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Covering properties in neutrosophic topological spaces

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Abstract. Single-valued neutrosophic set is being extensively used in solving real-life problems. Recently neutrosophic topological space was developed based on redefined single-valued neutrosophic set operations. The purpose of this article is to investigate some covering properties of these neutrosophic topological spaces.

Keywords: Neutrosophic Lindelöf ; Neutrosophic continuous function ; Neutrosophic compact ; Neutrosophic countably compact ; Single-valued Neutrosophic set.

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1. **Introduction**

In the year 1965, Zadeh [34] introduced the concept of a fuzzy set. But after some decades a new branch of philosophy, acknowledged as Neutrosophy, was developed and studied by Florentin Smarandache [22–24]. Smarandache [24] proved that the neutrosophic set was a generalization of the intuitionistic fuzzy set which was developed by K.Atanassov [1] in 1986 as an extension of a fuzzy set. Like an intuitionistic fuzzy set, an element in a neutrosophic set has the degree of membership and the degree of non-membership but it has another grade of membership known as the degree of indeterminacy and one very important point about the neutrosophic set is that all three neutrosophic components are independent of one another.

After Smarandache had introduced the concept of neutrosophy, it was studied by many researchers [7, 11, 29, 32]. In the year 2002, Smarandache [23] introduced the notion of neutrosophic topology on the non-standard interval. Lupiáñez [16–18] studied and investigated many properties of neutrosophic topological spaces. In the year 2012, Salama & Alblowi [25] introduced neutrosophic topological space as a generalization of intuitionistic fuzzy topological space developed by D.Coker [9] in 1997. Salma et.al. [26–28] studied generalized neutrosophic topological space, neutrosophic filters, and neutrosophic continuous functions. In the year 2016, Karatas and Kuru [15] redefined the single-valued neutrosophic set operations and introduced neutrosophic topology and then investigated some important properties of neutrosophic topological spaces. Later, various aspects of neutrosophic topology were developed by many researchers [2, 8, 12, 30, 31].

Neutrosophy, due to the fact of its flexibility and effectiveness, is attracting researchers throughout the world and is very useful not only in the development of science and technology but also in various other fields. For instance, Abdel-Basset et.al. [3–5] studied the applications of neutrosophic theory in a number of scientific fields. In 2014, Pramanik and Roy [19] studied the conflict between India and Pakistan over Jammu-Kashmir through neutrosophic game theory. Works on medical diagnosis [13, 33], decision-making problems [4, 5], image processing [14], etc. were also done in a neutrosophic environment. Recently some studies on COVID-19 [6, 10] had been done with the help of neutrosophic theory.

There are still many concepts to be developed in connection with neutrosophic topological spaces. Very recently Ray and Dey [20] introduced the idea of neutrosophic points on single-valued neutrosophic sets and studied various properties. The authors [21] also studied the relation of quasi-coincidence for neutrosophic sets. In this article, we investigate some covering properties of neutrosophic topological spaces.

2. Preliminaries

In this section we confer some basic concepts which will be helpful in the later sections.

2.1. Definition: [22]

Let X be the universe of discourse. A neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where the functions $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are real standard or non-standard subsets of $]^-0, 1^+[$, i.e., $\mathcal{T}_A : X \to\ ]^-0, 1^+[,\ \mathcal{I}_A : X \to\ ]^-0, 1^+[,\ \mathcal{F}_A : X \to\ ]^-0, 1^+[$ and $^-0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3^+$.

The neutrosophic set A is characterized by the truth-membership function $\mathcal{T}_A$, indeterminacy-membership function $\mathcal{I}_A$, falsehood-membership function $\mathcal{F}_A$.

2.2. Definition: [32]

Let X be the universe of discourse. A single-valued neutrosophic set A over X is defined as $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$, where $\mathcal{T}_A, \mathcal{I}_A, \mathcal{F}_A$ are functions from X to $[0, 1]$ and $0 \leq \mathcal{T}_A(x) + \mathcal{I}_A(x) + \mathcal{F}_A(x) \leq 3$.

The set of all single valued neutrosophic sets over X is denoted by $\mathcal{N}(X)$.

Throughout this article, a neutrosophic set (NS, for short) will mean a single-valued neutrosophic set.

2.3. **Definition: [15]**

Let $A, B \in \mathcal{N}(X)$. Then

(i) (Inclusion): If $\mathcal{T}_A(x) \leq \mathcal{T}_B(x), \mathcal{I}_A(x) \geq \mathcal{I}_B(x), \mathcal{F}_A(x) \geq \mathcal{F}_B(x)$ for all $x \in X$ then A is said to be a neutrosophic subset of B and which is denoted by $A \subseteq B$.

(ii) (Equality): If $A \subseteq B$ and $B \subseteq A$ then $A = B$.

(iii) (Intersection): The intersection of A and B, denoted by $A \cap B$, is defined as $A \cap B = \{\langle x, \mathcal{T}_A(x) \wedge \mathcal{T}_B(x), \mathcal{I}_A(x) \vee \mathcal{I}_B(x), \mathcal{F}_A(x) \vee \mathcal{F}_B(x)\rangle : x \in X\}$.

(iv) (Union): The union of A and B, denoted by $A \cup B$, is defined as $A \cup B = \{\langle x, \mathcal{T}_A(x) \vee \mathcal{T}_B(x), \mathcal{I}_A(x) \wedge \mathcal{I}_B(x), \mathcal{F}_A(x) \wedge \mathcal{F}_B(x)\rangle : x \in X\}$.

(v) (Complement): The complement of the NS A, denoted by A^c, is defined as $A^c = \{\langle x, \mathcal{F}_A(x), 1 - \mathcal{I}_A(x), \mathcal{T}_A(x)\rangle : x \in X\}$

(vi) (Universal Set): If $\mathcal{T}_A(x) = 1, \mathcal{I}_A(x) = 0, \mathcal{F}_A(x) = 0$ for all $x \in X$ then A is said to be neutrosophic universal set and which is denoted by $\tilde{X}$.

