Neutrosophic Duality
Ideas | Approaches | Accessibility | Availability

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Abstract

In this book, some notions are introduced about “Neutrosophic Duality”. Two chapters are devised as “Initial Notions”, and “Modified Notions”. Two manuscripts are cited as the references of these chapters which are my 79th, and 80th manuscripts. I’ve used my 79th, and 80th manuscripts to write this book.

In first chapter, there are some points as follow. New setting is introduced to study dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Maximum number of dominated vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of dominated vertices corresponded to dual-dominating set is called neutrosophic dual-dominating number. Forming sets from dominated vertices to figure out different types of number of vertices in the sets from dominated sets $n$ in the terms of maximum number of vertices to get maximum number to assign to neutrosophic graphs is key type of approach to have these notions namely dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having largest number of dominated vertices from different types of sets in the terms of maximum number and maximum neutrosophic number forming it to get maximum number to assign to a neutrosophic graph.

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices, $s$ and $n$, if $\mu(ns) = \sigma(n) \land \sigma(s)$, then $s$ dominates $n$ and $n$ dominates $s$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. If for every neutrosophic vertex $s$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus S$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(NTG)$; for given two vertices, $s$ and $n$, if $\mu(ns) = \sigma(n) \land \sigma(s)$, then $s$ dominates $n$ and $n$ dominates $s$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. If for every neutrosophic vertex $s$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus S$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(NTG)$. As concluding results, there are some statements,
Abstract

remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of dual-dominating number,” and “Setting of neutrosophic dual-dominating number,” for introduced results and used classes. This approach facilitates identifying sets which form dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of dominated vertices and neutrosophic cardinality of set of dominated vertices corresponded to dual-dominating set have eligibility to define dual-dominating number and neutrosophic dual-dominating number but different types of set of dominated vertices to define dual-dominating sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of set of dominated vertices in the terms of maximum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic dual-dominating notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this chapter. In second chapter, there are some points as follow. New setting is introduced to study dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Maximum number of resolved vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of resolved vertices correspondeed to dual-resolving set is called neutrosophic dual-resolving number. Forming sets from resolved vertices to figure out different types of number of vertices in the sets from resolved sets in the terms of maximum number of vertices to get maximum number to assign to neutrosophic graphs is key type of approach to have these notions namely dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having largest number of resolved vertices from different types of sets in the terms of maximum number and maximum neutrosophic number forming it to get maximum number to assign to a neutrosophic graph. Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices, $s$ and $s'$ if $d(s, n) \neq d(s', n)$, then $n$ resolves $s$ and $s'$ where $d$ is the minimum number of edges amid all paths from $s$ to $s'$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. If for every two neutrosophic vertices $s, s'$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus S$ such that $n$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S$ is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(NTG)$; for given two vertices, $s$ and $s'$ if $d(s, n) \neq d(s', n)$, then $n$ resolves $s$ and $s'$ where $d$ is the minimum number of edges amid all paths.
from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. If for every two neutrosophic vertices \( s, s' \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S \) is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}_n(NTG) \). As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of dual-resolving number,” and “Setting of neutrosophic dual-resolving number,” for introduced results and used classes. This approach facilitates identifying sets which form dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of resolved vertices and neutrosophic cardinality of set of resolved vertices corresponded to dual-resolving set have eligibility to define dual-resolving number and neutrosophic dual-resolving number but different types of set of resolved vertices to define dual-resolving sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of set of resolved vertices in the terms of maximum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic dual-resolving notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this chapter.


Two chapters are devised as “Initial Notions”, and “Modified Notions".
The author is going to express his gratitude and his appreciation about the brains and their hands which are showing the importance of words in the framework of every wisdom, knowledge, arts, and emotions which are streaming in the lines from the words, notions, ideas and approaches to have the material and the contents which are only the way to flourish the minds, to grow the notions, to advance the ways and to make the stable ways to be amid events and storms of minds for surviving from them and making the outstanding experiences about the tools and the ideas to be on the star lines of words and shining like stars, forever.
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CHAPTER 1

Initial Notions

The following sections are cited as follows, which is my 79th manuscript and I use prefix 79 as number before any labelling for items.


Dual-Dominating Numbers in Neutrosophic Style and Crisp Style Obtained From Classes of Neutrosophic Graphs

1.1 Abstract

New setting is introduced to study dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Maximum number of dominated vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of dominated vertices corresponded to dual-dominating set is called neutrosophic dual-dominating number. Forming sets from dominated vertices to figure out different types of number of vertices in the sets from dominated sets $n$ in the terms of maximum number of vertices to get maximum number to assign to neutrosophic graphs is key type of approach to have these notions namely dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs.

Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having largest number of dominated vertices from different types of sets in the terms of maximum number and maximum neutrosophic number forming it to get maximum number to assign to a neutrosophic graph.

Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices, $s$ and $n$, if $\mu(ns) = \sigma(n) \land \sigma(s),$ then $s$ dominates $n$ and $n$ dominates $s$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex $s$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus S$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(NTG)$; for given two vertices, $s$ and $n$, if $\mu(ns) = \sigma(n) \land \sigma(s),$ then $s$ dominates $n$ and $n$ dominates $s$. Let $S$ be a set of neutrosophic vertices
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[a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every neutrosophic vertex \( s \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S \) is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D^*_n(NTG) \). As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of dual-dominating number,” and “Setting of neutrosophic dual-dominating number,” for introduced results and used classes. This approach facilitates identifying sets which form dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of dominated vertices and neutrosophic cardinality of set of dominated vertices corresponded to dual-dominating set have eligibility to define dual-dominating number and neutrosophic dual-dominating number but different types of set of dominated vertices to define dual-dominating sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of set of dominated vertices in the terms of maximum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic dual-dominating notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

Keywords: Dual-Dominating Number, Neutrosophic Dual-Dominating

AMS Subject Classification: 05C17, 05C22, 05E45

1.2 Background

1.3 Motivation and Contributions

In this study, there’s an idea which could be considered as a motivation.

**Question 1.3.1.** Is it possible to use mixed versions of ideas concerning “dual-dominating number”, “neutrosophic dual-dominating number” and “Neutrosophic Graph” to define some notions which are applied to neutrosophic graphs?

It’s motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two paths have key roles to assign dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Thus they’re used to define new ideas which conclude to the structure of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. The concept of having largest number of dominated vertices in the terms of crisp setting and in the terms of neutrosophic setting inspires us to study the behavior of all dominated vertices in the way that, some types of numbers, dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, corresponded numbers conclude the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection “Preliminaries”, new notions of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as
individuals. In section “Preliminaries”, Maximum number of dominated vertices, is a number which is representative based on those vertices, have the key role in this way. General results are obtained and also, the results about the basic notions of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs, in section “Setting of dual-dominating number,” as individuals. In section “Setting of dual-dominating number,” dual-dominating number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of dual-dominating number,” and “Setting of neutrosophic dual-dominating number,” for introduced results and used classes. In section “Applications in Time Table and Scheduling”, two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications is featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

1.4 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited. Basic idea is about the model which is used. First definition introduces basic model.

Definition 1.4.1. (Graph).

$G = (V, E)$ is called a graph if $V$ is a set of objects and $E$ is a subset of $V \times V$ ($E$ is a set of 2-subsets of $V$) where $V$ is called vertex set and $E$ is called edge set. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

Definition 1.4.2. (Neutrosophic Graph And Its Special Case).

$NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3))$ is called a neutrosophic graph if it’s graph, $\sigma_i : V \rightarrow [0, 1]$, and $\mu_i : E \rightarrow [0, 1]$. We add one condition on it and we use special case of neutrosophic graph but with same name. The added condition is as follows, for every $v_i v_j \in E$,

$$\mu(v_i v_j) \leq \sigma(v_i) \land \sigma(v_j).$$

(i) : $\sigma$ is called neutrosophic vertex set.
1.4. Preliminaries

(ii) $\mu$ is called neutrosophic edge set.

(iii) $|V|$ is called order of NTG and it’s denoted by $\mathcal{O}(\text{NTG})$.

(iv) $\sum_{v \in V} \sum_{i=1}^{3} \sigma_i(v)$ is called neutrosophic order of NTG and it’s denoted by $\mathcal{O}_n(\text{NTG})$.

(v) $|E|$ is called size of NTG and it’s denoted by $\mathcal{S}(\text{NTG})$.

(vi) $\sum_{e \in E} \sum_{i=1}^{3} \mu_i(e)$ is called neutrosophic size of NTG and it’s denoted by $\mathcal{S}_n(\text{NTG})$.

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

**Definition 1.4.3.** Let $NTG : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then

(i) a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(\text{NTG})}$ is called path where $x_i, x_{i+1} \in E, i = 0, 1, \cdots, \mathcal{O}(\text{NTG}) - 1$;

(ii) strength of path $P : x_0, x_1, \cdots, x_{\mathcal{O}(\text{NTG})}$ is $\bigwedge_{i=0, \ldots, \mathcal{O}(\text{NTG})-1} \mu(x_i x_{i+1})$;

(iii) connectedness amid vertices $x_0$ and $x_1$ is

$$\mu^\infty(x_0, x_1) = \bigvee_{p : x_0, x_1, \cdots, x_t} \bigwedge_{i=0, \ldots, t-1} \mu(x_i x_{i+1});$$

(iv) a sequence of consecutive vertices $P : x_0, x_1, \cdots, x_{\mathcal{O}(\text{NTG})}, x_0$ is called cycle where $x_i x_{i+1} \in E, i = 0, 1, \cdots, \mathcal{O}(\text{NTG}) - 1$, $x_{\mathcal{O}(\text{NTG})} x_0 \in E$ and there are two edges $xy$ and $uv$ such that $\mu(xy) = \mu(uv) = \bigwedge_{i=0, \ldots, \mathcal{O}(\text{NTG})-1} \mu(u_i v_{i+1})$;

(v) it’s t-partite where $V$ is partitioned to $t$ parts, $V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t}$ and the edge $xy$ implies $x \in V_i^{s_i}$ and $y \in V_j^{s_j}$ where $i \neq j$. If it’s complete, then it’s denoted by $K_{s_1, s_2, \cdots, s_t}$ where $s_i$ is $\sigma$ on $V_i^{s_i}$ instead $V$ which mean $x \notin V_i$ induces $\sigma_i(x) = 0$. Also, $|V_i^{s_i}| = s_i$;

(vi) t-partite is complete bipartite if $t = 2$, and it’s denoted by $K_{s_1, s_2}$;

(vii) complete bipartite is star if $|V_1| = 1$, and it’s denoted by $S_{1, s_2}$;

(viii) a vertex in $V$ is center if the vertex joins to all vertices of a cycle. Then it’s wheel and it’s denoted by $W_{1, s_2}$;

(ix) it’s complete where $\forall uv \in V, \mu(uv) = \sigma(u) \wedge \sigma(v)$;

(x) it’s strong where $\forall uv \in E, \mu(uv) = \sigma(u) \wedge \sigma(v)$.

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.
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**Definition 1.4.4.** (Neutrosophic Graph And Its Special Case).

\( NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)) \) is called a neutrosophic graph if it’s graph, \( \sigma: V \rightarrow [0, 1] \), and \( \mu_i: E \rightarrow [0, 1] \). We add one condition on it and we use special case of neutrosophic graph but with same name. The added condition is as follows, for every \( v, v_j \in E \),

\[ \mu(v, v_j) \leq \sigma(v_j) \land \sigma(v_j). \]

\(|V|\) is called order of NTG and it’s denoted by \( O(NTG) \). \( \Sigma_{v \in V} \sigma(v) \) is called neutrosophic order of NTG and it’s denoted by \( O_n(NTG) \).

**Definition 1.4.5.** Let \( NTG : (V, E, \sigma, \mu) \) be a neutrosophic graph. Then it’s complete and denoted by \( CMT_\sigma \) if \( \forall x, y \in V \), \( xy \in E \) and \( \mu(xy) = \sigma(x) \land \sigma(y) \); a sequence of consecutive vertices \( P : x_0, x_1, \ldots, x_\ell \in V \) is called path and it’s denoted by \( PTH \) where \( x_0, x_1, \ldots, x_\ell \in V \) and \( \ell \) is the length of the path. A sequence of consecutive vertices \( P : x_0, x_1, \ldots, x_n \in V \) is called cycle and denoted by \( CYC \) where \( x_0, x_1, \ldots, x_n \in V \) and there are two edges \( xy \) and \( uv \) such that \( \mu(xy) = \mu(uv) = \sum_{i=0}^{n} \mu(v_i v_{i+1}) \); it’s \( t \)-partite where \( V \) is partitioned to \( t \) parts, \( V_1^{\sigma_1}, V_2^{\sigma_2}, \ldots, V_t^{\sigma_t} \) and the edge \( xy \) implies \( x \in V_i^{\sigma_i} \) and \( y \in V_j^{\sigma_j} \) where \( i \neq j \). If it’s complete, then it’s denoted by \( CMT_{\sigma_1, \sigma_2, \ldots, \sigma_t} \) where \( \sigma_i \) is \( \sigma \) on \( V_i^{\sigma_i} \) instead \( V \) which mean \( x \notin V_i \) induces \( \sigma_i(x) = 0 \). Also, \( |V_j^{\sigma_j}| = s_i \); \( t \)-partite is complete bipartite if \( t = 2 \), and it’s denoted by \( CMT_{\sigma_1, \sigma_2} \); complete bipartite is star if \( |V_1| = 1 \), and it’s denoted by \( STR_{1, \sigma_1} \); a vertex in \( V \) is center if the vertex joins to all vertices of a cycle. Then it’s wheel and it’s denoted by \( WHL_{1, \sigma_2} \).

**Remark 1.4.6.** Using notations which is mixed with literatures, are reviewed.

1.4.6.1. \( NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)), O(NTG), \) and \( O_n(NTG) \);\n
1.4.6.2. \( CMT_\sigma, \) \( PTH, \) \( CYC, \) \( STR_{1, \sigma_1}, \) \( CMT_{\sigma_1, \sigma_2}, \) \( CMT_{\sigma_1, \sigma_2, \ldots, \sigma_t}, \) \( WHL_{1, \sigma_2} \).

**Definition 1.4.7.** (Dual-Dominating Numbers).

Let \( NTG : (V, E, \sigma, \mu) \) be a neutrosophic graph. Then

1. for given two vertices, \( s \) and \( n \), if \( \mu(ns) = \sigma(n) \land \sigma(s) \), then \( s \) dominates \( n \) and \( n \) dominates \( s \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. If for every neutrosophic vertex \( s \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S \) is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(NTG) \);

2. for given two vertices, \( s \) and \( n \), if \( \mu(ns) = \sigma(n) \land \sigma(s) \), then \( s \) dominates \( n \) and \( n \) dominates \( s \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. If for every neutrosophic vertex \( s \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S \) is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(NTG) \).
1.5. Setting of dual-dominating number

For convenient usages, the word neutrosophic which is used in previous definition, won’t be used, usually.
In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

Example 1.4.8. In Figure [1.1], a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two vertices, \( s \) and \( n \), \( \mu(ns) = \sigma(n) \land \sigma(s) \). Thus \( s \) dominates \( n \) and \( n \) dominates \( s \);
(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;
(iii) for given two vertices, \( s \) and \( n \), \( \mu(ns) = \sigma(n) \land \sigma(s) \), then \( s \) dominates \( n \) and \( n \) dominates \( s \). Let \( S = V \setminus \{n\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = V \setminus \{n\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = V \setminus \{n\} \) is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(NTG) = O(NTG) - 1 \);
(iv) the corresponded set doesn’t have to be dominated by the set;
(v) \( V \) is exception when the set is considered in this notion;
(vi) for given two vertices, \( s \) and \( n \), \( \mu(ns) = \sigma(n) \land \sigma(s) \), then \( s \) dominates \( n \) and \( n \) dominates \( s \). Let \( S = V \setminus \{n\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = V \setminus \{n\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = V \setminus \{n\} \) is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(NTG) = O_n(NTG) - \sum_{i=1}^3 \sigma_i(n_i) = 5 \).

1.5 Setting of dual-dominating number

In this section, I provide some results in the setting of dual-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

Proposition 1.5.1. Let \( NTG : (V, E, \sigma, \mu) \) be a complete-neutrosophic graph. Then

\[
\mathcal{D}(CMT_\sigma) = \mathcal{O}(CMT_\sigma) - 1.
\]
1. Initial Notions

Figure 1.1: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

Proof. Suppose $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there’s one edge between two vertices. For given two vertices, $s$ and $n$, $\mu(ns) = \sigma(n) \land \sigma(s)$, then $s$ dominates $n$ and $n$ dominates $s$. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = V \setminus \{n\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(NTG) = O(NTG) - 1$. Thus

$$D(CMT_\sigma) = O(CMT_\sigma) - 1.$$ 

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

Example 1.5.2. In Figure 1.2, a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two vertices, $s$ and $n$, $\mu(ns) = \sigma(n) \land \sigma(s)$, Thus $s$ dominates $n$ and $n$ dominates $s$;

(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;

(iii) for given two vertices, $s$ and $n$, $\mu(ns) = \sigma(n) \land \sigma(s)$, then $s$ dominates $n$ and $n$ dominates $s$. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = V \setminus \{n\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum
1.5. Setting of dual-dominating number

Figure 1.2: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

Cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CMT_\sigma) = O(CMT_\sigma) - 1$;

(iv) the corresponded set doesn’t have to be dominated by the set;

(v) $V$ is exception when the set is considered in this notion;

(vi) for given two vertices, $s$ and $n$, $\mu(ns) = \sigma(n) \land \sigma(s)$, then $s$ dominates $n$ and $n$ dominates $s$. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]

If for every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = V \setminus \{n\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(CMT_\sigma) = O_n(CMT_\sigma) - \sum_{i=1}^{3} \sigma_i(n_4) = 5$.

