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- PhD, PostDocs, is an Emeritus Professor of Mathematics at the University of New Mexico, United States.
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- Postdoctoral in Applied Mathematics from Okayama University of Sciences, Japan .And The Guangdong University of Technology, Guangzhou, China.

His scientific achievements:

- He is the founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics since 1995.
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SuperHyperTopology, SuperHyperAlgebra, SuperHyperFunction, Neutrosophic SuperHyperAlgebra, degree of dependence and independence between neutrosophic components, refined neutrosophic set, neutrosophic over-under-off-set (with degrees of membership/ indeterminacy/nonmembership less than 0 and bigger than1), plithogenic set / logic / probability / statistics, symbolic plithogenic algebraic structures, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, extension of algebraic structures to NeutroAlgebra and AntiAlgebra, NeutroGeometry and AntiGeometry, NeutroTopology and AntiTopology, Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, Dezert-Smarandache Theory.

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- He presented papers and plenary lectures to tenths of international conferences and seminars around the world.
- He also published many books of poetry, dramas, children's stories, literary translations, essays, a novel, folklore collections, traveling memories, and art albums.
- He wrote articles and books in four languages: English, Romanian, French, and Spanish.
- His books were translated to Arabic, Chinese, Russian, Spanish, Greek, Portuguese, Italian, German, Serbo-Croatian, and Turkish, see
  https://fs.unm.edu/LiteratureLibrary.htm and
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In The Name of Allah, The Most Beneficent, The Most Merciful

So Exalted is Allah, the True Sovereign. Do not be hasty with the Quran before its inspiration is concluded to you, and say, "My Lord, increase me in knowledge"

(Allah Almighty is Truthful)

My dear colleagues

The pursuit of the best is the route of human interest at all times, and in our effort to deliver the finest while keeping up with the amazing scientific advancements experienced in our modern world, we bring to you this humble work titled:

(*Neutrosophic linear models and algorithms to find their optimal solution)*

Science serves as the foundation for managing life's concerns and human activities, and living without knowledge is a form of wandering and loss. When there are numerous answers and possibilities, using scientific methodology helps us comprehend the foundations of choice, decision-making, and selecting the proper solutions.

In this work, we give a study of linear models based on the concepts of neutrosophic science—a branch of knowledge founded on the ideas that there is no ultimate truth, no facts that can be definitively confirmed, and problems that go beyond simple right and wrong.

There is a third state between error and right, an indeterminate, undetermined, uncertain state. It is indeterminacy. Neutrosophic science gave each issue three dimensions, namely (T, I, F), correctness in degrees, indeterminacy in degrees, and error in degrees.

Neutrosophics is a relatively young science. It investigates the various spectrums that a person can imagine in a single issue, resulting in a more accurate description of the data of the issue under investigation and thus accurate results that leave no room for coincidence, assisting in making decisions that suit all of the circumstances experienced by the work environment of the system under investigation.

In this book, we present a study of linear models and algorithms to discover the optimal solution for them utilizing neutroscientific notions. We all know that the linear programming approach is one of the most significant methods of operations research, a field that arose as a result of the tremendous scientific progress that our modern world is experiencing. The group of scientific methodologies utilized is referred to as operations research It is a science that has gained broad success in numerous aspects of life by examining issues and finding for best solutions. This discipline is distinguished by the development of mathematical models, tools, and methodologies capable of expressing the concepts of efficiency and scarcity in a well-defined mathematical model for a given circumstance. It is capable of using scientific approaches to solve complicated issues in the management of huge systems in factories, institutions, and businesses, and it assists decision makers in these organizations in making optimum scientific judgments for the workflow.

Classical logic was used to handle these challenges, but the ideal solution was a specific value relevant to the conditions under which the data was obtained. It does not take into account the changes that may occur in the work environment. To obtain more accurate results and enjoy a margin of freedom, we present in this book a study of neutrosophic linear models and algorithms to find the optimal solution for them. The term “neutrosophic models” refers to models where the variables are values that are influenced by the environment. Examples of these variables are those found in the objective function, which, in the case of a maximization model, expresses profit and, in the case of a minimization model, expresses a cost. We take it in the form $Nc_j = c_j \pm \varepsilon_j$, where ε_{ij} is the indeterminacy, and it takes one of the forms $\varepsilon_j \in [\lambda_{j1}, \lambda_{j2}]$ or $\varepsilon_j \in \{\lambda_{j1}, \lambda_{j2}\}$ or otherwise, which is any neighborhood of the value c_{ij} that we obtain while adding the data on the issue then becomes the cost (or profit) matrix $Nc_j = [c_j \pm \varepsilon_j]$, and also the fixed values that represent the right side of the constraint swings, which express the available capabilities of capital, time, raw materials, etc., and they are also affected. In environmental conditions, we take it from the form $Nb_j = b_j + \delta_j$, where δ_j is the indeterminacy of the required quantities. It can take one of the forms $\delta_j \in [\mu_{i1}, \mu_{i2}]$ or $\delta_j \in \{\mu_{i1}, \mu_{i2}\}$, and the same situation applies to examples of variables in constraints that express quantities. The raw materials consumed in the production process are taken from the form $Na_{ij} = a_{ij} + \gamma_{ij}$, where γ_{ij} is the indeterminacy of the quantities necessary for the raw material i to produce one unit of product j. It can take one of the forms $\gamma_{ij} \in [\varphi_{ij1}, \varphi_{ij2}]$, or $\gamma_{ij} \in \{\varphi_{ij1}, \varphi_{ij2}\}$, which allows businesses a margin of error and aids in the acquisition of more accurate outcomes.

This book includes eight chapters:

Chapter I: Study of neutrosophic linear equations.

Chapter II: Neutrosophic Linear Models.

Chapter III: The graphical method for finding the optimal solution for neutrosophic linear models.

Chapter IV: The simplex direct neutrosophic algorithm for finding the optimal solution for linear models.

Chapter V: The modified simplex neutrosophic algorithm to find the optimal solution for linear models.

Chapter VI: The simplex algorithm with a synthetic basis to find the optimal solution for linear models.

Chapter VII: Neutrosophic Dual Linear Models and the Binary Algorithm.

Chapter VIII: Some applications to neutrosophic linear models.

We hope to God Almighty that this work will achieve the desired benefit from its preparation.

Authors

Florentin Smarandache and Maissam Ahmad Jdid

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Subject	Page

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Chapter I: Study of neutrosophic linear equations

Chapter I

Study of Neutrosophic linear equations

Introduction:

Given the significance of the linear programming method as one of the operations research methods, we felt it necessary to reformulate the systems of linear equations and some of the methods for solving them using the concepts of neutrosophic science, since research and studies using neutrosophics produced more accurate results than research employing classical logic.

1.1. Systems of linear equations according to classical logic:

The propositions of linear equations in which the number of equations equals *m* and the number of variables equals *n* are given according to classical logic in the following general form:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_m$$

and written in the following matrix form*:*

$$A.X = B$$

where:

$$A = \begin{bmatrix} a_{11} & a_{12} \dots a_{1n} \\ a_{21} & a_{22} \dots a_{2n} \\ \dots\dots\dots & \dots\dots\dots \\ a_{m1} & a_{m2} \dots a_{mn} \end{bmatrix} \quad B = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_m \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

where a_{ij} and b_i are real numbers for all values of

$i = 1,2, \dots, m$ and $j = 1,2, \dots, n$.

Three distinct examples of linear equation systems were identified.

First case:

There are the same number of equations and variables, i.e., $m = n$.

Second case:

There are more equations than variables, i.e., $m > n$

Third case:

There are fewer equations than there are variables, i.e., $m < n$. The following linear equation systems will be given utilizing neutrosophic science concepts. In this case, the real numbers a_{ij} and b_i will be treated as neutrosophic numbers, or as indefinite values of the form Nb_i and Na_{ij}. Perfectly determined, they can be any neighborhood of the real numbers a_{ij} and b_i, expressed in any of the following forms:

$Na_{ij} = a_{ij} + \varepsilon_{ij}$ and $Nb_i = b_i + \mu_i$ where $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$ or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$ or otherwise, then the systems of neutrosophic linear equations is written in the form below.

1.2. Systems of neutrosophic linear equations where the number of equations equals m and the number of variables equals n:

General form:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1\text{n}}x_n = Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2\text{n}}x_n = Nb_2$$

… … … … … … … … … … … … … … … ….

$$Na_{\mathrm{mn}}x_1 + Na_{\mathrm{mn}}x_2 + \cdots + Na_{mn}x_m = Nb_m$$

In the following matrix form:

$$NA.\mathrm{X} = NB$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} \mathrm{x}_1 \\ \mathrm{x}_2 \\ .. \\ \mathrm{x}_\mathrm{n} \end{bmatrix}$$

We examine the preceding equation systems in terms of the three previously described examples in order to establish their general solution.

First case:

There are the same number of equations and variables, i.e., $m = n$.

We write the systems of equations as follows:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1\mathrm{n}}x_n = Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2\mathrm{n}}x_n = Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$Na_{n1}x_1 + Na_{n2}x_2 + \cdots + Na_{nn}x_n = Nb_n$$

Or, in matrix form:

$$NA\,.X = NB$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots \\ Na_{n1} & Na_{n2} \dots Na_{nn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

The matrix is a square matrix whose determinant is $\Delta_N = |NA|$

Here we distinguish two cases:

1- $\Delta_N = 0$. This case gives rise to two cases:

a. If $\Delta_N = 0$ and $\Delta_{N_{x_j}} \neq 0$ where $\Delta_{N_{x_j}}$ is the determinant resulting from the determinant of the matrix of Δ_N after replacing the column containing the unknown x_j with the column of constants, then the systems have no solution.

b. If $\Delta_N = 0$ and $\Delta_{N_{x_j}} = 0$, this means that the systems of equations are not linearly independent, meaning that some are linearly related to each other. In order to handle this case, we eliminate one of the two equations that are linearly related; as a result, there are now m' equations instead of two, where $m' = m - 1$ and $m' < n$, which is the same as the second case that will be addressed later.

c. When $\Delta_N \neq 0$, that is, the systems of equations are linearly independent and the systems have a single solution, that can be found in multiple ways. We investigate the Gauss-Jordan method in this study because it serves as the foundation for the direct simplex algorithm that we employ to find the best solution for linear models.

1.3. Gauss-Jordan method for solving systems of neutrosophic linear equations where $m = n$:

To clarify the mathematical framework of the approach, we present the equations in the following matrix form:

$$\begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots & \dots \dots \dots \dots \dots \\ Na_{n1} & Na_{n2} \dots Na_{nn} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix} = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_n \end{bmatrix} \quad (1)$$

Or in the following abbreviated form:

$$NA\,.X = NB \quad (2)$$

Since $\Delta_N = |NA| \neq 0$, this means that the matrix NA has an inverse i.e., NA^{-1}. We multiply both sides of equation (2) by NA^{-1} and we find:

$$NA^{-1}.(NA\,.X) = NA^{-1}.NB$$

Hence, we get:

$$I.\mathrm{X} = NB'$$

which is written in the following detailed form:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ \dots & \dots & \dots \dots \dots \\ 0 & 0 & 0 \dots 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_n \end{bmatrix} \quad (3)$$

This process is the basis of the Gauss-Jordan method for solving a system of linear equations. In order to convert Figure (1) to Figure (2), we follow the following steps:

1- We express Figure (1) in the following table:

Variables / Equations	x_1	x_2	….	x_n	NB
1	Na_{11}	Na_{12}	….	Na_{1n}	Nb_1
2	Na_{21}	Na_{22}	…	Na_{2n}	Nb_2
….	…	…	…	…	…
n	Na_{n1}	Na_{n2}	…	Na_{nn}	Nb_n

Table No. (1): Table of equations

2- We convert the matrix NA to the unit matrix I by processing the rows of the table so that we make all non-diagonal elements in all its rows equal to zero and the diagonal elements equal to one. The steps below are used to eliminate the variable x_s from the equation t:

a- To make x_s equal to one, we divide all the elements of row t by Na_{ts}. This causes x_s to equal one and modifies the other expressions.
b- We set all elements of the column with x_s (except row t) equal to zero.
c- We calculate the rest of the elements of the new table from the following two relation:

$$\left.\begin{aligned} Na'_{ij} &= \left(Na_{ij} - Na_{is}\frac{Na_{tj}}{Na_{ts}}\right) = \frac{Na_{ij}Na_{ts} - Na_{is}Na_{tj}}{Na_{ts}} \\ Nb'_{i} &= \left(Nb_{i} - Na_{is}\frac{Nb_{t}}{Na_{ts}}\right) = \frac{Nb_{i}Na_{ts} - Na_{is}Nb_{t}}{Na_{ts}} \end{aligned}\right] \quad (4)$$

The element Na_{ts} is called the pivot element.
Following the previous process, the following table is produced:

Variables / Equations	Nx_1	Nx_2	….	Nx_n	NB'
1	1	0	….	0	Nb'_1
2	0	1	…	0	Nb'_2
….	…	…	…	…	…
n	0	0	…	1	Nb'_n

Table No. (2) Final solution table

The linear equation systems are expressed in the following matrix form:

$$I.NX = NB'$$

$$\begin{bmatrix} 1 & 0 & 0 \ldots 0 \\ 0 & 1 & 0 \ldots 0 \\ \ldots & \ldots & \ldots \ldots \ldots \\ 0 & 0 & 0 \ldots 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ \cdot\cdot \\ x_n \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ \cdot\cdot \\ Nb'_n \end{bmatrix} \quad \Longrightarrow$$

$$\begin{bmatrix} Nx_1 \\ Nx_2 \\ .. \\ Nx_n \end{bmatrix} = \begin{bmatrix} Nb_1' \\ Nb_2' \\ .. \\ Nb_n' \end{bmatrix}$$

$$\Rightarrow Nx_1 = Nb_1' , Nx_2 = Nb_2' , \dots , Nx_n = Nb_n'$$

Second case:

There are more equations than variables, i.e., $m > n$.

In this case, we form a new system from the set of equations in which the number of equations is equal to the number of variables by excluding a number of equations of $\mathrm{m} - \mathrm{n}$. Then, to make sure the equations that were excluded are satisfied; we solve the new systems in the same way as we solved the first case.

Third case: There are fewer equations than there are variables, i.e., $m < n$.

We are presented with a set of equations of the following form in this particular case.

$$\left.\begin{matrix} Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1\mathrm{m}}x_m + \cdots + Na_{1\mathrm{n}}x_n = Nb_1 \\ Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2\mathrm{m}}x_m + \cdots + Na_{2\mathrm{n}}x_n = Nb_2 \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \\ Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{\mathrm{mm}}x_m + \cdots + Na_{mn}x_n = Nb_m \end{matrix}\right\rVert \quad (5)$$

which is written in the following matrix form:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1m} \dots \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2m} \dots \dots Na_{1n} \\ \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots & \\ Na_{m1} & Na_{m2} \dots Na_{mm} \dots \dots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_n \end{bmatrix} X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_m \\ .. \\ .. \\ x_n \end{bmatrix}$$

In the following brief form:

$$NA_{(m.n)} . X_{(n.1)} = NB_{(m.1)} \quad (6)$$

1- We divide the matrix $NA_{(m.n)}$ into two matrices:

a- A square matrix of rank $(m.m)$ which we denote $NC_{(m.m)}$.

b- And a rectangular matrix of rank $(m.n-m)$ which we denote $ND_{(m.n-m)}$

2- The column matrix $X_{(n.1)}$ is divided into two matrices $X'_{(m.1)}$ and $X''_{(n-m.1)}$.