(vii) (Empty Set): If $\mathcal{T}_A(x) = 0, \mathcal{I}_A(x) = 1, \mathcal{F}_A(x) = 1$ for all $x \in X$ then A is said to be neutrosophic empty set and which is denoted by $\tilde{\emptyset}$.

2.4. **Definition: [25]**

Let $\{A_i : i \in \triangle\} \subseteq \mathcal{N}(X)$, where $\triangle$ is an index set. Then

(i) $\cup_{i\in\triangle} A_i = \{\langle x, \vee_{i\in\triangle}\mathcal{T}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{I}_{A_i}(x), \wedge_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

(ii) $\cap_{i\in\triangle} A_i = \{\langle x, \wedge_{i\in\triangle}\mathcal{T}_{A_i}(x), \vee_{i\in\triangle}\mathcal{I}_{A_i}(x), \vee_{i\in\triangle}\mathcal{F}_{A_i}(x)\rangle : x \in X\}$.

2.5. **Definition: [20]**

Let $\mathcal{N}(X)$ be the set of all neutrosophic sets over X. An NS $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ is called a neutrosophic point (NP, for short) iff for any element $y \in X$, $\mathcal{T}_P(y) = \alpha, \mathcal{I}_P(y) = \beta, \mathcal{F}_P(y) = \gamma$ for $y = x$ and $\mathcal{T}_P(y) = 0, \mathcal{I}_P(y) = 1, \mathcal{F}_P(y) = 1$ for $y \neq x$, where $0 < \alpha \leq 1, 0 \leq \beta < 1, 0 \leq \gamma < 1$. A neutrosophic point $P = \{\langle x, \mathcal{T}_P(x), \mathcal{I}_P(x), \mathcal{F}_P(x)\rangle : x \in X\}$ will be denoted by $P^x_{\alpha,\beta,\gamma}$ or $P < x, \alpha, \beta, \gamma >$ or simply by $x_{\alpha,\beta,\gamma}$. For the NP $x_{\alpha,\beta,\gamma}$, x will be called its support. The complement of the NP $P^x_{\alpha,\beta,\gamma}$ will be denoted by $(P^x_{\alpha,\beta,\gamma})^c$ or by $x^c_{\alpha,\beta,\gamma}$.

2.6. **Theorem: [28]**

Let $f : X \to Y$ be a function. Also let $A, A_i \in \mathcal{N}(X), i \in I$ and $B, B_j \in \mathcal{N}(Y), j \in J$. Then the following hold.

(i) $A_1 \subseteq A_2 \Leftrightarrow f(A_1) \subseteq f(A_2)$, $B_1 \subseteq B_2 \Leftrightarrow f^{-1}(B_1) \subseteq f^{-1}(B_2)$.

(ii) $A \subseteq f^{-1}(f(A))$ and if f is injective then $A = f^{-1}(f(A))$.

(iii) $f^{-1}(f(B)) \subseteq B$ and if f is surjective then $f^{-1}(f(B)) = B$.

(iv) $f^{-1}(\cup B_j) = \cup f^{-1}(B_j)$ and $f^{-1}(\cap B_j) = \cap f^{-1}(B_j)$.

(v) $f(\cup A_i) = \cup f(A_i)$, $f(\cap A_i) \subseteq \cap f(A_i)$ and if f is injective then $f(\cap A_i) = \cap f(A_i)$.

(vi) $f^{-1}(\tilde{\emptyset}_Y) = \tilde{\emptyset}_X$, $f^{-1}(\tilde{Y}) = \tilde{X}$.

(vii) $f(\tilde{\emptyset}_X) = \tilde{\emptyset}_Y$, $f(\tilde{X}) = \tilde{Y}$ if f is surjective.

2.7. **Definition: [28]**

Let X and Y be two non-empty sets and $f : X \to Y$ be a function. Also let $A \in \mathcal{N}(X)$ and $B \in \mathcal{N}(Y)$. Then

(1) Image of A under f is defined by
$f(A) = \{\langle y, f(\mathcal{T}_A)(y), f(\mathcal{I}_A)(y), (1 - f(1 - \mathcal{F}_A))(y)\rangle : y \in Y\}$, where

$$f(\mathcal{T}_A)(y) = \begin{cases} \sup\{\mathcal{T}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 0 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$f(\mathcal{I}_A)(y) = \begin{cases} \inf\{\mathcal{I}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

$$(1 - f(1 - \mathcal{F}_A))(y) = \begin{cases} \inf\{\mathcal{F}_A(x) : x \in f^{-1}(y)\} & \text{if } f^{-1}(y) \neq \emptyset \\ 1 & \text{if } f^{-1}(y) = \emptyset \end{cases}$$

(2) Pre-image of B under f is defined by
$f^{-1}(B) = \{\langle x, f^{-1}(\mathcal{T}_B)(x), f^{-1}(\mathcal{I}_B)(x), f^{-1}(\mathcal{F}_B)(x)\rangle : x \in X\}$

2.8. **Definition: [15]**

Let $\tau \subseteq \mathcal{N}(X)$. Then τ is called a neutrosophic topology on X if

(i) $\tilde{\emptyset}$ and $\tilde{X}$ belong to τ.

(ii) An arbitrary union of neutrosophic sets in τ is in τ.

(iii) The intersection of any two neutrosophic sets in τ is in τ.

If τ is a neutrosophic topology on X then the pair (X, τ) is called a neutrosophic topological space (NTS, for short) over X. The members of τ are called neutrosophic open sets in X. If for a neutrosophic set A, $A^c \in \tau$ then A is said to be a neutrosophic closed set in X.

