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Decision Making Based on Some similarity Measures under Interval Rough Neutrosophic Environment

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Abstract: This paper is devoted to propose cosine, Dice and Jaccard similarity measures of interval rough neutrosophic set and interval neutrosophic mean operator. Some of the properties of the proposed similarity measures have been established. We have proposed multi attribute decision making approaches based on proposed simlarity measures. To demonstrate the applicability and efficiency of the proposed approaches, a numerical example is solved and comparision has been done among the proposed the apprqaches.

Keywords: Tangent similarity measure, Single valued neutrosophic set, Cosine similarity measure, Medical diagnosis

1 Introduction

The concept of neutrosophic set was grounded by one of the greatest mathematician and philosopher Smarandache [1, 2, 3, 4, 5]. The root of neutrosophic set is the neutrosophy, a new branch of philosophy initiated by Smarandache [1]. Neutrosophy studies the ideas and notions that are neutral, indeterminate, unclear, vague, ambiguous, incomplete, contradictory, etc. Inherently, neutrosophic set is capable of dealing with uncertainty, indeterminate and inconsistent information. Smarandache endeavored to propagate the concept of neutrosophic set in all branches of sciences, social sciences and humanities. To use neutrosophic sets in practical fields such as real scientific and engineering applications, Wang et al.[6] extended the concept of neutrosophic set to single valued neutrosophic sets (SVNSs) and studied the set theoretic operators and various properties of SVNSs. Recently, single valued neutrosophic set has caught much attention to the researcher on various topics such as artificial intelligence [7], conflict resolution [8], education [9, 10], decision making [11-27] medical diagnosis [28], social problems [29, 30], etc. Smarandache's original ideas blossomed into a comprehensive corpus of methods and tools for dealing with membership degrees of truth, falsity, indeterminacy and non-probabilistic uncertainty. In essence, the basic concept of neutrosophic set is a generalization of classical set or crisp set [31, 32], fuzzy set [33], intuitionistic fuzzy set [34]. The field has experienced an enormous development, and Smarandache's seminal concept of neutrosophic set [1] has naturally evolved in different directions. Different sets were quickly proposed in the literature such as neutrosophic soft set [35], weighted neutrosophic soft sets [36], generalized neutrosophic soft set [37], Neutrosophic parametrized soft set [38], Neutrosophic soft expert sets [39, 40], neutrosophic refined sets [41, 42]. Neutrosophic soft multi-set [43], neutrosophic bipolar set (44), neutrosophic cubic set (45, 46), neutrosophic complex set (47), rough neutrosophic set (48, 49), interval rough neutrosophic set [50], Interval-valued neutrosophic soft rough sets [51, 52], etc.

Broumi et al. [48, 49] recently proposed new hybrid intelligent structure namely, rough neutrosophic set combing the concept of rough set theory [53] and the concept of neutrosophic set theory to deal with uncertainty and incomplete information. Rough neutrosophic set [48, 49] is the generalization of rough fuzzy sets [54], [55] and rough intuitionistic fuzzy sets [56]. Several studies of rough neutrosophic sets have been reported in the literature. Mondal and Pramanik [57] applied the concept of rough neutrosophic set in multi-attribute decision making based on grey relational analysis. Pramanik and Mondal [58] presented cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Pramanik and Mondal [59] also proposed some rough neutrosophic similarity measures namely Dice and Jaccard similarity measures of rough neutrosophic environment. Mondal and Pramanik [60] proposed rough neutrosophic multi attribute decision making based on rough score accuracy function. Pramanik and Mondal [61] presented cotangent similarity measure of rough neutrosophic sets and its application to medical diagnosis.

In 2015, Broumi and Smarandache [50] combined the concept of rough set theory [53] and interval neutrosophic set theory [62] and defined interval rough neutrosophic set.

In this paper, we develop some similarity measures namely, cCosine, Dice, Jaccard similarity measures based on interval rough neutrosophic sets [50].

Rest of the paper is organized in the following way. Section 2 describes preliminaries of neutrosophic sets and rough neutrosophic sets and interval rough neutrosophic sets. Section 3 presents definitions and propositions of the proposed functions. Section 4 is devoted to present multi attribute decision-making method based on similarity functions. In Section 5, we provide a numerical example of the proposed approaches. Section 6 presents the comparision of results of the three proposed approaches. Finally section 7 presents concluding remarks and future scopes of research.

2 Mathematical preliminaries

2.1 Neutrosophic set

Definition 2.1[1]

Let U be an universe of discourse. Then the neutrosophic set A can be presented of the form:

$A = \{< x{:}T_A(x), I_A(x), F_A(x)>, x \in U\}$, where the functions T, I, F: U→ $]^-0,1^+[$ define respectively the degree of membership, the degree of indeterminacy, and the degree of non-membership of the element $x \in U$ to the set A satisfying the following the condition.

$$^-0 \le \sup T_A(x) + \sup I_A(x) + \sup F_A(x) \le 3^+ \quad (1)$$

For two netrosophic sets (NSs), $A_{NS} = \{<x{:}\ T_A(x), I_A(x), F_A(x)> \mid x \in X\}$ and $B_{NS} = \{< x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$ the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$

(2) $A_{NS} = B_{NS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$

2.2 Single valued neutrosophic set (SVNS)

Definition 2.2 [6]

From philosophical point of view, the neutrosophic set assumes the value from real standard or non-standard subsets of $]^-0, 1^+[$. So instead of $]^-0, 1^+[$ one needs to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as scientific and engineering problems. Wang et. al [6] introduced single valued neutrosophic set (SVNS).

