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Entropy Based Grey Relational Analysis Method for Multi-Attribute Decision Making under Single Valued Neutrosophic Assessments

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Abstract. In this paper we investigate multi-attribute decision making problem with single-valued neutrosophic attribute values. Crisp values are inadequate to model real life situation due to imprecise information frequently used in decision making process. Neutrosophic set is one such tool that can handle these situations. The rating of all alternatives is expressed with single-valued neutrosophic set which is characterised by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Weight of each attribute is completely unknown to decision maker. We extend the grey relational analysis method to neutrosophic environment and apply it to multi-attribute decision making problem. Information entropy method is used to determine the unknown attribute weights. Neutrosophic grey relational coefficient is determined by using Hamming distance between each alternative to ideal neutrosophic estimates reliability solution and the ideal neutrosophic estimates un-reliability solution. Then neutrosophic relational degree is defined to determine the ranking order of all alternatives. Finally, an example is provided to illustrate the application of the proposed method.

Keywords: Neutrosophic set; Single-valued neutrosophic set; Grey relational analysis; Information Entropy; Multi-attribute decision making.

1 Introduction

Multiple attribute decision making (MADM) problems in the area of operation research, management science, economics, systemic optimization, urban planning and many other fields have achieved very much attention to the researchers during the last several decades. It is often used to solve various decision making and/or selection problems. These problems generally consist of choosing the most desirable alternative that has the highest degree of satisfaction from a set of alternatives with respect to their attributes. In this approach, the decision makers have to provide qualitative and/ or quantitative assessments for determining the performance of each alternative with respect to each attribute, and the relative importance of evaluation attribute.

In classical MADM methods, such as TOPSIS (Hwang & Yoon [1]), PROMETHEE (Brans et al. [2]), VIKOR (Opricovic [3-4]), ELECTRE (Roy [5]) the weight of each attributes and rating of each alternative are naturally considered with crisp numbers. However, in real complex situation, decision maker may prefer to evaluate the attributes by using linguistic variables rather than exact values due to his time pressure, lack of knowledge and lack of information processing capabilities about the problem domain. In such situations, the preference information of alternatives provided by the decision maker may be vague, imprecise or incomplete. Fuzzy set (Zadeh [6]) is one of such tool that utilizes this impreciseness in a mathematical form. MADM with imprecise information can be modelled quite well by using fuzzy set theory into the field of decision making.

Bellman and Zadeh [7] first investigated decision making problem in fuzzy environment. Chen [8] extended one of known classical MADM method, technique for order preference by similarity to ideal solution (TOPSIS). He developed a methodology for solving multi-criteria decision making problems in fuzzy environment. Zeng [9] solved fuzzy MADM problem with known attribute weight by using expected value operator of fuzzy variables. However, fuzzy set can only focus on the membership grade of vague parameters or events. It fails to handle non-membership degree and indeterminacy degree of imprecise parameters.

Atanassov [10] introduced intuitionistic fuzzy set (IFS). It is characterized by the membership degree, non-membership degree simultaneously. Impreciseness of the objectives can be well expressed by using IFS than fuzzy sets (Atanassov [11]). Therefore it has gained more and more attention to the researchers. Boran et.al [12] extended the TOPSIS method for multi-criteria intuitionistic decision making problem. Z.S. Xu[13] studied fuzzy multiple attribute decision making problems, in which all attribute values are given as intuitionistic fuzzy numbers and the preference information on alternatives can be provided by the decision maker. Z. Xu [14] proposed a solving method for MADM problem with interval-valued intuitionistic fuzzy decision making by using distance measure.

In IFSs, sum of membership degree and non-membership degree of a vague parameter is less than unity. Therefore, a certain amount of incomplete information or indeterminacy arises in an intuitionistic fuzzy set. It cannot handle all types of uncertainties successfully in different real physical problems such as problems involving incomplete information. Hence further generalizations of fuzzy as well as intuitionistic fuzzy sets are required.

Florentin Smarandache [15] introduced neutrosophic set (NS) and neutrosophic logic. It is actually generalization of different type of FSs and IFSs. The term "neutrosophy" means "knowledge of neutral thought". This "neutral" concept makes the differences between NSs and other sets like FSs, IFSs. Wang et al. [16] proposed single-valued neutrosophic set (SVNS) which is a sub-class of NSs. SVNS is characterized by truth membership degree (T), indeterminacy membership degree (I) and falsity membership degree (F) that are independent to each other. This is the key characteristic of NSs other than IFSs or fuzzy sets.
Such formulation is helpful for modelling MADM with neutrosophic set information for the most general ambiguity cases, including paradox. The assessment of attribute values by the decision maker takes the form of single-valued neutrosophic set. Ye [17] studied multi-criteria decision making problem under SVNS environment. He proposed a method for ranking of alternatives by using weighted correlation coefficient. Ye [18] also discussed single-valued neutrosophic cross entropy for multi-criteria decision making problems. He used similarity measure for interval valued neutrosophic set for solving multi-criteria decision making problems. Grey relational analysis (GRA) is widely used for MADM problems. Deng [19-20] developed the GRA method that is applied in various areas, such as economics, marketing, personal selection and agriculture. Zhang et al. [21] discussed GRA method for multi attribute decision making with interval numbers. An improved GRA method proposed by Rao & Singh [22] is applied for making a decision in manufacturing situations. Wei [23] studied the GRA method for intuitionistic fuzzy multi-criteria decision making. Therefore, it is necessary to pay attention to this issue for neutrosophic environment.

