{A}parameter}} \tilde{\cap} [ \langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle - \bigcup \left\{ \overline{\langle \mathcal{G}, \partial \rangle_{x_i}} : i \leq n \right\} ]$ . Then,  $\langle \mathcal{M}, \partial \rangle_n$  and  $\langle \omega, \partial \rangle_n$  are clearly *NSb* open sets and therefore, so are the sets  $\langle \mathcal{G}, \partial \rangle = \bigcup \{ \langle \mathcal{M}, \partial \rangle_n : n \in N \}$  and  $\langle \mathbb{H}, \partial \rangle = \langle \omega, \partial \rangle_n : n \in N$ . Now,  $\langle \Psi_1, \partial \rangle \in \bigcup \langle \Psi_3, \partial \rangle_{\left( \overline{y^{e'}_{(a',b',c')}} \right)^{\mathfrak{A}parameter}} : i \in N$  and  $\overline{\langle \Psi_3, \partial \rangle_{\left( \overline{y^{e'}_{(a',b',c')}} \right)^{\mathfrak{A}parameter}}} \tilde{\cap} \langle \Psi_1, \partial \rangle = (x^e_{(a,b,c)})^{\mathfrak{A}parameter}$ . So, it follows that  $\{ \langle \mathcal{M}, \partial \rangle_n : n \in N \}$  is *NSb* open covering of  $\langle \Psi_1, \partial \rangle$ . Therefore,  $\langle \Psi_1, \partial \rangle \in \bigcup \{ \langle \mathcal{M}, \partial \rangle_n : n \in N \} = \langle \mathcal{G}, \partial \rangle$ , Similarly,  $\langle \Psi_2, \partial \rangle \in \langle \mathbb{H}, \partial \rangle$ . Also,  $\langle \mathcal{M}, \partial \rangle_n \tilde{\cap} \langle \omega, \partial \rangle_n = \tilde{\emptyset}$  for each  $n$  that is  $\langle \mathcal{M}, \partial \rangle_1 \tilde{\cap} \langle \omega, \partial \rangle_1 = \tilde{\emptyset}$ ,  $\langle \mathcal{M}, \partial \rangle_2 \tilde{\cap} \langle \omega, \partial \rangle_2 = \tilde{\emptyset}$ ,  $\langle \mathcal{M}, \partial \rangle_3 \tilde{\cap} \langle \omega, \partial \rangle_3 = \tilde{\emptyset}$ ,  $\langle \mathcal{M}, \partial \rangle_4 \tilde{\cap} \langle \omega, \partial \rangle_4 = \tilde{\emptyset}$ ,  $\langle \mathcal{M}, \partial \rangle_5 \tilde{\cap} \langle \omega, \partial \rangle_5 = \tilde{\emptyset}$ ,  $\langle \mathcal{M}, \partial \rangle_6 \tilde{\cap} \langle \omega, \partial \rangle_6 = \tilde{\emptyset}$ ..... This guarantees that  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  is soft neutrosophic normal.

**Theorem 7.21.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  NSSTS and Suppose  $\langle \mathfrak{f}, \partial \rangle, \langle \mathfrak{g}, \partial \rangle$  be two NScontinuous function on a NS topological space  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  in to a NSSTS  $\langle Y^{crip}, \mathfrak{B}, \partial \rangle$  which is *NSb* Hausdorff. Then, soft set  $\{ (x^e_{(a,b,c)})^{\mathfrak{A}parameter} \in \mathcal{X}^{crip} : (\mathfrak{f})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} = (\mathfrak{g})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} \}$  is *NSb* closed of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$ .