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

**Proposition 1.5.3.** Let $NTG : (V, E, \sigma, \mu)$ be a path-neutrosophic graph. Then

$$D(PTH) = \left\lfloor \frac{2 \times O(PTH)}{3} \right\rfloor.$$ 

**Proof.** Suppose $PTH : (V, E, \sigma, \mu)$ is a path-neutrosophic graph. Let $x_1, x_2, \cdots, x_{O(PTH)}$ be a path-neutrosophic graph. For given two vertices, $x$ and $y$, there’s one path from $x$ to $y$. Let $S$ be an intended set which is dual-dominating set. Despite leaves $x_1$, and $x_{O(PTH)}$, two consecutive vertices belong to $S$. They could be dominated by previous vertex and upcoming vertex as if despite them so as maximal set $S$ is constructed. Thus $S = \{x_1', x_2', \cdots, x'_{O(PTH)}\}$ is the set $S$ is a set of vertices from path-neutrosophic graph $PTH : (V, E, \sigma, \mu)$ with new arrangements of vertices in which there are two consecutive vertices which aren’t neighbors. In this new arrangements, the notation of vertices from $x_i$ is changed to $x'_i$. Leaves doesn’t necessarily belong to $S$. Leaves are either belongs to $S$ or doesn’t belong to $S$. Adding only the vertices which aren’t consecutive contradicts with maximality of $S$ and maximum cardinality of $S$. It implies this construction is optimal. Thus, let

$$S = \{x_1, x_2, \cdots, x_{O(PTH)} - 1, x_{O(PTH)}\}.$$
1. Initial Notions

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus (S = \{x_1, x_2, \cdots, x_{ \lceil 2 \times O(PTH) \rceil - 1}, \frac{x_{ \lceil 2 \times O(PTH) \rceil}}{2} \}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{x_1, x_2, \cdots, x_{ \lceil 2 \times O(PTH) \rceil - 1}, \frac{x_{ \lceil 2 \times O(PTH) \rceil}}{2} \} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

\[
D(PTH) = \left\lfloor \frac{2 \times O(PTH)}{3} \right\rfloor.
\]

Thus

\[
D(PTH) = \left\lfloor \frac{2 \times O(PTH)}{3} \right\rfloor.
\]

**Example 1.5.4.** There are two sections for clarifications.

(a) In Figure [1.3], an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Let \( S = \{n_3, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 3 \);

(ii) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 3 \);

(iii) let \( S = \{n_3, n_4, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 3 \);

(iv) let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-dominating set.
In Figure (1.4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is let alongside triple pair of its values is called neutrosophic vertex. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1\} \) is called dual-dominating set. As if it, 3.3, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(PTH) = 3.7 \).

Let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(PTH) = 3.7 \).

In Figure (1.4), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

Let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D_n(PTH) = 4 \);

Let \( S = \{n_3, n_4, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 4 \);

Let \( S = \{n_3, n_4, n_1, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1, n_6\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1, n_6\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 4 \);

Let \( S = \{n_3, n_2, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which
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Figure 1.3: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

are consecutive vertices. For a neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_3, n_2, n_6 \}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{ n_3, n_2, n_6 \} \) is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 4 \):

(v) every set containing three consecutive vertices isn’t dual-dominating set. For instance, let \( S = \{ n_3, n_4, n_2 \} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,]. For a neutrosophic vertex \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_3, n_4, n_2 \}) \) such that \( n \) dominates \( n_3 \), then the set of neutrosophic vertices, \( S = \{ n_3, n_4, n_1 \} \) isn’t called dual-dominating set. So as maximum neutrosophic cardinality isn’t related to the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(PTH) = 6 \);

(vi) let \( S = \{ n_3, n_4, n_1, n_6 \} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_3, n_4, n_1, n_6 \}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{ n_3, n_4, n_1, n_6 \} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(PTH) = 6.3 \).

Proposition 1.5.5. Let \( NTG : (V, E, \sigma, \mu) \) be a cycle-neutrosophic graph where \( O(CYC) \geq 3 \). Then

\[
D(CYC) = \left\lfloor \frac{2 \times O(CYC)}{3} \right\rfloor.
\]

Proof. Suppose \( CYC : (V, E, \sigma, \mu) \) is a cycle-neutrosophic graph. For given two vertices, \( x \) and \( y \), there are only two paths with distinct edges from \( x \) to \( y \). Let

\[ x_1, x_2, \ldots, x_{O(CYC)-1}, x_O(CYC), x_1 \]
be a cycle-neutrosophic graph $CYC: (V,E,\sigma,\mu)$. Two consecutive vertices could belong to $S$ which is dual-dominating set related to dual-dominating number. Since these two vertices could be dominated by previous vertex and upcoming vertex despite them. If there are no vertices which are consecutive, then it contradicts with maximality of set $S$ and maximum cardinality of $S$. Thus, let

\[ S = \{x_1, x_2, \cdots, x_{\left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor - 1}, x_{\left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor}, x_1\} \]

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus S = \{x_1, x_2, \cdots, x_{\left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor - 1}, x_{\left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor}, x_1\}$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor - 1}, x_{\left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor}, x_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

\[ D(CYC) = \left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor. \]

Thus

\[ D(CYC) = \left\lfloor \frac{2 \times \mathcal{O}(CYC)}{3} \right\rfloor. \]

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 1.5.6.** There are two sections for clarifications.

(a) In Figure 1.5, an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.
1. Initial Notions

(i) Let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are only consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_2, n_5\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 4$;

(ii) Let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_4, n_1\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 4$;

(iii) Let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_4, n_1, n_6\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 4$;

(iv) Let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 4$;

(v) Let $S = \{n_1, n_2, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_1, n_2, n_4, n_5\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 4$;

(vi) Let $S = \{n_2, n_3, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_3, n_5, n_6\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(CYC) = 5.9$.

(b) In Figure 1.6, an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.
1.5. Setting of dual-dominating number

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_2\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 3$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_4\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_4, n_1\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 3$;

(iv) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_2, n_5\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CYC) = 3$;

(v) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_2, n_5\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5\}$ is called dual-dominating set. As if it, 5.1, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(CYC) = 5.7$;

(vi) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_3, n_4, n_1\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(CYC) = 5.7$. 

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**Proposition 1.5.7.** Let $NTG : (V, E, \sigma, \mu)$ be a star-neutrosophic graph with center $c$. Then

$$D(STR_{1, \sigma_2}) = O(STR_{1, \sigma_2}) - 1.$$  

**Proof.** Suppose $STR_{1, \sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, $c$, as one of its endpoints. All paths have one as their lengths, forever. $S = V \setminus \{c\}$ is a dual-dominating set related dual-dominating number. Since, let

$$S = V \setminus \{c\} = \{x_1, x_2, \ldots, x_{O(STR_{1, \sigma_2})-1}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex $x_i$ in $S$, there’s only one neutrosophic vertex $c$ in $V \setminus \{c\}$ such that $c$ dominates $x_i$, then the set of neutrosophic vertices, $S = V \setminus \{c\} = \{x_1, x_2, \ldots, x_{O(STR_{1, \sigma_2})-1}\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

$$D(STR_{1, \sigma_2}) = O(STR_{1, \sigma_2}) - 1.$$
1.5. Setting of dual-dominating number

Thus

\[ D(STR_{1,\sigma_2}) = \sigma(STR_{1,\sigma_2}) - 1. \]

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 1.5.8.** There is one section for clarifications. In Figure [17], a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex \( n_2 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \ \setminus (S = \{n_1, n_2\}) \) such that \( n \) dominates \( n_2 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2\} \) isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma_2}) = 4 \);

(ii) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \ \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma_2}) = 4 \);

(iii) let \( S = \{n_2, n_3, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \ \setminus (S = \{n_2, n_3, n_4, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4, n_5\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma_2}) = 4 \);

(iv) let \( S = \{n_1, n_3, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \ \setminus (S = \{n_1, n_3, n_4, n_5\}) \) such that \( n \) dominates \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3, n_4, n_5\} \) isn’t called dual-dominating set. So as its cardinality doesn’t relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma_2}) = 4 \);

(v) let \( S = \{n_1, n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For
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![Figure 1.7: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.](image)

1. Initial Notions

Figure 1.7: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

a neutrosophic vertex \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_1, n_3, n_2, n_5 \}) \) such that \( n \) dominates \( n_3 \), then the set of neutrosophic vertices, \( S = \{ n_1, n_3, n_2, n_5 \} \) isn’t called dual-dominating set. So as its cardinality doesn’t relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(S\mathcal{R}_{1,2}) = 4 \); 

\((vi)\) there’s only one dual-dominating set thus let \( S = \{ n_2, n_3, n_4, n_5 \} \) be a set of neutrosophic vertices \( [ \text{a vertex alongside triple pair of its values is called neutrosophic vertex.} ] \). For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_2, n_3, n_4, n_5 \}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{ n_2, n_3, n_4, n_5 \} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(S\mathcal{R}_{1,2}) = 5.7 \).

**Proposition 1.5.9.** Let \( NTG : (V, E, \sigma, \mu) \) be a complete-bipartite-neutrosophic graph. Then 

\[ D(CMC_{\sigma_1, \sigma_2}) = O(CMC_{\sigma_1, \sigma_2}) - 2. \]

**Proof.** Suppose \( CMC_{\sigma_1, \sigma_2} : (V, E, \sigma, \mu) \) is a complete-bipartite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding one vertex from each part. Let

\[ S = V \setminus \{ u, v \}_{u \in V_1, v \in V_2} = V_1 \setminus \{ u \} \cup V_2 \setminus \{ v \} = \{ x_1, x_2, \ldots, x_{O(CMC_{\sigma_1, \sigma_2})-2} \} \]

be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let

\[ S = V \setminus \{ u, v \}_{u \in V_1, v \in V_2} = V_1 \setminus \{ u \} \cup V_2 \setminus \{ v \} = \{ x_1, x_2, \ldots, x_{O(CMC_{\sigma_1, \sigma_2})-2} \} \]

be a set of neutrosophic vertices \( [ \text{a vertex alongside triple pair of its values is called neutrosophic vertex.} ] \). For every neutrosophic vertex \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in

\[ V \setminus (S = V \setminus \{ u, v \}_{u \in V_1, v \in V_2} = V_1 \setminus \{ u \}) \cup V_2 \setminus \{ v \} = \{ x_1, x_2, \ldots, x_{O(CMC_{\sigma_1, \sigma_2})-2} \} \]

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such that \( n \) dominates \( s \), then the set of neutrosophic vertices,
\[
S = V \setminus \{u, v\} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{\mathcal{O}(\text{CMC}_{\sigma_1, \sigma_2})-2}\}
\]
is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by
\[
D(\text{CMC}_{\sigma_1, \sigma_2}) = \mathcal{O}(\text{CMC}_{\sigma_1, \sigma_2}) - 2.
\]
Thus
\[
D(\text{CMC}_{\sigma_1, \sigma_2}) = \mathcal{O}(\text{CMC}_{\sigma_1, \sigma_2}) - 2.
\]

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 1.5.10.** There is one section for clarifications. In Figure [1.8], a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_2, n_4\}) \) such that \( n \) dominates \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_4\} \) isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(\text{CMC}_{\sigma_1, \sigma_2}) = 2 \);

(ii) let \( S = \{n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(\text{CMC}_{\sigma_1, \sigma_2}) = 2 \);

(iii) let \( S = \{n_2, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_1\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(\text{CMC}_{\sigma_1, \sigma_2}) = 2 \);

(iv) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic
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Figure 1.8: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC) = 2 \).

\((v)\) let \( S = \{n_4, n_3\} \) be a set of neutrosophic vertices \( \{a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.}\} \). For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_4, n_3\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_4, n_3\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC) = 2 \).

\((vi)\) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices \( \{a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.}\} \). For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_3\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(CMC) = 3.4 \).

**Proposition 1.5.11.** Let \( NTG : (V, E, \sigma, \mu) \) be a complete-t-partite-neutrosophic graph where \( t \geq 3 \). Then

\[
D(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) - 2.
\]

**Proof.** Suppose \( CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t} : (V, E, \sigma, \mu) \) is a complete-t-partite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding two vertices from two different parts. Let

\[
S = V \setminus \{u, v\}, u \in V_1, v \in V_2 = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.
\]
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Thus
\[ S = \{x_1, x_2, \ldots, x_{O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\} \]
be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let
\[ S = V \setminus \{u, v\} \cup V_1 \cup \ldots \cup V_t. \]
Thus
\[ S = \{x_1, x_2, \ldots, x_{O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\} \]
be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in
\[ V \setminus (S = V \setminus \{u, v\} \cup V_1 \cup \ldots \cup V_t). \]
Such that \( n \) dominates \( s \), then the set of neutrosophic vertices,
\[ S = \{x_1, x_2, \ldots, x_{O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\} \]
is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by
\[ D(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) - 2. \]
Thus
\[ D(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) - 2. \]

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 1.5.12.** There is one section for clarifications. In Figure 1.9, a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For a neutrosophic vertex \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_2, n_4\}) \) such that \( n \) dominates \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_4\} \) isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2. \)
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(ii) let \( S = \{n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1,\sigma_2,\ldots,\sigma_t}) = 2; \)

(iii) let \( S = \{n_2, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_1\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1,\sigma_2,\ldots,\sigma_t}) = 2; \)

(iv) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1,\sigma_2,\ldots,\sigma_t}) = 2; \)

(v) let \( S = \{n_4, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_4, n_3\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_4, n_3\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1,\sigma_2,\ldots,\sigma_t}) = 2; \)

(vi) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_3\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(CMC_{\sigma_1,\sigma_2,\ldots,\sigma_t}) = 3.4. \)

**Proposition 1.5.13.** Let \( NTG : (V,E,\sigma,\mu) \) be a wheel-neutrosophic graph. Then

\[
D(WHL_{1,\sigma_2}) = \mathcal{O}(WHL_{1,\sigma_2}) - 1.
\]

**Proof.** Suppose \( WHL_{1,\sigma_2} : (V,E,\sigma,\mu) \) is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, \( c \). \( S = V \setminus \{c\} \) is a dual-dominating set related dual-dominating number. Since, let

\[
S = V \setminus \{c\} = \{x_1, x_2, \ldots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}
\]
1.5. Setting of dual-dominating number

Figure 1.9: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex \( x_i \) in \( S \), there’s a neutrosophic vertex \( c \) in \( V \setminus \{ c \} = \{ x_1, x_2, \cdots, x_{O(WHL_{1,\sigma_2})-1} \} \) such that \( c \) dominates \( x_i \), then the set of neutrosophic vertices, \( S = V \setminus \{ c \} = \{ x_1, x_2, \cdots, x_{O(WHL_{1,\sigma_2})-1} \} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

\[
D(WHL_{1,\sigma_2}) = O(WHL_{1,\sigma_2}) - 1.
\]

Thus

\[
D(WHL_{1,\sigma_2}) = O(WHL_{1,\sigma_2}) - 1.
\]

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 1.5.14.** There is one section for clarifications. In Figure [1.10], a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{ n_1, n_2, n_3, n_5 \} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex \( n_2 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus \{ S = \{ n_1, n_2, n_3, n_5 \} \) such that \( n \) dominates \( n_2 \), then the set of neutrosophic vertices, \( S = \{ n_1, n_2, n_3, n_5 \} \) isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(WHL_{1,\sigma_2}) = 4; \)
1. Initial Notions

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s a neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_4\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(WHL_{1,\sigma_2}) = 4$;

(iii) let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n_1$ in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that $n_1$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(WHL_{1,\sigma_2}) = 4$;

(iv) let $S = \{n_2, n_3, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s a neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_3, n_4\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(WHL_{1,\sigma_2}) = 4$;

(v) let $S = \{n_2, n_3, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s a neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_3, n_5\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_5\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(WHL_{1,\sigma_2}) = 4$;

(vi) there’s only one dual-dominating set thus let $S = \{n_2, n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_3, n_4, n_5\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_3, n_4, n_5\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(WHL_{1,\sigma_2}) = 5.3$.

1.6 Setting of neutrosophic dual-dominating number

In this section, I provide some results in the setting of neutrosophic dual-dominating number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph,
1.6. Setting of neutrosophic dual-dominating number

Proposition 1.6.1. Let $NTG : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then

$$D_n(CMT_{\sigma}) = \mathcal{O}_n(CMT_{\sigma}) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

Proof. Suppose $CMT_{\sigma} : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $CMT_{\sigma} : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there’s one edge between two vertices. For given two vertices, $s$ and $n$, $\mu(ns) = \sigma(n) \land \sigma(s)$, then $s$ dominates $n$ and $n$ dominates $s$. Let $S = V \setminus \{n\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = V \setminus \{n\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = V \setminus \{n\}$ is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(NTG) = \mathcal{O}(NTG) - 1$. Thus

$$D_n(CMT_{\sigma}) = \mathcal{O}_n(CMT_{\sigma}) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).$$

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

Figure 1.10: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.
Example 1.6.2. In Figure 1.11, a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two vertices, \( s \) and \( n \), \( \mu(ns) = \sigma(n) \land \sigma(s) \). Thus \( s \) dominates \( n \) and \( n \) dominates \( s \);

(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum dominating set and minimal dominating set;

(iii) for given two vertices, \( s \) and \( n \), \( \mu(ns) = \sigma(n) \land \sigma(s) \), then \( s \) dominates \( n \) and \( n \) dominates \( s \). Let \( S = V \setminus \{n\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = V \setminus \{n\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = V \setminus \{n\} \) is called dual-dominating set. The maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMT) = O(CMT) - 1 \);

(iv) the corresponded set doesn’t have to be dominated by the set;

(v) \( V \) is exception when the set is considered in this notion;

(vi) for given two vertices, \( s \) and \( n \), \( \mu(ns) = \sigma(n) \land \sigma(s) \), then \( s \) dominates \( n \) and \( n \) dominates \( s \). Let \( S = V \setminus \{n\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] If for every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = V \setminus \{n\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = V \setminus \{n\} \) is called dual-dominating set. The maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(CMT) = O_n(CMT) - \sum_{i=1}^{3} \sigma_i(n_4) = 5 \).