Subsequently, the equation systems (5) are expressed in the matrix form shown below:

$$\left[NC_{(m.m)}, ND_{(m.n-m)}\right] . \begin{bmatrix} X'_{(m.1)} \\ X''_{(n-m.1)} \end{bmatrix} = NB_{(m.1)} \quad (7)$$

$$NC_{(m.m)}.X'_{(m.1)} + ND_{(m.n-m)}.X''_{(n-m.1)} = NB_{(m.1)}$$

We find that:

$$NC_{(m.m)}.X'_{(m.1)} = NB_{(m.1)} - ND_{(m.n-m)}.X''_{(n-m.1)} \quad (8)$$

Assuming that $|NC| \neq 0$, we multiply both sides in relation (8) by NC^{-1} and we find:

$$NC^{-1}.NC.X' = NC^{-1}.(NB - ND.X'')$$

$$I.X' = NC^{-1}.NB - NC^{-1}.ND.X'') \quad (9)$$

Assuming that $NC^{-1}.NB = NB'$ and $NC^{-1}.ND = ND'_{(m.n-m)}$ we find that:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ \dots & \dots & \dots \dots \dots \\ 0 & 0 & 0 \dots 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_m \end{bmatrix}$$

$$= \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_m \end{bmatrix} - \begin{bmatrix} Nd'_{11} & Nd'_{12} \dots Nd'_{1(n-m)} \\ Nd'_{21} & Nd'_{22} \dots Nd'_{1(n-m)} \\ \dots & \dots \dots \dots \\ Nd'_{m1} & Nd'_{m2} \dots Nd'_{m(n-m)} \end{bmatrix} . \begin{bmatrix} x_{m+1} \\ x_{m+2} \\ \dots \\ x_n \end{bmatrix} \quad (10)$$

which can be converted as follows into a set of linear equations:

$$Nx_1 = Nb'_1 - (Nd'_{11}x_{m+1} + Nd'_{12}x_{m+2} + \cdots + Nd'_{1(n-m)}x_n)$$
$$Nx_2 = Nb'_2 - (Nd'_{21}x_{m+1} + Nd'_{22}x_{m+2} + \cdots + Nd'_{2(n-m)}x_n)$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$Nx_m = Nb'_m - (Nd'_{m1}x_{m+1} + Nd'_{m2}x_{m+2} + \cdots + Nd'_{m(n-m)}x_n)$$

This means that we were able to calculate m in terms of $(n-m)$, $x_{m+1}, x_{m+2}, \dots, x_n$. We note that the values of the variables $x_1, x_2, \dots, x_m$, it relates to the values taken by the variables $x_{m+1}, x_{m+2}, \dots, x_n$, or in other words, what we give to the variables $x_{m+1}, x_{m+2}, \dots, x_n$, and that for every proposition of values such as $\beta_{m+1}, \beta_{m+2}, \dots, \beta_n$ for these variables we get a set of values for the variables $x_1, x_2, \dots, x_m$ is:

$$Nx_1 = Nb'_1 - (Nd'_{11}\beta_{m+1} + Nd'_{12}\beta_{m+2} + \cdots + Nd'_{1(n-m)}\beta_n)$$
$$Nx_2 = Nb'_2 - (Nd'_{21}\beta_{m+1} + Nd'_{22}\beta_{m+2} + \cdots + Nd'_{2(n-m)}\beta_n)$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$Nx_m = Nb'_m - (Nd'_{m1}\beta_{m+1} + Nd'_{m2}\beta_{m+2} + \cdots + Nd'_{m(n-m)}\beta_n)$$

Thus, we obtain a solution that includes all the variables of proposition (5)

Here is how the solution is structured:

$$(\beta_1, \beta_2, \dots, \beta_m, \beta_{m+1}, \beta_{m+2}, \dots, \beta_n)$$

But since the variables $x_{m+1}, x_{m+2}, \dots, x_n$ can take an infinite number of qualitative values (even if they are restricted by certain conditions), we obtain an infinite number of corresponding values for the variables $x_1, x_2, \dots, x_m$.

Thus, the set of equations (5) has an infinite number of acceptable solutions of the following form if $|NC| \neq 0$:

$$(x_1, x_2, \dots, x_m, x_{m+1}, x_{m+2}, \dots, x_n)$$

Thus, we obtain a solution that includes all variables of the proposition, which is the ordered solution:

$$(\beta_1, \beta_2, \dots, \beta_m, \beta_{m+1}, \beta_{m+2}, \dots, \beta_n)$$

1.4. Basic solutions of the neutrosophic linear equations:

Since proposition (5) has an infinite number of acceptable solutions, we will try to limit ourselves to a limited number by setting the variables $x_{m+1}, x_{m+2}, \dots, x_n$ equal to zero. Then proposition (9) takes the following form:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 \\ 0 & 1 & 0 \dots 0 \\ \dots & \dots & \dots \dots \dots \\ 0 & 0 & 0 \dots 1 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ \cdot\cdot \\ x_m \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ \cdot\cdot \\ Nb'_m \end{bmatrix} \qquad (11)$$

We get:

$$x_1 = Nb'_1 \ , x_2 = Nb'_2 \ , \dots, x_m = Nb'_m$$

Consequently, the complete solution is:

$$(Nb'_1, Nb'_2, \dots, Nb'_m, 0,0, \dots ,0)$$

Because it can be attributed to the rule with single normal vectors in the space R^m, we refer to this solution as the basic solution:

$$e_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \cdot\cdot \\ 0 \end{bmatrix} \quad e_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \cdot\cdot \\ 0 \end{bmatrix} \quad \dots \quad \dots \quad e_m = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \cdot\cdot \\ m \end{bmatrix}$$

The set of vectors $e_1, e_2, \dots, e_m$ form a rule because they are linearly independent, and the vector NB' can be expressed using the factorials $x_1, x_2, \dots, x_m$ as follows:

$$NB' = e_1x_1 + e_2x_2 + \cdots + e_mx_m$$

We call the variables $x_1, x_2, \dots, x_m$, basic variables and we call other variables $x_{m+1}, x_{m+2}, \dots, x_n$ free or non-basic variables because they take qualitative values.

The variables $x_1, x_2, \dots, x_m$ are chosen at random to serve as basic variables because, if we know that the following options exist for obtaining basic solutions, we can build alternative basic solutions:

$$C_n^m = \frac{n!}{m!\,(n-m)!}$$

There is a finite number of infinitely possible solutions.

Example 1:

The two linear equations below have a joint solution.

$$2x_1 + 7x_2 + 3x_3 + 2x_4 = [2,5]$$
$$3x_1 + 9x_2 + 4x_3 + x_4 = [3,7]$$
$$x_1 + 5x_2 + 3x_3 + 4x_4 = [4,8]$$

In the set of equations, the number of variables is $n = 4$ and the number of equations is $m = 3$. Therefore, the number of basic variables is equal to 3 and the number of non-basic free variables is $n - m = 1$. The number of possible solutions is calculated from the relation:

$$C_n^m = \frac{n!}{m!\,(n-m)!}$$

i.e.,

$$C_4^3 = \frac{4!}{3!\,(4-3)!} = 4$$

We write as follows:

$$(x_1, x_2, x_3, 0) \quad , (x_1, x_2, 0, x_4) \, , (x_1, 0, x_3, x_4), (0, x_2, x_3, x_4)$$

To obtain these solutions, we write the systems of equations in the following form:

$$2x_1 + 7x_2 + 3x_3 = [2,5] - 2x_4$$
$$3x_1 + 9x_2 + 4x_3 + x_4 = [3,7] - x_4$$
$$x_1 + 5x_2 + 3x_3 = [4,8] - 4x_4$$

The previous proposition is written in the following matrix form:

$$\begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} [2,5] \\ [3,7] \\ [4,8] \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} . [x_4]$$

$$C = \begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix} \quad X' = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad NB = \begin{bmatrix} [2,5] \\ [3,7] \\ [4,8] \end{bmatrix} \quad D = \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} \quad X^{"} = [x_4]$$

We calculate the determinant $|C|$.

We find:

$$|C| = \begin{vmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{vmatrix} = -3 \neq 0$$

We determine the matrix's reciprocal to identify the solutions:

$$C = \begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix}$$

We find:

$$C^{-1} = \begin{bmatrix} \frac{-7}{3} & 2 & \frac{-1}{3} \\ \frac{5}{3} & -1 & \frac{-1}{3} \\ -2 & 1 & 1 \end{bmatrix}$$

We compensate in the relation:

$$NC^{-1}.NC.X' = NC^{-1}.(NB - ND.X^{"})$$

We get*:*

$$\begin{bmatrix} \frac{-7}{3} & 2 & \frac{-1}{3} \\ \frac{5}{3} & -1 & \frac{-1}{3} \\ -2 & 1 & 1 \end{bmatrix} . \begin{bmatrix} 2 & 7 & 3 \\ 3 & 9 & 4 \\ 1 & 5 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-7}{3} & 2 & \frac{-1}{3} \\ \frac{5}{3} & -1 & \frac{-1}{3} \\ -2 & 1 & 1 \end{bmatrix} . \left(\begin{bmatrix} [2,5] \\ [3,7] \\ [4,8] \end{bmatrix} - \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} . [x_4] \right)$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} Nx_1 \\ Nx_2 \\ Nx_3 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{-7}{3} & 2 & \frac{-1}{3} \\ \frac{5}{3} & -1 & \frac{-1}{3} \\ -2 & 1 & 1 \end{bmatrix} . \begin{bmatrix} [2,5] \\ [3,7] \\ [4,8] \end{bmatrix} - \begin{bmatrix} \frac{-7}{3} & 2 & \frac{-1}{3} \\ \frac{5}{3} & -1 & \frac{-1}{3} \\ -2 & 1 & 1 \end{bmatrix} . \begin{bmatrix} 2 \\ 1 \\ 4 \end{bmatrix} . [x_4]$$

which can be transformed into the following systems of equations:

$$Nx_1 = \begin{bmatrix} \left[0, \frac{-1}{3}\right] \\ -\left[1, \frac{4}{3}\right] \\ [3,5] \end{bmatrix} - \begin{bmatrix} \frac{-16}{3} \\ 1 \\ 1 \end{bmatrix} . [x_4]$$

Setting the free variable x_4 equal to zero, we get:

$$Nx_1 = \begin{bmatrix} \left[0, \frac{-1}{3}\right] \\ -\left[1, \frac{4}{3}\right] \\ [3,5] \end{bmatrix}$$

i.e.,

$$x_1 = \left[0, \frac{-1}{3}\right], x_2 = -\left[1, \frac{4}{3}\right], x_3 = [3,5]$$

Thus, we obtain the first neutrosophic basic solution, which is:

$$(x_1, x_2, x_3, 0) = \left(\left[0, \frac{-1}{3}\right], -\left[1, \frac{4}{3}\right], [3,5], 0 \right)$$

We obtain other basic solutions in the same way.

Dissolved basic solutions:

If we get a value of zero for the variables we have selected as a basis, the fundamental solution is degenerate and invalid.

1.5. Gauss- Jordan method for solving a set of linear equations where $m < n$:

The following are the basic steps of the Gaussian-Jordan method, which are based on the previously mentioned mathematical principles:

1- We write the systems of equations (5) in the following matrix form:

$$I.X' + NC^{-1}.D.X'' = NC^{-1}.NB = NB'$$

$$[I\ ,NC^{-1}.ND]\,.\begin{bmatrix} X' \\ X'' \end{bmatrix} = NB' \qquad (12)$$

which is written in the following detailed form:

$$\begin{bmatrix} 1 & 0 & 0 \dots 0 & Nd'_{11} & Nd'_{12} \dots Nd'_{1(n-m)} \\ 0 & 1 & 0 \dots 0 & Nd'_{21} & Nd'_{22} \dots Nd'_{1(n-m)} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 \dots 1 & Nd'_{m1} & Nd'_{m2} \dots Nd'_{m(n-m)} \end{bmatrix} . \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix} = \begin{bmatrix} Nb'_1 \\ Nb'_2 \\ .. \\ Nb'_m \end{bmatrix} \qquad (13)$$

The transition from Figure (5) to Figure (12) is done the same steps we mentioned in the previous paragraph, but this method does not give us a basic solution unless we set the free variables equal to zero. If we do that, we only get the first solution. To obtain all solutions, we perform the following steps:

a. We organize the following table:

Variables / Equations	x_1	x_2	….	x_m	x_{m+1}	x_{m+2}	….	x_n	NB
1	a_{11}	a_{12}	….	a_{1m}	a_{1m+1}	a_{1m+2}	….	a_{1n}	Nb'_1
2	a_{21}	a_{22}	…	a_{2m}	a_{2m+1}	a_{2m+2}	….	a_{2n}	Nb'_2
….	…	…	…	…	…	…	…	…	…
m	a_{m1}	a_{m2}	…	a_{mm}	a_{mm+1}	a_{mm+2}	…	a_{mn}	Nb'_m

Table No. (3) The first table for the Gauss- Jordan method

b. We find the identity matrix $I_{m\times m}$ by processing the rows of the previous table in the same way as explained in the previous paragraph. To do this the specify variables that are entered in the base and let them be $x_1, x_2, \dots, x_m$. As a result of this processing, we obtain the following table:

Variables / Equations	x_1	x_2	….	x_m	x_{m+1}	x_{m+2}	….	x_n	NB'
1	1	0	….	0	Nd'_{11}	Nd'_{12}	….	Nd'_{1n-m}	Nb'_1
2	0	1	…	0	Nd'_{21}	Nd'_{22}	….	Nd'_{2n-m}	Nb'_2
….	…	…	…	…	…	…	…	…	…
m	0	0	…	1	Nd'_{m1}	Nd'_{22}	…	Nd'_{mn-m}	Nb'_2

Table No. (4): Table of the first basic solution

c. Setting all the free variables in Table (4) equal to zero, we obtain the following first basic solution:

$$(Nb'_1, Nb'_2, \dots, Nb'_m, 0,0, \dots ,0)$$

a. To obtain a second basic solution, we replace one of the basic variables, say x_m, with one of the non-basic variables x_{m+1}, by selecting the appropriate pivot element, and here it is Nd'_{m1}. We work to delete x_{m+1} from all equations except equation m. In this equation, we set the coefficient of this variable to one. We use the two relations (4) to carry out the necessary computations. We solve the subsequent second basic solution:

$$(Nb'_1, Nb'_2, \dots, Nb'_{m-1}, 0, Nb'_{m+1}, 0, \dots, 0)$$

We repeat the steps mentioned in step (d) to get additional basic solutions.