2.9. **Definition: [30]**

Let (X, τ) and (Y, σ) be two neutrosophic topological spaces and $f : X \to Y$ be a function. Then

(i) f is called a neutrosophic open function if $f(G) \in \sigma$ for all $G \in \tau$

(ii) f is called a neutrosophic continuous function if $f^{-1}(G) \in \tau$ for all $G \in \sigma$.

3. Main Results

3.1. Definition :

Let (X, τ) be a neutrosophic topological space. A subcollection $\mathcal{B}$ of τ is called a neutrosophic base (or simply, base) for τ iff for each $A \in \tau$, there exists a subcollection $\{A_i : i \in \Delta\} \subseteq \mathcal{B}$ such that $A = \cup\{A_i : i \in \triangle\}$, where $\triangle$ is an index set.

A subcollection $\mathcal{B}_*$ of τ is called a neutrosophic subbase (or simply, subbase) for τ iff the finite intersection of members of $\mathcal{B}_*$ forms a neutrosophic base for τ.

3.2. Definition :

An NTS (X, τ) is said to satisfy the second axiom of countability or is said to be neutrosophic C_{II} (or simply, C_{II}) space iff τ has a countable neutrosophic base, i.e., an NTS (X, τ) is said to be C_{II} space iff there exists a countable subcollection $\mathcal{B}$ of τ such that every member of τ can be expressed as the union of some members of $\mathcal{B}$.

3.3. Definition :

Let (X, τ) be an NTS. A collection $\{G_\lambda : \lambda \in \Delta\}$ of neutrosophic closed sets of X is said to have the finite intersection property (FIP, in short) iff every finite subcollection $\{G_{\lambda_k} : k = 1, 2, \cdots, n\}$ of $\{G_\lambda : \lambda \in \Delta\}$ satisfies the condition $\bigcap_{k=1}^{n} G_{\lambda_k} \neq \tilde{\emptyset}$, where $\triangle$ is an index set.

3.4. Definition :

Let (X, τ) be an NTS and $A \in \mathcal{N}(X)$. A collection $C = \{G_\lambda : \lambda \in \triangle\}$ of neutrosophic open sets of X is called a neutrosophic open cover (NOC, in short) of A if $A \subseteq \cup_{\lambda \in \triangle} G_\lambda$. We then say C covers A. In particular, C is said to be an NOC of X iff $\tilde{X} = \cup_{\lambda \in \triangle} G_\lambda$.

Let C be an NOC of the NS A and $C' \subseteq C$. Then C' is called a neutrosophic open subcover (NOSC, in short) of C if C' covers A.

An NOC of A is said to be countable (resp. finite) if it consists of a countable (resp. finite) number of neutrosophic open sets.

3.5. Definition :

An NS A in an NTS (X, τ) is said to be neutrosophic compact set iff every NOC of A has a finite NOSC. In particular, the space X is said to be neutrosophic compact space iff every NOC of X has a finite NOSC.

3.6. Definition :

An NTS (X, τ) is said to be neutrosophic countably compact space iff every countable NOC of X has a finite NOSC.

3.7. **Definition :**

An NTS (X,τ) is said to be neutrosophic Lindelöf iff every NOC of X has a countable NOSC.

3.8. **Example :**

Let $X = \{1,2\}$, $A = \{\langle 1,1,0,0\rangle, \langle 2,0,1,1\rangle\}$, $B = \{\langle 1,0,1,1\rangle, \langle 2,1,0,0\rangle\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}, A, B\}$. Clearly (X,τ) is an NTS. It is clear that (X,τ) is neutrosophic compact, neutrosophic countably compact as well as neutrosophic Lindelöf.

3.9. **Example :**

Let $X = \{a,b\}$ and $G_n = \{\langle a, \frac{n}{n+1}, \frac{1}{n+2}, \frac{1}{n+3}\rangle, \langle b, \frac{n+1}{n+2}, \frac{1}{n+3}, \frac{1}{n+4}\rangle\}, n \in \mathsf{N} = \{1,2,3,\cdots\}$ and $\tau = \{\tilde{X}, \tilde{\emptyset}\} \cup \{G_n : n \in \mathsf{N}\}$. Clearly (X,τ) is an NTS. Also it is easy to see that $\cup_{n\in\mathsf{N}} G_n = \tilde{X}$. Therefore $\{G_n : n \in \mathsf{N}\}$ is an NOC of X. Now $G_1 = \{\langle a, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}\rangle, \langle b, \frac{2}{3}, \frac{1}{4}, \frac{1}{5}\rangle\}$, $G_2 = \{\langle a, \frac{2}{3}, \frac{1}{4}, \frac{1}{5}\rangle, \langle b, \frac{3}{4}, \frac{1}{5}, \frac{1}{6}\rangle\}$, $G_3 = \{\langle a, \frac{3}{4}, \frac{1}{5}, \frac{1}{6}\rangle, \langle b, \frac{4}{5}, \frac{1}{6}, \frac{1}{7}\rangle\}$ and so on. Clearly $G_1 \cup G_2 = G_2$, $G_1 \cup G_3 = G_3$ and $G_1 \cup G_2 \cup G_3 = G_3$. So, for any finite subcollection $\{G_{n_k} : n_k \in M$, M is a finite subset of $\mathsf{N}\}$ of $\{G_n : n \in \mathsf{N}\}$, we have $\bigcup_{n_k} G_{n_k} = G_m \neq \tilde{X}$, where $m = \max\{n_k : n_k \in M\}$. Therefore (X,τ) is not a neutrosophic compact space.

3.10. **Theorem :**

Finite union of neutrosophic compact sets is neutrosophic compact.

Proof: Very obvious.