Let X be a space of points with generic element $x \in X$. A SVNS A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$, and a falsity membership function $F_A(x)$, for each point $x \in X$, $T_A(x)$, $I_A(x)$, $F_A(x) \in [0, 1]$. When X is continuous, a SVNS A can be written as follows:

$$A = \int_X \frac{< T_A(x), I_A(x), F_A(x) >}{x} : x \in X$$

When X is discrete, a SVNS A can be written as follows:

$$A = \sum_{i=1}^{n} \frac{< T_A(x_i), I_A(x_i), F_A(x_i) >}{x_i} : x_i \in X$$

For two SVNSs , $A_{SVNS} = \{<x{:}\ T_A(x), I_A(x), F_A(x)> \mid x \in X\}$ and $B_{SVNS} = \{<x, T_B(x), I_B(x), F_B(x)> \mid x \in X\}$ the two relations are defined as follows:

1. $A_{SVNS} \subseteq B_{SVNS}$ if and only if $T_A(x) \le T_B(x)$, $I_A(x) \ge I_B(x)$, $F_A(x) \ge F_B(x)$
2. $A_{SVNS} = B_{SVNS}$ if and only if $T_A(x) = T_B(x)$, $I_A(x) = I_B(x)$, $F_A(x) = F_B(x)$ for any $x \in X$

2.3 Interval neutrosophic sets

Definition 2.3.1 [62]

Let X be a space of points (bjects) with generic element $x \in X$. An interval neutrosophic set (INS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point $x \in X$., we have, $T_A(x), I_A(x), F_A(x) \in [0, 1]$.

For two IVNS,

$A_{INS} = \{< x, \left([T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)]\right) > \mid x \in X\}$ and

$B_{INS} = \{<x, \left([T_B^L(x), T_B^U(x)], [I_B^L(x), I_B^U(x)], [F_B^L(x), F_B^U(x)]\right) > \mid x \in X\}$ the two relations are defined as follows:

1. $A_{INS} \subseteq B_{INS}$ if and only if $T_A^L \le T_B^L$, $T_A^U \le T_B^U$; $I_A^L \ge I_B^L$, $I_A^U \ge I_B^U$; $F_A^L \ge F_B^L$, $F_A^U \ge F_B^U$
2. $A_{INS} = B_{INS}$ if and only if $T_A^L = T_B^L$, $T_A^U = T_B^U$; $I_A^L = I_B^L$, $I_A^U = I_B^U$; $F_A^L = F_B^L$, $F_A^U = F_B^U$

for all $x \in X$

2.4 Rough neutrosophic set

Definition 2.4.1 [48, 49]: Let Z be a non-zero set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of *P* in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P) = \left\langle \begin{matrix} < x, T_{\underline{N}(P)}(x), I_{\underline{N}(P)}(x), F_{\underline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{matrix} \right\rangle \quad (2)$$

$$\overline{N}(P) = \left\langle \begin{array}{l} < x, T_{\overline{N}(P)}(x), I_{\overline{N}(P)}(x),\ F_{\overline{N}(P)}(x) > / \\ z \in [x]_R, x \in Z \end{array} \right\rangle \quad (3)$$

Where, $T_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, T_P(z)$,

$I_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, I_P(z),\ F_{\underline{N}(P)}(x) = \wedge_z \in [x]_R\, F_P(z)$,

$T_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z), I_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, T_P(z)$,

$F_{\overline{N}(P)}(x) = \vee_z \in [x]_R\, I_P(z)$

So, $0 \le T_{\underline{N}(P)}(x) + I_{\underline{N}(P)}(x) + F_{\underline{N}(P)}(x) \le 3$

$0 \le T_{\overline{N}(P)}(x) + I_{\overline{N}(P)}(x) + F_{\overline{N}(P)}(x) \le 3$

The symbols $\vee$ and $\wedge$ denote "max" and "min'' operators respectively. $T_P(z)$, $I_P(z)$ and $F_P(z)$ are the membership, indeterminacy and non-membership of z with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z.

Thus NS mapping $\underline{N}$, $\overline{N}$: N(Z) $\rightarrow$ N(Z) are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair ($\underline{N}(P), \overline{N}(P)$) is called the rough neutrosophic set in (Z, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence clases of R if $\underline{N}(P) = \overline{N}(P)$; .e. $T_{\underline{N}(P)}(x) = T_{\overline{N}(P)}(x)$,

$I_{\underline{N}(P)}(x) = I_{\overline{N}(P)}(x),\ F_{\underline{N}(P)}(x) = F_{\overline{N}(P)}(x)$

for any x belongs to Z.

P is said to be a definable neutrosophic set in the approximation (Z, R). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

2.5 Interval neutrosophic rough sets [50]

Interval neutrosophic rough set [50] is the hybrid structure of rough sets and interval neutrosophic sets. According to Broumi and Smarandache [50] interval neutrosophic rough set is the generalizations of interval valued intuitionistic fuzzy rough set [63].

Definition 2.5.1 [53]

Let R be an equivalence relation on the universal set U. Then the pair (U, R) is called a Pawlak approximation space [5, 6]. An equivalence class of R containing x will be denoted by $[x]_R$ for X ⊆ U, the lower and upper approximation of X with respect to (U, R) are denoted by respectively R^*X and R_*X and are defined by

$R^*X = \{x \in U: [x]_R \subseteq X\}$,

$R^*X = \{x \in U: [x]_R \cap X \neq \emptyset\}$.

Now if $R^*X = R_*X$, then X is called definable; otherwise X is called a rough set.

Definition 2.5.2 [50]

Let U be a universe and X, a rough set in U. An intuitionistic fuzzy rough set A in U is characterized by a membership function μ_A:U→ [0, 1] and non-membership function ν_A: U→ [0, 1] such that $\mu_A(\underline{R}X) = 1$ and $\nu_A(\underline{R}X) = 0$

ie, $[\mu_A(x), \nu_A(x)] = [1, 0]$ if $x \in (\underline{R}X)$ and $\mu_A(U - \overline{R}X) = 0$

$\nu_A(U - \overline{R}X) = 1$

ie, $0 \le [\mu_A(\overline{R}X - \underline{R}X) + \nu_A(\overline{R}X - \underline{R}X)] \le 1$

2.5.1 Basic concept of rough approximations of an interval valued neutrosophic set and their properties

Definition 2.5.3 [50]

