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Single Valued Pentaparitioned Neutrosophic Off-Set / Over-Set / Under-Set

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Abstract: The main focus of this paper is to introduce the notion of single valued pentapartitioned neutrosophic off set / over set / under set. Besides, we establish several operations on single valued pentapartitioned neutrosophic off sets / over sets / under sets. Besides, we furnish some suitable examples to validate the results established in this article. Further, we establish some interesting results on single valued pentapartitioned neutrosophic off set / over set / under set.

Keywords: Neutrosophic Set; SV-PN-Set; SV-PN-off-set; SV-PN-over-set; SV-PN-under-set.

1. Introduction: In 1965, Zadeh [33] grounded the concept of fuzzy set, where every element has membership values between 0 and 1. Afterwards, Atanassov [1] introduced the notion of intuitionistic fuzzy set as an extension of fuzzy set. In 1998, Smarandache [27] presented the concept of neutrosophic set (in short N-S) by extending the idea of fuzzy set and intuitionistic fuzzy set to deal with the uncertainty events having indeterminacy. In every N-S, each member has three independent components namely truth, indeterminacy and false membership values. Later on, Wang et al. [32] grounded the notion of single valued neutrosophic set (in short SV-N-S), which is basically a subclass of N-S. One can use SV-N-S to represent indeterminate and incomplete information which makes trouble to take decision (or in selection) in the real world. Thereafter, many researchers of different countries used the notion of SV-N-S in their model (or algorithm) in the different branches of real world such as medical diagnosis, educational problem, social problems, decision-making problems, conflict resolution, image processing, etc. In 2013, Smarandache [28] introduced the idea of n-valued refined neutrosophic logic, and applied this notion in physics. In 2016, Smarandache [29] grounded a new concept of neutrosophic over-set, neutrosophic under-set, neutrosophic off-set, and studied their various properties.

In the year 2020, Mallick and Pramanik [25] grounded the idea of single valued pentapartitioned neutrosophic set by splitting the indeterminacy into three independent components namely contradiction, ignorance and unknown-membership, and studied several properties of them. In 2021, Das and Tripathy [17] grounded the notion of pentapartitioned neutrosophic topological space and formulated several results on it. Afterwards, Das et al. [12] established an MADM strategy based on tangent similarity measure. Later on, Majumder et al. [24] presented a cosine similarity measure based MADM strategy under the single valued pentapartitioned neutrosophic set environment. Recently, Das et al. [13] established a MADM strategy using grey relational analysis method under the single valued pentapartitioned neutrosophic set environment.

In this article, we introduce the notion of single valued pentapartitioned neutrosophic off set / over set / under set. Besides, we establish several operations on single valued pentapartitioned neutrosophic off sets / over sets / under sets. Besides, we furnish some suitable examples to validate the results established in this article. Further, we establish some interesting results on single valued pentapartitioned neutrosophic off set / over set / under set.

Research gap: No investigation on single valued pentapartitioned neutrosophic over-set / under-set / off-set has been reported in the recent literature.

Motivation: To fill the research gap, we introduce and study the notion of single valued pentapartitioned neutrosophic over-set/under-set/off-set.

The remaining part of this article has been divided into following three sections:

In section 2, we recall some basic definitions and properties related to N-Ss, single valued neutrosophic over-sets / under-sets / off-sets and single valued pentapartitioned neutrosophic sets. In section 3, we introduce the notion of single valued pentapartitioned neutrosophic over-set / under-set / off-set, and study some of their basic properties. In this section, we also formulated several interesting results on single valued pentapartitioned neutrosophic over-set / under-set / off-set. In section 4, we conclude the word done in this article.

2. Some Relevant Results:

In this section, we give some relevant definitions and results for our study of the main results of this paper.

The notion of N-S was defined by Smarandache [27] in the following way:

Assume that L be a non-empty set. Then D, an N-S over L is defined by:

$D=\{(\alpha,T_D(\alpha),I_D(\alpha),F_D(\alpha)):\alpha\in L\}$, where T_D, I_D, F_D are the truth, indeterminacy and false membership functions from the whole set L to [0, 1] respectively. So, $0 \leq T_D(\alpha) + I_D(\alpha) + F_D(\alpha) \leq 3$, for each $\alpha\in L$.

The notions of neutrosophic over-set, neutrosophic under-set, and neutrosophic off-set was also grounded by Smarandache [29] in the year 2016, and defined as follows:

Let L be a universal set. Then, a single valued neutrosophic over set D over L is defined by:

$D=\{(\alpha,T_D(\alpha),I_D(\alpha),F_D(\alpha)):\alpha\in L\}$, such that at least one member in D has at least one of the neutrosophic component that is greater than 1. Here, $T_D, I_D, F_D : L\rightarrow[0, \check{N}]$ are the truth, indeterminacy, and false membership functions respectively such that $0 < 1 < \check{N}$, and $\check{N}$ is the over-limit of D.

For example, D={(a,0.2,0.3,1.5), (b,0.9,1.3,0.2), (c,0.2,0.1,0.6)} is an neutrosophic over set defined over L. But K={(a,0.3,0.5,0.9), (b,0.8,0.4,0.9), (c,0.2,0.5,0.5)} is not an neutrosophic over set defined over L.

Let L be a universal set. Then, a single valued neutrosophic under set Y over L is defined by:

$Y=\{(\alpha,T_Y(\alpha),I_Y(\alpha),F_Y(\alpha)):\alpha\in L\}$, such that at least one member in Y has at least one of the neutrosophic component that is smaller than 0. That is, the truth, indeterminacy, and false membership functions T_Y, I_Y, F_Y are defined from L to $[\check{N}, 1]$ such that $\check{N} < 0 < 1$, and $\check{N}$ is said to be the under-limit of Y.
For example, Y={(a,0.2,-0.3,0.9), (b,-0.5,0.2,-0.2), (c,0.2,-0.1,0.6)} is an neutrosophic under-set defined over L. But Z={(a,0.3,0.5,0.9), (b,0.8,0.4,0.9), (c,0.2,0.5,0.5)} is not an neutrosophic under-set defined over L.