1.6. Non-negative basic solutions of systems of neutrosophic linear equations:

If all or some of the variables must be non-negative, then certain fundamental solutions are not sufficient since they violate the criterion. In this kind of circumstance, we need to look for good fundamental solutions among the basic solutions. Because the procedure in the example is not easily used, especially when there are numerous variables, the Gauss-Jordan method was devised to immediately find positive solutions.

The new method was called the simplex method, which is carried out according to the following steps:

1.7. The simplex method for finding non-negative basic solutions to a system of linear equations where $m < n$:

In the systems of equations (5):

1- By multiplying the equation with the negative second side by (-1), we are able to make all elements of the constant's column *NB* on the second side of the equations non-negative.

2- We put the coefficients of the new systems in a table.

3- We form a rule consisting of m variables by selecting the variable that we want to enter into the rule, for example, x_s, then we calculate the index.

$$\theta = Min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{Nb_t}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

We call the element Na_{ts} the pivot element, we delete the variable x_s from all equations according to the Gauss-Jordan method, except for the equation t, in which its coefficient is equal to one. We repeat the previous step until we form a base consisting of m variables.

4- Setting the non-basic variables equal to zero we obtain the following non-negative basic solution:

$$(Nb'_1, Nb'_2, \dots, Nb'_{m-1}, 0, Nb'_{m+1}, 0, \dots, 0)$$

5- We designate one of the variables as a basic variable, find the pivot element, and then carry out the same steps as for the variable x_s to obtain additional new non-negative basic solutions. We continue working until we have all of the non-negative basic solutions after we find a new one.

We explain the above using the following example:

Example 2:

$$x_1 - 3x_4 + 2x_5 = -[1,3]$$
$$x_2 + 2x_4 - 3x_5 = [2,8]$$

We multiply the first equation by (-1) until the condition $Nb_i > 0$ is met, and we obtain the following new systems:

$$-x_1 - 3x_3 - 2x_5 = [1,3]$$

$$x_2 + 2x_4 - 3x_5 = [2,8]$$

The stopping criterion is met if we are unable to locate a single free column that was not used for switching and that has at least one positive element. This indicates that all of the free column elements that were not utilized during the swap have negative values.

In the systems of equations, the number of variables is $n = 5$ and the number of equations is $m = 2$ Therefore, the number of basic variables is equal to 2 and the number of non-basic free variables is $n - m = 3$. The number of possible solutions is calculated from the relation:

$$C_n^m = \frac{n!}{m!\,(n-m)!}$$

i.e.,

$$C_5^2 = \frac{5!}{2!\,(5-2)!} = 10$$

We write as follows:

$$(x_1, x_2, 0,0,0)\ , (x_1, 0, x_3, 0,0), (x_3, 0,0, x_4, 0), (x_1, 0,0,0, x_5),$$
$$(0, x_2, x_3, 0,0)(0, x_2, 0, x_4, 0), (0, x_2, 0,0, x_5), (0,0, x_3, x_4, 0),$$
$$(0,0, x_3, 0, x_5), (0,0,0, x_4, x_5)$$

To obtain these solutions, we organize the following table:

Variables / Equations	x_1	x_2	x_3	x_4	x_5	NB
1	-1	0	0	3	-2	[1,3]
2	0	1	0	2	-3	[2,8]

Table No. (5): The first table for the simplex method

To find a basic solution to the set of equations, we select a variable, for example x_4, to be a basic variable, and to determine the appropriate anchor element, we calculate the index:

$$\theta = Min\left[\frac{Nb_i}{Na_{is}}\right] = Min\left[\frac{[1,3]}{3}, \frac{[2,8]}{2}\right] = \frac{[1,3]}{3}$$

The pivot is $a_{14} = 3$. The following table is produced after the necessary computations are made to remove the variable x_4 from the two equations:

Variables / Equations	x_1	x_2	x_3	x_4	x_5	NB'
x_4	$-\frac{1}{3}$	0	0	1	$-\frac{2}{3}$	$\left[\frac{1}{3},1\right]$
2	$\frac{2}{3}$	1	0	0	$\frac{5}{3}$	$\left[\frac{4}{3},6\right]$

Table No. (6) The second table for the simplex method

We choose another variable to be a basic variable. We note that the variable x_2 is ready to be a basic variable, and thus we get the following table:

Variables / Equations	x_1	x_2	x_3	x_4	x_5	NB'
x_4	$-\frac{1}{3}$	0	0	1	$-\frac{2}{3}$	$\left[\frac{1}{3},1\right]$
x_2	$\frac{2}{3}$	1	0	0	$\frac{5}{3}$	$\left[\frac{4}{3},6\right]$

Table No. (7): Final solution table

Thus, we obtain a base consisting of the variables x_2 , x_4. We set the free variables equal to zero, and we obtain the following non-negative neutrosophic basic solution:

$$\left(0,\left[\frac{4}{3},6\right],0,\left[\frac{1}{3},1\right],0\right)$$

We repeat the steps we took to find the previous solution to get additional solutions.

Conclusion:

In this study, we have examined the sets of neutrosophic linear equations that serve as the foundation for neutrosophic linear programming. Additionally, we have discussed the Gauss-Jordan method, which is regarded as the mathematical foundation for the simplex method, which finds positive basic solutions when there are constraints on some or all of the variables' values being positive. This method is then used to find the optimal solution for linear models using direct simplex. Our fundamental neutrosophic solutions that express indeterminate values are derived from the examples we have given on systems of neutrosophic equations. can be applied when the data supplied to the systems that follow these equation systems are dynamic. The margin of freedom provided by neutrosophic values might be advantageous in this case.

Chapter II: Neutrosophic Linear Models

Chapter II

Neutrosophic Linear Models

Introduction:

This chapter presents the formulas for neutrosophic linear mathematical models, which are linear models that include neutrosophic values in their mathematical relations, either in the constraint relation or the objective function relation. This allows the model to account for all possible changes in the operating environment of the system it represents, ensuring a safe workflow for the facility. To this end, we will treat the variables in the objective function as neutrosophic values, i.e., $Nc_j = c_j \pm \varepsilon_j$.

Also, the values that express the available capabilities are neutrosophic values, i.e., $Nb_i = b_i \pm \delta_i$ and , $Na_{ij} = a_{ij} \pm \mu_{ij}$ where $(j = 1,2, \dots, n\ , i = 1,2, \dots, m)$ are undefined values that have a margin of freedom and are taken according to the nature of the situation represented by the linear model–therefore, utilizing the subsequent investigation, we give the fundamental formulas of linear models:

2–1- Basic formulas of neutrosophic linear models:

Neutrosophic linear models can be classified according to the following formulas**:**

2-1-1-The general formula for the neutrosophic linear model:

The general neutrosophic formula for the linear mathematical model is given in abbreviated form as follows**:**

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max\ or\ Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_i \pm \delta_i \quad ; \ i = 1,2, \dots, m$$

$$x_j \geq 0$$

where $c_j + \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2, \dots, n$, $i = 1,2, \dots, m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables.

It is given in the following detailed form:

Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow (Max\ or\ Min)$$

Constraints:

$$Na_{\mathrm{i}1} x_1 + Na_{\mathrm{i}2} x_2 + \cdots + Na_{in} x_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} Nb_{\mathrm{i}} \quad i = 1,2, \dots, m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Linear models can also be expressed using matrices, and therefore the neutrosophic linear model given in the general form can be written using matrices as follows:

Find:

$$NZ = NC\ \mathrm{X} \longrightarrow (Max\ or\ Min)$$

Constraints:

$$NA\,X\begin{pmatrix}\geq\\ \leq\\ =\end{pmatrix}NB$$

$$X \geq 0$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \ \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad X = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

2-1-2- The canonical neutrosophic formula for the linear model:

If every variable is required to be non-negative and every constraint is provided in the form of an inequality that must be entered in the format (where ≤ is less than or equal to), then the linear program is considered canonical. The following is an abbreviated form of the neutrosophic canonical form:

$$NZ = \sum_{j=1}^{n}(c_j \pm \varepsilon_j)x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij}x_j \leq b_i \pm \delta_i \quad ; \ \ i = 1,2, \dots, m$$

$$x_j \geq 0$$

It is given in the following detailed form:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{N}a_{1n}x_n \leq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \leq Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \leq Nb_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

Linear models can also be expressed using matrices, and therefore the neutrosophic linear model given in the canonical form can be written using matrices as follows:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Max$$

Constraints:

$$NA\,\mathrm{X} \leq NB$$

$$\mathrm{X} \geq 0$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \ldots Na_{1n} \\ Na_{21} & Na_{22} \ldots Na_{2n} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ Na_{m1} & Na_{m2} \ldots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

2-1-3- The standard neutrosophic formula for the linear model:

When it comes to solving linear programming problems, the standard form is crucial. This is because solving a linear programming problem has been reduced to solving a set of linear equations made up of *n* equations with $n + m$ unknowns. Solving this sentence is helpful if it can be done, meaning that if it satisfies the requirement that $x_j \geq 0$ be non-negative, then the ideal values for the variables are those that satisfy the constraints and give the objective function the maximum or smallest value permissible by the problem text.

The standard neutrosophic formula is given in the following abbreviated form:

Find:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow (Max \ \ or \ Min)$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j = b_i \pm \delta_i \quad ; \ \ i = 1,2, \dots, m$$

$$x_j \geq 0$$

It is given in the following detailed form:
Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow (Max \ \ or \ Min)$$

Constraints:

$$Na_{11} x_1 + Na_{12} x_2 + \cdots + \mathrm{N}a_{1n} x_n = Nb_1$$

$$Na_{21} x_1 + Na_{22} x_2 + \cdots + Na_{2n} x_n = Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$Na_{m1} x_1 + Na_{m2} x_2 + \cdots + Na_{mn} x_n = Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Since matrices may also be used to define linear models, the neutrosophic linear model given in standard form can be stated as follows in matrices:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow (Max \;\; or \; Min)$$

Constraints:

$$NA\,\mathrm{X} = NB$$

$$\mathrm{X} \geq 0$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \; NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \;\; NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \; \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

With the exception of the non-negative constraints, which continue to be inequalities, all of the constraints in this case are of the equality type. All of the decision variables must also be non-negative, as must the right side of each equality constraint. In the standard neutrosophic form, the objective function can either be a minimization function or a maximization function.

2-1-4- The symmetrical formula of the neutrosophic linear model:

We say of a linear program that it is in the symmetrical form if all variables are constrained to be non-negative and if all constraints are given in the form of inequalities. the inequalities of the constraints of the maximization problem must be in the form ($\leq$) (less than or equal to), while the inequalities of the constraints in the minimization problem must be in the form ($\geq$) (greater than or equal to). Next, we utilize one of the two following formulas to construct the neutrosophic symmetric formula:

First figure:

The neutrosophic symmetric formula for the linear mathematical model is given in the abbreviated form as follows:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \leq b_i \pm \delta_i \quad ; \; i = 1,2, \dots, m$$

$$x_j \geq 0$$

It is given in the following detailed form:

Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow Max$$

Constraints:

$$Na_{11} x_1 + Na_{12} x_2 + \cdots + \mathrm{N} a_{1n} x_n \leq Nb_1$$

$$Na_{21} x_1 + Na_{22} x_2 + \cdots + Na_{2n} x_n \leq Nb_2$$

$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$

$$Na_{m1} x_1 + Na_{m2} x_2 + \cdots + Na_{mn} x_n \leq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Using matrices as follows:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Max$$

Constraints:

$$NA\,\mathrm{X} \leq NB$$

$$\mathrm{X} \geq 0$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots \dots \dots \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ \vdots \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ \vdots \\ Nc_n \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$

Second form:

The summary is as follows:

The neutrosophic symmetric formula for the linear mathematical model is given in the abbreviated form as follows:

$$NZ = \sum_{j=1}^{n} \left(c_j \pm \varepsilon_j\right) x_j \longrightarrow Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \geq b_i \pm \delta_i \quad ; \quad i = 1,2, \dots, m$$

$$x_j \geq 0$$

It is given in the following detailed form:
Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \dots + Nc_n x_n \longrightarrow Min$$

Constraints:

$$Na_{11} x_1 + Na_{12} x_2 + \dots + \mathrm{N}a_{1n} x_n \geq Nb_1$$

$$Na_{21} x_1 + Na_{22} x_2 + \dots + Na_{2n} x_n \geq Nb_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

Using matrices as follows:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Min$$

Constraints:

$$NA\,\mathrm{X} \geq NB$$

$$\mathrm{X} \geq 0$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots\dots\dots\dots\dots\dots\dots & \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

2-2- How to move from one formula to another:

A brief explanation of the neutrosophic linear models' formulas. It should be mentioned that we may use the following basic transformations to get from one formula to another:

- Converting the minimum value of the objective function f(x) to a maximum value by multiplying it by (-1) we get (- ($f(x)$)).
- If the inequalities were of the form (greater than or equal to) they will be converted to the form (less than or equal to) by multiplying both sides by (-1), and vice versa.

- The equality constraint can be converted into two inequalities of different direction.
- If the left side of an (inequality) constraint is given in absolute value, it can be converted into two regular inequalities.
- Constraint inequalities of the type (greater than or equal to) are converted to an equality constraint by subtracting an appropriate positive variable (i.e., artificial variable) from the left side of the inequality and this variable is entered into the objective function with zero coefficient.
- Constraint inequalities of the type (less than or equal to) are converted into an equality constraint by adding an appropriate positive variable (i.e., slack variable) to the left-hand side of the inequality and then this variable is entered into the objective function with zero coefficient.
- If one of the decision variables x is not constrained by the non-negative condition (that is, it can be negative, positive or zero), then it can be expressed as the difference between two non-negative variables x', x'' as follows $x = x' - x''$ and $x', x'' \geq 0$

2-3- Examples of the above:

The linear models in all examples are given in detailed form:

Example 1:

In its generic form, refer to the following as neutrosophic linear programming:

$$Min\ NL = (3 \pm \varepsilon_1)x_1 - (3 \pm \varepsilon_2)x_2 + (7 \pm \varepsilon_3)x_3$$

Constraints:

$$x_1 + x_2 + 3x_3 \leq 40 \pm \delta_1$$
$$x_1 + 9x_2 - 7x_3 \geq 50 \pm \delta_2$$
$$5x_1 + 3x_2 = 20 \pm \delta_3$$
$$|5x_2 + 8x_3| \leq 100 \pm \delta_4$$
$$x_1, x_2 \geq 0$$

where ε_j is indeterminate and could be

$$\varepsilon_j \in [\lambda_{1j}, \lambda_{2j}] \text{ or } \varepsilon_j \in \{\lambda_{1j}, \lambda_{2j}\} \; ; j = 1,2,3.$$

Also, the values that express the available possibilities δ_i are neutrosophic values. This means that it is indeterminate and could be

$$\delta_i \in [\mu_{1i}, \mu_{2i}] \text{ or } \delta_i \in \{\mu_{1i}, \mu_{2i}\} \; ; i = 1,2,3,4$$

To convert the above problem into the neutrosophic canonical form, we perform the following transformations:

• The objective function is a function of minimization that we turn into a function of maximization:

$$Min\ NL = (3 \pm \varepsilon_1)x_1 - (3 \pm \varepsilon_2)x_2 + (7 \pm \varepsilon_3)x_3$$

Transformed into:

$$Max\ NZ = -(3 \pm \varepsilon_1)x_1 + (3 \pm \varepsilon_2)x_2 - (7 \pm \varepsilon_3)x_3$$

• The second constraint is given (greater than or equal to) is converted into (less than or equal) by multiplying both sides by (-1) we get:

$$-x_1 - 9x_2 + 7x_3 \leq -(50 \pm \delta_2)$$

• Third constraint $5x_1 + 3x_2 = 20 \pm \delta_3$ transformed into two entries:

$$5x_1 + 3x_2 \leq 20 \pm \delta_3$$
$$5x_1 + 3x_2 \geq 20 \pm \delta_3$$

Then we turn the constraint $5x_1 + 3x_2 \geq 20 \pm \delta_3$ into:

$$-5x_1 - 3x_2 \leq -(20 \pm \delta_3)$$

• The constraint $|5x_2 + 8x_3| \leq 100 \pm \delta_4$ is equivalent to the two inequalities:

$$5x_2 + 8x_3 \leq 100 \pm \delta_4$$
$$-5x_2 - 8x_3 \leq 100 \pm \delta_4$$

• The variable x_3 is not restricted by the non-negative constraint, so it is replaced by the following assumption

$x_3 = x_3' - x_3''$where$x_3', x_3'' \geq 0$.

