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Interval-valued intuitionistic neutrosophic hypersoft TOPSIS method based on correlation coefficient

A.Bobin [1*] and V.Chinnadurai [2]

Department of Mathematics, [1]IFET College of Engineering (Autonomous), Tamilnadu, India.

[2] Department of Mathematics, Annamalai University, Tamilnadu, India.; chinnaduraiau@gmail.com

*Correspondence: bobinalbert@gmail.com

Abstract. In multi-criteria decision-making problems, we may have to deal with numbers that are in interval forms, like those of membership, non-membership grades and indeterminacy grades representing unique attributes of elements. When decision-makers come across such an environment, the decisions are harder to make and the most significant factor is that we need to combine these interval numbers to generate a single real number, which can be done using aggregation operators or score functions. To overcome this hindrance, we introduce the notion of interval-valued intuitionistic neutrosophic hypersoft set. This eventually helps the decision-maker to collect the data with no misconceptions. The primary aim of this study is to establish some operational laws for interval-valued intuitionistic neutrosophic hypersoft set. Also, we present the fundamental properties of two aggregation operators, interval-valued intuitionistic neutrosophic weighted average and interval-valued intuitionistic neutrosophic weighted geometric operators. Also, we propose an algorithm for the technique of order of preference by similarity to ideal solution (TOPSIS) method based on correlation coefficients to choose a suitable employee among the alternative using Leipzig leadership model in an organization for an upcoming new project. Finally, we present a comparative study with existing similarity measures to show the effectiveness of the proposed method.

Keywords: interval-valued neutrosophic set; intuitionistic set; hypersoft set.

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1. Introduction

Zadeh introduced the concept of fuzzy set (FS) [32] and FS has been widely used in various fields. The idea of intuitionistic FS (IFS) was presented by Atanassov [3], an extension of FS. Smarandache [24] developed the notion of the neutrosophic set (NS) characterized by the values of truth, indeterminacy, and falsity grades for each element of the set. Later, Wang et al. [27], [28] proposed the concepts of single-valued NS (SVNS) and interval-valued NS with a restricted condition for the membership values to overcome the constraints faced in NS. Chinnadurai et al. [9] discussed a solution to find out unique ranking among the alternatives. Chinnadurai and Bobin [10] proposed a concept to identify the profit and gains in decision-making problems by using Prospect theory. Chinnadurai and Bobin [11] introduced the concept of single valued neutrosophic N soft set. Also, Chinnadurai and Bobin [12] established the properties of interval-valued neutrosophic N soft set. Molodtsov [16] introduced the idea of the soft set (SS) to deal with uncertainties. Smarandache [25] presented the notion of the hypersoft set (HSS) to overcome the restriction faced in SS. Saeed [22] briefed the fundamental concepts of HSS. Ihsan et al. [21] used a hypersoft expert set for the recruitment process in MCDM problems.

Selvachandran et al. [23] presented a modified TOPSIS deviation method of SVNS. Wang and Chen [29] proposed a TOPSIS method, in which they got the optimal weights of attributes using linear programming of interval-valued IFS. Wang and Wan [30] investigated group decision making with interval-valued IFS. Nabeeh et al. [18] contributed to the personnel selection process among different alternatives by combining the analytical hierarchy method with the TOPSIS. Abdel-Basset et al. [1] combined type 2 NS and TOPSIS for supplier selection. Abdel-Basset et al. [2] proposed the use of bipolar neutrosophic numbers in the TOPSIS method for selecting smart medical devices. Endalkachew Teshome Ayele et al. [4] presented a method for traffic signal control using an interval-valued neutrosophic soft set. Christianto and Smarandache [13] proposed the idea of a third-way leadership model, a blend of hard-style and soft-style leadership. Harish et al. [14] used a combination of TOPSIS and Choquet integral method in hesitant FS to solve multi-criteria decision-making (MCDM) problems. Rana Muhammad Zulqarnain et al. [54] introduced the concept of intuitionistic fuzzy HSS and used the TOPSIS method based on correlation coefficient (CC). Rana Muhammad Zulqarnain et al. [55] studied the fundamental operations of interval-valued neutrosophic HSS. Saqlain Muhammad et al. [17] defined aggregation operators on neutrosophic HSS and studied some properties.

Rahman et al. [19] extended the concept of HSS to complex FS, complex IFS, and complex NS. Zulqarnain et al. [45] developed the TOPSIS method in a fuzzy environment and used it in the medical staff recruitment process. Zulqarnain et al. [46] established the concept of generalized TOPSIS method to solve MCDM problems. Zulqarnain and Dayan [43] presented a method for choosing the best criteria by using the fuzzy TOPSIS method. Zulqarnain et al. [44] proposed an idea for predicting diabetes using TOPSIS analysis. Zulqarnain et al. [34] used the TOPSIS method based on correlation coefficient and aggregation operators under intuitionistic fuzzy hypersoft set (IFHSS) environment. Zulqarnain et al. [39] used aggregation operators in the IFHSS environment to solve MCDM problems. Zulqarnain [48] developed a new TOPSIS method based on the correlation coefficient of interval-valued intuitionistic fuzzy soft sets in MCDM problems. Zulqarnain [53] established aggregation operators under Pythagorean fuzzy soft environment to solve MCDM problems. Zulqarnain et al. [50] developed aggregation operators of Pythagorean fuzzy soft sets for selecting green supplier chain management. Zulqarnain [49] discussed an application towards green supply chain management by using Pythagorean fuzzy soft set. Zulqarnain et al. [37] developed the concept of Pythagorean fuzzy hypersoft set (PFHSS). Zulqarnain et al. [47] discussed an idea of solving MCDM problems by using the generalized neutrosophic TOPSIS method. Zulqarnain et al. [38] used the concept of PFHSS in selecting the antivirus mask during the pandemic. Zulqarnain et al. [41] presented an application for solving MCDM problems using neutrosophic hypersoft matrices.

Zulqarnain et al. [42] discussed MCDM problems using the aggregation operators in the PFHSS environment. Zulqarnain [51] discussed an integrated TOPSIS model in a neutrosophic environment. Zulqarnain [52] proposed algorithms for a generalized multi-polar neutrosophic soft set to solve medical diagnoses. Zulqarnain et al. [33] proposed the generalized aggregate operators on neutrosophic HSS (NHSS) such as extended union, extended intersection, OR-operation, AND operation, etc., and established their properties. Samad et al. [40] extended the TOPSIS method based on correlation coefficient under NHSS environment in selecting an effective hand sanitizer during the pandemic. Rahman et al. [20] developed the concept of neutrosophic parametrized hypersoft set theory to solve MCDM problems. Zulqarnain et al. [35] discussed the concepts of the decision-making approach based on correlation coefficient under interval-valued neutrosophic hypersoft set (IVNHSS). Zulqarnain et al. [36] presented the fundamental operations on IVHSS and established their properties. Smarandache [26] proposed the notion of dependence and independence between the components of the FS and NS. Chinnadurai and Bobin [7], [8] defined the concepts of simplified intuitionistic neutrosophic SS (SINSS) and interval-valued intuitionistic neutrosophic SS (IVINSS) and studied some of their properties. In SINSS and IVINSS, the membership grades of truth and falsity depend on each other such that their sum cannot exceed one and the membership grade of indeterminacy is independent with a value less than or equal to one. Hence, in SINSS and IVINSS, the sum of the membership grades cannot exceed two.

All the above mentioned fuzzy hybrid sets cannot accommodate the membership grades of truth and falsity, which depend on each other such that their sum cannot exceed one and the membership grade of indeterminacy is independent with a value less than or equal to one. Therefore, to solve this problem, in this article, we present some aggregation operators for IVINHSS. We develop an algorithm to solve the decision-making problem based on the established operators. We have presented a numerical example to ensure the practicality of the developed algorithm. The main aim of the present study is to rank the alternatives based on interval-valued intuitionistic neutrosophic hypersoft set (IVINHSS) data using aggregation operators and also making use of the TOPSIS method based on CC. To the best of our knowledge, research on IVINHSS is confined to its theory and related development and applications. Therefore, the new method proposed in this paper can examine and provide a suitable solution to the decision-makers in ranking the alternatives. We present an MCDM approach based on TOPSIS, and the effectiveness of this method is showed through the selection of a suitable employee who can lead the project successfully. To prove the efficacy of the proposed method, a comparative analysis between the proposed and existing similarity measures (SMs) is given. Thus, the IVINHSS is a robust tool to predict uncertainties when the grades are in interval form for all truth, falsity, and indeterminacy grades for all the attributes.

The manuscript comprises the following sections. Section 2 briefs on existing definitions. Section 3 introduces the concept of IVINHSS and discusses some properties of CC and weighted CC of IVINHSS. Section 4 deals with the interval-valued intuitionistic neutrosophic hypersoft weighted average operator (IVINHSWAO) and interval-valued intuitionistic neutrosophic hypersoft weighted geometric operator (IVINHSWGO). Section 5 highlights the combination of CC with the TOPSIS method. Section 6 shows the significance of the proposed method with comparative analysis. Section 7 ends with a conclusion.

2. Preliminaries

We present some of the basic definitions required for this study. Let us consider the following notations throughout this study unless otherwise specified. Let $\mathcal{V}$ be the universe and $v_i \in \mathcal{V}$, $P(\mathcal{V})$ be the power set of $\mathcal{V}$, $\mathbb{N}$ represents natural numbers, $C[0,1]$ denotes the set of all closed sub intervals of [0,1] and $\mathcal{N}^U$ represent the collection of interval-valued intuitionistic NS (IVINS) over $\mathcal{V}$.

Definition 2.1. [32] A fuzzy set (FS) is a set of the form $\mathcal{F} = \big\{(v, \mathcal{T}_{\mathcal{F}}(v)) : v \in \mathcal{V}\big\}$, where $\mathcal{T}_{\mathcal{F}} : \mathcal{V} \to [0,1]$ defines the degree of membership of the element $v \in \mathcal{V}$.

Definition 2.2. [3] An intuitionistic FS (IFS) is an object of the form $\mathcal{C} = \big\{(v, \mathcal{T}_{\mathcal{C}}(v), \mathcal{F}_{\mathcal{C}}(v)) : v \in \mathcal{V}\big\}$, where $\mathcal{T}_{\mathcal{C}} : \mathcal{V} \to [0,1]$ and $\mathcal{F}_{\mathcal{C}} : \mathcal{V} \to [0,1]$ define the degree of membership and degree of non-membership of the element $v \in \mathcal{V}$, respectively and for every $v \in \mathcal{V}$, $0 \le \mathcal{T}_{\mathcal{C}}(v) + \mathcal{F}_{\mathcal{C}}(v) \le 1$, where $\pi_{\mathcal{C}}(v) = 1 - \mathcal{T}_{\mathcal{C}}(v) - \mathcal{F}_{\mathcal{C}}(v)$ represents the degree of hesitancy.

Definition 2.3. [27] A single valued neutrosophic set (SVNS) is an object of the form $\mathfrak{N} = \big\{\big\langle v, \mathcal{T}_{\mathfrak{N}}(v), \mathcal{I}_{\mathfrak{N}}(v), \mathcal{F}_{\mathfrak{N}}(v)\big\rangle : v \in \mathcal{V}\big\}$, where $\mathcal{T}_{\mathfrak{N}} : \mathcal{V} \to [0,1]$, $\mathcal{I}_{\mathfrak{N}} : \mathcal{V} \to [0,1]$ and $\mathcal{F}_{\mathfrak{N}} : \mathcal{V} \to [0,1]$ represent the degree of truth membership, degree of indeterminacy membership and degree of falsity membership of the element $v \in \mathcal{V}$, respectively and for every $v \in \mathcal{V}$,

$0 \leq \mathcal{T}_{\mathfrak{N}}(v) + \mathcal{I}_{\mathfrak{N}}(v) + \mathcal{F}_{\mathfrak{N}}(v) \leq 3$. $\mathfrak{N}^U$ denote the set of all single valued neutrosophic subsets of $\mathcal{V}$.

