MCGDM based on TOPSIS and VIKOR using Pythagorean neutrosophic soft with aggregation operators

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Abstract. Pythagorean neutrosophic soft set (PNSS set) is a new approach towards decision making under uncertainty. The PNSS set has much stronger abilities than the neutrosophic soft set and the Pythagorean fuzzy soft set. In this paper, we discuss aggregated operations for aggregating the PNSS decision matrix. The TOPSIS and VIKOR methods are strong approaches for multi criteria group decision making (MCGDM), which is various extensions of neutrosophic soft sets. In this approach, we propose a score function based on aggregating TOPSIS and VIKOR methods to the PNSS-positive ideal solution and the PNSS-negative ideal solution. Also, the TOPSIS and VIKOR methods provide the weights of decision-making. Afterward, a revised closeness is introduced to identify the optimal alternative.

Keywords: Pythagorean neutrosophic soft set, MCGDM, TOPSIS, VIKOR, aggregation operator.

1. Introduction

The classic article of 1965, Zadeh proposed fuzzy set theory \cite{39}. According to this definition a fuzzy set is a function described by a membership value. It takes degrees in real unit interval. But, later it has been seen that this definition is inadequate by considering not only the degree of membership but also the degree of non-membership. Neutrosophic set is a generalization of the fuzzy set and intuitionistic fuzzy set, where the truth-membership, indeterminacy-membership, and falsity-membership are represented independently. Atanassov\textsuperscript{3} described a set that is called an intuitionistic fuzzy set to handle mentioned ambiguity. Since this set has some problems in applications, Smarandache\textsuperscript{31} introduced neutrosophy to deal with...
the problems that involves indeterminate and inconsistent information. Yager [38] as being introduced by the concept of Pythagorean fuzzy sets. It has been extended from intuitionistic fuzzy sets and is distinguished by the requirement that the square sum of their degrees of membership and non-membership not exceed unity. A neutrosophic set is used to tackle uncertainty using the truth, indeterminacy, and falsity membership grades by Smarandache [30]. The theory of soft sets was proposed by [15]. Maji et al. proposed the concepts of the fuzzy soft set [13] and the intuitionistic fuzzy soft set [14]. These two theories are applied to solve various decision making problems. In recent years, Peng et al. [29] have extended the fuzzy soft set to the Pythagorean fuzzy soft set. Smarandache et al. [5,10] discussed the concept of Pythagorean neutrosophic set approach. A decision-making (DM) problem is the process of finding the best optional alternatives. In almost all such problems, the multiplicity of criteria for judging the alternatives is pervasive. That is, for many such problems, the decision maker wants to solve a multiple criteria decision making (MCDM) problem. A survey of the MCDM methods has been presented by Hwang and Yoon [7]. A MCDM problem can be expressed in matrix format as:

\[
\mathcal{P}_{n \times m} = \begin{pmatrix}
C_1 & C_2 & \ldots & C_m \\
A_1 & a_{11} & a_{12} & \ldots & a_{1m} \\
A_2 & a_{21} & a_{22} & \ldots & a_{2m} \\
\vdots & \vdots & \vdots & \ddots & \vdots \\
A_n & a_{n1} & a_{n2} & \ldots & a_{nm}
\end{pmatrix}
\]

where \(A_1, A_2, \ldots, A_n\) are possible alternatives among which decision makers must choose, \(C_1, C_2, \ldots, C_m\) are criteria with which alternative performance is measured, \(a_{ij}\) is the rating of alternative \(A_i\) in relation to criterion \(C_j\).

Many researchers have studied the TOPSIS and VIKOR methods for decision making problems, including Adeel et al. [1], Akram and Arshad [2], Boran et al. [4], Eraslan and Karaaslan [6], Peng and Dai [28], Xu and Zhang [36] and Zhang and Xu [40]. In 2021, Zulqarnain discussed the TOPSIS technique as it applies to interval valued intuitionistic fuzzy soft sets (IVIFSS) information, where the mechanisms are assumed in terms of IVIFSNs. To measure the degree of dependency of IVIFSS’s, [41] discussed a new correlation coefficient for IVIFSS’s and examined some properties of the developed correlation coefficient. To achieve the goal accurately, the TOPSIS technique may be extended to solve MADM problems. The basic idea of TOPSIS is rather straightforward. It simultaneously considers the distances to both positive ideal solutions (PIS) and negative ideal solutions (NIS), and a preference order is ranked according to their relative closeness and a combination of these two distance measures. The VIKOR method focuses on ranking and selecting from a set of alternatives, and determining compromise solutions for a problem with conflicting criteria, which can help
decision makers reach a final decision \cite{16,17}. Opricovic and Tzeng \cite{18} suggested using fuzzy logic for the VIKOR method. Tzeng et al. \cite{33} used and compared the VIKOR and TOPSIS methods in solving a public transportation problem. Newly, Pythagorean fuzzy logical with real life applications discussed many authors \cite{8,9,32,34,35,37}. Recently, Palanikumar et al. discussed various field of applications including algebraic structures \cite{11,12,19,27}.

2. Preliminaries

Definition 2.1. \cite{5} Let \( U \) be a non-empty set of the universe. A neutrosophic set \( A \) in \( U \) is an object having the following form : \( A = \{u, \sigma_A^T(u), \sigma_A^I(u), \sigma_A^F(u) | u \in U \} \), where \( \sigma_A^T(u), \sigma_A^I(u), \sigma_A^F(u) \) represents the degree of truth membership, degree of indeterminacy membership and degree of falsity membership of \( A \) respectively. The mapping \( \sigma_A^T, \sigma_A^I, \sigma_A^F : U \rightarrow [0,1] \) and \( 0 \leq \sigma_A^T(u) + \sigma_A^I(u) + \sigma_A^F(u) \leq 3 \).

Definition 2.2. \cite{10} Let \( U \) be a non-empty set of the universe, Pythagorean neutrosophic set (PNSS) \( A \) in \( U \) is an object having the following form : \( A = \{u, \sigma_A^T(u), \sigma_A^I(u), \sigma_A^F(u) | u \in U \} \), where \( \sigma_A^T(u), \sigma_A^I(u), \sigma_A^F(u) \) represents the degree of truth membership, degree of indeterminacy membership and degree of falsity membership of \( A \) respectively. The mapping \( \sigma_A^T, \sigma_A^I, \sigma_A^F : U \rightarrow [0,1] \) and \( 0 \leq (\sigma_A^T(u))^2 + (\sigma_A^I(u))^2 + (\sigma_A^F(u))^2 \leq 2 \). Since \( A = (\sigma_A^T, \sigma_A^I, \sigma_A^F) \) is called a Pythagorean neutrosophic number (PNNS).

Definition 2.3. The score function for any PNSS \( A = (\sigma_A^T, \sigma_A^I, \sigma_A^F) \) is defined as \( S(A) = \sigma_A^{2T} - \sigma_A^{2I} - \sigma_A^{2F} \), where \(-1 \leq S(A) \leq 1 \).

3. MCGDM based on PNSS sets

Definition 3.1. Let \( U \) be a non-empty set of the universe and \( E \) be a set of parameter. The pair \( (\Delta, A) \) or \( \Delta_A \) is called a Pythagorean neutrosophic soft set (PNSS set) on \( U \) if \( A \subseteq E \) and \( \Delta : A \rightarrow PNS^U \), where \( PNS^U \) is represent the aggregate of all Pythagorean neutrosophic subsets of \( U \). (ie) \( \Delta_A = \left\{ \left( e, \left\{ \sigma_{\Delta_A}^u \right\} : e \in A, u \in U \right\} \right\} \).

