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On Q-Neutrosophic Soft Fields

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Abstract: As an extension of neutrosophic soft sets, Q-neutrosophic soft sets were established to deal with two-dimensional indeterminate data. Different hybrid models of fuzzy sets were utilized to different algebraic structures, for example groups, rings, fields and lie-algebras. A field is an essential algebraic structure, which is widely used in algebra and several domains of mathematics. The motivation of the current work is to extend the thought of Q-neutrosophic soft sets to fields. In this paper, we define the notion of Q-neutrosophic soft fields. Structural characteristics of it are investigated. Moreover, the concepts of homomorphic image and pre-image of Q-neutrosophic soft fields are discussed. Finally, the Cartesian product of Q-neutrosophic soft fields is defined and some related properties are discussed.

Keywords: Neutrosophic soft field, Neutrosophic soft set, Q-neutrosophic soft field, Q-neutrosophic soft set.

1 Introduction

Fuzzy sets were established by Zadeh [1] as a tool to deal with uncertain data. Since then, fuzzy logic has been utilized in several real-world problems in uncertain environments. Consequently, numerous analysts discussed many results using distinct directions of fuzzy-set theory, for instance, interval valued fuzzy set [2] and intuitionistic fuzzy set [3]. These extensions can deal with uncertain real-world problems but it does not cope with indeterminate data. Thus, Smarandache [4] initiated the neutrosophic idea to overcome this problem. A neutrosophic set (NS) [5] is a mathematical notion serving issues containing inconsistent, indeterminate, and imprecise data. Molodtsov [6] introduced the concept of soft sets as another way to handle uncertainty. Since its initiation, a plenty of hybrid models of soft set have been produced, for example, fuzzy soft sets [7], neutrosophic soft sets (NSSs) [8]. Accordingly, NSSs became an important notion for more deep discussions [9–17]. NSSs were extended to Q-neutrosophic soft sets (Q-NSSs) [18] a new model that deals with two-dimensional uncertain data. Q-NSSs were further investigated and their basic operations and relations were discussed in [18, 19].

Different hybrid models of fuzzy sets and soft sets were utilized in different branches of mathematics, including algebra. This was started by Rosenfeld in 1971 [20] when he established the idea of fuzzy subgroup. Since then, the theories and approaches of fuzzy soft sets on different algebraic structures developed rapidly. Mukherjee and Bhattacharya [21] studied fuzzy groups, Sharma [22] discussed intuitionistic fuzzy groups. Recently, many researchers have applied different hybrid models of fuzzy sets and soft sets to several algebraic structures such as groups, semigroups, rings, fields and BCK/BCI-algebras [23–32]. NSs and NSSs have received more attention in studying the algebraic structure of set theories dealing with uncertainty. Cetkin and Aygun [33] established the concept of neutrosophic subgroup. Bera and Mahapatra introduced the notion of neutrosophic soft group [34], neutrosophic soft fields [35]. Moreover, two-dimensional hybrid models of fuzzy sets and soft sets were also applied to different algebraic structures. Solairaju and Nagarajan [36] introduced the notion of Q-fuzzy groups. Thiruveni and Solairaju defined the concept of neutrosophic Q-fuzzy subgroup [37], while Rasuli [38] established the notion of Q-fuzzy subring and anti Q-fuzzy subring. The concept of Q-NSSs was also implemented in the theories of groups and rings [39, 40].

Inspired by the above works and to utilize Q-NSSs to different algebraic structures, in the current paper, we continue the work presented in [41] about Q-neutrosophic soft fields (Q-NSFs) and investigate some of its structural characteristics; we give some theorems that simplifies the main definition, also we discuss the intersection and union of two Q-NSFs . The concepts of homomorphic image and pre-image of Q-NSFs are investigated. Also, we discuss the Cartesian product of Q-NSFs and discuss some related properties.

2 Preliminaries

In this section, we recall the basic definitions related to this work.

Definition 2.1 ([18]). Let X be a universal set, Q be a nonempty set and $A \subseteq E$ be a set of parameters. Let $\mu^l QNS(X)$ be the set of all multi Q-NSs on X with dimension $l = 1$. A pair (Γ_Q, A) is called a Q-NSS over X, where $\Gamma_Q : A \to \mu^l QNS(X)$ is a mapping, such that $\Gamma_Q(e) = \phi$ if $e \notin A$.

Definition 2.2 ([19]). The union of two Q-NSSs (Γ_Q, A) and (Ψ_Q, B) is the Q-NSS (Λ_Q, C) written as $(\Gamma_Q, A) \cup (\Psi_Q, B) = (\Lambda_Q, C)$, where $C = A \cup B$ and for all $c \in C, (x, q) \in X \times Q$, the truth-membership, indeterminacy-membership and falsity-membership of (Λ_Q, C) are as follows:

$$T_{\Lambda_{Q(c)}}(x,q) = \begin{cases} T_{\Gamma_{Q(c)}}(x,q) & \text{if } c \in A - B, \\ T_{\Psi_{Q(c)}}(x,q) & \text{if } c \in B - A, \\ \max\{T_{\Gamma_{Q(c)}}(x,q), T_{\Psi_{Q(c)}}(x,q)\} & \text{if } c \in A \cap B, \end{cases}$$

$$I_{\Lambda_{Q(c)}}(x,q) = \begin{cases} I_{\Gamma_{Q(c)}}(x,q) & \text{if } c \in A - B, \\ I_{\Psi_{Q(c)}}(x,q) & \text{if } c \in B - A, \\ \min\{I_{\Gamma_{Q(c)}}(x,q), I_{\Psi_{Q(c)}}(x,q)\} & \text{if } c \in A \cap B, \end{cases}$$

$$F_{\Lambda_{Q(c)}}(x,q) = \begin{cases} F_{\Gamma_{Q(c)}}(x,q) & \text{if } c \in A - B, \\ F_{\Psi_{Q(c)}}(x,q) & \text{if } c \in B - A, \\ \min\{F_{\Gamma_{Q(c)}}(x,q), F_{\Psi_{Q(c)}}(x,q)\} & \text{if } c \in A \cap B. \end{cases}$$

Definition 2.3 ([19]). The intersection of two Q-NSSs (Γ_Q, A) and (Ψ_Q, B) is the Q-NSS (Λ_Q, C) written as $(\Gamma_Q, A) \cap (\Psi_Q, B) = (\Lambda_Q, C)$, where $C = A \cap B$ and for all $c \in C$ and $(x, q) \in X \times Q$ the truth-membership, indeterminacy-membership and falsity-membership of (Λ_Q, C) are as follows:

$$\begin{aligned}
T_{\Lambda_{Q(c)}}(x,q) &= \min\{T_{\Gamma_{Q(c)}}(x,q), T_{\Psi_{Q(c)}}(x,q)\},\\
I_{\Lambda_{Q(c)}}(x,q) &= \max\{I_{\Gamma_{Q(c)}}(x,q), I_{\Psi_{Q(c)}}(x,q)\},\\
F_{\Lambda_{Q(c)}}(x,q) &= \max\{F_{\Gamma_{Q(c)}}(x,q), F_{\Psi_{Q(c)}}(x,q)\}.
\end{aligned}$$

3 Q-Neutrosophic Soft Fields

In this section, we define the notion of Q-NSF and discuss several related properties.