The aim of this paper is to extend the concept of GRA to develop a methodology for solving MADM problems with single valued neutrosophic set information. The information taken from expert's opinion about attribute values takes the form of single valued neutrosophic set. It is assumed that the information about attribute weights is completely unknown to decision maker. Entropy method is used for determining the unknown attribute weights. In this modified GRA method, the ideal neutrosophic estimates reliability solution and the ideal neutrosophic estimate un-reliability solution has been developed. Neutrosophic grey relational coefficient of each alternative is determined to rank the alternatives.

In order to do so, the remaining of this paper is organized as follows: Section 2 briefly introduce some preliminaries relating to neutrosophic set and the basics of single-valued neutrosophic set. In Section 3, Hamming distance between two single-valued neutrosophic sets is defined. Section 4 represents the model of MADM with SVNSs and discussion about modified GRA method to solve MADM problems. In section 5, an illustrative example is provided to show the effectiveness of the proposed model. Finally, section 6 presents the concluding remarks.

2 Preliminaries of Neutrosophic sets and Single valued neutrosophic set

Neutrosophic set is a part of neutrosophy, which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra (Smarandache [15]), and is a powerful general formal framework, which generalizes the above mentioned sets from philosophical point of view. Smarandache [15] gave the following definition of a neutrosophic set.

2.1 Definition of neutrosophic set

Definition 1 Let X be a space of points (objects) with generic element in X denoted by x. Then a neutrosophic set A in X is characterized by a truth membership function T_A, an indeterminacy membership function I_A and a falsity membership function F_A. The functions T_A, I_A and F_A are real standard or non-standard subsets of $]0^-, 1^+[$ that is
$T_A : X \to]0^-, 1^+[$; $I_A : X \to]0^-, 1^+[$; $F_A : X \to]0^-, 1^+[$

It should be noted that there is no restriction on the sum of $T_A(x)$, $I_A(x)$, $F_A(x)$ i.e. $0^- \le T_A(x) + I_A(x) + F_A(x) \le 3^+$

Definition 2 The complement of a neutrosophic set A is denoted by A^c and is defined by

$$T_{A^c}(x) = \{1^+\} - T_A(x);\; I_{A^c}(x) = \{1^+\} - I_A(x);$$

$$F_{A^c}(x) = \{1^+\} - F_A(x)$$

Definition 3 (Containment) A neutrosophic set A is contained in the other neutrosophic set B, $A \subseteq B$ if and only if the following result holds.

$$\inf T_A(x) \le \inf T_B(x)\,,\ \sup T_A(x) \le \sup T_B(x) \qquad (1)$$

$$\inf I_A(x) \ge \inf I_B(x)\,,\ \sup I_A(x) \ge \sup I_B(x) \qquad (2)$$

$$\inf F_A(x) \ge \inf F_B(x)\,,\ \sup F_A(x) \ge \sup F_B(x) \qquad (3)$$

for all x in *X*.

2.2 Some basics of single valued neutrosophic sets (SVNSs)

In this section we provide some definitions, operations and properties about single valued neutrosophic sets due to Wang et al. [16]. It will be required to develop the rest of the paper.

Definition 4 (Single-valued neutrosophic set). Let *X* be a universal space of points (objects), with a generic element of X denoted by x. A single-valued neutrosophic set $\tilde{\mathcal{N}} \subset X$ is characterized by a true membership function $T_{\tilde{\mathcal{N}}}(x)$, a falsity membership function $F_{\tilde{\mathcal{N}}}(x)$ and an indeterminacy function $I_{\tilde{\mathcal{N}}}(x)$ with $T_{\tilde{\mathcal{N}}}(x)$, $I_{\tilde{\mathcal{N}}}(x)$, $F_{\tilde{\mathcal{N}}}(x) \in [0, 1]$ for all x in *X*.

When X is continuous a SVNSs $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \int_x \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \right\rangle / x, \quad \forall x \in X.$$

and when X is discrete a SVNSs $\tilde{\mathcal{N}}$ can be written as

$$\tilde{\mathcal{N}} = \sum_{i=1}^{m} \left\langle T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x) \right\rangle / x, \quad \forall x \in X.$$

Actually, SVNS is an instance of neutrosophic set which can be used in real life situations like decision making, scientific and engineering applications. In case of SVNS, the degree of the truth membership $T_{\tilde{\mathcal{N}}}(x)$, the indeterminacy membership $I_{\tilde{\mathcal{N}}}(x)$ and the falsity membership $F_{\tilde{\mathcal{N}}}(x)$ values belong to [0, 1] instead of non standard unit interval] 0^-, 1^+ [as in the case of ordinary neutrosophic sets.

