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MCGDM Approach Using the Weighted Hyperbolic Sine Similarity Measure of Neutrosophic (Indeterminate Fuzzy) Multivalued Sets for the Teaching Quality Assessment of Teachers

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Abstract: A neutrosophic (indeterminate fuzzy) multivalued set (NMS) can be effectively described by neutrosophic number sequences with identical or different neutrosophic numbers $z_i = \mu_i + v_iI \subseteq [0, 1]$ ($i = 1, 2, \ldots, q$) for $\mu, v \in R$ and $I \in [I^-, I^+]$. Therefore, NMS is a stronger and more valuable tool for describing indeterminate fuzzy multivalued information. In this article, we propose the weighted hyperbolic sine similarity measure of NMSs to deal with the multi-criteria group decision-making (MCGDM) issue of teaching quality assessment with different indeterminate ranges of decision makers. To do so, first according to the hyperbolic sine function, we propose a hyperbolic sine similarity measure of NMSs and a weighted hyperbolic sine similarity measure of NMSs and investigate their desirable properties. Second, we develop a MCGDM approach with some indeterminate ranges in terms of the proposed weighted hyperbolic sine similarity measure of NMSs. Lastly, an illustrative example on the teaching quality assessment of teachers is presented to illustrate the applicability of the developed approach, then the developed approach is compared with the existing related approach to reveal the effectiveness of the developed approach for the teaching quality assessment of teachers in the environment of NMSs.

Keywords: neutrosophic (indeterminate fuzzy) multivalued set; neutrosophic number; hyperbolic sine similarity measure; group decision making; teaching quality assessment

1. Introduction

Fuzzy set (FS) [1] is represented by the degree of membership, which occurs only once for each element. Since FS can describe problems related to imprecise and ambiguous judgments, it has been used in various applications [2-7]. To express that an element occurs more than once with identical or different membership values, a fuzzy multiset (FM) [8-10] was proposed as the generalization of FS. Then, FMs were used for some applications, such as decision making and data analysis, clustering analysis, and medical diagnosis [11-16].

To describe the vagueness and indeterminacy of human judgments in real life environment, the neutrosophic number $z = \mu + vI$ for $\mu, v \in R$ and $I \in [I^-, I^+]$ introduced by Smarandache [17-19] can flexibly indicate indeterminate information according to an indeterminate range of I. Therefore, it is also regarded as a variable neutrosophic number, depending on indeterminate ranges of I. In a multi-criteria group decision-making (MCGDM) problem, to express multiple evaluation values of a criterion to an alternative given by multiple decision makers, Du and Ye [20] proposed a neutrosophic (indeterminate fuzzy) multivalued set (NMS), which is described by neutrosophic number sequences (NNS) with different and/or identical neutrosophic numbers ($z_i = [\mu_i + v_i I^-, \mu_i + v_i I^+] \subseteq [0, 1]$ (i = 1, 2, …, q) for μ, $v \in R$ and $I \in [I^-, I^+]$), as a particularly challenging generalization of FM, and then they developed the parameterized correlation coefficients (PCCs) of NMSs to perform MCGDM problems with some indeterminate ranges of decision makers. In indeterminate MCGDM problems, NMS implies its highlighting advantage in expressing indeterminate fuzzy multivalued problems with indeterminate ranges of $I \in [I^-, I^+]$. However, it is worth noting that the similarity measure is a key mathematical tool in decision-making problems, but unfortunately there is no similarity measure between NMSs in the current research. Therefore, this paper proposes a hyperbolic sine similarity measure (HSSM) between two NMSs and a weighted HSSM between two NMSs in view of the hyperbolic sine function, and then develops a MCGDM approach using the weighted HSSM to solve the assessment problem of teachers' teaching quality with some indeterminate ranges of decision makers in the setting of NMSs.

The rest of this article consists of the following sections. Section 2 reviews some notions of NMSs. In Section 3, the HSSM and weighted HSSM of NMSs are proposed according to the hyperbolic sine function. Section 4 develops a MCGDM approach using the weighted HSSM of NMSs along with specific indeterminate ranges of decision makers in the environment of NMSs. In Section 5, the developed MCGDM approach is applied in an illustrative example on the teaching quality assessment of teachers with some indeterminate ranges of decision makers. In Section 6, a comparative analysis with the related method is given to reveal the efficiency of the developed MCGDM approach in the environment of NMSs. Conclusions and future research are addressed in Section 7.