A single valued neutrosophic off-set K over a fixed set L is defined by:
$K=\{(\alpha,T_K(\alpha),I_K(\alpha),F_K(\alpha)):\alpha\in L\}$, such that some members of K has at least one of the neutrosophic component that is smaller than 0 and at least one of the neutrosophic component that is greater than 1. That is, the truth, indeterminacy, and false membership functions T_K, I_K, F_K are defined from L to $[\check{N}, \tilde{N}]$ such that $\check{N} < 0 < 1 < \tilde{N}$. Here, $\check{N}$ and $\tilde{N}$ are said to be the under-limit and over-limit of K respectively.

For example, K={(a,0.2,-0.3,1.6), (b,-0.5,0.2,0.2), (c,1.3,0.1,0.6)} is an neutrosophic off set defined over L. But L={(a,0.3,1.5,0.9), (b,0.8,0.4,0.9), (c,0.2,0.5,-0.5)} is not an neutrosophic off set defined over L.

Recently, Mallick and Pramanik [14] grounded the idea of pentapartitioned neutrosophic set (in short PNS) by extending the notions of N-S.

Suppose that L be a fixed set. Then D, a PNS over L is defined as follows:
$D=\{(\alpha,T_D(\alpha),C_D(\alpha),R_D(\alpha),U_D(\alpha),F_D(\alpha)):\alpha\in L\}$, where T_D, C_D, R_D, U_D, F_D: $L\rightarrow[0, 1]$ are the truth, contradiction, ignorance, unknown and false membership functions respectively. So,

$$0\leq T_D(\alpha)+C_D(\alpha)+R_D(\alpha)+U_D(\alpha)+F_D(\alpha)\leq 5.$$

Let $X=\{(\alpha,T_X(\alpha),C_X(\alpha),R_X(\alpha),U_X(\alpha),F_X(\alpha)):\alpha\in L\}$ and $Y=\{(\alpha,T_Y(\alpha),C_Y(\alpha),R_Y(\alpha), U_Y(\alpha),F_Y(\alpha)):\alpha\in L\}$ be two PNSs over L. Then,

(i) $X\subseteq Y \Leftrightarrow T_X(\alpha)\leq T_Y(\alpha)$, $C_X(\alpha)\leq C_Y(\alpha)$, $R_X(\alpha)\geq R_Y(\alpha)$, $U_X(\alpha)\geq U_Y(\alpha)$, $F_X(\alpha)\geq F_Y(\alpha)$, for all $\alpha\in L$.

(ii) $X\cup Y=\{(\alpha, \max\{T_X(\alpha), T_Y(\alpha)\}, \max\{C_X(\alpha), C_Y(\alpha)\}, \min\{R_X(\alpha), R_X(\alpha)\}, \min\{U_X(\alpha), U_X(\alpha)\}, \min\{F_X(\alpha), F_X(\alpha)\}):\alpha\in L\}$.

(iii) $X^c=\{(\alpha,T_X(\alpha),C_X(\alpha),1-R_X(\alpha),U_X(\alpha),F_X(\alpha)):\alpha\in L\}$.

(iv) $X\cap Y=\{(\alpha, \min\{T_X(\alpha), C_Y(\alpha)\}, \min\{C_X(\alpha), C_Y(\alpha)\}, \max\{R_X(\alpha), R_X(\alpha)\}, \max\{U_X(\alpha), U_X(\alpha)\}, \max\{F_X(\alpha), F_X(\alpha)\}):\alpha\in L\}$.

3. Pentapartitioned Neutrosophic Off-set / Over-set / Under-set:

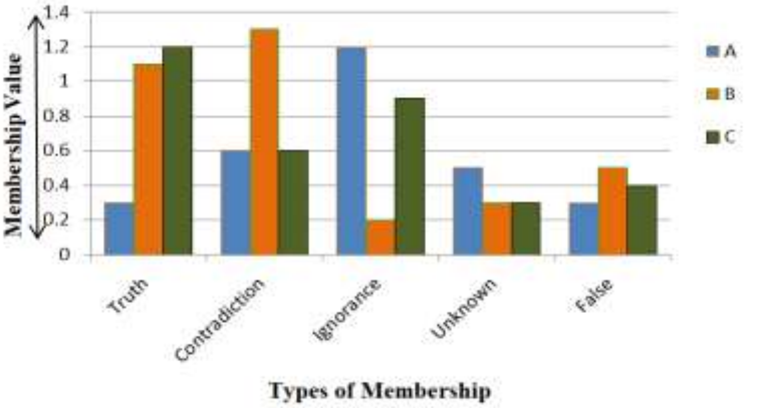
In this section, we introduce the notions of pentapartitioned neutrosophic off-set (in short PN-off-S) / pentapartitioned neutrosophic under-set (in short PN-under-S) / pentapartitioned neutrosophic over-set (in short PN-over-S). Then, we formulate and study some interesting results on them.

Definition 3.1. Let L be a universal set. Then D, a PN-over-S over L is defined by:
$D=\{(\alpha,T_D(\alpha),C_D(\alpha),G_D(\alpha),U_D(\alpha),F_D(\alpha)):\alpha\in L\}$, such that at least one member in D has at least one of the pentapartitioned neutrosophic component that is greater than 1 and no member has pentapartitioned neutrosophic components that are less than zero. Here, T_D, C_D, G_D, U_D, F_D : $L\rightarrow[0,$

Ň] are the truth, contradiction, ignorance, unknown and false membership functions respectively such that 1 < Ň, and Ň is said to be the over-limit of D.