Definition 2.4. [28] An interval valued neutrosophic set (IVNS) is a set of the form $\mathcal{R} = \{\langle v, [\underline{\mathcal{T}_{\mathcal{R}}}(v), \overline{\mathcal{T}_{\mathcal{R}}}(v)], [\underline{\mathcal{I}_{\mathcal{R}}}(v), \overline{\mathcal{I}_{\mathcal{R}}}(v)], [\underline{\mathcal{F}_{\mathcal{R}}}(v), \overline{\mathcal{F}_{\mathcal{R}}}(v)]\rangle : v \in \mathcal{V}\}$. IVNS can be represented as $\mathcal{R} = \{\langle v, \tilde{\mathcal{T}_{\mathcal{R}}}(v), \tilde{\mathcal{I}_{\mathcal{R}}}(v), \tilde{\mathcal{F}_{\mathcal{R}}}(v)\rangle : v \in \mathcal{V}\}$, where $\tilde{\mathcal{T}_{\mathcal{R}}} : \mathcal{V} \to C[0,1]$, $\tilde{\mathcal{I}_{\mathcal{R}}} : \mathcal{V} \to C[0,1]$ and $\tilde{\mathcal{F}_{\mathcal{R}}} : \mathcal{V} \to C[0,1]$ represent the degree of truth membership, degree of indeterminacy membership and degree of falsity membership in closed sub-intervals of the element $v \in \mathcal{V}$, respectively and for every $v \in \mathcal{V}$, $0 \leq \overline{T_{\mathcal{R}}}(v) + \overline{I_{\mathcal{R}}}(v) + \overline{F_{\mathcal{R}}}(v) \leq 3$. $\mathcal{R}^U$ denote the set of all interval valued neutrosophic subsets of $\mathcal{V}$.

Definition 2.5. [8] An IVINS in $\mathcal{V}$ is an object of the form $\Omega = \{\langle v, \alpha_{\Omega}(v), \beta_{\Omega}(v), \gamma_{\Omega}(v)\rangle\}$, where $\alpha_{\Omega}(v) : \mathcal{V} \to C[0,1]$, $\beta_{\Omega}(v) : \mathcal{V} \to C[0,1]$ and $\gamma_{\Omega}(v) : \mathcal{V} \to C[0,1]$. $\alpha_{\Omega}(v)$, $\beta_{\Omega}(v)$ and $\gamma_{\Omega}(v)$ are closed sub intervals of [0,1], representing the membership grades of truth, indeterminacy and falsity of the element $v \in \mathcal{V}$. The lower and upper ends of $\alpha_{\Omega}(v)$, $\beta_{\Omega}(v)$ and $\gamma_{\Omega}(v)$ are denoted, respectively by $\underline{\alpha}_{\Omega}(v)$, $\overline{\alpha}_{\Omega}(v)$, $\underline{\beta}_{\Omega}(v)$, $\overline{\beta}_{\Omega}(v)$, and $\underline{\gamma}_{\Omega}(v)$, $\overline{\gamma}_{\Omega}(v)$, where $0 \leq \overline{\alpha}_{\Omega}(v) + \overline{\gamma}_{\Omega}(v) \leq 1$ and $\underline{\alpha}_{\Omega}(v), \underline{\beta}_{\Omega}(v), \underline{\gamma}_{\Omega}(v) \geq 0$, $0 \leq \overline{\alpha}_{\Omega}(v) + \overline{\beta}_{\Omega}(v) + \overline{\gamma}_{\Omega}(v) \leq 2$, $\forall\ v \in \mathcal{V}$.

Definition 2.6. [16] A pair $(\Omega, \mathcal{E})$ is called a soft set (SS) over $\mathcal{V}$, if $\Omega : \mathcal{E} \to \mathcal{P}(\mathcal{V})$. Then for any $p \in \mathcal{E}$, $\Omega(p) = 1$ is equivalent to $v \in \Omega(p)$ and $\Omega(p) = 0$ is equivalent to $v \notin \Omega(p)$. Thus a SS is not a set, but a parameterized family of subsets of $\mathcal{V}$.

Definition 2.7. [25] Let $\Delta_1, \Delta_2, ..., \Delta_k$, be distinct attribute sets, whose corresponding sub-attributes are $\Delta_1 = \{\lambda_{11}, \lambda_{12}, ..., \lambda_{1f}\}, \Delta_2 = \{\lambda_{21}, \lambda_{22}, ..., \lambda_{2g}\}, ..., \Delta_k = \{\lambda_{k1}, \lambda_{k2}, ..., \lambda_{kh}\}$, where $1 \leq f \leq p$, $1 \leq g \leq q$, $1 \leq h \leq r$ and $p,\ q,\ r \in \mathbb{N}$, such that $\Delta_i \cap \Delta_j = \emptyset$, for each $i, j \in \{1, 2, ..., k\}$ and $i \neq j$. Then the Cartesian product of the distinct attribute sets $\Delta_1 \times \Delta_2 \times ... \times \Delta_k = \tilde{\Delta} = \{\lambda_{1f} \times \lambda_{2g} \times ... \times \lambda_{kh}\}$, represent a collection of multi- attributes. A pair $(\Omega, \tilde{\Delta})$ is called a hypersoft set (HSS) over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to P(\mathcal{V})$. HSS can be represented as $(\Omega, \tilde{\Delta}) = \left\{(\tilde{\lambda}, \Omega(\tilde{\lambda}))|\tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in P(\mathcal{V})\right\}$.

3. Interval-valued intuitionistic neutrosophic hypersoft set

We present the notion of interval-valued intuitionistic neutrosophic hypersoft set (IVINHSS). Also, we discuss some basic properties of correlation coefficient (CC) and weighted CC (WCC) on IVINHSS.

Definition 3.1. A pair $(\Omega, \tilde{\Delta})$ is called an IVINHSS over $\mathcal{V}$, where $\Omega : \tilde{\Delta} \to \mathcal{N}^U$. IVINHSS can be represented as $(\Omega, \tilde{\Delta}) = \left\{(\tilde{\lambda}, \Omega(\tilde{\lambda}))|\tilde{\lambda} \in \tilde{\Delta}, \Omega(\tilde{\lambda}) \in \mathcal{N}^U \in C[0,1]\right\}$, where $\Omega(\tilde{\lambda}) = \left\{\left\langle v, \alpha_{\Omega(\tilde{\lambda})}(v), \beta_{\Omega(\tilde{\lambda})}(v), \gamma_{\Omega(\tilde{\lambda})}(v)\right\rangle |v \in \mathcal{V}\right\}$, where $\alpha_{\Omega(\tilde{\lambda})}(v) : \mathcal{V} \to C[0,1]$, $\beta_{\Omega(\tilde{\lambda})}(v) : \mathcal{V} \to C[0,1]$ and $\gamma_{\Omega(\tilde{\lambda})}(v) : \mathcal{V} \to C[0,1]$. $\alpha_{\Omega(\tilde{\lambda})}(v)$, $\beta_{\Omega(\tilde{\lambda})}(v)$ and $\gamma_{\Omega(\tilde{\lambda})}(v)$ are closed sub intervals of

[0,1], representing the membership grades of truth, indeterminacy and falsity. The lower and upper ends of $\alpha_{\Omega(\tilde{\lambda})}(v)$, $\beta_{\Omega(\tilde{\lambda})}(v)$ and $\gamma_{\Omega(\tilde{\lambda})}(v)$ are denoted, respectively by $\underline{\alpha}_{\Omega(\tilde{\lambda})}(v)$, $\overline{\alpha}_{\Omega(\tilde{\lambda})}(v)$, $\underline{\beta}_{\Omega(\tilde{\lambda})}(v)$, $\overline{\beta}_{\Omega(\tilde{\lambda})}(v)$, and $\underline{\gamma}_{\Omega(\tilde{\lambda})}(v)$, $\overline{\gamma}_{\Omega(\tilde{\lambda})}(v)$, where $0 \leq \overline{\alpha}_{\Omega(\tilde{\lambda})}(v) + \overline{\gamma}_{\Omega(\tilde{\lambda})}(v) \leq 1$ and $\underline{\alpha}_{\Omega(\tilde{\lambda})}(v), \underline{\beta}_{\Omega(\tilde{\lambda})}(v), \underline{\gamma}_{\Omega(\tilde{\lambda})}(v) \geq 0$, $0 \leq \overline{\alpha}_{\Omega(\tilde{\lambda})}(v) + \overline{\beta}_{\Omega(\tilde{\lambda})}(v) + \overline{\gamma}_{\Omega(\tilde{\lambda})}(v) \leq 2$.

Example 3.2. Let $\mathcal{V} = \{v_1, v_2, v_3\}$ be a set of managers who evaluate an employee based on the Leipzig leadership model for an upcoming project. Let Δ_1, Δ_2, Δ_3 and Δ_4 be distinct attribute sets whose corresponding sub-attributes are represented as $\Delta_1 = \text{purpose} = \{\lambda_{11} = \text{achieve goals}\}$, $\Delta_2 = \text{entrepreneurial spirit} = \{\lambda_{21} = \text{quick decision}, \lambda_{22} = \text{logical decision}\}$,
$\Delta_3 = \text{responsibility} = \{\lambda_{31} = \text{inspire and motivate}, \lambda_{32} = \text{time management}\}$ and $\Delta_4 = \text{effectiveness} = \{\lambda_{41} = \text{successful accomplishment}\}$. Then $\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4$ is the distinct attribute set given by

$$\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4 = \{\lambda_{11}\} \times \{\lambda_{21}, \lambda_{22}\} \times \{\lambda_{31}, \lambda_{32}\} \times \{\lambda_{41}\}.$$
$$=\Big\{(\lambda_{11}, \lambda_{21}, \lambda_{31}, \lambda_{41}), (\lambda_{11}, \lambda_{21}, \lambda_{32}, \lambda_{41}), (\lambda_{11}, \lambda_{22}, \lambda_{31}, \lambda_{41}), (\lambda_{11}, \lambda_{22}, \lambda_{32}, \lambda_{41})\Big\}.$$
$$=\Big\{\tilde{\lambda_1}, \tilde{\lambda_2}, \tilde{\lambda_3}, \tilde{\lambda_4}\Big\}.$$

An IVINHSS $(\Omega, \tilde{\Delta})$ is a collection of subsets of $\mathcal{V}$, given by the managers for each employee based on the description in Table 1.

TABLE 1. Shows leadership skills of an employee in IVINHSS $(\Omega, \tilde{\Delta})$ form.

