Location Selection of Low-carbon Logistics Park Based on the Neutrosophic Numbers Multiple Attribute Decision Making

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Abstract: With the continuous advancement of science and technology, the decision-making environment faced by managers is increasingly complex, which puts forward higher requirements for managers. Due to the complexity of the multi-attribute decision-making problem, it is difficult for the decision-maker to make the correct choice. In addition, due to the influence of the educational background and limited knowledge of the managers, it is impossible to evaluate with simple statistical data. In essence, the location problem of low-carbon logistics parks is a MADM problem. Therefore, this paper establishes an optimization model to solve the multiple-attribute decision-making (MADM) problem with alternative preference and single-valued neutrosophic (SVN) information. Considering that the information of weight is unknown, a scientific model is built based on the minimum deviation method deriving the criterion weight. Furthermore, the above models and methods are extended to interval neutrosophic sets (INs). To verify the validity of the modified model, a numerical case for low-carbon logistics park site selection is taken as an example. Through the case study, we found that the method has strong operability and can make the most of the available information.

Keywords: Multiple-attribute decision-making (MADM); single-valued neutrosophic sets (SVNSs); interval neutrosophic sets (INs); preference information; low-carbon logistics park site selection

1. Introduction

Decision makers (DMs) often have difficulty expressing their preferences accurately when presented with inaccurate, uncertain expression when solving MADM issues[1-5]. Fuzzy Sets

(FSs) [6]are considered to solve the MADM problems [7, 8]. Intuitionistic fuzzy sets (IFS)[9], as an extension, have subsequently been widely used in solving MCDM problems. Since IFSs consider both membership and non-membership, they are more flexible and practical than traditional FSs[10-14]. In some practical cases, membership, non-membership and hesitation of an element of IFSs may not be a specific figure. Therefore, it is extended to IVFS [15-20]. To represent uncertainty and inconsistency in information, Smarandache [21] introduced neutrosophic sets (NS) as an alternative to IFSs and IVIFSs. To facilitate practical application, the SVNS [22] and INS [23] were proposed as subclasses of NS, and then Ye [24] introduce`ed SNS, SVNS and INS. According to the literature. NS is a generalization of FS, IFS, and IVIFS. In practice, SNS (SVNS and INS) are ideal for the expression of incomplete and uncertain information in practical applications. In recent years, SNSs (ins) and SVNSs (SVNSs) is the ideal choice for expressing incomplete, uncertain and inconsistent information. Sahin and Kucuk [25] gave the introduction of entropy measure of SVNSs. And correlation coefficient of SVNSs and the method of using SVNSs for decision making are introduced based on the preliminary knowledges. In a time-neutral environment, Broumi and Smarandache [26] built the correlation coefficients of INSs. While Zhang, Wang and Chen [27] developed INNWA operator and INNWG operator. Furthermore, Ye [28-32] introduced a similarity measure between SVNSs and INSs. Ye [33] examined interval-neutral MADM methods based on probability degree sequencing and ordered weighted aggregate operators. Ye and Jun [34] proposes an interval-neutral MADM with confidence information. Peng, Wang, Zhang and Chen [35] studied a transcendent approach to MADM problems with simplified neutral sets. With interval-value neutral sets, Zhang, Wang and Chen [36] devised a transcendent method to solve the MADM issues. In their paper, Tian, Zhang, Wang, Wang and Chen [37] examined the use of interval neutral set cross entropy. The SVNNWBM operator was proposed by Liu and Wang [38] using the Bonferroni mean, the WBM, and the normalized WBM. Liu, Chu, Li and Chen [39] combined Hamacher operator and generalized operator into NS, proposed the GNNHWA operator, GNNHOWA operator and GNNHHA operator. Zhao, Du and Guan [40] extended the GWA operator to work in line with the IVNS data. Liu and Wang [41] further proposed INPOWA operator. In their study, the preferred weighted average operator and priority weighted geometric operator for SNN [42] were then defined. Ye [43] proposed INWEA operator and DUNWEA operator based on exponential algorithm. Li, Liu and Chen [44] proposed some Heronian mean operators with SVNSs.

Knowledge explosion, information torrent, rapid technological change, rapid social change, rapid economic development, and so on, this is an era of change, but also an era of development. With the popularization of the Internet and the rapid development of the e-commerce industry, the way of shopping has gradually shifted from offline to online[45, 46]. Online shopping has become an indispensable part of contemporary people's lives, and the accompanying logistics system is an important part of it. support. The rapid development of SF Express, "Three Links and One Delivery" (Zhongtong, Shentong, Yuantong, Yunda Express), JD Logistics and other niche express delivery has driven economic growth, but their extensive logistics operations have also caused great damage to the environment. Influence. Logistics systems include a variety of activities, such as supplier production, transportation and distribution, that consume energy and emit carbon[47-50]. In the context of global warming and environmental deterioration, it is extremely urgent to develop low-carbon logistics[51-53]. Government departments have formulated a series of plans to implement them. The primary task of the logistics industry system from the perspective of low carbon is to carry out reasonable planning of logistics activities, build logistics parks and solve the problem of site selection of logistics parks[54-56]. The location of a low-carbon logistics park depends on factors such as the economic development of a certain place, market demand, low-carbon attributes of logistics and transportation routes, and whether carbon emissions meet environmental requirements[57-59]. In essence, the location problem of low-carbon logistics parks is a MADM problem. During the process of single valued neutral MADM with alternative preference information. The weights are not completely known or completely unknown. Nevertheless, none of the above methods are suitable for dealing with this situation. To overcome this limitation, it is necessary to find methods based on the minimum deviation method. The aim of this manuscript is to establish a method based on the least deviation method. We will introduce SVNSs in the next section of this paper. In Section 3, we build the MADM model under SVNSs, where the information about criterion weight is not completely known, and the attribute value and preference value of options are SVNNs. In Section 4, There is no complete information about criterion weight, and the attribute value and preference value are expressed as INNs. In Section 5, illustrative examples for low-carbon logistics park site selection are indicated. In Section 6, we summarize the full text.

2. Preliminaries

Definition 1[60]. Assume W be a set with an element in a fixed set W, which is denoted by ϖ. A NSs ν in W is defined by the function of truth-membership $\pi_\nu(\varpi)$, indeterminacy-membership $\vartheta_\nu(\varpi)$ and a falsity-membership function $\sigma_\nu(\varpi)$. The functions $\pi_\nu(\varpi)$, $\vartheta_\nu(\varpi)$ and $\sigma_\nu(\varpi)$ are real standard or nonstandard subsets of $]^-0,1^+[$, that's, $\pi_\nu(\varpi):W\to]^-0,1^+[$, $\vartheta_\nu(\varpi):W\to]^-0,1^+[$ and $\vartheta_\nu(\varpi):W\to]^-0,1^+[$. There is no restriction of $\pi_\nu(\varpi)$, $\vartheta_\nu(\varpi)$ and $\sigma_\nu(\varpi)$, so $0^-\le\sup\pi_\nu(\varpi)+\sup\vartheta_\nu(\varpi)+\sup\sigma_\nu(\varpi)\le 3^+$.

Definition 2[22]. Let W be a collection in fixed set W, denoted by ϖ. A SVNSs ν in W is defined as follows:

$$\nu=\left\{\left(\varpi,\pi_\nu(\varpi),\vartheta_\nu(\varpi),\sigma_\nu(\varpi)\right)W\right\} \tag{1}$$

Where $\pi_\nu(x)$, $\vartheta_\nu(\varpi)$ and $\sigma_\nu(\varpi)$ are in the value of $[0,1]$, that is, $\pi_\nu(\varpi):W\to[0,1]$, $\vartheta_\nu(\varpi):W\to[0,1]$ and $\sigma_\nu(\varpi):W\to[0,1]$. And the sum of $\pi_\nu(\varpi)$, $\vartheta_\nu(\varpi)$ and $\sigma_\nu(\varpi)$ meets the condition $0\le\pi_\nu(\varpi)+\vartheta_\nu(\varpi)+\sigma_\nu(\varpi)\le 3$. Then a simplification of ν is represented by $\nu=\left\{\left(\varpi,\pi_\nu(\varpi),\vartheta_\nu(\varpi),\sigma_\nu(\varpi)\right)\middle|\varpi\in W\right\}$, which is a SVNS.

For a SVNS $\left\{\left(\varpi,\pi_\nu(\varpi),\vartheta_\nu(\varpi),\sigma_\nu(\varpi)\right)\middle|\varpi\in W\right\}$, the ordered triple components $\left(\pi_\nu(\varpi),\vartheta_\nu(\varpi),\sigma_\nu(\varpi)\right)$, are defined as a SVNN, and each SVNN can be expressed as $\nu=(\pi_\nu,\vartheta_\nu,\sigma_\nu)$, where $\pi_\nu\in[0,1]$, $\vartheta_\nu\in[0,1]$, $\sigma_\nu\in[0,1]$, and $0\le\pi_\nu+\vartheta_\nu+\sigma_\nu\le 3$.

Definition 3[61]. Set $\nu=(\pi_\nu,\vartheta_\nu,\sigma_\nu)$ be a SVNN, a score function ψ is represented:

$$\psi(\nu)=\frac{(2+\pi_\nu-\vartheta_\nu-\sigma_\nu)}{3},\ \psi(\nu)\in[0,1] \tag{2}$$

Definition 4[61]. Set $\nu=(\pi_\nu,\vartheta_\nu,\sigma_\nu)$ be a SVNN, an accuracy function χ is represented:

$$\chi(\nu)=\pi_\nu-\sigma_\nu,\ \chi(\nu)\in[-1,1]\ . \tag{3}$$

Definition 5[61]. Let $\nu=(\pi_\nu,\vartheta_\nu,\sigma_\nu)$ and $\mu=(\pi_\mu,\vartheta_\mu,\sigma_\mu)$ be two SVNNs, $\psi(\nu)=\frac{(2+\pi_\nu-\vartheta_\nu-\sigma_\nu)}{3}$ and $\psi(\mu)=\frac{(2+\pi_\mu-\vartheta_\mu-\sigma_\mu)}{3}$ be the scores function, and let

$\chi(\nu)=\pi_\nu-\sigma_\nu$ and $\chi(\mu)=\pi_\mu-\sigma_\mu$ be the accuracy degrees, then if $\psi(\nu)<\psi(\nu)$, then $\nu<\mu$; if $\psi(\nu)=\psi(\mu)$, then

(1) if $\psi(\nu)=\psi(\nu)$, then $\nu=\mu$; (2) if $\psi(\nu)<\psi(\mu)$, then $\nu<\mu$.