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

Proposition 1.6.3. Let \( NTG : (V, E, \sigma, \mu) \) be a path-neutrosophic graph. Then

\[
D_n(PTH) = \max_{x \in S_{\{x_1, x_2, \ldots, x_n\}}} \sum_{i=1}^{3} \sigma_i(x)
\]
Thus called dual-dominating number and it’s denoted by $n$ oneneutrosophicvertex $n$ called neutrosophic vertex. For every neutrosophic vertex be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are construction is optimal. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\lfloor 2 \times \sigma(PTH) \rfloor - 1}, x_1 \}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus \{S = \{x_1, x_2, \cdots, x_{\lfloor 2 \times \sigma(PTH) \rfloor - 1}, x_1 \} \}$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\lfloor 2 \times \sigma(PTH) \rfloor - 1}, x_1 \}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

$$D_n(PTH) = \max_{x \in S = (x_1, x_2, \cdots, x_{\lfloor 2 \times \sigma(PTH) \rfloor - 1}, x_1)} \sum_{i=1}^{3} \sigma_i(x)$$

Thus

$$D_n(PTH) = \max_{x \in S = (x_1, x_2, \cdots, x_{\lfloor 2 \times \sigma(PTH) \rfloor - 1}, x_1)} \sum_{i=1}^{3} \sigma_i(x)$$

Example 1.6.4. There are two sections for clarifications.

(a) In Figure 12, an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus \{S = \{n_3, n_2\} \}$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(PTH) = 3$;
1. Initial Notions

(ii) Let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 3 \);

(iii) Let \( S = \{n_3, n_4, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 3 \);

(iv) Let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(PTH) = 3 \);

(v) Let \( S = \{n_3, n_4, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1\} \) is called dual-dominating set. As if it, 3.3, contradicts with the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(PTH) = 3.7 \);

(vi) Let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(PTH) = 3.7 \).

(b) In Figure 1.13, an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

(i) Let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-dominating set and this set is maximal. As if it contradicts
with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(PTH) = 4$;

(ii) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus \{S = \{n_3, n_4, n_1\}\}$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(PTH) = 4$;

(iii) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus \{S = \{n_3, n_4, n_1, n_6\}\}$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(PTH) = 4$;

(iv) let $S = \{n_3, n_2, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus \{S = \{n_3, n_2, n_6\}\}$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_6\}$ is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(PTH) = 4$;

(v) every set containing three consecutive vertices isn’t dual-dominating set. For instance, let $S = \{n_3, n_4, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For a neutrosophic vertex $n_3$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus \{S = \{n_3, n_4, n_2\}\}$ such that $n$ dominates $n_3$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ isn’t called dual-dominating set. So as maximum neutrosophic cardinality isn’t related to the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(PTH) = 6.3$;

(vi) let $S = \{n_3, n_4, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus \{S = \{n_3, n_4, n_1, n_6\}\}$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_6\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(PTH) = 6.3$.

**Proposition 1.6.5.** Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where
1. Initial Notions

Figure 1.12: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

\[ n_0(0.3,0.2,0.2) \rightarrow (0.3,0.2,0.1) \rightarrow n_2(0.9,0.8,0.1) \]

\[ n_1(0.2,0.5,0.7) \rightarrow (0.2,0.5,0.1) \rightarrow n_4(0.4,0.6,0.2) \]

\[ n_3(0.7,0.4,0.1) \]

Figure 1.13: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

\[ n_0(0.3,0.2,0.2) \rightarrow (0.2,0.2,0.2) \rightarrow n_2(0.2,0.4,0.5) \]

\[ n_1(0.6,0.8,0.8) \rightarrow (0.9,0.1,0.9) \rightarrow n_4(0.8,0.5,0.2) \]

\[ n_5(0.9,0.9,0.9) \]

\[ O(CYC) \geq 3. \]

Then

\[ D_n(CYC) = \max_{x \in S} \left\{ x, \frac{\max \left\{ x, \prod_{i=1}^{3} \sigma_i(x) \right\}}{x_{CYC}} \right\} \]

Proof. Suppose CYC : (V, E, σ, µ) is a cycle-neutrosophic graph. For given two vertices, x and y, there are only two paths with distinct edges from x to y. Let

\[ x_1, x_2, \ldots, x_{O(CYC)-1}, x_{O(CYC)}, x_1 \]

be a cycle-neutrosophic graph CYC : (V, E, σ, µ). Two consecutive vertices could belong to S which is dual-dominating set related to dual-dominating number. Since these two vertices could be dominated by previous vertex and upcoming vertex despite them. If there are no vertices which are consecutive, then it contradicts with maximality of set S and maximum cardinality of S. Thus, let

\[ S = \{ x_1, x_2, \ldots, x_{\frac{\max \left\{ x, \prod_{i=1}^{3} \sigma_i(x) \right\}}{x_{CYC}}} \} \]

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutro-
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Neutrosophic vertex \( s \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus (S = \{x_1, x_2, \ldots, x_{\lfloor CYC(x) \rfloor - 1}, x_{\lfloor CYC(x) \rfloor}, x_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{x_1, x_2, \ldots, x_{\lfloor CYC(x) \rfloor - 1}, x_{\lfloor CYC(x) \rfloor}, x_1\} \) is called dual-dominating set.

So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

\[
D_n(CYC) = \max_{x \in S = \{x_1, x_2, \ldots, x_{\lfloor CYC(x) \rfloor - 1}, x_{\lfloor CYC(x) \rfloor}, x_1\}} \sum_{i=1}^{3} \sigma_i(x)
\]

Thus

\[
D_n(CYC) = \max_{x \in S = \{x_1, x_2, \ldots, x_{\lfloor CYC(x) \rfloor - 1}, x_{\lfloor CYC(x) \rfloor}, x_1\}} \sum_{i=1}^{3} \sigma_i(x)
\]

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

Example 1.6.6. There are two sections for clarifications.

(a) In Figure (1.14), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CYC) = 4 \);

(ii) let \( S = \{n_3, n_4, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CYC) = 4 \);

(iii) let \( S = \{n_3, n_4, n_1, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1, n_6\}) \) such that \( n \) dominates \( s \), then the set of
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neutrosophic vertices, \( S = \{n_3, n_4, n_1, n_6\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CYC) = 4 \);

(iv) let \( S = \{n_2, n_3, n_5, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3, n_5, n_6\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_5, n_6\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CYC) = 4 \);

(v) let \( S = \{n_1, n_2, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_2, n_4, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_4, n_5\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CYC) = 4 \);

(vi) let \( S = \{n_2, n_3, n_5, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3, n_5, n_6\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_5, n_6\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(CYC) = 5.9 \).

(b) In Figure 1.15, an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_3, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_2\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_3, n_2\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CYC) = 3 \);

(ii) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set and this set is maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CYC) = 3 \);

(iii) let \( S = \{n_3, n_4, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n \) dominates \( s \), then the set of
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Figure 1.14: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_3, n_2, n_5 \}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{ n_3, n_2, n_5 \} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(CYC) = 5 \).7.

Proposition 1.6.7. Let \( NTG : (V, E, \sigma, \mu) \) be a star-neutrosophic graph with center \( c \). Then

\[
D_n(STR_{1, \sigma_2}) = O_n(STR_{1, \sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).
\]
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Figure 1.15: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

Proof. Suppose $STR_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a star-neutrosophic graph. An edge always has center, $c$, as one of its endpoints. All paths have one as their lengths, forever. $S = V \setminus \{c\}$ is a dual-dominating set related dual-dominating number. Since, let $S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{O(STR_{1,\sigma_2})-1}\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex $x_i$ in $S$, there's only one neutrosophic vertex $c$ in $V \setminus \{c\} = \{x_1, x_2, \cdots, x_{O(STR_{1,\sigma_2})-1}\}$ such that $c$ dominates $x_i$, then the set of neutrosophic vertices, $S = V \setminus \{c\} = \{x_1, x_2, \cdots, x_{O(STR_{1,\sigma_2})-1}\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

$$D_n(STR_{1,\sigma_2}) = O_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

Thus

$$D_n(STR_{1,\sigma_2}) = O_n(STR_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

Example 1.6.8. There is one section for clarifications. In Figure 1.16, a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive
vertices. For a neutrosophic vertex \( n_2 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_2\}) \) such that \( n \) dominates \( n_2 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2\} \) isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma}) = 4 \);

\( (ii) \) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma}) = 4 \);

\( (iii) \) let \( S = \{n_2, n_3, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3, n_4, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4, n_5\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma}) = 4 \);

\( (iv) \) let \( S = \{n_1, n_3, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_3, n_4, n_5\}) \) such that \( n \) dominates \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3, n_4, n_5\} \) isn’t called dual-dominating set. So as its cardinality doesn’t relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma}) = 4 \);

\( (v) \) let \( S = \{n_1, n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For a neutrosophic vertex \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_3, n_2, n_5\}) \) such that \( n \) dominates \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3, n_2, n_5\} \) isn’t called dual-dominating set. So as its cardinality doesn’t relate to the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(STR_{1,\sigma}) = 4 \);

\( (vi) \) there’s only one dual-dominating set thus let \( S = \{n_2, n_3, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3, n_4, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4, n_5\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(STR_{1,\sigma}) = 5.7 \).
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Figure 1.16: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

Proposition 1.6.9. Let $NTG : (V,E,\sigma,\mu)$ be a complete-bipartite-neutrosophic graph. Then

$$D_n(CMC_{\sigma_1,\sigma_2}) = O_n(CMC_{\sigma_1,\sigma_2}) - \min_{x \in V_1, y \in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Proof. Suppose $CMC_{\sigma_1,\sigma_2} : (V,E,\sigma,\mu)$ is a complete-bipartite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding one vertex from each part. Let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{O(CMC_{\sigma_1,\sigma_2})-2}\}$$

be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{O(CMC_{\sigma_1,\sigma_2})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s a neutrosophic vertex $n$ in

$$V \setminus S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{O(CMC_{\sigma_1,\sigma_2})-2}\}$$

such that $n$ dominates $s$, then the set of neutrosophic vertices,

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} = \{x_1, x_2, \cdots, x_{O(CMC_{\sigma_1,\sigma_2})-2}\}$$

is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

$$D_n(CMC_{\sigma_1,\sigma_2}) = O_n(CMC_{\sigma_1,\sigma_2}) - \min_{x \in V_1, y \in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$

Thus

$$D_n(CMC_{\sigma_1,\sigma_2}) = O_n(CMC_{\sigma_1,\sigma_2}) - \min_{x \in V_1, y \in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).$$
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The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. Some items are devised to make it more clear, next part gives one special case to apply definitions and results on it.

Example 1.6.10. There is one section for clarifications. In Figure (1.17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For a neutrosophic vertex \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus \{S = \{n_1, n_2, n_4\}\} \) such that \( n \) dominates \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_4\} \), isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1, \sigma_2}) = 2 \);

(ii) Let \( S = \{n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertex \( n \) in \( S \), there’s a neutrosophic vertex \( n \) in \( V \setminus \{S = \{n_2\}\} \) such that \( n \) dominates \( n \), then the set of neutrosophic vertices, \( S = \{n_2\} \), is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1, \sigma_2}) = 2 \);

(iii) Let \( S = \{n_2, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus \{S = \{n_2, n_1\}\} \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_1\} \), is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1, \sigma_2}) = 2 \);

(iv) Let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus \{S = \{n_2, n_4\}\} \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \), is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1, \sigma_2}) = 2 \);

(v) Let \( S = \{n_4, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus \{S = \{n_4, n_3\}\} \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_4, n_3\} \), is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC_{\sigma_1, \sigma_2}) = 2 \);
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Figure 1.17: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_1, n_3\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_{n}(CMC_{\sigma_1, \sigma_2}) = 3.4$.

**Proposition 1.6.11.** Let $NTG : (V, E, \sigma, \mu)$ be a complete-$t$-partite-neutrosophic graph where $t \geq 3$. Then

$$D_{n}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = O_{n}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) - \min_{x \in V_i, y \in V_j, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)),$$

**Proof.** Suppose $CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t} : (V, E, \sigma, \mu)$ is a complete-$t$-partite-neutrosophic graph. Every vertex in a part is dominated by another vertex in opposite part. Thus maximum cardinality implies excluding two vertices from two different parts. Let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\}$$

be a dual-dominating set related to the dual-dominating number. This construction gives the proof. Since let

$$S = V \setminus \{u, v\}_{u \in V_1, v \in V_2} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t.$$

Thus

$$S = \{x_1, x_2, \cdots, x_{O(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex $s$ in $S$, there’s a
neutralosrophic vertex $n$ in

\[
V \setminus S = V \setminus \{u, v\} = V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{l-1} \cup V_t.
\]

such that $n$ dominates $s$, then the set of neutralosrophic vertices,

\[
S = \{x_1, x_2, \cdots, \sigma(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) - 2\}
\]

Thus

\[
S = \{x_1, x_2, \cdots, \sigma(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) - 2\}
\]

is called dual-dominating set. So as the maximum neutralosrophic cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

\[
\mathcal{D}_n(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) = \mathcal{O}_n(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) - \min_{x \in V_{1,y \in V_j},i \neq j} 3 (\sigma_i(x) + \sigma_i(y)).
\]

Thus

\[
\mathcal{D}_n(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) = \mathcal{O}_n(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) - \min_{x \in V_{1,y \in V_j},i \neq j} 3 (\sigma_i(x) + \sigma_i(y)).
\]

The clarifications about results are in progress as follows. A complete-t-partite-neutralosrophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutralosrophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 1.6.12.** There is one section for clarifications. In Figure 1.18, a complete-t-partite-neutralosrophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutralosrophic vertices [a vertex alongside triple pair of its values is called neutralosrophic vertex.] which are consecutive vertices. For a neutralosrophic vertex $n_1$ in $S$, there’s no neutralosrophic vertex $n$ in $V \setminus (S = \{n_1, n_2, n_4\})$ such that $n$ dominates $n_1$, then the set of neutralosrophic vertices, $S = \{n_1, n_2, n_4\}$ isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) = 2$;

(ii) let $S = \{n_2\}$ be a set of neutralosrophic vertices [a vertex alongside triple pair of its values is called neutralosrophic vertex.] which aren’t consecutive vertices. For every neutralosrophic vertex $s$ in $S$, there’s a neutralosrophic vertex $n$ in $V \setminus (S = \{n_2\})$ such that $n$ dominates $s$, then the set of neutralosrophic vertices, $S = \{n_2\}$ is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CMC\sigma_1, \sigma_2, \cdots, \sigma_t) = 2$;
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Figure 1.18: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.

(iii) let $S = \{n_2, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_1\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_1\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(iv) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_4\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(v) let $S = \{n_4, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_4, n_3\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_4, n_3\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $s$ in $S$, there’s only one neutrosophic vertex $n$ in $V \setminus (S = \{n_1, n_3\})$ such that $n$ dominates $s$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by $D_n(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 3.4$. 

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1.6. Setting of neutrosophic dual-dominating number

Proposition 1.6.13. Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then

$$D_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, $c$. $S = V \setminus \{c\}$ is a dual-dominating set related dual-dominating number. Since, let $S = V \setminus \{c\} = \{x_1, x_2, \ldots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex $x_i$ in $S$, there’s a neutrosophic vertex $c$ in $V \setminus \{c\}$ such that $c$ dominates $x_i$, then the set of neutrosophic vertices, $S = V \setminus \{c\} = \{x_1, x_2, \ldots, x_{\mathcal{O}(WHL_{1,\sigma_2})-1}\}$ is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by

$$D_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

Thus

$$D_n(WHL_{1,\sigma_2}) = \mathcal{O}_n(WHL_{1,\sigma_2}) - \sum_{i=1}^{3} \sigma_i(c).$$

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

Example 1.6.14. There is one section for clarifications. In Figure (1.19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_3, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For a neutrosophic vertex $n_2$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus \{n_1, n_2, n_3, n_5\}$ such that $n$ dominates $n_2$, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_3, n_5\}$ isn’t called dual-dominating set and this set isn’t maximal. So as it doesn’t relate to maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by $D(WHL_{1,\sigma_2}) = 4$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertex $s$ in $S$, there’s a neutrosophic vertex $n$ in $V \setminus \{n_2, n_4\}$ such that $n$ dominates $s$, then the set of
neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(WHL_{1,\sigma_2}) = 4; \)

(iii) let \( S = \{n_2, n_3, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n_1 \) in \( V \setminus (S = \{n_2, n_3, n_4, n_5\}) \) such that \( n_1 \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4, n_5\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(WHL_{1,\sigma_2}) = 4; \)

(iv) let \( S = \{n_2, n_3, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(WHL_{1,\sigma_2}) = 4; \)

(v) let \( S = \{n_2, n_3, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_5\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(WHL_{1,\sigma_2}) = 4; \)

(vi) there’s only one dual-dominating set thus let \( S = \{n_2, n_3, n_4, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3, n_4, n_5\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4, n_5\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(WHL_{1,\sigma_2}) = 5.3; \)

1.7 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this
1.8. Case 1: Complete-t-partite Model alongside its dual-dominating number and its neutrosophic dual-dominating number

![Figure 1.19: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number.](79NTG19)

style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.