3.11. **Theorem :**

Let (X,τ) be an NTS. An NS $A = \{\langle x, \mathcal{T}_A(x), \mathcal{I}_A(x), \mathcal{F}_A(x)\rangle : x \in X\}$ in X is neutrosophic compact iff for every collection $C = \{G_\lambda : \lambda \in \triangle\}$ of neutrosophic open sets of X satisfying $\mathcal{T}_A(x) \leq \bigvee_{\lambda\in\triangle} \mathcal{T}_{G_\lambda}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{\lambda\in\triangle}(1 - \mathcal{I}_{G_\lambda}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{\lambda\in\triangle}(1 - \mathcal{F}_{G_\lambda}(x))$, there exists a finite subcollection $\{G_{\lambda_k} : k = 1,2,3,...,n\}$ such that $\mathcal{T}_A(x) \leq \bigvee_{k=1}^{n} \mathcal{T}_{G_{\lambda_k}}(x)$, $1 - \mathcal{I}_A(x) \leq \bigvee_{k=1}^{n}(1 - \mathcal{I}_{G_{\lambda_k}}(x))$ and $1 - \mathcal{F}_A(x) \leq \bigvee_{k=1}^{n}(1 - \mathcal{F}_{G_{\lambda_k}}(x))$.

Proof: Necessary Part : Let $C = \{G_\lambda : \lambda \in\triangle\}$ of neutrosophic open sets of X satisfying $\mathcal{T}_A(x) \leq \bigvee_{\lambda\in\triangle} \mathcal{T}_{G_\lambda}(x)$, $1-\mathcal{I}_A(x) \leq \bigvee_{\lambda\in\triangle}(1-\mathcal{I}_{G_\lambda}(x))$ and $1-\mathcal{F}_A(x) \leq \bigvee_{\lambda\in\triangle}(1-\mathcal{F}_{G_\lambda}(x))$. Now $1-\mathcal{I}_A(x) \leq \bigvee_{\lambda\in\triangle}(1-\mathcal{I}_{G_\lambda}(x)) \Rightarrow 1-\mathcal{I}_A(x) \leq 1-\mathcal{I}_{G_\beta}(x)$ for some $\beta \in\triangle \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_\beta}(x) \Rightarrow \mathcal{I}_A(x) \geq \bigwedge_{\lambda\in\triangle} \mathcal{I}_{G_\lambda}(x)$. Similarly $1 - \mathcal{F}_A(x) \leq \bigvee_{\lambda\in\triangle}(1 - \mathcal{F}_{G_\lambda}(x)) \Rightarrow \mathcal{F}_A(x) \geq \bigwedge_{\lambda\in\triangle} \mathcal{F}_{G_\lambda}(x)$. Therefore $A \subseteq \cup_{\lambda\in\triangle} G_\lambda$, i.e., C is an NOC of A. Since A is compact, so C has a finite NOSC $\{G_{\lambda_k} : k = 1,2,3,\cdots,n\}$. Therefore $A \subseteq \cup_{k=1}^{n} G_{\lambda_k}$. Then $\mathcal{T}_A(x) \leq \bigvee_{k=1}^{n} \mathcal{T}_{G_{\lambda_k}}(x)$, $\mathcal{I}_A(x) \geq \bigwedge_{k=1}^{n} \mathcal{I}_{G_{\lambda_k}}(x)$ and $\mathcal{F}_A(x) \geq \bigwedge_{k=1}^{n} \mathcal{F}_{G_{\lambda_k}}(x)$. Now $\mathcal{I}_A(x) \geq \bigwedge_{k=1}^{n} \mathcal{I}_{G_{\lambda_k}}(x) \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_{\lambda_m}}(x)$ for some $m, 1 \leq m \leq n \Rightarrow 1 - \mathcal{I}_A(x) \leq 1 - \mathcal{I}_{G_{\lambda_m}}(x), 1 \leq m \leq n \Rightarrow 1 - \mathcal{I}_A(x) \leq$

$\bigvee_{k=1}^{n}(1-\mathcal{I}_{G_{\lambda_k}}(x))$. Similarly we can show that $\mathcal{F}_A(x) \geq \bigwedge_{k=1}^{n}\mathcal{F}_{G_{\lambda_k}}(x) \Rightarrow 1-\mathcal{F}_A(x) \leq \bigvee_{k=1}^{n}(1-\mathcal{F}_{G_{\lambda_k}}(x))$. Thus $\mathcal{T}_A(x) \leq \bigvee_{k=1}^{n}\mathcal{T}_{G_{\lambda_k}}(x)$, $1-\mathcal{I}_A(x) \leq \bigvee_{k=1}^{n}(1-\mathcal{I}_{G_{\lambda_k}}(x))$ and $1-\mathcal{F}_A(x) \leq \bigvee_{k=1}^{n}(1-\mathcal{F}_{G_{\lambda_k}}(x))$.

