New Distance and Similarity Measures of Interval Neutrosophic Sets

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Abstract: In this paper we proposed a new distance and several similarity measures between interval neutrosophic sets.

Keywords: Neutrosophic set, Interval neutrosohic set, Similarity measure.

I. INTRODUCTION

The neutrsophic set, founded by F.Smarandache [1], has capability to deal with uncertainty, imprecise, incomplete and inconsistent information which exist in the real world. Neutrosophic set theory is a powerful tool in the formal framework, which generalizes the concepts of the classic set, fuzzy set [2], interval-valued fuzzy set [3], intuitionistic fuzzy set [4], interval-valued intuitionistic fuzzy set [5], and so on.

After the pioneering work of Smarandache, in 2005 Wang [6] introduced the notion of interval neutrosophic set (INS for short) which is a particular case of the neutrosophic set. INS can be described by a membership interval, a non-membership interval, and the indeterminate interval. Thus the interval value neutrosophic set has the virtue of being more flexible and practical than single value neutrosophic set. And the Interval Neutrosophic Set provides a more reasonable mathematical framework to deal with indeterminate and inconsistent information.

Many papers about neutrosophic set theory have been done by various researchers [7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20].

A similarity measure for neutrosophic set (NS) is used for estimating the degree of similarity between two neutrosophic sets. Several researchers proposed some similarity measures between NSs, such as S. Broumi and F. Smarandache [26], Jun Ye [11, 12], P. Majumdar and S.K.Smanta [23].

In the literature, there are few researchers who studied the distance and similarity measure of IVNS.

In 2013, Jun Ye [12] proposed similarity measures between interval neutrosophic set based on the Hamming and Euclidean distance, and developed a multicriteria decision–making method based on the similarity degree. S. Broumi and F. Smarandache [10] proposed a new similarity measure, called "cosine similarity measure of interval valued neutrosophic sets". On the basis of numerical computations, S. Broumi and F. Smarandache found out that their similarity measures are stronger and more robust than Ye's measures.

We all know that there are various distance measures in mathematics. So, in this paper, we will extend the generalized distance of single valued neutrosophic set proposed by Ye [12] to the case of interval neutrosophic set and we'll study some new similarity measures.

This paper is organized as follows. In section 2, we review some notions of neutrosophic set andinterval valued neutrosophic set. In section 3, some new similarity measures of interval valued neutrosophic sets and their proofs are introduced. Finally, the conclusions are stated in section 4.

II. PRELIMIAIRIES

This section gives a brief overview of the concepts of neutrosophic set, and interval valued neutrosophic set.

A. *Neutrosophic Sets*

1) *Definition [1]*

Let X be a universe of discourse, with a generic element in X denoted by x, then a neutrosophic set A is an object having the form:

$A = \{< x: T_A(x), I_A(x), F_A(x)>, x \in X\}$, where the functions $T, I, F : X \rightarrow\ ]^-0, 1^+[$ define respectively the degree of membership (or Truth), the degree of indeterminacy, and the degree of non-membership (or Falsehood) of the element $x \in X$ to the set A with the condition:

$$^-0 \leq T_A(x)+I_A(x)+F_A(x) \leq 3^+. \quad \textbf{(1)}$$

From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]^-0, 1^+[$. Therefore, instead of $]^-0, 1^+[$ we need to take the interval [0, 1] for technical applications, because $]^-0, 1^+[$ will be difficult to apply in the real applications such as in scientific and engineering problems.

For two NSs, A_{NS}= {<x, $T_A(x)$, $I_A(x)$, $F_A(x)$>|x ∈ X} **(2)**

and B_{NS} ={ <x, $T_B(x)$, $I_B(x)$, $F_B(x)$> | x ∈ X } the two relations are defined as follows:

(1) $A_{NS} \subseteq B_{NS}$ if and only if $T_A(x) \le T_B(x), I_A(x) \ge I_B(x), F_A(x) \ge F_B(x)$.

(2) $A_{NS} = B_{NS}$ if and only if, $T_A(x)$=$T_B(x)$, $I_A(x)$ =$I_B(x)$, $F_A(x)$ =$F_B(x)$.