2. Some Notions of NMSs

Definition 1 [20]. Let $Z = \{z_1, z_2, \ldots, z_m\}$ be a fixed set. A neutrosophic (indeterminate fuzzy) multivalued set (NMS) E on Z is denoted by $E = \left\{\left\langle z_k, e_E(z_k, I)\right\rangle \mid z_k \in Z, I \in [I^-, I^+]\right\}$, where $e_E(z_k, I)$ is the increasing neutrosophic number sequence $e_E(z_k, I) = \left(e^1(z_k, I), e^2(z_k, I), \ldots, e^{p_k}(z_k, I)\right)$ with identical and/or different neutrosophic numbers $e_E^i(z_k, I) = \mu_k^i + v_k^i I \subseteq e_E^{i+1}(z_k, I) = \mu_k^{i+1} + v_k^{i+1} I \subseteq [0,1]$ (i = 1, 2, …, p_k; k = 1, 2, …, m) of an element z_k to the set E for $I \in [I^-, I^+]$, μ_k^i, $v_k^i \in R$ and $z_k \in Z$.

For convenient expression, each element $e_E(z_k, I)$ (k = 1, 2, …, m) in Z is simply represented as the NNS $e_{Ek}(I) = (e_k^1(I), e_k^2(I), \ldots, e_k^{p_k}(I))$ for $I \in [I^-, I^+]$. If $e_k^i(I) = [\mu_k^i + v_k^i I^-, \mu_k^i + v_k^i I^+] \subseteq [0,1]$ or $e_k^i(I) = \mu_k^i + v_k^i I \in [0,1]$ (i = 1, 2, …, p_k; k = 1, 2, …, m) in $e_{Ek}(I)$, the NNS $e_{Ek}(I)$ can contain an interval-valued fuzzy sequence or a single-valued fuzzy sequence depending on a range/value of I. It is obvious that NMS contains the fuzzy multivalued set and interval-valued fuzzy multivalued set.

Definition 2 [20]. Let two NNSs be $e_{1k}(I) = (e_{1k}^1(I), e_{1k}^2(I), \ldots, e_{1k}^{p_k}(I))$ and $e_{2k}(I) = (e_{2k}^1(I), e_{2k}^2(I), \ldots, e_{2k}^{p_k}(I))$ with neutrosophic numbers $e_{1k}^i(I) = \mu_{1k}^i + v_{1k}^i I \subseteq [0,1]$ and $e_{2k}^i(I) = \mu_{2k}^i + v_{2k}^i I \subseteq [0,1]$ for $I \in [I^-, I^+]$ and $\mu_{1k}^i, v_{1k}^i, \mu_{2k}^i, v_{2k}^i \in R$ (i = 1, 2, …, p_k; k = 1, 2, …, m). Then, their relations are indicated below:

(1) $e_{1k}(I) \supseteq e_{2k}(I) \Leftrightarrow e_{1k}^i(I) = \mu_{1k}^i + v_{1k}^i I \supseteq e_{2k}^i(I) = \mu_{2k}^i + v_{2k}^i I$;

(2) $e_{1k}(I) = e_{2k}(I) \Leftrightarrow e_{1k}(I) \supseteq e_{2k}(I)$ and $e_{2k}(I) \supseteq e_{1k}(I)$;

(3)
$$e_{1k}(I) \cup e_{2k}(I) = \left(e_{1k}^1(I) \cup e_{2k}^1(I), e_{1k}^2(I) \cup e_{2k}^2(I), \ldots, e_{1k}^{p_k}(I) \cup e_{2k}^{p_k}(I)\right)$$
$$= \begin{pmatrix} [\mu_{1k}^1 + v_{1k}^1 I^- \vee \mu_{2k}^1 + v_{2k}^1 I^-, \mu_{1k}^1 + v_{1k}^1 I^+ \vee \mu_{2k}^1 + v_{2k}^1 I^+], \\ [\mu_{1k}^2 + v_{1k}^2 I^- \vee \mu_{2k}^2 + v_{2k}^2 I^-, \mu_{1k}^2 + v_{1k}^2 I^+ \vee \mu_{2k}^2 + v_{2k}^2 I^+], \ldots, \\ [\mu_{1k}^{p_k} + v_{1k}^{p_k} I^- \vee \mu_{2k}^{p_k} + v_{2k}^{p_k} I^-, \mu_{1k}^{p_k} + v_{1k}^{p_k} I^+ \vee \mu_{2k}^{p_k} + v_{2k}^{p_k} I^+] \end{pmatrix};$$