It should be noted that for a SVNS $\tilde{\mathcal{N}}$,

$$0 \le \sup T_{\tilde{\mathcal{N}}}(x) + \sup I_{\tilde{\mathcal{N}}}(x) + \sup F_{\tilde{\mathcal{N}}}(x) \le 3, \ \forall x \in X. \quad (4)$$

and for a neutrosophic set, the following relation holds

$$0^- \le \sup T_{\tilde{\mathcal{N}}}(x) + \sup I_{\tilde{\mathcal{N}}}(x) + \sup F_{\tilde{\mathcal{N}}}(x) \le 3^+, \ \forall x \in X. \quad (5)$$

For example, suppose ten members of a political party will critically review their specific agenda. Five of them agree with this agenda, three of them disagree and rest of two members remain undecided. Then by neutrosophic notation it can be expressed as $x\langle 0.5, 0.2, 0.3 \rangle$.

Definition 5 The complement of a neutrosophic set $\tilde{\mathcal{N}}$ is denoted by $\tilde{\mathcal{N}}^c$ and is defined by

$$T_{\tilde{\mathcal{N}}^c}(x) = F_{\tilde{\mathcal{N}}}(x); \ I_{\tilde{\mathcal{N}}^c}(x) = 1 - I_{\tilde{\mathcal{N}}}(x); \ F_{\tilde{\mathcal{N}}^c}(x) = T_{\tilde{\mathcal{N}}}(x)$$

Definition 6 A SVNS $\tilde{\mathcal{N}}_A$ is contained in the other SVNS $\tilde{\mathcal{N}}_B$, denoted as $\tilde{\mathcal{N}}_A \subseteq \tilde{\mathcal{N}}_B$, if and only if

$$T_{\tilde{\mathcal{N}}_A}(x) \le T_{\tilde{\mathcal{N}}_B}(x); \ I_{\tilde{\mathcal{N}}_A}(x) \ge I_{\tilde{\mathcal{N}}_B}(x); \ F_{\tilde{\mathcal{N}}_A}(x) \ge F_{\tilde{\mathcal{N}}_B}(x)$$

$\forall x \in X$.

Definition 7 Two single valued neutrosophic sets $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ are equal, i.e. $\tilde{\mathcal{N}}_A = \tilde{\mathcal{N}}_B$, if and only if $\tilde{\mathcal{N}}_A \subseteq \tilde{\mathcal{N}}_B$ and $\tilde{\mathcal{N}}_A \supseteq \tilde{\mathcal{N}}_B$.

Definition 8 (Union) The union of two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is a SVNS $\tilde{\mathcal{N}}_C$, written as $\tilde{\mathcal{N}}_C = \tilde{\mathcal{N}}_A \cup \tilde{\mathcal{N}}_B$. Its truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ by

$$T_{\tilde{\mathcal{N}}_C}(x) = \max(T_{\tilde{\mathcal{N}}_A}(x), T_{\tilde{\mathcal{N}}_B}(x));$$

$$I_{\tilde{\mathcal{N}}_C}(x) = \max(I_{\tilde{\mathcal{N}}_A}(x), I_{\tilde{\mathcal{N}}_B}(x));$$

$$F_{\tilde{\mathcal{N}}_C}(x) = \min(F_{\tilde{\mathcal{N}}_A}(x), F_{\tilde{\mathcal{N}}_B}(x)) \text{ for all x in X.}$$

Definition 9 (Intersection) The intersection of two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is a SVNS $\tilde{\mathcal{N}}_C$, written as $\tilde{\mathcal{N}}_C = \tilde{\mathcal{N}}_A \cap \tilde{\mathcal{N}}_B$, whose truth membership, indeterminacy-membership and falsity membership functions are related to those of $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ by $T_{\tilde{\mathcal{N}}_C}(x) = \min(T_{\tilde{\mathcal{N}}_A}(x), T_{\tilde{\mathcal{N}}_B}(x))$;

$$I_{\tilde{\mathcal{N}}_C}(x) = \min(I_{\tilde{\mathcal{N}}_A}(x), I_{\tilde{\mathcal{N}}_B}(x));$$

$$F_{\tilde{\mathcal{N}}_C}(x) = \max(F_{\tilde{\mathcal{N}}_A}(x), F_{\tilde{\mathcal{N}}_B}(x)) \text{ for all x in X.}$$

3 Distance between two neutrosophic sets.

Similar to fuzzy or intuitionistic fuzzy set, the general SVNS having the following pattern