Definition 6[61]. Let $\nu=(\pi_\nu,\vartheta_\nu,\sigma_\nu)$ and $\mu=(\pi_\mu,\vartheta_\mu,\sigma_\mu)$ be two SVNNs, and some basic operations are defined:

(1) $\nu\oplus\mu=(\pi_\nu+\pi_\mu-\pi_\nu\pi_\mu,\vartheta_\nu\vartheta_\mu,\sigma_\nu\sigma_\mu)$;
(2) $\nu\otimes\mu=(\pi_\nu\pi_\mu,\vartheta_\nu+\vartheta_\mu-\vartheta_\nu\vartheta_\mu,\sigma_\nu+\sigma_\mu-\sigma_\nu\sigma_\mu)$;
(3) $\lambda\nu=\left(1-(1-\pi_\nu)^\lambda,(\vartheta_\nu)^\lambda,(\sigma_\nu)^\lambda\right),\lambda>0$;
(4) $(\nu)^\lambda=\left((\pi_\nu)^\lambda,(\vartheta_\nu)^\lambda,1-(1-\sigma_\nu)^\lambda\right),\lambda>0$.

Based on Definition 6, the following properties are derived.

Theorem 1[22]. Let $\nu=(\pi_\nu,\vartheta_\nu,\sigma_\nu)$ and $\mu=(\pi_\mu,\vartheta_\mu,\sigma_\mu)$ be two SVNNs, $\lambda,\lambda_1,\lambda_2>0$, then

(1) $\nu\oplus\mu=\mu\oplus\nu$;
(2) $\nu\otimes\mu=\mu\otimes\nu$;
(3) $\lambda(\nu\oplus\mu)=\lambda\nu\oplus\lambda\mu$;
(4) $(\nu\otimes\mu)^\lambda=(\nu)^\lambda\otimes(\mu)^\lambda$;
(5) $\lambda_1\nu\oplus\lambda_2\nu=(\lambda_1+\lambda_2)\nu$;
(6) $(\nu)^{\lambda_1}\otimes(\nu)^{\lambda_2}=(\nu)^{(\lambda_1+\lambda_2)}$;
(7) $\left((\nu)^{\lambda_1}\right)^{\lambda_2}=(\nu)^{\lambda_1\lambda_2}$.

Definition 7[61]. Let $\nu_\alpha=(\pi_\alpha,\vartheta_\alpha,\sigma_\alpha)(\alpha=1,2,\cdots,\phi)$ be a collection of SVNNs, and let SVNWA: $Q^\phi\to Q$, if

$$\begin{aligned}\text{SVNWA}_\gamma(\nu_1,\nu_2,\cdots,\nu_\phi)&=\mathop{\oplus}_{\alpha=1}^{\phi}(\gamma_\alpha\nu_\alpha)\\&=\left(1-\prod_{\alpha=1}^{\phi}(1-\pi_\alpha)^{\gamma_\alpha},\prod_{\alpha=1}^{\phi}(\vartheta_\alpha)^{\gamma_\alpha},\prod_{\alpha=1}^{\phi}(\sigma_\alpha)^{\gamma_\alpha}\right)\end{aligned}\tag{4}$$

where $\gamma=(\gamma_1,\gamma_2,\cdots,\gamma_\phi)^T$ be the weight of $\nu_\alpha(\alpha=1,2,\cdots,\phi)$, and $\gamma_\alpha>0$, $\sum_{\alpha=1}^{\phi}\gamma_\alpha=1$, then SVNWA is called SVNWA operator.

Definition 8[25]. Let $\nu=\left(\pi_{\nu},\vartheta_{\nu},\sigma_{\nu}\right)$ and $\mu=\left(\pi_{\mu},\vartheta_{\mu},\sigma_{\mu}\right)$ be two SVNNs, then the Hamming distance is defined:

$$d\left(\nu,\mu\right)=\frac{1}{3}\left(\left|\pi_{\nu}-\pi_{\mu}\right|+\left|\vartheta_{\nu}-\vartheta_{\mu}\right|+\left|\sigma_{\nu}-\sigma_{\mu}\right|\right) \tag{5}$$

3. Models for SVN MADM issues

(1) Let $\varepsilon=\left\{\varepsilon_1,\varepsilon_2,\cdots,\varepsilon_{\phi}\right\}$ be a discrete set of alternatives;(2) Let $\varsigma=\left\{\varsigma_1,\varsigma_2,\cdots,\varsigma_{\varphi}\right\}$ be a set of attributes;(3) Let $\nu=\left(\nu_1,\nu_2,\cdots,\nu_{\phi}\right)$ be subjective preference and $\nu_{\alpha}=\left(\pi_{\nu_{\alpha}},\vartheta_{\nu_{\alpha}},\sigma_{\nu_{\alpha}}\right)$ are SVSNs, which is subjective preference on alternative $\varepsilon_{\alpha}\left(\alpha=1,2,\cdots,\phi\right)$.(4) The criterion weights is incompletely known. Let $\gamma=\left(\gamma_1,\gamma_2,\cdots,\gamma_{\varphi}\right)\in R$ be the weight of attributes, where $\gamma_{\beta}\geq 0$, $\beta=1,2,\cdots,\varphi$, $\sum_{\beta=1}^{\varphi}\gamma_{\beta}=1$, R is a set of criterion weight, constructed by the following forms [62, 63], for $\alpha\neq\beta$: **Form 1.** A weak sequence: $\gamma_{\alpha}\geq\gamma_{\beta}$; **Form 2.** A strict sequence: $\gamma_{\alpha}-\gamma_{\beta}\geq\ell_{\alpha}$, $\ell_{\alpha}>0$;**Form 3.** A sequence of differences: $\gamma_{\alpha}-\gamma_{\beta}\geq\gamma_{o}-\gamma_{\rho}$, for $\beta\neq o\neq\rho$;**Form 4.** A sequence with multiples: $\gamma_{\alpha}\geq\eta_{\alpha}\gamma_{\beta}$, $0\leq\eta_{\alpha}\leq 1$; **Form 5.** An interval form: $\ell_{\alpha}\leq\gamma_{\alpha}\leq\ell_{\alpha}+\varepsilon_{i}, 0\leq\ell_{\alpha}<\ell_{\alpha}+\varepsilon_{\alpha}\leq 1$. Suppose that $V=\left(\nu_{\alpha\beta}\right)_{\phi\times\varphi}=\left(\pi_{\alpha\beta},\vartheta_{\alpha\beta},\sigma_{\alpha\beta}\right)_{\phi\times\varphi}$ is SVN decision matrix, $\pi_{\alpha\beta}\in[0,1],\vartheta_{\alpha\beta}\in[0,1],\sigma_{\alpha\beta}\in[0,1]$, $0\leq\pi_{\alpha\beta}+\vartheta_{\alpha\beta}+\sigma_{\alpha\beta}\leq 3$, $\alpha=1,2,\cdots,\phi$, $\beta=1,2,\cdots,\varphi$.

Definition 9[61]. Let $V=\left(\nu_{\alpha\beta}\right)_{\phi\times\varphi}=\left(\pi_{\alpha\beta},\vartheta_{\alpha\beta},\sigma_{\alpha\beta}\right)_{\phi\times\varphi}$ is the SVN matrix, $\nu_{\alpha}=\left(\nu_{\alpha 1},\nu_{\alpha 2},\cdots,\nu_{\alpha\varphi}\right)$ be the attribute values for alternative ε_{α}, $\alpha=1,2,\cdots,\phi$, then we call

$$\begin{aligned}\nu_{\alpha}&=\left(\pi_{\alpha},\vartheta_{\alpha},\sigma_{\alpha}\right)=\mathrm{SVNWA}_{\gamma}\left(\nu_{\alpha 1},\nu_{\alpha 2},\cdots,\nu_{\alpha\varphi}\right)\\&=\bigoplus_{\beta=1}^{\varphi}\left(\gamma_{\beta}\nu_{\alpha\beta}\right)=\left(1-\prod_{\beta=1}^{\varphi}\left(1-\pi_{\alpha\beta}\right)^{\gamma_{\beta}},\prod_{\beta=1}^{\varphi}\left(\vartheta_{\alpha\beta}\right)^{\gamma_{\beta}},\prod_{\beta=1}^{\varphi}\left(\sigma_{\alpha\beta}\right)^{\gamma_{\beta}}\right)\end{aligned} \tag{6}$$

the overall value of ε_{α}, where $\gamma=\left(\gamma_1,\gamma_2,\cdots,\gamma_{\beta}\right)^{T}$ is the criterion attributes.

On the premise that the attribute weight is known, we can aggregate the weighted values into a total value through Eq. (6). According to the overall attribute value, we can make a final ranking of possible solutions, and finally choose the most suitable solution.

If the attribute weight information of the decision model is unknown, to reflect the subjective preference and objective information of the DM at the same time, the optimization decision model is established. However, there are some differences between DM's subjective preference and objective information. To make decision-making more scientific and reasonable, the selection of attribute weight vector should minimize the total deviation between objective information and DM subjective preference.