Definition 3.1. Let (Γ_Q, A) be a Q-NSS over a field $(F,+,.)$. Then, (Γ_Q, A) is said to be a Q-NSF over $(F,+,.)$ if for all $e \in A$, $\Gamma_Q(e)$ is a Q-neutrosophic subfield of $(F,+,.)$, where $\Gamma_Q(e)$ is a mapping given by $\Gamma_Q(e) : F \times Q \to [0,1]^3$.

Definition 3.2. Let $(F,+,.)$ be a field and (Γ_Q, A) be a Q-NSS over $(F,+,.)$. Then, (Γ_Q, A) is called a Q-NSF over $(F,+,.)$ if for all $x, y \in F, q \in Q$ and $e \in A$ it satisfies:

1. $T_{\Gamma_{Q(e)}}(x+y,q) \geq \min\left\{T_{\Gamma_{Q(e)}}(x,q), T_{\Gamma_{Q(e)}}(y,q)\right\}, I_{\Gamma_{Q(e)}}(x+y,q) \leq \max\left\{I_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(y,q)\right\}$ and $F_{\Gamma_{Q(e)}}(x+y,q) \leq \max\left\{F_{\Gamma_{Q(e)}}(x,q), F_{\Gamma_{Q(e)}}(y,q)\right\}$.
2. $T_{\Gamma_{Q(e)}}(-x,q) \geq T_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(-x,q) \leq I_{\Gamma_{Q(e)}}(x,q)$ and $F_{\Gamma_{Q(e)}}(-x,q) \leq F_{\Gamma_{Q(e)}}(x,q)$.
3. $T_{\Gamma_{Q(e)}}(x.y,q) \geq \min\left\{T_{\Gamma_{Q(e)}}(x,q), T_{\Gamma_{Q(e)}}(y,q)\right\}, I_{\Gamma_{Q(e)}}(x.y,q) \leq \max\left\{I_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(y,q)\right\}$ and $F_{\Gamma_{Q(e)}}(x.y,q) \leq \max\left\{F_{\Gamma_{Q(e)}}(x,q), F_{\Gamma_{Q(e)}}(y,q)\right\}$.
4. $T_{\Gamma_{Q(e)}}(x^{-1},q) \geq T_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(x^{-1},q) \leq I_{\Gamma_{Q(e)}}(x,q)$ and $F_{\Gamma_{Q(e)}}(x^{-1},q) \leq F_{\Gamma_{Q(e)}}(x,q)$.

Example 3.3. Let $F = (\mathbb{R},+,.)$ be the field of real numbers and $A = \mathbb{N}$ the set of natural numbers be the parametric set. Define a Q-NSS (Γ_Q, A) as follows for $q \in Q, x \in \mathbb{R}$ and $m \in \mathbb{N}$

$$T_{\Gamma_{Q(m)}}(x,q) = \begin{cases} 0 & \text{if } x \text{ is rational} \\ \frac{1}{9m} & \text{if } x \text{ is irrational} \end{cases},$$

$$I_{\Gamma_{Q(m)}}(x,q) = \begin{cases} 1-\frac{1}{3m} & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases},$$

$$F_{\Gamma_{Q(m)}}(x,q) = \begin{cases} 1+\frac{3}{m} & \text{if } x \text{ is rational} \\ 0 & \text{if } x \text{ is irrational} \end{cases}.$$

It is clear that $(\Gamma_Q, \mathbb{N})$ is a Q-NSF over F.

Proposition 3.4. *Let (Γ_Q, A) be a Q-NSF over $(F,+,.)$. Then, for the additive identity 0_F and the multiplicative identity 1_F, for all $x \in F, q \in Q$ and $e \in A$ the following hold*

1. $T_{\Gamma_{Q(e)}}(0_F,q) \geq T_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(0_F,q) \leq I_{\Gamma_{Q(e)}}(x,q)$ *and* $F_{\Gamma_{Q(e)}}(0_F,q) \leq F_{\Gamma_{Q(e)}}(x,q)$.

2. $T_{\Gamma_{Q(e)}}(1_F, q) \geq T_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(1_F, q) \leq I_{\Gamma_{Q(e)}}(x, q)$ *and* $F_{\Gamma_{Q(e)}}(1_F, q) \leq F_{\Gamma_{Q(e)}}(x, q)$, *for* $x \neq 0_F$.

3. $T_{\Gamma_{Q(e)}}(0_F, q) \geq T_{\Gamma_{Q(e)}}(1_F, q), I_{\Gamma_{Q(e)}}(0_F, q) \leq I_{\Gamma_{Q(e)}}(1_F, q)$ *and* $F_{\Gamma_{Q(e)}}(0_F, q) \leq F_{\Gamma_{Q(e)}}(1_F, q)$.

Proof. $\forall x \in F, q \in Q$ and $e \in A$

1. $T_{\Gamma_{Q(e)}}(0_F, q) = T_{\Gamma_{Q(e)}}(x - x, q) \geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(x, q)\right\} = T_{\Gamma_{Q(e)}}(x, q)$,
$I_{\Gamma_{Q(e)}}(0_F, q) = I_{\Gamma_{Q(e)}}(x - x, q) \leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(x, q)\right\} = I_{\Gamma_{Q(e)}}(x, q)$,
$F_{\Gamma_{Q(e)}}(0_F, q) = F_{\Gamma_{Q(e)}}(x - x, q) \leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(x, q)\right\} = F_{\Gamma_{Q(e)}}(x, q)$.

2. $T_{\Gamma_{Q(e)}}(1_F, q) = T_{\Gamma_{Q(e)}}(x.x^{-1}, q) \geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(x, q)\right\} = T_{\Gamma_{Q(e)}}(x, q)$,
$I_{\Gamma_{Q(e)}}(1_F, q) = I_{\Gamma_{Q(e)}}(x.x^{-1}, q) \leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(x, q)\right\} = I_{\Gamma_{Q(e)}}(x, q)$,
$F_{\Gamma_{Q(e)}}(1_F, q) = F_{\Gamma_{Q(e)}}(x.x^{-1}, q) \leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(x, q)\right\} = F_{\Gamma_{Q(e)}}(x, q)$.