The canonical neutrosophic form becomes:

$$Max\ NZ = -(3 \pm \varepsilon_1)x_1 + (3 \pm \varepsilon_2)x_2 - (7 \pm \varepsilon_3)(x_3' - x_3'')$$

Constraints:

$$x_1 + x_2 + 3(x_3' - x_3'') \leq 40 \pm \delta_1$$
$$-x_1 - 9x_2 + 7(x_3' - x_3'') \leq -(50 \pm \delta_2)$$
$$5x_1 + 3x_2 \leq 20 \pm \delta_3$$
$$-5x_1 - 3x_2 \leq -(20 \pm \delta_3)$$
$$5x_2 + 8(x_3' - x_3'') \leq 100 \pm \delta_4$$
$$-5x_2 - 8(x_3' - x_3'') \leq 100 \pm \delta_4$$

$$x_1, x_2, x_3', x_3'' \geq 0$$

Example 2:

A factory produces four types of products $S_1, S_2,, S_3, S_4$. For this purpose, the following raw materials are used: M_1, M_2, M_3 .

Keeping in mind that the profit is directly correlated with the quantity of units sold of the products, the factory management seeks to analyze the best way to organize production over a given time period (say, a month) and calculate the monthly production for each product in order to maximize profit. The following table displays the available amounts of raw materials required for each product as well as the profit:

Products / Materials	Product Type				Available Quantities
	S_1	S_2	S_3	S_4	
M_1	1.5	1	2.4	1	$3000 \pm \delta_1$
M_2	1	5	1	3.5	$9000 \pm \delta_2$
M_3	1.5	3	3.5	1	$7000 \pm \delta_3$
win one product	$4 \pm \varepsilon_1$	$8 \pm \varepsilon_2$	$5 \pm \varepsilon_3$	$6 \pm \varepsilon_4$	

Assuming that x_1, x_2, x_3, x_4 represent the number of units created from the types of goods during the course of the production period (a month, for example), the amount of raw material M_1 that is consumed in the creation of the four variations is as follows:

$$1.5x_1 + x_2 + 2.4x_3 + x_4$$

and it must not exceed $3000 \pm \delta_1$ from the available quantity, that is:

$$1.5x_1 + x_2 + 2.4x_3 + x_4 \leq 3000 \pm \delta_1 \qquad (1)$$

Likewise, the amount of raw material M_2 consumed in the production of the four types is:

$$x_1 + 5x_2 + x_3 + 3.5x_4 \leq 9000 \pm \delta_2 \qquad (2)$$

and the amount consumed of the raw material M_3 in the production of the four types is:

$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 \leq 7000 \pm \delta_3 \qquad (3)$$

In addition, the produced quantities must be non-negative, i.e.:

$$x_1, x_2, x_3, x_4 \geq 0 \qquad (4)$$

These are referred to as non-negative conditions.

Thus, we have identified all the constraints imposed on the variables of the problem.

We now define the objective function. If quantified units x_1, x_2, x_3, x_4 of species are produced in order, then the profit during the productive period will be:

$$NZ = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_4)x_4$$

It represents the objective function. Therefore, the mathematical model of the problem is:

$$Max\ NZ = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_4)x_4$$

Constraints:

$$1.5x_1 + x_2 + 2.4x_3 + x_4 \leq 3000 \pm \delta_1$$
$$x_1 + 5x_2 + x_3 + 3.5x_4 \leq 9000 \pm \delta_2$$
$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 \leq 7000 \pm \delta_3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

We have obtained a neutrosophical canonical linear model using the appropriate transformations, which can be written in the following neutrosophical standard form:

$$Max\ NZ = (4 \pm \varepsilon_1)x_1 + (8 \pm \varepsilon_2)x_2 + (5 \pm \varepsilon_3)x_3 + (6 \pm \varepsilon_3)x_4 + 0y_1 + 0y_2 + 0y_3$$

Constraints:

$$1.5x_1 + x_2 + 2.4x_3 + x_4 + y_1 = 3000 \pm \delta_1$$
$$x_1 + 5x_2 + x_3 + 3.5x_4 + y_2 = 9000 \pm \delta_2$$
$$1.5x_1 + 3x_2 + 3.5x_3 + x_4 + y_3 = 7000 \pm \delta_3$$
$$x_1, x_2, x_3, x_4 \geq 0$$

Conclusion:

The indeterminacy that we introduced into the data described by the linear model provides us with neutrosophical linear models that simulate reality and account for the majority of the changes that could occur in the operating environment of the system represented by the linear mathematical model, allowing us to continue studying linear programming topics such as identifying accompanying programs that need to be developed. The symmetrical mathematical model, solving linear models using the simplex approach, which involves creating models in the standard form, and other linear programming subjects are covered.

Chapter III: The graphical method for finding the optimal solution for neutrosophic linear models

Chapter III
The graphical method for finding the optimal solution for neutrosophic linear models

Introduction:

After discussing the linear models and their various formulas based on neutrosophic scientific principles, we provide the neutrosophic graphical approach that we apply to solve the neutrosophic linear models in this chapter.

One of the easiest approaches to tackling linear programming issues is the graphical method, which visualizes the model. But since linear programming issues frequently involve a lot of variables, it is insufficient to solve all of them, and the graphical technique can only be applied in the following situations:

- The number of unknowns is n = 1, or n = 2, or n = 3.
- In linear models whose constraints are equal constraints, if the number of unknowns and the number of equations meet one of the following conditions: n – m = 1 or n – m = 2 or n – m = 3.

Here, we may use the non-negative constraints that the linear model's variables have to turn the model into a function of one, two, or three variables. The graphical method for solving linear models where the constraints are equal and the difference between the number of unknowns and the number of constraints is equal to one, two, or three is reformulated in this study along with the graphical method for solving linear models using neutrosophics.

3.1. Graphical method for solving linear models

We find the optimal solution by following the steps below:

1. We determine the half-planes defined by the inequalities of the constraints by drawing the straight lines resulting from the transformation of the inequalities of the constraints. To do this, we specify two points that fulfill the constraint, and connect the two points to obtain the straight line that corresponds to the constraint. This straight line divides the plane into two halves to determine the half-plane that satisfies the constraint. We select a point at the top of the mapping from one of the two half-planes. We substitute the coordinates of this point into the inequality. If it is satisfied, then the region in which this point is located is the solution region. If it is not satisfied, then the opposite region is the solution region.
2. We define the common solution region, i.e., the region resulting from the intersection of the halves of the planes defined by constraint inequalities. This region must be non-empty so that we can proceed with the solution.
3. To represent the objective function, we note that its relation contains three unknowns, $Z,\ x_1\ , x_2$. Therefore, we need to know a value for Z that is unknown to us. Here we assume a value, let it be $Z_1 = 0$, draw the equation of the objective function Z_1 specify another value, let it be Z_2, and represent the equation. If we continue on in the same way, we will eventually have a sequence of parallel lines that

represent the objective function in addition to a line that is parallel to the original line.

4. We draw ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ where c_1 is coefficient of x_1 and c_2 is coefficient of x_2 in the objective function statement, and the direction of its increasing function is the direction of ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$, and the direction of its decreasing function is the opposite direction. This ray, i.e., the drawing is done according to the type of objective function (maximization or minimization). To put it more clearly, we find the optimal solution point by drawing the line representing Z_1 parallel to itself towards the ray $\vec{C} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$ to find the maximum value of the objective function (and reversing this direction to find the smallest value), until it passes through the last point of the common solution region and this point is the optimal solution point, which is located at the boundaries of the common solution region and any other displacement, no matter how small, takes it out of it.

3.2. Graphical method for finding the optimal solution for neutrosophic linear models

By using the concept of neutrosophic linear models, we can determine that the best solution a neutrosophic value appropriate under all circumstances may be reached by applying the same earlier procedures. We illustrate the above with the following example:

Example 1

A company produces two types of products A_1 ,A_2 and uses three types of raw materials B_1 ,B_2 , B_3 in the production process; the available quantities of each of the raw materials are B_i ; $i = 1,2,3$, the quantity required to produce one unit of each products A_j ; $j = 1,2$, and the profit derived from one unit of each of the products A_1 ,A_2 is shown in the following table:

products / raw materials	A_1	A_2	available quantities
B_1	6	4	36
B_2	2	3	12
B_3	5	0	10
profit	[6,8]	[2,4]	

Table Issue data

Requirement

Determine the quantities that must be produced of each product A_j ; $j = 1,2$, for the company to achieve maximum profit:

Ascertain the necessary production volumes for each product A_j ; $j = 1,2$, in order for the business to make the most profit possible:

Solution:

Suppose x_j is the quantity produced from the product, where $j = 1,2$, then we can formulate the following neutrosophic linear mathematical model:

$$Z = [6,8]x_1 + [2,4]x_2 \rightarrow Max$$

Constraints:

$$6x_1 + 4x_2 \leq 36 \quad (1)$$

$$2x_1 + 3x_2 \leq 12 \quad (2)$$

$$5x_1 \leq 15$$

$$x_1, x_2 \geq 0$$

Because the coefficients of the variables in the objective function are undetermined, the preceding model is a linear neutrosophic model. To identify the optimal solution for the preceding model, we will use a graphical method as shown below:

The first constraint

We draw the straight line representing the first constraint:

$$6x_1 + 4x_2 = 36$$

We impose:

$$x_1 = 0 \Rightarrow 4x_2 = 36 \Rightarrow x_2 = 9$$

We get the first point: $A(0,9)$.

We impose:

$$x_2 = 0 \Rightarrow 6x_1 = 36 \Rightarrow x_1 = 6$$

We get the second point: $B(6,0)$

We take a point at the top of the designation from one of the two halves of the resulting plane after having drawn the straight line through the two points $A(0,9)$ and $B(6,0)$. Let it be the point $O(0,0)$ and substitute it in the inequality of the first entry. We find that the inequality is satisfied i.e., the half of the plane to which the point $O(0,0)$ belongs is half of the solution plane of the first-constraint inequality.

We proceed in the same way for the second and third constraints and obtain the following graphical representation: Figure No. (1):

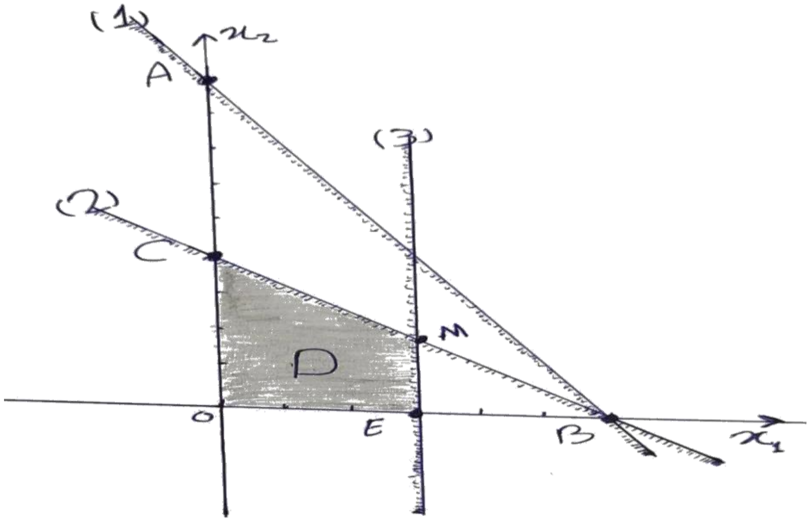

Figure No. (1) Graphic representation of the limitations of the linear model in Example 1

After we have shown the constraints, we notice that the common solution area is bounded by the polygon whose vertices are the points $(0,0)$, $E(3,0)$, M and $C(0,4)$.

The point M is the intersection point, and the second and third constraints coordinates are obtained by solving the following two equations:

$$2x_1 + 3x_2 = 12$$

$$5x_1 = 15$$

We find: $M(3,2)$

Substituting the coordinates of the vertex points into the objective function expression, we get:

$$Z_O = 0$$

$$Z_E \in [12,16]$$

$$Z_M \in [22,32]$$

$$Z_C \in [8,16]$$

This means that the highest value of function Z is reached at point $(3,2)$, i.e., the company must produce three units of the first product and two units of the second product, then it will achieve the maximum profit.

$$Max\ Z = Z_M \in [22,32]$$

Note:

When the number of points is small, we can easily substitute them in the objective function, and the point that gives the best value for the objective function is the optimal solution, but when there are a large number of constraints, we get a large number from the vertical points located on the perimeter of the common solution area. In the above scenario, calculating all of these points' coordinates and putting them into the objective function becomes problematic. As a result, as previously stated, we resort to the representation of the objective function and the calculation of the optimal solution point.

3.3. Non-negative constraints for optimal solution of some neutrosophic linear models using the graphical method

Example 2

Find the optimal solution for the following linear neutrosophic model:

$$Z = x_1 - x_2 - 3x_3 + x_4 + [2,5]x_5 - x_6 + 2x_7 - [10,15] \rightarrow Max$$

Constraints:

$$x_1 - x_2 + x_3 = 5 \qquad (1)$$

$$2x_1 - x_2 - x_3 - x_4 = -11 \qquad (2)$$

$$x_1 + x_2 - x_5 = -4 \qquad (3)$$

$$x_2 + x_6 = 6 \qquad (4)$$

$$2x_1 - 3x_2 - x_6 + 2x_7 = 8 \qquad (5)$$

$$x_1, x_2, x_3, x_4, x_5, x_6, x_7 \geq 0$$

Solution:

We note that the number of constraints is $m = 5$ and the number of variables is $n = 7$, which means that $n - m = 2$.