For $\chi_\beta \in \chi$, the deviation of alternative ε_α to DM's subjective preference is defined as follows:

$$K_{\alpha\beta}(\gamma) = \kappa\left(\nu_{\alpha\beta}, \nu_\alpha\right)\gamma_\beta, \alpha = 1, 2, \cdots, \phi, \beta = 1, 2, \cdots, \varphi. \tag{7}$$

Let $K_\alpha(\gamma) = \sum_{\beta=1}^{\varphi} K_{\alpha\beta}(\gamma) = \sum_{\beta=1}^{\varphi} \kappa\left(\nu_{\alpha\beta}, \nu_\alpha\right)\gamma_\beta, \alpha = 1, 2, \cdots, \phi$

Then $K_\alpha(\gamma)$ denote the deviation of ε_α to DM's subjective preference value ν_α.

According to the above analysis, we must select the criterion weight vector to minimize all deviations of possible solutions. To this end, we establish a linear programming model:

$$(\text{M-1})\begin{cases} \min K(\gamma) = \sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi} K_{\alpha\beta}(\gamma) = \sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi} \kappa\left(\nu_{\alpha\beta}, \nu_\alpha\right)\gamma_\beta \\ \text{Subject to} \quad \sum_{\beta=1}^{\varphi}\gamma_\beta = 1, \gamma_\beta \geq 0, \beta = 1, 2, \cdots, \varphi \end{cases}$$

where $\kappa\left(\nu_{\alpha\beta}, \nu_\alpha\right) = \frac{1}{3}\left(\left|\pi_{\alpha\beta} - \pi_{\nu_\alpha}\right| + \left|\vartheta_{\alpha\beta} - \vartheta_{\nu_\alpha}\right| + \left|\sigma_{\alpha\beta} - \sigma_{\nu_\alpha}\right|\right)$.

If the attribute weight information is completely unknown, another programming model is established:

$$(\text{M-2})\begin{cases} \min K(\gamma) = \sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi} K_{\alpha\beta}(\gamma) \\ = \frac{1}{3}\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}\left(\left|\pi_{\alpha\beta} - \pi_{\nu_\alpha}\right| + \left|\vartheta_{\alpha\beta} - \vartheta_{\nu_\alpha}\right| + \left|\sigma_{\alpha\beta} - \sigma_{\nu_\alpha}\right|\right)\gamma_\beta \\ s.t. \ \sum_{\beta=1}^{\varphi}\gamma_\beta^2 = 1, \gamma_\beta \geq 0, \beta = 1, 2, \cdots, \varphi \end{cases}$$

The Lagrange function is constructed as follows:

$$L(\gamma,\lambda)=\frac{1}{3}\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}\left(\left|\pi_{\alpha\varepsilon}-\pi_{v_\alpha}\right|+\left|\vartheta_{\alpha\beta}-\vartheta_{v_\alpha}\right|+\left|\sigma_{\alpha\beta}-\sigma_{v_\alpha}\right|\right)\gamma_\beta+\frac{\lambda}{6}\left(\sum_{\beta=1}^{\varphi}\gamma_\beta^2-1\right) \tag{8}$$

where λ is the Lagrange multiplier.

Differentiating Eq. (8) and setting these partial derivatives equal to zero:

$$\begin{cases}\dfrac{\partial L}{\partial\gamma_\beta}=\sum_{\alpha=1}^{m\phi}\left(\left|\pi_{\alpha\beta}-\pi_{v_\alpha}\right|+\left|\vartheta_{\alpha\beta}-\vartheta_{v_\alpha}\right|+\left|\sigma_{\alpha\beta}-\sigma_{v_\alpha}\right|\right)+\lambda\gamma_\beta=0\\ \dfrac{\partial L}{\partial\lambda}=\sum_{\beta=1}^{\varphi}\gamma_\beta^2-1=0\end{cases} \tag{9}$$

By solving Eq. (9), we get the attribute weights:

$$\gamma_\beta^*=\frac{\sum_{\alpha=1}^{\phi}\left(\left|\pi_{\alpha\beta}-\pi_{v_\alpha}\right|+\left|\vartheta_{\alpha\beta}-\vartheta_{v_\alpha}\right|+\left|\sigma_{\alpha\beta}-\sigma_{v_\alpha}\right|\right)}{\sqrt{\sum_{\beta=1}^{\varphi}\left[\left(\left|\pi_{\alpha\beta}-\pi_{v_\alpha}\right|+\left|\vartheta_{\alpha\beta}-\vartheta_{v_\alpha}\right|+\left|\sigma_{\alpha\beta}-\sigma_{\sigma_\alpha}\right|\right)\right]^2}} \tag{10}$$

By standardizing $\gamma_\beta^*\left(\beta=1,2,\cdots,\varphi\right)$ be a unit, we have

$$\gamma_\beta=\frac{\sum_{\alpha=1}^{\phi}\left(\left|\pi_{\alpha\beta}-\pi_{v_\alpha}\right|+\left|\vartheta_{\alpha\beta}-\vartheta_{v_\alpha}\right|+\left|\sigma_{\alpha\beta}-\sigma_{v_\alpha}\right|\right)}{\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}\left(\left|\pi_{\alpha\beta}-\pi_{v_\alpha}\right|+\left|\vartheta_{\alpha\beta}-\vartheta_{v_\alpha}\right|+\left|\sigma_{\alpha\beta}-\sigma_{v_\alpha}\right|\right)} \tag{11}$$

We propose a practical method to solve MADM with alternative preference and SVNs.

(Procedure one)

Step 1. Let $V=\left(v_{\alpha\beta}\right)_{\phi\times\varphi}=\left(\pi_{\alpha\beta},\vartheta_{\alpha\beta},\sigma_{\alpha\beta}\right)_{\phi\times\varphi}$ be a SVN matrix, $\gamma=\left(\gamma_1,\gamma_2,\cdots,\gamma_\varphi\right)$ be the criterion weight, where $\gamma_\beta\in[0,1]$, $\beta=1,2,\cdots,\varphi$, γ is a set of the known weight information. Let $v=\left(v_1,v_2,\cdots,v_\phi\right)$ be subjective preference, $v_\alpha=\left(\pi_{v_\alpha},\vartheta_{v_\alpha},\sigma_{v_\alpha}\right)$ are SVNNs, which are subjective preference values on alternatives $\varepsilon_\alpha\left(\alpha=1,2,\cdots,\phi\right)$.

Step 2. By solving the model (M-1), the partially known index values of the weight information are obtained.

Step 3. Utilize the weight $\gamma=\left(\gamma_1,\gamma_2,\cdots,\gamma_\varphi\right)$ and Eq. (7), we obtain the $\tilde{v}_\alpha$ of $\varepsilon_\alpha\left(\alpha=1,2,\cdots,\phi\right)$.

Step 4. Obtain the scores $\psi\left(\nu_{\alpha}\right)$ of $\nu_{\alpha}\left(\alpha=1,2,\cdots,\phi\right)$ to rank all the solutions $\varepsilon_{\alpha}\left(\alpha=1,2,\cdots,\phi\right)$. Then we calculate the $\chi\left(\nu_{\alpha}\right)$ and $\chi\left(\nu_{\beta}\right)$, and then rank the alternatives through $\chi\left(\nu_{\alpha}\right)$ and $\chi\left(\nu_{\beta}\right)$.

Step 5. Rank solutions $\varepsilon_{\alpha}\left(\alpha=1,2,\cdots,\phi\right)$ and select the best one through $\psi\left(\nu_{\alpha}\right)$ and $\chi\left(\nu_{\alpha}\right)\left(\alpha=1,2,\cdots,\phi\right)$.

Step 6. End.

4. Models for INS MADM problems

Definition 10[23]. Let W be a collection with element in fix set W, denoted by ϖ. An INSs $\tilde{\nu}$ in W is defined as follows:

$$\tilde{\nu}=\left\{\left(\varpi,\pi_{\tilde{\nu}}\left(\varpi\right),\vartheta_{\tilde{\nu}}\left(\varpi\right),\sigma_{\tilde{\nu}}\left(\varpi\right)\right)\middle|\varpi\in W\right\} \tag{12}$$

where $\pi_{\tilde{\nu}}\left(\varpi\right)$, $\vartheta_{\tilde{\nu}}\left(\varpi\right)$ and $\sigma_{\tilde{\nu}}\left(\varpi\right)$, which are interval values in the value of $\left[0,1\right]$, that is, $\pi_{\nu}\left(\varpi\right)\subseteq\left[0,1\right],\vartheta_{\nu}\left(\varpi\right)\subseteq\left[0,1\right]$ and $\sigma_{\nu}\left(\varpi\right)\subseteq\left[0,1\right]$. $0\leq\sup\left(\pi_{\tilde{\nu}}\left(\varpi\right)\right)+\sup\left(\vartheta_{\tilde{\nu}}\left(\varpi\right)\right)+\sup\left(\sigma_{\tilde{\nu}}\left(\varpi\right)\right)\leq 3$.

For a INSs $\left\{\left(\varpi,\pi_{\tilde{\nu}}\left(\varpi\right),\vartheta_{\tilde{\nu}}\left(\varpi\right),\sigma_{\tilde{\nu}}\left(\varpi\right)\right)\middle|\varpi\in W\right\}$, the ordered triple components $\left(\pi_{\tilde{\nu}}\left(\varpi\right),\vartheta_{\tilde{\nu}}\left(\varpi\right),\sigma_{\tilde{\nu}}\left(\varpi\right)\right)$, are described as an INNs, and each INN can be expressed as $\tilde{\nu}=\left(\tilde{\pi}_{\tilde{\nu}},\tilde{\vartheta}_{\tilde{\nu}},\tilde{\sigma}_{\tilde{\nu}}\right)=\left(\left[\tilde{\pi}_{\tilde{\nu}}^{X},\tilde{\pi}_{\tilde{\nu}}^{Y}\right],\left[\tilde{\vartheta}_{\tilde{\nu}}^{X},\tilde{\vartheta}_{\tilde{\nu}}^{Y}\right],\left[\tilde{\sigma}_{\tilde{\nu}}^{X},\tilde{\sigma}_{\tilde{\nu}}^{Y}\right]\right)$, where $\left[\tilde{\pi}_{\tilde{\nu}}^{X},\tilde{\pi}_{\tilde{\nu}}^{Y}\right]\subseteq\left[0,1\right],\left[\tilde{\vartheta}_{\tilde{\nu}}^{X},\tilde{\vartheta}_{\tilde{\nu}}^{Y}\right]\subseteq\left[0,1\right],\left[\tilde{\sigma}_{\tilde{\nu}}^{X},\tilde{\sigma}_{\tilde{\nu}}^{Y}\right]\subseteq\left[0,1\right]$, and $0\leq\tilde{\pi}_{\tilde{\nu}}^{Y}+\tilde{\vartheta}_{\tilde{\nu}}^{Y}+\tilde{\sigma}_{\tilde{\nu}}^{Y}\leq 3$.

