Separation axioms in neutrosophic supra topological space and neutrosophic supra bi-topological space

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Abstract

In this paper, as a new separation axioms in neutrosophic supra topological space, N_S-T_i- space (i=0,1,2) is built in this space. Moreover, SN.T_1T_2-open(closed) sets are defined in neutrosophic supra bi-topological spaces. Also SN_{Bi}-T_i-space (i=0,1,2) is built on this new neutrosophic sets. And their basic properties are presented. The relations between these new neutrosophic separation axioms is studied. Finally, many examples are presented.

Keywords: Neutrosophic supra topological spaces, Neutrosophic supra bi-topological spaces, SN.T_1T_2-open set, SN.T_1T_2-closed set, neutrosophic separation axioms, N_S-T_i- space, N_{SBi}-T_i- space (i=0,1,2).

1. Introduction

The idea of neutrosophic was invented and presented by F. Smarandache [1,2]. This science has many applications in all science, including topology, where neutrosophic topological space was defined by A. Salama and et al. in [3]. Also, the neutrosophic bi-topological space was defined by R. K. Al-Hamido [4] as an extension of neutrosophic topological spaces in 2019. The concept of neutrosophic supra bi-topological space has been studied in [5]. Also, the neutrosophic Tri-topological space was defined by R.K.Al-Hamido [6] as an extension of neutrosophic bi-topological spaces in 2018. Also, in 2018, R. K. Al-Hamido, extended neutrosophic bi-topological spaces to Neutrosophic Crisp Bi-Topological Spaces[7].

Later [8] studied the separations axioms but in neutrosophic crisp topological space via neutrosophic crisp points which is defined in this paper. Moreover, these new definitions of neutrosophic crisp points open the door to defined new types of separations axioms in neutrosophic crisp topological space. such as neutrosophic crisp semi separation axioms in[9] and separation axioms via neutrosophic crisp pre-open sets [10] in neutrosophic crisp topological spaces.

Based on neutrosophic crisp bi-topological space [7], separations axioms in neutrosophic crisp bi-topological space were grounded by R. K. Al-Hamido et al. in [11].

Khattak et al. [12] worked on soft b-separation axioms in NSTS. Suresh and Palaniammal [13] presented NS(WG) separation axioms in NTS.

Gunnuz Aras et al. [14] studied the separation axioms but in neutrosophic soft topological spaces(NSTS).

Mehmood et al. [15] worked on generalized neutrosophic separation axioms in NSTS.

Recently, Neutrosophic crisp set theory has been employed to model uncertainty in several areas of application such as image processing [16],[17], and in geographic information systems[18] and possible applications to database[19]. Also, neutrosophic sets [20] may have applications in the medical field [21-22].

Recently, in 2021, A.Acikgoza et al. studied separations axioms in neutrosophic topological space[23] for the first time.

Finally, R.Narmada et al. studied separation axioms in an ordered neutrosophic bitopological space in [24]. For more detail about neutrosophic topology see [25-32].

In this paper, we will defined new patterns from neutrosophic sets in neutrosophic supra bi-topological spaces, moreover we will defined separations axioms in neutrosophic supra topological space and in neutrosophic supra bi-topological space depending on SN.T_1T_2-open(closed) sets are defined in neutrosophic supra bi-topological spaces.

We will study the relationships among these new types of separations axioms, and we will also examine the relationship between separations axioms in neutrosophic supra topological space and neutrosophic supra bi-topological space.

2. Preliminaries

This section will discuss some basic definitions and properties of neutrosophic supra topology, which are helpful in sequel.

Definition 2.1.[8]

let X be a non-empty set, D be a neutrosophic set in X, then:

D is said to be neutrosophic quasi-coincident (neutrosophic q-coincident, for short) with *L*, denoted by *DqL* if and only if D $\not\subset L^c$. If *D* is not neutrosophic quasi-coincident with *L*, we denote by $D \tilde{q} L$.

Definition 2.2: [25]

A neutrosophic supra topology (NST) on a non-empty set X is a family Γ of neutrosophic subsets in X satisfying the following axioms.

1. 1_N and 0_N belong to Γ.

2. Γ is closed under arbitrary union.