**Step 1. (Definition)** Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

**Step 2. (Issue)** Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

**Step 3. (Model)** The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There’s one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table 1.1, clarifies about the assigned numbers to these situations.

Table 1.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

<table>
<thead>
<tr>
<th>Sections of $NTG$</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$n_3$</th>
<th>$n_4$</th>
<th>$n_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>(0.7, 0.9, 0.3)</td>
<td>(0.4, 0.2, 0.8)</td>
<td>(0.4, 0.2, 0.8)</td>
<td>(0.4, 0.2, 0.8)</td>
<td>(0.4, 0.2, 0.8)</td>
</tr>
<tr>
<td>Connections of $NTG$</td>
<td>$E_1$</td>
<td>$E_2$</td>
<td>$E_3$</td>
<td>$E_4$</td>
<td>$E_5$</td>
</tr>
<tr>
<td>Values</td>
<td>(0.4, 0.2, 0.3)</td>
<td>(0.5, 0.2, 0.3)</td>
<td>(0.3, 0.2, 0.3)</td>
<td>(0.3, 0.2, 0.3)</td>
<td>(0.3, 0.2, 0.3)</td>
</tr>
</tbody>
</table>
1. Initial Notions

Figure 1.20: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number

1.8 Case 1: Complete-t-partite Model alongside its dual-dominating number and its neutrosophic dual-dominating number

Step 4. (Solution) The neutrosophic graph alongside its dual-dominating number and its neutrosophic dual-dominating number as model, propose to use specific number. Every subject has connection with some subjects. Thus the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its dual-dominating number and its neutrosophic dual-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (1.20). This model is strong and even more it’s quasi-complete. And the study proposes using specific number which is called its dual-dominating number and its neutrosophic dual-dominating number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (1.20). In Figure (1.20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For a neutrosophic vertex $n_4$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus \{S = \{n_1, n_2, n_4\})$ such that $n$ dominates
1.9. Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number

(ii) let \( S = \{n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertex \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2\} \) is called dual-dominating set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC, \sigma_1, \sigma_2, \cdots, \sigma_t) = 2 \);

(iii) let \( S = \{n_2, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_1\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_1\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC, \sigma_1, \sigma_2, \cdots, \sigma_t) = 2 \);

(iv) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC, \sigma_1, \sigma_2, \cdots, \sigma_t) = 2 \);

(v) let \( S = \{n_4, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_4, n_3\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_4, n_3\} \) is called dual-dominating set. So as the maximum cardinality between all dual-dominating sets is called dual-dominating number and it’s denoted by \( D(CMC, \sigma_1, \sigma_2, \cdots, \sigma_t) = 2 \);

(vi) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertex \( s \) in \( S \), there’s only one neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_3\}) \) such that \( n \) dominates \( s \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-dominating set. So as the maximum neutrosophic cardinality between all dual-dominating sets is called neutrosophic dual-dominating number and it’s denoted by \( D_n(CMC, \sigma_1, \sigma_2, \cdots, \sigma_t) = 3.4 \).
1. Initial Notions

Figure 1.21: A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number

1.9 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its dual-dominating number and its neutrosophic dual-dominating number

Step 4. (Solution) The neutrosophic graph alongside its dual-dominating number and its neutrosophic dual-dominating number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its dual-dominating number and its neutrosophic dual-dominating number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure 1.21. This model is neutrosophic strong as individual and even more it’s complete. And the study proposes using specific number which is called its dual-dominating number and its neutrosophic dual-dominating number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure 1.21. There is one section for clarifications.

(i) For given two vertices, s and n, \( \mu(ns) = \sigma(n) \land \sigma(s) \). Thus s dominates n and n dominates s;

(ii) the existence of one vertex to do this function, dominating, is obvious thus this vertex form a set which is necessary and sufficient in the
1.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning its dual-dominating number and its neutrosophic dual-dominating number are defined in neutrosophic graphs. Thus,

**Question 1.10.1.** Is it possible to use other types of its dual-dominating number and its neutrosophic dual-dominating number?

**Question 1.10.2.** Are existed some connections amid different types of its dual-dominating number and its neutrosophic dual-dominating number in neutrosophic graphs?

**Question 1.10.3.** Is it possible to construct some classes of neutrosophic graphs which have “nice” behavior?

**Question 1.10.4.** Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?

**Problem 1.10.5.** Which parameters are related to this parameter?

**Problem 1.10.6.** Which approaches do work to construct applications to create independent study?

**Problem 1.10.7.** Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?
1. Initial Notions

1.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted.

This study uses two definitions concerning dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Maximum number of dominated vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of dominated vertices corresponded to dual-dominating set is called neutrosophic dual-dominating number. The connections of vertices which aren’t clarified by strong edges differ them from each other and put them in different categories to represent a number which is called dual-dominating number and neutrosophic dual-dominating number arising from dominated vertices in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table 1.2, some limitations and advantages of this study are pointed out.

Table 1.2: A Brief Overview about Advantages and Limitations of this Study

<table>
<thead>
<tr>
<th>Advantages</th>
<th>Limitations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Dual-Dominating Number of Model</td>
<td>1. Connections amid Classes</td>
</tr>
<tr>
<td>2. Neutrosophic Dual-Dominating Number of Model</td>
<td>2. Study on Families</td>
</tr>
<tr>
<td>3. Maximal Dual-Dominating Sets</td>
<td>3. Same Models in Family</td>
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<td>4. Dominated Vertices amid all Vertices</td>
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<td>5. Acting on All Vertices</td>
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Bibliography


CHAPTER 2

Modified Notions

The following sections are cited as follows, which is my 80th manuscript and I use prefix 80 as number before any labelling for items.


Dual-Resolving Numbers Excerpt from Some Classes of Neutrosophic Graphs With Some Applications

2.1 Abstract

New setting is introduced to study dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Maximum number of resolved vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of resolved vertices corresponded to dual-resolving set is called neutrosophic dual-resolving number. Forming sets from resolved vertices to figure out different types of number of vertices in the sets from resolved sets in the terms of maximum number of vertices to get maximum number to assign to neutrosophic graphs is key type of approach to have these notions namely dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Two numbers and one set are assigned to a neutrosophic graph, are obtained but now both settings lead to approach is on demand which is to compute and to find representatives of sets having largest number of resolved vertices from different types of sets in the terms of maximum number and maximum neutrosophic number forming it to get maximum number to assign to a neutrosophic graph.

Let $\text{NTG} : (V, E, \sigma, \mu)$ be a neutrosophic graph. Then for given two vertices, $s$ and $s'$ if $d(s, n) \neq d(s', n)$, then $n$ resolves $s$ and $s'$ where $d$ is the minimum number of edges amid all paths from $s$ to $s'$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices $s, s'$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus S$ such that $n$ resolves $s, s'$, then the set of neutrosophic vertices, $S$ is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(\text{NTG})$; for given two vertices, $s$ and $s'$ if $d(s, n) \neq d(s', n)$, then $n$ resolves $s$ and
2. Modified Notions

$s'$ where $d$ is the minimum number of edges amid all paths from $s$ to $s'$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. If for every two neutrosophic vertices $s, s'$ in $S$, there's at least one neutrosophic vertex $n$ in $V \setminus S$ such that $n$ resolves $s, s'$, then the set of neutrosophic vertices, $S$ is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}_n(NTG)$. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of dual-resolving number,” and “Setting of neutrosophic dual-resolving number,” for introduced results and used classes. This approach facilitates identifying sets which form dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. In both settings, some classes of well-known neutrosophic graphs are studied. Some clarifications for each result and each definition are provided. The cardinality of set of resolved vertices and neutrosophic cardinality of set of resolved vertices corresponded to dual-resolving set have eligibility to define dual-resolving number and neutrosophic dual-resolving number but different types of set of resolved vertices to define dual-resolving sets. Some results get more frameworks and perspective about these definitions. The way in that, different types of set of resolved vertices in the terms of maximum number to assign to neutrosophic graphs, opens the way to do some approaches. These notions are applied into neutrosophic graphs as individuals but not family of them as drawbacks for these notions. Finding special neutrosophic graphs which are well-known, is an open way to pursue this study. Neutrosophic dual-resolving notion is applied to different settings and classes of neutrosophic graphs. Some problems are proposed to pursue this study. Basic familiarities with graph theory and neutrosophic graph theory are proposed for this article.

**Keywords:** Dual-Resolving Number, Neutrosophic Dual-Resolving Number,

Classes of Neutrosophic Graphs

**AMS Subject Classification:** 05C17, 05C22, 05E45

2.2 Background

2.3. Motivation and Contributions

by Aronshtam and Ilani (2022), investigating the recoverable robust single machine scheduling problem under interval uncertainty in Ref. [Ref4] by Bold and Goerigk (2022), new bounds for the b-chromatic number of vertex deleted graphs in Ref. [Ref6] by Del-Vechio and Kouider (2022), bipartite completion of colored graphs avoiding chordless cycles of given lengths in Ref. [Ref7] by Elaine et al., infinite chromatic games in Ref. [Ref12] by Janczewski et al. (2022), edge-disjoint rainbow triangles in edge-colored graphs in Ref. [Ref13] by Li and Li (2022), rainbow triangles in arc-colored digraphs in Ref. [Ref14] by Li et al. (2022), a sufficient condition for edge 6-colorable planar graphs with maximum degree 6 in Ref. [Ref15] by Lu and Shi (2022), some comparative results concerning the Grundy and b-chromatic number of graphs in Ref. [Ref16] by Masih and Zaker (2022), color neighborhood union conditions for proper edge-pancyclicity of edge-colored complete graphs in Ref. [Ref21] by Wu et al. (2022), dimension and coloring alongside domination in neutrosophic hypergraphs in Ref. [Ref9] by Henry Garrett (2022), three types of neutrosophic alliances based on connectedness and (strong) edges in Ref. [Ref11] by Henry Garrett (2022), properties of SuperHyperGraph and neutrosophic SuperHyperGraph in Ref. [Ref10] by Henry Garrett (2022), are studied. Also, some studies and researches about neutrosophic graphs, are proposed as a book in Ref. [Ref8] by Henry Garrett (2022).

In this section, I use two subsections to illustrate a perspective about the background of this study.

2.3 Motivation and Contributions

In this study, there’s an idea which could be considered as a motivation.

**Question 2.3.1. Is it possible to use mixed versions of ideas concerning “dual-resolving number”, “neutrosophic dual-resolving number” and “Neutrosophic Graph” to define some notions which are applied to neutrosophic graphs?**

It’s motivation to find notions to use in any classes of neutrosophic graphs. Real-world applications about time table and scheduling are another thoughts which lead to be considered as motivation. Having connection amid two vertices have key roles to assign dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Thus they’re used to define new ideas which conclude to the structure of dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. The concept of having largest number of resolved vertices in the terms of crisp setting and in the terms of neutrosophic setting inspires us to study the behavior of all resolved vertices in the way that, some types of numbers, dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs, are the cases of study in the setting of individuals. In both settings, corresponded numbers conclude the discussion. Also, there are some avenues to extend these notions.

The framework of this study is as follows. In the beginning, I introduce basic definitions to clarify about preliminaries. In subsection “Preliminaries”, new notions of dual-resolving number and neutrosophic dual-resolving number
arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs, are highlighted, are introduced and are clarified as individuals. In section “Preliminaries”, Maximum number of resolved vertices, is a number which is representative based on those vertices, have the key role in this way. General results are obtained and also, the results about the basic notions of dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs, are elicited. Some classes of neutrosophic graphs are studied in the terms of dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs in section “Setting of dual-resolving number,” as individuals. In section “Setting of dual-resolving number,” dual-resolving number is applied into individuals. As concluding results, there are some statements, remarks, examples and clarifications about some classes of neutrosophic graphs namely path-neutrosophic graphs, cycle-neutrosophic graphs, complete-neutrosophic graphs, star-neutrosophic graphs, complete-bipartite-neutrosophic graphs, complete-t-partite-neutrosophic graphs, and wheel-neutrosophic graphs. The clarifications are also presented in both sections “Setting of dual-resolving number,” and “Setting of neutrosophic dual-resolving number,” for introduced results and used classes. In section “Applications in Time Table and Scheduling”, two applications are posed for quasi-complete and complete notions, namely complete-neutrosophic graphs and complete-t-partite-neutrosophic graphs concerning time table and scheduling when the suspicions are about choosing some subjects and the mentioned models are considered as individual. In section “Open Problems”, some problems and questions for further studies are proposed. In section “Conclusion and Closing Remarks”, gentle discussion about results and applications is featured. In section “Conclusion and Closing Remarks”, a brief overview concerning advantages and limitations of this study alongside conclusions is formed.

2.4 Preliminaries

In this subsection, basic material which is used in this article, is presented. Also, new ideas and their clarifications are elicited.

Basic idea is about the model which is used. First definition introduces basic model.

**Definition 2.4.1.** (Graph).

\( G = (V, E) \) is called a **graph** if \( V \) is a set of objects and \( E \) is a subset of \( V \times V \) (\( E \) is a set of 2-subsets of \( V \)) where \( V \) is called **vertex set** and \( E \) is called **edge set**. Every two vertices have been corresponded to at most one edge.

Neutrosophic graph is the foundation of results in this paper which is defined as follows. Also, some related notions are demonstrated.

**Definition 2.4.2.** (Neutrosophic Graph And Its Special Case).

\( NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)) \) is called a **neutrosophic graph** if it’s graph, \( \sigma_i : V \rightarrow [0, 1] \), and \( \mu_i : E \rightarrow [0, 1] \). We add one condition on it and we use **special case** of neutrosophic graph but with same name. The added condition is as follows, for every \( v_i v_j \in E \),

\[ \mu(v_i v_j) \leq \sigma(v_i) \land \sigma(v_j). \]
2.4. Preliminaries

(i) \( \sigma \) is called **neutrosophic vertex set**.

(ii) \( \mu \) is called **neutrosophic edge set**.

(iii) \(|V|\) is called **order** of NTG and it’s denoted by \( O(NTG) \).

(iv) \( \sum_{v \in V} \sum_{i=1}^{3} \sigma_i(v) \) is called **neutrosophic order** of NTG and it’s denoted by \( O_n(NTG) \).

(v) \(|E|\) is called **size** of NTG and it’s denoted by \( S(NTG) \).

(vi) \( \sum_{e \in E} \sum_{i=1}^{3} \mu_i(e) \) is called **neutrosophic size** of NTG and it’s denoted by \( S_n(NTG) \).

Some classes of well-known neutrosophic graphs are defined. These classes of neutrosophic graphs are used to form this study and the most results are about them.

**Definition 2.4.3.** Let \( NTG : (V, E, \sigma, \mu) \) be a neutrosophic graph. Then

(i) \( \) a sequence of consecutive vertices \( P : x_0, x_1, \cdots, x_O(NTG) \) is called **path** where \( x_ix_{i+1} \in E, \ i = 0, 1, \cdots, O(NTG) - 1; \)

(ii) \( \) **strength** of path \( P : x_0, x_1, \cdots, x_O(NTG) \) is \( \bigwedge_{i=0, \cdots, O(NTG)-1} \mu(x_ix_{i+1}); \)

(iii) \( \) connectedness amid vertices \( x_0 \) and \( x_t \) is

\[
\mu^\infty(x_0, x_t) = \bigvee_{P : x_0, x_1, \cdots, x_t} \bigwedge_{i=0, \cdots, t-1} \mu(x_ix_{i+1});
\]

(iv) \( \) a sequence of consecutive vertices \( P : x_0, x_1, \cdots, x_O(NTG), x_0 \) is called **cycle** where \( x_ix_{i+1} \in E, \ i = 0, 1, \cdots, O(NTG) - 1, \ x_O(NTG)x_0 \in E \) and there are two edges \( xy \) and \( uv \) such that \( \mu(xy) = \mu(uv) = \bigwedge_{i=0, 1, \cdots, n-1} \mu(v_iv_{i+1}); \)

(v) \( \) it’s **t-partite** where \( V \) is partitioned to \( t \) parts, \( V_1^{s_1}, V_2^{s_2}, \cdots, V_t^{s_t} \) and the edge \( xy \) implies \( x \in V_i^{s_i} \) and \( y \in V_j^{s_j} \) where \( i \neq j \). If it’s complete, then it’s denoted by \( K_{s_1, s_2, \cdots, s_t} \) where \( \sigma_i \) is \( \sigma \) on \( V_i^{s_i} \) instead \( V \) which mean \( x \not\in V_i \) induces \( \sigma_i(x) = 0 \). Also, \( |V_i^{s_i}| = s_i; \)

(vi) \( \) t-partite is **complete bipartite** if \( t = 2 \), and it’s denoted by \( K_{s_1, s_2}; \)

(vii) \( \) complete bipartite is **star** if \( |V_1| = 1 \), and it’s denoted by \( S_{1, s_2}; \)

(viii) \( \) a vertex in \( V \) is **center** if the vertex joins to all vertices of a cycle. Then it’s **wheel** and it’s denoted by \( W_{1, s_2}; \)

(ix) \( \) it’s **complete** where \( \forall uv \in V, \ \mu(uv) = \sigma(u) \land \sigma(v); \)

(x) \( \) it’s **strong** where \( \forall uv \in E, \ \mu(uv) = \sigma(u) \land \sigma(v). \)

To make them concrete, I bring preliminaries of this article in two upcoming definitions in other ways.
Remark 2.4.6. Let \( (dual-resolving numbers) \).