$\mathcal{V}$	$\tilde{\lambda_1}$	$\tilde{\lambda_2}$	$\tilde{\lambda_3}$	$\tilde{\lambda_4}$
v_1	$\langle[0.3, 0.4], [0.7, 0.8], [0.2, 0.3]\rangle$	$\langle[0.2, 0.4], [0.5, 0.6], [0.5, 0.6]\rangle$	$\langle[0.6, 0.7], [0.2, 0.1], [0.1, 0.2]\rangle$	$\langle[0.3, 0.4], [0.4, 0.5], [0.2, 0.3]\rangle$
v_2	$\langle[0.2, 0.4], [0.8, 0.9], [0.1, 0.3]\rangle$	$\langle[0.6, 0.7], [0.5, 0.6], [0.2, 0.3]\rangle$	$\langle[0.4, 0.5], [0.4, 0.6], [0.1, 0.2]\rangle$	$\langle[0.2, 0.5], [0.5, 0.6], [0.2, 0.4]\rangle$
v_3	$\langle[0.1, 0.2], [0.5, 0.7], [0.2, 0.3]\rangle$	$\langle[0.3, 0.4], [0.6, 0.7], [0.2, 0.4]\rangle$	$\langle[0.2, 0.3], [0.1, 0.3], [0.6, 0.7]\rangle$	$\langle[0.2, 0.3], [0.6, 0.8], [0.4, 0.6]\rangle$

3.1. *Correlation coefficient for IVINHSS*

Let the two IVINHSS over $\mathcal{V}$ be as given below.

$$(\Omega_1, \tilde{\Delta}_1) = \big\{(v_i, [\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i), \overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i)], [\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i), \overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i)], [\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i), \overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i)]\big\},$$
$$(\Omega_2, \tilde{\Delta}_2) = \big\{(v_i, [\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i), \overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i)], [\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i), \overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i)], [\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i), \overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i)]\big\}.$$

Definition 3.3. Let $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ be two IVINHSS. Then the interval-valued intuitionistic neutrosophic informational energies of $(\Omega_1, \tilde{\Delta}_1)$ and $(\Omega_2, \tilde{\Delta}_2)$ are represented as

$$\Phi(\Omega_1, \tilde{\Delta}_1) = \sum_{k=1}^{m} \sum_{i=1}^{n} \Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 \Big], \quad (1)$$

$$\Phi(\Omega_2,\tilde{\Delta}_2)=\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2$$
$$+(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]. \quad (2)$$

Definition 3.4. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then the correlation measure between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))$$
$$+(\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))$$
$$+(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big]. \quad (3)$$

Proposition 3.5. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then,*
(i) $\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_1,\tilde{\Delta}_1))=\Phi(\Omega_1,\tilde{\Delta}_1)$
(ii) $\mathcal{C}_{\mathcal{M}}((\Omega_2,\tilde{\Delta}_2),(\Omega_2,\tilde{\Delta}_2))=\Phi(\Omega_2,\tilde{\Delta}_2)$.

Proof. Straight forward □

Definition 3.6. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then, the CC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ is given by

$$\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}} \quad (4)$$

Proposition 3.7. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then, the following CC properties hold:*
(i) $0\le\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\le 1$;
(ii) $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=\mathcal{C}_C((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1)=(\Omega_2,\tilde{\Delta}_2)$, then $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))=1$.

Proof. (i)Obviously, $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\ge 0$. Now, we present the proof of $\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))\le 1$.

$$\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$$
$$=\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))$$
$$+(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big].$$

$$= \sum_{k=1}^{m} \Big[\Big((\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1))$$

$$+ (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) \Big)$$

$$+ \Big((\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) +$$

$$(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) \Big) + \dots$$

$$+ \Big((\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) +$$

$$(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) \Big) \Big].$$

By applying Cauchy-Schwarz inequality, we get

$$\mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))^2$$

$$\leq \sum_{k=1}^{m} \Big[\Big\{ (\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2$$

$$+ \dots + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2 \Big\} +$$

$$\Big\{ (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2$$

$$+ \dots + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2 \Big\} \Big] \times$$

$$\sum_{k=1}^{m} \Big[\Big\{ (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 +$$

$$\dots + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2$$

$$+ (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 +$$

$$\dots + (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2 \Big\} + \Big\{ (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + \dots + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2 \Big\} \Big].$$

$$\mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))^2$$

$$\leq \sum_{k=1}^{m} \sum_{i=1}^{n} \Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2$$

$$+ (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 \Big] \times \sum_{k=1}^{m} \sum_{i=1}^{n} \Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2$$

$$+ (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 \Big].$$

$\Rightarrow \mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))^2 \leq \Phi(\Omega_1, \tilde{\Delta}_1) \times \Phi(\Omega_2, \tilde{\Delta}_2).$

$\Rightarrow \mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leq \sqrt{\Phi(\Omega_1, \tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2, \tilde{\Delta}_2)}.$

$\Rightarrow \frac{\mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1, \tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2, \tilde{\Delta}_2)}} \leq 1.$

By using Definition 3.5, we get $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leq 1$.

Hence, $0 \leq \mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) \leq 1$. □

Proof. (ii) Straight forward. □

Proof. (iii) $\mathcal{C}_C((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2)) = \frac{\mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1, \tilde{\Delta}_1)} \times \sqrt{\Phi(\Omega_2, \tilde{\Delta}_2)}}$.

Since, $(\Omega_1, \tilde{\Delta}_1) = (\Omega_2, \tilde{\Delta}_2)$.

$\mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]}{\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]}\times\sqrt{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]}}.$$

$\Rightarrow \mathcal{C}_C((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1.$ □

Definition 3.8. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then, the CC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2\tilde{\Delta}_2)$ is defined as

$$\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\Big\{\Phi(\Omega_1,\tilde{\Delta}_1),\Phi(\Omega_2,\tilde{\Delta}_2)\Big\}}. \tag{5}$$

$\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$= \frac{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big]}{\max\Big\{\sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big], \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big\}}.$$

Proposition 3.9. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then, the following CC properties hold:*
(i) $0 \le \tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \le 1$;
(ii) $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \tilde{\mathcal{C}_C}((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$, *then* $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1$.

Proof. (i) Obviously, $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \ge 0$. Now, we present the proof of $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \le 1$.

$\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$

$$= \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))$$
$$+ (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i)) * (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big].$$
$$= \sum_{k=1}^{m}\Big[\Big((\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1))$$
$$+(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1)) + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1)) * (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1))\Big)$$
$$+\Big((\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2))$$
$$+(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2)) + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2)) * (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2))\Big) + ...$$
$$+\Big((\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n))$$
$$+(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n)) + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n)) * (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n))\Big)\Big].$$

By applying Cauchy-Schwarz inequality, we get

$$\mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))$$
$$\leq \Big\{ \sum_{k=1}^{m}\Big[\Big\{(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 +$$
$$(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}$$
$$+ \Big\{(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 +$$
$$(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_2))^2 + ... + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big] \times$$
$$\sum_{k=1}^{m}\Big[\Big\{(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 +$$
$$... + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}$$
$$+ \Big\{(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 +$$
$$... + (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\} + \Big\{(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_1))^2 + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_2))^2 + ... + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_n))^2\Big\}\Big]\Big\}^{\frac{1}{2}}.$$

$$\mathcal{C}_M((\Omega_1, \tilde{\Delta}_1), (\Omega_2, \tilde{\Delta}_2))$$
$$\leq \Big\{ \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2$$
$$+ (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big] \times \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2$$
$$+ (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big\}^{\frac{1}{2}}.$$
$$\leq \Big\{\Big(\max\Big\{ \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 +$$
$$(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big] \times \sum_{k=1}^{m}\sum_{i=1}^{n}\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 +$$
$$(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big\}\Big)^2\Big\}^{\frac{1}{2}}.$$

$$= \max\Bigg\{ \sum_{k=1}^{m}\sum_{i=1}^{n} \Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2$$

$$+ (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big] \times \sum_{k=1}^{m}\sum_{i=1}^{n} \Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 +$$

$$(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Bigg\}.$$

$$\Rightarrow \mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq \max\Bigg\{\Phi(\Omega_1,\tilde{\Delta}_1) \times \Phi(\Omega_2,\tilde{\Delta}_2)\Bigg\}.$$

$$\Rightarrow \frac{\mathcal{C}_M((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\Bigg\{\Phi(\Omega_1,\tilde{\Delta}_1)\times\Phi(\Omega_2,\tilde{\Delta}_2)\Bigg\}} \leq 1.$$

By using Definition 3.8, we get $\tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$.

Hence, $0 \leq \tilde{\mathcal{C}_C}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$.

Proofs of (ii) and (iii) are same as in Proposition 3.6. □

3.2. *Weighted correlation coefficient for IVINHSS*

We present the concept of weighted correlation coefficient (WCC) for IVINHSS. WCC facilitates decision-makers (DMs) to provide different weights for each alternative. Consider $\mathcal{D} = \{\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_m\}$ and $\mathcal{W} = \{\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_n\}$ as weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m}\mathcal{D}_k = 1, \sum_{i=1}^{n}\mathcal{W}_i = 1$.

Definition 3.10. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then, the WCC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\sqrt{\Phi(\Omega_1,\tilde{\Delta}_1)}\sqrt{\Phi(\Omega_2,\tilde{\Delta}_2)}} \tag{6}$$

$$\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$$

$$= \frac{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i) + \underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i) + \underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i) + \overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i) + \overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i) + \overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i)*\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i)\Big]\Big)}{\sqrt{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\Big)} \times \sqrt{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2 + (\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big)}}.$$

If $\mathcal{D} = \left\{\frac{1}{m}, \frac{1}{m}, ..., \frac{1}{m},\right\}$ and $\mathcal{W} = \left\{\frac{1}{n}, \frac{1}{n}, ..., \frac{1}{n},\right\}$, then WCC given in Eq.(6) reduces to CC as in Eq.(4).

Proposition 3.11. *Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then, the following WCC properties hold:*

(i) $0 \leq \mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \leq 1$;

(ii) $\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \mathcal{C}_{C_{\mathcal{W}}}((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$, *then* $\mathcal{C}_{C_{\mathcal{W}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1$.

Proof. Similar to Proposition 3.6. □

Definition 3.12. Let $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2,\tilde{\Delta}_2)$ be two IVINHSS. Then, the WCC between $(\Omega_1,\tilde{\Delta}_1)$ and $(\Omega_2\tilde{\Delta}_2)$ is defined as

$$\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \frac{\mathcal{C}_{\mathcal{M}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))}{\max\left\{\Phi(\Omega_1,\tilde{\Delta}_1),\Phi(\Omega_2,\tilde{\Delta}_2)\right\}}. \quad (7)$$

$$\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2))$$
$$= \frac{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i)) + (\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))+(\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))*(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))\Big]\Big)}{\max\Big\{\sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\underline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\overline{\alpha}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_1(\tilde{\lambda}_k)}(v_i))^2\Big]\Big), \sum_{k=1}^{m}\mathcal{D}_k\Big(\sum_{i=1}^{n}\mathcal{W}_i\Big[(\underline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\underline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\alpha}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\beta}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2+(\overline{\gamma}_{\Omega_2(\tilde{\lambda}_k)}(v_i))^2\Big]\Big)\Big\}}.$$

If $\mathcal{D} = \left\{\frac{1}{m},\frac{1}{m},...,\frac{1}{m},\right\}$ and $\mathcal{W} = \left\{\frac{1}{n},\frac{1}{n},...,\frac{1}{n},\right\}$, then WCC given in Eq.(7) reduces to CC as in Eq.(5).

Proposition 3.13. *Let* $(\Omega_1,\tilde{\Delta}_1)$ *and* $(\Omega_2,\tilde{\Delta}_2)$ *be two IVINHSS. Then, the following WCC properties hold:*
(i) $0 \le \mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) \le 1$;
(ii) $\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = \mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_2,\tilde{\Delta}_2),(\Omega_1,\tilde{\Delta}_1))$;
(iii) If $(\Omega_1,\tilde{\Delta}_1) = (\Omega_2,\tilde{\Delta}_2)$, *then* $\mathcal{C}_{\tilde{C_{\mathcal{W}}}}((\Omega_1,\tilde{\Delta}_1),(\Omega_2,\tilde{\Delta}_2)) = 1$.

Proof. Similiar to Proposition 3.6. □

4. Aggregation operators for IVINHSS

We now present the concept of interval-valued intuitionistic neutrosophic hypersoft weighted average operator(IVINHSWAO) and interval-valued intuitionistic neutrosophic hypersoft weighted geometric operator (IVINHSWGO) by using operational laws. Let κ represent the collection of interval-valued intuitionistic neutrosophic hypersoft numbers (IVINHSNs).

4.1. *Operational laws for IVINHSS*

Definition 4.1.