Assume that, (U, R) be a Pawlak approximation space, for an interval neutrosophic set

$$A = \{< x, [T_A^L(x), T_A^U(x)], [I_A^L(x), I_A^U(x)], [F_A^L(x), F_A^U(x)] > \mid x \in U\}.$$

The lower approximation $\underline{A}_R$ and the upper approximation $\overline{A}_R$ of A in the Pawlak approximation space (U, R) are expressed as follows:

$$\underline{A}_R = \{< x, [\wedge_{y\in[x]_R} \{T_A^L(y)\}, \wedge_{y\in[x]_R} \{T_A^U(y)\}], [\vee_{y\in[x]_R} \{I_A^L(y)\}, \vee_{y\in[x]_R} \{I_A^U(y)\}], [\vee_{y\in[x]_R} \{F_A^L(y)\}, \vee_{y\in[x]_R} \{F_A^U(y)\}] > \mid x \in U\}$$

$$\overline{A}_R = \{< x, [\vee_{y\in[x]_R} \{T_A^L(y)\}, \vee_{y\in[x]_R} \{T_A^U(y)\}], [\wedge_{y\in[x]_R} \{I_A^L(y)\}, \wedge_{y\in[x]_R} \{I_A^U(y)\}], [\wedge_{y\in[x]_R} \{F_A^L(y)\}, \wedge_{y\in[x]_R} \{F_A^U(y)\}] > \mid x \in U\}$$

The symbols ⋀ and ⋁ indicate "min" and "max" operators respectively. R denotes an equivalence relation for interval neutrosophic set A. Here $[x]_R$ is the equivalence class of the element x. It is obvious that

$[\wedge_{y\in[x]_R} \{T_A^L(y)\}, \wedge_{y\in[x]_R} \{T_A^U(y)\}] \subset [0,1]$

$[\vee_{y\in[x]_R} \{I_A^L(y)\}, \vee_{y\in[x]_R} \{I_A^U(y)\}] \subset [0,1]$

$[\vee_{y\in[x]_R} \{F_A^L(y)\}, \vee_{y\in[x]_R} \{F_A^U(y)\}] \subset [0,1]$

and

$$0 \le \wedge_{y\in[x]_R} \{T_A^U(y)\}] + \vee_{y\in[x]_R} \{I_A^U(y)\}] + \vee_{y\in[x]_R} \{F_A^U(y)\}] \le 3$$

Then $\underline{A}_R$ is an interval neutrosophic set (INS)

Similarly, we have

$[\vee_{y\in[x]_R} \{T_A^L(y)\}, \vee_{y\in[x]_R} \{T_A^U(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R} \{I_A^L(y)\}, \wedge_{y\in[x]_R} \{I_A^U(y)\}] \subset [0,1]$

$[\wedge_{y\in[x]_R} \{F_A^L(y)\}, \wedge_{y\in[x]_R} \{F_A^U(y)\}] \subset [0,1]$

and

$0 \le \vee_{y\in[x]_R} \{T_A^U(y)\}] + \wedge_{y\in[x]_R} \{I_A^U(y)\}] +$
$\wedge_{y\in[x]_R} \{F_A^U(y)\}] \le 3$

Then $\overline{A}_R$ is an interval neutrosophic set.

If $\underline{A}_R = \overline{A}_R$ then A is a definable set, otherwise A is an interval valued neutrosophic rough set. Here, $\underline{A}_R$ and $\overline{A}_R$ are called the lower and upper approximations of interval neutrosophic set with respect to approximation space (U, R) respectively. $\underline{A}_R$ and $\overline{A}_R$ are simply denoted by $\underline{A}$ and $\overline{A}$ respectively.

Proposition1 [50]: Let A and B be two interval neutrosophic sets and $\underline{A}$ and $\overline{A}$ the lower and upper approximation of interval neutrosophic set A with respect to approximation space (U, R) respectively. $\underline{B}$ and $\overline{B}$ are the lower and upper approximation of interval neutrosophic set B with respect to approximation space (U, R), respectively. Then the following relations hold good.

1. $\underline{A} \subseteq A \subseteq \overline{A}$
2. $\overline{A \cup B} = \overline{A} \cup \overline{B}$ and $\underline{A \cap B} = \underline{A} \cap \underline{B}$
3. $\overline{A \cap B} = \overline{A} \cap \overline{B}$ and $\underline{A \cup B} = \underline{A} \cup \underline{B}$
4. $\overline{\overline{A}} = \overline{\underline{A}} = \overline{A}$ and $\underline{\underline{A}} = \underline{\overline{A}} = \underline{A}$
5. $\underline{U} = U$ and $\overline{\phi} = \phi$
6. If $A \subseteq B$ then, $\underline{A} \subseteq \underline{B}$ and $\overline{A} \subseteq \overline{B}$
7. $\underline{A^c} = \overline{A}^c$ and $\overline{A^c} = \underline{A}^c$

Definition2.5.4 [50]

Assume that, (U, R) be a Pawlak approximation space and A and B are two interval neutrosophic sets over U.

If $\underline{A} = \underline{B}$ then A and B are called interval neutrosophic lower rough equal. If $\overline{A} = \overline{B}$, then A and B are called interval neutrosophic upper rough equal.

If $\underline{A} = \underline{B}$, $\overline{A} = \overline{B}$, then A and B are called interval neutrosophic rough equal.