Definition 2.4.7. \( WHL_1 \) and \( STR_1 \) is defined by \( \sigma \) and \( x \) implies \( t \)-partite where \( V \) is partitioned to \( t \) parts, \( V_1^{x_1}, V_2^{x_1} \). Also, \( x \in V_1^{x_1} \) and \( V_2^{x_1} \), \( i \neq j \). If it’s complete bipartite if \( t = 2 \), and it’s denoted by \( STR_{1,2} \); \( t \)-partite is complete bipartite if \( t = 2 \), and it’s denoted by \( STR_{1,2} \); a vertex in \( V \) is center if the vertex joins to all vertices of a cycle. Then it’s wheel and it’s denoted by \( WHL_{1,2} \).

Example 2.4.6. Using notations which is mixed with literatures, are reviewed.

2.4.6.1. \( NTG = (V, E, \sigma = (\sigma_1, \sigma_2, \sigma_3), \mu = (\mu_1, \mu_2, \mu_3)) \), \( O(NTG) \), and \( O_n(NTG) \):

2.4.6.2. \( CMT_\sigma, PTH, CYC, STR_{1,2}, CMT_{1,2}, CMT_{1,2} \), \( CMT_{\sigma_1, \sigma_2, \sigma_3} \), and \( WHL_{1,2} \).

Definition 2.4.7. (Dual-resolving numbers).

Let \( NTG = (V, E, \sigma, \mu) \) be a neutrosophic graph. Then

(i) for given two vertices, \( s \) and \( s' \) if \( d(s,n) \neq d(s',n) \), then \( n \) resolves \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices \( s, s' \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S \) is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(NTG) \);

(ii) for given two vertices, \( s \) and \( s' \) if \( d(s,n) \neq d(s',n) \), then \( n \) resolves \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. If for every two neutrosophic vertices \( s, s' \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S \) is called dual-resolving set. The maximum neutrosophic cardinality between all
dual-resolving sets is called **dual-resolving number** and it’s denoted by $R_n(NTG)$.

For convenient usages, the word neutrosophic which is used in previous definition, won’t be used, usually. In next part, clarifications about main definition are given. To avoid confusion and for convenient usages, examples are usually used after every part and names are used in the way that, abbreviation, simplicity, and summarization are the matters of mind.

**Example 2.4.8.** In Figure (2.1), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, $s,s'$, $d(s,n) = 1 = d(s',n)$. Thus $n$ doesn’t resolve $s$ and $s'$;

(ii) the existence of one neutrosophic vertex to do this function, resolving, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum resolving set and minimal resolving set as if it seems there’s no neutrosophic vertex to resolve so as to choose one vertex outside resolving set so as the function of resolving is impossible;

(iii) for given two vertices, $s$ and $s'$ if $d(s,n) = 1 = d(s',n)$, then $n$ doesn’t resolve $s$ and $s'$ where $d$ is the minimum number of edges amid all paths from $s$ to $s'$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices $s,s'$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus S$ such that $n$ resolves $s,s'$, then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(NTG) = 1$;

(iv) the corresponded set doesn’t have to be resolved by the set;

(v) $V$ isn’t used when the set is considered in this notion since $V \setminus \{v\}$ always works;

(vi) for given two vertices, $s$ and $s'$ if $d(s,n) = 1 = d(s',n)$, then $n$ doesn’t resolve $s$ and $s'$ where $d$ is the minimum number of edges amid all paths from $s$ to $s'$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices $s,s'$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus S$ such that $n$ resolves $s,s'$, then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_n(NTG) = 2$;

### 2.5 Setting of dual-resolving number

In this section, I provide some results in the setting of dual-resolving number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic
2. Modified Notions

Figure 2.1: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

**Proposition 2.5.1.** Let $\text{NTG} : (V, E, \sigma, \mu)$ be a complete-neutrosophic graph. Then

$$\mathcal{R}(\text{CMT}_\sigma) = 1.$$

**Proof.** Suppose $\text{CMT}_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph. By $\text{CMT}_\sigma : (V, E, \sigma, \mu)$ is a complete-neutrosophic graph, all vertices are connected to each other. So there’s one edge between two vertices. For given two vertices, $s$ and $s'$ if $d(s, n) = 1 = d(s', n)$, then $n$ doesn’t resolve $s$ and $s'$ where $d$ is the minimum number of edges amid all paths from $s$ to $s'$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices $s, s'$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus S$ such that $n$ resolves $s, s'$, then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(\text{NTG}) = 1$. Thus

$$\mathcal{R}(\text{CMT}_\sigma) = 1.$$

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 2.5.2.** In Figure (2.2), a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, $s, s'$, $d(s, n) = 1 = d(s', n)$. Thus $n$ doesn’t resolve $s$ and $s'$;
2.5. Setting of dual-resolving number

(ii) the existence of one neutrosophic vertex to do this function, resolving, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum resolving set and minimal resolving set as if it seems there’s no neutrosophic vertex to resolve so as to choose one vertex outside resolving set so as the function of resolving is impossible;

(iii) for given two vertices, \( s \) and \( s' \) if \( d(s, n) = 1 = d(s', n) \), then \( n \) doesn’t resolve \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices \( s, s' \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S = \{ s \} \) is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(NTG) = 1 \);

(iv) the corresponded set doesn’t have to be resolved by the set;

(v) \( V \) isn’t used when the set is considered in this notion since \( V \setminus \{ v \} \) always works;

(vi) for given two vertices, \( s \) and \( s' \) if \( d(s, n) = 1 = d(s', n) \), then \( n \) doesn’t resolve \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices \( s, s' \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S = \{ s \} \) is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}_n(NTG) = 2 \);

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

**Proposition 2.5.3.** Let \( NTG : (V, E, \sigma, \mu) \) be a path-neutrosophic graph. Then

\[
\mathcal{R}(PTH) = O(PTH) - 1.
\]
Thus sets is called dual-resolving number and it’s denoted by dual-resolving set. So as the maximum cardinality between all dual-resolving set of neutrosophic vertices, \( S \) be a set of neutrosophic vertices \[ a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.} \] of \( S \). They could be resolved by the leaf \( x_{O(PTH)} \), as if despite the leaf \( x_{O(PTH)} \), so as maximal set \( S \) is constructed. Thus \( S = \{x', x_2', \ldots, x'_{O(PTH)}-1\} \) is the set \( S \) is a set of vertices from path-neutrosophic graph \( PTH : (V, E, \sigma, \mu) \) with new arrangements of vertices in which there are all neutrosophic vertices which are either neighbors or not. In this new arrangements, the notation of vertices from \( x_i \) is changed to \( x'_i \). Leaves doesn’t necessarily belong to \( S \). Leaves are either belongs to \( S \) or doesn’t belong to \( S \) as if one leaf is outside. Adding all neutrosophic leaves contradicts with maximality of \( S \) and maximum cardinality of \( S \). It implies this construction is optimal. Thus, let \( S = \{x_1, x_2, \ldots, x_{O(PTH)}-2, x_{O(PTH)}-1\} \) be a set of neutrosophic vertices \[ a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.} .\] For every neutrosophic vertices \( s, s' \) in \( S \), there’s only one neutrosophic leaf \( x_{O(PTH)} \) in \( V \setminus (S = \{x_1, x_2, \ldots, x_{O(PTH)}-2, x_{O(PTH)}-1\}) \) such that \( x_{O(PTH)} \) resolves \( s, s' \) then the set of neutrosophic vertices, \( S = \{x_1, x_2, \ldots, x_{O(PTH)}-2, x_{O(PTH)}-1\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(PTH) = O(PTH) - 1 \).

Thus \[ R(PTH) = O(PTH) - 1. \]

\[ \square \]

**Example 2.5.4.** There are two sections for clarifications.

(a) In Figure (2.3), an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Let \( S = \{n_3, n_2\} \) be a set of neutrosophic vertices \[ a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.} .\] which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_3 \) in \( S \), there’s neutrosophic leaf \( n_1 \) in \( V \setminus (S = \{n_3, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_3, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(PTH) = 4 \);

(ii) \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices \[ a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.} .\] which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s neutrosophic leaf \( n_1 \) in \( V \setminus (S = \{n_4, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_4, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(PTH) = 4 \);
(iii) Let \( S = \{n_3, n_4, n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there’s only one neutrosophic leaf \( n_5 \) in \( V \setminus (S = \{n_3, n_4, n_1, n_2\}) \) such that \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1, n_2\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(PTH) = 4 \);

(iv) Let \( S = \{n_3, n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there’s a neutrosophic leaf \( n_1 \) in \( V \setminus (S = \{n_3, n_2, n_5\}) \) such that \( n_1 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_2, n_5\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(PTH) = 4 \);

(v) Let \( S = \{n_3, n_4, n_1\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there’s a neutrosophic leaf \( n_5 \) in \( V \setminus (S = \{n_3, n_4, n_1\}) \) such that \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(PTH) = 4 \);

(vi) Let \( S = \{n_3, n_4, n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there’s only one neutrosophic leaf \( n_5 \) in \( V \setminus (S = \{n_3, n_4, n_1, n_2\}) \) such that \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1, n_2\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}_n(PTH) = 5.1 \).

(b) In Figure 2.4, an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

(i) Let \( S = \{n_3, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex. which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_3 \) in \( S \), there’s neutrosophic leaf \( n_1 \) in \( V \setminus (S = \{n_3, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_3, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with

(ii) \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex. which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s neutrosophic leaf \( n_1 \) in \( V \setminus (S = \{n_4, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_4, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with
the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$;

(iii) let $S = \{n_3, n_4, n_1, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s only one neutrosophic leaf $n_6$ in $V \setminus (S = \{n_3, n_4, n_1, n_2, n_5\})$ such that $n_6$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_2, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$;

(iv) let $S = \{n_3, n_2, n_5, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s a neutrosophic leaf $n_1$ in $V \setminus (S = \{n_3, n_2, n_5, n_6\})$ such that $n_1$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_2, n_5, n_6\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$;

(v) let $S = \{n_3, n_1, n_6\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s a neutrosophic leaf $n_1$ in $V \setminus (S = \{n_3, n_1, n_6\})$ such that $n_1$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_1, n_6\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$;

(vi) let $S = \{n_3, n_4, n_5, n_2, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s only one neutrosophic leaf $n_6$ in $V \setminus (S = \{n_3, n_4, n_5, n_2, n_1\})$ such that $n_6$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_5, n_2, n_1\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_n(PTH) = 8.2$.

Proposition 2.5.5. Let $NTG : (V, E, \sigma, \mu)$ be a cycle-neutrosophic graph where $O(CYC) \geq 3$. Then

$$R(CYC) = O(CYC) - 2.$$ 

Proof. Suppose $CYC : (V, E, \sigma, \mu)$ is a cycle-neutrosophic graph. For given two vertices, $x$ and $y$, there are only two paths with distinct edges from $x$ to $y$. Let

$$x_1, x_2, \ldots, x_{O(CYC) - 1}, x_{O(CYC)}, x_1$$

be a cycle-neutrosophic graph $CYC : (V, E, \sigma, \mu)$. $O(CYC) - 2$ consecutive vertices could belong to $S$ which is dual-resolving set related to dual-resolving number where two neutrosophic vertices outside are consecutive. Since these two vertices could resolve all vertices. If there are no neutrosophic vertices which
2.5. Setting of dual-resolving number

Figure 2.3: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

Figure 2.4: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

are consecutive, then it contradicts with maximality of set $S$ and maximum cardinality of $S$. Thus, let

$$S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC) - 3}, x_{\mathcal{O}(CYC) - 2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s at least one neutrosophic vertex $n$ in $V \setminus S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC) - 3}, x_{\mathcal{O}(CYC) - 2}\}$ such that $n$ resolves $s$ and $s'$ then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\mathcal{O}(CYC) - 3}, x_{\mathcal{O}(CYC) - 2}\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

$$\mathcal{R}(CYC) = \mathcal{O}(CYC) - 2.$$
2. Modified Notions

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

Example 2.5.6. There are two sections for clarifications.

(a) In Figure (2.5), an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_3, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_3 \) in \( S \), there’s neutrosophic vertex \( n_1 \) in \( V \setminus (S = \{n_3, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_3, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(ii) \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s neutrosophic vertex \( n_1 \) in \( V \setminus (S = \{n_4, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_4, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(iii) let \( S = \{n_3, n_4, n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either neutrosophic vertex \( n_4 \) or neutrosophic vertex \( n_5 \) in \( V \setminus (S = \{n_3, n_4, n_1, n_2\}) \) such that either \( n_6 \) resolves \( s \) and \( s' \), or \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1, n_2\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(iv) let \( S = \{n_3, n_4, n_5, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either neutrosophic vertex \( n_1 \) or neutrosophic vertex \( n_2 \) in \( V \setminus (S = \{n_3, n_4, n_5, n_6\}) \) such that either \( n_1 \) resolves \( s \) and \( s' \), or \( n_2 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_5, n_6\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(v) let \( S = \{n_2, n_5, n_1, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either neutrosophic vertex \( n_3 \) or neutrosophic vertex \( n_4 \) in \( V \setminus (S = \{n_2, n_5, n_1, n_6\}) \) such
that either $n_3$ resolves $s$ and $s'$, or $n_4$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_2, n_5, n_1, n_6\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(\text{CYC}) = 4$;

(vi) let $S = \{n_3, n_1, n_6, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there are either neutrosophic vertex $n_5$ or neutrosophic vertex $n_4$ in $V \setminus (S = \{n_3, n_1, n_6, n_2\})$ such that either $n_5$ resolves $s$ and $s'$, or $n_4$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_1, n_6, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}_n(\text{CYC}) = 6.4$.

(b) In Figure 2.6, an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_3$ in $S$, there’s neutrosophic vertex $n_4$ in $V \setminus (S = \{n_3, n_2\})$ such that $n_4$ resolves $n_2$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn’t maximal. As it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(\text{CYC}) = 3$;

(ii) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s neutrosophic vertex $n_5$ in $V \setminus (S = \{n_4, n_2\})$ such that $n_5$ resolves $n_2$ and $n_4$, then the set of neutrosophic vertices, $S = \{n_4, n_2\}$ is called dual-resolving set and this set isn’t maximal. As it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(\text{CYC}) = 3$;

(iii) let $S = \{n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,]. For every neutrosophic vertices $s$ and $s'$ in $S$, there are either a neutrosophic vertex $n_1$ or neutrosophic vertex $n_2$ in $V \setminus (S = \{n_3, n_4, n_5\})$ such that either $n_1$ resolves $s$ and $s'$ or $n_2$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(\text{CYC}) = 3$;

(iv) let $S = \{n_1, n_2, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,]. For every neutrosophic vertices $s$ and $s'$ in $S$, there are either a neutrosophic vertex $n_3$ or neutrosophic vertex $n_4$ in $V \setminus (S = \{n_1, n_2, n_3\})$ such that either $n_3$ resolves $s$ and $s'$ or $n_4$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_3\}$ is
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Figure 2.5: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 3; \)

\((v)\) let \( S = \{n_1, n_2, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either a neutrosophic vertex \( n_4 \) or neutrosophic vertex \( n_5 \) in \( V \setminus (S = \{n_1, n_2, n_3\}) \) such that either \( n_4 \) resolves \( s \) and \( s' \) or \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_3\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 3; \)

\((vi)\) let \( S = \{n_2, n_3, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either a neutrosophic vertex \( n_1 \) or neutrosophic vertex \( n_5 \) in \( V \setminus (S = \{n_2, n_3, n_4\}) \) such that either \( n_1 \) resolves \( s \) and \( s' \) or \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4\} \) is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and its denoted by \( R_n(CYC) = 5.8. \)

**Proposition 2.5.7.** Let \( NTG : (V, E, \sigma, \mu) \) be a star-neutrosophic graph with center \( c \). Then

\[ R(STR_{1, \sigma_2}) = 2. \]

**Proof.** Suppose \( STR_{1, \sigma_2} : (V, E, \sigma, \mu) \) is a star-neutrosophic graph. An edge always has center, \( c \), as one of its endpoints. All paths have one as their lengths, forever. \( S = \{c, v\} \) is a dual-resolving set related dual-resolving number. Since, let

\[ S = \{c, v\} = V \setminus \{x_1, x_2, \ldots, x_{O(STR_{1, \sigma_2})-2}\} \]

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( v \) and \( c \) in \( S \), there’s
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Figure 2.6: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

A neutrosophic vertex $x$ in $V \setminus (S = \{c, v\}) = V \setminus \{x_1, x_2, \cdots, x_{\sigma(STR_1, \sigma_2)}\}$ such that $x$ resolves $v$ and $c$ then the set of neutrosophic vertices, $S = \{c, v\} = V \setminus \{x_1, x_2, \cdots, x_{\sigma(STR_1, \sigma_2)}\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

$$R(STR_1, \sigma_2) = 2.$$

Thus

$$R(STR_1, \sigma_2) = 2.$$

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 2.5.8.** There is one section for clarifications. In Figure 2.7, a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices $n_2$ and $n_1$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_1, n_2\})$ such that $n_3$ resolves $n_2$ and $n_1$, then the set of neutrosophic vertices, $S = \{n_1, n_2\}$ is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(STR_1, \sigma_2) = 2$;