Remark 3.2. If we write \( a_{ij} = \sigma_{\Delta_A}^T(e_j)(u_i), b_{ij} = \sigma_{\Delta_A}^I(e_j)(u_i) \) and \( c_{ij} = \sigma_{\Delta_A}^F(e_j)(u_i) \), where \( i = 1,2,\ldots,m \) and \( j = 1,2,\ldots,n \) then the PNSS set \( \Delta_A \) may be represented in matrix form as

\[
\Delta_A = \begin{bmatrix}
(a_{11}, b_{11}, c_{11}) & (a_{12}, b_{12}, c_{12}) & \cdots & (a_{1n}, b_{1n}, c_{1n}) \\
(a_{21}, b_{21}, c_{21}) & (a_{22}, b_{22}, c_{22}) & \cdots & (a_{2n}, b_{2n}, c_{2n}) \\
\vdots & \vdots & \ddots & \vdots \\
(a_{m1}, b_{m1}, c_{m1}) & (a_{m2}, b_{m2}, c_{m2}) & \cdots & (a_{mn}, b_{mn}, c_{mn})
\end{bmatrix}
\]

This matrix is called Pythagorean neutrosophic soft matrix (PNSSM).

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Definition 3.3. The cardinal set of the PNSS set $\Delta_A$ over $U$ is a PNSS set over $E$ and is defined as $c\Delta_A = \begin{pmatrix} e \end{pmatrix}$, where $\sigma_{\Delta_A}^T$, $\sigma_{\Delta_A}^\varphi$ and $\sigma_{\Delta_A}^\zeta$ are mapping from $E$ to unit interval respectively, where $\sigma_{\Delta_A}^T(e) = \frac{|\delta_A(e)|}{|U|}$, $\sigma_{\Delta_A}^\varphi(e) = \frac{|\zeta_A(e)|}{|U|}$ and $\sigma_{\Delta_A}^\zeta(e) = \frac{|\varphi_A(e)|}{|U|}$ where $|\delta_A(e)|$, $|\zeta_A(e)|$ and $|\varphi_A(e)|$ denote the scalar cardinalities of the PNSS sets $\delta_A(e)$, $\zeta_A(e)$ and $\varphi_A(e)$ respectively, and $|U|$ represents cardinality of the universe $U$. The collection of all cardinal sets of PNSS sets of $U$ is represented as $cPNS^U$. If $A \subseteq E = \{e_i : i = 1, 2, \ldots, n\}$, then $c\Delta_A \in cPNS^U$ may be represented in matrix form as $\{(a_{ij}, b_{ij}, c_{ij})\}_{1 \times n} = [(a_{11}, b_{11}, c_{11}), (a_{12}, b_{12}, c_{12}), \ldots, (a_{1n}, b_{1n}, c_{1n})]$, where $(a_{ij}, b_{ij}, c_{ij}) = \mu_{\Delta_A}(e_j)$, for $j = 1, 2, \ldots, n$. This matrix is termed as cardinal matrix of $c\Delta_A$ of $E$.

Definition 3.4. Let $\Delta_A \in PNS^U$ and $c\Delta_A \in cPNS^U$. The PNSS set aggregation operator $PNSS_{agg} : cPNS^U \times PNS^U \rightarrow PNS(U, E)$ is defined as

$$PNSS_{agg}(c\Delta_A, \Delta_A) = \begin{pmatrix} u \end{pmatrix} : u \in U \right) \left\{ (\sigma_{\Delta_A}^T(u), \sigma_{\Delta_A}^\varphi(u), \sigma_{\Delta_A}^\zeta(u)) : u \in U \right\}.$$ This collection is called aggregate Pythagorean neutrosophic set of PNSS set $\Delta_A$. The degree of truth membership $\sigma_{\Delta_A}^T(u) : U \rightarrow [0, 1]$ by $\sigma_{\Delta_A}^T(u) = \frac{1}{|E|} \sum_{e \in E} (\sigma_{\Delta_A}^T(e), \sigma_{\Delta_A}^T(e)) (u)$, degree of indeterminacy membership $\sigma_{\Delta_A}^\varphi(u) : U \rightarrow [0, 1]$ by $\sigma_{\Delta_A}^\varphi(u) = \frac{1}{|E|} \sum_{e \in E} (\sigma_{\Delta_A}^\varphi(e), \sigma_{\Delta_A}^\varphi(e)) (u)$ and degree of falsity membership $\sigma_{\Delta_A}^\zeta(u) : U \rightarrow [0, 1]$ by $\sigma_{\Delta_A}^\zeta(u) = \frac{1}{|E|} \sum_{e \in E} (\sigma_{\Delta_A}^\zeta(e), \sigma_{\Delta_A}^\zeta(e)) (u)$. The set $PNSS_{agg}(c\Delta_A, \Delta_A)$ is expressed in matrix form as

$$[\begin{array}{c} (a_{11}, b_{11}, c_{11}) \\ (a_{21}, b_{21}, c_{21}) \\ \vdots \\ (a_{m1}, b_{m1}, c_{m1}) \end{array}]\]_{m \times 1}$$

where $[(a_{i1}, b_{i1}, c_{i1})] = \mu_{\Delta_A}(u_i)$, for $i = 1, 2, \ldots, m$. This matrix is called PNSS aggregate matrix of $PNSS_{agg}(c\Delta_A, \Delta_A)$ over $U$.

Definition 3.5. Let $A = (\sigma_{ij}^T, \sigma_{ij}^\varphi, \sigma_{ij}^\zeta) \in PNSSM_{m \times n}$, then the choice matrix of PNSSM $A$ is given by $\mathcal{C}(A) = \left[ \begin{array}{c} \frac{\sum_{i=1}^n (\sigma_{ij}^T)^2}{n} \\ \frac{\sum_{i=1}^n (\sigma_{ij}^\varphi)^2}{n} \\ \frac{\sum_{i=1}^n (\sigma_{ij}^\zeta)^2}{n} \end{array} \right]_{m \times 1}$ ∀i when weights are equal.

Definition 3.6. Let $A = (\sigma_{ij}^T, \sigma_{ij}^\varphi, \sigma_{ij}^\zeta) \in PNSSM_{m \times n}$, then the weighted choice matrix of PNSSM $A$ is given by $\mathcal{C}_w(A) = \left[ \begin{array}{c} \frac{\sum_{i=1}^n w_j (\sigma_{ij}^T)^2}{\sum w_j} \\ \frac{\sum_{i=1}^n w_j (\sigma_{ij}^\varphi)^2}{\sum w_j} \\ \frac{\sum_{i=1}^n w_j (\sigma_{ij}^\zeta)^2}{\sum w_j} \end{array} \right]_{m \times 1}$ ∀i where $w_j > 0$ are weights (means weights are unequal).

Theorem 3.7. Let $\Delta_A$ be a PNSS set. Suppose that $M_{\Delta_A}, M_{c\Delta_A}, M_{\Delta_A}^*$ are matrices of $\Delta_A, c\Delta_A, \Delta_A^*$ respectively, then $M_{\Delta_A} \times M_{c\Delta_A} = M_{\Delta_A}^* \times |E|$, where $M_{c\Delta_A}$ is the transpose of $M_{\Delta_A}$.
The aggregate PNSS set \( \Delta \)

Step-1: Construct PNSS set \( \Delta_A \) over the universal \( U \).