Proposition2 [50]

Assume that (U, R) be a Pawlak approximation space and A and B two interval neutrosophic sets over U. then

1. $\underline{A} = \underline{B} \Rightarrow \underline{A \cap B} = \underline{A}$ and $\underline{A \cap B} = \underline{B}$
2. $\overline{A} = \overline{B} \Rightarrow \overline{A \cup B} = \overline{A}$ and $\overline{A \cup B} = \overline{B}$
3. $\overline{A} = \overline{A}^c$ and $\overline{B} = \overline{B}^c \Rightarrow \overline{A \cup B} = \overline{A^c \cup B^c}$
4. $\underline{A} = \underline{A}^c$ and $\underline{B} = \underline{B}^c \Rightarrow \underline{A \cap B} = \underline{A^c \cap B^c}$
5. $A \subseteq B$ and $\underline{B} = \phi$ then $\underline{A} = \phi$
6. $A \subseteq B$ and $\underline{B} = \underline{U}$ then $\underline{A} = \underline{U}$
7. $\underline{B} = \phi$ and $\underline{A} = \phi$ then $\underline{A \cap B} = \phi$
8. $\overline{A} = \overline{U}$ and $\overline{B} = \overline{U} \Rightarrow \overline{A \cup B} = \overline{U}$
9. $\overline{A} = \overline{U} \Rightarrow A = B$
10. $\overline{A} = \overline{\phi} \Rightarrow A = \phi$

3. Cosine, Dice, Jaccard similarity measures of interval rough neutrosophic environment

Cosine, Dice and Jaccard similarity measure are proposed in interval rough neutrosophic environment in the following subsections.

3.1 Cosine similarity measure of interval rough neutrosophic environment

Definition 3.1.1: Assume that there are two interval rough neutrosophic sets

$$A = \left\langle \begin{pmatrix} \{[\underline{T}_A(x_i)]^L, [\underline{T}_A(x_i)]^U\}, \\ \{[\underline{I}_A(x_i)]^L, [\underline{I}_A(x_i)]^U\}, \\ \{[\underline{F}_A(x_i)]^L, [\underline{F}_A(x_i)]^U\} \end{pmatrix}, \begin{pmatrix} \{[\overline{T}_A(x_i)]^L, [\overline{T}_A(x_i)]^U\}, \\ \{[\overline{I}_A(x_i)]^L, [\overline{I}_A(x_i)]^U\}, \\ \{[\overline{F}_A(x_i)]^L, [\overline{F}_A(x_i)]^U\} \end{pmatrix} \right\rangle \text{ and}$$

$$B = \left\langle \begin{pmatrix} \{[\underline{T}_B(x_i)]^L, [\underline{T}_B(x_i)]^U\}, \\ \{[\underline{I}_B(x_i)]^L, [\underline{I}_B(x_i)]^U\}, \\ \{[\underline{F}_B(x_i)]^L, [\underline{F}_B(x_i)]^U\} \end{pmatrix}, \begin{pmatrix} \{[\overline{T}_B(x_i)]^L, [\overline{T}_B(x_i)]^U\}, \\ \{[\overline{I}_B(x_i)]^L, [\overline{I}_B(x_i)]^U\}, \\ \{[\overline{F}_B(x_i)]^L, [\overline{F}_B(x_i)]^U\} \end{pmatrix} \right\rangle$$

in $X = \{x_1, x_2, \ldots, x_n\}$.

A cosine similarity measure between interval rough neutrosophic sets A and B is defined as follows:

$$C_{IRNS}(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{(\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i))}{\left(\sqrt{(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2}\right)\left(\sqrt{(\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2}\right)} \quad (4)$$

Where $\Delta T_A(x_i) =$

$$\left(\frac{[\underline{T}_A(x_i)]^L + [\underline{T}_A(x_i)]^U + [\overline{T}_A(x_i)]^L + [\overline{T}_A(x_i)]^U}{4}\right),$$

$\Delta T_B(x_i) =$

$$\left(\frac{[\underline{T}_B(x_i)]^L+[\underline{T}_B(x_i)]^U+[\overline{T}_B(x_i)]^L+[\overline{T}_B(x_i)]^U}{4}\right),$$

$$\Delta I_A(x_i) = \left(\frac{[\underline{I}_A(x_i)]^L+[\underline{I}_A(x_i)]^U+[\overline{I}_A(x_i)]^L+[\overline{I}_A(x_i)]^U}{4}\right),$$

$\Delta I_B(x_i) =$

$$\left(\frac{[\underline{I}_B(x_i)]^L+[\underline{I}_B(x_i)]^U+[\overline{I}_B(x_i)]^L+[\overline{I}_B(x_i)]^U}{4}\right),$$

$\Delta F_A(x_i) =$

$$\left(\frac{[\underline{F}_A(x_i)]^L+[\underline{F}_A(x_i)]^U+[\overline{F}_A(x_i)]^L+[\overline{F}_A(x_i)]^U}{4}\right),$$

$\Delta F_B(x_i) =$

$$\left(\frac{[\underline{F}_B(x_i)]^L+[\underline{F}_B(x_i)]^U+[\overline{F}_B(x_i)]^L+[\overline{F}_B(x_i)]^U}{4}\right).$$

Proposition 3

Let *A* and *B* be interval rough neutrosophic sets then

1. $0 \le C_{IRNS}(A,B) \le 1$
2. $C_{IRNS}(A,B) = C_{IRNS}(B,A)$
3. $C_{IRNS}(A, B) = 1$, iff A = B

Proofs :

1. It is obvious because all positive values of cosine function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously $C_{IRNS}(A, B) = 1$. On the other hand if $C_{IRNS}(A, B) = 1$ then,

$\Delta T_A(x_i) = \Delta T_B(x_i)$,

$\Delta I_A(x_i) = \Delta I_B(x_i)$,

$\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,

$[\underline{T}_A(x_i)]^L = [\underline{T}_B(x_i)]^L$,

$[\underline{T}_A(x_i)]^U = [\underline{T}_B(x_i)]^U$,

$[\overline{T}_A(x_i)]^L = [\overline{T}_B(x_i)]^L$,

$[\overline{T}_A(x_i)]^U = [\overline{T}_B(x_i)]^U$,

$[\underline{I}_A(x_i)]^L = [\underline{I}_B(x_i)]^L$,

$[\underline{I}_A(x_i)]^U = [\underline{I}_B(x_i)]^U$,

$[\overline{I}_A(x_i)]^L = [\overline{I}_B(x_i)]^L$,

$[\overline{I}_A(x_i)]^U = [\overline{I}_B(x_i)]^U$,

$[\underline{F}_A(x_i)]^L = [\underline{F}_B(x_i)]^L$,

$[\underline{F}_A(x_i)]^U = [\underline{F}_B(x_i)]^U$,

$[\overline{F}_A(x_i)]^L = [\overline{F}_B(x_i)]^L$,

$[\overline{F}_A(x_i)]^U = [\overline{F}_B(x_i)]^U$

This implies that A = B.