The pair (X, Γ) is called neutrosophic supra topological space (NSTS) in X. Moreover, members of Γ are known as neutrosophic supra open sets (NSOS).

The set of all neutrosophic supra open (closed) set is denoted NSOS(X) (NSCS(X)).

Definition 2.3. [5]

Let T_1,T_2 be two neutrosophic supra topology on a nonempty set X then (X,T_1,T_2) be a neutrosophic supra Bi-topological space (SBi-NTS for short).

3. Separation axioms in neutrosophic supra topological space

In this part, we have defined a new separation axioms in neutrosophic supra topological space, namely N_S-T_i-space (i=0,1,2), for first time.

Definition 3.1.
A neutrosophic set F in NSTS (X,T) is called N_S-T_0-space if for any pair of neutrosophic points (NP) $x\neq y\in X$, there exists an $U\in$ NOS(X) such that ($x\in U$ and $y\notin U$) or there exists $V\in$ NOS(X); ($y\in V$ and $x\notin V$).

Example 3.2.
Let X={n,m}, T= $\{ \{ n_{s,s,1-s} , m_{e,e,1-e} \} : s\in[0,1] , e\in[0,1] \}$
Then (X,T) is NSTS, (X,T) is N_S-T_0-space.

Definition 3.3.
A neutrosophic set F in NSTS (X,T) is called N_S-T_1-space if for any pair of neutrosophic points (NP) $x\neq y\in X$, there exists $U,V\in$ NOS(X); ($x\in U$ and $y\notin U$) and ($y\in V$ and $x\notin V$).

Example 3.4.
Let X={f,g},T= $\{ \{ f_{s,s,1-s} , g_{e,e,1-e} \}: s\in[0,1] , e\in[0,1) \}$.
Then (X,T) is NSTS, (X,T) is N_S-T_0-space. But, (X,T) is not N_S-T_1-space, because, $f_{1,1,0}$ and $g_{1,1,0}$ are neutrosophic points in (X, τ) ; $f_{1,1,0} \neq g_{1,1,0}$ and the only neutrosophic supra open set that contains $g_{1,1,0}$ is 1_N.

Definition 3.5.
A neutrosophic set F in NSTS (X,T) is called N_S-T_2-space if for any pair of neutrosophic points (NP) $x\neq y\in X$, there exists $U,V\in$ NOS(X); ($x\in U$ and $y\notin U$) and ($y\in V$ and $x\notin V$) ; $U\tilde{q}V$.

Theorem 3.6.
Let (X,T) be a NSTS, then:
If (X,T) is N_S-T_2-space then (X,T) is N_S-T_1-space.

Proof:
Let (X,T) is N_S-T_2-space, Then for any pair of neutrosophic points (NP) $x\neq y\in X$ there exists $U,V\in$ NOS(X); ($x\in U$ and $y\notin U$) and ($y\in V$ and $x\notin V$) ; $U\tilde{q}V$. so there exists $U,V\in$NOS(X); ($x\in U$ and $y\notin U$) and ($y\in V$ and $x\notin V$).
Therefore (X,T) is N_S-T_1-space.

Remark 3.7.
The converse of the theorem 3.6 is not true; see the following example:

Example 3.8.
In example 3.4, (X,T) is NSTS, X is N_S-T_0-space but not N_S-T_1-space, because, $f_{1,1,0}$ and $g_{1,1,0}$ are neutrosophic points in (X, T) ; $f_{1,1,0} \neq g_{1,1,0}$ and the only neutrosophic supra open set that contains $g_{1,1,0}$ is 1_N. Therefore, X is N_S-T_0-space but not N_S-T_2-space.

Theorem 3.9.
Let (X,T) be a NSTS, then:
If (X,T) is N_S-T_1-space, then (X,T) is N_S-T_0-space.

Proof:
Let (X,T) is N_S-T_1-space, Then for any pair of neutrosophic points (NP) $x\neq y\in X$, there exists $U,V\in$ NOS(X); ($x\in U$ and $y\notin U$) and ($y\in V$ and $x\notin V$), so there exists $U,V\in$NOS(X); ($x\in U$ and $y\notin U$) or ($y\in V$ and $x\notin V$).
Therefore (X,T) is N_S-T_0-space.