$\tilde{\mathcal{N}} = \{(x/(T_{\tilde{\mathcal{N}}}(x), I_{\tilde{\mathcal{N}}}(x), F_{\tilde{\mathcal{N}}}(x)) : x \in X\}$. For finite SVNSs can be represented by the ordered tetrads:

$$\tilde{\mathcal{N}} = \left\{ \begin{array}{l} (x_1/(T_{\tilde{\mathcal{N}}}(x_1), I_{\tilde{\mathcal{N}}}(x_1), F_{\tilde{\mathcal{N}}}(x_1)), \\ \ldots, x_m/(T_{\tilde{\mathcal{N}}}(x_m), I_{\tilde{\mathcal{N}}}(x_m), F_{\tilde{\mathcal{N}}}(x_m)) \end{array} \right\}, \ \forall x \in X$$

Definition 10 Let

$$\tilde{\mathcal{N}}_A = \left\{ \begin{array}{l} (x_1/(T_{\tilde{\mathcal{N}}_A}(x_1), I_{\tilde{\mathcal{N}}_A}(x_1), F_{\tilde{\mathcal{N}}_A}(x_1)), \\ \ldots, x_n/(T_{\tilde{\mathcal{N}}_A}(x_n), I_{\tilde{\mathcal{N}}_A}(x_n), F_{\tilde{\mathcal{N}}_A}(x_n)) \end{array} \right\}$$

and

$$\tilde{\mathcal{N}}_B = \left\{ \begin{array}{l} (x_1/(T_{\tilde{\mathcal{N}}_B}(x_1), I_{\tilde{\mathcal{N}}_B}(x_1), F_{\tilde{\mathcal{N}}_B}(x_1)), \\ \ldots, x_n/(T_{\tilde{\mathcal{N}}_B}(x_n), I_{\tilde{\mathcal{N}}_B}(x_n), F_{\tilde{\mathcal{N}}_B}(x_n)) \end{array} \right\} \quad (6)$$

be two single-valued neutrosophic sets (SVNSs) in $X = \{x_1, x_2, \ldots, x_n\}$.

Then the Hamming distance between two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is defined as follows:

$$d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A, \tilde{\mathcal{N}}_B\right) = \sum_{i=1}^{n} \left\{ \begin{array}{l} \left|T_{\tilde{\mathcal{N}}_A}(x_i) - T_{\tilde{\mathcal{N}}_B}(x_i)\right| + \left|I_{\tilde{\mathcal{N}}_A}(x_i) - I_{\tilde{\mathcal{N}}_B}(x_i)\right| \\ + \left|F_{\tilde{\mathcal{N}}_A}(x_i) - F_{\tilde{\mathcal{N}}_B}(x_i)\right| \end{array} \right\} \quad (7)$$

and normalized Hamming distance between two SVNSs $\tilde{\mathcal{N}}_A$ and $\tilde{\mathcal{N}}_B$ is defined as follows:

$$ {}^{N}d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A,\tilde{\mathcal{N}}_B\right)= \frac{1}{3n}\sum_{i=1}^{n}\left\{\left|T_{\tilde{\mathcal{N}}_A}(x_1)-T_{\tilde{\mathcal{N}}_B}(x_1)\right|+\left|I_{\tilde{\mathcal{N}}_A}(x_1)-I_{\tilde{\mathcal{N}}_B}(x_1)\right|+\left|F_{\tilde{\mathcal{N}}_A}(x_1)-F_{\tilde{\mathcal{N}}_B}(x_1)\right|\right\} \quad (8) $$

with the following two properties

1. $0 \le d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A,\tilde{\mathcal{N}}_B\right) \le 3n$ (9)
2. $0 \le {}^{N}d_{\tilde{\mathcal{N}}}\left(\tilde{\mathcal{N}}_A,\tilde{\mathcal{N}}_B\right) \le 1$ (10)

Proof: The proofs are obvious from the basic definition of SVNS.

4 GRA method for multiple attribute decision making problem with single valued neutrosophic information

Consider a multi-attribute decision making problem with *m* alternatives and *n* attributes. Let A_1, A_2, ..., A_m and $\mathbf{C_1}$, C_2, ..., C_n denote the alternatives and attributes respectively. The rating describes the performance of alternative A_i against attribute $\mathbf{C_j}$. For MADM weight vector W = {w_1, w_2,...,w_n} is assigned to the attributes. The weight $w_j > 0$ (j = 1, 2, ..., n) reflects the relative importance of attributes C_j (j = 1, 2, ..., m) to the decision making process. The weights of the attributes are usually determined on subjective basis. They represent the opinion of a single decision maker or synthesize the opinions of a group of experts using a group decision technique, as well. The values associated with the alternatives for MADM problems presented in the decision table.