B. Interval Valued Neutrosophic Sets

In actual applications, sometimes, it is not easy to express the truth-membership, indeterminacy-membership and falsity-membership by crisp value, and they may be easier to be expressed by interval numbers. Wang et al. [6] further defined interval neutrosophic sets (INS) shows as follows:

1) Definition [6]

Let X be a universe of discourse, with generic element in X denoted by x. An interval valued neutrosophic set (for short IVNS) A in X is characterized by truth-membership function$T_A(x)$, indeteminacy-membership function $I_A(x)$, and falsity-membership function $F_A(x)$. For each point x in X, we have that $T_A(x), I_A(x), F_A(x) \in [0,1]$.

For two IVNS, A_{IVNS}={<x,[$T_A^L(x)$,$T_A^U(x)$], [$I_A^L(x)$,$I_A^U(x)$], [$F_A^L(x)$,$F_A^U(x)$]> | x ∈ X } **(3)**

and B_{IVNS}= {<x, [$T_B^L(x)$,$T_B^U(x)$], [$I_B^L(x)$,$I_B^U(x)$], [$F_B^L(x)$,$F_B^U(x)$] > | x ∈ X } the two relations are defined as follows:

(1) $A_{IVNS} \subseteq B_{IVNS}$ if and only if $T_A^L(x) \le T_B^L(x), T_A^U(x) \le T_B^U(x), I_A^L(x) \ge I_B^L(x), I_A^U(x) \ge I_B^U(x), F_A^L(x) \ge F_B^L(x), F_A^U(x) \ge F_B^U(x)$.

(2) $A_{IVNS} = B_{IVNS}$ if and only if $T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$,$I_A^U(x_i) = I_B^U(x_i)$, $F_A^L(x_i) = F_B^L(x_i)$ and $F_A^U(x_i) = F_B^U(x_i)$ for any x ∈ X.

C. Defintion

Let A and B be two interval valued neutrosophic sets, then

i. $\mathbf{0 \le S(A,B) \le 1}$.

ii. $S(A,B) = S(B,A)$.

iii. $S(A,B) = 1$ if A= B, i.e $T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and $F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$, for i = 1, 2,…., n.

iv . A⊂ B ⊂ C ⇒ S(A,B) ≤ min (S(A,B), S(B,C).

III. New Distance Measure of Interval Valued Neutrosophic Sets

Let A and B be two single neutrosophic sets, then J. Ye [11] proposed a generalized single valued neutrosophic weighted distance measure between A and B as follows:

$$d_\lambda(A,B) = \left\{\frac{1}{3}\sum_{i=1}^{n} w_i\left[|T_A(x_i) - T_B(x_i)|^\lambda + |I_A(x_i) - I_B(x_i)|^\lambda + |F_A(x_i) - F_B(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}} \quad \mathbf{(4)}$$

where

$\lambda > 0$ and $T_A(x_i), I_A(x_i), F_A(x_i), T_B(x_i), I_B(x_i), F_B(x_i) \in [0, 1]$.

Based on the geometrical distance model and using the interval neutrosophic sets, we extended the distance (4) as follows:

$$d_\lambda(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n} w_i\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \mathbf{(5)}$$

The normalized generalized interval neutrosophic distance is

$$d_\lambda(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n} w_i\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \mathbf{(6)}$$

If w={$\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}$}, the distance (6) is reduced to the following distances:

$$d_\lambda(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n}\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \mathbf{(7)}$$

$$d_\lambda(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n}\left[|T_A^L(x_i) - T_B^L(x_i)|^\lambda + |T_A^U(x_i) - T_B^U(x_i)|^\lambda + |I_A^L(x_i) - I_B^L(x_i)|^\lambda + |I_A^U(x_i) - I_B^U(x_i)|^\lambda + |F_A^L(x_i) - F_B^L(x_i)|^\lambda + |F_A^U(x_i) - F_B^U(x_i)|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \mathbf{(8)}$$

Particular case.