(4)
$$e_{1j}(I)\cap e_{2j}(I)=\left(e_{1k}^{1}(I)\cap e_{2k}^{1}(I),e_{1k}^{2}(I)\cap e_{2k}^{2}(I),...,e_{1k}^{p_k}(I)\cap e_{2k}^{p_k}(I)\right)$$
$$=\begin{pmatrix}[\mu_{1k}^{1}+v_{1k}^{1}I^{-}\wedge\mu_{2k}^{1}+v_{2k}^{1}I^{-},\mu_{1k}^{1}+v_{1k}^{1}I^{+}\wedge\mu_{2k}^{1}+v_{2k}^{1}I^{+}],\\ [\mu_{1k}^{2}+v_{1k}^{2}I^{-}\wedge\mu_{2k}^{2}+v_{2k}^{2}I^{-},\mu_{1k}^{2}+v_{1k}^{2}I^{+}\wedge\mu_{2k}^{2}+v_{2k}^{2}I^{+}],...,\\ [\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{-}\wedge\mu_{2k}^{p_k}+v_{2k}^{p_k}I^{-},\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{+}\wedge\mu_{2k}^{p_k}+v_{2k}^{p_k}I^{+}]\end{pmatrix};$$

(5)
$$e_{1k}^{c}(I)=\begin{pmatrix}[1-(\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{+}),1-(\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{-})],[1-(\mu_{1k}^{p_k-1}+v_{1k}^{p_k-1}I^{+}),1-(\mu_{1k}^{p_k-1}+v_{1k}^{p_k-1}I^{-})],\\ ...,[1-(\mu_{1k}^{2}+v_{1k}^{2}I^{+}),1-(\mu_{1k}^{2}+v_{1k}^{2}I^{-})],[1-(\mu_{1k}^{1}+v_{1k}^{1}I^{+}),1-(\mu_{1k}^{1}+v_{1k}^{1}I^{-})]\end{pmatrix}$$

(Complement of $e_{1k}(I)$).

Suppose that there are two NMSs $E_1 = \{e_{11}(I), e_{12}(I), \ldots, e_{1m}(I)\}$ and $E_2 = \{e_{21}(I), e_{22}(I), \ldots, e_{2m}(I)\}$, where $e_{1k}(I)=\left(e_{1k}^{1}(I),e_{1k}^{2}(I),...,e_{1k}^{p_k}(I)\right)$ and $e_{2k}(I)=\left(e_{2k}^{1}(I),e_{2k}^{2}(I),...,e_{2k}^{p_k}(I)\right)$ ($k = 1, 2, \ldots, m$) are two collections of NNSs with neutrosophic numbers $e_{1k}^{i}(I)=\mu_{1k}^{i}+v_{1k}^{i}I\subseteq[0,1]$ and $e_{2k}^{i}(I)=\mu_{2k}^{i}+v_{2k}^{i}I\subseteq[0,1]$ for $I \in [I^-, I^+]$ and $\mu_{1k}^{i},v_{1k}^{i},\mu_{2k}^{i},v_{2k}^{i}\in R$ ($i = 1, 2, \ldots, p_k$; $k = 1, 2, \ldots, m$). Then, the importance of the NNS $e_{jk}(I)$ ($j = 1, 2$; $k = 1, 2, \ldots, m$) in E_1 and E_2 is specified by its weight $\varphi_k \in [0, 1]$ with $\sum_{k=1}^{m}\varphi_k=1$. Thus, Du and Ye [20] proposed the weighted PCCs of NMSs E_1 and E_2 with an indeterminate parameter $\rho \in [0, 1]$ below:

$$R_{w1}^{\rho}(E_1,E_2)=\frac{\sum_{k=1}^{m}\varphi_k\begin{Bmatrix}[\mu_{1k}^{1}+v_{1k}^{1}I^{-}+\rho v_{1k}^{1}(I^{+}-I^{-})][\mu_{2k}^{1}+v_{2k}^{1}I^{-}+\rho v_{2k}^{1}(I^{+}-I^{-})]\\ +[\mu_{1j}^{2}+v_{1j}^{2}I^{-}+\rho v_{1j}^{2}(I^{+}-I^{-})][\mu_{2j}^{2}+v_{2j}^{2}I^{-}+\rho v_{2j}^{2}(I^{+}-I^{-})]+\\ ...+[\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{-}+\rho v_{1k}^{p_k}(I^{+}-I^{-})][\mu_{2k}^{p_k}+v_{2k}^{p_k}I^{-}+\rho v_{2k}^{p_k}(I^{+}-I^{-})]\end{Bmatrix}}{\sqrt{\sum_{k=1}^{m}\varphi_k\begin{Bmatrix}[\mu_{1j}^{1}+v_{1j}^{1}I^{-}+\rho v_{1j}^{1}(I^{+}-I^{-})]^2\\ +[\mu_{1j}^{2}+v_{1j}^{2}I^{-}+\rho v_{1j}^{2}(I^{+}-I^{-})]^2+\\ ...+[\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{-}+\rho v_{1k}^{p_k}(I^{+}-I^{-})]^2\end{Bmatrix}}\times\sqrt{\sum_{k=1}^{m}\varphi_k\begin{Bmatrix}[\mu_{2j}^{1}+v_{2j}^{1}I^{-}+\rho v_{2j}^{1}(I^{+}-I^{-})]^2\\ +[\mu_{2j}^{2}+v_{2j}^{2}I^{-}+\rho v_{2j}^{2}(I^{+}-I^{-})]^2+\\ ...+[\mu_{2k}^{p_k}+v_{2k}^{p_k}I^{-}+\rho v_{2k}^{p_k}(I^{+}-I^{-})]^2\end{Bmatrix}}}, \quad (1)$$

$$R_{w2}^{\rho}(E_1,E_2)=\frac{\sum_{k=1}^{m}\varphi_k\begin{Bmatrix}[\mu_{1k}^{1}+v_{1k}^{1}I^{-}+\rho v_{1k}^{1}(I^{+}-I^{-})][\mu_{2k}^{1}+v_{2k}^{1}I^{-}+\rho v_{2k}^{1}(I^{+}-I^{-})]\\ +[\mu_{1j}^{2}+v_{1j}^{2}I^{-}+\rho v_{1j}^{2}(I^{+}-I^{-})][\mu_{2j}^{2}+v_{2j}^{2}I^{-}+\rho v_{2j}^{2}(I^{+}-I^{-})]+\\ ...+[\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{-}+\rho v_{1k}^{p_k}(I^{+}-I^{-})][\mu_{2k}^{p_k}+v_{2k}^{p_k}I^{-}+\rho v_{2k}^{p_k}(I^{+}-I^{-})]\end{Bmatrix}}{\max\left\{\sum_{k=1}^{m}\varphi_k\begin{Bmatrix}[\mu_{1j}^{1}+v_{1j}^{1}I^{-}+\rho v_{1j}^{1}(I^{+}-I^{-})]^2\\ +[\mu_{1j}^{2}+v_{1j}^{2}I^{-}+\rho v_{1j}^{2}(I^{+}-I^{-})]^2+\\ ...+[\mu_{1k}^{p_k}+v_{1k}^{p_k}I^{-}+\rho v_{1k}^{p_k}(I^{+}-I^{-})]^2\end{Bmatrix},\sum_{k=1}^{m}\varphi_k\begin{Bmatrix}[\mu_{2j}^{1}+v_{2j}^{1}I^{-}+\rho v_{2j}^{1}(I^{+}-I^{-})]^2\\ +[\mu_{2j}^{2}+v_{2j}^{2}I^{-}+\rho v_{2j}^{2}(I^{+}-I^{-})]^2+\\ ...+[\mu_{2k}^{p_k}+v_{2k}^{p_k}I^{-}+\rho v_{2k}^{p_k}(I^{+}-I^{-})]^2\end{Bmatrix}\right\}}.$$

(2)

3. Hyperbolic Sine Similarity Measures of NMSs

According to the hyperbolic sine function, this section proposes the HSSM and weighted HSSM between two NMSs.