3. Follows directly by applying 1. □

Theorem 3.5. *A Q-NSS* (Γ_Q, A) *over the field* $(F, +, .)$ *is a Q-NSF if and only if for all* $x, y \in F, q \in Q$ *and* $e \in A$

1. $T_{\Gamma_{Q(e)}}(x - y, q) \geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(y, q)\right\}, I_{\Gamma_{Q(e)}}(x - y, q) \leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(y, q)\right\}, F_{\Gamma_{Q(e)}}(x - y, q) \leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(y, q)\right\}$.

2. $T_{\Gamma_{Q(e)}}(x.y^{-1}, q) \geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(y, q)\right\}, I_{\Gamma_{Q(e)}}(x.y^{-1}, q) \leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(y, q)\right\}, F_{\Gamma_{Q(e)}}(x.y^{-1}, q) \leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(y, q)\right\}$.

Proof. Suppose that (Γ_Q, A) is a Q-NSF over $(F, +, .)$. Then,

$$\begin{aligned}
T_{\Gamma_{Q(e)}}(x - y, q) &\geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(-y, q)\right\} \geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(y, q)\right\},\\
I_{\Gamma_{Q(e)}}(x - y, q) &\leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(-y, q)\right\} \leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(y, q)\right\},\\
F_{\Gamma_{Q(e)}}(x - y, q) &\leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(-y, q)\right\} \leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(y, q)\right\}.
\end{aligned}$$

Also,

$$\begin{aligned}
T_{\Gamma_{Q(e)}}(x.y^{-1}, q) &\geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(y^{-1}, q)\right\} \geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(y, q)\right\},\\
I_{\Gamma_{Q(e)}}(x.y^{-1}, q) &\leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(y^{-1}, q)\right\} \leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(y, q)\right\},\\
F_{\Gamma_{Q(e)}}(x.y^{-1}, q) &\leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(y^{-1}, q)\right\} \leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(y, q)\right\}.
\end{aligned}$$

Conversely, Suppose that conditions 1 and 2 are satisfied. We show that for each $e \in A$, (Γ_Q, A) is a Q-neutrosophic subfield

$$\begin{aligned}
T_{\Gamma_{Q(e)}}(-x, q) = T_{\Gamma_{Q(e)}}(0_F - x, q) &\geq \min\left\{T_{\Gamma_{Q(e)}}(0_F, q), T_{\Gamma_{Q(e)}}(x, q)\right\}\\
&\geq \min\left\{T_{\Gamma_{Q(e)}}(x, q), T_{\Gamma_{Q(e)}}(x, q)\right\} = T_{\Gamma_{Q(e)}}(x, q),\\
I_{\Gamma_{Q(e)}}(-x, q) = I_{\Gamma_{Q(e)}}(0_F - x, q) &\leq \max\left\{I_{\Gamma_{Q(e)}}(0_F, q), I_{\Gamma_{Q(e)}}(x, q)\right\}\\
&\leq \max\left\{I_{\Gamma_{Q(e)}}(x, q), I_{\Gamma_{Q(e)}}(x, q)\right\} = I_{\Gamma_{Q(e)}}(x, q),\\
F_{\Gamma_{Q(e)}}(-x, q) = F_{\Gamma_{Q(e)}}(0_F - x, q) &\leq \max\left\{F_{\Gamma_{Q(e)}}(0_F, q), F_{\Gamma_{Q(e)}}(x, q)\right\}\\
&\leq \max\left\{F_{\Gamma_{Q(e)}}(x, q), F_{\Gamma_{Q(e)}}(x, q)\right\} = F_{\Gamma_{Q(e)}}(x, q)\}
\end{aligned}$$

also,

$$\begin{aligned}
T_{\Gamma_{Q(e)}}(x+y,q) = T_{\Gamma_{Q(e)}}(x-(-y),q) &\geq \min\left\{T_{\Gamma_{Q(e)}}(x,q), T_{\Gamma_{Q(e)}}(y,q)\right\},\\
I_{\Gamma_{Q(e)}}(x+y,q) = I_{\Gamma_{Q(e)}}(x-(-y),q) &\leq \max\left\{I_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(y,q)\right\},\\
F_{\Gamma_{Q(e)}}(x+y,q) = F_{\Gamma_{Q(e)}}(x-(-y),q) &\leq \max\left\{F_{\Gamma_{Q(e)}}(x,q), F_{\Gamma_{Q(e)}}(y,q)\right\}.
\end{aligned}$$

Next,

$$\begin{aligned}
T_{\Gamma_{Q(e)}}(x^{-1},q) = T_{\Gamma_{Q(e)}}(1_F.x^{-1},q) &\geq \min\left\{T_{\Gamma_{Q(e)}}(1_F,q), T_{\Gamma_{Q(e)}}(x,q)\right\}\\
&\geq \min\left\{T_{\Gamma_{Q(e)}}(x,q), T_{\Gamma_{Q(e)}}(x,q)\right\} = T_{\Gamma_{Q(e)}}(x,q),\\
I_{\Gamma_{Q(e)}}(x^{-1},q) = I_{\Gamma_{Q(e)}}(1_F.x^{-1},q) &\leq \max\left\{I_{\Gamma_{Q(e)}}(1_F,q), I_{\Gamma_{Q(e)}}(x,q)\right\}\\
&\leq \max\left\{I_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(x,q)\right\} = I_{\Gamma_{Q(e)}}(x,q),\\
F_{\Gamma_{Q(e)}}(x^{-1},q) = F_{\Gamma_{Q(e)}}(1_F.x^{-1},q) &\leq \max\left\{F_{\Gamma_{Q(e)}}(1_F,q), F_{\Gamma_{Q(e)}}(x,q)\right\}\\
&\leq \max\left\{F_{\Gamma_{Q(e)}}(x,q), F_{\Gamma_{Q(e)}}(x,q)\right\} = F_{\Gamma_{Q(e)}}(x,q)\}
\end{aligned}$$

and

$$\begin{aligned}
T_{\Gamma_{Q(e)}}(x.y,q) = T_{\Gamma_{Q(e)}}(x(y^{-1})^{-1},q) &\geq \min\left\{T_{\Gamma_{Q(e)}}(x,q), T_{\Gamma_{Q(e)}}(y,q)\right\},\\
I_{\Gamma_{Q(e)}}(x.y,q) = I_{\Gamma_{Q(e)}}(x(y^{-1})^{-1},q) &\leq \max\left\{I_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(y,q)\right\},\\
F_{\Gamma_{Q(e)}}(x.y,q) = F_{\Gamma_{Q(e)}}(x(y^{-1})^{-1},q) &\leq \max\left\{F_{\Gamma_{Q(e)}}(x,q), F_{\Gamma_{Q(e)}}(y,q)\right\}.
\end{aligned}$$

This completes the proof. □

Theorem 3.6. *Let* (Γ_Q, A) *and* (Ψ_Q, B) *be two Q-NSFs over* $(F, +, .)$. *Then,* $(\Gamma_Q, A) \cap (\Psi_Q, B)$ *is also Q-NSF over* $(F, +, .)$.