Sufficient Part : Let $C = \{G_\lambda : \lambda \in\triangle\}$ be an NOC of A. Then $A \subseteq \cup_{\lambda\in\triangle}G_\lambda$, i.e., $\mathcal{T}_A(x) \leq \bigvee_{\lambda\in\triangle}\mathcal{T}_{G_\lambda}(x)$, $\mathcal{I}_A(x) \geq \bigwedge_{\lambda\in\triangle}\mathcal{I}_{G_\lambda}(x)$ and $\mathcal{F}_A(x) \geq \bigwedge_{\lambda\in\triangle}\mathcal{F}_{G_\lambda}(x)$. Now $\mathcal{I}_A(x) \geq \bigwedge_{\lambda\in\triangle}\mathcal{I}_{G_\lambda}(x) \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_\alpha}(x)$ for some $\alpha \Rightarrow 1-\mathcal{I}_A(x) \leq 1-\mathcal{I}_{G_\alpha}(x) \Rightarrow 1-\mathcal{I}_A(x) \leq \bigvee_{\lambda\in\triangle}(1-\mathcal{I}_{G_\lambda}(x))$. Similarly $\mathcal{F}_A(x) \geq \bigwedge_{\lambda\in\triangle}\mathcal{F}_{G_\lambda}(x) \Rightarrow 1-\mathcal{F}_A(x) \leq \bigvee_{\lambda\in\triangle}(1-\mathcal{F}_{G_\lambda}(x))$. Thus the collection C satisfies the condition $\mathcal{T}_A(x) \leq \bigvee_{\lambda\in\triangle}\mathcal{T}_{G_\lambda}(x)$, $1-\mathcal{I}_A(x) \leq \bigvee_{\lambda\in\triangle}(1-\mathcal{I}_{G_\lambda}(x))$ and $1-\mathcal{F}_A(x) \leq \bigvee_{\lambda\in\triangle}(1-\mathcal{F}_{G_\lambda}(x))$. By the hypothesis, there exists a finite subcollection $\{G_{\lambda_k} : k = 1,2,3,...,n\}$ such that $\mathcal{T}_A(x) \leq \bigvee_{k=1}^{n}\mathcal{T}_{G_{\lambda_k}}(x)$, $1-\mathcal{I}_A(x) \leq \bigvee_{k=1}^{n}(1-\mathcal{I}_{G_{\lambda_k}}(x))$ and $1-\mathcal{F}_A(x) \leq \bigvee_{k=1}^{n}(1-\mathcal{F}_{G_{\lambda_k}}(x))$. Now $1-\mathcal{I}_A(x) \leq \bigvee_{k=1}^{n}(1-\mathcal{I}_{G_{\lambda_k}}(x)) \Rightarrow 1-\mathcal{I}_A(x) \leq 1-\mathcal{I}_{G_{\lambda_m}}(x)$ for some $m \Rightarrow \mathcal{I}_A(x) \geq \mathcal{I}_{G_{\lambda_m}}(x) \Rightarrow \mathcal{I}_A(x) \geq \bigwedge_{k=1}^{n}\mathcal{I}_{G_{\lambda_k}}(x)$. Similarly we shall have $\mathcal{F}_A(x) \geq \bigwedge_{k=1}^{n}\mathcal{F}_{G_{\lambda_k}}(x)$. Therefore $A \subseteq \cup_{k=1}^{n}G_{\lambda_k}$, i.e., the NOC C of A has a finite NOSC $\{G_{\lambda_k} : k = 1,2,3,\cdots,n\}$. Thus A is neutrosophic compact.

Hence proved.

3.12. **Theorem :**

Let (X,τ) be an NTS. Then X is neutrosophic compact iff for every collection $C = \{G_\lambda : \lambda \in\triangle\}$ of neutrosophic open sets of X satisfying $\bigvee_{\lambda\in\triangle}\mathcal{T}_{G_\lambda}(x) = 1$, $\bigvee_{\lambda\in\triangle}(1-\mathcal{I}_{G_\lambda}(x)) = 1$ and $\bigvee_{\lambda\in\triangle}(1-\mathcal{F}_{G_\lambda}(x)) = 1$, there exists a finite subcollection $\{G_{\lambda_k} : k = 1,2,3,...,n\}$ such that $\bigvee_{k=1}^{n}\mathcal{T}_{G_{\lambda_k}}(x) = 1$, $\bigvee_{k=1}^{n}(1-\mathcal{I}_{G_{\lambda_k}}(x)) = 1$ and $\bigvee_{k=1}^{n}(1-\mathcal{F}_{G_{\lambda_k}}(x)) = 1$.

Proof: Immediate from 3.11.

3.13. **Theorem :**

Let β be a neutrosophic base for an NTS (X,τ). Then X is neutrosophic compact iff every NOC of X by the members of β has a finite NOSC.

Proof: Necessary Part : Obvious.

Sufficient Part : Let $\beta = \{B_\alpha : \alpha \in\triangle\}$ be the neutrosophic base. Also let $\mathcal{C} = \{G_\lambda : \lambda \in\triangle\}$ be an NOC of X. Then each member G_λ of $\mathcal{C}$ is the union of some members of β and the totality of such members of β is evidently an NOC of X. By the hypothesis, this collection of members of β has a finite NOSC $\mathcal{D} = \{B_{\alpha_j} : j = 1,2,3,\cdots,n\}$, say. Clearly for each B_{α_j} in $\mathcal{D}$, we can find a G_{λ_j} in $\mathcal{C}$ such that $B_{\alpha_j} \subseteq G_{\lambda_j}$. Therefore the finite subcollection $\{G_{\lambda_j} : j = 1,2,3,\cdots,n\}$ of $\mathcal{C}$ is an NOC of X. Therefore X is neutrosophic compact.

3.14. **Theorem :**

If the NTS (X,τ) is C_{II} then neutrosophic compactness and neutrosophic countably compactness are equivalent.

Proof: First we show that if (X,τ) is neutrosophic compact then it is neutrosophic countably compact. Let $\mathcal{A} = \{A_i : i \in\triangle\}$ be a countable NOC of X. Since X is compact, so $\mathcal{A}$ has a finite NOSC. Therefore X is neutrosophic countably compact. Next we show that if (X,τ) is neutrosophic countably compact then it is neutrosophic compact. Let $\mathcal{A} = \{A_i : i \in\triangle\}$ be any NOC of X. Since X is C_{II}, so there exists a countable base $\mathcal{B} = \{B_n : n = 1,2,3,\cdots\}$ for τ. Then each $A_i \in \mathcal{A}$ can be expressed as the union of some members of $\mathcal{B}$. Let $A_i = \bigcup_{k=1}^{i_0} B_{n_k}$, where $B_{n_k} \in \mathcal{B}$ and i_0 may be infinity. Clearly $\mathcal{B}_0 = \{B_{n_k}\}$ is an NOC of X. Also $\mathcal{B}_0$ is countable as $\mathcal{B}_0 \subseteq \mathcal{B}$. Since X is countably compact, so $\mathcal{B}_0$ has a finite NOSC $\mathcal{B}_1$, say. Since by construction, each member of $\mathcal{B}_1$ is contained in one member A_i, so these A_i's form a finite subfamily of $\mathcal{A}$ and certainly a cover of X. Thus the NOC $\mathcal{A}$ of X has a finite NOSC. Therefore X is neutrosophic compact. Hence Proved.