Remark 3.10.
The converse of the theorem 3.9 is not true, see the following example:

Example 3.11.

In example 3.5, (X,T) is NSTS, X is N_S-T_0-space but not N_S-T_1-space, because, $f_{1,1,0}$ and $g_{1,1,0}$ are neutrosophic points in (X, τ) ; $f_{1,1,0} \neq g_{1,1,0}$ and the only neutrosophic supra open set that contains $g_{1,1,0}$ is 1_N.

Remark 3.12.

Let (X,T) be a NSTS, then:

(X,T) is N_S-T_2-space $\Rightarrow$ (X,T) is N_S-T_1-space $\Rightarrow$ (X,T) is N_S-T_0-space.

Proof:

Proof following from theorem 3.9 and theorem 3.6.

4. Separation axioms in neutrosophic supra bi-topological space

In this part, we have defined for the first time a new separation axioms in neutrosophic supra topological space, which named N_{SBi}-T_i-space (i=0,1,2).

Definition 4.1.

A neutrosophic set A in SBi-NTS (X,T_1,T_2) is called " $N_S.T_1T_2$-open set " if it is a neutrosophic open set in (X,T_1) or in (X,T_2).

- A neutrosophic set B in SBi-NTS (X,T_1,T_2) is called " $N_S.T_1T_2$-closed set " iff its complement is " $N_S.T_1T_2$-open set ".
- The set of all " $N_S.T_1T_2$-open (closed) sets " is denoted to be" $N_S.T_1T_2$-NOS ($N_S.T_1T_2$-NCS)".

Definition 4.2.

A SBi-NTS (X,T_1,T_2) is called N_{SBi}-T_0-space if :$\forall x \neq y \in X$, $\exists$ U$\in$ $N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) or $\exists$ V$\in$ $N_S.T_1T_2$-NOS; ($y \in V$ and $x \notin V$).

Example 4.3.

Let X={n,m}, A_1={< n, 0.4, 0.4, 0.4>, < m, 0.5, 0.5, 0.5>}, A_2={< n, 0.3, 0.3, 0.3>, < m, 0.6, 0.6, 0.6>},

T_1={ 0_N, A_1, A_2, $A_1 \vee A_2$, 1_N}, T_2={ { $n_{s,s,1-s}$, $m_{e,e,1-e}$ } : s∈[0,1] , e∈[0,1] }

Then (X,T_1,T_2) is SBi-NTS, (X,T_1,T_2) is N_{SBi}-T_0-space.

Definition 4.4.

A SBi-NTS (X,T_1,T_2) is called N_{SBi}-T_1-space if :$\forall x \neq y \in X$, $\exists$ U,V$\in$ $N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$).

Example 4.5.

Let X={f,g}, A_1={< f, 0.4, 0.4, 0.4>, < g, 0.5, 0.5, 0.5>}, A_2={< f, 0.3, 0.3, 0.3>, < g, 0.6, 0.6, 0.6>},

T_1={ 0_N, A_1, A_2, $A_1 \vee A_2$, 1_N}, T_2={ {$f_{s,s,1-s}$, $g_{e,e,1-e}$ } : s∈[0,1] , e∈[0,1) }

Then (X,T_1,T_2) is SBi-NTS, (X,T_1,T_2) is N_{SBi}-T_0-space. But (X,T_1,T_2) is not N_{SBi}-T_1-space.

Definition 4.6.

A SBi-NTS (X,T_1,T_2) is called N_{SBi}-T_2-space if :$\forall x \neq y \in X$, $\exists$ U,V$\in$ $N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$) ; $U \tilde{q} V$.

Theorem 4.7.

Let (X,T_1,T_2) be a SBi-NTS, then:

(X,T_1,T_2) is N_{SBi}-T_0-space $\Leftrightarrow$ (X,T_1) is N_S-T_0-space or (X,T_2) is N_S-T_0-space.

Proof :

$\Rightarrow$:

Let (X,T_1,T_2) is N_{SBi}-T_0-space then, $\forall x \neq y \in X$, $\exists$ U,V$\in$ $N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$) so there existe U,V$\in$ T_1-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$) or there existe U,V$\in$ T_2-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$) therefore (X,T_1) is N_S-T_0-space or (X,T_2) is N_S-T_0-space.