Definition 11[64]. Let $\tilde{\nu}=\left(\left[\tilde{\pi}_{\tilde{\nu}}^{X},\tilde{\pi}_{\tilde{\nu}}^{Y}\right],\left[\tilde{\vartheta}_{\tilde{\nu}}^{X},\tilde{\vartheta}_{\tilde{\nu}}^{Y}\right],\left[\tilde{\sigma}_{\tilde{\nu}}^{X},\tilde{\sigma}_{\tilde{\nu}}^{Y}\right]\right)$ be an INN, a score function ψ is represented:

$$\psi\left(\tilde{\nu}\right)=\frac{\left(2+\tilde{\pi}_{\tilde{\nu}}^{X}-\tilde{\vartheta}_{\tilde{\nu}}^{X}-\tilde{\sigma}_{\tilde{\nu}}^{X}\right)+\left(2+\tilde{\pi}_{\tilde{\nu}}^{Y}-\tilde{\vartheta}_{\tilde{\nu}}^{Y}-\tilde{\sigma}_{\tilde{\nu}}^{Y}\right)}{6},\ \psi\left(\tilde{\nu}\right)\in\left[0,1\right]. \tag{13}$$

Definition 12[64]. Let $\tilde{\nu}=\left(\left[\tilde{\pi}_{\tilde{\nu}}^{X},\tilde{\pi}_{\tilde{\nu}}^{Y}\right],\left[\tilde{\vartheta}_{\tilde{\nu}}^{X},\tilde{\vartheta}_{\tilde{\nu}}^{Y}\right],\left[\tilde{\sigma}_{\tilde{\nu}}^{X},\tilde{\sigma}_{\tilde{\nu}}^{Y}\right]\right)$ be an INN, an accuracy function χ is represented:

$$\chi\left(\tilde{\nu}\right)=\frac{\left(\tilde{\pi}_{\tilde{\nu}}^{X}+\tilde{\pi}_{\tilde{\nu}}^{Y}\right)-\left(\tilde{\sigma}_{\tilde{\nu}}^{X}+\tilde{\sigma}_{\tilde{\nu}}^{Y}\right)}{2},\ \chi\left(\tilde{\nu}\right)\in\left[-1,1\right]. \tag{14}$$

Tang [64] gave an order relation between two INNs.

Definition 13[64]. Let $\tilde{\nu} = \left(\left[\tilde{\pi}_{\tilde{\nu}}^{X}, \tilde{\pi}_{\tilde{\nu}}^{Y}\right], \left[\tilde{\vartheta}_{\tilde{\nu}}^{X}, \tilde{\vartheta}_{\tilde{\nu}}^{Y}\right], \left[\tilde{\sigma}_{\tilde{\nu}}^{X}, \tilde{\sigma}_{\tilde{\nu}}^{Y}\right]\right)$ and $\tilde{\mu} = \left(\left[\tilde{\pi}_{\tilde{\mu}}^{X}, \tilde{\pi}_{\tilde{\mu}}^{Y}\right], \left[\tilde{\vartheta}_{\tilde{\mu}}^{X}, \tilde{\vartheta}_{\tilde{\mu}}^{Y}\right], \left[\tilde{\sigma}_{\tilde{\mu}}^{X}, \tilde{\sigma}_{\tilde{\mu}}^{Y}\right]\right)$ be two INNs, $\psi(\tilde{\nu}) = \frac{\left(2+\tilde{\pi}_{\tilde{\nu}}^{X} - \tilde{\vartheta}_{\tilde{\nu}}^{X} - \tilde{\sigma}_{\tilde{\nu}}^{X}\right) + \left(2+\tilde{\pi}_{\tilde{\nu}}^{Y} - \tilde{\vartheta}_{\tilde{\nu}}^{Y} - \tilde{\sigma}_{\tilde{\nu}}^{Y}\right)}{6}$ and $\psi(\tilde{\mu}) = \frac{\left(2+\tilde{\pi}_{\tilde{\mu}}^{X} - \tilde{\vartheta}_{\tilde{\mu}}^{X} - \tilde{\sigma}_{\tilde{\mu}}^{X}\right) + \left(2+\tilde{\pi}_{\tilde{\mu}}^{Y} - \tilde{\vartheta}_{\tilde{\mu}}^{Y} - \tilde{\sigma}_{\tilde{\mu}}^{Y}\right)}{6}$ be the scores of $\tilde{\nu}$ and $\tilde{\mu}$, respectively, and let $\chi(\tilde{\nu}) = \frac{\left(\tilde{\pi}_{\tilde{\nu}}^{X} + \tilde{\pi}_{\tilde{\nu}}^{Y}\right) - \left(\tilde{\sigma}_{\tilde{\nu}}^{X} + \tilde{\sigma}_{\tilde{\nu}}^{Y}\right)}{2}$ and $\chi(\tilde{\mu}) = \frac{\left(\tilde{\pi}_{\tilde{\mu}}^{X} + \tilde{\pi}_{\tilde{\mu}}^{Y}\right) - \left(\tilde{\sigma}_{\tilde{\mu}}^{X} + \tilde{\sigma}_{\tilde{\mu}}^{Y}\right)}{2}$ be the accuracy degrees of $\tilde{\nu}$ and $\tilde{\mu}$, then if $\psi(\tilde{\nu}) < \psi(\tilde{\mu})$, then $\tilde{\nu} < \tilde{\mu}$; if $\psi(\tilde{\nu}) = \psi(\tilde{\mu})$, then

(2) if $\psi(\tilde{\nu}) = \psi(\tilde{\mu})$, then $\tilde{\nu} = \tilde{\mu}$; (2) if $\chi(\tilde{\nu}) < \chi(\tilde{\mu})$, then $\tilde{\mu} < \tilde{\mu}$.

Definition 14[27]. Let $\tilde{\nu}_1 = \left(\left[\tilde{\pi}_1^{X}, \tilde{\pi}_1^{Y}\right], \left[\tilde{\vartheta}_1^{X}, \tilde{\vartheta}_1^{Y}\right], \left[\tilde{\sigma}_1^{X}, \tilde{\sigma}_1^{Y}\right]\right)$ and $\tilde{\nu}_2 = \left(\left[\tilde{\pi}_2^{X}, \tilde{\pi}_2^{Y}\right], \left[\tilde{\vartheta}_2^{X}, \tilde{\vartheta}_2^{Y}\right], \left[\tilde{\sigma}_2^{X}, \tilde{\sigma}_2^{Y}\right]\right)$ be two INNs, and some basic operations are defined:

$$(1)\ \tilde{\nu}_1 \oplus \tilde{\nu}_2 = \begin{pmatrix} \left[\tilde{\pi}_1^{X} + \tilde{\pi}_1^{X} - \tilde{\pi}_1^{X}\tilde{\pi}_1^{X}, \tilde{\pi}_1^{Y} + \tilde{\pi}_1^{Y} - \tilde{\pi}_1^{Y}\tilde{\pi}_1^{Y}\right], \\ \left[\tilde{\vartheta}_1^{X}\tilde{\vartheta}_2^{X}, \tilde{\vartheta}_1^{Y}\tilde{\vartheta}_2^{Y}\right], \left[\tilde{\sigma}_1^{X}\tilde{\sigma}_2^{X}, \tilde{\sigma}_1^{Y}\tilde{\sigma}_2^{Y}\right] \end{pmatrix};$$

$$(2)\ \tilde{\nu}_1 \otimes \tilde{\nu}_2 = \begin{pmatrix} \left[\tilde{\pi}_1^{X}\tilde{\pi}_2^{X}, \tilde{\pi}_1^{Y}\tilde{\pi}_2^{Y}\right], \\ \left[\tilde{\vartheta}_1^{X} + \tilde{\vartheta}_1^{X} - \tilde{\vartheta}_1^{X}\tilde{\vartheta}_1^{X}, \tilde{\vartheta}_1^{Y} + \tilde{\vartheta}_1^{Y} - \tilde{\vartheta}_1^{Y}\tilde{\vartheta}_1^{Y}\right], \\ \left[\tilde{\sigma}_1^{X} + \tilde{\sigma}_1^{X} - \tilde{\sigma}_1^{X}\tilde{\sigma}_1^{X}, \tilde{\sigma}_1^{Y} + \tilde{\sigma}_1^{Y} - \tilde{\sigma}_1^{Y}\tilde{\sigma}_1^{Y}\right] \end{pmatrix};$$

$$(3)\ \lambda\tilde{\nu}_1 = \begin{pmatrix} \left[1-\left(1-\tilde{\pi}_1^{X}\right)^{\lambda}, 1-\left(1-\tilde{\pi}_1^{Y}\right)^{\lambda}\right], \\ \left[\left(\tilde{\vartheta}_1^{X}\right)^{\lambda}, \left(\tilde{\vartheta}_1^{Y}\right)^{\lambda}\right], \left[\left(\tilde{\sigma}_1^{X}\right)^{\lambda}, \left(\tilde{\sigma}_1^{Y}\right)^{\lambda}\right] \end{pmatrix}, \lambda > 0;$$

$$(4)\ \left(\tilde{\nu}_1\right)^{\lambda} = \begin{pmatrix} \left[\left(\tilde{\pi}_1^{X}\right)^{\lambda}, \left(\tilde{\pi}_1^{Y}\right)^{\lambda}\right], \left[\left(\tilde{\vartheta}_1^{X}\right)^{\lambda}, \left(\tilde{\vartheta}_1^{Y}\right)^{\lambda}\right], \\ \left[1-\left(1-\tilde{\sigma}_1^{X}\right)^{\lambda}, 1-\left(1-\tilde{\sigma}_1^{Y}\right)^{\lambda}\right] \end{pmatrix}, \lambda > 0.$$