(ii) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which aren’t consecutive vertices. For every neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_4\})$ such that $n$ resolves $n_2$
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and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) isn’t called dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(STR_{1,\sigma_2}) = 2; \)

(iii) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices \[a vertex alongside triple pair of its values is called neutrosophic vertex,\] which are consecutive vertices. For neutrosophic vertices \( n_3 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_4 \) in \( V \setminus (S = \{n_1, n_3\}) \) such that \( n_4 \) resolves \( n_3 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(STR_{1,\sigma_2}) = 2; \)

(iv) let \( S = \{n_2, n_5\} \) be a set of neutrosophic vertices \[a vertex alongside triple pair of its values is called neutrosophic vertex,\] which aren’t consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_5 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_5\}) \) such that \( n \) resolves \( n_2 \) and \( n_5 \), then the set of neutrosophic vertices, \( S = \{n_2, n_5\} \) isn’t called dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(STR_{1,\sigma_2}) = 2; \)

(v) let \( S = \{n_1, n_5\} \) be a set of neutrosophic vertices \[a vertex alongside triple pair of its values is called neutrosophic vertex,\] which are consecutive vertices. For neutrosophic vertices \( n_5 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus (S = \{n_1, n_5\}) \) such that \( n_3 \) resolves \( n_5 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_5\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(STR_{1,\sigma_2}) = 2; \)

(vi) let \( S = \{n_1, n_4\} \) be a set of neutrosophic vertices \[a vertex alongside triple pair of its values is called neutrosophic vertex,\] which are consecutive vertices. For neutrosophic vertices \( n_4 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus (S = \{n_1, n_4\}) \) such that \( n_3 \) resolves \( n_4 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_4\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}_n(STR_{1,\sigma_2}) = 3.7. \)

**Proposition 2.5.9.** Let \( NTG: (V, E, \sigma, \mu) \) be a complete-bipartite-neutrosophic graph. Then

\[
\mathcal{R}(CMC_{\sigma_1,\sigma_2}) = 2.
\]

**Proof.** Suppose \( CMC_{\sigma_1,\sigma_2}: (V, E, \sigma, \mu) \) is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is resolved by any given vertex. Thus maximum cardinality implies including one vertex
2.5. Setting of dual-resolving number

Figure 2.7: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

from each part. Let

\[ S = V \setminus \{ x_1, x_2, \ldots, x_{\text{CMC}_{\sigma_1, \sigma_2}} \} = \{ u, v \} \cup \{ u, v \} \cup \{ u, v \} \Rightarrow \]

be a dual-resolving set related to the dual-resolving number. This construction gives the proof. Since let

\[ S = V \setminus \{ x_1, x_2, \ldots, x_{\text{CMC}_{\sigma_1, \sigma_2}} \} = \{ u, v \} \cup \{ u, v \} \cup \{ u, v \} \Rightarrow \]

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \( u \) and \( v \) in \( S \), there’s a neutrosophic vertex \( n \) in

\[ V \setminus (S = V \setminus \{ x_1, x_2, \ldots, x_{\text{CMC}_{\sigma_1, \sigma_2}} \} = \{ u, v \} \cup \{ u, v \} \cup \{ u, v \}) \]

such that \( n \) resolves \( u \) and \( v \), then the set of neutrosophic vertices,

\[ S = V \setminus \{ x_1, x_2, \ldots, x_{\text{CMC}_{\sigma_1, \sigma_2}} \} = \{ u, v \} \cup \{ u, v \} \cup \{ u, v \} \]

is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

\[ \mathcal{R}(\text{CMC}_{\sigma_1, \sigma_2}) = 2. \]

Thus

\[ \mathcal{R}(\text{CMC}_{\sigma_1, \sigma_2}) = 2. \]

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 2.5.10.** There is one section for clarifications. In Figure 2.8, a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.
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(i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_4$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus (S = \{n_1, n_2, n_4\})$ such that $n$ resolves $n_4$ and $n_1$, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4\}$ isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(CMC_{\sigma_1, \sigma_2}) = 2$;

(ii) let $S = \{n_2, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_2$ and $n_3$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_3\})$ such that $n$ resolves $n_2$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_2, n_3\}$ isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(CMC_{\sigma_1, \sigma_2}) = 2$;

(iii) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_3$ in $S$, there’s a neutrosophic vertex $n_2$ in $V \setminus (S = \{n_1, n_3\})$ such that $n_2$ resolves $n_1$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(CMC_{\sigma_1, \sigma_2}) = 2$;

(iv) let $S = \{n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_2$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_1, n_2\})$ such that $n_3$ resolves $n_1$ and $n_2$, then the set of neutrosophic vertices, $S = \{n_1, n_2\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(CMC_{\sigma_1, \sigma_2}) = 2$;

(v) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_2, n_4\})$ such that $n_3$ resolves $n_2$ and $n_4$, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}(CMC_{\sigma_1, \sigma_2}) = 2$;

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_3$ in $S$, there’s a neutrosophic vertex $n_2$ in $V \setminus (S = \{n_1, n_3\})$ such that $n_2$ resolves $n_1$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-resolving set. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}_n(CMC_{\sigma_1, \sigma_2}) = 3.4$.

**Proposition 2.5.11.** Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then

\[
\mathcal{R}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2.
\]
2.5. Setting of dual-resolving number

Figure 2.8: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

Proof. Suppose $CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is resolved by any given vertex. Thus maximum cardinality implies including two vertices from two different parts. Let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$ 

Thus

$$S = V \setminus \{x_1, x_2, \ldots, x_{\sigma(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\}$$

be a dual-resolving set related to the dual-resolving number. This construction gives the proof. Since let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$ 

Thus

$$S = V \setminus \{x_1, x_2, \ldots, x_{\sigma(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices $u$ and $v$ in $S$, there’s a neutrosophic vertex $n$ in

$$V \setminus (S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$ 

such that $n$ resolves $u$ and $v$, then the set of neutrosophic vertices,

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\}_{u \in V_1, v \in V_2}.$$ 

Thus

$$S = V \setminus \{x_1, x_2, \ldots, x_{\sigma(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t})-2}\}$$

is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

$$\mathcal{R}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2.$$
Thus

\[ \mathcal{R}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2. \]

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case.

Example 2.5.12. There is one section for clarifications. In Figure 2.5, a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \ \setminus \ (S = \{n_1, n_2, n_4\}) \) such that \( n \) resolves \( n_4 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_4\} \) isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2; \)

(ii) let \( S = \{n_2, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \ \setminus \ (S = \{n_2, n_3\}) \) such that \( n \) resolves \( n_2 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_2, n_3\} \) isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2; \)

(iii) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_3 \) in \( S \), there’s a neutrosophic vertex \( n_2 \) in \( V \ \setminus \ (S = \{n_1, n_3\}) \) such that \( n_2 \) resolves \( n_1 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2; \)

(iv) let \( S = \{n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_2 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \ \setminus \ (S = \{n_1, n_2\}) \) such that \( n_3 \) resolves \( n_1 \) and \( n_2 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2\} \) is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2; \)

(v) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \ \setminus \ (S = \{n_2, n_4\}) \) such that \( n_3 \) resolves \( n_2 \) and \( n_4 \), then the
2.5. Setting of dual-resolving number

Figure 2.9: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$.

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_3$ in $S$, there’s a neutrosophic vertex $n_2$ in $V \setminus (S = \{n_1, n_3\})$ such that $n_2$ resolves $n_1$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-resolving set. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_n(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 3.4$.

Proposition 2.5.13. Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then

$$R(WHL_{1,\sigma_2}) = 2.$$  

Proof. Suppose $WHL_{1,\sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, e. $S = \{c, s\}$ is a dual-resolving set related dual-resolving number. Since, let

$$S = V \setminus \{x_1, x_2, \ldots, x_{\mathcal{O}(WHL_{1,\sigma_2})-2}\} = \{c, s\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices $c$ and $s$ in $S$, there’s a neutrosophic vertex $n$ in $V \setminus (S = V \setminus \{x_1, x_2, \ldots, x_{\mathcal{O}(WHL_{1,\sigma_2})-2}\} = \{c, s\})$ such that $n$ resolves $c$ and $s$, then the set of neutrosophic vertices, $S = V \setminus \{x_1, x_2, \ldots, x_{\mathcal{O}(WHL_{1,\sigma_2})-2}\} = \{c, s\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

$$R(WHL_{1,\sigma_2}) = 2.$$
Thus

\[ R(WHL_{1,\sigma_2}) = 2. \]

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 2.5.14.** There is one section for clarifications. In Figure (2.10), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus (S = \{n_1, n_2\}) \) such that \( n_3 \) resolves \( n_2 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(WHL_{1,\sigma_2}) = 2; \)

(ii) Let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) isn’t called dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(WHL_{1,\sigma_2}) = 2; \)

(iii) Let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices \( n_3 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_4 \) in \( V \setminus (S = \{n_1, n_3\}) \) such that \( n_4 \) resolves \( n_3 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(WHL_{1,\sigma_2}) = 2; \)

(iv) Let \( S = \{n_2, n_5\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which aren’t consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_5 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_5\}) \) such that \( n \) resolves \( n_2 \) and \( n_5 \), then the set of neutrosophic vertices, \( S = \{n_2, n_5\} \) isn’t called dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(WHL_{1,\sigma_2}) = 2; \)
2.6 Setting of neutrosophic dual-resolving number

In this section, I provide some results in the setting of neutrosophic dual-resolving number. Some classes of neutrosophic graphs are chosen. Complete-neutrosophic graph, path-neutrosophic graph, cycle-neutrosophic graph, star-neutrosophic graph, bipartite-neutrosophic graph, t-partite-neutrosophic graph, and wheel-neutrosophic graph, are both of cases of study and classes which the results are about them.

**Proposition 2.6.1.** Let \( NTG : (V, E, \sigma, \mu) \) be a complete-neutrosophic graph. Then

\[
\begin{align*}
(v) & \text{ let } S = \{n_1, n_5\} \text{ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices } n_5 \text{ and } n_1 \text{ in } S, \text{ there’s a neutrosophic vertex } n_3 \text{ in } V \setminus (S = \{n_1, n_5\}) \text{ such that } n_3 \text{ resolves } n_5 \text{ and } n_1, \text{ then the set of neutrosophic vertices, } S = \{n_1, n_5\} \text{ is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by } R(WHL_{1, \sigma_2}) = 2; \\
(vi) & \text{ let } S = \{n_1, n_4\} \text{ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex,] which are consecutive vertices. For neutrosophic vertices } n_4 \text{ and } n_1 \text{ in } S, \text{ there’s a neutrosophic vertex } n_3 \text{ in } V \setminus (S = \{n_1, n_4\}) \text{ such that } n_3 \text{ resolves } n_4 \text{ and } n_1, \text{ then the set of neutrosophic vertices, } S = \{n_1, n_4\} \text{ is called dual-resolving set and this set is maximal. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by } R_n(WHL_{1, \sigma_2}) = 3.7.
\end{align*}
\]
2. Modified Notions

\[ R_n(CMT_\sigma) = \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x). \]

**Proof.** Suppose \( CMT_\sigma : (V, E, \sigma, \mu) \) is a complete-neutrosophic graph. By \( CMT_\sigma : (V, E, \sigma, \mu) \) is a complete-neutrosophic graph, all vertices are connected to each other. So there’s one edge between two vertices. For given two vertices, \( s \) and \( s' \) if \( d(s, n) = 1 = d(s', n) \), then \( n \) doesn’t resolve \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every two neutrosophic vertices \( s, s' \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \). Let \( S \) be a set of neutrosophic vertices, \( S = \{ s \} \) is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

\[ R_n(CMT_\sigma) = \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x). \]

Thus

\[ R_n(CMT_\sigma) = \max_{x \in V} \sum_{i=1}^{3} \sigma_i(x). \]

The clarifications about results are in progress as follows. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 2.6.2.** In Figure 2.11, a complete-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) For given two neutrosophic vertices, \( s, s' \), \( d(s, n) = 1 = d(s', n) \). Thus \( n \) doesn’t resolve \( s \) and \( s' \);

(ii) the existence of one neutrosophic vertex to do this function, resolving, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum resolving set and minimal resolving set as if it seems there’s no neutrosophic vertex to resolve so as to choose one vertex outside resolving set so as the function of resolving is impossible;

(iii) for given two vertices, \( s \) and \( s' \) if \( d(s, n) = 1 = d(s', n) \), then \( n \) doesn’t resolve \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every two neutrosophic vertices \( s, s' \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S = \{ s \} \) is called dual-resolving set. The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(NTG) = 1 \);
2.6. Setting of neutrosophic dual-resolving number

Figure 2.11: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

(iv) the corresponded set doesn’t have to be resolved by the set;

(v) \( V \) isn’t used when the set is considered in this notion since \( V \setminus \{v\} \) always works;

(vi) for given two vertices, \( s \) and \( s' \) if \( d(s,n) = 1 = d(s',n) \), then \( n \) doesn’t resolve \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices \([a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.]}\). For every two neutrosophic vertices \( s, s' \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S = \{s\} \) is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R_n(NTG) = 2 \);

Another class of neutrosophic graphs is addressed to path-neutrosophic graph.

**Proposition 2.6.3.** Let \( NTG : (V, E, \sigma, \mu) \) be a path-neutrosophic graph. Then

\[
R_n(PTH) = \mathcal{O}_n(PTH) - \min_{x \in V} \sum_{i=1}^{3} \sigma_i(x).
\]

**Proof.** Suppose \( PTH : (V, E, \sigma, \mu) \) is a path-neutrosophic graph. Let \( x_1, x_2, \cdots, x_{\mathcal{O}(PTH)} \) be a path-neutrosophic graph. For given two vertices, \( x \) and \( y \), there’s one path from \( x \) to \( y \). Let \( S \) be an intended set which is dual-resolving set. Despite one leaf \( x_{\mathcal{O}(PTH)} \), all neutrosophic vertices belong to \( S \). They could be resolved by the leaf \( x_{\mathcal{O}(PTH)} \), as if despite the leaf \( x_{\mathcal{O}(PTH)} \), so as maximal set \( S \) is constructed. Thus \( S = \{x_1', x_2', \cdots, x_{\mathcal{O}(PTH)-1}'\} \) is the set \( S \) is a set of vertices from path-neutrosophic graph \( PTH : (V, E, \sigma, \mu) \) with new arrangements of vertices in which there are all neutrosophic vertices which are either neighbors or not. In this new arrangements, the notation of vertices from \( x_i \) is changed to \( x_i' \). Leaves doesn’t necessarily belong to \( S \). Leaves are either belongs to \( S \) or doesn’t belong to \( S \) as if one leaf is outside. Adding all neutrosophic leaves contradicts with maximality of \( S \) and maximum cardinality of \( S \). It implies this construction is optimal. Thus, let

\[
S = \{x_1, x_2, \cdots, x_{\mathcal{O}(PTH)-2}, x_{\mathcal{O}(PTH)-1}\}
\]
be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s, s'$ in $S$, there’s only one neutrosophic leaf $x_{\sigma(P)}$ in $V \setminus (S = \{x_1, x_2, \cdots, x_{\sigma(P)}\})$ such that $x_{\sigma(P)}$ resolves $s, s'$ then the set of neutrosophic vertices, $S = \{x_1, x_2, \cdots, x_{\sigma(P)}\}$ is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_{n}(P) = O_{n}(P) - \min_{x \in V} \sum_{i=1}^{3} \sigma_{i}(x)$.

Thus $R_{n}(P) = O_{n}(P) - \min_{x \in V} \sum_{i=1}^{3} \sigma_{i}(x)$.

**Example 2.6.4.** There are two sections for clarifications.

(a) In Figure 2.12, an odd-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New viewpoint implies different kinds of definitions to get more scrutiny and more discernment.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_3$ in $S$, there’s neutrosophic leaf $n_1$ in $V \setminus (S = \{n_3, n_2\})$ such that $n_1$ resolves $n_2$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(P) = 4$;

(ii) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s neutrosophic leaf $n_1$ in $V \setminus (S = \{n_4, n_2\})$ such that $n_1$ resolves $n_2$ and $n_4$, then the set of neutrosophic vertices, $S = \{n_4, n_2\}$ is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(P) = 4$;

(iii) let $S = \{n_3, n_4, n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s only one neutrosophic leaf $n_5$ in $V \setminus (S = \{n_3, n_4, n_1, n_2\})$ such that $n_5$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(P) = 4$;

(iv) let $S = \{n_3, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s a neutrosophic leaf

\[ R_{n}(P) = O_{n}(P) - \min_{x \in V} \sum_{i=1}^{3} \sigma_{i}(x). \]
2.6. Setting of neutrosophic dual-resolving number

In Figure (2.13), an even-path-neutrosophic graph is illustrated. Some applied in this section.

Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertices $s$ and $s'$ in $S$, there’s a neutrosophic leaf $n_5$ in $V \setminus (S = \{n_3, n_4, n_1\})$ such that $n_5$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_3$ in $S$, there’s neutrosophic leaf $n_1$ in $V \setminus (S = \{n_3, n_2\})$ such that $n_1$ resolves $n_2$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$.

(ii) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s neutrosophic leaf $n_1$ in $V \setminus (S = \{n_4, n_2\})$ such that $n_1$ resolves $n_2$ and $n_4$, then the set of neutrosophic vertices, $S = \{n_4, n_2\}$ is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$.