(i) If $\lambda = 1$ then the distances (7) and (8) are reduced to the following Hamming distance and respectively normalized Hamming distance defined by Ye Jun [11]:

$$d_H(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)| + |T_A^U(x_i) - T_B^U(x_i)| + |I_A^L(x_i) - I_B^L(x_i)| + |I_A^U(x_i) - I_B^U(x_i)| + |F_A^L(x_i) - F_B^L(x_i)| + |F_A^U(x_i) - F_B^U(x_i)|]\right\}, \quad \textbf{(9)}$$

$$d_{NH}(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)| + |T_A^U(x_i) - T_B^U(x_i)| + |I_A^L(x_i) - I_B^L(x_i)| + |I_A^U(x_i) - I_B^U(x_i)| + |F_A^L(x_i) - F_B^L(x_i)| + |F_A^U(x_i) - F_B^U(x_i)|]\right\}. \quad \textbf{(10)}$$

(ii) If $\lambda = 2$ then the distances (7) and (8) are reduced to the following Euclidean distance and respectively normalized Euclidean distance defined by Ye Jun [12]:

$$d_E(A,B) = \left\{\frac{1}{6}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)|^2 + |T_A^U(x_i) - T_B^U(x_i)|^2 + |I_A^L(x_i) - I_B^L(x_i)|^2 + |I_A^U(x_i) - I_B^U(x_i)|^2 + |F_A^L(x_i) - F_B^L(x_i)|^2 + |F_A^U(x_i) - F_B^U(x_i)|^2]\right\}^{\frac{1}{2}}, \quad \textbf{(11)}$$

$$d_{NE}(A,B) = \left\{\frac{1}{6n}\sum_{i=1}^{n}[|T_A^L(x_i) - T_B^L(x_i)|^2 + |T_A^U(x_i) - T_B^U(x_i)|^2 + |I_A^L(x_i) - I_B^L(x_i)|^2 + |I_A^U(x_i) - I_B^U(x_i)|^2 + |F_A^L(x_i) - F_B^L(x_i)|^2 + |F_A^U(x_i) - F_B^U(x_i)|^2]\right\}^{\frac{1}{2}}. \quad \textbf{(12)}$$

IV. New Similarity Measures of Interval Valued Neutrosophic Set

A. Similarity measure based on the geometric distance model

Based on distance (4), we define the similarity measure between the interval valued neutrosophic sets A and B as follows:

$$S_{DM}(A,B) = 1- \left\{\frac{1}{6n}\sum_{i=1}^{n}\left[\left|T_A^L(x_i) - T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) - T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) - I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) - I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) - F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) - F_B^U(x_i)\right|^\lambda\right]\right\}^{\frac{1}{\lambda}}, \quad \textbf{(13)}$$

where $\lambda > 0$ and $S_{DM}(A,B)$ is the degree of similarity of A and B .

If we take the weight of each element $x_i \in X$ into account, then

$$S_{DM}^{w}(A,B)=1- \left\{\frac{1}{6}\sum_{i=1}^{n} w_i \left[\left|T_A^L(x_i) - T_B^L(x_i)\right|^\lambda + \left|T_A^U(x_i) - T_B^U(x_i)\right|^\lambda + \left|I_A^L(x_i) - I_B^L(x_i)\right|^\lambda + \left|I_A^U(x_i) - I_B^U(x_i)\right|^\lambda + \left|F_A^L(x_i) - F_B^L(x_i)\right|^\lambda + \left|F_A^U(x_i) - F_B^U(x_i)\right|^\lambda\right]\right\}^{\frac{1}{\lambda}}. \quad \textbf{(14)}$$

If each elements has the same importance, i.e. w = $\{\frac{1}{n}, \frac{1}{n}, \dots, \frac{1}{n}\}$, then similarity (14) reduces to (13).

By (definition C) it can easily be known that $S_{DM}(A,B)$ satisfies all the properties of the definition..

Similarly, we define another similarity measure of A and B, as:

$$S(A,B) = 1 - \left[\frac{\sum_{i=1}^{n}\left(\left|T_A^L(x_i)-T_B^L(x_i)\right|^\lambda+\left|T_A^U(x_i)-T_B^U(x_i)\right|^\lambda+\left|I_A^L(x_i)-I_B^L(x_i)\right|^\lambda+\left|I_A^U(x_i)-I_B^U(x_i)\right|^\lambda+\left|F_A^L(x_i)-F_B^L(x_i)\right|^\lambda+\left|F_A^U(x_i)-F_B^U(x_i)\right|^\lambda\right)}{\sum_{i=1}^{n}\left(\left|T_A^L(x_i)+T_B^L(x_i)\right|^\lambda+\left|T_A^U(x_i)+T_B^U(x_i)\right|^\lambda+\left|I_A^L(x_i)+I_B^L(x_i)\right|^\lambda+\left|I_A^U(x_i)+I_B^U(x_i)\right|^\lambda+\left|F_A^L(x_i)+F_B^L(x_i)\right|^\lambda+\left|F_A^U(x_i)+F_B^U(x_i)\right|^\lambda\right)}\right]^{\frac{1}{\lambda}}. \quad \textbf{(15)}$$