Example 3.1. Assume that L={a, b, c} be a fixed set. Then, D={(a,0.3,0.6,1.2,0.5,0.3), (b,1.1,1.3,0.2,0.3, 0.5), (c,1.2,0.6,0.9,0.3,0.4)} is a PN-over-S defined over L. But K={(a,0.1,0.4,0.4,0.6,0.8), (b,0.9,0.5,0.8,0.6, 0.8), (c,0.1,0.6,0.7,0.8,0.9)} is not a PN-over-S defined over L.

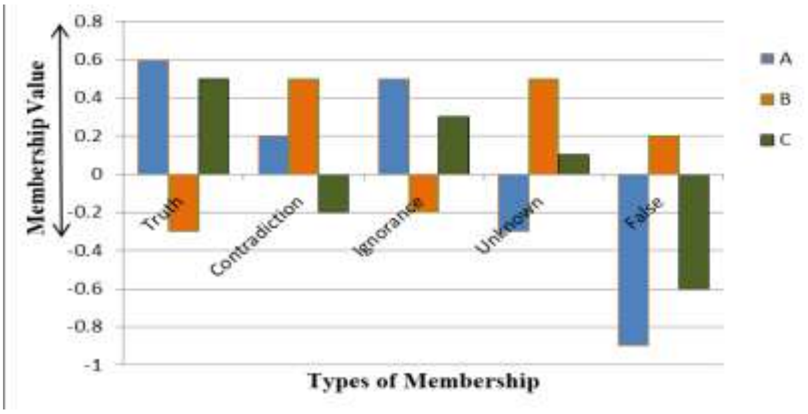
The pictorial representation of **Example 3.1** is given as follows:



Definition 3.2. Suppose that L be a fixed universal set. Then Y, a PN-under-S over L is defined by: Y={(α,T_Y(α),C_Y(α),G_Y(α),U_Y(α),F_Y(α)):α∈L}, such that at least one member in Y has at least one of the neutrosophic component that is smaller than 0 and no member has pentapartitioned neutrosophic components that are greater than one. That is, the truth, contradiction, ignorance, unknown, and false membership functions T_Y, C_Y, G_Y,U_Y, F_Y are defined from L to [Ň,1] such that Ň<0, and Ň is said to be the under-limit of Y.

Example 3.2. Assume that L={a, b, c} be a fixed set. Then, Y={(a,0.6,0.2,0.5,-0.3,-0.9), (b,-0.3,0.5,-0.2, 0.5,0.2), (c,0.5,-0.2,0.3,0.1,-0.6)} is a PN-under-S over L. But Z={(a,0.3,0.2,0.8,0.5,0.9), (b,0.9,0.8,0.5,0.4, 0.9), (c,0.2,0.5,0.3,0.5,0.5)} is not a PN-under-S over L.

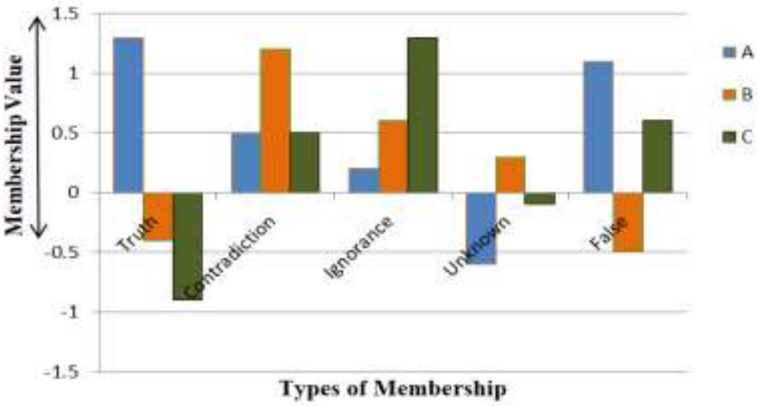
The pictorial representation of **Example 3.2** is given as follows:



Definition 3.3. Assume that L be a fixed non-empty set. Then K, a PN-off-S over L is defined by: $K=\{(\alpha,T_K(\alpha),C_K(\alpha),G_K(\alpha),U_K(\alpha),F_K(\alpha)):\alpha\in L\}$, such that some members of K has at least one of the pentapartitioned neutrosophic component that is smaller than 0 and at least one of the pentapartitioned neutrosophic component that is greater than 1. That is, the truth, contradiction, ignorance, unknown, and false membership functions T_K, C_K, G_K, U_K, F_K are defined from L to $[\check{N}, \tilde{N}]$ such that $\check{N} < 0 < 1 < \tilde{N}$. Here, $\check{N}$ and $\tilde{N}$ are called the under-limit and over-limit of K respectively.

Example 3.3. Assume that L={a, b, c} be a fixed set. Then, K={(a,1.3,0.5,0.2,-0.6,1.1), (b,-0.4,1.2,0.6, 0.3,-0.5), (c,-0.9,0.5,1.3,-0.1,0.6)} is a PN-off-S defined over L. But L={(a,0.3,0.3,0.4,0.4, 0.9), (b,0.9,0.4,0.1,0.2,0.3), (c,0.6,0.4,0.4,0.3,0.3)} is not a PN-off-S defined over L.

The pictorial representation of **Example 3.3** is given as follows:



Definition 3.4. Assume that L be a fixed set. Then, null PN-over-S (0_P) and the absolute PN-over-S (1_P) over L is defined by:

(i) $0_P = \{(\alpha,0,\tilde{N},\tilde{N},\tilde{N},0): \alpha\in L\}$;

(ii) $1_P = \{(\alpha,\tilde{N},0,0,0,\tilde{N}): \alpha\in L\}$.