Proof. Let $(\Gamma_Q, A) \cap (\Psi_Q, B) = (\Lambda_Q, A \cap B)$. Now, $\forall x, y \in F, q \in Q$ and $e \in A \cap B$,

$$\begin{aligned}
T_{\Lambda_{Q(e)}}(x-y,q) &= \min\Big\{T_{\Gamma_{Q(e)}}(x-y,q), T_{\Psi_{Q(e)}}(x-y,q)\Big\}\\
&\geq \min\Big\{\min\left\{T_{\Gamma_{Q(e)}}(x,q), T_{\Gamma_{Q(e)}}(y,q)\right\}, \min\left\{T_{\Psi_{Q(e)}}(x,q), T_{\Psi_{Q(e)}}(y,q)\right\}\Big\}\\
&= \min\Big\{\min\left\{T_{\Gamma_{Q(e)}}(x,q), T_{\Psi_{Q(e)}}(x,q)\right\}, \min\left\{T_{\Gamma_{Q(e)}}(y,q), T_{\Psi_{Q(e)}}(y,q)\right\}\Big\}\\
&= \min\Big\{T_{\Lambda_{Q(e)}}(x,q), T_{\Lambda_{Q(e)}}(y,q)\Big\},
\end{aligned}$$

also,

$$\begin{aligned}
I_{\Lambda_{Q(e)}}(x-y,q) &= \max\Big\{I_{\Gamma_{Q(e)}}(x-y,q), I_{\Psi_{Q(e)}}(x-y,q)\Big\}\\
&\leq \max\Big\{\max\left\{I_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(y,q)\right\}, \max\left\{I_{\Psi_{Q(e)}}(x,q), I_{\Psi_{Q(e)}}(y,q)\right\}\Big\}\\
&= \max\Big\{\max\left\{I_{\Gamma_{Q(e)}}(x,q), I_{\Psi_{Q(e)}}(x,q)\right\}, \max\left\{I_{\Gamma_{Q(e)}}(y,q), I_{\Psi_{Q(e)}}(y,q)\right\}\Big\}\\
&= \max\Big\{I_{\Lambda_{Q(e)}}(x,q), I_{\Lambda_{Q(e)}}(y,q)\Big\},
\end{aligned}$$

similarly, $F_{\Lambda_{Q(e)}}(x-y,q) \leq \max\Big\{F_{\Lambda_{Q(e)}}(x,q), F_{\Lambda_{Q(e)}}(y,q)\Big\}$. Next,

$$\begin{aligned}
T_{\Lambda_{Q(e)}}(x.y^{-1},q) &= \min\Big\{T_{\Gamma_{Q(e)}}(x.y^{-1},q), T_{\Psi_{Q(e)}}(x.y^{-1},q)\Big\} \\
&\geq \min\Big\{\min\big\{T_{\Gamma_{Q(e)}}(x,q), T_{\Gamma_{Q(e)}}(y,q)\big\}, \min\big\{T_{\Psi_{Q(e)}}(x,q), T_{\Psi_{Q(e)}}(y,q)\big\}\Big\} \\
&= \min\Big\{\min\big\{T_{\Gamma_{Q(e)}}(x,q), T_{\Psi_{Q(e)}}(x,q)\big\}, \min\big\{T_{\Gamma_{Q(e)}}(y,q), T_{\Psi_{Q(e)}}(y,q)\big\}\Big\} \\
&= \min\Big\{T_{\Lambda_{Q(e)}}(x,q), T_{\Lambda_{Q(e)}}(y,q)\Big\},
\end{aligned}$$

also,

$$\begin{aligned}
I_{\Lambda_{Q(e)}}(x.y^{-1},q) &= \max\Big\{I_{\Gamma_{Q(e)}}(x.y^{-1},q), I_{\Psi_{Q(e)}}(x.y^{-1},q)\Big\} \\
&\leq \max\Big\{\max\big\{I_{\Gamma_{Q(e)}}(x,q), I_{\Gamma_{Q(e)}}(y,q)\big\}, \max\big\{I_{\Psi_{Q(e)}}(x,q), I_{\Psi_{Q(e)}}(y,q)\big\}\Big\} \\
&= \max\Big\{\max\big\{I_{\Gamma_{Q(e)}}(x,q), I_{\Psi_{Q(e)}}(x,q)\big\}, \max\big\{I_{\Gamma_{Q(e)}}(y,q), I_{\Psi_{Q(e)}}(y,q)\big\}\Big\} \\
&= \max\Big\{I_{\Lambda_{Q(e)}}(x,q), I_{\Lambda_{Q(e)}}(y,q)\Big\}
\end{aligned}$$

similarly, we can show $F_{\Lambda_{Q(e)}}(x.y^{-1},q) \leq \max\Big\{F_{\Lambda_{Q(e)}}(x,q), F_{\Lambda_{Q(e)}}(y,q)\Big\}$. This completes the proof. □

Remark 3.7. For two Q-NSFs (Γ_Q, A) and (Ψ_Q, B) over $(F,+,.)$, $(\Gamma_Q, A) \cup (\Psi_Q, B)$ is not generally a Q-NSF.
For example, let $F = (\mathbb{Q},+,.), E = 2\mathbb{Z}$. Consider two Q-NSFs (Γ_Q, E) and (Ψ_Q, E) over F as follows: for $x \in \mathbb{Q}, q \in Q$ and $m \in \mathbb{Z}$

$$T_{\Gamma_{Q(4m)}}(x,q) = \begin{cases} 0.50 & \text{if } x = 4tm, \exists t \in \mathbb{Z}, \\ 0 & \text{otherwise,} \end{cases}$$

$$I_{\Gamma_{Q(4m)}}(x,q) = \begin{cases} 0 & \text{if } x = 4tm, \exists t \in \mathbb{Z}, \\ 0.25 & \text{otherwise,} \end{cases}$$

$$F_{\Gamma_{Q(4m)}}(x,q) = \begin{cases} 0.40 & \text{if } x = 4tm, \exists t \in \mathbb{Z}, \\ 0.10 & \text{otherwise,} \end{cases}$$

and

$$T_{\Psi_{Q(4m)}}(x,q) = \begin{cases} 0.70 & \text{if } x = 6tm, \exists t \in \mathbb{Z}, \\ 0 & \text{otherwise,} \end{cases}$$

$$I_{\Psi_{Q(4m)}}(x,q) = \begin{cases} 0 & \text{if } x = 6tm, \exists t \in \mathbb{Z}, \\ 0.50 & \text{otherwise,} \end{cases}$$

$$F_{\Psi_{Q(4m)}}(x,q) = \begin{cases} 0.20 & \text{if } x = 6tm, \exists t \in \mathbb{Z}, \\ 0.40 & \text{otherwise.} \end{cases}$$