Let $\Omega_{e_{11}} = \langle[\underline{\alpha}_{11}, \overline{\alpha}_{11}], [\underline{\beta}_{11}, \overline{\beta}_{11}], [\underline{\gamma}_{11}, \overline{\gamma}_{11}]\rangle$ and $\Omega_{e_{12}} = \langle[\underline{\alpha}_{12}, \overline{\alpha}_{12}], [\underline{\beta}_{12}, \overline{\beta}_{12}], [\underline{\gamma}_{12}, \overline{\gamma}_{12}]\rangle$ be two IVINHSS and δ a positive integer. Then,

(i) $\Omega_{e_{11}} \oplus \Omega_{e_{12}} = \langle[\underline{\alpha}_{11} + \underline{\alpha}_{12} - \underline{\alpha}_{11}\underline{\alpha}_{12}, \overline{\alpha}_{11} + \overline{\alpha}_{12} - \overline{\alpha}_{11}\overline{\alpha}_{12}], [\underline{\beta}_{11} + \underline{\beta}_{12} - \underline{\beta}_{11}\underline{\beta}_{12}, \overline{\beta}_{11} + \overline{\beta}_{12} - \overline{\beta}_{11}\overline{\beta}_{12}], [\underline{\gamma}_{11}\underline{\gamma}_{12}, \overline{\gamma}_{11}\overline{\gamma}_{12}]\rangle$;

(ii) $\Omega_{e_{11}} \otimes \Omega_{e_{12}} = \langle[\underline{\alpha}_{11}\underline{\alpha}_{12}, \overline{\alpha}_{11}\overline{\alpha}_{12}], [\underline{\beta}_{11}\underline{\beta}_{12}, \overline{\beta}_{11}\overline{\beta}_{12}], [\underline{\gamma}_{11} + \underline{\gamma}_{12} - \underline{\gamma}_{11}\underline{\gamma}_{12}, \overline{\gamma}_{11} + \overline{\gamma}_{12} - \overline{\gamma}_{11}\overline{\gamma}_{12}]\rangle$;

(iii) $\delta\Omega_{e_{11}} = \langle[(1-(1-\underline{\alpha}_{11})^{\delta}, (1-(1-\overline{\alpha}_{11})^{\delta}], [(1-(1-\underline{\beta}_{11})^{\delta}, (1-(1-\overline{\beta}_{11})^{\delta}], [(\underline{\gamma}_{11})^{\delta}, (\overline{\gamma}_{11})^{\delta}]\rangle$;

(iv) $(\Omega_{e_{11}})^{\delta} = \langle[(\underline{\alpha}_{11})^{\delta}, (\overline{\alpha}_{11})^{\delta}], [(\underline{\beta}_{11})^{\delta}, (\overline{\beta}_{11})^{\delta}], [(1-(1-\underline{\gamma}_{11})^{\delta}, (1-(1-\overline{\gamma}_{11})^{\delta}]\rangle$.

4.2. *Interval-valued intuitionistic neutrosophic hypersoft weighted average operator*

Definition 4.2. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1, \sum_{i=1}^{n} \mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = \langle[\underline{\alpha}_{ik}, \overline{\alpha}_{ik}], [\underline{\beta}_{ik}, \overline{\beta}_{ik}], [\underline{\gamma}_{ik}, \overline{\gamma}_{ik}]\rangle$ be an IVINHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, $\mathcal{A} : \kappa^n \to \kappa$, IVINHSWAO is represented as

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \bigoplus_{k=1}^{m} \mathcal{D}_k \left(\bigoplus_{i=1}^{n} \mathcal{W}_i \Omega_{e_{ik}} \right).$$

Theorem 4.3. *Let $\Omega_{e_{ik}} = \langle[\underline{\alpha}_{ik}, \overline{\alpha}_{ik}], [\underline{\beta}_{ik}, \overline{\beta}_{ik}], [\underline{\gamma}_{ik}, \overline{\gamma}_{ik}]\rangle$ be an IVINHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, the aggregated value of IVINHSWAO is also an IVINHSN, which is given by*

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \Big\langle \Big[1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{n} \Big(1 - \underline{\alpha}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{n} \Big(1 - \overline{\alpha}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{n} \Big(1 - \underline{\beta}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k},$$
$$1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{n} \Big(1 - \overline{\beta}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{m} \Big(\prod_{i=1}^{n} \Big(\underline{\gamma}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{m} \Big(\prod_{i=1}^{n} \Big(\overline{\gamma}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \Big] \Big\rangle.$$

Proof. If $n = 1$, then $\mathcal{W}_1 = 1$. By using Definition 4.1, we get

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{1m}}) = \bigoplus_{k=1}^{m} \mathcal{D}_k \Omega_{e_{1k}}.$$
$$= \Big\langle \Big[1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{1} \Big(1 - \underline{\alpha}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{1} \Big(1 - \overline{\alpha}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{1} \Big(1 - \underline{\beta}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k},$$
$$1 - \prod_{k=1}^{m} \Big(\prod_{i=1}^{1} \Big(1 - \overline{\beta}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{m} \Big(\prod_{i=1}^{1} \Big(\underline{\gamma}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{m} \Big(\prod_{i=1}^{1} \Big(\overline{\gamma}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big] \Big\rangle.$$

If $m = 1$, then $\mathcal{D}_1 = 1$. By using Definition 4.2, we get

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{21}}, ..., \Omega_{e_{n1}}) = \bigoplus_{i=1}^{n} \mathcal{W}_i \Omega_{e_{i1}}.$$
$$= \Big\langle \Big[1 - \prod_{k=1}^{1} \Big(\prod_{i=1}^{n} \Big(1 - \underline{\alpha}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{1} \Big(\prod_{i=1}^{n} \Big(1 - \overline{\alpha}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[1 - \prod_{k=1}^{1} \Big(\prod_{i=1}^{n} \Big(1 - \underline{\beta}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k},$$
$$1 - \prod_{k=1}^{1} \Big(\prod_{i=1}^{n} \Big(1 - \overline{\beta}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{1} \Big(\prod_{i=1}^{n} \Big(\underline{\gamma}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{1} \Big(\prod_{i=1}^{n} \Big(\overline{\gamma}_{ik}\Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big] \Big\rangle.$$

Hence, the results hold for $n = 1$ and $m = 1$.

Now, if $m = l_1 + 1$ and $n = l_2$, then,

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{l_2(l_1+1)}}) = \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \bigg(\bigoplus_{i=1}^{l_2} \mathcal{W}_i \Omega_{e_{ik}} \bigg).$$

$$= \Big\langle \Big[1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \underline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \overline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \underline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \overline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(\underline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(\overline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big] \Big\rangle.$$

Similarly, if $m = l_1$, $n = l_2 + 1$, then,

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{(l_2+1)l_1}}) = \bigoplus_{k=1}^{l_1} \mathcal{D}_k \bigg(\bigoplus_{i=1}^{l_2+1} \mathcal{W}_i \Omega_{e_{ik}} \bigg).$$

$$= \Big\langle \Big[1 - \prod_{k=1}^{l_1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \underline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \overline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[1 - \prod_{k=1}^{l_1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \underline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \overline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{l_1} \Big(\prod_{i=1}^{l_2+1} \Big(\underline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1} \Big(\prod_{i=1}^{l_2+1} \Big(\overline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big] \Big\rangle.$$

Now, if $m = l_1 + 1$, $n = l_2 + 1$, then,

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{(l_2+1)(l_1+1)}})$$
$$= \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \Big(\bigoplus_{i=1}^{l_2+1} \mathcal{W}_i \Omega_{e_{ik}} \Big).$$
$$= \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \Big(\bigoplus_{i=1}^{l_2} \mathcal{W}_i \Omega_{e_{ik}} \Big) \bigoplus_{k=1}^{l_1+1} \mathcal{D}_k \Big(\mathcal{W}_{l_2+1} \Omega_{e_{(l_2+1)k}} \Big).$$

$$\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{(l_2+1)(l_1+1)}})$$
$$= \Big\langle \Big[1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \underline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \overline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big] \oplus \Big[1 - \prod_{k=1}^{l_1+1} \Big(\Big(1 - \underline{\alpha}_{(l_2+1)k} \Big)^{\mathcal{W}_{(l_2+1)}} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\Big(1 - \overline{\alpha}_{(l_2+1)k} \Big)^{\mathcal{W}_{(l_2+1)}} \Big)^{\mathcal{D}_k} \Big], \Big[1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \underline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(1 - \overline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big]$$
$$\oplus \Big[1 - \prod_{k=1}^{l_1+1} \Big(\Big(1 - \underline{\beta}_{(l_2+1)k} \Big)^{\mathcal{W}_{(l_2+1)}} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\Big(1 - \overline{\beta}_{(l_2+1)k} \Big)^{\mathcal{W}_{(l_2+1)}} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(\underline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2} \Big(\overline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \Big] \oplus \Big[\prod_{k=1}^{l_1+1} \Big(\Big(\underline{\gamma}_{(l_2+1)k} \Big)^{\mathcal{W}_{(l_2+1)}} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1} \Big(\Big(\overline{\gamma}_{(l_2+1)k} \Big)^{\mathcal{W}_{(l_2+1)}} \Big)^{\mathcal{D}_k} \Big\rangle.$$
$$= \Big\langle \Big[1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \underline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \overline{\alpha}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \underline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2+1} \Big(1 - \overline{\beta}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2+1} \Big(\underline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{l_1+1} \Big(\prod_{i=1}^{l_2+1} \Big(\overline{\gamma}_{ik} \Big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big] \Big\rangle.$$

Hence, the results hold for $n = l_2 + 1$ and $m = l_1 + 1$.

Therefore, by induction method, the result is true $\forall\ m, n \geq 1$.

Since

$0 \leq \overline{\alpha}_{ik} + \overline{\gamma}_{ik} \leq 1$ and $0 \leq \overline{\beta}_{ik} \leq 1$.

$\Leftrightarrow 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\underline{\gamma}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} +$
$\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\overline{\gamma}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leq 1$ and $1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leq 1.$

$\Leftrightarrow 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} +$
$\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leq 1$ and $1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leq 1.$

$\Leftrightarrow 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} +$
$\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} + 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k} \leq 2.$

Therefore, the aggregated value given by IVINHSWAO is also an IVINHSN. □

Example 4.4. Let us consider the same values mentioned in Example 3.2. Also, let $\mathcal{W}_i = \{0.25, 0.35, 0.40\}$ and $\mathcal{D}_k = \{0.30, 0.20, 0.40, 0.10\}$ be the weight of managers and attributes, respectively. Then,

$$\begin{aligned}\mathcal{A}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{34}}) &= \Big\langle \Big[1 - \prod_{k=1}^{4}\Big(\prod_{i=1}^{3}\left(1-\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \Big[1 - \prod_{k=1}^{4}\Big(\prod_{i=1}^{3}\left(1-\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big], \Big[1 - \prod_{k=1}^{4}\Big(\prod_{i=1}^{3}\left(1-\underline{\beta}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \\ &\Big[1 - \prod_{k=1}^{4}\Big(\prod_{i=1}^{3}\left(1-\overline{\beta}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big], \Big[\prod_{k=1}^{4}\Big(\prod_{i=1}^{3}\left(\underline{\gamma}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \Big[\prod_{k=1}^{4}\Big(\prod_{i=1}^{3}\left(\overline{\gamma}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big]\Big\rangle. \\ &= \langle [0.32, 0.45], [0.49, 0.64], [0.20, 0.34] \rangle.\end{aligned}$$

4.3. *Interval-valued intuitionistic neutrosophic hypersoft weighted geometric operator*