Definition 3.5. Assume that L be a fixed set. Then, null PN-under-S (0_P) and the absolute PN-under-S (1_P) over L is defined by:

(i) $0_P = \{(\alpha,\check{N},1,1,1,\check{N}): \alpha\in L\}$;

(ii) $1_P = \{(\alpha,1,\check{N},\check{N},\check{N},1): \alpha\in L\}$.

Definition 3.6. Assume that L be a fixed set. Then, null PN-off-S (0_P) and the absolute PN-off-S (1_P) over L is defined by:

(i) $0_P = \{(\alpha,\check{N},\tilde{N},\tilde{N},\tilde{N},\check{N}): \alpha\in L\}$;

(ii) $1_P = \{(\alpha,\tilde{N},\check{N},\check{N},\check{N},\tilde{N}): \alpha\in L\}$.

Definition 3.7. Assume that $K=\{(\alpha,T_K(\alpha),C_K(\alpha),R_K(\alpha),U_K(\alpha),F_K(\alpha)):\alpha\in L\}$ and $Y=\{(\alpha,T_Y(\alpha),C_Y(\alpha),R_Y(\alpha), U_Y(\alpha),F_Y(\alpha)):\alpha\in L\}$ be two PN-over-Ss / PN-under-Ss / PN-off-Ss. Then, the intersection and union of K and Y is defined by

(i) $K\cap Y=\{(\alpha, \min\{T_K(\alpha),T_Y(\alpha)\}, \max\{C_K(\alpha), C_Y(\alpha)\}, \max\{R_K(\alpha), R_Y(\alpha)\}, \max\{U_K(\alpha), U_Y(\alpha)\}, \min\{F_K(\alpha), F_Y(\alpha)): \alpha\in L\}$;

(ii) $K \cup Y=\{(\alpha, \max\{T_K(\alpha), T_Y(\alpha)\}, \min\{C_K(\alpha), C_Y(\alpha)\}, \min\{R_K(\alpha), R_Y(\alpha)\}, \min\{U_K(\alpha), U_Y(\alpha)\}, \max\{F_K(\alpha), F_Y(\alpha)): \alpha \in L\}$.

Example 3.4. Assume that, L={c, d} be a fixed set. Suppose that K={(c,0.9,0.8,1.3,0.4,1.5), (d,0.2,1.3,1.7, 0.2,0.9)} and Y={(c,0.6,0.3,1.6,1.2,0.8), (d,0.8,0.3,0.8,1.5,0.7)} be two PN-over-Ss over L. Then,

(i) $K \cap Y$={(c,0.6,0.8,1.6,1.2,0.8), (d,0.2,1.3,1.7,1.5,0.7)};

(ii) $K \cup Y$={(c,0.9,0.3,1.3,0.4,1.5), (d,0.8,0.3,0.8,0. 2,0.9)}.

Example 3.5. Assume that, L={c, d} be a fixed set. Suppose that K={(c,0.6,0.6,-0.2,0.5,-0.9), (d,0.5,-0.5, 0.4,0.6,-0.1)} and Y={(c,-0.4,0.7,0.5,0.4,-0.8), (d,0.8,-0.5,-0.3,0.5,0.8)} be two PN-under-Ss over L. Then,

(i) $K \cap Y$={(c,-0.4,0.7,0.5,0.5,-0.9), (d,0.5,-0.5,0.4,0.6,-0.1)};

(ii) $K \cup Y$={(c,0.6,0.6,-0.2,0.4,-0.8), (d,0.8,-0.5,-0.3,0.5,0.8)}.

Example 3.6. Assume that, L={c, d} be a universe of discourse. Let K={(c,0.8,-0.6,0.7,0.6,1.1), (d,1.5,-0.2, 0.9,0.7,0.4)} and Y={(c,1.8,0.9,-0.9,0.7,1.2), (d,0.1,0.7,1.5,-0.6,0.9)} be two PN-off-Ss. Then,

(i) $K \cap Y$={(c,0.8,0.9,0.7,0.7,1.1), (d,0.1,0.7,1.5,0.7,0.4)};

(ii) $K \cup Y$={(c,1.8,-0.6,-0.9,0.6,1.2), (d,1.5,-0.2,0.9,-0.6,0.9)}.

Definition 3.8. Let K and Y be two PN-off-Ss / PN-under-Ss / PN-over-Ss over L. Then,

(i) $K \subseteq Y$ if and only if $T_K(\alpha) \leq T_Y(\alpha)$, $C_K(\alpha) \geq C_Y(\alpha)$, $R_K(\alpha) \geq R_Y(\alpha)$, $U_K(\alpha) \geq U_Y(\alpha)$, $F_K(\alpha) \leq F_Y(\alpha), \forall \alpha \in L$;

(ii) $K^c=\{(\alpha,1-T_K(\alpha),1-C_K(\alpha),1-R_K(\alpha),1-U_K(\alpha),1-F_K(\alpha)):\alpha \in L\}$.

Example 3.7. Suppose that L={c, d} be a non-empty set. Assume that K={(c,1.5,1.9,0.9,0.9,0.5), (d,1.7, 1.3,1.3,0.4,0.3)} and Y={(c,1.6,1.2,0.8,0.3,0.6), (d,1.8,0.8,1.2,0.3,0.9)} be two PN-over-Ss. Then, $K \subseteq Y$, and K^c={(c,-0.5,-0.9,0.1,0.1,0.5), (d,-0.7,-0.3,-0.3,0.6,0.7)} and Y^c={(c,-0.6,-0.2,0.2,0.7,0.4), (d,-0.8,0.2,-0.2, 0.7,0.1)}.

Example 3.8. Suppose that L={c, d} be a non-empty set. Assume that K={(c,0.8,0.7,-0.8,0.3,0.9) (d,-0.9, 0.8,-0.2,0.3,-0.1)} and Y={(c,0.9,-0.5,-0.9,0.1,0.9), (d,0.7,0.5,-0.3,-0.5,0.1)} be two PN-under-Ss over L. Then, $K \subseteq Y$, and K^c={(c,0.2,0.3,1.8,0.7,0.1), (d,1.9,0.2,1.2,0.7,1.1)} and Y^c={(c,0.1,1.5,1.9,0.9,0.1), (d,0.3, 0.5,1.3,1.5,0.9)}.