As a result, using the graphical method and the non-negative constraints, identify the best solution for the previous model using the steps below.

1- We calculate five variables in terms of only two variables.
2- Given that the linear model's variables fulfill the non-negative requirements, we can derive five inequalities of the type greater than or equal to from the variables we computed.
3- The objective function with only two variables is obtained by substituting the five variables.
4- We write the new model, which is a linear model with two variables, so that the optimal solution can be found graphically.

We apply the previous steps to Example 2.

We find:

$$x_3 = 5 - x_1 + x_2 \qquad (1)'$$

$$x_4 = 3x_1 - 2x_2 + 6 \qquad (2)'$$

$$x_5 = x_1 + x_2 + 4 \qquad (3)'$$

$$x_6 = 6 - x_2 \qquad (4)'$$

$$x_7 = 7 - x_1 + x_2 \qquad (5)'$$

Substituting in the objective function, we get:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25]$$

Since $x_3, x_4, x_5, x_6, x_7 \geq 0$ from (1)', (2)', (3)', (4)', (5)', we get the following set of constraints:

$$5 - x_1 + x_2 \geq 0$$

$$-3x_1 + 2x_2 - 3 \geq 0$$

$$x_1 + x_2 + 4 \geq 0$$

$$6 - x_2 \geq 0$$

$$7 - x_1 + x_2 \geq 0$$

Neutrosophic linear mathematical model:

Find:

$$Z = [1,4]x_1 + [3,6]x_2 + [8,25] \to Max$$

Constraints:

$$5 - x_1 + x_2 \geq 0$$

$$3x_1 - 2x_2 + 6 \geq 0$$

$$x_1 + x_2 + 4 \geq 0$$

$$6 - x_2 \geq 0$$

$$7 - x_1 + x_2 \geq 0$$

$$x_1, x_2 \geq 0$$

Because the model includes two variables, the best solution may be identified visually by following the procedures outlined in Example (1).

The required graphic representation is found in Figure No. (2):

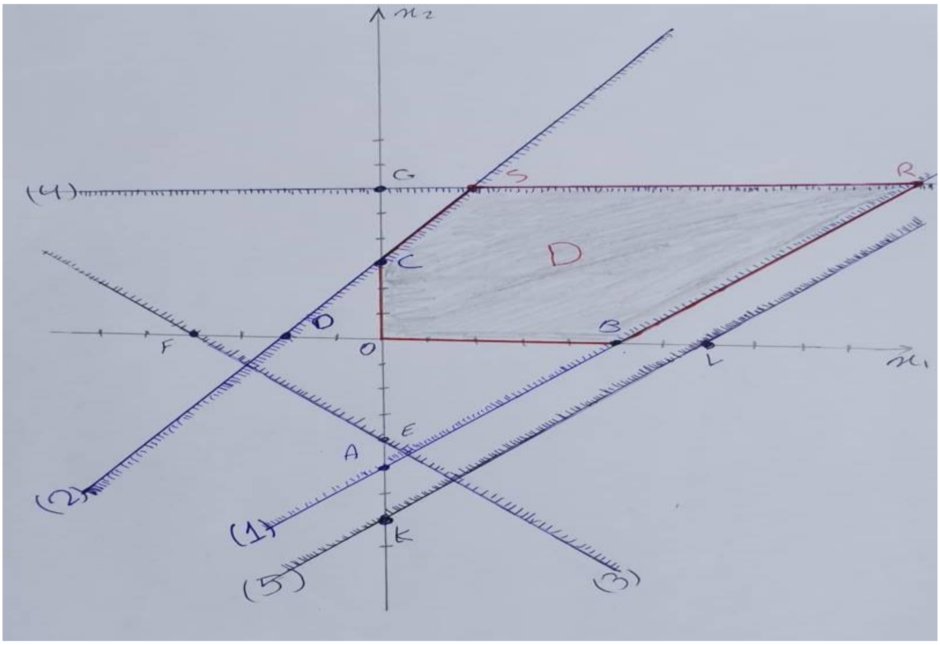

Figure No. (2): Graphical representation of the constraints of the linear model in Example 2

Region D is the region of joint solutions and is defined by the polygon $OBRSC$, where $O(0,0)$, $(5,0)$, $C(0,3)$, and for the two points R, S we find that the point R is the point of intersection of the first and fourth entries.

We obtain its coordinates by solving the set of equations:

$$5 - x_1 + x_2 = 0$$

$$6 - x_2 = 0$$

We get: $R(11,6)$.

The point S is the point of intersection of the second and fourth entries.

We obtain its coordinates by solving the set of equations:

$$3x_1 - 2x_2 + 6 = 0$$

$$6 - x_2 = 0$$

We get: $S(2,6)$.

Since the optimal solution is located at one of the vertices of the common solution region, we substitute the coordinates of these points with the objective function:

At point $O(0,0)$

$$Z_O = 0$$

At point $B(5,0)$

$$Z_B = [13,45]$$

At point $R(11,6)$

$$Z_R \in [37,105]$$

At point $S(2,6)$

$$Z_S \in [28,69]$$

At point $C(0,3)$

$$Z_C \in [17,43]$$

The greatest value of the objective function is at the point $R(11,6)$ that is $x_1 = 11$ and $x_2 = 6$.

We calculate the values of the remaining variables by (1)', (2)', (3)', (4)', (5)'.

We find: $x_3 = 0$, $x_4 = 27$, $x_5 = 21$, $x_6 = 0$, $x_7 = 2$.

Substituting in the objective function of the original model we obtain the maximum value of the Z function, which is:

$$MaxZ \in [68,126]$$

Important Notes:

1- A vertical point in space R^n is covered by the graphical solution. The ideal solution pertains to a vertical point, which is the outcome of several lines or planes intersecting, therefore the number of non-existent components is at least $n - m$ components.

2- Certain conditions that are irrelevant to the solution process might be included in the model.

3- When one of the sides of the common solution area that passes through the ideal solution point is parallel to the straight-line Z=0, the ideal solution can be a single point or an infinite number of points. Thus, when the objective function is represented by a straight line, this line will apply to the parallel side, and all of the infinitely many points on that side will be perfect solutions.

4- We say that the objective function has an endless number of acceptable solutions that offer us greater values of Z if the region of acceptable solutions is open in terms of growing the function Z, meaning that we cannot stop at a particular perfect solution.

5- When the requirements clash, there is no ideal (acceptable) solution and the zone of alternatives is an empty set (the problem is impossible to solve).

Conclusion:

This chapter addressed both the graphical approach and a method that is rarely discussed in classical operations research references: employing non-negative constraints to graphically identify the optimal solution for specific neutrosophic linear models. It should be emphasized, however, that certain neutrosophic linear models contain two variables. In particular cases, reaching the common solution region or determining the optimal solution once the common solution has been located may be challenging, hence the simplex neutrosophic method is recommended. Because the main goal is to arrive at the optimal solution, the researcher must select the appropriate strategy for the model he seeks to solve.

Chapter IV: The simplex direct neutrosophic algorithm for finding the optimal solution for linear models

Introduction.

4-1- The neutrosophic linear models set in the symmetrical form and of the Max type.

4-2- The neutrosophic linear models are in symmetric form and are of type Min.

Conclusion.

Chapter IV

The simplex direct neutrosophic algorithm for finding the optimal solution for linear models

Introduction:

Linear programming is a method for choosing decisions and approving the optimal program for independent activities while taking available resources into account Linear programming is used to solve problems with specific goals, such as maximizing profit, minimizing cost, or saving the most time or effort... etc., noting that the linear programming problem, which consists of a linear function and knowledge of a set of inequalities or equations (constraints), is characterized by the presence of a large number of acceptable non-negative solutions, and what is required is to find the optimal solution from a set of solutions. We depend on the information obtained when we explored non-negative solutions to the system of neutrosophic linear equations (in the first chapter) to arrive at this solution. Then we used the simplex method, which serves as the mathematical foundation for the direct simplex algorithm utilized to determine the optimal solution for the linear models presented in this chapter.

Direct simplex algorithm for solving neutrosophic linear models:

The direct simplex algorithm consists of three stages:

a- The stage of converting the imposed model into an equivalent systematic form.
b- The stage of converting the regular form into a basic form to obtain the non-negative basic solutions.

c- The stage of searching for the optimal solution required from among the non-negative basic solutions.

We will utilize the direct simplex method in this chapter to determine the best solution for the neutrosophic linear models described in the second chapter of this book, and we will identify the following cases:

1- The neutrosophic linear models are in symmetric form and are of type Max.
2- The neutrosophic linear models are in symmetric form and are of type Min.
3- The neutrosophic linear models are given in the general form.

Using the direct simplex method to find the optimal solution:

4-1- The neutrosophic linear models set in the symmetrical form and of type Max:

The neutrosophic linear model of type Max is written in the symmetrical form, as we mentioned in the second chapter, in the following form:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{N}a_{1n}x_n \leq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \leq Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \leq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

The study is carried out according to the following steps:

1- We write the model in standard form; we get:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0.y_1 + 0.y_2 + \cdots + 0.y_m \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \mathrm{N}a_{1n}x_n + y_1 = Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + y_2 = Nb_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + y_m = Nb_m$$

$$x_1, x_2, \dots, x_n, y_1, y_2, \dots, y_m \geq 0$$

2- We convert the model to the basic form. We can see here that the extra variables serve as a starting point for searching for the optimal solution. As a result, the model information is organized in the table below:

Variables / **basic**	x_1	x_2	……	x_n	y_1	y_1	...	y_m	**Available quantities**
y_1	Na_{11}	Na_{12}	...	Na_{1n}	1	0	...	0	Nb_1
y_2	Na_{21}	Na_{22}	...	Na_{2n}	0	1	...	0	Nb_2
...	...	...	...	...	...	...	...	...	...
y_m	Na_{m1}	Na_{m2}	...	Na_{mn}	0	0	...	1	Nb_m
objective function	Nc_1	Nc_2	...	Nc_n	0	0	0	0	$Z - 0$

Table No. (1): Basic information of the model

We have a first base consisting of the variables $y_1, y_2, \dots, y_m$, then the variables $x_1, x_2, \dots, x_n$ are non-basic variables and we move to the next step:

3- We determine the appropriate variable from the equations and insert it into the rule by studying examples of the variables in the row of the objective function Z. Since the objective function is a maximization function, we select the largest positive values in the row of the objective function. In other words, we take:

$$Max(Nc_1, Nc_2, \dots, Nc_n) = Nc_s$$

For example, let it be Nc_s corresponding to the variable x_s.

Thus, we have determined the pivot column. This means that the variable x_s will enter the base to determine the variable that will exit from the base, and therefore the pivot line. We calculate the following indicator:

$$\theta \in Min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{b_{tN}}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

The element located at the intersection of the pivot column with the pivot row is the pivot element.

- We divide the pivot row by the pivot element, we get:

$$\frac{Na_{t1}}{Na_{ts}}, \frac{Na_{t2}}{Na_{ts}}, \dots, \frac{Na_{ts-1}}{Na_{ts}}, 1, \frac{Na_{ts+1}}{Na_{ts}}, \dots, \frac{Na_{tn}}{Na_{ts}}, \dots, \frac{Nb_t}{Na_{ts}}$$

- We make all the elements of the pivot column equal to zeros, except for the pivot element, which is equal to one.
- We perform the appropriate calculations to calculate the current of the new table using the following relations:

$$Na'_{ij} = Na_{ij} - Na_{tj}\frac{Na_{is}}{Na_{ts}} = \frac{Na_{ij}Na_{ts} - Na_{tj}Na_{is}}{Na_{ts}}$$

$$Nb'_i = Nb_i - Nb_t\frac{Na_{is}}{Na_{ts}} = \frac{Nb_iNa_{ts} - Nb_tNa_{is}}{Na_{ts}}$$

$$Nc_j' = Nc_j - Nc_s \frac{Na_{tj}}{Na_{ts}} = \frac{Nc_j Na_{ts} - Nc_s Na_{tj}}{Na_{ts}}$$

We get the following table:

Variables / basic	x_1	x_2	…	x_{s-1}	x_s	…	x_n	y_1	y_1	…	y_m	Available quantities
y_1	Na'_{11}	Na'_{12}	…	Na'_{1s-1}	0	…	Na'_{1n}	1	0	…	0	Nb'_1
y_2	Na_{21}	Na_{22}	…	Na'_{2s-1}	0	…	Na_{2n}	0	1	…	0	Nb'_2
…	…	…	…	…	0	…	…	…	…	…	…	…
x_s	$\frac{Na_{t1}}{Na_{ts}}$	$\frac{Na_{t2}}{Na_{ts}}$	…	$\frac{Na_{ts-1}}{Na_{ts}}$	1	…	$\frac{Na_{tn}}{Na_{ts}}$	0	0	…	0	$\frac{Nb_t}{Na_{ts}}$
…	…	…	…	…	0	…	…	…	…	…	…	…
y_m	Na_{m1}	Na_{m2}	…	Na'_{ms-1}	0	…	Na_{mn}	0	0	…	1	Nb'_m
objective function	Nc'_1	Nc'_2	…	Nc'_{s-1}	0	…	Nc'_n	0	0	0	0	NZ'

Table No. (2) The first step in the simplex direct neutrosophic algorithm

- We apply the stopping criterion of the Simplex algorithm to the objective function row in Table No. (2).

Stopping criterion:

Because the objective function is of the maximize type, the objective function row in the table must not have any positive values (but if the objective function is of the minimization type, the objective function row in the new table must not contain any negative values). If the requirement is not fulfilled, we return to step (3) and continue the process until the stopping criterion is met and the desired optimal solution is obtained. As a result, we have new non-negative neutrosophic basic solutions as well as non-basic (free) solutions equal to zero. The ideal solution is expressed as follows:

$$\left(Nb_1', Nb_2', \dots, Nb_m', 0,0, \dots ,0\right)$$

The following table represents the final solution if the basic solutions are: $(x_1, x_2, \dots, x_m)$

variables basic	x_1	x_2	…	x_m	x_{m+1}	…	x_n	y_1	y_2	…	y_m	Available quantities
x_1	1	0	…	0	Na'_{1m+1}	…	Na'_{1n}	$N\beta_{11}$	$N\beta_{11}$	…	$N\beta_{1m}$	Nb'_1
x_2	0	1	…	0	Na'_{2m+1}	…	Na_{2n}	$N\beta_{21}$	$N\beta_{22}$	…	$N\beta_{2m}$	Nb'_2
…	…	…	…	0	0	…	…	…	…	…	…	…
x_m	0	0	…	1	$Na'_{m,m+1}$	…	Na_{mn}	$N\beta_{m1}$	$N\beta_{m2}$	…	$N\beta_{mm}$	Nb'_m
objective function	0	0	…	0	Nc'_{m+1}	…	Nc'_n	Nq_1	Nq_2	…	Nq_m	NZ'

Table No. (3) The final solution in the simplex direct neutrosophic algorithm

where $N\beta_{ii}$ and Nq_1 are the examples of the additional variables in the constraints and in the objective function after performing the aforementioned iterative operations, the optimal solution is:

$$x_1 = Nb'_1, x_2 = Nb'_2, \dots, x_m = Nb'_m$$

which gives the maximum value of the following objective function:

$$NZ' = Nc_1 Nb'_1 + Nc_2 Nb'_2 + \cdots + Nc_m Nb'_m$$

We explain the above using the following example:

Example 1:

Problem Statements Classical Values:

A corporation manufactures two types of products A and B from four raw materials F_1, F_2, F_3, F_4. The following table shows the amounts required from each of these materials to produce one unit of each of the two products, the accessible quantities of raw materials, and the profit returned from one unit of both products:

Raw Materials \ Products	Required quantity per unit		Available quantities of the raw materials
	A	**B**	
F_1	2	3	19
F_2	2	1	13
F_3	0	3	15
F_4	3	0	18
Profit Returned per unit	7	5	

Table No. (4) Classic data for the issue

Requirement:

Finding the ideal production plan that maximizes the company's profit from products A and B.