**Proof:** Let If  $\{ (x^e_{(a,b,c)})^{\mathfrak{A}parameter} \in \mathcal{X}^{crip} : (\mathfrak{f})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} = (\mathfrak{g})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} \}$  is a NS set of function. If  $\{ (x^e_{(a,b,c)})^{\mathfrak{A}parameter} \in \mathcal{X}^{crip} : (\mathfrak{f})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} = (\mathfrak{g})(x) \}^c = \tilde{\emptyset}$ , it is clearly *NSb* open and therefore,  $\{ (x^e_{(a,b,c)})^{\mathfrak{A}parameter} \in \mathcal{X}^{crip} : (\mathfrak{f})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} = (\mathfrak{g})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} \}$  is *NSb* closed, that is nothing is proved in this case. Let us consider the case when  $\{ (x^e_{(a,b,c)})^{\mathfrak{A}parameter} \in \mathcal{X}^{crip} : (\mathfrak{f})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} = (\mathfrak{g})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} \}^c \neq (x^e_{(a,b,c)})^{\mathfrak{A}parameter}$ . And let  $\rho \in \{ (x^e_{(a,b,c)})^{\mathfrak{A}parameter} \in \mathcal{X}^{crip} : (\mathfrak{f})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} = (\mathfrak{g})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} \}^c$ . Then  $\rho$  does not belong  $\{ (x^e_{(a,b,c)})^{\mathfrak{A}parameter} \in \mathcal{X}^{crip} : (\mathfrak{f})(x^e_{(a,b,c)})^{\mathfrak{A}parameter} = (\mathfrak{g})(x) \}$ . Result in  $(\mathfrak{f})(\rho) \neq (\mathfrak{g})(\rho)$ . Now,  $\langle Y^{crip}, \mathfrak{B}, \partial \rangle$  being



NSbHausdorff space so there exists NSbopen sets  $\langle \mathcal{G}, \partial \rangle$  and  $\langle \mathcal{H}, \partial \rangle$  of  $(\mathcal{F})(\rho)$  and  $(\mathcal{G})(\rho)$  respectively such that  $\langle \mathcal{G}, \partial \rangle$  and  $\langle \mathcal{H}, \partial \rangle$  such that these NS sets such that the possibility of one rules out the possibility of other. By soft continuity of  $\langle \mathcal{F}, \partial \rangle, \langle \mathcal{G}, \partial \rangle, \langle \mathcal{F}, \partial \rangle^{-1}$  as well as  $\langle \mathcal{G}, \partial \rangle^{-1}$  is NSbopen nhd of  $\rho$  and therefore, so is  $\langle \mathcal{F}, \partial \rangle^{-1} \tilde{\cap} \langle \mathcal{G}, \partial \rangle^{-1}$  is contained in  $\{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \mathcal{X}^{crip} : (\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = (\mathcal{G})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}$ , for,  $(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in (\langle \mathcal{F}, \partial \rangle^{-1} \tilde{\cap} \langle \mathcal{G}, \partial \rangle^{-1}) \Rightarrow (\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \langle \mathcal{G}, \partial \rangle$  and  $(\mathcal{G})(\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \neq (\mathcal{G})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})$  because  $\langle \mathcal{G}, \partial \rangle$  and  $\langle \mathcal{H}, \partial \rangle$  are mutually exclusive. This implies that  $x$  does not belong to  $\{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \mathcal{X}^{crip} : (\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) =$

$$(\mathcal{G})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}. \text{Therefore } \left\{ \begin{array}{l} \rho \in (\mathcal{F})^{-1}(\langle \mathcal{G}, \partial \rangle) \tilde{\cap} (\mathcal{G})^{-1}(\langle \mathcal{G}, \partial \rangle) \\ \in \\ \left\{ (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \mathcal{X}^{crip} \right\}^c \\ : (\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = \\ (\mathcal{G})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \end{array} \right\} \text{ This shows that}$$

$$\left\{ (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \mathcal{X}^{crip} : \begin{array}{l} (\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = \\ (\mathcal{G})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \end{array} \right\}^c \text{ is nhd of each of its points. So, } \left\{ (x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \mathcal{X}^{crip} : \begin{array}{l} (\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = \\ (\mathcal{G})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \end{array} \right\}^c$$

NSb open and hence  $\{(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) \in \mathcal{X}^{crip} : (\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = (\mathcal{G})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}$  is NSb closed.

**Theorem 7.22.** Let  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  NSSTS such that it is NSb Hausdorff space and let  $(\mathcal{F})$  be soft continuous function of  $\langle \mathcal{X}^{crip}, \mathfrak{A}, \partial \rangle$  into itself. Then, the NS set of fixed points under  $(\mathcal{F})$  is a NSb closed set.