Step-2: Compute the cardinalities and find the cardinal set \( c\Delta_A \) of \( \Delta_A \).

Step-3: Find aggregate PNSS set \( \Delta_A^* \) of \( \Delta_A \).

Step-4: Compute the value of score function by \( S(u) = \sigma_u^{2T} - \sigma_u^{2T} - \sigma_u^{2F}, \forall u \in U \).

Step-5: Compute \( S(u) \) is maximum is the best alternative.

Example 3.8. Suppose that an automobile company produces ten different types of cars \( U = \{ C_1, C_2, ..., C_{10} \} \) and lets a set of parameters \( E = \{ e_1, e_2, ..., e_5 \} \) represent fuel economy, acceleration, top speed, ride comfort, and good power steering, respectively. Suppose that a customer has to decide which car purchase? Following the discussion, each car is evaluated using a subset of parameters \( A = \{ e_1, e_2, e_4 \} \subset E \). We apply the above algorithm as follows.

Step-1: We Construct PNSS set \( \Delta_A \) of \( U \) is defined as below:

\[
\Delta_A = \left\{ \left( e_1, \left\{ \frac{C_1}{0.55,0.75,0.6}, \frac{C_2}{0.8,0.75,0.65} \frac{C_3}{0.75,0.75,0.55}, \frac{C_4}{0.9,0.5,0.8}, \frac{C_5}{0.65,0.6,0.65} \right\} \right), \right. \\
\left( e_2, \left\{ \frac{C_1}{0.6,0.75,0.55}, \frac{C_2}{0.65,0.55,0.8}, \frac{C_3}{0.55,0.65,0.6}, \frac{C_4}{0.65,0.7,0.7}, \frac{C_5}{0.5,0.8,0.55} \right\} \right), \right. \\
\left( e_4, \left\{ \frac{C_1}{0.75,0.7,0.7}, \frac{C_2}{0.5,0.6,0.75}, \frac{C_3}{0.6,0.65,0.8}, \frac{C_4}{0.7,0.75,0.7}, \frac{C_5}{0.9,0.55,0.55} \right\} \right). \\
\frac{e_3}{0.36}, \frac{e_2}{0.295}, \frac{e_4}{0.345} \}
\]

Step-2: The cardinal set of \( \Delta_A \) as \( c\Delta_A = \left\{ \frac{e_1}{0.36,0.33,0.32}, \frac{e_2}{0.295,0.345,0.315} \right\} \). \( \frac{e_4}{0.345,0.325,0.365} \}

Step-3: The aggregate PNSS set \( \Delta_A^* \) of \( \Delta_A \) is \( M_{\Delta_A} = \frac{M_{\Delta_A} \times M_{\Delta_A}^*}{|E|} \).

\[
\begin{bmatrix}
0.55 & 0 & 0 & 0 & 0 \\
0.6 & 0 & 0 & 0 & 0 \\
0.65 & 0.75 & 0 \\
0.8 & 0 & 0 & 0.5 & 0 \\
0.55 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0.6 & 0 \\
0.7 & 0 & 0 & 0 & 0 \\
0 & 0.65 & 0 & 0.7 & 0 \\
0.9 & 0 & 0 & 0.9 & 0 \\
0.65 & 0.5 & 0 & 0 & 0
\end{bmatrix}
= \frac{1}{5}
\begin{bmatrix}
0.36 & 0.295 & 0.345 & 0.75 & 0.33 \\
0 & 0.7 & 0 & 0.6 & 0.345 \\
0 & 0 & 0 & 0.65 & 0.325 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

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\[
\begin{bmatrix}
0.6 & 0 & 0 & 0 & 0 \\
0 & 0.5 & 0 & 0 & 0 \\
0 & 0.8 & 0.7 & 0 & 0 \\
0.65 & 0 & 0 & 0.75 & 0 \\
0 & 0.6 & 0 & 0 & 0 \\
0 & 0 & 0.8 & 0 & 0 \\
0.55 & 0 & 0 & 0 & 0 \\
0 & 0.7 & 0 & 0.7 & 0 \\
0.8 & 0 & 0 & 0.7 & 0 \\
0.6 & 0.55 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.6 & 0 & 0 & 0 & 0 \\
0.55 & 0 & 0 & 0 & 0 \\
0.7 & 0 & 0.7 & 0 \\
0.8 & 0 & 0.7 & 0 \\
0.6 & 0.55 & 0 & 0 & 0 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
\begin{bmatrix}
0.32 \\
0.315 \\
0 \\
0.365 \\
0 \\
\end{bmatrix}
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.495 \\
0.05175 \\
0.0315 \\
0.0504 \\
0.0495 \\
0.0504 \\
0.0495 \\
0.0504 \\
0.0495 \\
0.0504 \\
\end{bmatrix}
\]

\[
\begin{bmatrix}
0.384 \\
0.0495 \\
0.0315 \\
0.1023 \\
0.0384 \\
0.0495 \\
0.0315 \\
0.1023 \\
0.0384 \\
0.0495 \\
\end{bmatrix}
\]

Hence, \( \Delta_A^* = \big\{ \begin{array}{l}
C_1 (0.00236, 0.00091) \\
C_2 (0.00242, 0.00091) \\
C_3 (0.00915, 0.00091) \\
C_4 (0.00806, 0.00091) \\
C_5 (0.00239, 0.00091) \\
C_6 (0.00348, 0.00091) \\
C_7 (0.00115, 0.00091) \\
C_8 (0.01097, 0.00091) \\
C_9 (0.00091, 0.00091) \\
C_{10} (0.0085, 0.00091) \\
\end{array} \big\}.

**Step-4**: The values of the score function \( S(C_i) \) for each element of \( U \) are tabulated as follows.

<table>
<thead>
<tr>
<th>Car</th>
<th>( S(C_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( C_1 )</td>
<td>-0.00236</td>
</tr>
<tr>
<td>( C_2 )</td>
<td>-0.00242</td>
</tr>
<tr>
<td>( C_3 )</td>
<td>-0.00915</td>
</tr>
<tr>
<td>( C_4 )</td>
<td>-0.00806</td>
</tr>
<tr>
<td>( C_5 )</td>
<td>-0.00239</td>
</tr>
<tr>
<td>( C_6 )</td>
<td>-0.00348</td>
</tr>
<tr>
<td>( C_7 )</td>
<td>-0.00115</td>
</tr>
<tr>
<td>( C_8 )</td>
<td>-0.01097</td>
</tr>
<tr>
<td>( C_9 )</td>
<td>0.00091</td>
</tr>
<tr>
<td>( C_{10} )</td>
<td>-0.0085</td>
</tr>
</tbody>
</table>

**Figure 1** Graphical representation using MCGDM based on PNSS.

**Step 5**: Since \( \max_i S(C_i) = 0.00091 \) which corresponds to \( C_9 \). Therefore in this case the most suitable car \( C_9 \) for the customer would be purchased.
Algorithm-II

**Step-1:** Construct Pythagorean neutrosophic soft matrix (PNSS matrix) on the basis of the parameters.

**Step-2:** Case-I (Equal weights) Compute the choice matrix for the positive membership, neutral membership and negative membership of PNSS matrix.

Case-II (Unequal weights) Compute the choice matrix for the positive membership, neutral membership and negative membership of PNSS matrix.