Example 3.9. Suppose that L={c, d} be a non-empty set. Assume that K={(c,0.8,-0.7,1.5,0.6,1.5), (d,1.7, 0.5,-0.1,0.3,0.5)} and Y={(c,0.8,-0.8,0.9,0.5,1.7), (d,1.8,0.2,-0.5,0.2,0.8)} be two PN-off-Ss over L. Then, $K \subseteq Y$, and K^c={(c,0.2,1.7,-0.5,0.4,-0.5), (d,-0.7,0.5,1.1,0.7,0.5)} and Y^c={(c,0.2,1.8,0.1,0.5,-0.7), (d,-0.8,0.8, 1.5,0.8,0.2)}.

Proposition 3.1. Assume that K and Y be two PN-off-Ss / PN-under-Ss / PN-over-Ss over L. Then,

(i) $K \cup Y = Y \cup K$;

(ii) $K \cap Y = Y \cap K$.

Proof. It is known that, $K \cup Y = \{(\alpha, \max(T_K(\alpha), T_Y(\alpha)), \min(C_K(\alpha), C_Y(\alpha)), \min(R_K(\alpha), R_Y(\alpha)), \min(U_K(\alpha), U_Y(\alpha)), \max(F_K(\alpha), F_Y(\alpha)): \alpha \in L\} = \{(\alpha, \max(T_Y(\alpha), T_K(\alpha)), \min(C_Y(\alpha), C_K(\alpha)), \min(R_Y(\alpha), R_K(\alpha)), \min(U_Y(\alpha), U_K(\alpha)), \max(F_Y(\alpha), F_K(\alpha)): \alpha \in L\} = Y \cup K$.

Therefore, $K \cup Y = Y \cup K$.

Similarly, it can be established that $K \cap Y = Y \cap K$.

Proposition 3.2. Let K_1, K_2 and K_3 be three PN-off-Ss / PN-under-Ss / PN-over-Ss over L. Then, $K_1 \cup (K_2 \cup K_3) = (K_1 \cup K_2) \cup K_3$ and $K_1 \cap (K_2 \cap K_3) = (K_1 \cap K_2) \cap K_3$.

Proof. Suppose that, $\alpha_i \in K_1 \cup (K_2 \cup K_3)$. Therefore,

$\alpha_i \in K_1 \cup \{(\alpha_i, \max(T_{K_2}(\alpha_i), T_{K_3}(\alpha_i)), \min(C_{K_2}(\alpha_i), C_{K_3}(\alpha_i)), \min(R_{K_2}(\alpha_i), R_{K_3}(\alpha_i)), \min(U_{K_2}(\alpha_i), U_{K_3}(\alpha_i)), \max(F_{K_2}(\alpha_i), F_{K_3}(\alpha_i)): \alpha_i \in L\}$

$\Rightarrow \alpha_i \in \{(\alpha_i, \max(T_{K_1}(\alpha_i), T_{K_2}(\alpha_i), T_{K_3}(\alpha_i)), \min(C_{K_1}(\alpha_i), C_{K_2}(\alpha_i), C_{K_3}(\alpha_i)), \min(R_{K_1}(\alpha_i), R_{K_2}(\alpha_i), R_{K_3}(\alpha_i)), \min(U_{K_1}(\alpha_i), U_{K_2}(\alpha_i), U_{K_3}(\alpha_i)), \max(F_{K_1}(\alpha_i), F_{K_2}(\alpha_i), F_{K_3}(\alpha_i)): \alpha_i \in L\}$

$\Rightarrow \alpha_i \in \{(\alpha_i, \max(T_{K_1}(\alpha_i), T_{K_2}(\alpha_i)), \min(C_{K_1}(\alpha_i), C_{K_2}(\alpha_i)), \min(R_{K_1}(\alpha_i), R_{K_2}(\alpha_i)), \min(U_{K_1}(\alpha_i), U_{K_2}(\alpha_i)), \max(F_{K_1}(\alpha_i), F_{K_2}(\alpha_i)): \alpha_i \in L\} \cup K_3$

$\Rightarrow \alpha_i \in (K_1 \cup K_2) \cup K_3$

$\Rightarrow K_1 \cup (K_2 \cup K_3) \subset (K_1 \cup K_2) \cup K_3$ (1)

Assume that, $\beta_i \in (K_1 \cup K_2) \cup K_3$. Therefore,

$\beta_i \in \{(\beta_i, \max(T_{K_1}(\beta_i), T_{K_2}(\beta_i)), \min(C_{K_1}(\beta_i), C_{K_2}(\beta_i)), \min(R_{K_1}(\beta_i), R_{K_2}(\beta_i)), \min(U_{K_1}(\beta_i), U_{K_2}(\beta_i)), \max(F_{K_1}(\beta_i), F_{K_2}(\beta_i)): \beta_i \in L\} \cup K_3$

$\Rightarrow \beta_i \in \{(\beta_i, \max(T_{K_1}(\beta_i), T_{K_2}(\beta_i), T_{K_3}(\beta_i)), \min(C_{K_1}(\beta_i), C_{K_2}(\beta_i), C_{K_3}(\beta_i)), \min(R_{K_1}(\beta_i), R_{K_2}(\beta_i), R_{K_3}(\beta_i)), \min(U_{K_1}(\beta_i), U_{K_2}(\beta_i), U_{K_3}(\beta_i)), \max(F_{K_1}(\beta_i), F_{K_2}(\beta_i), F_{K_3}(\beta_i)) : \beta_i \in L\}$