Theorem 2[27]. Let $\tilde{\nu}_1 = \left(\left[\tilde{\pi}_1^{X}, \tilde{\pi}_1^{Y}\right], \left[\tilde{\vartheta}_1^{X}, \tilde{\vartheta}_1^{Y}\right], \left[\tilde{\sigma}_1^{X}, \tilde{\sigma}_1^{Y}\right]\right)$ and $\tilde{\nu}_2 = \left(\left[\tilde{\pi}_2^{X}, \tilde{\pi}_2^{Y}\right], \left[\tilde{\vartheta}_2^{X}, \tilde{\vartheta}_2^{Y}\right], \left[\tilde{\sigma}_2^{X}, \tilde{\sigma}_2^{Y}\right]\right)$ be two INNs, $\lambda, \lambda_1, \lambda_2 > 0$, then

(1) $\tilde{v}_1 \oplus \tilde{v}_2 = \tilde{v}_2 \oplus \tilde{v}_1$;

(2) $\tilde{v}_1 \otimes \tilde{v}_2 = \tilde{v}_2 \otimes \tilde{v}_1$;

(3) $\lambda(\tilde{v}_1 \oplus \tilde{v}_2) = \lambda \tilde{v}_1 \oplus \lambda \tilde{v}_2$;

(4) $(\tilde{v}_1 \otimes \tilde{v}_2)^{\lambda} = (\tilde{v}_1)^{\lambda} \otimes (\tilde{v}_2)^{\lambda}$;

(5) $\lambda_1 \tilde{v}_1 \oplus \lambda_2 \tilde{v}_1 = (\lambda_1 + \lambda_2)\tilde{v}_1$;

(6) $(\tilde{v}_1)^{\lambda_1} \otimes (\tilde{v}_1)^{\lambda_2} = (\tilde{v}_1)^{(\lambda_1+\lambda_2)}$;

(7) $\left((\tilde{v}_1)^{\lambda_1}\right)^{\lambda_2} = (\tilde{v}_1)^{\lambda_1\lambda_2}$.

Definition 15[27]. Let $\tilde{v}_\beta = \left(\left[\tilde{\pi}_\beta^X, \tilde{\pi}_\beta^Y\right], \left[\tilde{\vartheta}_\beta^X, \tilde{\vartheta}_\beta^Y\right], \left[\tilde{\sigma}_\beta^X, \tilde{\sigma}_\beta^Y\right]\right)(\beta = 1, 2, \cdots, \varphi)$ $(\beta = 1, 2, \cdots, \varphi)$ be a collection of INNs, and let INWA: $Q^{\varphi} \to Q$, if

$$\begin{aligned} \text{INWA}_\gamma(\tilde{v}_1, \tilde{v}_2, \cdots, \tilde{v}_\varphi) &= \overset{\varphi}{\underset{\beta=1}{\oplus}}(\gamma_\beta \tilde{v}_\beta) \\ &= \left(\begin{array}{l} \left[1 - \prod_{\beta=1}^{\varphi}\left(1 - \tilde{\pi}_\beta^X\right)^{\gamma_\beta}, 1 - \prod_{\beta=1}^{\varphi}\left(1 - \tilde{\pi}_\beta^Y\right)^{\gamma_\beta}\right], \\ \left[\prod_{\beta=1}^{\varphi}\left(\tilde{\vartheta}_\beta^X\right)^{\gamma_\beta}, \prod_{\beta=1}^{\varphi}\left(\tilde{\vartheta}_\beta^Y\right)^{\gamma_\beta}\right], \left[\prod_{\beta=1}^{\varphi}\left(\tilde{\sigma}_\beta^X\right)^{\gamma_\beta}, \prod_{\beta=1}^{\varphi}\left(\tilde{\sigma}_\beta^Y\right)^{\gamma_\beta}\right] \end{array} \right) \end{aligned} \quad (15)$$

where $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_\varphi)^T$ be the criterion weight, and $\gamma_\beta > 0$, $\sum_{\beta=1}^{n} \gamma_\beta = 1$, then INWA is called INWA operator.

Definition 16[64]. Let $\tilde{v}_1 = \left(\left[\tilde{\pi}_1^X, \tilde{\pi}_1^Y\right], \left[\tilde{\vartheta}_1^X, \tilde{\vartheta}_1^Y\right], \left[\tilde{\sigma}_1^X, \tilde{\sigma}_1^Y\right]\right)$ and $\tilde{v}_2 = \left(\left[\tilde{\pi}_2^X, \tilde{\pi}_2^Y\right], \left[\tilde{\vartheta}_2^X, \tilde{\vartheta}_2^Y\right], \left[\tilde{\sigma}_2^X, \tilde{\sigma}_2^Y\right]\right)$ be two INNs, then the normalized Hamming distance between $\tilde{v}_1 = \left(\left[\tilde{\pi}_1^X, \tilde{\pi}_1^Y\right], \left[\tilde{\vartheta}_1^X, \tilde{\vartheta}_1^Y\right], \left[\tilde{\sigma}_1^X, \tilde{\sigma}_1^Y\right]\right)$ and $\tilde{v}_2 = \left(\left[\tilde{\pi}_2^X, \tilde{\pi}_2^Y\right], \left[\tilde{\vartheta}_2^X, \tilde{\vartheta}_2^Y\right], \left[\tilde{\sigma}_2^X, \tilde{\sigma}_2^Y\right]\right)$ is defined:

$$\kappa(\tilde{v}_1, \tilde{v}_2) = \frac{1}{6}\left(\begin{array}{l} \left|\tilde{\pi}_1^X - \tilde{\pi}_2^X\right| + \left|\tilde{\pi}_1^Y - \tilde{\pi}_2^Y\right| + \left|\tilde{\vartheta}_1^X - \tilde{\vartheta}_2^X\right| + \\ \left|\tilde{\vartheta}_1^Y - \tilde{\vartheta}_2^Y\right| + \left|\tilde{\sigma}_1^X - \tilde{\sigma}_2^X\right| + \left|\tilde{\sigma}_1^Y - \tilde{\sigma}_2^Y\right| \end{array} \right) \quad (16)$$

Let ε, ς and γ be presented as in section 3. Suppose that $\tilde{V} = (\tilde{v}_{\alpha\beta})_{\phi\times\varphi} = \left(\left[\tilde{\pi}_{\alpha\beta}^X, \tilde{\pi}_{\alpha\beta}^Y\right], \left[\tilde{\vartheta}_{\alpha\beta}^X, \tilde{\vartheta}_{\alpha\beta}^Y\right], \left[\tilde{\sigma}_{\alpha\beta}^X, \tilde{\sigma}_{\alpha\beta}^Y\right]\right)_{\phi\times\varphi}$ is the INN matrix,

$\left[\tilde{\pi}_{\alpha\beta}^{X},\tilde{\pi}_{\alpha\beta}^{Y}\right]\subseteq[0,1]$, $\left[\tilde{\vartheta}_{\alpha\beta}^{X},\tilde{\vartheta}_{\alpha\beta}^{Y}\right]\subseteq[0,1]$, $\left[\tilde{\sigma}_{\alpha\beta}^{X},\tilde{\sigma}_{\alpha\beta}^{Y}\right]\subseteq[0,1]$, $0\le\tilde{\pi}_{\alpha\beta}^{Y}+\tilde{\vartheta}_{\alpha\beta}^{Y}+\tilde{\sigma}_{\alpha\beta}^{Y}\le 3$, $\alpha=1,2,\cdots,\phi$, $\beta=1,2,\cdots,\varphi$. The subjective preference information on alternatives is known, and $\tilde{v}_{\alpha}=\left(\left[\tilde{\pi}_{\tilde{v}_\alpha}^{X},\tilde{\pi}_{\tilde{v}_\alpha}^{Y}\right],\left[\tilde{\vartheta}_{\tilde{v}_\alpha}^{X},\tilde{\vartheta}_{\tilde{v}_\alpha}^{Y}\right],\left[\tilde{\sigma}_{\tilde{v}_\alpha}^{X},\tilde{\sigma}_{\tilde{v}_\alpha}^{Y}\right]\right)$ are INNs, which is subjective preference values on alternative $\varepsilon_{\alpha}\left(\alpha=1,2,\cdots,\phi\right)$.