If we take the weight of each element $x_i \in X$ into account, then

$$S(A,B) = 1 - \left[\frac{\sum_{i=1}^{n} w_i\left(\left|T_A^L(x_i)-T_B^L(x_i)\right|^\lambda+\left|T_A^U(x_i)-T_B^U(x_i)\right|^\lambda+\left|I_A^L(x_i)-I_B^L(x_i)\right|^\lambda+\left|I_A^U(x_i)-I_B^U(x_i)\right|^\lambda+\left|F_A^L(x_i)-F_B^L(x_i)\right|^\lambda+\left|F_A^U(x_i)-F_B^U(x_i)\right|^\lambda\right)}{\sum_{i=1}^{n} w_i\left(\left|T_A^L(x_i)+T_B^L(x_i)\right|^\lambda+\left|T_A^U(x_i)+T_B^U(x_i)\right|^\lambda+\left|I_A^L(x_i)+I_B^L(x_i)\right|^\lambda+\left|I_A^U(x_i)+I_B^U(x_i)\right|^\lambda+\left|F_A^L(x_i)+F_B^L(x_i)\right|^\lambda+\left|F_A^U(x_i)+F_B^U(x_i)\right|^\lambda\right)}\right]^{\frac{1}{\lambda}}. \quad \textbf{(16)}$$

It also has been proved that all conditions of the definition are satisfied. If each elements has the same importance, and then the similarity (16) reduces to (15).

B. Similarity measure based on the interval valued neutrosophic theoretic approach:

In this section, following the similarity measure between two neutrosophic sets defined by P. Majumdar in [24], we extend Majumdar's definition to interval valued neutrosophic sets.

Let A and B be two interval valued neutrosophic sets, then we define a similarity measure between A and B as follows:

$$S_{TA}(A,B)=\frac{\sum_{i=1}^{n}\{\min\{T_A^L(x_i),T_B^L(x_i)\}+\min\{T_A^U(x_i),T_B^U(x_i)\}+\min\{I_A^L(x_i),I_B^L(x_i)\}+\min\{I_A^U(x_i),I_B^U(x_i)\}+\min\{F_A^L(x_i),F_B^L(x_i)\}+\min\{F_A^U(x_i),F_B^U(x_i)\}}{\sum_{i=1}^{n}\{\max\{T_A^L(x_i),T_B^L(x_i)\}+\max\{T_A^U(x_i),T_B^U(x_i)\}+\max\{I_A^L(x_i),I_B^L(x_i)\}+\max\{I_A^U(x_i),I_B^U(x_i)\}+\max\{F_A^L(x_i),F_B^L(x_i)\}+\max\{F_A^U(x_i),F_B^U(x_i)\}} \quad \textbf{(17)}$$

1) Proposition

Let A and B be two interval valued neutrosophic sets, then

iv. $\mathbf{0 \le S_{TA}(A,B) \le 1}$.

v. $S_{TA}(A,B) = S_{TA}(A,B)$.

vi. $S(A,B) = 1$ if A = B i.e.

$T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$ and $F_A^L(x_i) = F_B^L(x_i)$, $F_A^U(x_i) = F_B^U(x_i)$ for i = 1, 2, …., n.

iv. $A \subset B \subset C \Rightarrow S_{TA}(A,B) \le \min(S_{TA}(A,B), S_{TA}(B,C))$.

Proof. Properties (i) and (ii) follow from the definition.

(iii) It is clearly that if A = B $\Rightarrow S_{TA}(A,B) = 1$

$\Rightarrow \sum_{i=1}^{n}\{\min\{T_A^L(x_i),T_B^L(x_i)\}+\min\{T_A^U(x_i),T_B^U(x_i)\}+\min\{I_A^L(x_i),I_B^L(x_i)\}+\min\{I_A^U(x_i),I_B^U(x_i)\}+\min\{F_A^L(x_i),F_B^L(x_i)\}+\min\{F_A^U(x_i),F_B^U(x_i)\} = \sum_{i=1}^{n}\{max\{T_A^L(x_i),T_B^L(x_i)\}+max\{T_A^U(x_i),T_B^U(x_i)\}+max\{I_A^L(x_i),I_B^L(x_i)\}+max\{I_A^U(x_i),I_B^U(x_i)\}+max\{F_A^L(x_i),F_B^L(x_i)\}+max\{F_A^U(x_i),F_B^U(x_i)\}$