**Step-3:** Choose alternative with maximum score value.

Case-I: By Example 3.8

\[
C(A) = \begin{bmatrix}
0.0605 & 0.1125 & 0.072 \\
0.072 & 0.1125 & 0.05 \\
0.197 & 0.1585 & 0.226 \\
0.178 & 0.17 & 0.197 \\
0.0605 & 0.0845 & 0.072 \\
0.072 & 0.0845 & 0.128 \\
0.098 & 0.1125 & 0.0605 \\
0.1825 & 0.2105 & 0.196 \\
0.324 & 0.1105 & 0.226 \\
0.1345 & 0.2 & 0.1325
\end{bmatrix}
\]

Score value =

<table>
<thead>
<tr>
<th>Car</th>
<th>(S(C_i))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(C_1)</td>
<td>-0.01418</td>
</tr>
<tr>
<td>(C_2)</td>
<td>-0.00997</td>
</tr>
<tr>
<td>(C_3)</td>
<td>-0.03739</td>
</tr>
<tr>
<td>(C_4)</td>
<td>-0.03603</td>
</tr>
<tr>
<td>(C_5)</td>
<td>-0.00866</td>
</tr>
<tr>
<td>(C_6)</td>
<td>-0.01834</td>
</tr>
<tr>
<td>(C_7)</td>
<td>-0.00671</td>
</tr>
<tr>
<td>(C_8)</td>
<td>-0.04942</td>
</tr>
<tr>
<td>(C_9)</td>
<td>0.04169</td>
</tr>
<tr>
<td>(C_{10})</td>
<td>-0.03947</td>
</tr>
</tbody>
</table>

Case-II: Weights \((w_j) = \{0.16, 0.19, 0.25, 0.22, 0.18\}\).

By Example 3.8

\[
C_w(A) = \begin{bmatrix}
0.0484 & 0.09 & 0.0576 \\
0.0684 & 0.106875 & 0.0475 \\
0.200425 & 0.165275 & 0.2294 \\
0.1574 & 0.1576 & 0.19135 \\
0.057475 & 0.080275 & 0.0684 \\
0.0792 & 0.09295 & 0.1408 \\
0.0784 & 0.09 & 0.0484 \\
0.188075 & 0.21685 & 0.2009 \\
0.3078 & 0.10655 & 0.2102 \\
0.1151 & 0.1792 & 0.115075
\end{bmatrix}
\]

Score value =

<table>
<thead>
<tr>
<th>Car</th>
<th>(S(C_i))</th>
</tr>
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<tbody>
<tr>
<td>(C_1)</td>
<td>-0.00908</td>
</tr>
<tr>
<td>(C_2)</td>
<td>-0.009</td>
</tr>
<tr>
<td>(C_3)</td>
<td>-0.03831</td>
</tr>
<tr>
<td>(C_4)</td>
<td>-0.03668</td>
</tr>
<tr>
<td>(C_5)</td>
<td>-0.00782</td>
</tr>
<tr>
<td>(C_6)</td>
<td>-0.02219</td>
</tr>
<tr>
<td>(C_7)</td>
<td>-0.0043</td>
</tr>
<tr>
<td>(C_8)</td>
<td>-0.05201</td>
</tr>
<tr>
<td>(C_9)</td>
<td>0.0392</td>
</tr>
<tr>
<td>(C_{10})</td>
<td>-0.03211</td>
</tr>
</tbody>
</table>

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Algorithm-III

**Step-1:** Obtain the aggregated Pythagorean neutrosophic weighted averaging (PNSWA) numbers \( \mathcal{C}(A) = (\sum_{j=1}^{n} w_j \sigma_{ij}^{T}, \sum_{j=1}^{n} w_j \sigma_{ij}^{I}, \sum_{j=1}^{n} w_j \sigma_{ij}^{F}) \).

**Step-2:** Compute the score function of \( S(\mathcal{C}_i) \).

**Step-3:** Select the optimal alternative by \( \max_i S(\mathcal{C}_i) \) value.

Weights \( (w_j) = \{0.16, 0.19, 0.25, 0.22, 0.18\} \).

By Example 3.8:

\[
\mathcal{C}(A) = \begin{bmatrix}
0.088 & 0.12 & 0.096 \\
0.114 & 0.1425 & 0.095 \\
0.2885 & 0.2585 & 0.306 \\
0.238 & 0.244 & 0.269 \\
0.1045 & 0.1235 & 0.114 \\
0.132 & 0.143 & 0.176 \\
0.112 & 0.12 & 0.088 \\
0.2775 & 0.298 & 0.287 \\
0.342 & 0.201 & 0.282 \\
0.199 & 0.248 & 0.2005 \\
\end{bmatrix}
\]

Score value:

\[
\begin{align*}
\text{Car} & \quad S(\mathcal{C}_i) \\
\mathcal{C}_1 & \quad -0.01587 \\
\mathcal{C}_2 & \quad -0.01634 \\
\mathcal{C}_3 & \quad -0.07723 \\
\mathcal{C}_4 & \quad -0.07525 \\
\mathcal{C}_5 & \quad -0.01733 \\
\mathcal{C}_6 & \quad -0.034 \\
\mathcal{C}_7 & \quad -0.0096 \\
\mathcal{C}_8 & \quad -0.09417 \\
\mathcal{C}_9 & \quad -0.00296 \\
\mathcal{C}_{10} & \quad -0.0621
\end{align*}
\]

3.1. Analysis for PNSS-Methods:

Analysis of final ranking as follows:

<table>
<thead>
<tr>
<th>Methods</th>
<th>Ranking of alternatives</th>
<th>Optimal alternatives</th>
</tr>
</thead>
<tbody>
<tr>
<td>Algorithm - I</td>
<td>( \mathcal{C}_8 \leq \mathcal{C}_3 \leq \mathcal{C}_4 \leq \mathcal{C}_6 \leq \mathcal{C}_2 \leq \mathcal{C}_5 \leq \mathcal{C}<em>7 \leq \mathcal{C}</em>{10} \leq \mathcal{C}_9 )</td>
<td>( \mathcal{C}_9 )</td>
</tr>
<tr>
<td>Algorithm - II Case - (i)</td>
<td>( \mathcal{C}<em>8 \leq \mathcal{C}</em>{10} \leq \mathcal{C}_3 \leq \mathcal{C}_4 \leq \mathcal{C}_6 \leq \mathcal{C}_1 \leq \mathcal{C}_2 \leq \mathcal{C}_5 \leq \mathcal{C}_7 \leq \mathcal{C}_9 )</td>
<td>( \mathcal{C}_9 )</td>
</tr>
<tr>
<td>Algorithm - II Case - (ii)</td>
<td>( \mathcal{C}_8 \leq \mathcal{C}_3 \leq \mathcal{C}<em>4 \leq \mathcal{C}</em>{10} \leq \mathcal{C}_6 \leq \mathcal{C}_1 \leq \mathcal{C}_2 \leq \mathcal{C}_5 \leq \mathcal{C}_7 \leq \mathcal{C}_9 )</td>
<td>( \mathcal{C}_9 )</td>
</tr>
<tr>
<td>Algorithm - III</td>
<td>( \mathcal{C}_8 \leq \mathcal{C}_3 \leq \mathcal{C}<em>4 \leq \mathcal{C}</em>{10} \leq \mathcal{C}_6 \leq \mathcal{C}_5 \leq \mathcal{C}_2 \leq \mathcal{C}_1 \leq \mathcal{C}_7 \leq \mathcal{C}_9 )</td>
<td>( \mathcal{C}_9 )</td>
</tr>
</tbody>
</table>

Therefore most suitable car \( \mathcal{C}_9 \) for the customer would be purchased.