Table 1 Decision table of attribute values

$$ D=\left\langle d_{ij}\right\rangle_{m\times n}= \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{cccc} C_1 & C_2 & \dots & C_n \end{array} \\ \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & d_{22} & \dots & d_{2n} \\ \dots & \dots & \dots & \dots \\ \dots & \dots & \dots & \dots \\ d_{m1} & d_{m2} & \dots & d_{mn} \end{bmatrix} \end{array} \quad (11) $$

GRA is one of the derived evaluation methods for MADM based on the concept of grey relational space. The main procedure of GRA method is firstly translating the performance of all alternatives into a comparability sequence. This step is called data pre-processing. According to these sequences, a reference sequence (ideal target sequence) is defined. Then, the grey relational coefficient between all comparability sequences and the reference sequence for different values of distinguishing coefficient are calculated. Finally, based on these grey relational coefficients, the grey relational degree between the reference sequence and every comparability sequences is calculated. If an alternative gets the highest grey relational grade with the reference sequence, it means that the comparability sequence is most similar to the reference sequence and that alternative would be the best choice (Fung [24]). The steps of improved GRA under SVNS are described below:

Step 1 Determine the most important criteria.

Generally, there are many criteria or attributes in decision making problems where some of them are important and others may not be so important. So it is crucial, to select the proper criteria or attribute for decision making situations. The most important criteria may be chosen with help of experts' opinions or by some others method that are technically sound.

Step 2 Data pre-processing

Assuming for a multiple attribute decision making problem having *m* alternatives and *n* attributes, the general form of decision matrix can be presented as shown in Table-1. It may be mentioned here that the original GRA method can effectively deal mainly with quantitative attributes. However, there exists some difficulty in the case of qualitative attributes. In the case of a qualitative attribute (i.e. quantitative value is not available); an assessment value is taken as SVNSs.

Step 3 Construct the decision matrix with SVNSs

For multi-attribute decision making problem, the rating of alternative A_i (i = 1, 2,…m) with respect to attribute C_j (j = 1, 2,…n) is assumed as SVNS. It can be represented with the following looks

$$ A_i=\left\{ C_1 \big/ \left\langle T_{i1},I_{i1},F_{i1}\right\rangle, C_2 \big/ \left\langle T_{i2},I_{i2},F_{i2}\right\rangle,\dots, C_n \big/ \left\langle T_{in},I_{in},F_{in}\right\rangle : C_j \in C \right\}. $$

$$ =\left\langle C_j \big/ \left\langle T_{ij},I_{ij},F_{ij}\right\rangle : C_j\in C\right\rangle \text{ for } j = 1, 2,\dots, n. $$

T_{ij}, I_{ij}, F_{ij} are the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i satisfying the attribute C_j, respectively where $0 \le T_{ij} \le 1$, $0 \le I_{ij} \le 1$, $0 \le F_{ij} \le 1$ and $0 \le T_{ij}+I_{ij}+F_{ij} \le 3$. The decision matrix can be taken in the form:

Table 2 Decision table with SVNSs

$$D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n} = \begin{array}{c} \\ A_1 \\ A_2 \\ . \\ . \\ A_m \end{array} \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_n \end{array} \\ \begin{bmatrix} \langle T_{11}, I_{11}, F_{11}\rangle & \langle T_{12}, I_{12}, F_{12}\rangle & \ldots & \langle T_{1n}, I_{1n}, F_{1n}\rangle \\ \langle T_{21}, I_{21}, F_{21}\rangle & \langle T_{22}, I_{22}, F_{22}\rangle & \ldots & \langle T_{2n}, I_{2n}, F_{2n}\rangle \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \langle T_{m1}, I_{m1}, F_{m1}\rangle & \langle T_{m2}, I_{m2}, F_{m2}\rangle & \ldots & \langle T_{mn}, I_{mn}, F_{mn}\rangle \end{bmatrix} \end{array} \quad (12)$$

Step 4: Determine the weights of criteria.

In the decision-making process, decision makers may often face with unknown attribute weights. It may happens that the importance of the decision makers are not equal. Therefore, we need to determine reasonable attribute weight for making a proper decision. Many methods are available to determine the unknown attribute weight in the literature such as maximizing deviation method (Wu and Chen [25]), entropy method (Wei and Tang [26]; Xu and Hui [27]), optimization method (Wang and Zhang [28-29]) etc. In this paper, we propsoe information entropy method.

4.1 Entropy method:

Entropy has an important contribution for measuring uncertain information (Shannon [30-31]). Zadeh [32] introduced the fuzzy entropy for the first time. Similarly Bustince and Burrillo [33] introduced the intuitionistic fuzzy entropy. Szmidt and Kacprzyk [34] extended the axioms of De Luca and Termini's [35] non-probabilistic entropy in the setting of fuzzy set theory into intuitionistic fuzzy information entropy. Vlachos and Sergiadis [36] also studied intuitionistic fuzzy information entropy. Majumder and Samanta [37] developed some similarity and entropy measures for SVNSs. The entropy measure can be used to determine the attributes weights when it is unequal and completely unknown to decision maker. Hwan and Yoon (1981) developed a method to determine the attribute weights based on information entropy.