$\Rightarrow \beta_i \in K_1 \cup \{(\beta_i, \max(T_{K_2}(\beta_i), T_{K_3}(\beta_i)), \min(C_{K_2}(\beta_i), C_{K_3}(\beta_i)), \min(R_{K_2}(\beta_i), R_{K_3}(\beta_i)), \min(U_{K_2}(\beta_i), U_{K_3}(\beta_i)), \max(F_{K_2}(\beta_i), F_{K_3}(\beta_i)) : \beta_i \in L\}$

$\Rightarrow \beta_i \in K_1 \cup (K_2 \cup K_3)$

$\Rightarrow (K_1 \cup K_2) \cup K_3 \subset K_1 \cup (K_2 \cup K_3)$ (2)

From eqs (1) and (2), we have, $K_1 \cup (K_2 \cup K_3) = (K_1 \cup K_2) \cup K_3$.

Similarly, it can be established that, $K_1 \cap (K_2 \cap K_3) = (K_1 \cap K_2) \cap K_3$.

Proposition 3.3. Let K_1, K_2 and K_3 be three PN-off-Ss / PN-under-Ss / PN-over-Ss over L. Then, $K_1 \cup (K_2 \cap K_3) = (K_1 \cup K_2) \cap (K_1 \cup K_3)$ and $K_1 \cap (K_2 \cup K_3) = (K_1 \cap K_2) \cup (K_1 \cap K_3)$.

Proof. Suppose that $\alpha_i \in K_1 \cup (K_2 \cap K_3)$. Therefore,

$\alpha_i \in K_1 \cup \{(\alpha_i, \min(T_{K_2}(\alpha_i), T_{K_3}(\alpha_i)), \max(C_{K_2}(\alpha_i), C_{K_3}(\alpha_i)), \max(R_{K_2}(\alpha_i), R_{K_3}(\alpha_i)), \max(U_{K_2}(\alpha_i), U_{K_3}(\alpha_i)), \min(F_{K_2}(\alpha_i), F_{K_3}(\alpha_i)): \alpha_i \in L\}$

$\Rightarrow \alpha_i \in \{(\alpha_i, \max(T_{K_1}(\alpha_i), \min(T_{K_2}(\alpha_i), T_{K_3}(\alpha_i))), \min(C_{K_1}(\alpha_i), \max(C_{K_2}(\alpha_i), C_{K_3}(\alpha_i))), \min(R_{K_1}(\alpha_i), \max(R_{K_2}(\alpha_i), R_{K_3}(\alpha_i))), \min(U_{K_1}(\alpha_i), \max(U_{K_2}(\alpha_i), U_{K_3}(\alpha_i))), \max(F_{K_1}(\alpha_i), \min(F_{K_2}(\alpha_i), F_{K_3}(\alpha_i))): \alpha_i \in L\}$

$\Rightarrow \alpha_i \in \{(\alpha_i, \max(T_{K_1}(\alpha_i), T_{K_2}(\alpha_i)), \min(C_{K_1}(\alpha_i), C_{K_2}(\alpha_i)), \min(R_{K_1}(\alpha_i), R_{K_2}(\alpha_i)), \min(U_{K_1}(\alpha_i), U_{K_2}(\alpha_i)), \max(F_{K_1}(\alpha_i), F_{K_2}(\alpha_i)): \alpha_i \in L\} \cap \{(\alpha_i, \max(T_{K_1}(\alpha_i), T_{K_3}(\alpha_i)), \min(C_{K_1}(\alpha_i), C_{K_3}(\alpha_i)), \min(R_{K_1}(\alpha_i), R_{K_3}(\alpha_i)), \min(U_{K_1}(\alpha_i), U_{K_3}(\alpha_i)), \max(F_{K_1}(\alpha_i), F_{K_3}(\alpha_i)): \alpha_i \in L\}$

$\Rightarrow \alpha_i \in (K_1 \cup K_2) \cap (K_1 \cup K_3)$

$\Rightarrow K_1 \cup (K_2 \cap K_3) \subset (K_1 \cup K_2) \cap (K_1 \cup K_3)$ (1)

Assume that, $\beta_i \in (K_1 \cup K_2) \cap (K_1 \cup K_3)$. Therefore,

$\beta_i \in \{(\beta_i, \max(T_{K_1}(\beta_i), T_{K_2}(\beta_i)), \min(C_{K_1}(\beta_i), C_{K_2}(\beta_i)), \min(R_{K_1}(\beta_i), R_{K_2}(\beta_i)), \min(U_{K_1}(\beta_i), U_{K_2}(\beta_i)), \max(F_{K_1}(\beta_i), F_{K_2}(\beta_i)): \beta_i \in L\} \cap \{(\beta_i, \max(T_{K_1}(\beta_i), T_{K_3}(\beta_i)), \min(C_{K_1}(\beta_i), C_{K_3}(\beta_i)), \min(R_{K_1}(\beta_i), R_{K_3}(\beta_i)), \min(U_{K_1}(\beta_i), U_{K_3}(\beta_i)), \max(F_{K_1}(\beta_i), F_{K_3}(\beta_i)): \beta_i \in L\}$

$\Rightarrow \beta_i \in \{(\beta_i, \max(T_{K_1}(\beta_i), \min(T_{K_2}(\beta_i), T_{K_3}(\beta_i))), \min(C_{K_1}(\beta_i), \max(C_{K_2}(\beta_i), C_{K_3}(\beta_i))), \min(R_{K_1}(\beta_i), \max(R_{K_2}(\beta_i), R_{K_3}(\beta_i))), \min(U_{K_1}(\beta_i), \max(U_{K_2}(\beta_i), U_{K_3}(\beta_i))), \max(F_{K_1}(\beta_i), \min(F_{K_2}(\beta_i), F_{K_3}(\beta_i))): \beta_i \in L\}$

$\Rightarrow \beta_i \in K_1 \cup (K_2 \cap K_3)$

$\Rightarrow(K_1\cup K_2)\cap(K_1\cup K_3)\subset K_1\cup(K_2\cap K_3)$ (2)

From eqs. (1) and (2), we have $K_1\cup(K_2\cap K_3) = (K_1\cup K_2)\cap(K_1\cup K_3)$.