Definition 4.5. Let $\mathcal{D}_k$ and $\mathcal{W}_i$ be weight vectors for alternatives and experts, respectively, such that $\mathcal{D}_k, \mathcal{W}_i > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1, \sum_{i=1}^{n} \mathcal{W}_i = 1$ and $\Omega_{e_{ik}} = (\alpha_{ik}, \beta_{ik}, \gamma_{ik})$ be an IVINHSN, where $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. Then, $\mathcal{G} : \kappa^n \to \kappa$, IVINHSWGO is defined as

$$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) = \bigotimes_{k=1}^{m}\left(\bigotimes_{i=1}^{n}\left(\Omega_{e_{ik}}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}.$$

Theorem 4.6. *Let* $\Omega_{e_{ik}} = \left\langle [\underline{\alpha}_{ik}, \overline{\alpha}_{ik}], [\underline{\beta}_{ik}, \overline{\beta}_{ik}], [\underline{\gamma}_{ik}, \overline{\gamma}_{ik}] \right\rangle$ *be an IVINHSN, where* $i = \{1, 2, ...n\}$, $k = \{1, 2, ...m\}$. *Then, the aggregated value of IVINHSWGO is also an IVINHSN, which is given by*

$$\begin{aligned}\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{nm}}) &= \Big\langle \Big[\prod_{k=1}^{m}\Big(\prod_{i=1}^{n}\left(\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \prod_{k=1}^{m}\Big(\prod_{i=1}^{n}\left(\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \Big], \Big[\prod_{k=1}^{m}\Big(\prod_{i=1}^{n}\left(\underline{\beta}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \prod_{k=1}^{m}\Big(\prod_{i=1}^{n}\left(\overline{\beta}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, \Big], \\ &\Big[1 - \prod_{k=1}^{m}\Big(\prod_{i=1}^{n}\left(1-\underline{\gamma}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m}\Big(\prod_{i=1}^{n}\left(1-\overline{\gamma}_{ik}\right)^{\mathcal{W}_i}\Big)^{\mathcal{D}_k}\Big]\Big\rangle.\end{aligned}$$

Proof. Similar to Theorem 4.3. □

Example 4.7. Let us consider the same values mentioned in Example 3.2 and the weight of managers and attributes be as in Example 4.4. Then,

$$\mathcal{G}(\Omega_{e_{11}}, \Omega_{e_{12}}, ..., \Omega_{e_{34}})$$
$$= \Big\langle \Big[\prod_{k=1}^{4} \Big(\prod_{i=1}^{3} \big(\underline{\alpha}_{ik} \big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{4} \Big(\prod_{i=1}^{3} \big(\overline{\alpha}_{ik} \big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big], \Big[\prod_{k=1}^{4} \Big(\prod_{i=1}^{3} \big(\underline{\beta}_{ik} \big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, \prod_{k=1}^{4} \Big(\prod_{i=1}^{3} \big(\overline{\beta}_{ik} \big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big],$$
$$\Big[1 - \prod_{k=1}^{4} \Big(\prod_{i=1}^{3} \big(1 - \underline{\gamma}_{ik} \big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{4} \Big(\prod_{i=1}^{3} \big(1 - \overline{\gamma}_{ik} \big)^{\mathcal{W}_i} \Big)^{\mathcal{D}_k} \Big] \Big\rangle.$$
$$= \langle [0.26, 0.40], [0.37, 0.50], [0.28, 0.41] \rangle .$$

5. MCDM problems based on TOPSIS and CC method

TOPSIS method helps to find the best alternative based on minimum and maximum distance from the interval-valued intuitionistic neutrosophic positive ideal solution (IVINPIS) and interval-valued intuitionistic neutrosophic negative ideal solution (IVINNIS). Also, when TOPSIS method is combined with CC instead of similarity measures, it provides reliable results for predicting the closeness coefficients. We present an algorithm and a case study to illustrate the IVINHSS TOPSIS method based on CC.

5.1. *Algorithm to solve MCDM problems with IVINHSS data based on TOPSIS and CC method*

Let $\mathcal{A} = \left\{\mathcal{A}^1, \mathcal{A}^2, ..., \mathcal{A}^x\right\}$ be a set of selected employees and $\mathcal{V} = \{v_1, v_2, ..., v_n\}$ be a set of managers responsible to evaluate the employees with weights $\mathcal{W}_i = (\mathcal{W}_1, \mathcal{W}_2, ..., \mathcal{W}_n)$, such that $\mathcal{W}_i > 0$ and $\sum_{i=1}^{n} \mathcal{W}_i = 1$. Let $\tilde{\Delta} = \left\{\tilde{\lambda}_1, \tilde{\lambda}_2, ..., \tilde{\lambda}_m\right\}$ be a set of multi-valued sub-attributes with weights $\mathcal{D}_k = (\mathcal{D}_1, \mathcal{D}_2, ..., \mathcal{D}_m)$, such that $\mathcal{D}_k > 0$ and $\sum_{k=1}^{m} \mathcal{D}_k = 1$. The evaluation of employees $\mathcal{A}^t$, $(t = 1, 2, ..., x)$ performed by the managers v_i, $(i = 1, 2, ..., n)$ based on the multi-valued sub-attributes $\tilde{\lambda}_k$, $(k = 1, 2, ..., m)$ are given in IVINHSS form and represented as $\Omega^t_{ik} = \left\langle [\underline{\alpha}_{ik}, \overline{\alpha}_{ik}], [\underline{\beta}_{ik}, \overline{\beta}_{ik}], [\underline{\gamma}_{ik}, \overline{\gamma}_{ik}] \right\rangle$, such that $0 \leq \overline{\alpha}^t_{ik} + \overline{\gamma}^t_{ik} \leq 1$ and $0 \leq \overline{\alpha}^t_{ik} + \overline{\beta}^t_{ik} + \overline{\gamma}^t_{ik} \leq 2$ $\forall$ i, k. The managing experts aid to accommodate the multi-sub attributes values in IVINHSS form.

Step 1. Construct the matrix for each multi-valued sub-attributes in IVINHSS form as below:

$[\mathcal{A}^t, \tilde{\Delta}]_{n\times m} = [\mathcal{A}^t]_{n\times m}$

$$= \begin{matrix} \\ v_1 \\ v_2 \\ \vdots \\ v_n \end{matrix} \begin{matrix} \tilde{\lambda}_1 & \tilde{\lambda}_2 & \cdots & \tilde{\lambda}_m \\ \left[\begin{matrix} \left\langle [\underline{\alpha}^t_{11}, \overline{\alpha}^t_{11}], [\underline{\beta}^t_{11}, \overline{\beta}^t_{11}], [\underline{\gamma}^t_{11}, \overline{\gamma}^t_{11}] \right\rangle & \left\langle [\underline{\alpha}^t_{12}, \overline{\alpha}^t_{12}], [\underline{\beta}^t_{12}, \overline{\beta}^t_{12}], [\underline{\gamma}^t_{12}, \overline{\gamma}^t_{12}] \right\rangle & \cdots & \left\langle [\underline{\alpha}^t_{1m}, \overline{\alpha}^t_{1m}], [\underline{\beta}^t_{1m}, \overline{\beta}^t_{1m}], [\underline{\gamma}^t_{1m}, \overline{\gamma}^t_{1m}] \right\rangle \\ \left\langle [\underline{\alpha}^t_{21}, \overline{\alpha}^t_{21}], [\underline{\beta}^t_{21}, \overline{\beta}^t_{21}], [\underline{\gamma}^t_{21}, \overline{\gamma}^t_{21}] \right\rangle & \left\langle [\underline{\alpha}^t_{22}, \overline{\alpha}^t_{22}], [\underline{\beta}^t_{22}, \overline{\beta}^t_{22}], [\underline{\gamma}^t_{22}, \overline{\gamma}^t_{22}] \right\rangle & \cdots & \left\langle [\underline{\alpha}^t_{2m}, \overline{\alpha}^t_{2m}], [\underline{\beta}^t_{2m}, \overline{\beta}^t_{2m}], [\underline{\gamma}^t_{2m}, \overline{\gamma}^t_{2m}] \right\rangle \\ \vdots & \vdots & \ddots & \vdots \\ \left\langle [\underline{\alpha}^t_{n1}, \overline{\alpha}^t_{n1}], [\underline{\beta}^t_{n1}, \overline{\beta}^t_{n1}], [\underline{\gamma}^t_{n1}, \overline{\gamma}^t_{n1}] \right\rangle & \left\langle [\underline{\alpha}^t_{n2}, \overline{\alpha}^t_{n2}], [\underline{\beta}^t_{n2}, \overline{\beta}^t_{n2}], [\underline{\gamma}^t_{n2}, \overline{\gamma}^t_{n2}] \right\rangle & \cdots & \left\langle [\underline{\alpha}^t_{nm}, \overline{\alpha}^t_{nm}], [\underline{\beta}^t_{nm}, \overline{\beta}^t_{nm}], [\underline{\gamma}^t_{nm}, \overline{\gamma}^t_{nm}] \right\rangle \end{matrix}\right] \end{matrix}$$

Step 2. Obtain the weighted decision matrix for each multi-valued sub-attributes,

$[\tilde{A}^t_{ik}]_{n\times m}$

$$= \left\langle \left[1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\alpha}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right], \left[1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\underline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, 1 - \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(1-\overline{\beta}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right], \left[\prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\underline{\gamma}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}, \prod_{k=1}^{m}\left(\prod_{i=1}^{n}\left(\overline{\gamma}_{ik}\right)^{\mathcal{W}_i}\right)^{\mathcal{D}_k}\right]\right\rangle$$

$$= \left\langle \left[\tilde{\underline{\alpha}}_{ik}, \tilde{\overline{\alpha}}_{ik}\right], \left[\tilde{\underline{\beta}}_{ik}, \tilde{\overline{\beta}}_{ik}\right], \left[\tilde{\underline{\gamma}}_{ik}, \tilde{\overline{\gamma}}_{ik}\right]\right\rangle.$$

Step 3. Determine the IVINPIS and IVINNIS for weighted IVINHSS as below:

$$\tilde{\mathcal{A}}^{+} = \left\langle \left[\tilde{\underline{\alpha}}^{+}, \tilde{\overline{\alpha}}^{+}\right], \left[\tilde{\underline{\beta}}^{+}, \tilde{\overline{\beta}}^{+}\right], \left[\tilde{\underline{\gamma}}^{+}, \tilde{\overline{\gamma}}^{+}\right]\right\rangle_{n\times m} = \left\langle \left[\tilde{\underline{\alpha}}^{(\vee_{ij})}, \tilde{\overline{\alpha}}^{(\vee_{ij})}\right], \left[\tilde{\underline{\beta}}^{(\wedge_{ij})}, \tilde{\overline{\beta}}^{(\wedge_{ij})}\right], \left[\tilde{\underline{\gamma}}^{(\wedge_{ij})}, \tilde{\overline{\gamma}}^{(\wedge_{ij})}\right]\right\rangle,$$

$$\tilde{\mathcal{A}}^{-} = \left\langle \left[\tilde{\underline{\alpha}}^{-}, \tilde{\overline{\alpha}}^{-}\right], \left[\tilde{\underline{\beta}}^{-}, \tilde{\overline{\beta}}^{-}\right], \left[\tilde{\underline{\gamma}}^{-}, \tilde{\overline{\gamma}}^{-}\right]\right\rangle_{n\times m} = \left\langle \left[\tilde{\underline{\alpha}}^{(\wedge_{ij})}, \tilde{\overline{\alpha}}^{(\wedge_{ij})}\right], \left[\tilde{\underline{\beta}}^{(\wedge_{ij})}, \tilde{\overline{\beta}}^{(\wedge_{ij})}\right], \left[\tilde{\underline{\gamma}}^{(\vee_{ij})}, \tilde{\overline{\gamma}}^{(\vee_{ij})}\right]\right\rangle,$$

where $\vee_{ij} = \arg\max_t \left\{\varphi^t_{ij}\right\}$ and $\wedge_{ij} = \arg\min_t \left\{\varphi^t_{ij}\right\}$.