In this paper we propose an entropy method for determining attribute weight. According to Majumder and Samanta [37], the entropy measure of a SVNS $\tilde{\mathcal{N}}_A = \langle T_{\tilde{\mathcal{N}}_A}(x_1), I_{\tilde{\mathcal{N}}_A}(x_1), F_{\tilde{\mathcal{N}}_A}(x_1)\rangle$ is

$$E_i(\tilde{\mathcal{N}}_A) = 1 - \frac{1}{n}\sum_{i=1}^{m}\left(T_{\tilde{\mathcal{N}}_A}(x_i) + F_{\tilde{\mathcal{N}}_A}(x_i)\right)\left| I_{\tilde{\mathcal{N}}_A}(x_i) - I^C_{\tilde{\mathcal{N}}_A}(x_i)\right| \quad (13)$$

which has the following properties:

1. $E_i(\tilde{\mathcal{N}}_A) = 0$ if $\tilde{\mathcal{N}}_A$ is a crisp set and $I_{\tilde{\mathcal{N}}_A}(x_i) = 0$ $\forall x \in X$.
2. $E_i(\tilde{\mathcal{N}}_A) = 1$ if $\langle T_{\tilde{\mathcal{N}}_A}(x_1), I_{\tilde{\mathcal{N}}_A}(x_1), F_{\tilde{\mathcal{N}}_A}(x_1)\rangle = \langle 0.5, 0.5, 0.5\rangle$ $\forall x \in X$.
3. $E_i(\tilde{\mathcal{N}}_A) \ge E_i(\tilde{\mathcal{N}}_B)$ if $\tilde{\mathcal{N}}_A$ is more uncertain than $\tilde{\mathcal{N}}_B$ i.e. $T_{\tilde{\mathcal{N}}_A}(x_1) + F_{\tilde{\mathcal{N}}_A}(x_1) \le T_{\tilde{\mathcal{N}}_B}(x_1) + F_{\tilde{\mathcal{N}}_B}(x_1)$ and $\left| I_{\tilde{\mathcal{N}}_A}(x_i) - I_{\tilde{\mathcal{N}}_{A^c}}(x_i)\right| \le \left| I_{\tilde{\mathcal{N}}_B}(x_i) - I_{\tilde{\mathcal{N}}_{B^c}}(x_i)\right|$
4. $E_i(\tilde{\mathcal{N}}_A) = E_i(\tilde{\mathcal{N}}_{A^c})$ $\forall x \in X$.

In order to obtain the entropy value E_j of the j-th attribute C_j (j = 1, 2,…, n), equation (13) can be written as :

$$E_j = 1 - \frac{1}{n}\sum_{i=1}^{m}\left(T_{ij}(x_i) + F_{ij}(x_i)\right)\left| I_{ij}(x_i) - I^C_{ij}(x_i)\right|$$

for i = 1, 2,..,m; j = 1, 2,…,n. (14)

It is also noticed that $E_j \in [0,1]$. Due to Hwang and Yoon [1], and Wang and Zhang [29] the entropy weight of the j-th attibute C_j is presented by

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n}(1 - E_j)} \quad (15)$$

We get weight vector W= (w_1, w_2,…,w_n)^T of attributes C_j (j = 1,2,…, n) with $w_j \ge 0$ and $\sum_{j=1}^{n} w_j = 1$

Step 5. Determine the ideal neutrosophic estimates reliability solution (INERS) and the ideal neutrosophic estimates un-reliability solution (INEURS) for neutrosophic decision matrix.

For a neutrosophic decision making matrix $D_{\tilde{\mathcal{N}}} = [q_{\tilde{\mathcal{N}}ij}]_{m\times n} = \langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}$, T_{ij}, I_{ij}, F_{ij} are the degrees of membership, degree of indeterminacy and degree of non membership of the alternative A_i of A satisfying the attribute C_j of C. The neutrosophic estimate reliability estimation can be easily determined from the concept of SVNS cube proposed by Dezert [38].

Definition 11 From the neutrosophic cube, the membership grade represents the estimates reliability. The ideal neutrosophic estimates reliability solution (INERS) $Q^+_{\tilde{\mathcal{N}}} = [q^+_{\tilde{\mathcal{N}}1}, q^+_{\tilde{\mathcal{N}}2}, ..., q^+_{\tilde{\mathcal{N}}n}]$ is a solution in which every component $q^+_{\tilde{\mathcal{N}}j} = \langle T^+_j, I^+_j, F^+_j\rangle$, where $T^+_j = \max_i\{T_{ij}\}$, $I^+_j = \min_i\{I_{ij}\}$ and $F^+_j = \min_i\{F_{ij}\}$ in the neutrosophic decision matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij}\rangle_{m\times n}$ for i = 1, 2, .., m; j = 1, 2, …, n.

Definition 12 Similarly, in the neutrosophic cube maximum un-reliability happens when the indeterminacy membership grade and the degree of falsity membership reaches maximum simultaneously. Therefore, the ideal neutrosophic estimates un-reliability solution (INEURS) $Q^-_{\tilde{\mathcal{N}}} = [q^-_{\tilde{\mathcal{N}}1}, q^-_{\tilde{\mathcal{N}}2}, ..., q^-_{\tilde{\mathcal{N}}n}]$ can be taken as a solution in the form $q^-_{\tilde{\mathcal{N}}j} = \langle T^-_j, I^-_j, F^-_j \rangle$, where $T^-_j = \min_i\{T_{ij}\}$, $I_j = \max_i\{I_{ij}\}$ and $F^-_j = \max_i\{F_{ij}\}$ in the neutrosophic decision matrix $D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{m\times n}$ for i = 1, 2,..,m; j = 1, 2,…,n.