$\Leftarrow$:

Let (X,T_1) be a N_S-T_0-space or (X,T_2) be a N_S-T_0-space. Then, for every $x \neq y \in X$, there existe $U,V \in T_1$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$)or $U,V \in T_2$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$), so there exist $U,V \in N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) or ($y \in V$ and $x \notin V$). Therefore (X,T_1,T_2) is N_{SBi}-T_0-space.

Theorem 4.8.

Let (X,T_1,T_2) be a SBi-NTS, then:

(X,T_1,T_2) is N_{SBi}-T_i-space $\Leftrightarrow$ (X,T_1) is N_S-T_i-space or (X,T_2) is N_S-T_i-space (i=1,2).

Proof:

In the same way of proof theorem 4.7.

Theorem 4.9.

Let (X,T_1,T_2) be a SBi-NTS, then:

(X,T_1,T_2) is N_{SBi}-T_2-space $\Rightarrow$ (X,T_1,T_2) is N_{SBi}-T_1-space

Proof:

Let (X,T_1,T_2) is N_{SBi}-T_2-space, Then $\forall x \neq y \in X$, $\exists$ $U,V \in N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$) ; Uq̃V. so there existe $U,V \in N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$).

Therefore (X,T_1,T_2) is N_{SBi}-T_1-space.

Remark 4.10.

The converse of the theorem 4.9 is not true, see the following example:

Example 4.11.

In example 4.5, (X,T_1,T_2) is SBi-NTS, X is N_{SBi}-T_0-space but not N_{SBi}-T_1-space.

Therefore X is N_{SBi}-T_0-space but not N_{SBi}-T_2-space.

Theorem 4.12.

Let (X,T_1,T_2) be a SBi-NTS, then:

(X,T_1,T_2) is N_{SBi}-T_1-space $\Rightarrow$ (X,T_1,T_2) is N_{SBi}-T_0-space.

Proof:

Let (X,T_1,T_2) is N_{SBi}-T_1-space, Then $\forall x \neq y \in X$, $\exists$ $U,V \in N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) and ($y \in V$ and $x \notin V$), so there existe $U,V \in N_S.T_1T_2$-NOS; ($x \in U$ and $y \notin U$) or ($y \in V$ and $x \notin V$).

Therefore (X,T_1,T_2) is N_{SBi}-T_0-space.

Remark 4.13.

The converse of the theorem 4.12 is not true, see the following example:

Example 4.14.

In example 4.5, (X,T_1,T_2) is SBi-NTS, X is N_{SBi}-T_0-space but not N_{SBi}-T_1-space.

Remark 4.15.

Let (X,T_1,T_2) be a SBi-NTS, then:

(X,T_1,T_2) is N_{SBi}-T_2-space $\Rightarrow$ (X,T_1,T_2) is N_{SBi}-T_1-space $\Rightarrow$ (X,T_1,T_2) is N_{SBi}-T_0-space.

Proof:

Proof following from theorem 4.9 and theorem 4.12.

Theorem 4.16.

Let (X,T_1,T_2) be a SBi-NTS, then:

If (X,T_1) is N_S-T_i-space and (X,T_2) is N_S-T_i-space, then(X,T_1,T_2) is N_{SBi}-T_i-space(i=0,1,2).

Proof :

From the theorem 4.7 and theorem 4.8.

Remark 4.17:

If (X,T_1,T_2) is N_{SBi}-T_i-space(i=0,1,2) then may be(X,T_1) or (X,T_2) is not N_S-T_i-space, so the converse of the Theorem 4.16 is not true.

5. Conclusion

In this paper, we have defined for first time a new separation axioms in neutrosophic supra topological space and neutrosophic supra bi-topological space which namely NS-Ti-space and NSBi-Ti-space (i=0,1,2).

In the future, using these notions, various classes of separation axioms in neutrosophic supra topological space and neutrosophic supra bi-topological space as NS-Ti-space and NSBi-Ti-space (i=3,4,5) , and many researchers can be studied.

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