3.15. **Theorem :**

If the NTS (X,τ) is C_{II} then it is neutrosophic Lindelöf.

Proof: Let $\mathcal{A} = \{A_i : i \in\triangle\}$ be an NOC of X. Since X is C_{II}, so there exists a countable base $\mathcal{B} = \{B_n : n = 1,2,3,\cdots\}$ for τ. Then each $A_i \in \mathcal{A}$ can be expressed as the union of some members of $\mathcal{B}$. Let $A_i = \bigcup_{k=1}^{i_0} B_{n_k}$, where $B_{n_k} \in \mathcal{B}$ and i_0 may be infinity. Let $\mathcal{B}_0 = \{B_{n_k}\}$. Then $\mathcal{B}_0$ is an NOC of X. Also $\mathcal{B}_0$ is countable as $\mathcal{B}_0 \subseteq \mathcal{B}$. By construction, each member of $\mathcal{B}_0$ is contained in one A_i . So, these A_i's form a countable NOSC of $\mathcal{A}$. Thus the NOC $\mathcal{A}$ of X has a countable NOSC. Therefore X is neutrosophic Lindelöf. Hence proved.

3.16. **Theorem :**

An NTS (X,τ) is neutrosophic compact iff every collection of neutrosophic closed sets with the FIP has a non-empty intersection.

Proof: Necessary part : Let $\mathcal{A} = \{N_i : i \in\triangle\}$ be an arbitrary collection of neutrosophic closed sets with the FIP. We show that $\cap_{i\in\triangle} N_i \neq \tilde{\emptyset}$. On the contrary, suppose that $\cap_{i\in\triangle} N_i = \tilde{\emptyset}$. Then $(\cap_{i\in\triangle} N_i)^c = (\tilde{\emptyset})^c \Rightarrow \cup_{i\in\triangle} N_i^c = \tilde{X}$. Therefore $\mathcal{B} = \{N_i^c : N_i \in \mathcal{A}\}$ is an NOC of X and so $\mathcal{B}$ has a finite NOSC $\{N_{i_1}^c, N_{i_2}^c, ..., N_{i_k}^c\}$, say. Then $\cup_{j=1}^k N_{i_j}^c = \tilde{X} \Rightarrow \cap_{j=1}^k N_{i_j} = \tilde{\emptyset}$, which is a contradiction as $\mathcal{A}$ has FIP. Therefore $\cap_{i\in\triangle} N_i \neq \tilde{\emptyset}$.

Sufficient part : Let $\mathcal{C} = \{G_i : i \in\triangle\}$ be an NOC of X. Suppose that $\mathcal{C}$ has no finite NOSC. Then for every finite subcollection $\{G_{i_1}, G_{i_2}, ..., G_{i_k}\}$ of $\mathcal{C}$, we have $\cup_{j=1}^k G_{i_j} \neq \tilde{X} \Rightarrow \cap_{j=1}^k G_{i_j}^c \neq \tilde{\emptyset}$. Therefore $\{G_i^c : G_i \in \mathcal{C}\}$ is a collection of neutrosophic closed sets having the FIP. By the assumption, $\cap_{i\in\triangle} G_i^c \neq \tilde{\emptyset} \Rightarrow \cup_{i\in\triangle} G_i \neq \tilde{X}$. This implies that $\mathcal{C}$ is not an NOC of X, which is a contradiction. Therefore $\mathcal{C}$ must have a finite NOSC. Therefore X is neutrosophic compact.

Hence proved.

3.17. **Theorem :**

Let (X, τ_1) and (Y, τ_2) be two NTSs and let $f : X \to Y$ be a neutrosophic continuous function. If A is neutrosophic compact in (X, τ_1) then $f(A)$ is neutrosophic compact in (Y, τ_2).

Proof: Let $\mathcal{B} = \{G_\lambda : \lambda \in \triangle\}$ be an NOC of $f(A)$, where $G_\lambda = \{\langle y, \mathcal{T}_{G_\lambda}(y), \mathcal{I}_{G_\lambda}(y), \mathcal{F}_{G_\lambda}(y)\rangle : y \in Y\}$. Then $f(A) \subseteq \cup_{\lambda\in\triangle} G_\lambda \Rightarrow f^{-1}(f(A)) \subseteq f^{-1}(\cup_{\lambda\in\triangle} G_\lambda) \Rightarrow f^{-1}(f(A)) \subseteq \cup_{\lambda\in\Delta} f^{-1}(G_\lambda) \Rightarrow A \subseteq \cup_{\lambda\in\Delta} f^{-1}(G_\lambda)[\because A \subseteq f^{-1}(f(A))]$. Since G_λ is open in Y, so $f^{-1}(G_\lambda)$ is open in X as f is continuous. Therefore $C = \{f^{-1}(G_\lambda) : \lambda \in \Delta\}$ is an NOC of A. Since A is compact, so C has a finite NOSC $\{f^{-1}G_{\lambda_1}, f^{-1}G_{\lambda_2}, \cdots, f^{-1}G_{\lambda_n}\}$. Therefore $A \subseteq \cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) \Rightarrow f(A) \subseteq f(\cup_{i=1}^{n} f^{-1}(G_{\lambda_i})) \Rightarrow f(A) \subseteq \cup_{i=1}^{n} f(f^{-1}(G_{\lambda_i})) \Rightarrow f(A) \subseteq \cup_{i=1}^{n} G_{\lambda_i}$. Thus the NOC $\mathcal{B}$ of $f(A)$ has a finite NOSC. Therefore $f(A)$ is neutrosophic compact. Hence proved.