**Proof:** Let  $\delta = \{(\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}$ . If  $\delta^c = \tilde{\emptyset}$ , Then is NSb open and

therefore  $\{(\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}$  NSb closed. So, let

$$\{(\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}^c \neq \tilde{\emptyset} \quad \text{and} \quad \text{let} \quad (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \in$$

$$\{(\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}^c \quad . \text{Then,}$$

$(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  does not belong to  $\{(\mathcal{F})(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}) = (x^e_{(a,b,c)}, \mathfrak{A}^{parameter})\}$  and

therefore  $(\mathcal{F})(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}) \neq (y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$ . Now,  $(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter})$  and

$(f)\left(\left(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right)$  being two distinct points of the NSb Hasdorff space  $\langle \mathcal{X}^{crisp}, \mathfrak{T}, \partial \rangle$ , so there exists NSb open sets  $\langle \mathcal{G}, \partial \rangle$  and  $\langle \mathfrak{H}, \partial \rangle$  such that  $\left(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right) \in \langle \mathcal{G}, \partial \rangle, (f)\left(\left(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right)\right) \in \langle \mathfrak{H}, \partial \rangle$  and  $\langle \mathcal{G}, \partial \rangle, \langle \mathfrak{H}, \partial \rangle$  are disjoint. Also, by the NS continuity of  $(f)$ ,  $(f)^{-1}(\langle \mathfrak{H}, \partial \rangle)$  is NS b open set containing  $y$ . We pretend that  $\langle \mathcal{G}, \partial \rangle \tilde{\cap} (f)^{-1}(\langle \mathfrak{H}, \partial \rangle) \in \{(f)\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\}^c$ . Snice,  $\mu \in \langle \mathcal{G}, \partial \rangle \tilde{\cap} (f)^{-1}(\langle \mathfrak{H}, \partial \rangle) \Rightarrow \mu \in \langle \mathcal{G}, \partial \rangle \& \mu \in (f)^{-1} \Rightarrow \mu \in \langle \mathcal{G}, \partial \rangle \& (f)(\mu) \in \langle \mathfrak{H}, \partial \rangle \Rightarrow \mu \neq (f)(\mu)$ . As  $\langle \mathcal{G}, \partial \rangle \tilde{\cap} \langle \mathfrak{H}, \partial \rangle = \tilde{\emptyset} \Rightarrow \mu$  does not belong to  $\{(f)\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\} \Rightarrow \mu \in \{(f)\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\}^c$ . Therefore,  $\left(y^{e'}_{(a',b',c')}, \mathfrak{A}^{parameter}\right) \in \langle \mathcal{G}, \partial \rangle \tilde{\cap} (f)^{-1}(\langle \mathfrak{H}, \partial \rangle) \in \{(f)\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\}^c$ . Thus,  $\{(f)\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\}^c$  is the NS nhd of each of its points. So,  $\{(f)\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\}^c$  is NSb open and hence  $\{(f)\left(\left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\right) = \left(x^e_{(a,b,c)}, \mathfrak{A}^{parameter}\right)\}$  is NS b-closed.

### 8. Conclusion

In this paper, neutrosophic soft points with one point greater than the other and their properties, generalized neutrosophic soft open set known as b-open set, neutrosophic soft separation axioms theoretically and with support of suitable examples with respect to soft points, neutrosophic soft  $b_0$ -space engagement with generalized neutrosophic soft closed set, neutrosophic soft  $b_2$ -space engagement with generalized neutrosophic soft open set are addressed. In continuation, neutrosophic soft  $b_0$ -space behave as neutrosophic soft  $b_2$ -space with the plantation of some extra condition on soft  $b_0$ -space, neutrosophic soft  $b_3$ -space and related theorems, neutrosophic soft  $b_4$ -space, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft separation axioms, monotonous behavior of neutrosophic soft function with connection of different neutrosophic soft close sets are reflected. Secondly, long touched has been given to neutrosophic soft countability connection with bases and sub-bases, neutrosophic soft product spaces and its engagement through different generalized neutrosophic soft open set and close sets, neutrosophic soft coordinate spaces and its engagement through different generalized neutrosophic soft open set and close sets, Finally, neutrosophic soft countability and its relationship with Bolzano Weirstrass Property through engagement of compactness, neutrosophic soft strongly spaces and its

related theorems, neutrosophic soft sequences and its relation with neutrosophic soft compactness, neutrosophic soft Lindelof space and related theorems are supposed to address.

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### Conflicts of Interest

The authors declare that they have no conflict of interest.

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