If we consider the weight w_i of each element x_i, a weighted interval rough cosine similarity measure between interval rough neutrosophic sets A and B can be defined as follows:

$C_{WIRNS}(A,B) =$

$$\sum_{i=1}^{n} w_i \frac{(\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i))}{\left(\sqrt{(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2}.\sqrt{(\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2}\right)} \quad (5)$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $C_{WIRNS}(A, B) = C_{IRNS}(A, B)$.

The weighted interval rough cosine similarity measure between two interval rough neutrosophic sets A and B also satisfies the following properties:

Proposition4

1. $0 \le C_{WIRNS}(A,B) \le 1$
2. $C_{WIRNS}(A,B) = C_{WIRNS}(B,A)$
3. $C_{WIRNS}(A, B) = 1$, iff A = B

Proof :

The proofs of above properties are similar to the profs of the propertyies of the proposition (3).

3.2 Dice similarity measure of interval rough neutrosophic environment

Definition 3.2.2

A Dice similarity measure between interval rough neutrosophic sets A and B (defined in 3.1.1) is defined as follows:

$DIC_{IRNS}(A,B) =$

$$\frac{1}{n}\sum_{i=1}^{n} \frac{2.[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\left[(\Delta T_A(x_i))^2+(\Delta I_A(x_i))^2+(\Delta F_A(x_i))^2 + (\Delta T_B(x_i))^2+(\Delta I_B(x_i))^2+(\Delta F_B(x_i))^2\right]} \quad (6)$$

Where, $\Delta T_A(x_i) =$

$$\left(\frac{[\underline{T}_A(x_i)]^L+[\underline{T}_A(x_i)]^U+[\overline{T}_A(x_i)]^L+[\overline{T}_A(x_i)]^U}{4}\right),$$

$\Delta T_B(x_i) =$

$$\left(\frac{[\underline{T}_B(x_i)]^L+[\underline{T}_B(x_i)]^U+[\overline{T}_B(x_i)]^L+[\overline{T}_B(x_i)]^U}{4}\right),$$

$$\Delta I_A(x_i) = \left(\frac{[\underline{I}_A(x_i)]^L + [\underline{I}_A(x_i)]^U + [\bar{I}_A(x_i)]^L + [\bar{I}_A(x_i)]^U}{4}\right),$$

$$\Delta I_B(x_i) = \left(\frac{[\underline{I}_B(x_i)]^L + [\underline{I}_B(x_i)]^U + [\bar{I}_B(x_i)]^L + [\bar{I}_B(x_i)]^U}{4}\right),$$

$$\Delta F_A(x_i) = \left(\frac{[\underline{F}_A(x_i)]^L + [\underline{F}_A(x_i)]^U + [\bar{F}_A(x_i)]^L + [\bar{F}_A(x_i)]^U}{4}\right),$$

$$\Delta F_B(x_i) = \left(\frac{[\underline{F}_B(x_i)]^L + [\underline{F}_B(x_i)]^U + [\bar{F}_B(x_i)]^L + [\bar{F}_B(x_i)]^U}{4}\right).$$

Proposition 5

Let *A* and *B* be interval rough neutrosophic sets then

1. $0 \le DIC_{IRNS}(A, B) \le 1$
2. $DIC_{IRNS}(A, B) = DIC_{IRNS}(B, A)$
3. $DIC_{IRNS}(A, B) = 1$, iff A = B

Proofs :

1. It is obvious because all positive values of Dice function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously $DIC_{IRNS}(A, B) = 1$. On the other hand if $DIC_{IRNS}(A, B) = 1$ then,

$\Delta T_A(x_i) = \Delta T_B(x_i)$,

$\Delta I_A(x_i) = \Delta I_B(x_i)$,

$\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,

$[\underline{T}_A(x_i)]^L = [\underline{T}_B(x_i)]^L$,

$[\underline{T}_A(x_i)]^U = [\underline{T}_B(x_i)]^U$,

$[\bar{T}_A(x_i)]^L = [\bar{T}_B(x_i)]^L$,

$[\bar{T}_A(x_i)]^U = [\bar{T}_B(x_i)]^U$,

$[\underline{I}_A(x_i)]^L = [\underline{I}_B(x_i)]^L$,

$[\underline{I}_A(x_i)]^U = [\underline{I}_B(x_i)]^U$,

$[\bar{I}_A(x_i)]^L = [\bar{I}_B(x_i)]^L$,

$[\bar{I}_A(x_i)]^U = [\bar{I}_B(x_i)]^U$,

$[\underline{F}_A(x_i)]^L = [\underline{F}_B(x_i)]^L$,

$[\underline{F}_A(x_i)]^U = [\underline{F}_B(x_i)]^U$,

$[\bar{F}_A(x_i)]^L = [\bar{F}_B(x_i)]^L$,

$[\bar{F}_A(x_i)]^U = [\bar{F}_B(x_i)]^U$

This implies that A = B.

If we consider the weight w_i of each element x_i, a weighted interval rough Dice similarity measure between interval rough neutrosophic sets A and B is defined as follows:

$$DIC_{WIRNS}(A, B) = \sum_{i=1}^{n} w_i \frac{2.[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\left[\begin{array}{l}(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2 \\ + (\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2\end{array}\right]} \quad (7)$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $DIC_{WIRNS}(A, B) = DIC_{IRNS}(A, B)$.

The weighted interval rough Dice similarity measure between two interval rough neutrosophic sets A and B also satisfies the following properties:

Proposition6

1. $0 \le DIC_{WIRNS}(A, B) \le 1$
2. $DIC_{WIRNS}(A, B) = DIC_{WIRNS}(B, A)$
3. $DIC_{WIRNS}(A, B) = 1$, iff A = B

Proof :

The proofs of above properties are similar to the proofs of the properties of the proposition (5).