Definition 3. Set two NMSs as $E_1 = \{e_{11}(I), e_{12}(I), \ldots, e_{1m}(I)\}$ and $E_2 = \{e_{21}(I), e_{22}(I), \ldots, e_{2m}(I)\}$, where $e_{1k}(I)=\left(e_{1k}^{1}(I),e_{1k}^{2}(I),...,e_{1k}^{p_k}(I)\right)$ and $e_{2k}(I)=\left(e_{2k}^{1}(I),e_{2k}^{2}(I),...,e_{2k}^{p_k}(I)\right)$ ($k = 1, 2, \ldots, m$) are two collections of NNSs with neutrosophic numbers $e_{1k}^{i}(I)=\mu_{1k}^{i}+v_{1k}^{i}I\subseteq[0,1]$ and $e_{2k}^{i}(I)=\mu_{2k}^{i}+v_{2k}^{i}I\subseteq[0,1]$ for $I \in [I^-, I^+]$ and $\mu_{1k}^{i},v_{1k}^{i},\mu_{2k}^{i},v_{2k}^{i}\in R$ ($i = 1, 2, \ldots, p_k$; $k = 1, 2, \ldots, m$). Thus, HSSM between two NMSs E_1 and E_2 is expressed below:

$$Sh(E_1,E_2)=1-\frac{1}{m}\sum_{k=1}^{m}\sinh\left[\frac{\ln(1+\sqrt{2})}{2p_k}\sum_{i=1}^{p_k}\begin{pmatrix}\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{2k}^i+v_{2k}^iI^-)\right|\\+\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{2k}^i+v_{2k}^iI^+)\right|\end{pmatrix}\right]. \quad (3)$$

Proposition 1. The HSSM $Sh(E_1, E_2)$ reveals the following properties:

(C1) $Sh(E_1, E_2) = Sh(E_2, E_1)$;

(C2) $0 \le Sh(E_1, E_2) \le 1$;

(C3) $Sh(E_1, E_2) = 1$ if only if $E_1 = E_2$;

(C4) If $E_1 \subseteq E_2 \subseteq E_3$ for any NMSs E_1, E_2, E_3, then $Sh(E_1, E_2) \ge Sh(E_1, E_3)$ and $Sh(E_2, E_3) \ge Sh(E_1, E_3)$.

Proof:

(C1) It is straightforward.

(C2) Since the values of $\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{2k}^i+v_{2k}^iI^-)\right|$ and $\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{2k}^i+v_{2k}^iI^+)\right|$ (i = 1, 2, …, p_k; k = 1, 2, …, m) are between 0 and 1, the value of the hyperbolic sine function in Eq. (3) falls in the interval [0, 1], and then the value of Eq. (3) also falls in the interval [0, 1]. Therefore, there is $0 \le Sh(E_1, E_2) \le 1$.

(C3) If $E_1 = E_2$, this reveals $e_{1k}(I) = e_{2k}(I)$, and then there is $e_{1k}^i(I)=\mu_{1k}^i+v_{1k}^iI$ = $e_{2k}^i(I)=\mu_{2k}^i+v_{2k}^iI$ (i = 1, 2, …, p_k; k = 1, 2, …, m) for $I \in [I^-, I^+]$. Thus, there are $\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{2k}^i+v_{2k}^iI^-)\right|=0$ and $\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{2k}^i+v_{2k}^iI^+)\right|=0$. Hence, $Sh(E_1, E_2) = 1$ exists.

If $Sh(E_1, E_2) = 1$, this reveals $\sinh(x) = 0$ in Eq. (3), then there are $\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{2k}^i+v_{2k}^iI^-)\right|=0$ and $\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{2k}^i+v_{2k}^iI^+)\right|=0$ (i = 1, 2, …, p_k; k = 1, 2, …, m). Thus, there is $e_{1k}^i(I)=\mu_{1k}^i+v_{1k}^iI$ = $e_{2k}^i(I)=\mu_{2k}^i+v_{2k}^iI$. Therefore, there exists $e_{1k}(I) = e_{2k}(I)$. It is obvious that $E_1 = E_2$ exists.