Step 4. Determine the CC for each alternative from IVINPIS and IVINNIS.

$$\chi^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^+) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^+)}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{(\Phi\tilde{A}^+)}} \text{ and}$$

$$\lambda^t = \mathcal{C}_C(\tilde{A}^t, \tilde{A}^-) = \frac{\mathcal{C}_{\mathcal{M}}(\tilde{A}^t, \tilde{A}^-)}{\sqrt{\Phi(\tilde{A}^t)} * \sqrt{\Phi(\tilde{A}^-)}}$$

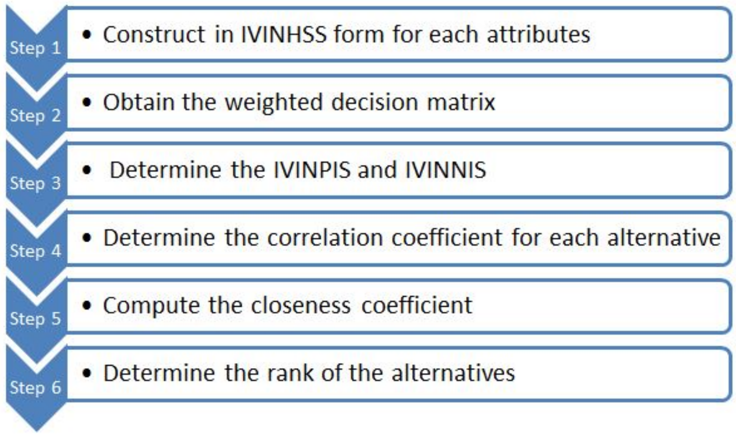
Step 5. Compute the closeness coefficient of neutrosophic ideal solution as below:

$$\epsilon^t = \frac{1-\lambda^t}{2-\chi^t-\lambda^t}$$

Step 6. Arrange the ϵ^t values in descending order and determine the rank of the alternatives $\mathcal{A}^t$, $(t = 1, 2, ..., x)$. The one with the maximum value is the suitable employee to lead the new project.

The graphical representation of the proposed method is given in Figure 1:

FIGURE 1. Flowchart of the proposed method



5.2. *Application based on TOPSIS and CC method*

Let $\mathcal{A} = \{\mathcal{A}^1, \mathcal{A}^2, \mathcal{A}^3, \mathcal{A}^4\}$ be a set of employees and let $\mathcal{V} = \{v_1, v_2, v_3\}$ be a set of managers who evaluate the employees based on the Leipzig leadership model for an upcoming project with weights $\mathcal{W}_i = (0.35, 0.15, 0.30, 0.20)$. Let Δ_1, Δ_2, Δ_3 and Δ_4 be distinct attribute sets whose corresponding sub-attributes are represented as Δ_1 = purpose = $\{\lambda_{11}$ = achieve goals$\}$, Δ_2 = entrepreneurial spirit = $\{\lambda_{21}$ = quick decision, λ_{22} = logical decision$\}$, Δ_3 = responsibility = $\{\lambda_{31}$ = inspire and motivate, λ_{32} = time management$\}$ and Δ_4 = effectiveness = $\{\lambda_{41}$ = successful accomplishment$\}$. Then $\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4$ is the distinct attribute set given by

$$\tilde{\Delta} = \Delta_1 \times \Delta_2 \times \Delta_3 \times \Delta_4 = \{\lambda_{11}\} \times \{\lambda_{21}, \lambda_{22}\} \times \{\lambda_{31}, \lambda_{32}\} \times \{\lambda_{41}\}.$$

$$= \Big\{(\lambda_{11}, \lambda_{21}, \lambda_{31}, \lambda_{41}), (\lambda_{11}, \lambda_{21}, \lambda_{32}, \lambda_{41}), (\lambda_{11}, \lambda_{22}, \lambda_{31}, \lambda_{41}), (\lambda_{11}, \lambda_{22}, \lambda_{32}, \lambda_{41})\Big\}.$$

$$= \Big\{\tilde{\lambda_1}, \tilde{\lambda_2}, \tilde{\lambda_3}, \tilde{\lambda_4}\Big\} \text{ with weights } \mathcal{D}_k = (0.20, 0.25, 0.30, 0.25).$$

This study aims to find an employee who can successfully lead the project.

Step 1. Construct $\mathcal{A}^1$, $\mathcal{A}^2$, $\mathcal{A}^3$ and $\mathcal{A}^4$ matrices for each multi-valued sub-attributes in IVINHSS form.

TABLE 2. Representation of values in IVINHSS form for $\mathcal{A}^1$.

$\mathcal{A}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$
v_1	$\langle[0.43, 0.55], [0.91, 0.95], [0.31, 0.36]\rangle$	$\langle[0.43, 0.52], [0.58, 0.81], [0.12, 0.21]\rangle$	$\langle[0.67, 0.71], [0.77, 0.81], [0.19, 0.29]\rangle$
v_2	$\langle[0.32, 0.45], [0.71, 0.78], [0.22, 0.29]\rangle$	$\langle[0.54, 0.63], [0.34, 0.44], [0.15, 0.24]\rangle$	$\langle[0.45, 0.48], [0.62, 0.72], [0.25, 0.35]\rangle$
v_3	$\langle[0.29, 0.53], [0.81, 0.89], [0.31, 0.41]\rangle$	$\langle[0.37, 0.41], [0.66, 0.71], [0.29, 0.35]\rangle$	$\langle[0.49, 0.51], [0.49, 0.59], [0.39, 0.42]\rangle$
v_4	$\langle[0.34, 0.43], [0.61, 0.82], [0.42, 0.53]\rangle$	$\langle[0.48, 0.59], [0.31, 0.42], [0.21, 0.41]\rangle$	$\langle[0.42, 0.47], [0.57, 0.61], [0.39, 0.45]\rangle$

$\mathcal{A}^1$	$\tilde{\lambda}_4$
v_1	$\langle[0.15, 0.19], [0.49, 0.51], [0.32, 0.34]\rangle$
v_2	$\langle[0.24, 0.29], [0.65, 0.72], [0.51, 0.55]\rangle$
v_3	$\langle[0.33, 0.39], [0.94, 0.98], [0.44, 0.45]\rangle$
v_4	$\langle[0.48, 0.49], [0.78, 0.84], [0.26, 0.34]\rangle$

TABLE 3. Representation of values in IVINHSS form for $\mathcal{A}^2$.

$\mathcal{A}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$
v_1	$\langle[0.61, 0.65], [0.25, 0.35], [0.22, 0.31]\rangle$	$\langle[0.44, 0.59], [0.59, 0.71], [0.11, 0.12]\rangle$	$\langle[0.44, 0.51], [0.42, 0.45], [0.21, 0.25]\rangle$
v_2	$\langle[0.39, 0.41], [0.91, 0.99], [0.41, 0.59]\rangle$	$\langle[0.59, 0.64], [0.66, 0.76], [0.21, 0.31]\rangle$	$\langle[0.54, 0.62], [0.31, 0.36], [0.32, 0.38]\rangle$
v_3	$\langle[0.32, 0.42], [0.82, 0.88], [0.41, 0.49]\rangle$	$\langle[0.48, 0.54], [0.21, 0.37], [0.29, 0.32]\rangle$	$\langle[0.49, 0.54], [0.49, 0.59], [0.25, 0.29]\rangle$
v_4	$\langle[0.34, 0.44], [0.66, 0.77], [0.33, 0.38]\rangle$	$\langle[0.69, 0.74], [0.68, 0.79], [0.19, 0.21]\rangle$	$\langle[0.58, 0.66], [0.69, 0.71], [0.33, 0.34]\rangle$

$\mathcal{A}^2$	$\tilde{\lambda}_4$
v_1	$\langle[0.21, 0.28], [0.57, 0.59], [0.41, 0.43]\rangle$
v_2	$\langle[0.28, 0.31], [0.67, 0.68], [0.57, 0.61]\rangle$
v_3	$\langle[0.41, 0.46], [0.77, 0.81], [0.23, 0.29]\rangle$
v_4	$\langle[0.21, 0.29], [0.69, 0.71], [0.44, 0.49]\rangle$

TABLE 4. Representation of values in IVINHSS form for $\mathcal{A}^3$.

$\mathcal{A}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$
v_1	$\langle[0.55, 0.56], [0.68, 0.78], [0.32, 0.37]\rangle$	$\langle[0.48, 0.55], [0.68, 0.87], [0.11, 0.28]\rangle$	$\langle[0.51, 0.54], [0.55, 0.62], [0.30, 0.32]\rangle$
v_2	$\langle[0.42, 0.46], [0.45, 0.55], [0.41, 0.48]\rangle$	$\langle[0.39, 0.45], [0.81, 0.91], [0.29, 0.31]\rangle$	$\langle[0.47, 0.49], [0.35, 0.42], [0.21, 0.42]\rangle$
v_3	$\langle[0.53, 0.55], [0.66, 0.76], [0.24, 0.42]\rangle$	$\langle[0.51, 0.65], [0.38, 0.42], [0.24, 0.29]\rangle$	$\langle[0.32, 0.34], [0.31, 0.41], [0.35, 0.41]\rangle$
v_4	$\langle[0.31, 0.43], [0.35, 0.45], [0.14, 0.29]\rangle$	$\langle[0.35, 0.48], [0.31, 0.49], [0.31, 0.38]\rangle$	$\langle[0.63, 0.64], [0.22, 0.32], [0.15, 0.21]\rangle$

$\mathcal{A}^3$	$\tilde{\lambda}_4$
v_1	$\langle[0.51, 0.53][0.41, 0.44][0.21, 0.24]\rangle$
v_2	$\langle[0.42, 0.43][0.45, 0.49][0.32, 0.34]\rangle$
v_3	$\langle[0.05, 0.12][0.65, 0.69][0.45, 0.49]\rangle$
v_4	$\langle[0.21, 0.26][0.72, 0.79][0.22, 0.23]\rangle$

TABLE 5. Representation of values in IVINHSS form for $\mathcal{A}^4$.

$\mathcal{A}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$	$\tilde{\lambda}_3$
v_1	$\langle[0.61, 0.71], [0.36, 0.55][0.09, 0.21]\rangle$	$\langle[0.31, 0.39], [0.67, 0.77], [0.29, 0.39]\rangle$	$\langle[0.27, 0.34], [0.17, 0.27], [0.32, 0.35]\rangle$
v_2	$\langle[0.44, 0.54], [0.46, 0.66][0.12, 0.25]\rangle$	$\langle[0.41, 0.57], [0.87, 0.92], [0.39, 0.41]\rangle$	$\langle[0.39, 0.41], [0.39, 0.41], [0.41, 0.49]\rangle$
v_3	$\langle[0.34, 0.44], [0.66, 0.77][0.33, 0.39]\rangle$	$\langle[0.53, 0.64], [0.64, 0.77], [0.21, 0.28]\rangle$	$\langle[0.14, 0.15], [0.49, 0.59], [0.62, 0.68]\rangle$
v_4	$\langle[0.52, 0.66], [0.35, 0.49][0.14, 0.25]\rangle$	$\langle[0.47, 0.56], [0.41, 0.45], [0.27, 0.34]\rangle$	$\langle[0.25, 0.29], [0.46, 0.66], [0.31, 0.34]\rangle$

$\mathcal{A}^4$	$\tilde{\lambda}_4$
v_1	$\langle[0.37, 0.39], [0.81, 0.91], [0.49, 0.51]\rangle$
v_2	$\langle[0.41, 0.42], [0.38, 0.42], [0.29, 0.31]\rangle$
v_3	$\langle[0.52, 0.59], [0.65, 0.69], [0.23, 0.29]\rangle$
v_4	$\langle[0.31, 0.36], [0.42, 0.51], [0.61, 0.62]\rangle$

Step 2. Obtain $\tilde{A}^1$, $\tilde{A}^2$, $\tilde{A}^3$ and $\tilde{A}^4$, the weighted matrices for each multi-valued sub-attributes.