3.3 Jaccard similarity measure of interval rough neutrosophic environment

Definition 3.3.1 A Jaccard similarity measure between interval rough neutrosophic sets A and B (defined in 3.1.1) is defined as follows:

$$JAC_{IRNS}(A, B) = \frac{1}{n}\sum_{i=1}^{n} \frac{[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\left[\begin{array}{l}(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2 + \\ (\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2 \\ + \Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) \\ + \Delta F_A(x_i)\Delta F_B(x_i)\end{array}\right]} \quad (8)$$

where

$$\Delta T_A(x_i) = \left(\frac{[\underline{T}_A(x_i)]^L + [\underline{T}_A(x_i)]^U + [\bar{T}_A(x_i)]^L + [\bar{T}_A(x_i)]^U}{4}\right),$$

$$\Delta T_B(x_i) = \left(\frac{[\underline{T}_B(x_i)]^L + [\underline{T}_B(x_i)]^U + [\overline{T}_B(x_i)]^L + [\overline{T}_B(x_i)]^U}{4}\right),$$

$$\Delta I_A(x_i) = \left(\frac{[\underline{I}_A(x_i)]^L + [\underline{I}_A(x_i)]^U + [\overline{I}_A(x_i)]^L + [\overline{I}_A(x_i)]^U}{4}\right),$$

$$\Delta I_B(x_i) = \left(\frac{[\underline{I}_B(x_i)]^L + [\underline{I}_B(x_i)]^U + [\overline{I}_B(x_i)]^L + [\overline{I}_B(x_i)]^U}{4}\right),$$

$$\Delta F_A(x_i) = \left(\frac{[\underline{F}_A(x_i)]^L + [\underline{F}_A(x_i)]^U + [\overline{F}_A(x_i)]^L + [\overline{F}_A(x_i)]^U}{4}\right),$$

$$\Delta F_B(x_i) = \left(\frac{[\underline{F}_B(x_i)]^L + [\underline{F}_B(x_i)]^U + [\overline{F}_B(x_i)]^L + [\overline{F}_B(x_i)]^U}{4}\right).$$

Proposition 7

Let *A* and *B* be interval rough neutrosophic sets then

1. $0 \le JAC_{IRNS}(A, B) \le 1$
2. $JAC_{IRNS}(A, B) = JAC_{IRNS}(B, A)$
3. $JAC_{IRNS}(A, B) = 1$, iff A = B

Proofs :

1. It is obvious because all positive values of Jaccard function are within 0 and 1.
2. It is obvious that the proposition is true.
3. When A = B, then obviously $JAC_{IRNS}(A, B) = 1$. On the other hand if $JAC_{IRNS}(A, B) = 1$ then,

$\Delta T_A(x_i) = \Delta T_B(x_i)$,

$\Delta I_A(x_i) = \Delta I_B(x_i)$,

$\Delta F_A(x_i) = \Delta F_B(x_i)$ ie,

$[\underline{T}_A(x_i)]^L = [\underline{T}_B(x_i)]^L$,

$[\underline{T}_A(x_i)]^U = [\underline{T}_B(x_i)]^U$,

$[\overline{T}_A(x_i)]^L = [\overline{T}_B(x_i)]^L$,

$[\overline{T}_A(x_i)]^U = [\overline{T}_B(x_i)]^U$,

$[\underline{I}_A(x_i)]^L = [\underline{I}_B(x_i)]^L$,

$[\underline{I}_A(x_i)]^U = [\underline{I}_B(x_i)]^U$,

$[\overline{I}_A(x_i)]^L = [\overline{I}_B(x_i)]^L$,

$[\overline{I}_A(x_i)]^U = [\overline{I}_B(x_i)]^U$,

$[\underline{F}_A(x_i)]^L = [\underline{F}_B(x_i)]^L$,

$[\underline{F}_A(x_i)]^U = [\underline{F}_B(x_i)]^U$,

$[\overline{F}_A(x_i)]^L = [\overline{F}_B(x_i)]^L$,

$[\overline{F}_A(x_i)]^U = [\overline{F}_B(x_i)]^U$

This implies that A = B.

If we consider the weight w_i of each element x_i, a weighted interval rough Jaccard similarity measure between interval rough neutrosophic sets A and B can be defined as follows:

$$JAC_{WIRNS}(A, B) = \sum_{i=1}^{n} w_i \frac{[\Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) + \Delta F_A(x_i)\Delta F_B(x_i)]}{\left[\begin{array}{l}(\Delta T_A(x_i))^2 + (\Delta I_A(x_i))^2 + (\Delta F_A(x_i))^2 + \\ (\Delta T_B(x_i))^2 + (\Delta I_B(x_i))^2 + (\Delta F_B(x_i))^2 \\ + \Delta T_A(x_i)\Delta T_B(x_i) + \Delta I_A(x_i)\Delta I_B(x_i) \\ + \Delta F_A(x_i)\Delta F_B(x_i)\end{array}\right]} \quad (9)$$

$w_i \in [0,1]$, i = 1, 2,…, n and $\sum_{i=1}^{n} w_i = 1$. If we take $w_i = \frac{1}{n}$, i = 1, 2,…, n, then $JAC_{WIRNS}(A, B) = JAC_{IRNS}(A, B)$

The weighted interval rough Jaccard similarity measure between two interval rough neutrosophic sets A and B also satisfies the following properties:

Proposition 8

1. $0 \le JAC_{WIRNS}(A, B) \le 1$
2. $JAC_{WIRNS}(A, B) = JAC_{WIRNS}(B, A)$
3. $JAC_{WIRNS}(A, B) = 1$, iff A = B

Proof :

The proofs of above properties are similar to the proofs of the properties of proposition (7).

4. Decision making based on cosine, Dice and Jaccard hamming similarity operator under interval rough neutrosophic environment

In this section, we apply interval rough similarity measures between IRNSs to the multi-criteria decision making problem. Assume that, A ={ A_1, A_2,..., A_m }be a set of alternatives and C = {C_1, C_2, ..., C_n} be the set of attributes.