Similarly, it can be established that, $K_1\cap(K_2\cup K_3) = (K_1\cap K_2)\cup(K_1\cap K_3)$.

Proposition 3.4. Let K_1 be a PN-off-Ss / PN-under-Ss / PN-over-Ss over L. Then, $K_1\cap K_1^c = 0_{PN}$.

Proof: Suppose that, $\alpha_i\in K_1\cap K_1^c$. This implies,

$\alpha_i\in\{(\alpha_i, T_{K_1}(\alpha_i), C_{K_1}(\alpha_i), R_{K_1}(\alpha_i), U_{K_1}(\alpha_i), F_{K_1}(\alpha_i)): \alpha_i\in L\} \cap \{(\alpha_i, 1\text{-}T_{K_1}(\alpha_i), 1\text{-}C_{K_1}(\alpha_i), 1\text{-}R_{K_1}(\alpha_i), 1\text{-}U_{K_1}(\alpha_i), 1\text{-}F_{K_1}(\alpha_i)): \alpha_i\in L\}$

$\Rightarrow \alpha_i\in\{(\alpha_i, \min(T_{K_1}(\alpha_i), 1\text{-}T_{K_1}(\alpha_i)), \max(C_{K_1}(\alpha_i), 1\text{-}C_{K_1}(\alpha_i)), \max(R_{K_1}(\alpha_i), 1\text{-}R_{K_1}(\alpha_i)), \max(U_{K_1}(\alpha_i), 1\text{-}U_{K_1}(\alpha_i)), \min(F_{K_1}(\alpha_i), 1\text{-}F_{K_1}(\alpha_i))): \alpha_i\in L\}$

$\Rightarrow \alpha_i\in 0_{PN}$

Therefore, $K_1\cap K_1^c\subset 0_{PN}$ (3)

Again, consider $\beta_i\in 0_{PN}$

$\Rightarrow\beta_i\in \{\min(T_{K_1}(\beta_i),(1\text{-}T_{K_1}(\beta_i)), \max(C_{K_1}(\beta_i),U_{K_1}(\beta_i)), \max(R_{K_1}(\beta_i),(1\text{-}R_{K_1}(\beta_i)), \max(U_{K_1}(\beta_i),C_{K_1}(\beta_i)), \min(F_{K_1}(\beta_i),(1\text{-}F_{K_1}(\beta_i))\}$

$\Rightarrow\beta_i\in \{T_{K_1}(\beta_i),C_{K_1}(\beta_i),R_{K_1}(\beta_i),U_{K_1}(\beta_i),F_{K_1}(\beta_i)\}\cap\{(1\text{-}T_{K_1}(\beta_i)),U_{K_1}(\beta_i),(1\text{-}R_{K_1}(\beta_i)),C_{K_1}(\beta_i),(1\text{-}F_{K_1}(\beta_i))\}$

$\Rightarrow\beta_i\in K_1\cap K_1^c$.

Therefore, $0_{PN}\in K_1\cap K_1^c$ (4)

From the equation (3) and (4) we can conclude that,

$K_1\cap K_1^c = 0_{PN}$

Proposition 3.5. Let K_1 and K_2 be two PN-off-Ss / PN-under-Ss / PN-over-Ss over L. Then,

(i) $(K_1\cup K_2)^c=K_1^C\cap K_2^C$

(ii) $(K_1\cap K_2)^c=K_1^C\cup K_2^C$

Proof: Suppose that, $\alpha_i\in((K_1\cup K_2)^c$

$\Rightarrow\alpha_i\in \{\max(T_{K_1}(\alpha_i),T_{K_2}(\alpha_i)), \min(C_{K_1}(\alpha_i),C_{K_2}(\alpha_i)), \min(R_{K_1}(\alpha_i),R_{K_2}(\alpha_i)), \min(U_{K_1}(\alpha_i),U_{K_2}(\alpha_i)), \max(F_{K_1}(\alpha_i),F_{K_2}(\alpha_i))\}^c$

$\Rightarrow\alpha_i\in \{\min((1\text{-}T_{K_1}(\alpha_i)),(1\text{-}T_{K_2}(\alpha_i))), \max(U_{K_1}(\alpha_i),U_{K_2}(\alpha_i)), \max((1\text{-}R_{K_1}(\alpha_i)),(1\text{-}R_{K_2}(\alpha_i))), \max(C_{K_1}(\alpha_i), C_{K_2}(\alpha_i)), \min((1\text{-}F_{K_1}(\alpha_i)),(1\text{-}F_{K_2}(\alpha_i)))\}$

$\Rightarrow\alpha_i\in \{(1\text{-}T_{K_1}(\alpha_i)), C_{K_1}(\alpha_i),(1\text{-}R_{K_1}(\alpha_i)), U_{K_1}(\alpha_i),(1\text{-}F_{K_1}(\alpha_i))\}\cap\{(1\text{-}T_{K_2}(\alpha_i)), C_{K_2}(\alpha_i),(1\text{-}R_{K_2}(\alpha_i)), U_{K_2}(\alpha_i), (1\text{-}F_{K_1}(\alpha_i))\}$.