Definition 17[27]. Let $\tilde{V}=\left(\tilde{v}_{\alpha\beta}\right)_{\phi\times\varphi}=\left(\left[\tilde{\pi}_{\alpha\beta}^{X},\tilde{\pi}_{\alpha\beta}^{Y}\right],\left[\tilde{\vartheta}_{\alpha\beta}^{X},\tilde{\vartheta}_{\alpha\beta}^{Y}\right],\left[\tilde{\sigma}_{\alpha\beta}^{X},\tilde{\sigma}_{\alpha\beta}^{Y}\right]\right)_{\phi\times\varphi}$ is the INN matrix, $\tilde{v}_{\alpha}=\left(\tilde{v}_{\alpha1},\tilde{v}_{\alpha2},\cdots,\tilde{v}_{\alpha\varphi}\right)$ be the vector of attribute values for ε_{α}, $\alpha=1,2,\cdots,\phi$, then we call

$$\begin{aligned}\tilde{v}_{\alpha}&=\left(\left[\tilde{\pi}_{\alpha}^{X},\tilde{\pi}_{\alpha}^{Y}\right],\left[\tilde{\vartheta}_{\alpha}^{X},\tilde{\vartheta}_{\alpha}^{Y}\right],\left[\tilde{\sigma}_{\alpha}^{X},\tilde{\sigma}_{\alpha}^{Y}\right]\right)\\&=\mathrm{INWA}_{\gamma}\left(\tilde{v}_{\alpha1},\tilde{v}_{\alpha2},\cdots,\tilde{v}_{\alpha\varphi}\right)=\mathop{\oplus}_{\beta=1}^{\varphi}\left(\gamma_{\beta}\tilde{v}_{\alpha\beta}\right)\\&=\left(\begin{array}{l}\left[1-\prod_{\beta=1}^{\varphi}\left(1-\tilde{\pi}_{\alpha\beta}^{X}\right)^{\gamma_\beta},1-\prod_{\beta=1}^{\varphi}\left(1-\tilde{\pi}_{\alpha\beta}^{Y}\right)^{\gamma_\beta}\right],\\\left[\prod_{\beta=1}^{\varphi}\left(\tilde{\vartheta}_{\alpha\beta}^{X}\right)^{\gamma_\beta},\prod_{\beta=1}^{\varphi}\left(\tilde{\vartheta}_{\alpha\beta}^{Y}\right)^{\gamma_\beta}\right],\left[\prod_{\beta=1}^{\varphi}\left(\tilde{\sigma}_{\alpha\beta}^{X}\right)^{\gamma_\beta},\prod_{\beta=1}^{\varphi}\left(\tilde{\sigma}_{\alpha\beta}^{Y}\right)^{\gamma_\beta}\right]\end{array}\right)\end{aligned}\tag{17}$$

the overall value of ε, where $\gamma=\left(\gamma_1,\gamma_2,\cdots,\gamma_{\varphi}\right)^{T}$ is the criterion weight.

When the attribute weight information is completely known, aggregate all the weighted attribute values corresponding to each alternative into a whole using Eq. (17).

If the decision model is difficult to obtain attribute weight, sometimes the criterion weight information is completely unknown. To reflect the subjective preference and objective information of decision-makers, an optimization model is established to obtain the weight of attributes. However, there are some differences between DM's subjective preference and objective information. To make the decision more reasonable, the selection of criterion weight vector is to minimize the total deviation between objective information and DM subjective preference.

The least deviation method was used to calculate the difference between DM's subjective preference and objective information. For the $\varsigma_{\beta}\in\varsigma$, the deviation of alternative ε_{α} to DM's subjective preference is described as follows:

$$K_{\alpha\beta}\left(\gamma\right)=\kappa\left(\tilde{v}_{\alpha\beta},\tilde{v}_{\alpha}\right)\gamma_{\beta},\alpha=1,2,\cdots,\phi,\beta=1,2,\cdots,\varphi.\tag{18}$$

Let $K_\alpha(\gamma)=\sum_{\beta=1}^{\varphi}K_{\alpha\beta}(\gamma)=\sum_{\beta=1}^{\varphi}\kappa\left(\tilde{v}_{\alpha\beta},\tilde{v}_\alpha\right)\gamma_\beta,\alpha=1,2,\cdots,\phi$ Based on the above analysis, we must choose weights to minimize all deviations from all alternatives. To this end, we establish a linear programming model:

$$(\text{M-3})\quad\begin{cases}\min K(\gamma)=\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}K_{\alpha\beta}(\gamma)=\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}\kappa\left(\tilde{v}_{\alpha\beta},\tilde{v}_\alpha\right)\gamma_\beta\\ \text{Subject to}\quad \sum_{\beta=1}^{\varphi}\gamma_\beta=1,\gamma_\beta\geq 0,\beta=1,2,\cdots,\varphi\end{cases}$$

where $\kappa\left(\tilde{v}_{\alpha\beta},\tilde{v}_\alpha\right)=\frac{1}{6}\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X-\tilde{\pi}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\pi}_{\alpha\beta}^Y-\tilde{\pi}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\vartheta}_{\alpha\beta}^X-\tilde{\vartheta}_{\tilde{v}_\alpha}^X\right|\\+\left|\tilde{\vartheta}_{\alpha\beta}^Y-\tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\sigma}_{\alpha\beta}^X-\tilde{\sigma}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\sigma}_{\alpha\beta}^Y-\tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}$.

By solving the model (M-3), we get the $\gamma=\left(\gamma_1,\gamma_2,\cdots,\gamma_\varphi\right)$, which is used as attributes weight.

If the information about criterion weights is completely unknown, we build another programming model:

$$(\text{M-4})\begin{cases}\min K(\gamma)=\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}K_{\alpha\beta}(\gamma)\\ =\frac{1}{6}\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X-\tilde{\pi}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\pi}_{\alpha\beta}^Y-\tilde{\pi}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\vartheta}_{\alpha\beta}^X-\tilde{\vartheta}_{\tilde{v}_\alpha}^X\right|+\\ \left|\tilde{\vartheta}_{\alpha\beta}^Y-\tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\sigma}_{\alpha\beta}^X-\tilde{\sigma}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\sigma}_{\alpha\beta}^Y-\tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}\gamma_\beta\\ s.t.\ \sum_{\beta=1}^{\varphi}\gamma_\beta^2=1,\gamma_\beta\geq 0,\beta=1,2,\cdots,\varphi\end{cases}$$

To solve this model, we build the Lagrange function:

$$L(\gamma,\lambda)=\frac{1}{6}\sum_{\alpha=1}^{\phi}\sum_{\beta=1}^{\varphi}\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X-\tilde{\pi}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\pi}_{\alpha\beta}^Y-\tilde{\pi}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\vartheta}_{\alpha\beta}^X-\tilde{\vartheta}_{\tilde{v}_\alpha}^X\right|+\\ \left|\tilde{\vartheta}_{\alpha\beta}^Y-\tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\sigma}_{\alpha\beta}^X-\tilde{\sigma}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\sigma}_{\alpha\beta}^Y-\tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}\gamma_\beta+\frac{\lambda}{12}\left(\sum_{\beta=1}^{\varphi}\gamma_\beta^2-1\right)\quad(19)$$

where λ is the Lagrange multiplier.

Differentiating Eq. (19) with respect to $\gamma_\beta\left(\beta=1,2,\cdots,\varphi\right)$ and λ, and setting these partial derivatives equal to zero,

$$\begin{cases}\frac{\partial L}{\partial\gamma_\beta}=\sum_{\alpha=1}^{\phi}\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X-\tilde{\pi}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\pi}_{\alpha\beta}^Y-\tilde{\pi}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\vartheta}_{\alpha\beta}^X-\tilde{\vartheta}_{\tilde{v}_\alpha}^X\right|\\+\left|\tilde{\vartheta}_{\alpha\beta}^Y-\tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right|+\left|\tilde{\sigma}_{\alpha\beta}^X-\tilde{\sigma}_{\tilde{v}_\alpha}^X\right|+\left|\tilde{\sigma}_{\alpha\beta}^Y-\tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}+\lambda\gamma_\beta=0\\ \frac{\partial L}{\partial\lambda}=\sum_{\beta=1}^{\varphi}\gamma_\beta^2-1=0\end{cases}\quad(20)$$

By solving Eq. (20), we get the attribute weights:

$$\gamma_\beta^* = \frac{\sum_{\alpha=1}^{\phi}\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X - \tilde{\pi}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\pi}_{\alpha\beta}^Y - \tilde{\pi}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\vartheta}_{\alpha\beta}^X - \tilde{\vartheta}_{\tilde{v}_\alpha}^X\right| + \\ \left|\tilde{\vartheta}_{\alpha\beta}^Y - \tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\sigma}_{\alpha\beta}^X - \tilde{\sigma}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\sigma}_{\alpha\beta}^Y - \tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}}{\sqrt{\sum_{\beta=1}^{\varphi}\left[\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X - \tilde{\pi}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\pi}_{\alpha\beta}^Y - \tilde{\pi}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\vartheta}_{\alpha\beta}^X - \tilde{\vartheta}_{\tilde{v}_\alpha}^X\right| + \\ \left|\tilde{\vartheta}_{\alpha\beta}^Y - \tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\sigma}_{\alpha\beta}^X - \tilde{\sigma}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\sigma}_{\alpha\beta}^Y - \tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}\right]^2}} \tag{21}$$

By normalizing $\gamma_\beta^*\left(\beta = 1, 2, \cdots, \varphi\right)$ be a unit, we have

$$w_j = \frac{\sum_{\alpha=1}^{\phi}\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X - \tilde{\pi}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\pi}_{\alpha\beta}^Y - \tilde{\pi}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\vartheta}_{\alpha\beta}^X - \tilde{\vartheta}_{\tilde{v}_\alpha}^X\right| + \\ \left|\tilde{\vartheta}_{\alpha\beta}^Y - \tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\sigma}_{\alpha\beta}^X - \tilde{\sigma}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\sigma}_{\alpha\beta}^Y - \tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}}{\sum_{\beta=1}^{\varphi}\sum_{\alpha=1}^{\phi}\begin{pmatrix}\left|\tilde{\pi}_{\alpha\beta}^X - \tilde{\pi}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\pi}_{\alpha\beta}^Y - \tilde{\pi}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\vartheta}_{\alpha\beta}^X - \tilde{\vartheta}_{\tilde{v}_\alpha}^X\right| + \\ \left|\tilde{\vartheta}_{\alpha\beta}^Y - \tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right| + \left|\tilde{\sigma}_{\alpha\beta}^X - \tilde{\sigma}_{\tilde{v}_\alpha}^X\right| + \left|\tilde{\sigma}_{\alpha\beta}^Y - \tilde{\sigma}_{\tilde{v}_\alpha}^Y\right|\end{pmatrix}} \tag{22}$$