Step 6 Calculate neutrosophic grey relational coefficient of each alternative from INERS and INEURS.

Grey relational coefficient of each alternative from INERS is:

$$\chi^+_{ij} = \frac{\min_i \min_j \Delta^+_{ij} + \rho \max_i \max_j \Delta^+_{ij}}{\Delta^+_{ij} + \rho \max_i \max_j \Delta^+_{ij}}, \text{ where}$$

$$\Delta^+_{ij} = d\left(q^+_{\tilde{\mathcal{N}}j}, q_{\tilde{\mathcal{N}}ij}\right), \text{ for i= 1, 2,…,m. and j=1, 2,…,n.} \quad (16)$$

Grey relational coefficient of each alternative from INEURS is:

$$\chi^-_{ij} = \frac{\min_i \min_j \Delta^-_{ij} + \rho \max_i \max_j \Delta^-_{ij}}{\Delta^-_{ij} + \rho \max_i \max_j \Delta^-_{ij}}, \text{ where, } \Delta^-_{ij} =$$

$$d\left(q_{\tilde{\mathcal{N}}ij}, q^-_{\tilde{\mathcal{N}}j}\right), \text{ for i= 1, 2,…,m. and j=1, 2,…,n.} \quad (17)$$

$\rho \in [0, 1]$ is the distinguishable coefficient or the identification coefficient used to adjust the range of the comparison environment, and to control level of differences of the relation coefficients. When ρ =1, the comparison environment is unaltered; when ρ = 0, the comparison environment disappears. Smaller value of distinguishing coefficient will yield in large range of grey relational coefficient. Generally, ρ = 0.5 is considered for decision- making situation.

Step 7. Calculate of neutrosophic grey relational coefficient.

Calculate the degree of neutrosophic grey relational coefficient of each alternative from INERS and INEURS using the following equation respectively:

$$\chi^+_i = \sum_{j=1}^{n} w_j \chi^+_{ij} \quad (18)$$

and $\chi^-_i = \sum_{j=1}^{n} w_j \chi^-_{ij}$ for i = 1, 2,…, m. (19)

Step 8. Calculate the neutrosophic relative relational degree.

We calculate the neutrosophic relative relational degree of each alternative from ITFPIS with the help of following equations:

$$R_i = \frac{\chi^+_i}{\chi^+_i + \chi^-_i}, \text{ for i = 1, 2,…, m.} \quad (20)$$

Step 9. Rank the alternatives.

According to the relative relational degree, the ranking order of all alternatives can be determined. The highest value of R_i yields the most important alternative.

5 . Illustrative Examples

In this section, a multi-attribute decision-making problem is considered to demonstrate the application as well as the effectiveness of the proposed method. We consider the decision-making problem adapted from Ye [39]. Suppose there is an investment company, which wants to invest a sum of money to the best one from these four possible alternatives (1) A_1 is a car company; (2) A_2 is a food company; (3) A_3 is a computer company; and (4) A_4 is an arms company. The investment company must take a decision according to the following three criteria: (1) C_1 is the risk analysis; (2) C_2 is the growth analysis; and (3) C_3 is the environmental impact analysis. Thus, when the four possible alternatives with respect to the above three criteria are evaluated by the expert, we can obtain the following single-valued neutrosophic decision matrix:

$$D_{\tilde{\mathcal{N}}} = \langle T_{ij}, I_{ij}, F_{ij} \rangle_{4\times 3} = \begin{array}{c} \\ A_1 \\ A_2 \\ A_3 \\ A_4 \end{array} \begin{array}{ccc} C_1 & C_2 & C_3 \\ \end{array} \begin{bmatrix} \langle 0.4, 0.2, 0.3 \rangle & \langle 0.4, 0.2, 0.3 \rangle & \langle 0.2, 0.2, 0.5 \rangle \\ \langle 0.6, 0.1, 0.2 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.5, 0.2, 0.2 \rangle \\ \langle 0.3, 0.2, 0.3 \rangle & \langle 0.5, 0.2, 0.3 \rangle & \langle 0.5, 0.3, 0.2 \rangle \\ \langle 0.7, 0.0, 0.1 \rangle & \langle 0.6, 0.1, 0.2 \rangle & \langle 0.4, 0.3, 0.2 \rangle \end{bmatrix} \quad (21)$$

Step1: Determine the weights of attribute

Entropy value E_j of the j-th (j = 1, 2, 3) attributes can be determined from SVN decision matrix $D_{\tilde{\mathcal{N}}}$ (21) and equation (14) as: E_1 = 0.50; E_2 = 0.2733 and E_3 = 0.5467.