(iii) let $S = \{n_3, n_4, n_1, n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertices $s$ and $s'$ in $S$, there’s only one neutrosophic leaf $n_6$ in $V \setminus (S = \{n_3, n_4, n_1, n_2, n_5\})$ such that $n_6$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_2, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 5$.

(b) In Figure (2.13), an even-path-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows. New definition is applied in this section.

(v) let $S = \{n_3, n_4, n_1\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s a neutrosophic leaf $n_5$ in $V \setminus (S = \{n_3, n_4, n_1\})$ such that $n_5$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 4$.

(vi) let $S = \{n_3, n_4, n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices $s$ and $s'$ in $S$, there’s only one neutrosophic leaf $n_5$ in $V \setminus (S = \{n_3, n_4, n_1, n_2\})$ such that $n_5$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_1, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(PTH) = 4$.
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Figure 2.12: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

\(\text{(iv)}\) Let \(S = \{n_3, n_2, n_5, n_6\}\) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \(s\) and \(s'\) in \(S\), there’s a neutrosophic leaf \(n_1\) in \(V \setminus (S = \{n_3, n_2, n_5, n_6\})\) such that \(n_1\) resolves \(s\) and \(s'\), then the set of neutrosophic vertices, \(S = \{n_3, n_2, n_5, n_6\}\) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \(R_{n}(PTH) = 5\).

\(\text{(v)}\) Let \(S = \{n_3, n_4, n_1, n_6\}\) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \(s\) and \(s'\) in \(S\), there’s a neutrosophic leaf \(n_5\) in \(V \setminus (S = \{n_3, n_4, n_1, n_6\})\) such that \(n_5\) resolves \(s\) and \(s'\), then the set of neutrosophic vertices, \(S = \{n_3, n_4, n_1, n_6\}\) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \(R_{n}(PTH) = 5\).

\(\text{(vi)}\) Let \(S = \{n_3, n_4, n_5, n_2, n_1\}\) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \(s\) and \(s'\) in \(S\), there’s only one neutrosophic leaf \(n_6\) in \(V \setminus (S = \{n_3, n_4, n_5, n_2, n_1\})\) such that \(n_6\) resolves \(s\) and \(s'\), then the set of neutrosophic vertices, \(S = \{n_3, n_4, n_5, n_2, n_1\}\) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \(R_{n}(PTH) = 8.2\).

**Proposition 2.6.5.** Let \(NTG : (V, E, \sigma, \mu)\) be a cycle-neutrosophic graph where \(O(CYC) \geq 3\). Then

\[
R_{n}(CYC) = O_{n}(CYC) - \min_{x, y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).
\]

**Proof.** Suppose \(CYC : (V, E, \sigma, \mu)\) is a cycle-neutrosophic graph. For given two
2.6. Setting of neutrosophic dual-resolving number

Figure 2.13: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

vertices, \( x \) and \( y \), there are only two paths with distinct edges from \( x \) to \( y \). Let 

\[
x, x_2, \ldots, x_{O(CYC)-1}, x_{O(CYC)}, x_1
\]

be a cycle-neutrosophic graph \( CYC : (V, E, \sigma, \mu) \). \( O(CYC) - 2 \) consecutive vertices could belong to \( S \) which is dual-resolving set related to dual-resolving number where two neutrosophic vertices outside are consecutive. Since these two vertices could resolve all vertices. If there are no neutrosophic vertices which are consecutive, then it contradicts with maximality of set \( S \) and maximum cardinality of \( S \). Thus, let 

\[
S = \{x_1, x_2, \ldots, x_{O(CYC)-1}, x_{O(CYC)}-2\}
\]

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there’s at least one neutrosophic vertex \( n \) in \( V \) \( \setminus \) \( (S = \{x_1, x_2, \ldots, x_{O(CYC)-3}, x_{O(CYC)}-2\}) \) such that \( n \) resolves \( s \) and \( s' \) then the set of neutrosophic vertices, \( S = \{x_1, x_2, \ldots, x_{O(CYC)-3}, x_{O(CYC)}-2\} \) is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by 

\[
R_n(CYC) = O_n(CYC) - \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).
\]

Thus 

\[
R_n(CYC) = O_n(CYC) - \min_{x,y \in V} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)).
\]

The clarifications about results are in progress as follows. An odd-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. An even-cycle-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.
Example 2.6.6. There are two sections for clarifications.

(a) In Figure [2.14], an even-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_3, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_3 \) in \( S \), there’s neutrosophic vertex \( n_1 \) in \( V \setminus (S = \{n_3, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_3, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(ii) \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s neutrosophic vertex \( n_1 \) in \( V \setminus (S = \{n_4, n_2\}) \) such that \( n_1 \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_4, n_2\} \) is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(iii) let \( S = \{n_3, n_4, n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either neutrosophic vertex \( n_6 \) or neutrosophic vertex \( n_5 \) in \( V \setminus (S = \{n_3, n_4, n_1, n_2\}) \) such that either \( n_6 \) resolves \( s \) and \( s' \), or \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_1, n_2\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(iv) let \( S = \{n_3, n_4, n_5, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either neutrosophic vertex \( n_1 \) or neutrosophic vertex \( n_2 \) in \( V \setminus (S = \{n_3, n_4, n_5, n_6\}) \) such that either \( n_1 \) resolves \( s \) and \( s' \), or \( n_2 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_3, n_4, n_5, n_6\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(v) let \( S = \{n_2, n_5, n_1, n_6\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either neutrosophic vertex \( n_3 \) or neutrosophic vertex \( n_4 \) in \( V \setminus (S = \{n_2, n_5, n_1, n_6\}) \) such that either \( n_3 \) resolves \( s \) and \( s' \), or \( n_4 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_2, n_5, n_1, n_6\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CYC) = 4 \);

(vi) let \( S = \{n_3, n_1, n_6, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] For
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every neutrosophic vertices $s$ and $s'$ in $S$, there are either neutrosophic vertex $n_5$ or neutrosophic vertex $n_4$ in $V \setminus (S = \{n_3, n_1, n_6, n_2\})$ such that either $n_5$ resolves $s$ and $s'$, or $n_4$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_1, n_6, n_2\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_n(CYC) = 6.4$.

(b) In Figure 2.15, an odd-cycle-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_3, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_3$ in $S$, there’s neutrosophic vertex $n_4$ in $V \setminus (S = \{n_3, n_2\})$ such that $n_4$ resolves $n_2$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CYC) = 3$;

(ii) $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For every neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s neutrosophic vertex $n_5$ in $V \setminus (S = \{n_3, n_2\})$ such that $n_5$ resolves $n_2$ and $n_4$, then the set of neutrosophic vertices, $S = \{n_3, n_2\}$ is called dual-resolving set and this set isn’t maximal. As if it contradicts with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CYC) = 3$;

(iii) let $S = \{n_3, n_4, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices $s$ and $s'$ in $S$, there are either a neutrosophic vertex $n_1$ or neutrosophic vertex $n_2$ in $V \setminus (S = \{n_3, n_4, n_5\})$ such that either $n_1$ resolves $s$ and $s'$ or $n_2$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_3, n_4, n_5\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CYC) = 3$;

(iv) let $S = \{n_1, n_2, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices $s$ and $s'$ in $S$, there are either a neutrosophic vertex $n_3$ or neutrosophic vertex $n_4$ in $V \setminus (S = \{n_1, n_2, n_5\})$ such that either $n_3$ resolves $s$ and $s'$ or $n_4$ resolves $s$ and $s'$, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_3\}$ is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CYC) = 3$;

(v) let $S = \{n_1, n_2, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices $s$ and $s'$ in $S$, there are either...
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Figure 2.14: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

Figure 2.15: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

a neutrosophic vertex \( n_4 \) or neutrosophic vertex \( n_5 \) in \( V \setminus (S = \{n_1, n_2, n_3\}) \) such that either \( n_4 \) resolves \( s \) and \( s' \) or \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_3\} \) is called dual-resolving set. So as the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}(CYC) = 3 \);

(vi) let \( S = \{n_2, n_3, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every neutrosophic vertices \( s \) and \( s' \) in \( S \), there are either a neutrosophic vertex \( n_1 \) or neutrosophic vertex \( n_5 \) in \( V \setminus (S = \{n_2, n_3, n_4\}) \) such that either \( n_1 \) resolves \( s \) and \( s' \) or \( n_5 \) resolves \( s \) and \( s' \), then the set of neutrosophic vertices, \( S = \{n_2, n_3, n_4\} \) is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( \mathcal{R}_n(CYC) = 5.8 \).

Proposition 2.6.7. Let \( NTG : (V, E, \sigma, \mu) \) be a star-neutrosophic graph with
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center c. Then

\[ R_n(STR_1,\sigma_2) = \max_{y \in V} \sum_{i=1}^{3} (\sigma_i(c) + \sigma_i(y)). \]

Proof. Suppose \( STR_1,\sigma_2 : (V, E, \sigma, \mu) \) is a star-neutrosophic graph. An edge always has center, c, as one of its endpoints. All paths have one as their lengths, forever. \( S = \{c, v\} \) is a dual-resolving set related dual-resolving number. Since, let

\[ S = \{c, v\} = V \setminus \{x_1, x_2, \cdots, x_{\sigma(STR_1,\sigma_2)}\} \]

be a set of neutrosophic vertices \( [a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.}] \). For every neutrosophic vertices \( v \) and \( c \) in \( S \), there’s a neutrosophic vertex \( x \) in \( V \setminus S = V \setminus \{x_1, x_2, \cdots, x_{\sigma(STR_1,\sigma_2)}\} \) such that \( x \) resolves \( v \) and \( c \) then the set of neutrosophic vertices, \( S = \{c, v\} = V \setminus \{x_1, x_2, \cdots, x_{\sigma(STR_1,\sigma_2)}\} \) is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

\[ R_n(STR_1,\sigma_2) = \max_{y \in V} \sum_{i=1}^{3} (\sigma_i(c) + \sigma_i(y)). \]

Thus

\[ R_n(STR_1,\sigma_2) = \max_{y \in V} \sum_{i=1}^{3} (\sigma_i(c) + \sigma_i(y)). \]

The clarifications about results are in progress as follows. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A star-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

Example 2.6.8. There is one section for clarifications. In Figure (2.16), a star-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2\} \) be a set of neutrosophic vertices \( [a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.}] \) which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus S = \{n_1, n_2\} \) such that \( n_3 \) resolves \( n_2 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(STR_1,\sigma_2) = 2 \);

(ii) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices \( [a \text{ vertex alongside triple pair of its values is called neutrosophic vertex.}] \) which aren’t consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus S = \{n_2, n_4\} \) such that \( n \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) isn’t called...
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dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(STR_{1,\sigma_2}) = 2; \)

(iii) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices \([a vertex alongside triple pair of its values is called neutrosophic vertex.\)] which are consecutive vertices. For neutrosophic vertices \( n_3 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_4 \) in \( V \setminus (S = \{n_1,n_3\}) \) such that \( n_4 \) resolves \( n_3 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(STR_{1,\sigma_2}) = 2; \)

(iv) let \( S = \{n_2, n_5\} \) be a set of neutrosophic vertices \([a vertex alongside triple pair of its values is called neutrosophic vertex.\)] which aren’t consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_5 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2,n_5\}) \) such that \( n \) resolves \( n_2 \) and \( n_5 \), then the set of neutrosophic vertices, \( S = \{n_2, n_5\} \) isn’t called dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(STR_{1,\sigma_2}) = 2; \)

(v) let \( S = \{n_1, n_5\} \) be a set of neutrosophic vertices \([a vertex alongside triple pair of its values is called neutrosophic vertex.\)] which are consecutive vertices. For neutrosophic vertices \( n_5 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus (S = \{n_1,n_5\}) \) such that \( n_3 \) resolves \( n_5 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_5\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(STR_{1,\sigma_2}) = 2; \)

(vi) let \( S = \{n_1, n_4\} \) be a set of neutrosophic vertices \([a vertex alongside triple pair of its values is called neutrosophic vertex.\)] which are consecutive vertices. For neutrosophic vertices \( n_4 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus (S = \{n_1,n_4\}) \) such that \( n_3 \) resolves \( n_4 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_4\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R_n(STR_{1,\sigma_2}) = 3.7. \)

**Proposition 2.6.9.** Let \( NTG : (V,E,\sigma,\mu) \) be a complete-bipartite-neutrosophic graph. Then

\[
R_n(CMC_{\sigma_1,\sigma_2}) = \max_{x\in V_1, y\in V_2} \sum_{i=1}^{3}(\sigma_i(x) + \mu_i(y)).
\]

**Proof.** Suppose \( CMC_{\sigma_1,\sigma_2} : (V,E,\sigma,\mu) \) is a complete-bipartite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is resolved

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Figure 2.16: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

by any given vertex. Thus maximum cardinality implies including one vertex from each part. Let

\[ S = V \setminus \{x_1, x_2, \ldots, x_{\text{CMC} \sigma_1, \sigma_2}\} = \{u, v\}_{u \in V_1, v \in V_2} \]

be a dual-resolving set related to the dual-resolving number. This construction gives the proof. Since let

\[ S = V \setminus \{x_1, x_2, \ldots, x_{\text{CMC} \sigma_1, \sigma_2}\} = \{u, v\}_{u \in V_1, v \in V_2} \]

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \( u \) and \( v \) in \( S \), there’s a neutrosophic vertex \( n \) in

\[ V \setminus (S = V \setminus \{x_1, x_2, \ldots, x_{\text{CMC} \sigma_1, \sigma_2}\}) = \{u, v\}_{u \in V_1, v \in V_2} \]

such that \( n \) resolves \( u \) and \( v \), then the set of neutrosophic vertices,

\[ S = V \setminus \{x_1, x_2, \ldots, x_{\text{CMC} \sigma_1, \sigma_2}\} = \{u, v\}_{u \in V_1, v \in V_2} \]

is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

\[ R_n(\text{CMC} \sigma_1, \sigma_2) = \max_{x \in V_1, y \in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)). \]

Thus

\[ R_n(\text{CMC} \sigma_1, \sigma_2) = \max_{x \in V_1, y \in V_2} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)). \]

The clarifications about results are in progress as follows. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more senses about new notions. A complete-bipartite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.
Example 2.6.10. There is one section for clarifications. In Figure (2.17), a complete-bipartite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2, n_4\} \) be a set of neutrosophic vertices \([\text{a vertex alongside triple pair of its values is called neutrosophic vertex}]\) which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_1, n_2, n_4\}) \) such that \( n \) resolves \( n_4 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2, n_4\} \) isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2}) = 2; \)

(ii) let \( S = \{n_2, n_3\} \) be a set of neutrosophic vertices \([\text{a vertex alongside triple pair of its values is called neutrosophic vertex}]\) which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{n_2, n_3\}) \) such that \( n \) resolves \( n_2 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_2, n_3\} \) isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2}) = 2; \)

(iii) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices \([\text{a vertex alongside triple pair of its values is called neutrosophic vertex}]\) which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_3 \) in \( S \), there’s a neutrosophic vertex \( n_2 \) in \( V \setminus (S = \{n_1, n_3\}) \) such that \( n_2 \) resolves \( n_1 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2}) = 2; \)

(iv) let \( S = \{n_1, n_2\} \) be a set of neutrosophic vertices \([\text{a vertex alongside triple pair of its values is called neutrosophic vertex}]\) which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_2 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus (S = \{n_1, n_2\}) \) such that \( n_3 \) resolves \( n_1 \) and \( n_2 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2\} \) is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2}) = 2; \)

(v) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices \([\text{a vertex alongside triple pair of its values is called neutrosophic vertex}]\) which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus (S = \{n_2, n_4\}) \) such that \( n_3 \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2}) = 2; \)

(vi) let \( S = \{n_1, n_3\} \) be a set of neutrosophic vertices \([\text{a vertex alongside triple pair of its values is called neutrosophic vertex}]\) which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_3 \) in \( S \), there’s a neutrosophic vertex \( n_2 \) in \( V \setminus (S = \{n_1, n_3\}) \) such that \( n_2 \) resolves \( n_1 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{n_1, n_3\} \) is called dual-resolving set. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R_n(CMC_{\sigma_1, \sigma_2}) = 3.4. \)
2.6. Setting of neutrosophic dual-resolving number

Proposition 2.6.11. Let $NTG : (V, E, \sigma, \mu)$ be a complete-t-partite-neutrosophic graph where $t \geq 3$. Then

$$R_n(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = \max_{x \in V_i, y \in V_j, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_j(y)).$$

Proof. Suppose $CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t} : (V, E, \sigma, \mu)$ is a complete-t-partite-neutrosophic graph. Every vertex in a part and another vertex in opposite part is resolved by any given vertex. Thus maximum cardinality implies including two vertices from two different parts. Let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\} \cup V_1 \cup V_2.$$ 

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}} - 2\}$$

be a dual-resolving set related to the dual-resolving number. This construction gives the proof. Since let

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\} \cup V_1 \cup V_2.$$ 

Thus

$$S = V \setminus \{x_1, x_2, \cdots, x_{CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}} - 2\}$$

be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices $u$ and $v$ in $S$, there’s a neutrosophic vertex $n$ in

$$V \setminus (S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\} \cup V_1 \cup V_2.$$ 

such that $n$ resolves $u$ and $v$, then the set of neutrosophic vertices,

$$S = V \setminus (V_1 \setminus \{u\} \cup V_2 \setminus \{v\} \cup V_3 \cup \cdots \cup V_{t-1} \cup V_t) = \{u, v\} \cup V_1 \cup V_2.$$ 

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Thus

\[ S = V \setminus \{ x_1, x_2, \cdots, x_{\sigma(CMC_{\sigma_1, \sigma_2, \cdots, \sigma_t})-2} \} \]

is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by

\[ R_n(CMC_{\sigma_1, \sigma_2, \cdots, \sigma_t}) = \max_{x \in V, y \in V, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)). \]

Thus

\[ R_n(CMC_{\sigma_1, \sigma_2, \cdots, \sigma_t}) = \max_{x \in V, y \in V, i \neq j} \sum_{i=1}^{3} (\sigma_i(x) + \sigma_i(y)). \]

The clarifications about results are in progress as follows. A complete-t-partite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A complete-t-partite-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 2.6.12.** There is one section for clarifications. In Figure 2.18, a complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{ n_1, n_2, n_4 \} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_1, n_2, n_4 \}) \) such that \( n \) resolves \( n_4 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{ n_1, n_2, n_4 \} \) isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2, \cdots, \sigma_t}) = 2; \)

(ii) Let \( S = \{ n_2, n_3 \} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_3 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus (S = \{ n_2, n_3 \}) \) such that \( n \) resolves \( n_3 \) and \( n_2 \), then the set of neutrosophic vertices, \( S = \{ n_2, n_3 \} \) isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2, \cdots, \sigma_t}) = 2; \)

(iii) Let \( S = \{ n_1, n_3 \} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices \( n_1 \) and \( n_3 \) in \( S \), there’s a neutrosophic vertex \( n_2 \) in \( V \setminus (S = \{ n_1, n_3 \}) \) such that \( n_2 \) resolves \( n_1 \) and \( n_3 \), then the set of neutrosophic vertices, \( S = \{ n_1, n_3 \} \) is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by \( R(CMC_{\sigma_1, \sigma_2, \cdots, \sigma_t}) = 2; \)
2.6. Setting of neutrosophic dual-resolving number

Figure 2.18: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

(iiv) let $S = \{n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_2$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_1, n_2\})$ such that $n_3$ resolves $n_1$ and $n_2$, then the set of neutrosophic vertices, $S = \{n_1, n_2\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(v) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_2, n_4\})$ such that $n_3$ resolves $n_2$ and $n_4$, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(vii) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_3$ in $S$, there’s a neutrosophic vertex $n_2$ in $V \setminus (S = \{n_1, n_3\})$ such that $n_2$ resolves $n_1$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-resolving set. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $\mathcal{R}_n(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 3.4$.