$\Rightarrow \sum_{i=1}^{n}\{[\min\{T_A^L(x_i),T_B^L(x_i)\}-\max\{T_A^L(x_i),T_B^L(x_i)\}]+[\min\{T_A^U(x_i),T_B^U(x_i)\}-\max\{T_A^U(x_i),T_B^U(x_i)\}]+[\min\{I_A^L(x_i),I_B^L(x_i)\}-\max\{I_A^L(x_i),I_B^L(x_i)\}]+[\min\{I_A^U(x_i),I_B^U(x_i)\}-\max\{I_A^U(x_i),I_B^U(x_i)\}]+[\min\{F_A^L(x_i),F_B^L(x_i)\}-\max\{F_A^L(x_i),F_B^L(x_i)\}]+[\min\{F_A^U(x_i),F_B^U(x_i)\}-\max\{F_A^U(x_i),F_B^U(x_i)\}]\} = 0$.

Thus for each x, one has that

$[\min\{T_A^L(x_i),T_B^L(x_i)\}-\max\{T_A^L(x_i),T_B^L(x_i)\}] = 0$
$[\min\{T_A^U(x_i),T_B^U(x_i)\}-\max\{T_A^U(x_i),T_B^U(x_i)\}] = 0$
$[\min\{I_A^L(x_i),I_B^L(x_i)\}-\max\{I_A^L(x_i),I_B^L(x_i)\}] = 0$
$[\min\{I_A^U(x_i),I_B^U(x_i)\}-\max\{I_A^U(x_i),I_B^U(x_i)\}] = 0$
$[\min\{F_A^L(x_i),F_B^L(x_i)\}-\max\{F_A^L(x_i),F_B^L(x_i)\}] = 0$
$[\min\{F_A^U(x_i),F_B^U(x_i)\}-\max\{F_A^U(x_i),F_B^U(x_i)\}] = 0$

hold.

Thus $T_A^L(x_i) = T_B^L(x_i)$, $T_A^U(x_i) = T_B^U(x_i)$, $I_A^L(x_i) = I_B^L(x_i)$, $I_A^U(x_i) = I_B^U(x_i)$, $F_A^L(x_i) = F_B^L(x_i)$ and $F_A^U(x_i) = F_B^U(x_i) \Rightarrow A=B$

(iv) Now we prove the last result.

Let $A \subset B \subset C$, then we have

$T_A^L(x) \le T_B^L(x) \le T_C^L(x)$, $T_A^U(x) \le T_B^U(x) \le T_C^L(x)$, $I_A^L(x) \ge I_B^L(x) \ge I_C^L(x)$, $I_A^U(x) \ge I_B^U(x) \ge I_C^U(x)$, $F_A^L(x) \ge F_B^L(x) \ge F_C^L(x)$, $F_A^U(x) \ge F_B^U(x) \ge F_C^U(x)$ for all $x \in X$.

Now

$T_A^L(x) + T_A^U(x) + I_A^L(x) + I_A^U(x) + F_B^L(x) + F_B^U(x) \ge T_A^L(x) + T_A^U(x) + I_A^L(x) + I_A^U(x) + F_C^L(x) + F_C^U(x)$

and

$T_B^L(x) + T_B^U(x) + I_B^L(x) + I_B^U(x) + F_A^L(x) + F_A^U(x) \ge T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_A^L(x) + F_A^U(x)$.

$$S(A,B) = \frac{T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_B^L(x)+F_B^U(x)}{T_B^L(x)+T_B^U(x)+I_B^L(x)+I_B^U(x)+F_A^L(x)+F_A^U(x)} \ge \frac{T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_C^L(x)+F_C^U(x)}{T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_A^L(x)+F_A^U(x)} = S(A,C).$$