3.18. **Theorem :**

Let (X, τ_1) and (Y, τ_2) be two NTSs and let $f : X \to Y$ is a neutrosophic continuous onto function. If (X, τ_1) is neutrosophic compact then (Y, τ_2) is neutrosophic compact.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $\mathcal{B} = \{G_\lambda : \lambda \in \Delta\}$ be an NOC of Y, where $G_\lambda = \{\langle y, \mathcal{T}_{G_\lambda}(y), \mathcal{I}_{G_\lambda}(y), \mathcal{F}_{G_\lambda}(y)\rangle : y \in Y\}$. Then $\cup_{\lambda\in\triangle} G_\lambda = \tilde{Y} \Rightarrow f^{-1}(\cup_{\lambda\in\Delta} G_\lambda) = f^{-1}(\tilde{Y}) \Rightarrow \cup_{\lambda\in\Delta} f^{-1}(G_\lambda) = \tilde{X}$. Since G_λ is open in Y, so $f^{-1}(G_\lambda)$ is open in X as f is continuous. Therefore $C = \{f^{-1}(G_\lambda) : \lambda \in \Delta\}$ is an NOC of X. Since X is compact, so C has a finite NOSC $\{f^{-1}G_{\lambda_1}, f^{-1}G_{\lambda_2}, \cdots, f^{-1}G_{\lambda_n}\}$. Therefore $\cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) = \tilde{X} \Rightarrow f(\cup_{i=1}^{n} f^{-1}(G_{\lambda_i})) = f(\tilde{X}) \Rightarrow \cup_{i=1}^{n} f(f^{-1}(G_{\lambda_i})) = \tilde{Y} \Rightarrow \cup_{i=1}^{n} G_{\lambda_i} = \tilde{Y}$.. Thus the NOC $\mathcal{B}$ of Y has a finite NOSC. Therefore Y is neutrosophic compact. Hence proved.

3.19. **Theorem :**

Let (X, τ_1) and (Y, τ_2) be two NTSs and let $f : X \to Y$ is a neutrosophic continuous onto function. If X is neutrosophic countably compact then Y is also neutrosophic countably compact.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $\mathcal{A} = \{G_\lambda : \lambda \in \Delta\}$ be a countable NOC of Y, where $G_\lambda = \{\langle y, \mathcal{T}_{G_\lambda}(y), \mathcal{I}_{G_\lambda}(y), \mathcal{F}_{G_\lambda}(y)\rangle : y \in Y\}$. Then $\cup_{\lambda\in\triangle} G_\lambda = \tilde{Y} \Rightarrow f^{-1}(\cup_{\lambda\in\Delta} G_\lambda) = f^{-1}(\tilde{Y}) \Rightarrow \cup_{\lambda\in\Delta} f^{-1}(G_\lambda) = \tilde{X}$. Since G_λ is open in Y, so $f^{-1}(G_\lambda)$ is open in X as f is continuous. Therefore $C = \{f^{-1}(G_\lambda) : \lambda \in \Delta\}$ is an NOC of X. Obviously C is countable as A is countable. Again since X is neutrosophic countably compact, so C has a finite NOSC $\{f^{-1}G_{\lambda_1}, f^{-1}G_{\lambda_2}, \cdots, f^{-1}G_{\lambda_n}\}$. Therefore $\cup_{i=1}^{n} f^{-1}(G_{\lambda_i}) = \tilde{X} \Rightarrow f(\cup_{i=1}^{n} f^{-1}(G_{\lambda_i})) = f(\tilde{X}) \Rightarrow \cup_{i=1}^{n} f(f^{-1}(G_{\lambda_i})) = \tilde{Y} \Rightarrow \cup_{i=1}^{n} G_{\lambda_i} = \tilde{Y}$. Thus $\mathcal{A}$ has a finite NOSC. Hence Y is neutrosophic countably compact.

3.20. **Theorem :**

Let (X, τ_1) and (Y, τ_2) be two NTSs and let $f : X \to Y$ is a neutrosophic continuous onto function. If X is neutrosophic Lindelöf then Y is also neutrosophic Lindelöf.

Proof: Since f is onto, so $f(\tilde{X}) = \tilde{Y}$. Let $\mathcal{A} = \{A_i : i \in\triangle\}$, be an NOC of Y. Then $\tilde{Y} = \bigcup_{i\in\triangle} A_i \Rightarrow f^{-1}(\tilde{Y}) = f^{-1}(\bigcup_{i\in\triangle} A_i) \Rightarrow \tilde{X} = \bigcup_{i\in\triangle} f^{-1}(A_i) \Rightarrow \{f^{-1}(A_i) : i \in\triangle\}$ is an NOC of X. Since X is neutrosophic Lindelöf, so $\{f^{-1}(A_i) : i \in\triangle\}$ has a countable NOSC $\mathcal{B} = \{f^{-1}(A_{i_k}) : k = 1, 2, 3, \ldots\}$. Therefore $\tilde{X} = \bigcup_{k=1}^{i_0} f^{-1}(A_{i_k})$, where i_0 may be infinity. This gives $f(\tilde{X}) = f[\bigcup_{k=1}^{i_0} f^{-1}(A_{i_k})] \Rightarrow \tilde{Y} = \bigcup_{k=1}^{i_0}[f(f^{-1}(A_{i_k})] \Rightarrow \tilde{Y} = \bigcup_{k=1}^{i_0} A_{i_k} \Rightarrow \{A_{i_k} : k = 1, 2, 3, \cdots\}$ an NOC of Y. Since $\mathcal{B}$ is countable, so $\{A_{i_k} : k = 1, 2, 3, \cdots\}$ is also countable. Therefore the NOC $\mathcal{A}$ of Y has a countable NOSC $\{A_{i_k} : k = 1, 2, 3, \cdots\}$, i.e., Y is neutrosophic Lindelöf. Hence proved.