Then the corresponding entropy weights w_1, w_2, w_3 of all attributes according to equation (15) are obtained by w_1 = 0.2958; w_2 = 0.4325 and w_3 = 0.2697 such that $\sum_{j=1}^{3} w_j = 1$.

Step1: Determine the ideal neutrosophic estimates reliability solution (INERS):

$$Q^{+}_{\tilde{N}} = [q^{+}_{\tilde{N}_1}, q^{+}_{\tilde{N}_2}, q^{+}_{\tilde{N}_3}] =$$

$$\left[\left\langle \max_i\{T_{i1}\}, \min_i\{I_{i1}\}, \min_i\{F_{i1}\}\right\rangle, \left\langle \max_i\{T_{i2}\}, \min_i\{I_{i2}\}, \min_i\{F_{i2}\}\right\rangle, \left\langle \max_i\{T_{i3}\}, \min_i\{I_{i3}\}, \min_i\{F_{i3}\}\right\rangle\right]$$

$$= \left[\langle 0.7, 0.0, 0.1\rangle, \langle 0.6, 0.1, 0.2\rangle, \langle 0.5, 0.2, 0.2\rangle\right]$$

Step 2: Determine the ideal neutrosophic estimates un-reliability solution (INEURS):

$$Q^{+}_{\tilde{N}} = [q^{-}_{\tilde{N}_1}, q^{-}_{\tilde{N}_2}, q^{-}_{\tilde{N}_3}] =$$

$$\left[\left\langle \min_i\{T_{i1}\}, \max_i\{I_{i1}\}, \max_i\{F_{i1}\}\right\rangle, \left\langle \min_i\{T_{i2}\}, \max_i\{I_{i2}\}, \max_i\{F_{i2}\}\right\rangle, \left\langle \min_i\{T_{i3}\}, \max_i\{I_{i3}\}, \max_i\{F_{i3}\}\right\rangle\right]$$

$$= \left[\langle 0.4, 0.2, 0.3\rangle, \langle 0.4, 0.2, 0.3\rangle, \langle 0.2, 0.3, 0.5\rangle\right]$$

Step 3: Calculation of the neutrosophic grey relational co-efficient of each alternative from INERS and INEURS.

By using Equation (16) the neutrosophic grey relational coefficient of each alternative from INERS can be obtained as:

$$\left[\chi^{+}_{ij}\right]_{4\times 3} = \begin{bmatrix} 0.3636 & 0.5000 & 0.4000 \\ 0.5714 & 1.0000 & 1.0000 \\ 0.3333 & 0.5714 & 0.8000 \\ 1.0000 & 1.0000 & 0.6666 \end{bmatrix} \quad (22)$$

Similarly, from Equation (17) the neutrosophic grey relational coefficient of each alternative from INEURS is

$$\left[\chi^{-}_{ij}\right]_{4\times 3} = \begin{bmatrix} 1.0000 & 1.0000 & 0.7778 \\ 0.4667 & 0.4667 & 0.3333 \\ 0.7778 & 0.7778 & 0.3684 \\ 0.3333 & 0.4667 & 0.4111 \end{bmatrix} \quad (23)$$

Step 4: Determine the degree of neutrosophic grey relational co-efficient of each alternative from INERS and INEURS. The required neutrosophic grey relational co-efficient corresponding to INERS is obtained by using equations (18) as:

$$\chi^{+}_{1} = 0.43243;\ \chi^{+}_{2} = 0.87245;\ \chi^{+}_{3} = 0.56222;$$
$$\chi^{+}_{4} = 0.91004 \quad (24)$$

and corresponding to INEURS is obtained with the help of equation (19) as:

$$\chi^{-}_{1} = 0.9111;\ \chi^{-}_{2} = 0.4133;\ \chi^{-}_{3} = 0.6140;$$
$$\chi^{-}_{4} = 0.3978 \quad (25)$$

Step 5: Thus neutrosophic relative degree of each alternative from INERS can be obtained with the help of equation (20) as: R_1= 0.31507; R_2= 0.66949; R_3= 0.54275 and R_4= 0.68835.

Step 6: The ranking order of all alternatives can be determined according the value of neutrosophic relational degree i.e. $R_4 > R_2 > R_3 > R_1$. It is seen that the highest value of neutrosophic relational degree is R_4 therefore A_4 i.e. Arms Company is the best alternative for investment purpose.

6 Conclusion

In practical applications for MADM process, the assessments of all attributes are convenient to use the linguistic variables rather than numerical values. In most ambiguity cases, SVNS plays an important role to model MADM problem. In this paper, we study about SVNS based MADM in which all the attribute weight information is unknown. Entropy based modified GRA analysis method is proposed to solve this MADM problem. Neutrosophic grey relation coefficient is proposed for solving multiple attribute decision-making problems. Finally, an illustrative example is provided to show the feasibility of the developed approach. This proposed method can also be applied in the application of the multiple attribute decision-making with interval valued neutrosophic set and to other domains, such as decision making, pattern recognition, medical diagnosis and clustering analysis.

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