Again, similarly we have

$T_B^L(x) + T_B^U(x) + I_B^L(x) + I_B^U(x) + F_C^L(x) + F_C^U(x) \ge T_A^L(x) + T_A^U(x) + I_A^L(x) + I_A^U(x) + F_C^L(x) + F_C^U(x)$
$T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_A^L(x) + F_A^U(x) \ge T_C^L(x) + T_C^U(x) + I_C^L(x) + I_C^U(x) + F_B^L(x) + F_B^U(x)$

$$S(B,C) = \frac{T_B^L(x)+T_B^U(x)+I_B^L(x)+I_B^U(x)+F_C^L(x)+F_C^U(x)}{T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_B^L(x)+F_B^U(x)} \ge \frac{T_A^L(x)+T_A^U(x)+I_A^L(x)+I_A^U(x)+F_C^L(x)+F_C^U(x)}{T_C^L(x)+T_C^U(x)+I_C^L(x)+I_C^U(x)+F_A^L(x)+F_A^U(x)} = S(A,C)$$

$\Rightarrow S_{TA}(A,B) \le \min(S_{TA}(A,B), S_{TA}(B,C))$.

Hence the proof of this proposition.

If we take the weight of each element $x_i \in X$ into account, then

$$S(A,B)=\frac{\sum_{i=1}^{n} w_i\{\min\{T_A^L(x_i),T_B^L(x_i)\}+\min\{T_A^U(x_i),T_B^U(x_i)\}+\min\{I_A^L(x_i),I_B^L(x_i)\}+\min\{I_A^U(x_i),I_B^U(x_i)\}+\min\{F_A^L(x_i),F_B^L(x_i)\}+\min\{F_A^U(x_i),F_B^U(x_i)\}}{\sum_{i=1}^{n} w_i\{\max\{T_A^L(x_i),T_B^L(x_i)\}+\max\{T_A^U(x_i),T_B^U(x_i)\}+\max\{I_A^L(x_i),I_B^L(x_i)\}+\max\{I_A^U(x_i),I_B^U(x_i)\}+\max\{F_A^L(x_i),F_B^L(x_i)\}+\max\{F_A^U(x_i),F_B^U(x_i)\}}$$

(18)

Particularly, if each element has the same importance, then (18) is reduced to (17), clearly this also satisfies all the properties of the definition.

C. Similarity measure based on matching function by using interval neutrosophic sets:

Chen [24] and Chen et al. [25] introduced a matching function to calculate the degree of similarity between fuzzy sets. In the following, we extend the matching function to deal with the similarity measure of interval valued neutrosophic sets.

Let A and B be two interval valued neutrosophic sets, then we define a similarity measure between A and B as follows:

$$S_{MF}(A,B) = \frac{\sum_{i=1}^{n}\left(\left(T_A^L(x_i)\cdot T_B^L(x_i)\right)+\left(T_A^U(x_i)\cdot T_B^U(x_i)\right)+\left(I_A^L(x_i)\cdot I_B^L(x_i)\right)+\left(I_A^U(x_i)\cdot I_B^U(x_i)\right)+\left(F_A^L(x_i)\cdot F_B^L(x_i)\right)+\left(F_A^U(x_i)\cdot F_B^U(x_i)\right)\right)}{\max\left(\sum_{i=}^{n}(T_A^L(x_i)^2+T_A^U(x_i)^2+I_A^L(x_i)^2+I_A^U(x_i)^2+F_A^L(x_i)^2+F_A^U(x_i)^2),\ \sum_{i=}^{n}(T_B^L(x_i)^2+T_B^U(x_i)^2+I_B^L(x_i)^2+I_B^U(x_i)^2+F_B^L(x_i)^2+F_B^U(x_i)^2)\right)} \quad (19)$$

Proof.

i. $\mathbf{0 \leq S_{MF}(A,B) \leq 1}$.

The inequality $S_{MF}(A,B) \geq 0$ is obvious. Thus, we only prove the inequality S(A, B) ≤ 1.

$$S_{MF}(A,B)= \sum_{i=1}^{n}\left(\left(T_A^L(x_i)\cdot T_B^L(x_i)\right)+\left(T_A^U(x_i)\cdot T_B^U(x_i)\right)+\left(I_A^L(x_i)\cdot I_B^L(x_i)\right)+\left(I_A^U(x_i)\cdot I_B^U(x_i)\right)+\left(F_A^L(x_i)\cdot F_B^L(x_i)\right)+\left(F_A^U(x_i)\cdot F_B^U(x_i)\right)\right)$$