3.21. **Theorem : (Alexander subbase lemma)**

Let β be a subbase of an NTS (X, τ). Then X is neutrosophic compact iff for every collection of neutrosophic closed sets chosen from β^c having the FIP, there is a non-empty intersection.

Proof: Necessary part : Immediate.

Sufficient Part : On the contrary, let us suppose that X is not compact. Then there exists a collection $\mathcal{C} = \{G_i : i \in I\}$, where $G_i = \{\langle x, \mathcal{T}_{G_i}(x), \mathcal{I}_{G_i}(x), \mathcal{F}_{G_i}(x)\rangle : x \in X\}$, of neutrosophic closed sets of X having the FIP such that $\cap_{i\in\triangle} G_i = \tilde{\emptyset}$. The collection of all such collections $\mathcal{C}$ can be arranged in an order by using the classical inclusion($\subseteq$) and the collection will certainly have an upper bound. Therefore by Zorn's lemma, there will be a maximal collection of all the collections $\mathcal{C}$. Let $\mathcal{M} = \{M_j : j \in J\}$ be the maximal collection, where $M_j = \{\langle x, \mathcal{T}_{M_j}(x), \mathcal{I}_{M_j}(x), \mathcal{F}_{M_j}(x)\rangle : x \in X\}$. This collection $\mathcal{M}$ has the following properties : (i) $\tilde{\emptyset} \notin \mathcal{M}$ (ii) $P \in \mathcal{M}, P \subseteq Q \Rightarrow Q \in \mathcal{M}$ (iii) $P, Q \in \mathcal{M} \Rightarrow P \cap Q \in \mathcal{M}$ (iv) $\cap(\mathcal{M} \cap \beta^c) = \tilde{\emptyset}$. Clearly the property (iv) delivers a contradiction to the hypothesis. Therefore X is compact.

Hence proved.

3.22. **Definition :**

An NTS (X, τ) is said to be neutrosophic locally compact iff for every NP $x_{\alpha,\beta,\gamma}$ in X, there exists neutrosophic τ-open set G such that $x_{\alpha,\beta,\gamma} \in G$ and G is neutrosophic compact in X.

3.23. **Theorem :**

Every neutrosophic compact space is neutrosophic locally compact space.

Proof: Let (X, τ) be a neutrosophic compact space and let $x_{\alpha,\beta,\gamma}$ be an NP in X. Since X is neutrosophic compact and since $\tilde{X}$ is a neutrosophic open set containing $x_{\alpha,\beta,\gamma}$, so, X is a neutrosophic locally compact space.

3.24. **Remark :**

Every neutrosophic locally compact space need not be neutrosophic compact space. We establish it by the following example.

Let $X = \mathsf{N} = \{1, 2, 3, \cdots\}$. For $n \in \mathsf{N}$, we define $G_n = \{\langle x, \mathcal{T}_{G_n}(x), \mathcal{I}_{G_n}(x), \mathcal{F}_{G_n}(x) : x \in X\}$, where $\mathcal{T}_{G_n}(x) = 1, \mathcal{I}_{G_n}(x) = 0, \mathcal{F}_{G_n}(x) = 0$ if $x \leq n$ and $\mathcal{T}_{G_n}(x) = 0, \mathcal{I}_{G_n}(x) = 1, \mathcal{F}_{G_n}(x) = 1$ if $x > n$. Let τ be the set consisting of $\tilde{\emptyset}$, $\tilde{X}$ and the neutrosophic sets G_n, $n \in \mathsf{N}$. Obviously (X, τ) is an NTS and it is also clear that (X, τ) is a neutrosophic locally compact space but not a neutrosophic compact space.

3.25. **Theorem :**

Let f be a neutrosophic continuous function from a neutrosophic locally compact space (X, τ) onto an NTS (Y, σ). If f is neutrosophic open function then (Y, σ) is also neutrosophic locally compact space.

Proof: Let $y_{p,q,r}$ be any NP in Y. Also let $x_{\alpha,\beta,\gamma}$ be an NP in X such that $x_{\alpha,\beta,\gamma} \in f^{-1}(y_{p,q,r})$. Then $f(x_{\alpha,\beta,\gamma}) = y_{p,q,r}$. Since $x_{\alpha,\beta,\gamma} \in X$, and X neutrosophic locally compact, so there exists a τ-open set G such that $x_{\alpha,\beta,\gamma} \in G$ and G is neutrosophic compact in X. Now $x_{\alpha,\beta,\gamma} \in G \Rightarrow f(x_{\alpha,\beta,\gamma}) \in f(G) \Rightarrow y_{p,q,r} \in f(G)$. Since f is neutrosophic continuous and G is neutrosophic compact in X, so $f(G)$ is neutrosophic compact in Y. Again since f is a neutrosophic open function, so is $f(G)$ is a σ-open set. Thus for any any NP $y_{p,q,r}$ in Y, there exists a σ-open set $f(G)$ such that $y_{p,q,r} \in f(G)$ and $f(G)$ is neutrosophic compact in Y. Therefore (Y, σ) is neutrosophic locally compact space.

4. Conclusions :

In this article, we have defined neutrosophic compactness, neutrosophic countably compactness, neutrosophic Lindelöfness and investigated various covering properties. Especially we have shown that if a neutrosophic topological space is neutrosophic C_{II} then neutrosophic compactness and neutrosophic countably compactness are equivalent. We have proved that the neutrosophic compactness is preserved under neutrosophic continuous function. We have also stated and proved the neutrosophic version of "Alexander subbase lemma". Lastly, we have defined neutrosophic locally compact space and put forward two propositions with proofs. Hope that the findings in this article will assist the research fraternity to move forward for the development of different aspects of neutrosophic topology.

5. Conflict of Interest

We certify that there is no actual or potential conflict of interest in relation to this article.

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