4. MCGDM based on PNSS-TOPSIS aggregating operator

Algorithm-IV (PNSS-TOPSIS)

**Step-1:** Assume that \( \mathcal{D} = \{ \mathcal{D}_i : i \in \mathbb{N} \} \) is a finite set of decision makers/experts, \( \mathcal{C} = \{ \mathcal{C}_i : i \in \mathbb{N} \} \) is the finite collection of alternatives and \( D = \{ e_i : i \in \mathbb{N} \} \) is a finite family of parameters/criterion.

**Step-2:** By selecting the linguistic terms and constructing weighted parameter matrix \( \mathcal{P} \) can
be written as

\[
P = [w_{ij}]_{n \times m} = \begin{bmatrix}
    w_{11} & w_{12} & \cdots & w_{1m} \\
    w_{21} & w_{22} & \cdots & w_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    w_{n1} & w_{n2} & \cdots & w_{nm}
\end{bmatrix}
\]

Where \( w_{ij} \) is the weight assigned by the expert \( D_i \) to the alternative \( P_j \) by considering linguistic variables.

**Step-3:** Construct weighted normalized decision matrix using the following

\[
\hat{N} = [\hat{n}_{ij}]_{n \times m} = \begin{bmatrix}
    \hat{n}_{11} & \hat{n}_{12} & \cdots & \hat{n}_{1m} \\
    \hat{n}_{21} & \hat{n}_{22} & \cdots & \hat{n}_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    \hat{n}_{n1} & \hat{n}_{n2} & \cdots & \hat{n}_{nm}
\end{bmatrix}
\]

where \( \hat{n}_{ij} = \frac{w_{ij}}{\sqrt{\sum_{i=1}^{n} w_{ij}^2}} \) is the normalized criteria rating and obtaining the weighted vector \( W = (m_1, m_2, \ldots, m_m) \), where \( m_i = \frac{w_i}{\sqrt{\sum_{i=1}^{n} w_{ii}}} \) is the relative weight of the \( j^{th} \) criterion and \( w_j = \frac{\sum_{i=1}^{n} \hat{n}_{ij}}{n} \).

**Step-4:** Construct PNSS decision matrix can be calculate as follows

\[
\mathcal{D}_i = [x_{ijk}]_{l \times m} = \begin{bmatrix}
    x_{i11} & x_{i12} & \cdots & x_{i1m} \\
    x_{i21} & x_{i22} & \cdots & x_{i2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{ij1} & x_{ij2} & \cdots & x_{ijm} \\
    \vdots & \vdots & \ddots & \vdots \\
    x_{il1} & x_{il2} & \cdots & x_{ilm}
\end{bmatrix}
\]

Where \( x_{ijk} \) is a PNSS element for \( i^{th} \) decision maker so that \( \mathcal{D}_i \) for each \( i \). Then obtain the aggregating matrix \( \mathcal{A} = \frac{\mathcal{D}_1 + \mathcal{D}_2 + \cdots + \mathcal{D}_n}{n} = [y_{jk}]_{l \times m} \).

**Step-5:** Find the weighted PNSS decision matrix by

\[
\mathcal{Y} = [z_{jk}]_{l \times m} = \begin{bmatrix}
    z_{11} & z_{12} & \cdots & z_{1m} \\
    z_{21} & z_{22} & \cdots & z_{2m} \\
    \vdots & \vdots & \ddots & \vdots \\
    z_{j1} & z_{j2} & \cdots & z_{jm} \\
    \vdots & \vdots & \ddots & \vdots \\
    z_{l1} & z_{l2} & \cdots & z_{lm}
\end{bmatrix}
\]
Where \( z_{jk} = m_k \times y_{jk} \).

**Step-6:** Calculate PNSSV-PIS and PNSSV-NIS. Now, 
\[
\text{PNSSV-PIS} = [z_1^+, z_2^+ , ..., z_l^+] = \{(\vee_k z_{jk}, \wedge_k z_{jk}, \vee_k z_{jk}) : k = 1, 2, ..., m\}
\] and 
\[
\text{PNSSV-NIS} = [z_1^-, z_2^-, ..., z_m^-] = \{(\wedge_k z_{jk}, \vee_k z_{jk}, \vee_k z_{jk}) : k = 1, 2, ..., m\},
\] where \( \vee \) stands for PNSS union and \( \wedge \) represents PNSS intersection.

**Step-7:** Compute PNSS-Euclidean distances of each alternative from PNSSV-PIS and PNSSV-NIS. Now, 
\[
(\text{d}_{j^+})^2 = \sum_{k=1}^{m} \left[ (\sigma_{jk}^T - \sigma_{j^+}^T)^2 + (\sigma_{jk}^I - \sigma_{j^+}^I)^2 + (\sigma_{jk}^F - \sigma_{j^+}^F)^2 \right]
\] and 
\[
(\text{d}_{j^-})^2 = \sum_{k=1}^{m} \left[ (\sigma_{jk}^T - \sigma_{j^-}^T)^2 + (\sigma_{jk}^I - \sigma_{j^-}^I)^2 + (\sigma_{jk}^F - \sigma_{j^-}^F)^2 \right],
\] where \( j = 1, 2, ..., n \).

**Step-8:** Calculate the relative closeness of each alternative to the ideal solution by 
\[
C^*(z_j) = \frac{d_{j^-}}{d_{j^+} + d_{j^-}} \in [0, 1].
\]

**Step-9:** The rank of alternatives in decreasing or increasing order of their relative closeness coefficients. The bigger \( C^*(z_j) \), the more desirable alternative \( z_j \).

**Step-10:** The best alternative is the one with the highest relative closeness to the ideal solution.

**Example 4.1.** Assume that a firm plans to invest some money in stock exchange by purchasing some shares of best five companies. In order to minimize the risk factor, they decide to invest their money 30%, 25%, 20%, 15% and 10% in accordance with the top ranked five companies.

**Step-1:** Assume that \( \mathcal{D} = \{ D_i : i = 1, 2, 3, 4, 5 \} \) is a finite set of decision makers/experts, \( \mathcal{C} = \{ z_i : i = 1, 2, ..., 10 \} \) is the collection of companies/alternatives and \( D = \{ e_i : i = 1, 2, ..., 5 \} \) is a finite family of parameters/criterion, where \( e_1 = \text{Momentum} \), \( e_2 = \text{Value} \), \( e_3 = \text{Growth} \), \( e_4 = \text{Volatility} \), \( e_5 = \text{Quality} \).

**Step-2:** Forms a Linguistic terms for judging alternatives as given below:

<table>
<thead>
<tr>
<th>Linguistic terms</th>
<th>Fuzzy weights</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Good Testing (VGT)</td>
<td>0.95</td>
</tr>
<tr>
<td>Good Testing (GT)</td>
<td>0.80</td>
</tr>
<tr>
<td>Average Testing (AT)</td>
<td>0.65</td>
</tr>
<tr>
<td>Poor Testing (PT)</td>
<td>0.50</td>
</tr>
<tr>
<td>Very Poor Testing (VPT)</td>
<td>0.35</td>
</tr>
</tbody>
</table>

Construct weighted parameter matrix
\[
\mathcal{P} = [w_{ij}]_{5 \times 5} = \begin{bmatrix}
GC & VGC & PC & VPC & AC \\
AC & GC & VPC & PC & GC \\
PC & AC & VGC & VGC & VPC \\
VGC & PC & AC & GC & PC \\
AC & VPC & VGC & GC & VPC
\end{bmatrix}
\]

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Where $w_{ij}$ is the weight provided by the specialist $P_i$ to each parameter $P_j$.