TABLE 6. Representation of weighted values in IVINHSS form for $\tilde{A}^1$.

$\tilde{A}^1$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
v_1	$\langle[0.0386, 0.0062], [0.1552, 0.0229], [0.9213, 0.9922]\rangle$	$\langle[0.0480, 0.0071], [0.0731, 0.0625], [0.8307, 0.0529]\rangle$
v_2	$\langle[0.0116, 0.0014], [0.0365, 0.0036], [0.9556, 0.9972]\rangle$	$\langle[0.0287, 0.0029], [0.0155, 0.0053], [0.9314, 0.0635]\rangle$
v_3	$\langle[0.0204, 0.0031], [0.0949, 0.0090], [0.9322, 0.9964]\rangle$	$\langle[0.0341, 0.0027], [0.0778, 0.0290], [0.9114, 0.0956]\rangle$
v_4	$\langle[0.0165, 0.0019], [0.0370, 0.0057], [0.9659, 0.9980]\rangle$	$\langle[0.0322, 0.0037], [0.0184, 0.0051], [0.9250, 0.1197]\rangle$

$\tilde{A}^1$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle[0.1099, 0.0143], [0.1430, 0.0745], [0.8400, 0.0904]\rangle$	$\langle[0.0142, 0.0021], [0.0573, 0.0273][0.9052, 0.0913]\rangle$
v_2	$\langle[0.0266, 0.0023], [0.0427, 0.0139], [0.9396, 0.1162]\rangle$	$\langle[0.0103, 0.0010], [0.0387, 0.0116][0.9751, 0.1737]\rangle$
v_3	$\langle[0.0589, 0.0044], [0.0589, 0.0251], [0.9188, 0.1414]\rangle$	$\langle[0.0296, 0.0026], [0.1903, 0.0887][0.9403, 0.1301]\rangle$
v_4	$\langle[0.0322, 0.0032], [0.0494, 0.0104], [0.9451, 0.1591]\rangle$	$\langle[0.0322, 0.0028], [0.0730, 0.0169][0.9349, 0.0955]\rangle$

TABLE 7. Representation of weighted values in IVINHSS form for $\tilde{A}^2$.

$\tilde{A}^2$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
v_1	$\langle[0.0638, 0.0134], [0.0200, 0.0055], [0.8995, 0.9852]\rangle$	$\langle[0.0495, 0.0142], [0.0751, 0.0062], [0.8244, 0.0284]\rangle$
v_2	$\langle[0.0148, 0.0016], [0.0697, 0.0136], [0.9737, 0.9985]\rangle$	$\langle[0.0329, 0.0038], [0.0397, 0.0246], [0.9432, 0.0864]\rangle$
v_3	$\langle[0.0229, 0.0025], [0.0978, 0.0097], [0.9480, 0.9968]\rangle$	$\langle[0.0479, 0.0045], [0.0176, 0.0113], [0.9114, 0.0874]\rangle$
v_4	$\langle[0.0165, 0.0020], [0.0423, 0.0049], [0.9567, 0.9969]\rangle$	$\langle[0.0569, 0.0056], [0.0554, 0.0164], [0.9204, 0.0549]\rangle$

$\tilde{A}^2$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle[0.0591, 0.0136], [0.0556, 0.0036], [0.8489, 0.0747]\rangle$	$\langle[0.0205, 0.0053], [0.0712, 0.0045], [0.9250, 0.1188]\rangle$
v_2	$\langle[0.0344, 0.0043], [0.0166, 0.0093], [0.9501, 0.1304]\rangle$	$\langle[0.0123, 0.0014], [0.0408, 0.0197], [0.9792, 0.2049]\rangle$
v_3	$\langle[0.0589, 0.0054], [0.0589, 0.0259], [0.8828, 0.0929]\rangle$	$\langle[0.0388, 0.0036], [0.1044, 0.0398], [0.8957, 0.0780]\rangle$
v_4	$\langle[0.0508, 0.0054], [0.0679, 0.0156], [0.9357, 0.1125]\rangle$	$\langle[0.0118, 0.0015], [0.0569, 0.0131], [0.9598, 0.1488]\rangle$

TABLE 8. Representation of weighted values in IVINHSS form for $\tilde{A}^3$.

$\tilde{A}^3$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
v_1	$\langle[0.0544, 0.0089], [0.0767, 0.0164], [0.9234, 0.9893]\rangle$	$\langle[0.0557, 0.0109], [0.0949, 0.0384], [0.8244, 0.0731]\rangle$
v_2	$\langle[0.0163, 0.0021], [0.0178, 0.0026], [0.9737, 0.9977]\rangle$	$\langle[0.0184, 0.0025], [0.0604, 0.0107], [0.9547, 0.0864]\rangle$
v_3	$\langle[0.0443, 0.0071], [0.0627, 0.0126], [0.9180, 0.9924]\rangle$	$\langle[0.0521, 0.0116], [0.0353, 0.0086], [0.8985, 0.0756]\rangle$
v_4	$\langle[0.0148, 0.0017], [0.0171, 0.0018], [0.9244, 0.9964]\rangle$	$\langle[0.0214, 0.0025], [0.0184, 0.0029], [0.9432, 0.1046]\rangle$

$\tilde{A}^3$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	$\langle[0.0722, 0.0126], [0.0805, 0.0221], [0.8813, 0.1014]\rangle$	$\langle[0.0606, 0.0103], [0.0452, 0.0111], [0.8724, 0.0614]\rangle$
v_2	$\langle[0.0282, 0.0033], [0.0192, 0.0030], [0.9322, 0.1472]\rangle$	$\langle[0.0203, 0.0023], [0.0222, 0.0030], [0.9582, 0.0962]\rangle$
v_3	$\langle[0.0342, 0.0056], [0.0329, 0.0099], [0.9099, 0.1353]\rangle$	$\langle[0.0039, 0.0015], [0.0758, 0.0182], [0.9419, 0.1432]\rangle$
v_4	$\langle[0.0580, 0.0046], [0.0148, 0.0020], [0.8925, 0.0633]\rangle$	$\langle[0.0118, 0.0012], [0.0617, 0.0067], [0.9271, 0.0587]\rangle$

Table 9. Representation of weighted values in IVINHSS form for $\tilde{\mathcal{A}}^4$.

$\tilde{\mathcal{A}}^4$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
v_1	⟨[0.0638, 0.0157], [0.0308, 0.0102], [0.8449, 0.9803]⟩	⟨[0.0320, 0.0079], [0.0925, 0.0113], [0.8974, 0.0992]⟩
v_2	⟨[0.0173, 0.0027], [0.0184, 0.0038], [0.9384, 0.9953]⟩	⟨[0.0196, 0.0037], [0.0737, 0.0116], [0.9654, 0.1165]⟩
v_3	⟨[0.0247, 0.0029], [0.0627, 0.0073], [0.9357, 0.9954]⟩	⟨[0.0551, 0.0063], [0.0738, 0.0228], [0.8896, 0.0740]⟩
v_4	⟨[0.0290, 0.0063], [0.0171, 0.0039], [0.9244, 0.9920]⟩	⟨[0.0313, 0.0060], [0.0261, 0.0026], [0.9367, 0.0916]⟩

$\tilde{\mathcal{A}}^4$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	⟨[0.0326, 0.0080], [0.0194, 0.0030], [0.8873, 0.1035]⟩	⟨[0.0397, 0.0079], [0.1353, 0.0184], [0.9395, 0.1399]⟩
v_2	⟨[0.0220, 0.0028], [0.0220, 0.0030], [0.9607, 0.1727]⟩	⟨[0.0196, 0.0024], [0.0178, 0.0026], [0.9547, 0.0834]⟩
v_3	⟨[0.0135, 0.0013], [0.0589, 0.0167], [0.9579, 0.2738]⟩	⟨[0.0536, 0.0055], [0.0758, 0.0182], [0.8957, 0.0770]⟩
v_4	⟨[0.0172, 0.0030], [0.0363, 0.0056], [0.9322, 0.1089]⟩	⟨[0.0184, 0.0033], [0.0269, 0.0031], [0.9756, 0.2004]⟩

Step 3. Determine the IVINPIS and IVINNIS from the weighted matrices, $\tilde{\mathcal{A}}^1$, $\tilde{\mathcal{A}}^2$, $\tilde{\mathcal{A}}^3$ and $\tilde{\mathcal{A}}^4$.

Table 10. Representation of IVINPIS ($\tilde{\mathcal{A}}^+$) from the weighted matrices.

$\tilde{\mathcal{A}}^+$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
v_1	⟨[0.0638, 0.0157], [0.0200, 0.0055], [0.8449, 0.9803]⟩	⟨[0.0557, 0.0142], [0.0731, 0.0062], [0.8244, 0.0284]⟩
v_2	⟨[0.0173, 0.0027], [0.0178, 0.0026], [0.9384, 0.9953]⟩	⟨[0.0329, 0.0038], [0.0155, 0.0053], [0.9314, 0.0635]⟩
v_3	⟨[0.0443, 0.0071], [0.0627, 0.0073], [0.9180, 0.9924]⟩	⟨[0.0551, 0.0116], [0.0176, 0.0086], [0.8896, 0.0740]⟩
v_4	⟨[0.0290, 0.0063], [0.0171, 0.0018], [0.9244, 0.9920]⟩	⟨[0.0569, 0.0060], [0.0184, 0.0026], [0.9204, 0.0549]⟩

$\tilde{\mathcal{A}}^+$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	⟨[0.1099, 0.0143], [0.0194, 0.0030], [0.8400, 0.0747]⟩	⟨[0.0606, 0.0103], [0.0452, 0.0045], [0.8724, 0.0614]⟩
v_2	⟨[0.0344, 0.0043], [0.0166, 0.0030], [0.9322, 0.1162]⟩	⟨[0.0203, 0.0024], [0.0178, 0.0026], [0.9547, 0.0834]⟩
v_3	⟨[0.0589, 0.0056], [0.0329, 0.0099], [0.8828, 0.0929]⟩	⟨[0.0536, 0.0055], [0.0758, 0.0182], [0.8957, 0.0770]⟩
v_4	⟨[0.0580, 0.0054], [0.0148, 0.0020], [0.8925, 0.0633]⟩	⟨[0.0322, 0.0033], [0.0269, 0.0031], [0.9271, 0.0587]⟩

Table 11. Representation of IVINPIS ($\tilde{\mathcal{A}}^-$) from the weighted matrices.