Let $(\Gamma_Q, A) \cup (\Psi_Q, B) = (\Lambda_Q, E)$. For $m = 2, x = 8, y = 12$ we have

$$T_{\Lambda_Q(8)}(8-12,q) = T_{\Lambda_Q(8)}(-4,q) = \max\left\{T_{\Gamma_Q(8)}(-4,q), T_{\Psi_Q(8)}(-4,q)\right\} = \max\{0,0\} = 0$$

and

$$\begin{aligned}
\min\left\{T_{\Lambda_Q(8)}(8,q),T_{\Lambda_Q(8)}(12,q)\right\} \\
&= \min\left\{\max\left\{T_{\Gamma_Q(8)}(8,q), T_{\Psi_Q(8)}(8,q)\right\}, \max\left\{T_{\Gamma_Q(8)}(12,q), T_{\Psi_Q(8)}(12,q)\right\}\right\} \\
&= \min\left\{\max\left\{0.50, 0\right\}, \max\left\{0, 0.7\right\}\right\} \\
&= \min\left\{0.50, 0.70\right\} = 0.50.
\end{aligned}$$

Hence, $T_{\Lambda_Q(8)}(8-12,q) < \min\left\{T_{\Lambda_Q(8)}(8,q), T_{\Lambda_Q(8)}(12,q)\right\}$. Thus, the union is not a Q-NSF.

4 Q-Neutrosophic Soft Homomorphism

In this section, we define the Q-neutrosophic soft function, then define the image and pre-image of a Q-NSS under a Q-neutrosophic soft function. In continuation, we introduce the notion of Q-neutrosophic soft homomorphism along with some of it's properties.

Definition 4.1. Let $g : X \times Q \to Y \times Q$ and $h : A \to B$ be two functions where A and B are parameter sets. Then, the pair (g, h) is called a Q-neutrosophic soft function from $X \times Q$ to $Y \times Q$.

Definition 4.2. Let (Γ_Q, A) and (Ψ_Q, B) be two Q-NSSs defined over $X \times Q$ and $Y \times Q$, respectively, and (g, h) be a Q-neutrosophic soft function from $X \times Q$ to $Y \times Q$. Then,

1. The image of (Γ_Q, A) under (g, h), denoted by $(g, h)(\Gamma_Q, A)$, is a Q-NSS over $Y \times Q$ and is defined by:

$$(g,h)(\Gamma_Q, A) = \Big(g(\Gamma_Q), h(A)\Big) = \left\{\left\langle b, g(\Gamma_Q)(b) : b \in h(A)\right\rangle\right\},$$

   where for all $b \in h(A), y \in Y$ and $q \in Q$,

$$T_{g(\Gamma_Q)(b)}(y,q) = \begin{cases} \max_{g(x,q)=(y,q)} \max_{h(a)=b}[T_{\Gamma_Q(a)}(x,q)] & \text{if } (x,q) \in g^{-1}(y,q), \\ 0 & \text{otherwise,} \end{cases}$$

$$I_{g(\Gamma_Q)(b)}(y,q) = \begin{cases} \min_{g(x,q)=(y,q)} \min_{h(a)=b}[I_{\Gamma_Q(a)}(x,q)] & \text{if } (x,q) \in g^{-1}(y,q), \\ 1 & \text{otherwise,} \end{cases}$$

$$F_{g(\Gamma_Q)(b)}(y,q) = \begin{cases} \min_{g(x,q)=(y,q)} \min_{h(a)=b}[F_{\Gamma_Q(a)}(x,q)] & \text{if } (x,q) \in g^{-1}(y,q), \\ 1 & \text{otherwise,} \end{cases}$$

2. The preimage of (Ψ_Q, B) under (g, h), denoted by $(g, h)^{-1}(\Psi_Q, B)$, is a Q-NSS over X and is defined

by:

$$(g,h)^{-1}(\Psi_Q,B)=\Big(g^{-1}(\Psi_Q),h^{-1}(B)\Big)=\Big\{\Big\langle a,g^{-1}(\Psi_Q)(a):a\in h^{-1}(B)\Big\rangle\Big\},$$

where for all $a\in h^{-1}(B), x\in X$ and $q\in Q$,

$$\begin{aligned}T_{g^{-1}(\Psi_Q)(a)}(x,q)&=T_{\Psi_Q[h(a)]}(g(x,q)),\\ I_{g^{-1}(\Psi_Q)(a)}(x,q)&=I_{\Psi_Q[h(a)]}(g(x,q)),\\ F_{g^{-1}(\Psi_Q)(a)}(x,q)&=F_{\Psi_Q[h(a)]}(g(x,q)).\end{aligned}$$

If g and h are injective (surjective), then (g,h) is injective (surjective).

Definition 4.3. Let (g,h) be a Q-neutrosophic soft function from $X\times Q$ to $Y\times Q$. If g is a homomorphism from $X\times Q$ to $Y\times Q$, then (g,h) is said to be a Q-neutrosophic soft homomorphism. If g is an isomorphism from $X\times Q$ to $Y\times Q$ and h is a one-to-one mapping from A to B, then (g,h) is said to be a Q-neutrosophic soft isomorphism.