(C4) Since $E_1 \subseteq E_2 \subseteq E_3$, there are $e_{1k}(I) \subseteq e_{2k}(I) \subseteq e_{3k}(I)$, then there is also $e_{1k}^i(I)=\mu_{1k}^i+v_{1k}^iI$ $\subseteq$ $e_{2k}^i(I)=\mu_{2k}^i+v_{2k}^iI$ $\subseteq$ $e_{3k}^i(I)=\mu_{3k}^i+v_{3k}^iI$. Therefore, there are $\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{2k}^i+v_{2k}^iI^-)\right|$ $\le$ $\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{3k}^i+v_{3k}^iI^-)\right|$, $\left|(\mu_{2k}^i+v_{2k}^iI^-)-(\mu_{3k}^i+v_{3k}^iI^-)\right|$ $\le$ $\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{3k}^i+v_{3k}^iI^-)\right|$, $\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{2k}^i+v_{2k}^iI^+)\right|$ $\le$ $\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{3k}^i+v_{3k}^iI^+)\right|$, and $\left|(\mu_{2k}^i+v_{2k}^iI^+)-(\mu_{3k}^i+v_{3k}^iI^+)\right|$ $\le$ $\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{3k}^i+v_{3k}^iI^+)\right|$. Since the hyperbolic sine function $\sinh(x)$ for $x \ge 0$ is an increasing function, there are $Sh(E_1, E_2) \ge Sh(E_1, E_3)$ and $Sh(E_2, E_3) \ge Sh(E_1, E_3)$ corresponding to Eq. (3).

When the weight of $e_{ik}(I)$ (i = 1,2; k = 1, 2, …, m) is specified by φ_k with $\varphi_k \in [0, 1]$ and $\sum_{k=1}^{m}\varphi_k=1$, we give the weighted HSSM of NMSs:

$$Sh_W(E_1,E_2)=1-\sum_{k=1}^{m}\varphi_k\sinh\left[\frac{\ln(1+\sqrt{2})}{2p_k}\sum_{i=1}^{p_k}\begin{pmatrix}\left|(\mu_{1k}^i+v_{1k}^iI^-)-(\mu_{2k}^i+v_{2k}^iI^-)\right|\\+\left|(\mu_{1k}^i+v_{1k}^iI^+)-(\mu_{2k}^i+v_{2k}^iI^+)\right|\end{pmatrix}\right]. \quad (4)$$

It is obvious that the weighted HSSM $Sh_W(E_1, E_2)$ also reveals the following properties:

(C1) $Sh_W(E_1, E_2) = Sh_W(E_2, E_1)$;

(C2) $0 \le Sh_W(E_1, E_2) \le 1$;

(C3) $Sh_W(E_1, E_2) = 1$ if only if $E_1 = E_2$;

(C4) If $E_1 \subseteq E_2 \subseteq E_3$ for NMSs E_1, E_2, E_3, then $Sh_W(E_1, E_2) \ge Sh_W(E_1, E_3)$ and $Sh_W(E_2, E_3) \ge Sh_W(E_1, E_3)$.

4. MCGDM Approach Using the Weighted HSSM of NMSs

The section develops a MCGDM approach using the weighted HSSM of NMSs with some indeterminate ranges of decision makers in the environment of NMSs.

When performing a MCGDM issue, a set of alternatives $F = \{F_1, F_2, \ldots, F_q\}$ is preliminarily provided and assessed by a set of criteria $Z = \{z_1, z_2, \ldots, z_m\}$. The weight vector of Z is given by $\varphi = (\varphi_1, \varphi_2, \ldots, \varphi_m)$. Thus, we can carry out the MCGDM issue in terms of the following steps.

Step 1: Every alternative F_j ($j = 1, 2, \ldots, q$) is assessed over the criteria z_k ($k = 1, 2, \ldots, m$) by a group of p decision makers/experts, and then their evaluation values are represented by the NNSs $e_{jk}(I) = \left(e_{jk}^1(I), e_{jk}^2(I), \ldots, e_{jk}^p(I)\right)$ for $e_{jk}^i(I) = \mu_{jk}^i + v_{jk}^i I \subseteq [0,1]$ ($i = 1, 2, \ldots, p$; $j = 1, 2, \ldots, q$; $k = 1, 2, \ldots, m$) and $I \in \{[I_1^-, I_1^+], [I_2^-, I_2^+], \ldots, [I_s^-, I_s^+]\}$. Thus, all the NNSs $E_j = \{e_{j1}(I), e_{j2}(I), \ldots, e_{jm}(I)\}$ ($j = 1, 2, \ldots, q$) is constructed as the NMS decision matrix $E = (e_{jk}(I))_{q\times m}$.