**Step-3:** The normalized weighted decision matrix is

$$\tilde{N} = [\tilde{n}_{ij}]_{5 \times 5} = \begin{bmatrix} 0.4926 & 0.6214 & 0.3101 & 0.219 & 0.5208 \\ 0.4002 & 0.5233 & 0.2171 & 0.3128 & 0.641 \\ 0.3079 & 0.4251 & 0.5892 & 0.5943 & 0.2804 \\ 0.585 & 0.327 & 0.4031 & 0.5005 & 0.4006 \\ 0.4002 & 0.2289 & 0.5892 & 0.5005 & 0.2804 \end{bmatrix}.$$  

And weighted vector is $W = (0.1231, 0.1308, 0.124, 0.1251, 0.1603)$.

**Step-4:** The aggregated decision matrix $\mathcal{A}$ can be written as

$$\mathcal{A} = \frac{P_1 + P_2 + P_3 + P_4 + P_5}{5} = \begin{bmatrix} (0.78, 0.48, 0.7) & (0.7, 0.45, 0.6) & (0.68, 0.6, 0.65) & (0.65, 0.75, 0.9) & (0.78, 0.57, 0.6) \\ (0.8, 0.7, 0.9) & (0.65, 0.75, 0.85) & (0.64, 0.66, 0.64) & (0.69, 0.8, 0.67) & (0.68, 0.81, 0.7) \\ (0.75, 0.65, 0.75) & (0.72, 0.68, 0.42) & (0.72, 0.87, 0.45) & (0.74, 0.7, 0.59) & (0.62, 0.56, 0.85) \\ (0.8, 0.95, 0.62) & (0.9, 0.8, 0.65) & (0.85, 0.8, 0.41) & (0.81, 0.8, 0.56) & (0.9, 0.69, 0.75) \\ (0.8, 0.55, 0.95) & (0.55, 0.65, 0.9) & (0.62, 0.61, 0.68) & (0.69, 0.54, 0.67) & (0.68, 0.62, 0.7) \\ (0.84, 0.83, 0.62) & (0.9, 0.8, 0.45) & (0.9, 0.43, 0.73) & (0.83, 0.49, 0.8) & (0.9, 0.68, 0.45) \\ (0.79, 0.65, 0.75) & (0.75, 0.55, 0.65) & (0.78, 0.65, 0.55) & (0.65, 0.75, 0.9) & (0.8, 0.57, 0.6) \\ (0.75, 0.7, 0.68) & (0.76, 0.7, 0.42) & (0.8, 0.43, 0.43) & (0.47, 0.8, 0.85) & (0.83, 0.5, 0.55) \\ (0.85, 0.61, 0.74) & (0.66, 0.58, 0.65) & (0.7, 0.62, 0.78) & (0.4, 0.9, 0.64) & (0.58, 0.77, 0.6) \\ (0.9, 0.55, 0.65) & (0.63, 0.62, 0.8) & (0.69, 0.72, 0.55) & (0.83, 0.6, 0.49) & (0.62, 0.49, 0.78) \end{bmatrix}$$

$$= [y_{jk}]_{10 \times 5}$$

**Step-5:** The weighted PNSS decision matrix $\mathcal{V}$ can be written as $\mathcal{V} = m_k \times y_{jk} =$

$$\begin{bmatrix} (0.0961, 0.0591, 0.0862) & (0.0916, 0.0589, 0.0785) & (0.0843, 0.0744, 0.0806) & (0.0813, 0.0938, 0.1126) & (0.125, 0.0913, 0.0962) \\ (0.0985, 0.0862, 0.1108) & (0.085, 0.0981, 0.1112) & (0.0794, 0.0819, 0.0794) & (0.0863, 0.1001, 0.0838) & (0.109, 0.1298, 0.1122) \\ (0.0924, 0.08, 0.0924) & (0.0942, 0.089, 0.0549) & (0.0893, 0.1079, 0.0558) & (0.0926, 0.0876, 0.0738) & (0.0994, 0.0897, 0.1362) \\ (0.0985, 0.117, 0.0764) & (0.1177, 0.1047, 0.085) & (0.1054, 0.0992, 0.0509) & (0.1013, 0.1001, 0.0701) & (0.1442, 0.1106, 0.1202) \\ (0.0985, 0.0677, 0.117) & (0.0719, 0.085, 0.1177) & (0.0769, 0.0757, 0.0843) & (0.0863, 0.0676, 0.0838) & (0.109, 0.0994, 0.1122) \\ (0.1034, 0.1022, 0.0764) & (0.1177, 0.1047, 0.0589) & (0.1116, 0.0533, 0.0905) & (0.1039, 0.0613, 0.1001) & (0.1442, 0.109, 0.0721) \\ (0.0973, 0.08, 0.0924) & (0.0987, 0.0719, 0.085) & (0.0967, 0.0806, 0.0682) & (0.0813, 0.0938, 0.1126) & (0.1282, 0.0913, 0.0962) \\ (0.0924, 0.0862, 0.0837) & (0.0994, 0.0916, 0.0549) & (0.0992, 0.0533, 0.0533) & (0.0588, 0.1001, 0.1064) & (0.133, 0.0801, 0.0881) \\ (0.1047, 0.0751, 0.0911) & (0.0863, 0.0759, 0.085) & (0.0886, 0.0769, 0.0967) & (0.05, 0.1126, 0.0801) & (0.0929, 0.1234, 0.0962) \\ (0.1108, 0.0677, 0.08) & (0.0824, 0.0811, 0.1047) & (0.0856, 0.0893, 0.0682) & (0.1039, 0.0751, 0.0613) & (0.0994, 0.0785, 0.125) \end{bmatrix}$$

$$= [z_{jk}]_{10 \times 5}.$$
Step-6: We find PNSSV-PIS and PNSSV-NIS can be written as

\[
\begin{align*}
z^+ & \quad \text{PNSSV} - \text{PIS} & \quad z^- & \quad \text{PNSSV} - \text{NIS} \\
1 & \quad (0.125, 0.0589, 0.0785) & 1^- & \quad (0.0813, 0.0938, 0.1126) \\
2 & \quad (0.109, 0.0819, 0.0794) & 2^- & \quad (0.0794, 0.1298, 0.1122) \\
3 & \quad (0.0994, 0.08, 0.0549) & 3^- & \quad (0.0893, 0.1079, 0.1362) \\
4 & \quad (0.1442, 0.0992, 0.0509) & 4^- & \quad (0.0985, 0.1117, 0.1202) \\
5 & \quad (0.109, 0.0676, 0.0838) & 5^- & \quad (0.0719, 0.0994, 0.1177) \\
6 & \quad (0.1442, 0.0533, 0.0589) & 6^- & \quad (0.1034, 0.109, 0.1001) \\
7 & \quad (0.1282, 0.0719, 0.0682) & 7^- & \quad (0.0813, 0.0938, 0.1126) \\
8 & \quad (0.133, 0.0533, 0.0533) & 8^- & \quad (0.0588, 0.1001, 0.1064) \\
9 & \quad (0.1047, 0.0751, 0.0801) & 9^- & \quad (0.05, 0.1234, 0.0967) \\
10 & \quad (0.1108, 0.0677, 0.0613) & 10^- & \quad (0.0824, 0.0893, 0.125) \\
\end{align*}
\]

Step-7: We found PNSS euclidean distances of each alternative from PNSSV-PIS and PNSSV-NIS.