We represent the quantities produced by the product A with the symbol x_1, and the quantities created by the product B with the symbol x_2. After developing and solving the necessary mathematical model, we find that $x_1 = 5, x_2 = 3$, and hence the maximum profit $MaxZ = 50$ of monetary unit.

Problem Statements neutrosophic Values:

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two products A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Products / Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	B	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Table No. (5) Neutrosophic data for the issue

Requirement:

Finding the ideal production plan that maximizes the company's profit from products A and B.

Represent the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2, the problem will be reformulated from the neutrosophic perspective as follow:

$$NZ = [5,8]\, x_1 + [3,6]\, x_2 \longrightarrow Max$$

Constraints:

$$2x_1 + 3x_2 + y_1 = [14,20]$$
$$2x_1 + x_2 + y_2 = [10,16]$$
$$3x_2 + y_3 = [12,18]$$
$$3x_1 + y_4 = [15,21]$$

$$x_1, x_2 \geq 0$$

The preceding program must be rewritten in an equivalent way by including slack variables:

$$NZ = [5,8]\, x_1 + [3,6]\, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4 \longrightarrow Max$$

Constraints:

$$2x_1 + 3x_2 + y_1 = [14,20]$$
$$2x_1 + x_2 + y_2 = [10,16]$$
$$3x_2 + y_3 = [12,18]$$
$$3x_1 + y_4 = [15,21]$$
$$x_1, x_2, y_1, y_2, y_3, y_4 \geq 0$$

We arrange the previous information in the following table:

Variables / basic	x_1	x_1	y_1	y_2	y_3	y_4	Available quantities
y_1	2	3	1	0	0	0	[14,20]
y_2	2	1	0	1	0	0	[10,16]
y_3	0	3	0	0	1	0	[12,18]
y_4	3	0	0	0	0	1	[15,21]
objective function	[5,8]	[3,6]	0	0	0	0	$NZ - 0$

Table No. (6): The first step in the simplex method

- We note that the additional variables form an initial base consisting of the variables (y_1, y_2, y_3, y_4). Then we consider the variables (x_1, x_2) are non-basic variables and we move to the next step:
- We determine the appropriate variable from the equations and insert it into the rule by studying the examples of the variables included in the expression for the objective function NZ. Since the objective function is a maximization function, we choose the variable with the largest positive examples from the last row in the table, that is from the row of the objective function. In other words, we take

$$\text{Max}([5,8],[3,6]) = [5,8]$$

It is clear that versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables. The following calculation has been performed to indicate which basic variables should be expelled:

$$\theta \in Min\left[\frac{[14,20]}{2},\frac{[10,16]}{2},\frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 which is the pivot element, divide the elements of the row versus to y_4 yields:

$$\frac{3}{3},\frac{0}{3},\frac{0}{3},\frac{0}{3},\frac{0}{3},\frac{1}{3},\frac{[15,21]}{3} = [5,7]$$

Then we make all the elements of the pivot column equal to zero, except for the pivot element, which is equal to one. We perform the appropriate calculations using the following relations:

$$Na'_{ij} = Na_{ij} - Na_{tj}\frac{Na_{is}}{Na_{ts}} = \frac{Na_{ij}Na_{ts} - Na_{tj}Na_{is}}{Na_{ts}}$$

$$Nb_i^{'} = Nb_i - Nb_t\frac{Na_{is}}{Na_{ts}} = \frac{Nb_iNa_{ts} - Nb_tNa_{is}}{Na_{ts}}$$

$$Nc'_j = Nc_j - Nc_s\frac{Na_{tj}}{Na_{ts}} = \frac{Nc_jNa_{ts} - Nc_sNa_{tj}}{Na_{ts}}$$

We obtain the following table:

variables / basic	x_1	x_2	y_1	y_2	y_3	y_4	Available quantities
y_1	0	3	1	0	0	$\frac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\frac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	[3,6]	0	0	0	$[\frac{-8}{3},\frac{-5}{3}]$	$NZ - [25,56]$

For No. (7), the second step is the simplex method

The variable x_2 should be added to the basic variables as there is still a non-negative value in the row of the objective function (i.e., [3,6]) that is corresponding to the x_2 column. Which fundamental variable ought to be eliminated now? To get the solution to this question, take these steps:

$$\theta \in \text{Min}\left[\frac{[4,6]}{3},\frac{[0,4]}{1},\frac{[12,18]}{3}\right] = \frac{[4,6]}{3} = [\frac{4}{3},2]$$

which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3 , the required calculations yield the following table:

Variables / basic	x_1	x_2	y_1	y_2	y_3	y_4	Available quantities
x_2	0	1	$\frac{1}{3}$	0	0	$\frac{-2}{9}$	$[\frac{4}{3},2]$
y_2	0	0	$\frac{1}{3}$	1	0	$\frac{-4}{9}$	$[\frac{-4}{3},2]$
y_3	0	0	-1	0	1	$\frac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	0	[-2, -1]	0	0	$[\frac{-6}{9},-1]$	$NZ - [29,68]$

Table No. (8) Final solution

The objective function's row makes it evident that every element is either zero or a neutrosophic negative integer. This indicates that we have arrived at the optimal solution, which is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

The following results from substituting the aforementioned optimal solution into the objective maximum function:

$$MaxNZ \in [5,8].[5,7] + [3,6].\left[\frac{4}{3},2\right] = [25,56] + [4,12] = [29,68]$$

which is identical to the previous result.

Substituting the optimal solution into the constraints we find:

$$2[5,7] + 3\left[\frac{4}{3},2\right] + 0 = [14,20]$$

$$2[5,7] + \left[\frac{4}{3},2\right] + \left[\frac{-4}{3},2\right] = [10,16]$$

$$3\left[\frac{4}{3},2\right] + [8,12] = [12,18]$$

$$3[5,7] + 0 = [15,21]$$

We observe that the optimal solution satisfies all constraints.

We summarize the previous results in the following table:

Classical logic								
issue data						results		
c_1	c_2	b_1	b_2	b_3	b_4	x_1	x_2	$Max\ Z$
7	5	19	13	15	18	5	3	50
Neutrosophic logic								
issue data						results		
c_{1N}	c_{2N}	b_{1N}	b_{2N}	b_{3N}	b_{4N}	x_{1N}	x_{2N}	$Max\ NZ$
[5,8]	[3,6]	[14,20]	[10,16]	[12,18]	[15,21]	[5,7]	$[\frac{4}{3},2]$	[29,68]

Table No. (9) Comparison between the results of solving the problem, classical data, and neutrosophical data

4-2- The neutrosophic linear models are in symmetric form and are of type Min.

The neutrosophic linear model of type Min is written in the symmetrical form, as we mentioned in the second chapter, in the following form:

Find:

$$NL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Min$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n \geq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \geq Nb_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

We can search for the optimal solution by following one of the following methods:

- As we examined in the second chapter, the objective function may be converted to a function of the maximizing type by multiplying its line by (-1), which yields the optimal solution for the prior linear model. The model is then written in standard form. Since all of the extra variables are preceded by a negative sign, we can see that there isn't a pre-made beginning rule in this case. Instead, we must first look for an initial solution, then refine it until we get the optimal one by going back through the same processes.
- Additionally, we may identify the dual model, which will undoubtedly resemble a symmetry of the maximization kind. Next, we can solve it optimally as we previously did, or by applying the dual method to solve both the model and the dual model, which we shall discuss in the book's seventh chapter.
- In such models, it is preferable to use the synthetic simplex algorithm, which will be presented in the book's sixth chapter.
- Additionally, we can find the solution without changing the objective function. However, we must alter the previously described steps in one way: to find the anchor column, we must choose the element that is the most negative; this element's column serves as the anchor column. We then follow the previously stated solution, with the stopping criterion being that all of the objective function's line elements must be either positive or zero.

1- **Neutrosophic linear models are given in the general form**:

The neutrosophic linear model is written in the following general form:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow (Max\ or\ Min)$$

Constraints:

$$Na_{i1}x_1 + Na_{i2}x_2 + \cdots + Na_{in}x_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} Nb_i \quad i = 1,2,\dots,m$$

$$x_1, x_2, \dots, x_n \geq 0$$

In the beginning, the model is written in standard form; extra variables are rarely used while writing the model in basic form. Here, we observe that while some of the extra variables fit into the definition to be considered fundamental variables, some do not. Furthermore, no equivalent additional variables exist if there are certain limits on the equality type. As a result, there are no basic variables. We must first construct a foundation upon which to launch our exploration for the optimal answer. Additionally, using a simplex with an artificial base — which will be covered in this book's sixth chapter — is preferred in this situation.

Important Notes:

If some of the examples corresponding to the free variables in the objective function line in the final table for the maximization type are positive, this indicates that we have not reached the required ideal solution and must delete the free variables associated to the positive value. We return to step 2 and complete the essential steps. This is something we keep mentioning — Operations till we achieve an objective function line with just zeros or negative (positive) values. Alternatively, we may encounter one of the following scenarios:

a- There is no ideal solution because the solution region is open in the direction of increasing NZ, and we infer this from the absence of a positive element in the fulcrum.

b- There is an infinite number of optimal solutions because the levels of the objective function NZ are parallel to one of the sides or surfaces of the common solution region. We deduce this from the presence of a zero in the final row of the table of the last optimal solution, which corresponds to one of the free variables. Then, by adjusting the variable, we can achieve another optimal solution. We will receive another basic solution as a result of changing one of the basic variables.

c- If there is no optimal solution, this happens because the constraints conflict with each other. We infer this from the absence of any positive element except for the constant Nb_i in one of the lines. This indicates that, in cases where restrictions conflict, the left side takes a positive value and the right side takes a negative value.

d- After finding the optimal solution, we must ensure that it meets all of the requirements and returns the same value for the objective function by substituting the objective function and the constraints.

Conclusion:

We draw the following conclusions from the earlier research and the data presented in Table (9): when we solve using classical data, the values we obtain are specific and do not account for changes that might occur in the operating environment of the system represented by the mathematical model. In contrast, when we use neutrosophic data, we obtain areas of any indeterminate values, and this indeterminacy is more accurate, simulates reality, and takes into account most of the changes that may occur in the operating environment of the system represented by the linear mathematical model.

As a result, neutrosophical data provide us with a more general and comprehensive study than known classical data, i.e., working with known classical data is no longer sufficient at present, because the development of science and the instability in the status of the facility's work environment has placed before us a large number of cases that require quick and accurate treatment to avoid losses that the facility may be exposed to, which cannot be treated.

Neutrosophy, meanwhile, delivers greater comprehensiveness in analyzing the results and assists in getting the essential accuracy. On the one hand, we emphasize the importance of selecting the proper method to solve the model under consideration from among the algorithms presented in this book in order to save effort and time in looking for the optimal solution.

Chapter V: Modified Neutrosophic Simplex algorithm to find the optimal solution for linear models

Chapter V

Modified Neutrosophic Simplex algorithm to find the optimal solution for linear models

Introduction:

In this chapter, we present the modified neutrosophic simplex algorithm, which was developed to address a problem we encountered when using the direct simplex algorithm: the large number of calculations required in each step of the solution, which takes a long time and effort.

5-1- Steps of the modified simplex neutrosophic algorithm:

We explain the steps of the modified simplex algorithm using the following neutrosophic linear mathematical model:

$$Max\ Z = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le Nb_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le Nb_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n \le Nb_3$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le Nb_m$$

$$x_1, x_2, \dots, x_n \ge 0$$

To find the optimal solution for this linear neutrosophic model using the modified simplex algorithm.

1- We write the neutrosophic linear model in standard form, and we get the following model:

$$\max Z = c_{1N}x_1 + c_{2N}x_2 + \cdots + c_{nN}x_n + 0y_1 + 0y_2 + \cdots + 0y_m$$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + y_1 = b_{1N}$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + y_2 = b_{2N}$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + y_3 = b_{3N}$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + y_m = b_{mN}$$

$$x_1, x_2, \dots, x_n, y_1, y_2, y_3, \dots, y_m \geq 0$$

2- We convert the regular linear model to the basic form and insert the coefficients in a short table with the basic variables in the first column and the non- basic variables in the top row.

   a. We define the pivot column, which is the column corresponding to the largest positive value in the objective function row because the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s.

   b. We define the pivot row; The pivot row is determined with following indicator:

$$\theta = min\left[\frac{b_{iN}}{a_{is}}\right] = \frac{b_{tN}}{a_{ts}} > 0; \quad a_{is} > 0, b_{iN} > 0$$

Let this row be the line of the base variable y_t

Then the pivot element is the element resulting from the intersection of the fulcrum column and the pivot row, i.e., We explain the second step in the following table:

Non-Basic variables / Basic Variables	x_1	x_2	…..	x_s	….	x_n	Nb_i
y_1	a_{11}	a_{12}	……	a_{1s}	…..	a_{1n}	Nb_1
y_2	a_{21}	a_{22}	…….	a_{1s}	….	a_{2n}	Nb_2
……..	…….	……..	…….	……	…..	……	…….
y_t	a_{t1}	a_{t2}	……	a_{ts}	……	a_{tn}	Nb_t
……	……	……	….	…..	…..	……	…….
y_m	a_{m1}	a_{m2}	…….	a_{1s}	….	a_{mn}	Nb_m
NZ	Nc_1	Nc_2	……	Nc_s	…..	Nc_n	$NZ - Nc_o$

Table No. 1: Anchor element table

The pivot element is the one formed by the junction of the fulcrum column and the pivot row. The second phase is explained in the table below.