Based on the above model, a practical method is proposed to solve INN-MADM with alternative preference information. The method includes the following steps:

(Procedure two)

Step 1. Let $\tilde{V} = \left(\tilde{v}_{\alpha\beta}\right)_{\phi\times\varphi} = \left(\left[\tilde{\pi}_{\alpha\beta}^X, \tilde{\pi}_{\alpha\beta}^Y\right], \left[\tilde{\vartheta}_{\alpha\beta}^X, \tilde{\vartheta}_{\alpha\beta}^Y\right], \left[\tilde{\sigma}_{\alpha\phi}^X, \tilde{\sigma}_{\alpha\beta}^Y\right]\right)_{\phi\times\varphi}$ be an INN matrix, where $\tilde{v}_{\alpha\beta} = \left[\tilde{\pi}_{\alpha\beta}^X, \tilde{\pi}_{\alpha\beta}^Y\right], \left[\tilde{\vartheta}_{\alpha\beta}^X, \tilde{\vartheta}_{\alpha\beta}^Y\right], \left[\tilde{\sigma}_{\alpha\beta}^X, \tilde{\sigma}_{\alpha\beta}^Y\right]$, for $\varepsilon_\alpha \in \varepsilon$ with respect to $\varsigma_\beta \in \varsigma$, $\gamma = \left(\gamma_1, \gamma_2, \cdots, \gamma_\varphi\right)$ be the weight of attributes, where $\gamma_\beta \in [0,1]$, $\beta = 1, 2, \cdots, \varphi$, which is constructed by the forms 1-5. Let $\tilde{v} = \left(\tilde{v}_1, \tilde{v}_2, \cdots, v_\phi\right)$ be subjective preference, $\tilde{v}_\alpha = \left(\left[\tilde{\pi}_{\tilde{v}_\alpha}^X, \tilde{\pi}_{\tilde{v}_\alpha}^Y\right], \left[\tilde{\vartheta}_{\tilde{v}_\alpha}^X, \tilde{\vartheta}_{\tilde{v}_\alpha}^Y\right], \left[\tilde{\sigma}_{\tilde{v}_\alpha}^X, \tilde{\sigma}_{\tilde{v}_\alpha}^Y\right]\right)$ are INNs, which are subjective preference on alternatives $\varepsilon_\alpha\left(\alpha = 1, 2, \cdots, \phi\right)$.

Step 2. By solving the model (M-3), the partially known index values of the weight is obtained. If the criterion weight is unknown, then we can obtain the criterion weights by Eq. (22).

Step 3. Utilize $\gamma = \left(\gamma_1, \gamma_2, \cdots, \gamma_\varphi\right)$ and Eq. (17), we obtain the $\tilde{v}_\alpha$ of $\varepsilon_\alpha\left(\alpha = 1, 2, \cdots, \phi\right)$.

Step 4. Compute out the scores $\psi\left(\tilde{v}_\alpha\right)$ of $\tilde{v}_\alpha\left(\alpha = 1, 2, \cdots, \phi\right)$ to rank all the alternatives $\varepsilon_\alpha\left(\alpha = 1, 2, \cdots, \phi\right)$ then rank the alternatives ε_α and ε_β through $\chi\left(\tilde{v}_\alpha\right)$ and $\chi\left(\tilde{v}_\beta\right)$.

Step 5. Rank all the alternatives $\varepsilon_\alpha \left(\alpha = 1, 2, \cdots, m\right)$ and select the best one(s) through $\psi\left(\tilde{v}_\alpha\right)$ and $\chi\left(\tilde{v}_\alpha\right)\left(\alpha = 1, 2, \cdots, \phi\right)$.

Step 6. End.

5. Case Study

With the expansion of the e-commerce Internet, online shopping has been enthusiastically sought after by people, and the logistics industry has also risen rapidly. Logistics promotes economic growth and is increasingly prominent in the national economic status. However, as an indispensable part of the logistics industry, logistics parks have many difficult problems. Usually occupies a large scale, and the construction investment cost is high. Once completed, it is not easy to relocate, and today's environmental problems are becoming more and more serious. The basic criteria for planning and building a low-carbon logistics park It is "low energy consumption, high efficiency". The location problem of low-carbon logistics parks can be regarded as a MADM problem. Generally, multiple decision-makers give corresponding evaluations to a limited number of alternatives under the influence of different factors, and use scientific decision-making methods to evaluate the relevant ones. The evaluation information is processed, so as to sort the different alternatives and make a reasonable choice. In this section, we apply the constructed model to a real-world example, taking the low-carbon logistics park site selection as an example. Through market research, a panel of five possible low-carbon logistics park sites $\varepsilon_\alpha \left(\alpha = 1, 2, 3, 4, 5\right)$ was selected. The experts selected four indexes to evaluate five low-carbon logistics park sites: ① ς_1 is transportation and warehousing investments; ② ς_2 is regional goods material turnover; ③ ς_3 is land use; ④ ς_4 is degree of environmental protection. Five possible low-carbon logistics park sites $\varepsilon_\alpha \left(\alpha = 1, 2, 3, 4, 5\right)$ will use the SVNNs by the decision maker under the above four attributes, as listed in the following matrix.

$$\tilde{V}=\begin{bmatrix} (0.5, 0.8, 0.1) & (0.6, 0.3, 0.3) & (0.3, 0.6, 0.1) & (0.5,0.7,0.2) \\ (0.7, 0.2, 0.1) & (0.7, 0.2, 0.2) & (0.7, 0.2, 0.4) & (0.8,0.2,0.1) \\ (0.6, 0.7, 0.2) & (0.5, 0.7, 0.3) & (0.5, 0.3, 0.1) & (0.6,0.3,0.2) \\ (0.8, 0.1, 0.3) & (0.6, 0.3, 0.4) & (0.3, 0.4, 0.2) & (0.5,0.6,0.1) \\ (0.6, 0.4, 0.4) & (0.4, 0.8, 0.1) & (0.7, 0.6, 0.1) & (0.5,0.8,0.2) \end{bmatrix}$$

DMs' subjective preference value on alternative:

$$\tilde{v}_1 = (0.6, 0.5, 0.2), \tilde{v}_2 = (0.7, 0.2, 0.1), \tilde{v}_3 = (0.3, 0.4, 0.3)$$
$$\tilde{v}_4 = (0.9, 0.3, 0.2), \tilde{v}_5 = (0.5, 0.6, 0.4)$$

Next, developed method is used to select the best location of low-carbon logistics park.

Case 1: Criterion weight information is known as follows:

$$\gamma = \{0.18 \le \gamma_1 \le 0.23, 0.20 \le \gamma_2 \le 0.24, 0.25 \le \gamma_3 \le 0.30,$$
$$0.25 \le \gamma_4 \le 0.33, \gamma_\beta \ge 0, \beta = 1, 2, 3, 4, \sum_{\beta=1}^{4} \gamma_\beta = 1\}$$

Step 1. The single-objective programming model is obtained as follows:

$$\min K(\gamma) = 0.6333\gamma_1 + 0.6667\gamma_2 + 0.8333\gamma_3 + 0.7000\gamma_4$$

Solving this model, we get the weight of attributes: $\gamma = (0.2300\ \ 0.2400\ \ 0.2500\ \ 0.2800\ )^T$

Step 2. Utilize the weight $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_\varphi)$ and by Eq. (6), we obtain the $\tilde{v}_\alpha$ of the low-carbon logistics park site $\varepsilon_\alpha\ (\alpha = 1, 2, \cdots, \phi)$.

$$\tilde{v}_1 = (0.4845, 0.5667, 0.1581), \tilde{v}_2 = (0.7322, 0.2000, 0.1670)$$
$$\tilde{v}_3 = (0.5538, 0.4468, 0.1854), \tilde{v}_4 = (0.5824, 0.3040, 0.2135)$$
$$\tilde{v}_5 = (0.5633, 0.6348, 0.1670)$$

Step 3. Calculate the scores $\psi(\tilde{v}_\alpha)$ of $\tilde{v}_\alpha\ (\alpha = 1, 2, \cdots, \phi)$

$$\psi(\tilde{v}_1) = 0.5866, \psi(\tilde{v}_2) = 0.7884, \psi(\tilde{v}_3) = 0.6406$$
$$\psi(\tilde{v}_4) = 0.6883, \psi(\tilde{v}_5) = 0.5872$$

Step 4. Rank all the low-carbon logistics park sites $\varepsilon_\alpha\ (\alpha = 1, 2, 3, 4, 5)$ through $\psi(\tilde{v}_\alpha)$ $(\alpha = 1, 2, \cdots, 5)$: $\varepsilon_2 \succ \varepsilon_4 \succ \varepsilon_3 \succ \varepsilon_5 \succ \varepsilon_1$, and thus the most desirable low-carbon logistics park site is ε_5.

Case 2: When the weight is unknown, we use another method to get the optimal location of low-carbon logistics park.

Step 1. Get the weight of attributes:

$$\gamma = (0.2235\ \ 0.2353\ \ 0.2941\ \ 0.2471\ )^T$$

Step 2. Utilize the $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_\varphi)$ and Eq. (6), we obtain the overall values $\tilde{v}_\alpha$ of the low-carbon logistics park site $\varepsilon_\alpha\ (\alpha = 1, 2, \cdots, \phi)$.

$$\tilde{v}_1=(0.4762,0.5647,0.1537),\tilde{v}_2=(0.7286,0.2000,0.1770)$$
$$\tilde{v}_3=(0.5498,0.4425,0.1794),\tilde{v}_4=(0.5732,0.3031,0.2172)$$
$$\tilde{v}_5=(0.5727,0.6296,0.1618)$$

Step 3. Compute out the scores $\psi(\tilde{v}_\alpha)\ (\alpha=1,2,\cdots,\phi)$ of $\tilde{v}_\alpha\ (\alpha=1,2,\cdots,\phi)$.

$$\psi(\tilde{v}_1)=0.5860,\psi(\tilde{v}_2)=0.7839,\psi(\tilde{v}_3)=0.6426$$
$$\psi(\tilde{v}_4)=0.6843,\psi(\tilde{v}_5)=0.5938$$

Step 4. Rank all the solutions through $\psi(\tilde{v}_\alpha):\varepsilon_2\succ\varepsilon_4\succ\varepsilon_3\succ\varepsilon_5\succ\varepsilon_1$, and thus the most desirable low-carbon logistics park site is ε_2.