Step 2: The ideal solution is given by the ideal NMS:

$$E^* = \left\{ \underbrace{\left(\overbrace{[1,1],[1,1],\ldots,[1,1]}^{p}\right), \left(\overbrace{[1,1],[1,1],\ldots,[1,1]}^{p}\right), \ldots, \left(\overbrace{[1,1],[1,1],\ldots,[1,1]}^{p}\right)}_{m} \right\}.$$

Thus, the weighted HSSM of the NMSs E_j and E^* for F_j ($j = 1, 2, \ldots, q$) is given by the following equation:

$$Sh_W(E_j, E^*) = 1 - \sum_{k=1}^{m} \varphi_k \sinh\left[\frac{\ln(1+\sqrt{2})}{2p} \sum_{i=1}^{p} \left(2 - (\mu_{jk}^i + v_{jk}^i I^-) - (\mu_{1k}^i + v_{1k}^i I^+)\right)\right]. \quad (5)$$

Step 3: In terms of the weighted HSSM values, the alternatives are sorted in descending order, and the best one is chosen.

Step 4: End.

5. Illustrative Example on the Teaching Quality Evaluation of Teachers

In the process of university education, the teaching quality of teachers is a key issue, because it will affect students' career choices, employment, and professional status. Establishing a teaching quality evaluation system in colleges and universities is an effective operating mechanism and management strategy to improve teaching quality. Since the teaching quality evaluation of teachers is a MCGDM issue with some indeterminacy, this section applies the developed MCGDM approach to an illustrative example on the teaching quality assessment of teachers to reveal the applicability and efficiency of the developed MCGDM approach in the environment of NMSs.

A university hopes to select one teacher with the best teaching quality from the School of Mechanical and Electrical Engineering. The school preliminarily provides four potential teachers, which are indicated by a set of alternatives $F = \{F_1, F_2, F_3, F_4\}$. To assess their teaching quality, they must satisfy the requirements of four criteria: teaching ability (z_1), teaching method (z_2), teaching attitude (z_3), and student satisfaction (z_4). Then, the weight vector of the four criteria is specified as $\varphi = (0.3, 0.25, 0.2, 0.25)$. The decision steps are described below.

First, the evaluation values of each alternative with respect to the four criteria are given by three experts/decision makers and expressed as the NNSs $e_{jk}(I) = \left(e_{jk}^1(I), e_{jk}^2(I), e_{jk}^3(I)\right)$ for $e_{jk}^i(I) = \mu_{jk}^i + v_{jk}^i I \subseteq [0,1]$ ($i = 1, 2, 3$; $j, k = 1, 2, 3, 4$) and $I \in \{[0, 0.1], [0, 0.3], [0, 0.6]\}$. Then, the NMS decision matrix $E = (e_{jk}(I))_{4\times 4}$ is tabulated in Table 1.

Next, using Eq. (5) for $I \in \{[0, 0.1], [0, 0.3], [0, 0.6]\}$, the weighted HSSM values of the NMSs E_j and E^* for F_j ($j = 1, 2, 3, 4$) and the decision results are given in Table 2.

In view of the decision results in Table 2, all sorting orders are the same and reveal their robustness corresponding to some indeterminate ranges of *I*. Then, the best teacher is F_4.

Table 1. The decision matrix of NMSs

	z_1	z_2	z_3	z_4
F_1	(0.5+0.1*I*, 0.7+0.2*I*, 0.8+0.1*I*)	(0.6+0.3*I*, 0.7+0.2*I*, 0.8+0.1*I*)	(0.7+0.2*I*, 0.8+0.2*I*, 0.9+0.1*I*)	(0.6+0.1*I*, 0.7+0.2*I*, 0.7+0.1*I*)
F_2	(0.7+0.3*I*, 0.8+0.1*I*, 0.8+0.1*I*)	(0.7+0.2*I*, 0.7+0.1*I*, 0.8+0.2*I*)	(0.6+0.1*I*, 0.7+0.1*I*, 0.8+0.2*I*)	(0.7+0.2*I*, 0.8+0.2*I*, 0.8+0.2*I*)
F_3	(0.6+0.4*I*, 0.7+0.2*I*, 0.8+0.1*I*)	(0.5+0.2*I*, 0.6+0.1*I*, 0.6+0.1*I*)	(0.7+0.1*I*, 0.8+0.2*I*, 0.8+0.1*I*)	(0.7+0.1*I*, 0.8+0.2*I*, 0.8+0.1*I*)
F_4	(0.7+0.3*I*, 0.8+0.1*I*, 0.8+0.2*I*)	(0.7+0.2*I*, 0.8+0.1*I*, 0.8+0.1*I*)	(0.8+0.3*I*, 0.8+0.2*I*, 0.8+0.1*I*)	(0.7+0.2*I*, 0.8+0.3*I*, 0.8+0.2*I*)