1. We put opposite the pivot element a_{ts} the reciprocal of $\frac{1}{a_{ts}}$.
2. We calculate the elements of the row corresponding to the pivot row (except the pivot row element) by dividing the elements of the pivot row by the anchor element a_{ts}
3. We calculate all the elements of the column opposite the fulcrum (except the fulcrum element) by dividing the elements of the fulcrum column by the fulcrum element a_{ts} and then multiplying them by(-1)
4. We calculate the other elements from the following relation:

$$a'_{ij} = a_{ij} - a_{tj}\frac{a_{is}}{a_{ts}} = \frac{a_{ij}a_{ts} - a_{tj}a_{is}}{a_{ts}} \qquad (1)$$

$$Nb'_i = Nb_i - Nb_t\frac{a_{is}}{a_{ts}} = \frac{Nb_i a_{ts} - Nb_t a_{is}}{a_{ts}} \qquad (2)$$

$$Nc'_j = Nc_j - Nc_s\frac{a_{tj}}{a_{ts}} = \frac{Nc_j a_{ts} - Nc_s a_{tj}}{a_{ts}} \qquad (3)$$

We obtain the following table:

Non-Basic Variables / Basic Variables	x_1	x_2	…..	y_t	….	x_n	Nb_i'
y_1	a_{11}'	a_{12}'	……	$\frac{-a_{1s}}{a_{ts}}$	…..	a_{1n}'	Nb_1'
y_2	a_{21}'	a_{22}'	…….	$\frac{-a_{2s}}{a_{ts}}$	….	a_{21}'	Nb_2'
……..	…….	……..	…….	……	…..	……	…….
x_s	$\frac{a_{t1}}{a_{ts}}$	$\frac{a_{t2}}{a_{ts}}$	……	$\frac{1}{a_{ts}}$	……	$\frac{a_{tn}}{a_{ts}}$	$\frac{Nb_t}{a_{ts}}$
……	……	……	….	…..	…..	……	…….
y_m	a_{m1}'	a_{m1}'	…….	$\frac{-a_{ms}}{a_{ts}}$	….	a_{mn}'	Nb_m'
NZ	Nc_1'	Nc_2'	……	$\frac{-Nc_s}{a_{ts}}$	…..	Nc_n'	$NZ - Nc_0'$

Table No. 2: The first stage in searching for the optimal solution

We apply the stopping criterion of the Simplex algorithm to the objective function row in Table (2) below:

Given that the objective function is of the maximize type, the objective function row in the table must not include any positive value (but if the objective function is of the minimize type, the objective function row in the new table must not contain any negative value), assuming that the criterion is of the maximize type, we return to step No. (3) and repeat the same steps until the stopping criterion is met and we obtain the desired ideal solution.

We explain the above using the following example:

Example:

A company produces two types of products A, B using four raw materials F_1, F_2, F_3, F_4. The quantities needed from each of these materials to produce one unit of each of the two products A, B, the available quantities of the raw materials, and the profit returned from one unit of both products are shown in the following table:

Products / Raw Materials	Required quantity per unit		Available quantities of the raw materials
	A	***B***	
F_1	2	3	[14,20]
F_2	2	1	[10,16]
F_3	0	3	[12,18]
F_4	3	0	[15,21]
Profit Returned per unit	[5,8]	[3,6]	

Table No. 3: Issue data

Requirement:

Finding the ideal production plan that maximizes the company's profit from products A and B.

Represent the quantities produced from the product A with the symbol x_1, and from the product B with the symbol x_2. The problem will be redefined from a neutrosophical standpoint as follows:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2$$

Constraints:

$$2x_1 + 3x_2 \leq [14,20]$$
$$2x_1 + x_2 \leq [10,16]$$
$$3x_2 \leq [12,18]$$
$$3x_1 \leq [15,21]$$

$$x_1 \geq 0, x_2 \geq 0$$

We apply the modified simplex algorithm:

1- The standard form of the previous linear model is:

$$\max Z \in [5,8]\, x_1 + [3,6]\, x_2 + 0y_1 + 0y_2 + 0y_3 + 0y_4$$

Constraints:

$$2x_1 + 3x_2 + y_1 = [14,20]$$
$$2x_1 + x_2 + y_2 = [10,16]$$
$$3x_2 + y_3 = [12,18]$$
$$3x_1 + y_4 = [15,21]$$

$$x_1, x_2 \geq 0, y_1, y_2, y_3, y_4 \geq 0$$

2- We organize the previous information in the following modified simplex table:

Non-basic var. / **Basic var.**	x_1	x_1	b_i
y_1	2	3	[14,20]
y_2	2	1	[10,16]
y_3	0	3	[12,18]
y_4	3	0	[15,21]
objective function	[5,8]	[3,6]	$Z - 0$

Table No.4: Simplex table according to the modified Neutrosophic simplex algorithm

We know $[a, b] \leq [c, d]$ if $a \leq c$ and $b \leq d$,Therefore.

It is clear that $\max([5,8], [3,6]) = [5,8]$ versus to the column of x_1 , meaning that the variable x_1 should be placed instead of one of the basic variables.

The following calculation has been performed to indicate which basic variables should be expelled:

$$\theta \in min\left[\frac{[14,20]}{2}, \frac{[10,16]}{2}, \frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

The value of θ indicates that the row versus to the variable y_4, and the element positioned in the cross row/column is 3 where is the pivot element, divide the elements of the row versus to y_4 yields.

3- We calculate the elements of the new table using relations (1), (2), (3), we obtain the following table:

Basic var. \ Non-basic var.	x_1	x_2	b_i'
y_1	$\frac{-2}{3}$	**3**	[4,6]
y_2	$\frac{-2}{3}$	1	[0,4]
y_3	0	3	[12,18]
x_1	$\frac{1}{3}$	0	[5,7]
objective function	$\left[\frac{-8}{3}, \frac{-5}{3}\right]$	[3,6]	$Z - [25,56]$

Table No.5: Table of the first step in searching for the optimal solution

4- We apply a stopping criterion in the algorithm. We find:

There is still a non-negative number in the objective function's row. (i.e., [3,6]).

This means that we have not yet reached the optimal solution, so we repeat the previous steps as follows:

Where is versus to the x_2 column, this leads to the fact that the variable x_2 should be entered into the basic variables. The question now is, which fundamental variable should be eliminated?

To solve this question, perform the following calculation:

$[a, b] \leq [c, d]$ if $a \leq c$ and $b \leq d$. Therefore,

$$\theta \in \min\left[\frac{[4,6]}{3}, \frac{[0,4]}{1}, \frac{[12,18]}{3}\right] = \frac{[4,6]}{3} = [\frac{4}{3}, 2]$$

which is versus to the slack variable y_1, the pivot element equal 3, hence the row versus to y_1 should be divided by 3. The necessary computations result in the following tables:

Non-basic var. / basic var.	x_1	x_2	b_i'
x_2	$\frac{-2}{9}$	$\mathbf{\frac{1}{3}}$	$[\frac{4}{3},2]$
y_2	$\frac{-4}{9}$	$\frac{-1}{3}$	$[\frac{-4}{3},2]$
y_3	$\frac{2}{3}$	$\mathbf{-1}$	$[8,12]$
x_1	1	0	$[5,7]$
objective function	$[-6,-1]$	$[-2,-1]$	$Z-[29,68]$

Table No. 6: Final solution table

We apply the algorithm stopping criterion. We discover that the condition has been satisfied, and hence we have arrived at the optimal solution.

The optimal solution for the linear model is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

The value of the objective function corresponds to:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3},2\right] = [25,56] + [4,12] = [29,68]$$

It is clear from the row of the objective function that all the elements are neutrosophic negative numbers, this means that we have reached to the optimal solution is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function, the result is:

This means that the company must produce the quantity $x_1^* \in [5,7]$ of product A and quantity $x_2^* \in \left[\frac{4}{3}, 2\right]$ of product B, thereby achieving a maximum profit of:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3}, 2\right] = [25,56] + [4,12] = [29,68]$$

To compare between the modified Simplex method and the direct Simplex method, we solved the same example using the direct Simplex algorithm. Below are the solution tables:

Non-basic var. / Basic var.	x_1	x_1	y_1	y_2	y_3	y_4	b_i
y_1	2	3	1	0	0	0	[14,20]
y_2	2	1	0	1	0	0	[10,16]
y_3	0	3	0	0	1	0	[12,18]
y_4	3	0	0	0	0	1	[15,21]
objective function	[5,8]	[3,6]	0	0	0	0	$Z - 0$

Table No. 7: Simplex table according to the direct neutrosophic simplex algorithm

Non-basic var. / Basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
y_1	0	**3**	1	0	0	$\frac{-2}{3}$	[4,6]
y_2	0	1	0	1	0	$\frac{-2}{3}$	[0,4]
y_3	0	3	0	0	1	0	[12,18]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	[3,6]	0	0	0	$[\frac{-8}{3}, \frac{-5}{3}]$	$Z - [25,56]$

Table No. 8: Table of the first step in searching for the optimal solution

Non-basic var. / basic var.	x_1	x_2	y_1	y_2	y_3	y_4	b_i
x_2	0	1	$\frac{1}{3}$	0	0	$\frac{-2}{9}$	$[\frac{4}{3},2]$
y_2	0	0	$\frac{1}{3}$	1	0	$\frac{-4}{9}$	$[\frac{-4}{3},2]$
y_3	0	0	-1	0	1	$\frac{2}{3}$	[8,12]
x_1	1	0	0	0	0	$\frac{1}{3}$	[5,7]
objective function	0	0	[-2, -1]	0	0	$[\frac{-6}{9},-1]$	$Z-[29,68]$

Table No. 9: Final solution table

The row of the objective function clearly shows that all of the components are either zero or neutrosophic negative values, indicating that we have arrived at the optimal solution, which is:

$$x_1^* \in [5,7], x_2^* \in \left[\frac{4}{3},2\right], y_2^* \in \left[\frac{-4}{3},2\right], y_3^* \in [8,12], y_1^* = y_4^* = 0$$

Substitute the above optimal solution into the objective maximum function:

$$\max Z \in [5,8].[5,7] + [3,6].\left[\frac{4}{3},2\right] = [25,56] + [4,12] = [29,68]$$

Conclusion:

We are able to observe from the previous study that we obtained the same optimal solution as when we used the direct simplex method, but with a much smaller number of calculations, as shown by comparing the solution tables using the modified simplex method, Tables No. (4), No. (5), No. (6), with solution tables using the direct simplex method, Tables No. (7), No. (8), No. (9). To save time and effort, we emphasize the importance of adopting the modified simplex approach to identify the best solution for linear models, especially when there are a lot of variables and restrictions in the model.

Chapter VI: Finding a rule solution for linear models using artificial variables

Introduction.

6-1- Artificial base simplex algorithm.

6-2- Processing the model and all constraints of type equals.

6-3- Processing model constraints mixed.

Conclusion.

Chapter VI

Finding a rule solution for linear models using artificial variables

Introduction:

In this chapter, we show the simplex approach with neutrosophic artificial variables, which is favored for usage in linear models when there is no ready-made base to utilize to find the best solution.

As an initial phase in the study, artificial variables are introduced to the constraints in a number equal to the number of constraints that do not contain a basic variable. Concerning the optimal solution, we must eliminate all artificial variables and convert them to non-basic variables so that they take the value zero and therefore do not impact the linear model's perfect solution. The following study is used to explain the preceding:

6-1- Artificial base simplex algorithm:

The end result of solving linear models is to find the optimal solution among a collection of acceptable solutions. This is accomplished by the use of a basic solution that is enhanced using the direct simplex method, and it consists of three main phases.

1. The stage of converting the imposed model into an equivalent systematic form.
2. The stage of converting the regular form into a basic form to obtain the non-negative basic solutions.

3. The stage of searching for the ideal solution required from among the non-negative basic solutions.

As a result, the process of searching for the optimal solution does not begin until a basic solution is obtained. However, in many linear models, obtaining the basic solution is difficult, so the simplex method with an artificial base was proposed, in which a base is formed consisting of a set of artificial variables that are non-negative is added to constraints that do not contain a basic variable, thus obtaining the basic solution. Then we use the direct simplex technique to refine it till we have the best answer. Using neutrosophic principles, we will reformulate the simplex method with an artificial basis to discover the best solution for linear models when obtaining a basis solution is problematic.

Text of the issue:

Find the optimal solution for the following neutrosophic linear model:

$$Max\ Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n + NC_0$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots..$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

where:

$$NC_j = C_j \pm \varepsilon_j\ ,\quad Nb_i = b_i \pm \delta_i,\quad a_{ij},$$

$$j = 1,2, \dots, n\ , i = 1,2, \dots, m$$

are constants having set or interval values according to the nature of the given problem, x_j are decision variables. It is worth noting that the index N subscribes to coefficients with neutrosophic values. The objective function coefficients $NC_1, NC_2, \dots, NC_n$, They are neutrosophic values of the form

That is, $Nc_j \in [\lambda_{j1}, \lambda_{j2}]$, where $\lambda_{j1}, \lambda_{j2}$ are the upper and the lower bounds of the objective variables x_j respectively, $j = 1,2, \dots, n$. the right-hand side of the inequality constraints $Nb_1, Nb_2, \dots, Nb_m$.

$Nb_i \in [\mu_{i1}, \mu_{i2}]$, here, μ_{i1}, μ_{i2} are the upper and the lower bounds of the constraint $i = 1,2, \dots, m$.

In the previous model, we note that the number of variables is n and the number of constraints is m, and this model is in the standard form.

We move to the second stage, which is to find a basic solution. We apply the simplex method with an artificial basis in this case, where is represented by:

1- We create an artificial basic form from the standard form by adding a non-negative artificial variable ε_i to the left side of each of the constraint equations. Thus, we form a base consisting of the non-negative variables $\varepsilon_1, \varepsilon_2, \dots, \varepsilon_m$
2- Because the artificial variables are put into constraints that were initially linear equations, these variables must have the value zero in order for the linear constraints to be unaffected.
3- As a result, we must shift all of them off the base until they become non-base variables, and we utilize the direct simplex technique to do this.

4- We introduce these variables into the objective function with the value M (where M is a sufficiently large positive number that is at least greater than any $|Nc_j|$) and preceded by a minus sign (because the objective function is a maximization function) in order to avoid transferring them back to the base variables.

5- We obtain the following basic form of the neutrosophic linear model:

$$Max\, Z = NC_1x_1 + NC_2x_2 + \cdots + NC_nx_n - M\varepsilon_1 - M\,\varepsilon_2 - \cdots - M\,\varepsilon_m + NC_0$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n + \varepsilon_1 = Nb_1$$
$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n + \varepsilon_2 = Nb_2$$
$$a_{31}x_1 + a_{32}x_2 + \cdots + a_{3n}x_n + \varepsilon_3 = Nb_3$$
$$\dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots \dots$$
$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n + \varepsilon_m = Nb_m$$

$$x_j \geq 0\ , \varepsilon_i > 0\,, Nb_i > 0\,; j = 1,2,\dots,n\ and\ i = 1,2,\dots,m$$

6- After obtaining the basic solution, we use the direct simplex algorithm to improve this solution to reach the optimal solution. Therefore, we arrange the previous information in a table as follows:

Basic \ **Variables**	x_1	x_2	....	x_n	ε_1	ε_2	....	ε_m	b_i
ε_1	a_{11}	a_{12}	....	a_{1n}	1	0	....	0	b_1
ε_2	a_{21}	a_{22}	...	a_{2n}	0	1	0	0	b_2
....	...	...	...	...	...	...	...	...	...
ε_m	a_{m1}	a_{m2}	...	a_{mn}	0	0	...	1	b_m
Objective function	NC_1	NC_2	....	NC_n	$-M$	$-M$	...	$-M$	$Z - NC_0$

Table No. (1) General data of the model

We eliminate the artificial variables. Here we study the constants b_i corresponding to the artificial variables and select the largest of them, let it be b_t corresponding to the variable ε_t and we consider its row to be the pivot row. Then we determine the pivot element in it by dividing the elements of the objective function row (elements NC_j) by the elements of the ε_t row and then we take the smallest positive ratio θ where:

$$\theta = \underset{j}{\text{Min}}\left[\frac{NC_j}{a_{tj}} > 0\right] = \frac{NC_s}{a_{ts}}$$

where $a_{tj} > 0$, then the pivot element is a_{ts}, and we exchange the variables x_s and ε_t, according to the direct neutrosophic Simplex algorithm instructions.