**Proposition 2.6.13.** Let $NTG : (V, E, \sigma, \mu)$ be a wheel-neutrosophic graph. Then

$$\mathcal{R}_n(WHL_{1, \sigma_2}) = \max_{y \in V} \sum_{i=1}^{3} (\sigma_i(c) + \sigma_i(y)).$$

**Proof.** Suppose $WHL_{1, \sigma_2} : (V, E, \sigma, \mu)$ is a wheel-neutrosophic graph. The argument is elementary. All vertices of a cycle join to one vertex, $c$. $S = \{c, s\}$
is a dual-resolving set related dual-resolving number. Since, let
\[ S = V \setminus \{x_1, x_2, \cdots, x_O(WHL_{1,s_2}) - 2\} = \{c, s\} \]
be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex]. For every neutrosophic vertices \( c \) and \( s \) in \( S \), there’s a neutrosophic vertex \( n \) in \( V \setminus \{S = V \setminus \{x_1, x_2, \cdots, x_O(WHL_{1,s_2}) - 2\} = \{c, s\}\} \) such that \( n \) resolves \( c \) and \( s \), then the set of neutrosophic vertices, \( S = V \setminus \{x_1, x_2, \cdots, x_O(WHL_{1,s_2}) - 2\} = \{c, s\} \) is called dual-resolving set. So as the maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by
\[
\mathcal{R}(WHL_{1,s_2}) = \max_{y \in V} \sum_{i=1}^{3} (\sigma_i(c) + \sigma_i(y)).
\]
Thus
\[
\mathcal{R}(WHL_{1,s_2}) = \max_{y \in V} \sum_{i=1}^{3} (\sigma_i(c) + \sigma_i(y)).
\]

The clarifications about results are in progress as follows. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it. To make it more clear, next part gives one special case to apply definitions and results on it. Some items are devised to make more sense about new notions. A wheel-neutrosophic graph is related to previous result and it’s studied to apply the definitions on it, too.

**Example 2.6.14.** There is one section for clarifications. In Figure (2.19), a wheel-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let \( S = \{n_1, n_2\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices \( n_2 \) and \( n_1 \) in \( S \), there’s a neutrosophic vertex \( n_3 \) in \( V \setminus \{S = \{n_1, n_2\}\} \) such that \( n_3 \) resolves \( n_2 \) and \( n_1 \), then the set of neutrosophic vertices, \( S = \{n_1, n_2\} \) is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by
\[
\mathcal{R}(WHL_{1,s_2}) = 2;
\]

(ii) let \( S = \{n_2, n_4\} \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertices \( n_2 \) and \( n_4 \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus \{S = \{n_2, n_4\}\} \) such that \( n \) resolves \( n_2 \) and \( n_4 \), then the set of neutrosophic vertices, \( S = \{n_2, n_4\} \) isn’t called dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by
\[
\mathcal{R}(WHL_{1,s_2}) = 2;
\]
2.7. Applications in Time Table and Scheduling

(iii) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_3$ and $n_1$ in $S$, there’s a neutrosophic vertex $n_4$ in $V \setminus (S = \{n_1, n_3\})$ such that $n_4$ resolves $n_3$ and $n_1$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(WHL_{1,\sigma_2}) = 2$;

(iv) let $S = \{n_2, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which aren’t consecutive vertices. For every neutrosophic vertices $n_2$ and $n_5$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_5\})$ such that $n$ resolves $n_2$ and $n_5$, then the set of neutrosophic vertices, $S = \{n_2, n_5\}$ isn’t called dual-resolving set. So as it doesn’t relate to dual-resolving number. As if it doesn’t contradict with the maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(WHL_{1,\sigma_2}) = 2$;

(v) let $S = \{n_1, n_5\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_5$ and $n_1$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_1, n_5\})$ such that $n_3$ resolves $n_5$ and $n_1$, then the set of neutrosophic vertices, $S = \{n_1, n_5\}$ is called dual-resolving set and this set is maximal. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(WHL_{1,\sigma_2}) = 2$;

(vi) let $S = \{n_1, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_4$ and $n_1$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_1, n_4\})$ such that $n_3$ resolves $n_4$ and $n_1$, then the set of neutrosophic vertices, $S = \{n_1, n_4\}$ is called dual-resolving set and this set is maximal. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_n(WHL_{1,\sigma_2}) = 3.7$.

2.7 Applications in Time Table and Scheduling

In this section, two applications for time table and scheduling are provided where the models are either complete models which mean complete connections are formed as individual and family of complete models with common neutrosophic vertex set or quasi-complete models which mean quasi-complete connections are formed as individual and family of quasi-complete models with common neutrosophic vertex set.

Designing the programs to achieve some goals is general approach to apply on some issues to function properly. Separation has key role in the context of this style. Separating the duration of work which are consecutive, is the matter and it has importance to avoid mixing up.
2. Modified Notions

Figure 2.19: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number.

**Step 1. (Definition)** Time table is an approach to get some attributes to do the work fast and proper. The style of scheduling implies special attention to the tasks which are consecutive.

**Step 2. (Issue)** Scheduling of program has faced with difficulties to differ amid consecutive sections. Beyond that, sometimes sections are not the same.

**Step 3. (Model)** The situation is designed as a model. The model uses data to assign every section and to assign to relation amid sections, three numbers belong unit interval to state indeterminacy, possibilities and determinacy. There’s one restriction in that, the numbers amid two sections are at least the number of the relations amid them. Table (2.1), clarifies about the assigned numbers to these situations.

Table 2.1: Scheduling concerns its Subjects and its Connections as a neutrosophic graph in a Model.

<table>
<thead>
<tr>
<th>Sections of $NTG$</th>
<th>$n_1$</th>
<th>$n_2$</th>
<th>$\cdots$</th>
<th>$n_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Values</td>
<td>(0.7, 0.9, 0.3)</td>
<td>(0.4, 0.2, 0.8)</td>
<td>(\cdots)</td>
<td>(0.4, 0.2, 0.8)</td>
</tr>
<tr>
<td>Connections of $NTG$</td>
<td>$E_1$</td>
<td>$E_2$</td>
<td>(\cdots)</td>
<td>$E_6$</td>
</tr>
<tr>
<td>Values</td>
<td>(0.4, 0.2, 0.3)</td>
<td>(0.5, 0.2, 0.3)</td>
<td>(\cdots)</td>
<td>(0.3, 0.2, 0.3)</td>
</tr>
</tbody>
</table>

**2.8 Case 1: Complete-t-partite Model alongside its dual-resolving number and its neutrosophic dual-resolving number**

**Step 4. (Solution)** The neutrosophic graph alongside its dual-resolving number and its neutrosophic dual-resolving number as model, propose to use specific number. Every subject has connection with some subjects. Thus
2.8. Case 1: Complete-t-partite Model alongside its dual-resolving number and its neutrosophic dual-resolving number

Figure 2.20: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number

the connection is applied as possible and the model demonstrates quasi-full connections as quasi-possible. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is star, the number is different. Also, it holds for other types such that complete, wheel, path, and cycle. The collection of situations is another application of its dual-resolving number and its neutrosophic dual-resolving number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are five subjects which are represented as Figure (2.20). This model is strong and even more it’s quasi-complete. And the study proposes using specific number which is called its dual-resolving number and its neutrosophic dual-resolving number. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to this model and situation to compare them with same situations to get more precise. Consider Figure (2.20).

In Figure (2.20), an complete-t-partite-neutrosophic graph is illustrated. Some points are represented in follow-up items as follows.

(i) Let $S = \{n_1, n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_4$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus (S = \{n_1, n_2, n_4\})$ such that $n$ resolves $n_4$ and $n_1$, then the set of neutrosophic vertices, $S = \{n_1, n_2, n_4\}$ isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(ii) let $S = \{n_2, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.] which are consecutive vertices. For neutrosophic vertices $n_2$ and $n_3$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus (S = \{n_2, n_3\})$ such that $n$
resolves $n_2$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_2, n_3\}$
isn’t called dual-resolving set. So as it doesn’t relate to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(iii) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_3$ in $S$, there’s a neutrosophic vertex $n_2$ in $V \setminus (S = \{n_1, n_3\})$ such that $n_2$ resolves $n_1$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(iv) let $S = \{n_1, n_2\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_2$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_1, n_2\})$ such that $n_3$ resolves $n_1$ and $n_2$, then the set of neutrosophic vertices, $S = \{n_1, n_2\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(v) let $S = \{n_2, n_4\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_2$ and $n_4$ in $S$, there’s a neutrosophic vertex $n_3$ in $V \setminus (S = \{n_2, n_4\})$ such that $n_3$ resolves $n_2$ and $n_4$, then the set of neutrosophic vertices, $S = \{n_2, n_4\}$ is called dual-resolving set. So as it relates to maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 2$;

(vi) let $S = \{n_1, n_3\}$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex] which are consecutive vertices. For neutrosophic vertices $n_1$ and $n_3$ in $S$, there’s a neutrosophic vertex $n_2$ in $V \setminus (S = \{n_1, n_3\})$ such that $n_2$ resolves $n_1$ and $n_3$, then the set of neutrosophic vertices, $S = \{n_1, n_3\}$ is called dual-resolving set. So as it relates to maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_n(CMC_{\sigma_1, \sigma_2, \ldots, \sigma_t}) = 3.4$.

2.9 Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number

Step 4. (Solution) The neutrosophic graph alongside its dual-resolving number and its neutrosophic dual-resolving number as model, propose to use specific number. Every subject has connection with every given subject in deemed way. Thus the connection applied as possible and the model demonstrates full connections as possible between parts but with different
2.9. Case 2: Complete Model alongside its A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number

Figure 2.21: A Neutrosophic Graph in the Viewpoint of its dual-resolving number and its neutrosophic dual-resolving number

view where symmetry amid vertices and edges are the matters. Using the notion of strong on the connection amid subjects, causes the importance of subject goes in the highest level such that the value amid two consecutive subjects, is determined by those subjects. If the configuration is complete multipartite, the number is different. Also, it holds for other types such that star, wheel, path, and cycle. The collection of situations is another application of its dual-resolving number and its neutrosophic dual-resolving number when the notion of family is applied in the way that all members of family are from same classes of neutrosophic graphs. As follows, There are four subjects which are represented in the formation of one model as Figure (2.21). This model is neutrosophic strong as individual and even more it’s complete. And the study proposes using specific number which is called its dual-resolving number and its neutrosophic dual-resolving number for this model. There are also some analyses on other numbers in the way that, the clarification is gained about being special number or not. Also, in the last part, there is one neutrosophic number to assign to these models as individual. A model as a collection of situations to compare them with another model as a collection of situations to get more precise. Consider Figure (2.21). There is one section for clarifications.

(i) For given two neutrosophic vertices, \( s, s' \), \( d(s, n) = 1 = d(s', n) \). Thus \( n \) doesn’t resolve \( s \) and \( s' \);

(ii) the existence of one neutrosophic vertex to do this function, resolving, is obvious thus this vertex form a set which is necessary and sufficient in the term of minimum resolving set and minimal resolving set as if it seems there’s no neutrosophic vertex to resolve so as to choose one vertex outside resolving set so as the function of resolving is impossible;

(iii) for given two vertices, \( s \) and \( s' \) if \( d(s, n) = 1 = d(s', n) \), then \( n \) doesn’t resolve \( s \) and \( s' \) where \( d \) is the minimum number of edges amid all paths from \( s \) to \( s' \). Let \( S \) be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices \( s, s' \) in \( S \), there’s no neutrosophic vertex \( n \) in \( V \setminus S \) such that \( n \) resolves \( s, s' \), then the set of neutrosophic vertices, \( S = \{ s \} \) is called dual-resolving set.
2. Modified Notions

The maximum cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R(NTG) = 1$;

$(iv)$ the corresponded set doesn’t have to be resolved by the set;

$(v)$ $V$ isn’t used when the set is considered in this notion since $V \setminus \{v\}$ always works;

$(vi)$ for given two vertices, $s$ and $s'$ if $d(s, n) = 1 = d(s', n)$, then $n$ doesn’t resolve $s$ and $s'$ where $d$ is the minimum number of edges amid all paths from $s$ to $s'$. Let $S$ be a set of neutrosophic vertices [a vertex alongside triple pair of its values is called neutrosophic vertex.]. For every two neutrosophic vertices $s, s'$ in $S$, there’s no neutrosophic vertex $n$ in $V \setminus S$ such that $n$ resolves $s, s'$, then the set of neutrosophic vertices, $S = \{s\}$ is called dual-resolving set. The maximum neutrosophic cardinality between all dual-resolving sets is called dual-resolving number and it’s denoted by $R_n(NTG) = 2$;

2.10 Open Problems

In this section, some questions and problems are proposed to give some avenues to pursue this study. The structures of the definitions and results give some ideas to make new settings which are eligible to extend and to create new study. Notion concerning its dual-resolving number and its neutrosophic dual-resolving number are defined in neutrosophic graphs. Thus,

**Question 2.10.1.** Is it possible to use other types of its dual-resolving number and its neutrosophic dual-resolving number?

**Question 2.10.2.** Are existed some connections amid different types of its dual-resolving number and its neutrosophic dual-resolving number in neutrosophic graphs?

**Question 2.10.3.** Is it possible to construct some classes of neutrosophic graphs which have “nice” behavior?

**Question 2.10.4.** Which mathematical notions do make an independent study to apply these types in neutrosophic graphs?

**Problem 2.10.5.** Which parameters are related to this parameter?

**Problem 2.10.6.** Which approaches do work to construct applications to create independent study?

**Problem 2.10.7.** Which approaches do work to construct definitions which use all definitions and the relations amid them instead of separate definitions to create independent study?

2.11 Conclusion and Closing Remarks

In this section, concluding remarks and closing remarks are represented. The drawbacks of this article are illustrated. Some benefits and advantages of this study are highlighted. This study uses two definitions concerning dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic
graphs assigned to neutrosophic graphs. Maximum number of resolved vertices, is a number which is representative based on those vertices. Maximum neutrosophic number of resolved vertices corresponded to dual-resolving set is called neutrosophic dual-resolving number. The connections of vertices which aren’t clarified by minimum number of edges amid them differ them from each other and put them in different categories to represent a number which is called dual-resolving number and neutrosophic dual-resolving number arising from resolved vertices in neutrosophic graphs assigned to neutrosophic graphs. Further studies could be about changes in the settings to compare these notions amid different settings of neutrosophic graphs theory. One way is finding some relations amid all definitions of notions to make sensible definitions. In Table 2.2, some limitations and advantages of this study are pointed out.

Table 2.2: A Brief Overview about Advantages and Limitations of this Study

<table>
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<tr>
<th>Advantages</th>
<th>Limitations</th>
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</thead>
<tbody>
<tr>
<td>1. Dual-Resolving Number of Model</td>
<td>1. Connections amid Classes</td>
</tr>
<tr>
<td>2. Neutrosophic Dual-Resolving Number of Model</td>
<td>2. Study on Families</td>
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<td>3. Maximal Dual-Resolving Sets</td>
<td>3. Same Models in Family</td>
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<td>4. Resolved Vertices amid all Vertices</td>
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<td>5. Acting on All Vertices</td>
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