$$= T_A^L(x_1)\cdot T_B^L(x_1)+T_A^L(x_2)\cdot T_B^L(x_2)+\ldots+T_A^L(x_n)\cdot T_B^L(x_n)+T_A^U(x_1)\cdot T_B^U(x_1)+T_A^U(x_2)\cdot T_B^U(x_2)+\ldots+T_A^U(x_n)\cdot T_B^U(x_n)+$$
$$I_A^L(x_1)\cdot I_B^L(x_1)+I_A^L(x_2)\cdot I_B^L(x_2)+\ldots+I_A^L(x_n)\cdot I_B^L(x_n)+I_A^U(x_1)\cdot I_B^U(x_1)+I_A^U(x_2)\cdot I_B^U(x_2)+\ldots+I_A^U(x_n)\cdot I_B^U(x_n)+$$
$$F_A^L(x_1)\cdot F_B^L(x_1)+F_A^L(x_2)\cdot F_B^L(x_2)+\ldots+F_A^L(x_n)\cdot F_B^L(x_n)+F_A^U(x_1)\cdot T_B^U(x_1)+F_A^U(x_2)\cdot F_B^U(x_2)+\ldots+F_A^U(x_n)\cdot F_B^U(x_n).$$

According to the Cauchy–Schwarz inequality:

$$(x_1\cdot y_1+x_2\cdot y_2+\cdots+x_n\cdot y_n)^2 \leq ({x_1}^2+{x_2}^2+\cdots+{x_n}^2)\cdot({y_1}^2+{y_2}^2+\cdots+{y_n}^2)$$

where $(x_1, x_2, \ldots, x_n) \in R^n$ and $(y_1, y_2, \ldots, y_n) \in R^n$

we can obtain

$$[S_{MF}(A,B)]^2 \leq \sum_{i=1}^{n}\left(T_A^L(x_i)^2+T_A^U(x_i)^2+I_A^L(x_i)^2+I_A^U(x_i)^2+F_A^L(x_i)^2+F_A^U(x_i)^2\right)\cdot$$
$$\sum_{i=1}^{n}\left(T_B^L(x_i)^2+T_B^U(x_i)^2+I_B^L(x_i)^2+I_B^U(x_i)^2+F_B^L(x_i)^2+F_B^U(x_i)^2\right)= S(A,A)\cdot S(B,B)$$

Thus $S_{MF}(A,B) \leq [S(A,A)]^{\frac{1}{2}}\cdot[S(B,B)]^{\frac{1}{2}}$.

Then $S_{MF}(A,B) \leq \max\{S(A,A), S(B,B)]$.

Therefore $S_{MF}(A, B) \leq 1$.

If we take the weight of each element $x_i \in X$ into account, then

$$S_{MF}^{w}(A,B)= \frac{\sum_{i=1}^{n} w_i\left(\left(T_A^L(x_i)\cdot T_B^L(x_i)\right)+\left(T_A^U(x_i)\cdot T_B^U(x_i)\right)+\left(I_A^L(x_i)\cdot I_B^L(x_i)\right)+\left(I_A^U(x_i)\cdot I_B^U(x_i)\right)+\left(F_A^L(x_i)\cdot F_B^L(x_i)\right)+\left(F_A^U(x_i)\cdot F_B^U(x_i)\right)\right)}{\max\left(\sum_{i=}^{n} w_i\,(T_A^L(x_i)^2+T_A^U(x_i)^2+I_A^L(x_i)^2+I_A^U(x_i)^2+F_A^L(x_i)^2+F_A^U(x_i)^2),\ \sum_{i=}^{n} w_i\,(T_B^L(x_i)^2+T_B^U(x_i)^2+I_B^L(x_i)^2+I_B^U(x_i)^2+F_B^L(x_i)^2+F_B^U(x_i)^2)\right)} \quad (20)$$

Particularly, if each element has the same importance, then the similarity (20) is reduced to (19). Clearly this also satisfies all the properties of definition.

The larger the value of S(A,B), the more the similarity between A and B.