Example 4.4. Let $A=\mathbb{N}$ (the set of natural numbers) be the parametric set and $F=(\mathbb{Z}_5,+,.)$ be a field. Define a Q-NSS (Γ_Q,A) as follows, for any $a\in A, q\in Q$ and $x\in\mathbb{Z}_5$,

$$T_{\Gamma_Q(a)}(x,q)=\begin{cases}0 & \text{if } x\in\{\bar{1},\bar{3}\}\\ \frac{1}{3a} & \text{if } x\in\{\bar{0},\bar{2},\bar{4}\}\end{cases},$$

$$I_{\Gamma_Q(a)}(x,q)=\begin{cases}1-\frac{1}{a} & \text{if } x\in\{\bar{1},\bar{3}\}\\ 0 & \text{if } x\in\{\bar{0},\bar{2},\bar{4}\}\end{cases},$$

$$F_{\Gamma_Q(a)}(x,q)=\begin{cases}\frac{3}{a+1} & \text{if } x\in\{\bar{1},\bar{3}\}\\ 0 & \text{if } x\in\{\bar{0},\bar{2},\bar{4}\}\end{cases}.$$

Now, let $g:\mathbb{Z}_5\times Q\to\mathbb{Z}_5\times Q$ and $h:\mathbb{N}\to\mathbb{N}$ be given by $g(x,q)=3x+1$ and $h(a)=a^2$. Then for $b\in\mathbb{N}^2, y\in 3\mathbb{Z}_5+1$, the image of (Γ_Q,A) under (g,h) as follows :

$$T_{g(\Gamma_Q)(b)}(y,q)=\begin{cases}0 & \text{if } y\in\{\bar{0},\bar{2},\bar{4}\}\\ \frac{1}{3\sqrt{b}} & \text{if } y\in\{\bar{1},\bar{3}\}\end{cases},$$

$$I_{g(\Gamma_Q)(b)}(y,q)=\begin{cases}1-\frac{1}{\sqrt{b}} & \text{if } y\in\{\bar{0},\bar{2},\bar{4}\}\\ 0 & \text{if } y\in\{\bar{1},\bar{3}\}\end{cases},$$

$$F_{g(\Gamma_Q)(b)}(y,q)=\begin{cases}\frac{1}{1+\sqrt{b}} & \text{if } y\in\{\bar{0},\bar{2},\bar{4}\}\\ 0 & \text{if } y\in\{\bar{1},\bar{3}\}\end{cases}.$$

Theorem 4.5. *Let (Γ_Q,A) be a Q-NSF over F_1 and $(g,h):F_1\times Q\to F_2\times Q$ be a Q-neutrosophic soft homomorphism. Then, $(g,h)(\Gamma_Q,A)$ is a Q-NSF over F_2.*

Proof. Let $b\in h(A)$ and $y_1,y_2\in F_2$. For $g^{-1}(y_1,q)=\phi$ or $g^{-1}(y_2,q)=\phi$, the proof is straight forward.

So, assume there exists $x_1, x_2 \in F_1$ such that $g(x_1, q) = (y_1, q)$ and $g(x_2, q) = (y_2, q)$. Then,

$$\begin{aligned}
T_{g(\Gamma_Q)(b)}(y_1 - y_2, q) &= \max_{g(x,q)=(y_1-y_2,q)} \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x, q)\right] \\
&\geq \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_1 - x_2, q)\right] \\
&\geq \max_{h(a)=b} \left[\min\left\{T_{\Gamma_Q(a)}(x_1, q), T_{\Gamma_Q(a)}(-x_2, q)\right\}\right] \\
&\geq \max_{h(a)=b} \left[\min\left\{T_{\Gamma_Q(a)}(x_1, q), T_{\Gamma_Q(a)}(x_2, q)\right\}\right] \\
&= \min\left\{\max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_1, q)\right], \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_2, q)\right]\right\}
\end{aligned}$$

$$\begin{aligned}
T_{g(\Gamma_Q)(b)}(y_1.y_2^{-1}, q) &= \max_{g(x,q)=(y_1.y_2^{-1},q)} \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x, q)\right] \\
&\geq \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_1.x_2^{-1}, q)\right] \\
&\geq \max_{h(a)=b} \left[\min\left\{T_{\Gamma_Q(a)}(x_1, q), T_{\Gamma_Q(a)}(x_2^{-1}, q)\right\}\right] \\
&\geq \max_{h(a)=b} \left[\min\left\{T_{\Gamma_Q(a)}(x_1, q), T_{\Gamma_Q(a)}(x_2, q)\right\}\right] \\
&= \min\left\{\max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_1, q)\right], \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_2, q)\right]\right\}
\end{aligned}$$

Since, the inequality is satisfied for each $x_1, x_2 \in F_1$, satisfying $g(x_1, q) = (y_1, q)$ and $g(x_2, q) = (y_2, q)$. Then,

$$\begin{aligned}
T_{g(\Gamma_Q)(b)}(y_1 - y_2, q) &\geq \min\left\{\max_{g(x_1,q)=(y_1,q)} \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_1, q)\right], \max_{g(x_2,q)=(y_1,q)} \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_2, q)\right]\right\} \\
&= \min\left\{T_{g(\Gamma_Q)(b)}(y_1, q), T_{g(\Gamma_Q)(b)}(y_2, q)\right\}.
\end{aligned}$$

$$\begin{aligned}
T_{g(\Gamma_Q)(b)}(y_1.y_2^{-1}, q) &\geq \min\left\{\max_{g(x_1,q)=(y_1,q)} \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_1, q)\right], \max_{g(x_2,q)=(y_1,q)} \max_{h(a)=b} \left[T_{\Gamma_Q(a)}(x_2, q)\right]\right\} \\
&= \min\left\{T_{g(\Gamma_Q)(b)}(y_1, q), T_{g(\Gamma_Q)(b)}(y_2, q)\right\}.
\end{aligned}$$

Similarly, we show that
$I_{g(\Gamma_Q)(b)}(y_1 - y_2, q) \leq \max\left\{I_{g(\Gamma_Q)(b)}(y_1, q), I_{g(\Gamma_Q)(b)}(y_2, q)\right\}$,
$I_{g(\Gamma_Q)(b)}(y_1.y_2^{-1}, q) \leq \max\left\{I_{g(\Gamma_Q)(b)}(y_1, q), I_{g(\Gamma_Q)(b)}(y_2, q)\right\}$,
$F_{g(\Gamma_Q)(b)}(y_1 - y_2, q) \leq \max\left\{F_{g(\Gamma_Q)(b)}(y_1, q), F_{g(\Gamma_Q)(b)}(y_2, q)\right\}$,
$F_{g(\Gamma_Q)(b)}(y_1.y_2^{-1}, q) \leq \max\left\{F_{g(\Gamma_Q)(b)}(y_1, q), F_{g(\Gamma_Q)(b)}(y_2, q)\right\}$.