$\Rightarrow\alpha_i\in K_1^C\cap K_2^C$

$\Rightarrow(K_1\cup K_2)^c\subset K_1^C\cap K_2^C$ (5)

Assume that, $\beta_i\in K_1^C\cap K_2^C$

$\Rightarrow\beta_i\in \{(1\text{-}T_{K_1}(\beta_i)), C_{K_1}(\beta_i),(1\text{-}R_{K_1}(\beta_i)), U_{K_1}(\beta_i),(1\text{-}F_{K_1}(\beta_i))\}\cap\{(1\text{-}T_{K_2}(\beta_i)), C_{K_2}(\beta_i),(1\text{-}R_{K_2}(\beta_i)), U_{K_2}(\beta_i), (1\text{-}F_{K_1}(\beta_i))\}$

$\Rightarrow\beta_i\in \{\min((1\text{-}T_{K_1}(\beta_i)),(1\text{-}T_{K_2}(\beta_i))), \max(U_{K_1}(\beta_i),U_{K_2}(\beta_i)), \max((1\text{-}R_{K_1}(\beta_i)),(1\text{-}R_{K_2}(\beta_i))), \max(C_{K_1}(\beta_i), C_{K_2}(\beta_i)), \min((1\text{-}F_{K_1}(\beta_i)),(1\text{-}F_{K_2}(\beta_i)))\}$

$\Rightarrow\beta_i\in \{\max(T_{K_1}(\beta_i),T_{K_2}(\beta_i)), \min(C_{K_1}(\beta_i),C_{K_2}(\beta_i)), \min(R_{K_1}(\beta_i),R_{K_2}(\beta_i)), \min(U_{K_1}(\beta_i),U_{K_2}(\beta_i)), \max(F_{K_1}(\beta_i),F_{K_2}(\beta_i))\}^c$

$\Rightarrow\beta_i\in (K_1\cup K_2)^c$

Therefore, $(K_1 \cap K_2)^c \subset K_1^C \cup K_2^C$. (6)

From the equation (5) and (6) we can conclude that,

$(K_1 \cup K_2)^c = K_1^C \cap K_2^C$.

Assume that, $\alpha_i \in (K_1 \cap K_2)^c$

$\Rightarrow \alpha_i \in \{\min(T_{K_1}(\alpha_i), T_{K_2}(\alpha_i)), \max(C_{K_1}(\alpha_i), C_{K_2}(\alpha_i)), \max(R_{K_1}(\alpha_i), R_{K_2}(\alpha_i)), \max(U_{K_1}(\alpha_i), U_{K_2}(\alpha_i)), \min(F_{K_1}(\alpha_i), F_{K_2}(\alpha_i))\}^c$

$\Rightarrow \alpha_i \in \{\max((1-T_{K_1}(\alpha_i)),(1-T_{K_2}(\alpha_i))), \min(U_{K_1}(\alpha_i), U_{K_2}(\alpha_i)), \min((1-R_{K_1}(\alpha_i)),(1-R_{K_2}(\alpha_i))), \max(C_{K_1}(\alpha_i), C_{K_2}(\alpha_i)), \max((1-F_{K_1}(\alpha_i)),(1-F_{K_2}(\alpha_i)))\}$

$\Rightarrow \alpha_i \in \{(1-T_{K_1}(\alpha_i)), U_{K_1}(\alpha_i),(1-R_{K_1}(\alpha_i)), C_{K_1}(\alpha_i),(1-F_{K_1}(\alpha_i))\} \cup \{(1-T_{K_2}(\alpha_i)), U_{K_2}(\alpha_i),(1-R_{K_2}(\alpha_i)), C_{K_2}(\alpha_i), (1-F_{K_1}(\alpha_i))\}$

$\Rightarrow \alpha_i \in (K_1^C \cup K_2^C)$

Therefore, $(K_1 \cap K_2)^c \subset K_1^C \cup K_2^C$ (7)

Assume that, $\beta_i \in (K_1^C \cup K_2^C)$

$\Rightarrow \beta_i \in \{(1-T_{K_1}(\beta_i)), U_{K_1}(\beta_i),(1-R_{K_1}(\beta_i)), C_{K_1}(\beta_i),(1-F_{K_1}(\beta_i))\} \cup \{(1-T_{K_2}(\beta_i)), U_{K_2}(\beta_i),(1-R_{K_2}(\beta_i)), C_{K_2}(\beta_i), (1-F_{K_1}(\beta_i))\}$

$\Rightarrow \beta_i \in \{\max((1-T_{K_1}(\beta_i)),(1-T_{K_2}(\beta_i))), \min(U_{K_1}(\beta_i), U_{K_2}(\beta_i)), \min((1-R_{K_1}(\beta_i)),(1-R_{K_2}(\beta_i))), \max(C_{K_1}(\beta_i), C_{K_2}(\beta_i)), \max((1-F_{K_1}(\beta_i)),(1-F_{K_2}(\beta_i)))\}$

$\Rightarrow \beta_i \in \{\min(T_{K_1}(\beta_i), T_{K_2}(\beta_i)), \max(C_{K_1}(\beta_i), C_{K_2}(\beta_i)), \max(R_{K_1}(\beta_i), R_{K_2}(\beta_i)), \max(U_{K_1}(\beta_i), U_{K_2}(\beta_i)), \min(F_{K_1}(\beta_i), F_{K_2}(\beta_i))\}^c$

$\Rightarrow \beta_i \in (K_1 \cap K_2)^c$

Therefore, $(K_1^C \cup K_2^C) \subset (K_1 \cap K_2)^c$ (8)

From eq. (7) and eq. (8), we can conclude that $(K_1 \cap K_2)^c = K_1^C \cup K_2^C$.

6. Conclusions: In this article, we have introduced the notion of single valued pentapartitioned neutrosophic over-set / under-set / off-set. Besides, we have studied several properties on them. In the future, we hoped that the notion of some algebraic structures like Groups, Field, etc. can be easily applied to the proposed sets. Furthermore, the notion of proposed sets can also be applied to real life decision making problems [5, 12, 13, 19, 22, 24, etc.].

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