V. Comparison of New Similarity Measure of IVNS With The Existing Measures.

Let A and B be two interval valued neutrosophic sets in the universe of discourse X = {x_1, x_2 ,.., x_n}.The new similarity $S_{TA}(A,B)$ of IVNS and the existing similarity measures of interval valued neutrosophic sets (examples 1 and 2) introduced in [10, 12, 23] are listed as follows:

Pinaki similarity I:

this similarity measure was proposed as concept of association coefficient of the neutrosophic sets as follows

$$S_{PI}=\frac{\sum_{i=1}^{n}\{\min\{T_A(x_i),T_B(x_i)\}+\min\{I_A(x_i),I_B(x_i)\}+\min\{F_A(x_i),F_B(x_i)\}\}}{\sum_{i=1}^{n}\{\max\{T_A(x_i),T_B(x_i)\}+\max\{I_A(x_i),I_B(x_i)\}+\max\{F_A(x_i),F_B(x_i)\}\}} \quad (21)$$

Broumi and Smarandache cosine similarity:

$$C_N(A,B) = \frac{1}{n}\sum_{i=1}^{n} \frac{(T_A^L(x_i) + T_A^U(x_i))\,(T_B^L(x_i)+T_B^U(x_i)) + (I_A^L(x_i) + I_A^U(x_i))\,(I_B^L(x_i) + I_B^U(x_i)) + (F_A^L(x_i) + F_A^U(x_i))\,(F_B^L(x_i) + F_B^U(x_i))}{\sqrt{(T_A^L(x_i) + T_A^U(x_i))^2+(I_A^L(x_i)+I_A^U(x_i))^2+(F_A^L(x_i) + F_A^U(x_i))^2}\ \sqrt{(T_B^L(x_i) + T_B^U(x_i))^2+(I_B^L(x_i)+I_B^U(x_i))^2+(F_B^L(x_i) + F_B^U(x_i))^2}} \quad \textbf{(22)}$$

Ye similarity

$$S_{ye}(A, B) = 1-\frac{1}{6}\sum_{i=1}^{n} [|\inf T_A(x_i) - \inf T_B(x_i)| + |\sup T_A(x_i) - \sup T_B(x_i)| + |\inf I_A(x_i) - \inf I_B(x_i)| + |\sup I_A(x_i) - \sup I_B(x_i)| + |\inf F_A(x_i) - \inf F_B(x_i)| + |\sup F_A(x_i) - \sup F_B(x_i)|]. \quad \textbf{(23)}$$

Example 1

Let A = {<x, (a, 0.2 , 0.6 , 0.6), (b, 0.5, 0.3 , 0.3), (c, 0.6 , 0.9 , 0.5)>}

and B = {<x, (a, 0.5 , 0.3 , 0.8), (b, 0.6 , 0.2 , 0.5), (c, 0.6 , 0.4 , 0.4)>}.

Pinaki similarity I = 0.6.

S_{ye}(A, B) = 0.38 (with w_i =1).

Cosine similarity $\mathbf{C_N(A, B)}$ = 0.95.

$S_{TA}(A,B)$ = **0.8.**

Example 2:

Let A= {<x, (a, [0.2 , 0.3] ,[0.2, 0.6], [0.6 , 0.8]), (b, [0.5 , 0.7], [0.3, 0.5], [0.3 , 0.6]), (c, [0.6 , 0.9] ,[0.3, 0.9], [0.3, 0.5])>} and

B={<x, (a, [0.5 , 0.3] ,[0.3, 0.6], [0.6 , 0.8]), (b, [0.6, 0.8] ,[0.2, 0.4], [0.5 , 0.6]), (c, [0.6 , 0.9] ,[0.3, 0.4], [0.4 , 0.6])>}.

Pinaki similarity I = NA.

S_{ye}(A, B) = 0.7 (with w_i =1).

Cosine similarity $\mathbf{C_N(A, B)}$ = 0.92.

$S_{TA}(A,B)$ = **0.9.**

On the basis of computational study Jun Ye [12] has shown that their measure is more effective and reasonable. A similar kind of study with the help of the proposed new measure based on theoretic approach, it has been done and it is found that the obtained results are more refined and accurate. It may be observed from the above examples that the values of similarity measures are closer to 1 with $S_{TA}(A,B)$ which is this proposed similarity measure.

VI. Conclusions

Few distance and similarity measures have been proposed in literature for measuring the distance and the degree of similarity between interval neutrosophic sets. In this paper, we proposed a new method for distance and similarity measure for measuring the degree of similarity between two weighted interval valued neutrosophic sets, and we have extended the work of Pinaki, Majumdar and S. K. Samant and Chen. The results of the proposed similarity measure and existing similarity measure are compared.

In the future, we will use the similarity measures which are proposed in this paper in group decision making

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