$\tilde{\mathcal{A}}^-$	$\tilde{\lambda}_1$	$\tilde{\lambda}_2$
v_1	⟨[0.0386, 0.0062], [0.0200, 0.0055], [0.9234, 0.9922]⟩	⟨[0.0320, 0.0071], [0.0731, 0.0062], [0.8974, 0.0992]⟩
v_2	⟨[0.0116, 0.0014], [0.0178, 0.0026], [0.9737, 0.9985]⟩	⟨[0.0184, 0.0025], [0.0155, 0.0053], [0.9654, 0.1165]⟩
v_3	⟨[0.0204, 0.0025], [0.0178, 0.0026], [0.9480, 0.9968]⟩	⟨[0.0341, 0.0027], [0.0176, 0.0086], [0.9114, 0.0956]⟩
v_4	⟨[0.0148, 0.0017], [0.0171, 0.0018], [0.9659, 0.9980]⟩	⟨[0.0214, 0.0025], [0.0176, 0.0026], [0.9432, 0.1197]⟩

$\tilde{\mathcal{A}}^-$	$\tilde{\lambda}_3$	$\tilde{\lambda}_4$
v_1	⟨[0.0326, 0.0080], [0.0194, 0.003], [0.8873, 0.1035]⟩	⟨[0.0142, 0.0021], [0.0452, 0.0045], [0.93950.1399]⟩
v_2	⟨[0.0220, 0.0023], [0.0166, 0.003], [0.9607, 0.1727]⟩	⟨[0.0103, 0.0010], [0.0178, 0.0026], [0.97920.2049]⟩
v_3	⟨[0.0135, 0.0013], [0.0166, 0.003], [0.9579, 0.2738]⟩	⟨[0.0039, 0.0015], [0.0178, 0.0026], [0.94190.1432]⟩
v_4	⟨[0.0172, 0.0030], [0.0148, 0.002], [0.9451, 0.1591]⟩	⟨[0.0118, 0.0012], [0.0269, 0.0031], [0.97560.2004]⟩

Step 4. Determine the CC for the alternatives by using the values of IVINPIS and IVINNIS.

$$\chi^1 = 0.9968, \chi^2 = 0.9984, \chi^3 = 0.9988 \text{ and } \chi^4 = 0.9968.$$

$$\lambda^1 = 0.9957, \lambda^2 = 0.9972, \lambda^3 = 0.9971 \text{ and } \lambda^4 = 0.9984.$$

Step 5. Compute the closeness coefficient of neutrosophic ideal solution as below.

$$\epsilon^1 = 0.5733, \epsilon^2 = 0.6364, \epsilon^3 = 0.7073 \text{ and } \epsilon^4 = 0.3333.$$

Step 6. Arrange the values in descending order.

$$\epsilon^3 > \epsilon^2 > \epsilon^1 > \epsilon^4.$$

$$\Rightarrow \mathcal{A}^3 > \mathcal{A}^2 > \mathcal{A}^1 > \mathcal{A}^4.$$

Hence, $\mathcal{A}^3$ is the best among the group who can lead the project successfully.

6. **Comparative Analysis**

We combine the proposed interval-valued intuitionistic neutrosophic TOPSIS method with existing SMs to show the reliability, validity and effectiveness of the proposed TOPSIS method based on CC.

Example 6.1. Consider the same IVINHSS values and weights mentioned in Section 5.2. We now combine the proposed TOPSIS method, with the SMs given below to rank the alternatives.

(i) $\mathcal{S}_Y(\Omega_1, \Omega_2)$ [5]

$$= 1 - \frac{1}{n}\sum_{i=1}^{n} w_j \Big[|\underline{\alpha}_{\Omega_1(q_i)}(v_j) - \underline{\alpha}_{\Omega_2(q_i)}(v_j)| + |\overline{\alpha}_{\Omega_1(q_i)}(v_j) - \overline{\alpha}_{\Omega_2(q_i)}(v_j)| + |\underline{\beta}_{\Omega_1(q_i)}(v_j) - \underline{\beta}_{\Omega_2(q_i)}(v_j)| + |\overline{\beta}_{\Omega_1(q_i)}(v_j) - \overline{\beta}_{\Omega_2(q_i)}(v_j)| + |\underline{\gamma}_{\Omega_1(q_i)}(v_j) - \underline{\gamma}_{\Omega_2(q_i)}(v_j)| + |\overline{\gamma}_{\Omega_1(q_i)}(v_j) - \overline{\gamma}_{\Omega_2(q_i)}(v_j)| \Big].$$

(ii) $\mathcal{S}_T(\Omega_1, \Omega_2)$ [6]

$$= \frac{\sum_{i=1}^{n} \Big(min(\underline{\alpha}_{\Omega_1(q_i)}(v_j), \underline{\alpha}_{\Omega_2(q_i)}(v_j)) + min(\overline{\alpha}_{\Omega_1(q_i)}(v_j), \overline{\alpha}_{\Omega_2(q_i)}(v_j)) + min(\underline{\beta}_{\Omega_1(q_i)}(v_j), \underline{\beta}_{\Omega_2(q_i)}(v_j)) + min(\overline{\beta}_{\Omega_1(q_i)}(v_j), \overline{\beta}_{\Omega_2(q_i)}(v_j)) + min(\underline{\gamma}_{\Omega_1(q_i)}(v_j), \underline{\gamma}_{\Omega_2(q_i)}(v_j)) + min(\overline{\gamma}_{\Omega_1(q_i)}(v_j), \overline{\gamma}_{\Omega_2(q_i)}(v_j)) \Big)}{\sum_{i=1}^{n} \Big(max(\underline{\alpha}_{\Omega_1(q_i)}(v_j), \underline{\alpha}_{\Omega_2(q_i)}(v_j)) + max(\overline{\alpha}_{\Omega_1(q_i)}(v_j), \overline{\alpha}_{\Omega_2(q_i)}(v_j)) + max(\underline{\beta}_{\Omega_1(q_i)}(v_j), \underline{\beta}_{\Omega_2(q_i)}(v_j)) + max(\overline{\beta}_{\Omega_1(q_i)}(v_j), \overline{\beta}_{\Omega_2(q_i)}(v_j)) + max(\underline{\gamma}_{\Omega_1(q_i)}(v_j), \underline{\gamma}_{\Omega_2(q_i)}(v_j)) + max(\overline{\gamma}_{\Omega_1(q_i)}(v_j), \overline{\gamma}_{\Omega_2(q_i)}(v_j)) \Big)},$$

(iii) $\mathcal{S}_H(\Omega_1, \Omega_2)$ [6]

$$= \frac{1}{6}\sum_{i=1}^{n} w_j \Big[|\underline{\alpha}_{\Omega_1(q_i)}(v_j) - \underline{\alpha}_{\Omega_2(q_i)}(v_j)| + |\overline{\alpha}_{\Omega_1(q_i)}(v_j) - \overline{\alpha}_{\Omega_2(q_i)}(v_j)| + |\underline{\beta}_{\Omega_1(q_i)}(v_j) - \underline{\beta}_{\Omega_2(q_i)}(v_j)| + |\overline{\beta}_{\Omega_1(q_i)}(v_j) - \overline{\beta}_{\Omega_2(q_i)}(v_j)| + |\underline{\gamma}_{\Omega_1(q_i)}(v_j) - \underline{\gamma}_{\Omega_2(q_i)}(v_j)| + |\overline{\gamma}_{\Omega_1(q_i)}(v_j) - \overline{\gamma}_{\Omega_2(q_i)}(v_j)| \Big].$$

(iv) $\mathcal{S}_E(\Omega_1, \Omega_2)$ [6]

$$= \Big(\sum_{i=1}^{n} w_j \Big[|\underline{\alpha}_{\Omega_1(q_i)}(v_j) - \underline{\alpha}_{\Omega_2(q_i)}(v_j)|^2 + |\overline{\alpha}_{\Omega_1(q_i)}(v_j) - \overline{\alpha}_{\Omega_2(q_i)}(v_j)|^2 + |\underline{\beta}_{\Omega_1(q_i)}(v_j) - \underline{\beta}_{\Omega_2(q_i)}(v_j)|^2 + |\overline{\beta}_{\Omega_1(q_i)}(v_j) - \overline{\beta}_{\Omega_2(q_i)}(v_j)|^2 + |\underline{\gamma}_{\Omega_1(q_i)}(v_j) - \underline{\gamma}_{\Omega_2(q_i)}(v_j)|^2 + |\overline{\gamma}_{\Omega_1(q_i)}(v_j) - \overline{\gamma}_{\Omega_2(q_i)}(v_j)|^2 \Big] \Big)^{\frac{1}{2}}.$$

(v) $\mathcal{S}_{C_1}(\Omega_1, \Omega_2)$ [31]

$$= \frac{1}{n}\sum_{i=1}^{n} \mathrm{Cos}\Big[\frac{\pi}{4}\Big(|\underline{\alpha}_{\Omega_1(q_i)}(v_j) - \underline{\alpha}_{\Omega_2(q_i)}(v_j)| \vee |\underline{\beta}_{\Omega_1(q_i)}(v_j) - \underline{\beta}_{\Omega_2(q_i)}(v_j)| \vee |\underline{\gamma}_{\Omega_1(q_i)}(v_j) - \underline{\gamma}_{\Omega_2(q_i)}(v_j)|$$
$$+ |\overline{\alpha}_{\Omega_1(q_i)}(v_j) - \overline{\alpha}_{\Omega_2(q_i)}(v_j)| \vee |\overline{\beta}_{\Omega_1(q_i)}(v_j) - \overline{\beta}_{\Omega_2(q_i)}(v_j)| \vee |\overline{\gamma}_{\Omega_1(q_i)}(v_j) - \overline{\gamma}_{\Omega_2(q_i)}(v_j)|\Big)\Big].$$

(vi) $\mathcal{S}_{C_2}(\Omega_1, \Omega_2)$ [31]

$$= \frac{1}{n}\sum_{i=1}^{n} \mathrm{Cos}\Big[\frac{\pi}{12}\Big(|\underline{\alpha}_{\Omega_1(q_i)}(v_j) - \underline{\alpha}_{\Omega_2(q_i)}(v_j)| + |\overline{\alpha}_{\Omega_1(q_i)}(v_j) - \overline{\alpha}_{\Omega_2(q_i)}(v_j)| + |\underline{\beta}_{\Omega_1(q_i)}(v_j) - \underline{\beta}_{\Omega_2(q_i)}(v_j)|$$
$$+ |\overline{\beta}_{\Omega_1(q_i)}(v_j) - \overline{\beta}_{\Omega_2(q_i)}(v_j)| + |\underline{\gamma}_{\Omega_1(q_i)}(v_j) - \underline{\gamma}_{\Omega_2(q_i)}(v_j)| + |\overline{\gamma}_{\Omega_1(q_i)}(v_j) - \overline{\gamma}_{\Omega_2(q_i)}(v_j)|\Big)\Big].$$

TABLE 12. Comparison of existing similarity measures with proposed method.

Determination of rank using existing similarity measures
$\mathcal{S}_Y(\psi_1, \psi_2)$ [5]$\Rightarrow \mathcal{A}^1 = \mathcal{A}^4 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^3 = 0.49$
$\mathcal{S}_T(\psi_1, \psi_2)$ [6]$\Rightarrow \mathcal{A}^1 = \mathcal{A}^4 = 0.50$ and $\mathcal{A}^2 = \mathcal{A}^3 = 0.49$
$\mathcal{S}_{C_1}(\psi_1, \psi_2)$ [31]$\Rightarrow \mathcal{A}^1 = \mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.50$
$\mathcal{S}_{C_2}(\psi_1, \psi_2)$ [31]$\Rightarrow \mathcal{A}^1 = \mathcal{A}^2 = \mathcal{A}^3 = \mathcal{A}^4 = 0.50$

Analysis : From Table 12, it is evident that, when SMs of $\mathcal{S}_Y(\psi_1, \psi_2)$ [5], $\mathcal{S}_T(\psi_1, \psi_2)$ [6], $\mathcal{S}_{C_1}(\psi_1, \psi_2)$ [31] and $\mathcal{S}_{C_2}(\psi_1, \psi_2)$ [31] are used in the proposed TOPSIS method instead of CC, it is not possible to identify the best alternative. However, the best alternative is identified in the proposed method when CC is used. Hence, it is evident that the proposed TOPSIS method based on CC is more reliable and effective than SMs.

7. Conclusions

In this work, we have introduced the notion of IVINHSS and established some of its properties. The aim of this research is to introduce new operational laws for IVINHSS. Also, we have presented the aggregation operators for IVINHSS by using the operational laws and established some of their properties. We have proposed aggregation operators and an application based on the TOPSIS method to identify a suitable employee, who can handle the project successfully using the Leipzig leadership model. To study the closeness coefficients, we have applied CC instead of SMs in the proposed TOPSIS method. We have presented a comparative study between the proposed method and the existing SMS to prove the reliability of the proposed model. In the future, we can extend this structure to several aggregate operators, combine IVINHSS with N soft set and in various decision-making problems.

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