The proposed decision making approach is described using the following steps..

Step 1: **Construct of the decision matrix with interval rough neutrosophic number**

The decision maker fomrs a decision matrix with respect to m alternatives and n attributes in terms of interval rough neutrosophic numbers (see the Table 1).

Table1: Interval rough neutrosophic decision matrix

$$D=\left\langle \underline{d}_{ij}^{L},\overline{d}_{ij}^{U}\right\rangle_{m\times n}=$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \left\langle \underline{d}_{11}^{L},\overline{d}_{11}^{U}\right\rangle & \left\langle \underline{d}_{12}^{L},\overline{d}_{12}^{U}\right\rangle & \ldots & \left\langle \underline{d}_{1n}^{L},\overline{d}_{1n}^{U}\right\rangle \\ A_2 & \left\langle \underline{d}_{21}^{L},\overline{d}_{21}^{U}\right\rangle & \left\langle \underline{d}_{22}^{L},\overline{d}_{22}^{U}\right\rangle & \ldots & \left\langle \underline{d}_{2n}^{L},\overline{d}_{2n}^{U}\right\rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \left\langle \underline{d}_{m1}^{L},\overline{d}_{m1}^{U}\right\rangle & \left\langle \underline{d}_{m2}^{L},\overline{d}_{m2}^{U}\right\rangle & \ldots & \left\langle \underline{d}_{mn}^{L},\overline{d}_{mn}^{U}\right\rangle \end{array} \quad (10)$$

Here $\left\langle \underline{d}_{ij}^{L},\overline{d}_{ij}^{U}\right\rangle$ is the interval rough neutrosophic number according to the i-th alternative and the j-th attribute.

Step 2: **Determine interval rough neutrosophic mean operator (IRNMO)**

$$\left\langle \Delta T(x_i),\Delta I(x_i),\Delta F(x_i)\right\rangle=$$

$$\left(\begin{array}{l}\dfrac{[\underline{T}(x_i)]^L+[\underline{T}(x_i)]^U+[\overline{T}(x_i)]^L+[\overline{T}(x_i)]^U}{4},\\ \dfrac{[\underline{I}(x_i)]^L+[\underline{I}(x_i)]^U+[\overline{I}(x_i)]^L+[\overline{I}(x_i)]^U}{4},\\ \dfrac{[\underline{F}(x_i)]^L+[\underline{F}(x_i)]^U+[\overline{F}(x_i)]^L+[\overline{F}(x_i)]^U}{4}\end{array}\right) \quad (11)$$

i= 1, 2, ..., n.

Step 3: **Determine the weights of the attributes**

Assume that the weight of the attributes C_j (j= 1, 2, ..., n) considered by the decision-maker is w_j (j = 1, 2, … , n). Where, all w_j ∈belongs to [0, 1]

And $\sum_{j=1}^{n} w_j=1$.

Step 4: **Determine the benefit type attributes and cost type attributes**

The evaluation attribute can be categorized into two types: benefit attribute and cost attribute. In the proposed decision-making method, an ideal alternative can be identified by using a maximum operator for the benefit attribute and a minimum operator for the cost attribute to determine the best value of each criterion among all the alternatives. Therefore, we define an ideal alternative as follows.

$A^*=\{C_1^*, C_2^*, \ldots, C_m^*\}$.

Where benefit attribute

$$C_j^*=\left[\max_i T_{C_j}^{(A_i)},\min_i I_{C_j}^{(A_i)},\min_i F_{C_j}^{(A_i)}\right] \quad (12)$$

The cost attribute

$$C_j^*=\left[\min_i T_{C_j}^{(A_i)},\max_i I_{C_j}^{(A_i)},\max_i F_{C_j}^{(A_i)}\right] \quad (13)$$

Step 5: **Determine the weighted interval rough neutrosophic similarity measure of the alternatives**

Using the equations (5), (7), and (9), the weighted interval rough neutrosophic similarity functions can be written as follows.

$$C_{WIRNS}(A,B)=\sum_{j=1}^{n} w_j C_{IRNS}(A,B) \quad (14)$$

$$DIC_{WIRNS}(A,B)=\sum_{j=1}^{n} w_j DIC_{IRNS}(A,B) \quad (15)$$

$$JAC_{WIRNS}(A,B)=\sum_{j=1}^{n} w_j JAC_{IRNS}(A,B) \quad (16)$$

Step 6: **Rank the alternatives**

Through the weighted interval rough neutrosophic similarity measure between each alternative and the ideal alternative, the ranking order of all alternatives can be determined based on the descending order of similarity measures.

Step 7: End

5. Numerical Example

Assume that a decision maker intends to select the most suitable laptop for random use from the four initially chosen laptops (S_1, S_2, S_3) by considering four attributes namely: features C_1, reasonable Price C_2, Customer care C_3, risk factor C_4. Based on the proposed approach discussed in section 4, the considered problem is solved by the following steps:

Step 1: **Construct the decision matrix with interval rough neutrosophic number**

The decision maker forms a decision matrix with respect to three alternatives and four attributes in terms of interval rough neutrosophic numbers as follows.

Table2. Decision matrix with interval rough neutrosophic number

$$d_S = \left\langle \underline{N}(P)^L, \overline{N}(P)^U \right\rangle_{3\times 4} =$$