<table>
<thead>
<tr>
<th>Alternative ( z_i )</th>
<th>( d_i^+ )</th>
<th>( d_i^- )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.0979</td>
<td>0.0906</td>
</tr>
<tr>
<td>2</td>
<td>0.0899</td>
<td>0.0964</td>
</tr>
<tr>
<td>3</td>
<td>0.0979</td>
<td>0.1446</td>
</tr>
<tr>
<td>4</td>
<td>0.1166</td>
<td>0.1174</td>
</tr>
<tr>
<td>5</td>
<td>0.0863</td>
<td>0.0859</td>
</tr>
<tr>
<td>6</td>
<td>0.1282</td>
<td>0.1025</td>
</tr>
<tr>
<td>7</td>
<td>0.0983</td>
<td>0.0851</td>
</tr>
<tr>
<td>8</td>
<td>0.1408</td>
<td>0.1381</td>
</tr>
<tr>
<td>9</td>
<td>0.0906</td>
<td>0.1217</td>
</tr>
<tr>
<td>10</td>
<td>0.0937</td>
<td>0.1101</td>
</tr>
</tbody>
</table>

Step-8: We calculate closeness coefficients of each alternative from PNSSV-PIS and PNSSV-NIS.

<table>
<thead>
<tr>
<th>Alternative ( z_i )</th>
<th>( C_i^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.4807</td>
</tr>
<tr>
<td>2</td>
<td>0.5174</td>
</tr>
<tr>
<td>3</td>
<td>0.5962</td>
</tr>
<tr>
<td>4</td>
<td>0.5017</td>
</tr>
<tr>
<td>5</td>
<td>0.4988</td>
</tr>
<tr>
<td>6</td>
<td>0.4444</td>
</tr>
<tr>
<td>7</td>
<td>0.4639</td>
</tr>
<tr>
<td>8</td>
<td>0.4951</td>
</tr>
<tr>
<td>9</td>
<td>0.5734</td>
</tr>
<tr>
<td>10</td>
<td>0.5404</td>
</tr>
</tbody>
</table>

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Step-9: The order of the alternatives for $C_i^*$ is $z_3 \geq z_9 \geq z_{10} \geq z_2 \geq z_4 \geq z_5 \geq z_8 \geq z_1 \geq z_7 \geq z_6$.

**Figure 2** Graphical representation using MCGDM based on TOPSIS.

Step-10: The above ranking, it conclude that the firm should $z_3$ invest 30%, $z_9$ invest 25%, $z_{10}$ invest 20%, $z_2$ invest 15% and $z_4$ invest 10%.

5. MCGDM based on PNSS-VIKOR aggregating operator

**Algorithm-V (PNSS-VIKOR)**

**Step-1:** Assume that $D = \{D_i : i \in \mathbb{N}\}$ is a finite set of decision makers/experts, $C = \{z_i : i \in \mathbb{N}\}$ is the finite collection of alternatives and $D = \{e_i : i \in \mathbb{N}\}$ is a finite family of parameters/criterion.

**Step-2:** By selecting the linguistic terms and constructing weighted parameter matrix $P$ can be written as

$$P = \begin{bmatrix} w_{11} & w_{12} & \cdots & w_{1m} \\ w_{21} & w_{22} & \cdots & w_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ w_{n1} & w_{n2} & \cdots & w_{nm} \end{bmatrix}$$

Where $w_{ij}$ is the weight assigned by the expert $D_i$ to the alternative $P_j$ by considering linguistic variables.

**Step-3:** Construct weighted normalized decision matrix using the following

$$\hat{N} = [\hat{n}_{ij}]_{n \times m} = \begin{bmatrix} \hat{n}_{11} & \hat{n}_{12} & \cdots & \hat{n}_{1m} \\ \hat{n}_{21} & \hat{n}_{22} & \cdots & \hat{n}_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ \hat{n}_{n1} & \hat{n}_{n2} & \cdots & \hat{n}_{nm} \end{bmatrix}$$

where $\hat{n}_{ij} = \frac{w_{ij}}{\sqrt{\sum_{i=1}^{n} w_{ij}^2}}$ is the normalized criteria rating and obtaining the weighted vector $W = (m_1, m_2, \ldots, m_m)$, where $m_i = \frac{w_i}{\sqrt{\sum_{i=1}^{n} w_i^2}}$ is the relative weight of the $j^{th}$ criterion and $w_j = \sum_{i=1}^{n} \hat{n}_{ij}$. 

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Step-4: Construct PNSS decision matrix can be calculated as

\[ D_i = [x_{jk}^i]_{l \times m} = \begin{bmatrix} x_{11}^i & x_{12}^i & \cdots & x_{1m}^i \\ x_{21}^i & x_{22}^i & \cdots & x_{2m}^i \\ \vdots & \vdots & \ddots & \vdots \\ x_{j1}^i & x_{j2}^i & \cdots & x_{jm}^i \\ \vdots & \vdots & \ddots & \vdots \\ x_{l1}^i & x_{l2}^i & \cdots & x_{lm}^i \end{bmatrix} \]

Where \( x_{jk}^i \) is a PNSS element for \( i^{th} \) decision maker so that \( D_i \) for each \( i \). Then obtain the aggregating matrix \( A = \frac{D_1 + D_2 + \ldots + D_n}{n} = [y_{jk}]_{l \times m} \).

Step-5: Construct the weighted PNSS decision matrix by

\[ Y = [z_{jk}]_{l \times m} = \begin{bmatrix} z_{11} & z_{12} & \cdots & z_{1m} \\ z_{21} & z_{22} & \cdots & z_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ z_{j1} & z_{j2} & \cdots & z_{jm} \\ \vdots & \vdots & \ddots & \vdots \\ z_{l1} & z_{l2} & \cdots & z_{lm} \end{bmatrix} \]

Where \( z_{jk} = m_k \times y_{jk} \).

Step-6: Calculate the values of PNSSV-PIS and PNSSV-NIS. Now, PNSSV-PIS = \( [z_1^+, z_2^+, \ldots, z_I^+] = \{(\vee_k z_{jk}, \wedge_k z_{jk}, \& k z_{jk}) : j = 1, 2, \ldots, I\} \) and PNSSV-PIS = \( [z_1^-, z_2^-, \ldots, z_I^-] = \{(\vee_k z_{jk}, \& k z_{jk}, \vee_k z_{jk}) : j = 1, 2, \ldots, I\} \), where \( \vee \) stands for PNSS union and \( \wedge \) represents PNSS intersection.

Step-7: Find the values of utility \( \mathcal{I}_i \), individual regret \( \mathcal{R}_i \) and compromise \( \mathcal{D}_i \), where \( \mathcal{I}_i = \sum_{j=1}^{m} m_j \left( \frac{d(z_{ij}, z_{ij}^+)}{d(z_{ij}, z_{ij}^-)} \right) \), \( \mathcal{R}_i = \max_{j=1}^{m} m_j \left( \frac{d(z_{ij}, z_{ij}^+)}{d(z_{ij}, z_{ij}^-)} \right) \) and \( \mathcal{D}_i = \kappa \left( \mathcal{I}_i - \mathcal{I}_i^- \right) + (1 - \kappa) \left( \mathcal{R}_i - \mathcal{R}_i^- \right) \).