If the five possible low-carbon logistics park sites $\varepsilon_\alpha\ (\alpha=1,2,3,4,5)$ are to be evaluated using the INNs, as listed in the following matrix.

$$\tilde{V}=\begin{bmatrix}
([0.5,0.6],[0.8,0.9],[0.1,0.2]) & ([0.6,0.7],[0.3,0.4],[0.3,0.4]) & ([0.3,0.4],[0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.7,0.8],[0.1,0.2])\\
([0.7,0.9],[0.2,0.3],[0.1,0.2]) & ([0.7,0.8],[0.1,0.2],[0.2,0.3]) & ([0.7,0.9],[0.2,0.3],[0.4,0.5]) & ([0.8,0.9],[0.2,0.3],[0.1,0.2])\\
([0.6,0.7],[0.7,0.8],[0.2,0.3]) & ([0.5,0.6],[0.7,0.8],[0.3,0.4]) & ([0.5,0.6],[0.3,0.4],[0.1,0.2]) & ([0.6,0.7],[0.3,0.4],[0.2,0.3])\\
([0.8,0.9],[0.1,0.2],[0.3,0.4]) & ([0.6,0.7],[0.3,0.4],[0.4,0.5]) & ([0.3,0.4],[0.4,0.5],[0.2,0.3]) & ([0.5,0.6],[0.6,0.7],[0.1,0.2])\\
([0.6,0.7],[0.4,0.5],[0.4,0.5]) & ([0.4,0.5],[0.8,0.9],[0.1,0.2]) & ([0.7,0.8],[0.6,0.7],[0.1,0.2]) & ([0.5,0.6],[0.8,0.9],[0.2,0.3])
\end{bmatrix}$$

DM's subjective preference value on alternative is:

$$\tilde{v}_1=([0.6,0.7],[0.5,0.6],[0.2,0.3]),\tilde{v}_2=([0.7,0.8],[0.2,0.3],[0.1,0.2])$$
$$\tilde{v}_3=([0.3,0.4],[0.4,0.5],[0.3,0.4]),\tilde{v}_4=([0.9,1.0],[0.3,0.4],[0.2,0.3])$$
$$\tilde{v}_5=([0.5,0.6],[0.6,0.7],[0.4,0.5])$$

Case 1: The attribute weights are partially known,

$$\gamma=\{0.18\le\gamma_1\le0.23,0.20\le\lambda_2\le0.24,0.25\le\gamma_3\le0.30,$$
$$0.25\le\gamma_4\le0.33,\gamma_\beta\ge0,\beta=1,2,3,4,\textstyle\sum_{\beta=1}^{4}\gamma_\beta=1\}$$

Step 1. Establish the ingle-objective programming model:

$$\min K(\gamma)=0.6500\gamma_1+0.7000\gamma_2+0.8500\gamma_3+0.7333\gamma_4$$

Solving this model, we get the weight of attributes:

$$\gamma = \begin{pmatrix} 0.2300 & 0.2400 & 0.2500 & 0.2800 \end{pmatrix}^T$$

Step 2. Utilize the $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_\varphi)$ and Eq. (17), we obtain the $\tilde{v}_\alpha$ of the low-carbon logistics park sites $\varepsilon_\alpha \left(\alpha = 1, 2, \cdots, \phi\right)$.

$$\begin{aligned}
\tilde{v}_1 &= ([0.4845, 0.5168], [0.5667, 0.6731], [0.1302, 0.2362]) \\
\tilde{v}_2 &= ([0.7322, 0.8556], [0.1693, 0.2722], [0.1670, 0.2772]) \\
\tilde{v}_3 &= ([0.5538, 0.6323], [0.4468, 0.5540], [0.1854, 0.2905]) \\
\tilde{v}_4 &= ([0.5824, 0.6960], [0.3040, 0.4218], [0.2135, 0.3234]) \\
\tilde{v}_5 &= ([0.5633, 0.4059], [0.6348, 0.7383], [0.1670, 0.2766])
\end{aligned}$$

Step 3. Calculate the scores $\psi(\tilde{\gamma}_\alpha)$ of $\varepsilon_\alpha \left(\alpha = 1, 2, \cdots, \phi\right)$

$$\begin{aligned}
&\psi(\tilde{v}_1) = 0.5658, \psi(\tilde{v}_2) = 0.7837, \psi(\tilde{v}_3) = 0.6183 \\
&\psi(\tilde{v}_4) = 0.6693, \psi(\tilde{v}_5) = 0.5254
\end{aligned}$$

Step 4. Rank all the low-carbon logistics park sites $\varepsilon_\alpha \left(\alpha = 1, 2, 3, 4, 5\right)$ through scores $\psi(\tilde{v}_\alpha)$ $(\alpha = 1, 2, \cdots, 5)$ of $\tilde{v}_\alpha \left(\alpha = 1, 2, \cdots, \phi\right)$: $\varepsilon_2 \succ \varepsilon_4 \succ \varepsilon_3 \succ \varepsilon_1 \succ \varepsilon_5$, and most desirable alternative is ε_2.

Case 2: If attribute weights are completely unknown, we utilize an alternative approach to obtain the best low-carbon logistics park sites.

Step 1. Utilize the Eq. (22) to get the weight of attributes:

$$\gamma = \begin{pmatrix} 0.2216 & 0.2386 & 0.2898 & 0.2500 \end{pmatrix}^T$$

Step 2. Utilize the $\gamma = (\gamma_1, \gamma_2, \cdots, \gamma_\varphi)$ and Eq. (17), we obtain the $\tilde{v}_\alpha$ of the low-carbon logistics park site $\varepsilon_\alpha \left(\alpha = 1, 2, \cdots, \phi\right)$.

$$\begin{aligned}
\tilde{v}_1 &= ([0.4774, 0.5169], [0.5633, 0.6696], [0.1300, 0.2360]) \\
\tilde{v}_2 &= ([0.7289, 0.8546], [0.1695, 0.2723], [0.1763, 0.2873]) \\
\tilde{v}_3 &= ([0.5499, 0.6328], [0.4431, 0.5503], [0.1802, 0.2857]) \\
\tilde{v}_4 &= ([0.5734, 0.6960], [0.3040, 0.4209], [0.2171, 0.3264]) \\
\tilde{v}_5 &= ([0.5714, 0.4078], [0.6312, 0.7346], [0.1617, 0.2712])
\end{aligned}$$

Step 3. Calculate the scores of $\tilde{v}_\alpha \left(\alpha = 1, 2, \cdots, \phi\right)$.

$$\psi\left(\tilde{v}_1\right) = 0.5659, \psi\left(\tilde{v}_2\right) = 0.7797, \psi\left(\tilde{v}_3\right) = 0.6206,$$
$$\psi\left(\tilde{v}_4\right) = 0.6668, \psi\left(\tilde{v}_5\right) = 0.5301$$

Step 4. Rank all the low-carbon logistics park sites $\varepsilon_\alpha \left(\alpha = 1, 2, 3, 4, 5\right)$ through $\psi\left(\tilde{v}_\alpha\right)$: $\varepsilon_2 \succ \varepsilon_4 \succ \varepsilon_3 \succ \varepsilon_1 \succ \varepsilon_5$, and thus the most optimal low-carbon logistics park site is ε_2.

6. Conclusion

Under the background of increasingly standardized logistics market and increasingly fierce market competition, there is an increasing demand for establishing and improving logistics functions and information-based logistics centers. In order to respond to the new needs of economic and social development and advocate the concept of green, low-carbon and sustainable development, low-carbon logistics is the only way for the development of the logistics industry. The planning and construction of logistics parks are considered to be an important part of promoting the development of modern logistics. In the planning process of the logistics park, the layout and location function are the important basic parts that affect the overall development of the logistics park. Choosing a reasonable location is particularly important for building a logistics center. One of the most important parts of logistics park planning is the quantitative optimization of the logistics park location problem. In recent years, the location theory has developed rapidly, and there are many types of locations. The rapid development of the location theory of logistics parks is attributed to the informatization of today's science and technology, which provides a powerful tool for feasibility analysis and rational decision-making. The logistics park location problem is also regarded as a MADM problem. In this manuscript, we studied the SVN-MADM problem with alternative preference information. In the fuzzy background, the weight information of indicators is often uncertain, and based on this, the minimum deviation method is used to determine the weight of indicators. On the other hand, in the process of MADM, in order to obtain comprehensive evaluation information, The SVNWA operator is used to aggregate all decision information. Calculate the value of the scoring function and the accuracy function and rank the alternatives. On the basis of guaranteeing the validity, the calculation steps are relatively simple, thus realizing the operability. Furthermore, the above models and methods are extended to INNs. Finally, illustrative examples for low-carbon logistics park site selection demonstrates the extension of the model from theory to practical application. The constructed models and methods can be applied to other MADM problems, such as investment risk assessment, selection of commodity suppliers, selection of factory locations, etc. In the future research, we shall continue to focus on the detailed research of decision-making methods and aggregation operators by fusing TODIM method[65-67], QUALIFLEX method [68-71], ARAS method [72-75], WASPAS method [76-79], Maclaurin symmetric mean (MSM) [80], Muirhead mean (MM) [81-84] and power average (PA) [85, 86] operators to Neutrosophic numbers and propose some new MAGDM methods.

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