	C_1	C_2	C_3	C_4
A_1	$\left\langle \begin{pmatrix}[0.6, 0.7],[0.3, 0.5],\\ [0.3, 0.4]\end{pmatrix}, \begin{pmatrix}[0.8, 0.9],[0.1, 0.3],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.7],[0.3, 0.4],\\ [0.1, 0.2]\end{pmatrix}, \begin{pmatrix}[0.7, 0.9],[0.3, 0.5],\\ [0.3, 0.4]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.6],[0.4, 0.5],\\ [0.4, 0.6]\end{pmatrix}, \begin{pmatrix}[0.7, 0.8],[0.2, 0.4],\\ [0.3, 0.4]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.8, 0.9],[0.3, 0.4],\\ [0.5, 0.6]\end{pmatrix}, \begin{pmatrix}[0.7, 0.8],[0.3, 0.5],\\ [0.3, 0.5]\end{pmatrix} \right\rangle$
A_2	$\left\langle \begin{pmatrix}[0.7, 0.8],[0.2, 0.3],\\ [0.0, 0.2]\end{pmatrix}, \begin{pmatrix}[0.7, 0.9],[0.1, 0.2],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.6, 0.7],[0.1, 0.2],\\ [0.0, 0.2]\end{pmatrix}, \begin{pmatrix}[0.6, 0.7],[0.1, 0.3],\\ [0.1, 0.3]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.7],[0.2, 0.3],\\ [0.1, 0.2]\end{pmatrix}, \begin{pmatrix}[0.6, 0.9],[0.3, 0.5],\\ [0.2, 0.4]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.7, 0.8],[0.3, 0.5],\\ [0.1, 0.3]\end{pmatrix}, \begin{pmatrix}[0.5, 0.7],[0.5, 0.6],\\ [0.2, 0.3]\end{pmatrix} \right\rangle$
A_3	$\left\langle \begin{pmatrix}[0.6, 0.7],[0.3, 0.4],\\ [0.0, 0.3]\end{pmatrix}, \begin{pmatrix}[0.6, 0.9],[0.1, 0.2],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.5, 0.7],[0.2, 0.4],\\ [0.2, 0.4]\end{pmatrix}, \begin{pmatrix}[0.6, 0.8],[0.1, 0.3],\\ [0.1, 0.2]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.6, 0.8],[0.2, 0.4],\\ [0.3, 0.4]\end{pmatrix}, \begin{pmatrix}[0.6, 0.8],[0.2, 0.5],\\ [0.3, 0.5]\end{pmatrix} \right\rangle$	$\left\langle \begin{pmatrix}[0.4, 0.7],[0.2, 0.4],\\ [0.4, 0.5]\end{pmatrix}, \begin{pmatrix}[0.5, 0.8],[0.2, 0.5],\\ [0.0, 0.2]\end{pmatrix} \right\rangle$

(17)

Step 2: **Determine the interval rough neutrosophic mean operator (IRNMO)**

Using IRNMO, the transferred decision matrix is as follows.

Table 3: Transformed decision matrix

	C_1	C_2	C_3	C_4
A_1	$\langle 0.750, 0.300, 0.250\rangle$	$\langle 0.700, 0.375, 0.250\rangle$	$\langle 0.650, 0.375, 0.425\rangle$	$\langle 0.800, 0.375, 0.475\rangle$
A_2	$\langle 0.775, 0.200, 0.125\rangle$	$\langle 0.650, 0.175, 0.150\rangle$	$\langle 0.675, 0.350, 0.225\rangle$	$\langle 0.675, 0.475, 0.225\rangle$
A_3	$\langle 0.700, 0.250, 0.150\rangle$	$\langle 0.650, 0.250, 0.225\rangle$	$\langle 0.700, 0.325, 0.375\rangle$	$\langle 0.600, 0.325, 0.275\rangle$

(18)

Step 3: Determine the weights of attributes

The weight vectors considered by the decision maker are 0.35, 0.25, 0.25 and 0.15 respectively.

Step 4: Determine the benefit type attribute and cost type attribute

Here three benefit type attributes C_1, C_2, C_3 and one cost type attribute C_4. Using equation (12), (13) and (18) we calculate the ideal alternative as follows.

A^* = [(0.775, 0.200, 0.125), (0.700, 0.175, 0.150), (0.700, 0.325, 0.225),(0.600, 0.475, 0.475)]

Step 5: Calculate the weighted interval rough neutrosophic similarity scores of the alternatives

Calculated values of weighted interval rough neutrosophic similarity values presented as follows.

$$C_{WIRNS}(A^*, A_1) = 0.9754$$

$$C_{WIRNS}(A^*, A_2) = 0.9979$$

$$C_{WIRNS}(A^*, A_3) = 0.9878$$

$$DIC_{WIRNS}(A^*, A_1) = 0.9716$$

$$DIC_{WIRNS}(A^*, A_2) = 0.9971$$

$$DIC_{WIRNS}(A^*, A_3) = 0.9835$$

$$JAC_{WIRNS}(A^*, A_1) = 0.9448$$

$$JAC_{WIRNS}(A^*, A_2) = 0.9943$$

$$JAC_{WIRNS}(A^*, A_3) = 0.9678$$

Step 6: Rank the alternatives

Ranking the alternatives is prepared based on the descending order of similarity measures (see the table 6). Highest value reflects the best alternative.

Hence, the laptop A_2 is the best alternative for random use.

6. Comparision between three proposed approaches

Weighted interval rough similarity measures	**Measured value**	**Ranking order**
Weighted interval rough cosine similarity	$C_{WIRNS}(A_1, A^*) = 0.9754$ $C_{WIRNS}(A_2, A^*) = 0.9979$ $C_{WIRNS}(A_3, A^*) = 0.9878$	$A_2 \succ A_3 \succ A_1$

measure		
Weighted interval rough Dice similarity measure	$D_{WIRNS}(A_1, A^*) = 0.9716$ $D_{WIRNS}(A_2, A^*) = 0.9971$ $D_{WIRNS}(A_3, A^*) = 0.9835$	$A_2 \succ A_3 \succ A_1$
Weighted interval rough Jaccard similarity measure	$J_{WIRNS}(A_1, A^*) = 0.9448$ $J_{WIRNS}(A_2, A^*) = 0.9943$ $J_{WIRNS}(A_3, A^*) = 0.9678$	$A_2 \succ A_3 \succ A_1$

Conclusion

In this paper, we have proposed cosine, Dice and Jaccard similarity measures of interval rough neutrosophic set and proved some of their basic properties. We have presented an application, namely selection of best laptop for random use. The thrust of the concept presented in the paper will be in pattern recognition, medical diagnosis, personnel selection, etc. in interval neutrosophic environment..

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