Where \( \mathcal{I}_i^+ = \max_i \mathcal{I}_i \), \( \mathcal{I}_i^- = \min_i \mathcal{I}_i \), \( \mathcal{R}_i^+ = \max_i \mathcal{R}_i \) and \( \mathcal{R}_i^- = \min_i \mathcal{R}_i \). The real number \( \kappa \) is called a coefficient of decision mechanism. The role of \( \kappa \) is that if compromise solution is to be selected by majority if \( \kappa > 0.5 \); for consensus if \( \kappa = 0.5 \) and \( \kappa < 0.5 \) represents veto. Let \( m_j \) represents the weight of the \( j^{th} \) criteria.

Step-8: The rank of choices and derive compromise solution. Arrange \( \mathcal{I}_i \), \( \mathcal{R}_i \) and \( \mathcal{D}_i \) in increasing order to make these three ranking lists. The alternative \( z_\alpha \) will be declared compromise solution if it ranks the best in \( \mathcal{D}_i \) (having least value) and satisfies the following two requirements simultaneously:

- \([C - 1]\) acceptable: If \( z_\alpha \) and \( z_\beta \) represent top alternatives in \( \mathcal{D}_i \), then \( \mathcal{D}(z_\beta) - \mathcal{D}(z_\alpha) \geq \frac{1}{n-1} \), where \( n \) is the number of parameters.
- \([C - 2]\) acceptable: The alternative \( z_\alpha \) should be best ranked by \( \mathcal{I}_i \) and \( \mathcal{R}_i \).

If above two conditions are not met simultaneously, then there exist multiple compromise solutions:

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(i) If only condition $C - 1$ is satisfied, then both alternatives $z_{\alpha}$ and $z_{\beta}$ are called the compromise solutions:

(ii) If condition $C - 1$ is not satisfied, then the alternatives $z_{\alpha}, z_{\beta}, ..., z_{\zeta}$ are called the compromise solutions, where $z_{\zeta}$ is founded by $Q(z_{\zeta}) - Q(z_{\alpha}) \geq \frac{1}{n-1}$.

**Example 5.1.** We resolve Example 4.1 using VIKOR method. The first five steps are the same as in Example 4.1. So we start with step 6.

**Step-6:** We compute PNSSV-PIS and PNSSV-NIS are listed as follows.

<table>
<thead>
<tr>
<th>Alternative ($z$)</th>
<th>$z^+$ PNSSV - PIS</th>
<th>$z^-$ PNSSV - NIS</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1^+$</td>
<td>(0.1108, 0.0591, 0.0764)</td>
<td>$z_1^-$ (0.0924, 0.117, 0.117)</td>
</tr>
<tr>
<td>$z_2^+$</td>
<td>(0.1177, 0.0589, 0.0549)</td>
<td>$z_2^-$ (0.0719, 0.1047, 0.1177)</td>
</tr>
<tr>
<td>$z_3^+$</td>
<td>(0.1116, 0.0533, 0.0509)</td>
<td>$z_3^-$ (0.0769, 0.1079, 0.0967)</td>
</tr>
<tr>
<td>$z_4^+$</td>
<td>(0.1039, 0.0613, 0.0613)</td>
<td>$z_4^-$ (0.05, 0.1126, 0.1126)</td>
</tr>
<tr>
<td>$z_5^+$</td>
<td>(0.1442, 0.0785, 0.0721)</td>
<td>$z_5^-$ (0.0929, 0.1298, 0.1362)</td>
</tr>
</tbody>
</table>

**Step-7:** Taking $\kappa = 0.5$, we found that the values of utility $S_i$, individual regret $R_i$ and compromise $Q_i$ for each alternative $z_i$.

<table>
<thead>
<tr>
<th>Alternative ($z$)</th>
<th>$S_i$</th>
<th>$R_i$</th>
<th>$Q_i$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$z_1$</td>
<td>0.2972</td>
<td>0.0897</td>
<td>0.2208</td>
</tr>
<tr>
<td>$z_2$</td>
<td>0.457</td>
<td>0.1225</td>
<td>0.9271</td>
</tr>
<tr>
<td>$z_3$</td>
<td>0.3763</td>
<td>0.1309</td>
<td>0.7881</td>
</tr>
<tr>
<td>$z_4$</td>
<td>0.4024</td>
<td>0.0997</td>
<td>0.5843</td>
</tr>
<tr>
<td>$z_5$</td>
<td>0.4104</td>
<td>0.1189</td>
<td>0.7732</td>
</tr>
<tr>
<td>$z_6$</td>
<td>0.3065</td>
<td>0.0737</td>
<td>0.1049</td>
</tr>
<tr>
<td>$z_7$</td>
<td>0.3031</td>
<td>0.0897</td>
<td>0.2364</td>
</tr>
<tr>
<td>$z_8$</td>
<td>0.2666</td>
<td>0.1033</td>
<td>0.2591</td>
</tr>
<tr>
<td>$z_9$</td>
<td>0.4212</td>
<td>0.1196</td>
<td>0.8079</td>
</tr>
<tr>
<td>$z_{10}$</td>
<td>0.3184</td>
<td>0.1148</td>
<td>0.4958</td>
</tr>
</tbody>
</table>

**Step-8:** The rank of alternatives for $Q_i$: $z_6 \leq z_1 \leq z_7 \leq z_8 \leq z_{10} \leq z_4 \leq z_5 \leq z_3 \leq z_9 \leq z_2$. Now, $Q(z_1) - Q(z_6) = 0.1159 \geq \frac{1}{4}$. Thus, the condition $C-1$ is not satisfied. Further $Q(z_{10}) - Q(z_6) = 0.3909 \geq \frac{1}{4}$. Therefore, we decide $z_6, z_1, z_7, z_8, z_{10}$ are multiple compromise solutions. Hence the firm should invest 30% on $z_6$, 25% on $z_1$, 20% on $z_7$, 15% on $z_8$ and 10% on $z_{10}$.
6. Analysis and discussion

We used the above example to analyse the two methods in the literature. The ranking results of all ten alternatives were obtained using these two approaches. These two methods assume a scale component for each criterion. The normalisation approach is different in these two methods. The TOPSIS method utilises a vector normalisation approach and the VIKOR method utilises a linear normalisation approach. The TOPSIS method uses “$n$”- dimensional Euclidean distance that by itself could constitute some balance between total and individual contentment, but the VIKOR method uses a different way by which weight “$\kappa$” is introduced. The major difference between the two methods is in the aggregation function. We can find the ranking of values using an aggregating function. The best ranked alternative by VIKOR is closest to the ideal solution. However, the best ranked alternative by TOPSIS is the one using the ranking index, which does not mean the closest to the ideal solution. Hence, the advantage of the VIKOR method gives a compromise solution.

7. Conclusion:

In this communication, we studied various properties of PNSSS and PNSSM that occur in investment decision making. We proposed the first four algorithms, followed by MCGDM under PNSS. The last two algorithms are based on PNSS linguistic TOPSIS and VIKOR approaches using aggregation operators. Again, we interact with the PNSS aggregation operator and score function values based on some technique. Also, we made use of various sorts of statistical charts to imagine the rankings of different alternatives under consideration. We have analyzed an application of the new approach in a DM problem regarding the selection of particulars where we can see the different conclusions obtained by using different types of aggregation operators.

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References


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31. Ting Yu Chen; An Interval Valued Pythagorean Fuzzy Compromise Approach with Correlation Based Closeness Indices for Multiple Criteria Decision Analysis of Bridge Construction Methods, Complexity; 2018, pp. 1–29.


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