□

Theorem 4.6. *Let (Ψ_Q, B) be a Q-NSF over F_2 and (g, h) be a Q-neutrosophic soft homomorphism from $F_1 \times Q$ to $F_2 \times Q$. Then, $(g, h)^{-1}(\Psi_Q, B)$ is a Q-NSF over over F_1.*

Proof. For $a \in h^{-1}(B)$ and $x_1, x_2 \in F_1$, we have

$$\begin{aligned} T_{g^{-1}(\Psi_Q)(a)}(x_1 - x_2, q) &= T_{\Psi_Q[h(a)]}(g(x_1 - x_2, q)) \\ &= T_{\Psi_Q[h(a)]}(g(x_1, q) - g(x_2, q)) \\ &\geq \min\left\{T_{\Psi_Q[h(a)]}(g(x_1, q)), T_{\Psi_Q[h(a)]}(-g(x_2, q))\right\} \\ &\geq \min\left\{T_{\Psi_Q[h(a)]}(g(x_1, q)), T_{\Psi_Q[h(a)]}(g(x_2, q))\right\} \\ &= \min\left\{T_{g^{-1}(\Psi_Q)(a)}(x_1, q), T_{g^{-1}(\Psi_Q)(a)}(x_2, q)\right\} \end{aligned}$$

and

$$\begin{aligned} T_{g^{-1}(\Psi_Q)(a)}(x_1.x_2^{-1}, q) &= T_{\Psi_Q[h(a)]}(g(x_1.x_2^{-1}, q)) \\ &= T_{\Psi_Q[h(a)]}(g(x_1, q).g(x_2^{-1}, q)) \\ &\geq \min\left\{T_{\Psi_Q[h(a)]}(g(x_1, q)), T_{\Psi_Q[h(a)]}(g(x_2, q)^{-1})\right\} \\ &\geq \min\left\{T_{\Psi_Q[h(a)]}(g(x_1, q)), T_{\Psi_Q[h(a)]}(g(x_2, q))\right\} \\ &= \min\left\{T_{g^{-1}(\Psi_Q)(a)}(x_1, q), T_{g^{-1}(\Psi_Q)(a)}(x_2, q)\right\} \end{aligned}$$

Similarly, we can obtain

$$\begin{aligned} I_{g^{-1}(\Psi_Q)(a)}(x_1 - x_2, q) &\leq \max\left\{I_{g^{-1}(\Psi_Q)(a)}(x_1, q), I_{g^{-1}(\Psi_Q)(a)}(x_2, q)\right\}, \\ I_{g^{-1}(\Psi_Q)(a)}(x_1.x_2^{-1}, q) &\leq \max\left\{I_{g^{-1}(\Psi_Q)(a)}(x_1, q), I_{g^{-1}(\Psi_Q)(a)}(x_2, q)\right\}, \\ F_{g^{-1}(\Psi_Q)(a)}(x_1 - x_2, q) &\leq \max\left\{F_{g^{-1}(\Psi_Q)(a)}(x_1, q), F_{g^{-1}(\Psi_Q)(a)}(x_2, q)\right\}, \\ F_{g^{-1}(\Psi_Q)(a)}(x_1.x_2^{-1}, q) &\leq \max\left\{F_{g^{-1}(\Psi_Q)(a)}(x_1, q), F_{g^{-1}(\Psi_Q)(a)}(x_2, q)\right\}. \end{aligned}$$

Thus, the theorem is proved. □

5 Cartesian Product of Q-Neutrosophic Soft Fields

In this section, we define the Cartesian product of Q-NSFs and prove that it is also a Q-NSF.

Definition 5.1. Let (Γ_Q, A) and (Ψ_Q, B) be two Q-NSFs over $(F_1, +, .)$ and $(F_2, +, .)$, respectively. Then, their Cartesian product $(\Lambda_Q, A \times B) = (\Gamma_Q, A) \times (\Psi_Q, B)$, where $\Lambda_Q(a, b) = \Gamma_Q(a) \times \Psi_Q(b)$ for $(a, b) \in A \times B$. Analytically, for $x \in F_1, y \in F_2$ and $q \in Q$

$$\Lambda_Q(a, b) = \left\{\left\langle\big((x, y), q\big), T_{\Lambda_Q(a,b)}\big((x, y), q\big), I_{\Lambda_Q(a,b)}\big((x, y), q\big), F_{\Lambda_Q(a,b)}\big((x, y), q\big)\right\rangle\right\}, \text{ where}$$

$$T_{\Lambda_Q(a,b)}\big((x,y),q\big) = \min\big\{T_{\Gamma_Q(a)}\big(x,q\big), T_{\Psi_Q(b)}\big(y,q\big)\big\},$$
$$I_{\Lambda_Q(a,b)}\big((x,y),q\big) = \max\big\{I_{\Gamma_Q(a)}\big(x,q\big), I_{\Psi_Q(b)}\big(y,q\big)\big\},$$
$$F_{\Lambda_Q(a,b)}\big((x,y),q\big) = \max\big\{F_{\Gamma_Q(a)}\big(x,q\big), F_{\Psi_Q(b)}\big(y,q\big)\big\}.$$

Theorem 5.2. *Let (Γ_Q, A) and (Ψ_Q, B) be two Q-NSFs over $(F_1, +, .)$ and $(F_2, +, .)$, respectively. Then, their Cartesian product $(\Gamma_Q, A) \times (\Psi_Q, B)$ is a Q-NSF over $(F_1 \times F_2)$.*

Proof. Let $(\Lambda_Q, A \times B) = (\Gamma_Q, A) \times (\Psi_Q, B)$, where $\Lambda_Q(a,b) = \Gamma_Q(a) \times \Psi_Q(b)$ for $(a,b) \in A \times B$. Then, for $\big((x_1,y_1),q\big), \big((x_2,y_2),q\big) \in (F_1 \times F_2) \times Q$ we have,

$$\begin{aligned}
&T_{\Lambda_Q(a,b)}\Big(\big((x_1,y_1)-(x_2,y_2),q\big)\Big)\\
&= T_{\Lambda_Q(a,b)}\big((x_1-x_2, y_1-y_2),q\big)\\
&= \min\big\{T_{\Gamma_Q(a)}\big((x_1-x_2),q\big), T_{\Psi_Q(b)}\big((y_1-y_2),q\big)\big\}\\
&\geq \min\Big\{\min\big\{T_{\Gamma_Q(a)}\big(x_1,q\big), T_{\Gamma_Q(a)}\big(-x_2,q\big)\big\}, \min\big\{T_{\Psi_Q(b)}\big(y_1,q\big), T_{\Psi_Q(b)}\big(-y_2,q\big)\big\}\Big\}\\
&\geq \min\Big\{\min\big\{T_{\Gamma_Q(a)}\big(x_1,q\big), T_{\Gamma_Q(a)}\big(x_2,q\big)\big\}, \min\big\{T_{\Psi_Q(b)}\big(y_1,q\big), T_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}\\
&= \min\Big\{\min\big\{T_{\Gamma_Q(a)}\big(x_1,q\big), T_{\Psi_Q(b)}\big(y_1,q\big)\big\}, \min\big\{T_{\Gamma_Q(a)}\big(x_2,q\big), T_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}\\
&= \min\Big\{T_{\Lambda_Q(a,b)}\big((x_1,y_1),q\big), T_{\Lambda_Q(a,b)}\big((x_2,y_2),q\big)\Big\}
\end{aligned}$$