Table 2. The decision results for $I \in$ {[0, 0.1], [0, 0.3], [0, 0.6]}

I	$Shw(E_1, E^*)$, $Shw(E_2, E^*)$, $Shw(E_3, E^*)$, $Shw(E_4, E^*)$	Sorting order	The best teacher
I = [0, 0.1]	0.7409, 0.7809, 0.7362, 0.8075	$F_4 > F_2 > F_1 > F_3$	F_4
I = [0, 0.3]	0.7551, 0.7960, 0.7511, 0.8247	$F_4 > F_2 > F_1 > F_3$	F_4
I = [0, 0.6]	0.7764, 0.8187, 0.7733, 0.8503	$F_4 > F_2 > F_1 > F_3$	F_4

6. Comparison with the Related MCGDM Approach

This section compares the developed MCGDM approach with the related MCGDM approach [20] to reveal the efficiency of the developed MCGDM approach by the illustrative example on the teaching quality evaluation of teachers in the setting of NMSs.

Using Eqs. (1) and (2), the values of the weighted PCCs $R^{\rho}_{w1}(E_j, E^*)$ and $R^{\rho}_{w2}(E_j, E^*)$ for *I* = [0, 1] and ρ = 0.1, 0.3, 0.6 and their decision results are shown in Tables 3 and 4.

Table 3. The decision results corresponding to $R^{\rho}_{w1}(E_j, E^*)$ for *I* = [0, 1] and ρ = 0.1, 0.3, 0.6

ρ	$R^{\rho}_{w1}(E_1, E^*)$, $R^{\rho}_{w1}(E_2, E^*)$, $R^{\rho}_{w1}(E_3, E^*)$, $R^{\rho}_{w1}(E_4, E^*)$	Sorting order	The best teacher
ρ = 0.1	0.9898, 0.9967, 0.9908, 0.9986	$F_4 > F_2 > F_3 > F_1$	F_4
ρ = 0.3	0.9907, 0.9967, 0.9920, 0.9987	$F_4 > F_2 > F_3 > F_1$	F_4
ρ = 0.6	0.9914, 0.9962, 0.9926, 0.9984	$F_4 > F_2 > F_3 > F_1$	F_4

Table 4. The decision results corresponding to $R^{\rho}_{w2}(E_j, E^*)$ for *I* = [0, 1] and ρ = 0.1, 0.3, 0.6

ρ	$R^{\rho}_{w2}(E_1, E^*)$, $R^{\rho}_{w2}(E_2, E^*)$, $R^{\rho}_{w2}(E_3, E^*)$, $R^{\rho}_{w2}(E_4, E^*)$	Sorting order	The best teacher
ρ = 0.1	0.7173, 0.7618, 0.7130, 0.7925	$F_4 > F_2 > F_1 > F_3$	F_4
ρ = 0.3	0.7487, 0.7955, 0.7457, 0.8308	$F_4 > F_2 > F_1 > F_3$	F_4
ρ = 0.6	0.7957, 0.8460, 0.7947, 0.8883	$F_4 > F_2 > F_1 > F_3$	F_4

In view of the sorting results in Tables 2-4, the sorting orders in Tables 2 and 4 are the same, but slightly different from the sorting orders in Table 3. However, the best teacher is always F_4 among all decision results. It is obvious that the developed MCGDM approach is effective in the MCGDM example with some indeterminate ranges of decision makers.

7. Conclusions

According to the hyperbolic sine function, this article proposed the HSSM and weighted HSSM between NMSs. Then, a MCGDM approach with some indeterminate ranges was developed in terms of the weighted HSSM of NMSs. Next, the developed MCGDM approach was applied to an illustrative example on the teaching quality evaluation of teachers in the setting of NMSs. Through the comparison of the developed MCGDM approach with the related MCGDM approach, the results revealed the efficiency of the developed MCGDM approach for the teaching quality evaluation of teachers in the setting of NMSs. However, the proposed HSSMs and MCGDM approach will also be used for pattern recognition, clustering analysis, and medical diagnosis in the environment of NMSs, which are considered as the future research targets.

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