also,

$$\begin{aligned}
&I_{\Lambda_Q(a,b)}\Big(\big((x_1,y_1)-(x_2,y_2),q\big)\Big)\\
&= I_{\Lambda_Q(a,b)}\big((x_1-x_2, y_1-y_2),q\big)\\
&= \max\big\{I_{\Gamma_Q(a)}\big((x_1-x_2),q\big), I_{\Psi_Q(b)}\big((y_1-y_2),q\big)\big\}\\
&\leq \max\Big\{\max\big\{I_{\Gamma_Q(a)}\big(x_1,q\big), I_{\Gamma_Q(a)}\big(-x_2,q\big)\big\}, \max\big\{I_{\Psi_Q(b)}\big(y_1,q\big), I_{\Psi_Q(b)}\big(-y_2,q\big)\big\}\Big\}\\
&\leq \max\Big\{\max\big\{I_{\Gamma_Q(a)}\big(x_1,q\big), I_{\Gamma_Q(a)}\big(x_2,q\big)\big\}, \max\big\{I_{\Psi_Q(b)}\big(y_1,q\big), I_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}\\
&= \max\Big\{\max\big\{I_{\Gamma_Q(a)}\big(x_1,q\big), I_{\Psi_Q(b)}\big(y_1,q\big)\big\}, \max\big\{I_{\Gamma_Q(a)}\big(x_2,q\big), I_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}\\
&= \max\Big\{I_{\Lambda_Q(a,b)}\big((x_1,y_1),q\big), I_{\Lambda_Q(a,b)}\big((x_2,y_2),q\big)\Big\},
\end{aligned}$$

similarly, $F_{\Lambda_Q(a,b)}\Big(\big((x_1,y_1)-(x_2,y_2),q\big)\Big) \leq \max\Big\{F_{\Lambda_Q(a,b)}\big((x_1,y_1),q\big), F_{\Lambda_Q(a,b)}\big((x_2,y_2),q\big)\Big\}$. Next,

$$\begin{aligned}
&T_{\Lambda_Q(a,b)}\Big(\big((x_1,y_1).(x_2,y_2)^{-1},q\big)\Big)\\
&= T_{\Lambda_Q(a,b)}\big((x_1.x_2^{-1}, y_1.y_2^{-1}),q\big)\\
&= \min\big\{T_{\Gamma_Q(a)}\big((x_1.x_2^{-1}),q\big), T_{\Psi_Q(b)}\big((y_1.y_2^{-1}),q\big)\big\}\\
&\geq \min\Big\{\min\big\{T_{\Gamma_Q(a)}\big(x_1,q\big), T_{\Gamma_Q(a)}\big(x_2^{-1},q\big)\big\}, \min\big\{T_{\Psi_Q(b)}\big(y_1,q\big), T_{\Psi_Q(b)}\big(y_2^{-1},q\big)\big\}\Big\}
\end{aligned}$$

$$\geq \min\Big\{\min\big\{T_{\Gamma_Q(a)}\big(x_1,q\big),T_{\Gamma_Q(a)}\big(x_2,q\big)\big\},\min\big\{T_{\Psi_Q(b)}\big(y_1,q\big),T_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}$$
$$=\min\Big\{\min\big\{T_{\Gamma_Q(a)}\big(x_1,q\big),T_{\Psi_Q(b)}\big(y_1,q\big)\big\},\min\big\{T_{\Gamma_Q(a)}\big(x_2,q\big),T_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}$$
$$=\min\Big\{T_{\Lambda_Q(a,b)}\big((x_1,y_1),q\big),T_{\Lambda_Q(a,b)}\big((x_2,y_2),q\big)\Big\},$$

$$\begin{aligned}
&I_{\Lambda_Q(a,b)}\Big(\big((x_1,y_1).(x_2,y_2)^{-1},q\big)\Big)\\
&\quad=I_{\Lambda_Q(a,b)}\big((x_1.x_2^{-1},y_1.y_2^{-1}),q\big)\\
&\quad=\max\big\{I_{\Gamma_Q(a)}\big((x_1.x_2^{-1}),q\big),I_{\Psi_Q(b)}\big((y_1.y_2^{-1}),q\big)\big\}\\
&\quad\leq\max\Big\{\max\big\{I_{\Gamma_Q(a)}\big(x_1,q\big),I_{\Gamma_Q(a)}\big(x_2^{-1},q\big)\big\},\max\big\{I_{\Psi_Q(b)}\big(y_1,q\big),I_{\Psi_Q(b)}\big(y_2^{-1},q\big)\big\}\Big\}\\
&\quad\leq\max\Big\{\max\big\{I_{\Gamma_Q(a)}\big(x_1,q\big),I_{\Gamma_Q(a)}\big(x_2,q\big)\big\},\max\big\{I_{\Psi_Q(b)}\big(y_1,q\big),I_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}\\
&\quad=\max\Big\{\max\big\{I_{\Gamma_Q(a)}\big(x_1,q\big),I_{\Psi_Q(b)}\big(y_1,q\big)\big\},\max\big\{I_{\Gamma_Q(a)}\big(x_2,q\big),I_{\Psi_Q(b)}\big(y_2,q\big)\big\}\Big\}\\
&\quad=\max\Big\{I_{\Lambda_Q(a,b)}\big((x_1,y_1),q\big),I_{\Lambda_Q(a,b)}\big((x_2,y_2),q\big)\Big\},
\end{aligned}$$

similarly, $F_{\Lambda_Q(a,b)}\Big(\big((x_1,y_1),q\big).\big((x_2,y_2)^{-1},q\big)\Big) \leq \max\Big\{F_{\Lambda_Q(a,b)}\big((x_1,y_1),q\big),F_{\Lambda_Q(a,b)}\big((x_2,y_2),q\big)\Big\}$. This completes the proof. □

6 Conclusions

In this study, we have introduced the concept of Q-neutrosophic soft fields. We have investigated some of its structural characteristics. Also, we have discussed the concepts of homomorphic image and pre-image of Q-neutrosophic soft fields. Moreover, we have defined the Cartesian product of Q-neutrosophic soft fields and discussed some related properties. The proposed notion enriches knowledge on neutrosophic sets in the branch of algebra. Also, it illuminates the way for more further deep discussion in algebra under neutrosophic and Q-neutrosophic soft environment for example, by establishing the notions of n-valued neutrosophic soft fields Q-neutrosophic soft modules and more.

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