We repeat step (7) until all artificial variables are removed and a normal basis consisting of the basic variables is obtained.

After eliminating the artificial variables, we revert to using the direct neutrosophic simplex technique.

6-2- Processing the model and all constraints of type equals:

Using the following example, we show how to use the simplex method with a synthetic rule to discover the best solution for linear models with all equal constraints:

Example 1:

Find the ideal solution for the following linear model:

$$Max\ Z = -12x_1 + [6,9]x_2 + 3x_3$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4,6]$$

$$6x_1 - 3x_2 + 3x_3 = [-12, -9]$$

$$x_1, x_2, x_3 \geq 0$$

Solution:

1- We convert the model to the standard form, multiply the second equation by (-1) and we obtain the following model:

Find a rule solution for the following neutrosophic linear model:

$$Max\, Z = -12x_1 + [6{,}9]x_2 + 3x_3$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4{,}6]$$

$$-6x_1 + 3x_2 - 3x_3 = [9{,}12]$$

$$x_1, x_2, x_3 \geq 0$$

2- We add the artificial variables and enter them into the objective function with a capital letter M preceded by a minus sign. Here we take $M = 15$.

Find a rule solution for the following neutrosophic linear model:

$$Max\, Z = -12x_1 + [6{,}9]x_2 + 3x_3 - 15\varepsilon_1 - 15\varepsilon_2$$

Constraints:

$$8x_1 - x_2 + 4x_3 = [4{,}6]$$

$$-6x_1 + 3x_2 - 3x_3 = [9{,}12]$$

$$x_1, x_2, x_3, \varepsilon_1, \varepsilon_2 \geq 0$$

We arrange the previous information in the following table:

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	8	-1	4	1	0	$[4,6]$
ε_2	-6	3	-3	0	1	$[9,12]$
Objective function	-12	$[6,9]$	3	-15	-15	$Z-0$

Table No. (2) :Artificial base table

Since the rule is artificial, we study the constants b_i and find that the largest of them belongs to the interval $[9,12]$ corresponding to the variable ε_2. Therefore, we divide the objective function row by the positive elements in the ε_2 row and calculate the index θ, and we find that:

$$\theta = \underset{j}{\text{Min}}\left[\frac{[6,9]}{3}\right] = \frac{[6,9]}{3}$$

Thus, the pivot element is (3) corresponding to x_2. Therefore, we replace x_2 with ε_2, then the variable x_2 becomes a basice variable and ε_2 comes out of the base. We perform the necessary calculations and obtain the following table:

Basic \ Variables	x_1	x_2	x_3	ε_1	ε_2	b_i
ε_1	6	0	3	1	$\frac{1}{3}$	$[7,10]$
x_2	-2	1	-1	0	$\frac{1}{3}$	$[3,4]$
Objective function	$[0,6]$	0	$[9,12]$	-15	$[-18,-17]$	$Z-[18,36]$

Table No. (3): The first change table in the base

Because the artificial variable ε_1 remains in the base, we do another replacement, this time using the pivot line as the line opposite it. We compute the index θ to identify the pivot column and find:

$$\theta = \underset{j}{\text{Min}} \left[\frac{[0,6]}{6}, \frac{[9,12]}{3} \right] \in \frac{[0,6]}{6}$$

Thus, the pivot element is (6) corresponding to x_1, so we move x_1 to the base instead of ε_1, so we get the following table:

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
x_1	1	0	$\frac{1}{2}$	$\frac{3}{6}$	$\frac{1}{18}$	$\left[\frac{7}{6}, \frac{10}{6}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3}, \frac{22}{3}\right]$
Objective function	0	0	9	$[-18, -15]$	$\left[-18, \frac{-50}{3}\right]$	$Z - [18,46]$

Table No. (4): The second change in the base

We can see from the preceding table that the basis variables x_1 and $, x_2$ have an initial solution for the linear model, which gives us the following rule solution:

$$\left(x_1 \in \left[\frac{7}{6}, \frac{10}{6}\right], x_2 \in \left[\frac{16}{3}, \frac{22}{3}\right], x_3 = 0, \varepsilon_1 = 0, \varepsilon_2 = 0 \right)$$

But it is clear from the table that this solution is not the ideal solution because in the objective function row there is a positive value corresponding to the variable x_3. Therefore, we apply the direct simplex algorithm to improve the basic solution. We obtain the ideal solution from the following table:

Variables / Basic	x_1	x_2	x_3	ε_1	ε_2	b_i
x_3	2	0	1	1	$\frac{1}{9}$	$\left[\frac{7}{3}, \frac{10}{3}\right]$
x_2	0	1	0	$\frac{1}{3}$	$\frac{4}{9}$	$\left[\frac{16}{3}, \frac{22}{3}\right]$
objective function	-18	0	0	$[-27, -24]$	$\left[-19, \frac{-53}{3}\right]$	$Z - [39,76]$

Table No. (5): The optimal solution for the model

Optimal solution for the linear model:

$$x_1 = 0, x_2 \in \left[\frac{16}{3}, \frac{22}{3}\right], x_3 \in \left[\frac{7}{3}, \frac{10}{3}\right], \varepsilon_1 = 0, \varepsilon_2 = 0$$

In this solution, the objective function takes its greatest value, which is:

$$Z \in [39,76]$$

By transferring the constraints and the objective function statement, we can verify the solution. We remark that the values in the ideal solution of the preceding linear model are neutrosophic values.

6-3- Processing model constraints mixed:

Using the following example, we show how to use the simplex method with a synthetic rule to discover the best solution for linear models with mixed constraints:

Example 2:

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3$$

Constraints:

$$x_1 - 2x_2 + x_3 \leq [3,7]$$

$$-4x_1 + x_2 + 2x_3 \geq [9,6]$$

$$2x_1 - x_3 = 1$$

$$x_1, x_2, x_3 \geq 0$$

Converting this model to standard form the problem becomes:

Find the ideal solution for the following linear model:

$$Min\, Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$

$$-4x_1 + x_2 + 2x_3 - y_2 = [9,6]$$

$$2x_1 - x_3 = 1$$

$$x_1, x_2, x_3, y_1, y_2 \geq 0$$

The variable y_1 in the first constraint is a basic variable, and since there are no other basic variables, we add artificial variables to the second and third restrictions and enter them into the objective function in sufficiently positive times because the model is a minimization model, and thus we obtain the following basic form:

Because the variable y_1 in the first constraint is a basic variable, and there are no other basic variables, we add artificial variables to the second and third constraints and enter them into the objective function in sufficiently positive times because the model is a minimization model, yielding the basic form:

Find the ideal solution for the following linear model:

$$Min\,Z = -3x_1 + [8,10]x_2 + [0,6]x_3 + 0y_1 + 0y_2 + 12\varepsilon_1 + 12\varepsilon_2$$

Constraints:

$$x_1 - 2x_2 + x_3 + y_1 = [3,7]$$

$$-4x_1 + x_2 + 2x_3 - y_2 + \varepsilon_1 = [9,6]$$

$$2x_1 - x_3 + \varepsilon_2 = 1$$

$$x_1, x_2, x_3, y_1, y_2, \varepsilon_1, \varepsilon_2 \geq 0$$

To insert the basic variables and delete the artificial variables from the base, we use the identical procedures as in Example 1.

We apply the direct simplex method to determine the best solution after acquiring the optimal solution.

Important Notes:

1- If the row ε_i does not include a positive element and $b_t > 0$, this indicates a conflict of constraints and the problem is unsolvable.

2- If we cannot find a positive ratio $\frac{NC_j}{a_{tj}}$, we calculate the largest negative ratio θ' where:

$$\theta' = Max\left[\frac{NC_j}{a_{tj}} < 0\right] = \frac{NC_s}{a_{ts}}$$

where $a_{tj} > 0$, so a_{ts} is the pivot element and it is definitely a positive element.

Conclusion:

In the previous research, we introduced the synthetic simplex method, which is an essential method for determining the optimal solution for neutrosophic linear models in the event that a rule solution cannot be found. We found that the optimal solution that we obtained has neutrosophic values, indeterminate values, perfectly defined, belonging to a field that represents its minimum. The linear model can reflect the maximum value of the objective function, which is proportionate to the conditions surrounding the system's operational environment.

Chapter VII: Neutrosophic Dual Linear Models and the Binary Algorithm

Chapter VII

Neutrosophic Conjugate Linear Models and the Dual Algorithm

Introduction:

In our practical life, we encounter many problems that are formulated in the form of linear mathematical models consisting of an objective function and a set of constraints in the form of equations or inequalities .The linear model is stated in a number of formulas that differ according to the kind of objective function and the form of the constraints. The linear model formulas are described in the second chapter of this book, and as previously mentioned, each of these formulas has a purpose. For example, when we want to find the optimal solution for a linear model, we must first put it in the standard form. One of the most significant theories in linear programming, the dual theory, uses symmetric formulas as we previously mentioned. Its basic tenet is that for every linear model, there exists a conjugate linear model. This is because solving the original linear model yields a solution to the dual model, meaning that solving the linear programming model actually produces solutions for two linear models.

In this chapter, we present a study of the neutrosophic dual models and the binary simplex algorithm that works to find the optimal solution for the two models. The original and the dual ones at the same time. This algorithm is significant because it is used in numerous operations research fields such as integer programming techniques, certain nonlinear programming algorithms, and sensitivity analysis in linear programming.

7-1- Neutrosophic companion models:

7-1-1- The matrix form of the neutrosophic conjugate models:

To use matrices to discover the related model for a given neutrosophic linear model, we first put the neutrosophic linear model in symmetrical form. As we learned in the second chapter, the linear model is in the symmetrical form if all variables are constrained to be non-negative and if all constraints are given in the form of inequalities (and the inequalities of the maximization model constraints must be written in the form (≤ less than or equal to), whereas the inequalities of the minimization model constraints must be written in the form (≥greater than or equal to). then the linear model is written in one of two cases:

The first case: The original model is symmetrical and of the maximization type:

Original model:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Max$$

Constraints:

$$NA\,\mathrm{X} \le NB$$

$$\mathrm{X} \ge 0$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \dots Na_{1n} \\ Na_{21} & Na_{22} \dots Na_{2n} \\ \dots & \dots \dots \dots \dots \dots \\ Na_{m1} & Na_{m2} \dots Na_{mn} \end{bmatrix} \quad NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} \quad NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \quad \mathrm{Y} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

The dual linear model:

Find:

$$NL = NB\,\mathrm{Y} \longrightarrow Min$$

Constraints:

$$NA^T\mathrm{Y} \geq NC$$

$$\mathrm{Y} \geq 0$$

where:

$$NA^T = \begin{bmatrix} Na_{11} & Na_{21} \ldots Na_{m1} \\ Na_{12} & Na_{22} \ldots Na_{m2} \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ Na_{1n} & Na_{2n} \ldots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \mathrm{Y} = \begin{bmatrix} y_1 \\ y_2 \\ .. \\ y_m \end{bmatrix}$$

The second case: The model is symmetrical and miniaturized:

Original model:

Find:

$$NZ = NC\,\mathrm{X} \longrightarrow Min$$

Constraints:

$$NA\mathrm{X} \geq NB$$

$$\mathrm{X} \geq 0$$

where:

$$NA = \begin{bmatrix} Na_{11} & Na_{12} \ldots Na_{1n} \\ Na_{21} & Na_{22} \ldots Na_{2n} \\ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\ Na_{m1} & Na_{m2} \ldots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix} NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix} \mathrm{X} = \begin{bmatrix} x_1 \\ x_2 \\ .. \\ x_n \end{bmatrix}$$

The dual linear model:

Find:

$$NL = NB\ \mathrm{Y} \longrightarrow Max$$

Constraints:

$$NA^T \mathrm{Y} \leq NC$$

$$\mathrm{Y} \geq 0$$

where:

$$NA^T = \begin{bmatrix} Na_{11} & Na_{21} \dots Na_{m1} \\ Na_{12} & Na_{22} \dots Na_{m2} \\ \dots \dots \dots \dots \dots \dots \dots & \\ Na_{1n} & Na_{2n} \dots Na_{mn} \end{bmatrix} NB = \begin{bmatrix} Nb_1 \\ Nb_2 \\ .. \\ Nb_m \end{bmatrix}\ NC = \begin{bmatrix} Nc_1 \\ Nc_2 \\ .. \\ Nc_n \end{bmatrix}\ \mathrm{Y} = \begin{bmatrix} y_1 \\ y_2 \\ .. \\ y_m \end{bmatrix}$$

We summarize the process of finding neutrosophic dual models using matrices in the following steps:

1. We define a new non-negative variable for each constraint of the original model
2. We make the wind (cost) vector in the original model a column vector of constants in the companion model
3. We make the constants column vector in the original model the cost (profit) vector in the companion model
4. We transform a matrix of the parsimony of the variables of the constraints in the original model into the parsimony of the variables in the dual model
5. We reverse the direction of the constraint inequalities
6. We reverse the direction of the examples, that is, we change the increase to the maximum limit to a decrease to the minimum limit, and vice versa.

7-1-2- Finding neutrosophic dual models using the double table:

We previously found that we can write linear models in three forms:

The matrix form is as shown in the previous paragraph.

The following short form:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow (Max \ \ or \ \ Min)$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} b_i \pm \delta_i \quad ; \ i = 1,2, \dots, m$$

$$x_j \geq 0$$

The detailed figure follows:

Find:

$$NZ = Nc_1 x_1 + Nc_2 x_2 + \cdots + Nc_n x_n \longrightarrow (Max \ \ or \ \ Min)$$

Constraints:

$$Na_{i1} x_1 + Na_{i2} x_2 + \cdots + Na_{in} x_n \begin{pmatrix} \geq \\ \leq \\ = \end{pmatrix} Nb_i \quad ; i = 1,2, \dots, m$$

$$x_1, x_2, \dots, x_n \geq 0$$

To find the dual model, we put the neutrosophic linear model in the symmetrical form, and here we distinguish two cases:

First case:

The original model is symmetrical and of the maximization type:

$$NZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Max$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \leq b_i \pm \delta_i \quad ; \; i = 1,2, \dots, m$$

$$x_j \geq 0$$

Second case:

The model is symmetrical and miniaturized:

$$NL = \sum_{j=1}^{n} (c_j \pm \varepsilon_j) x_j \longrightarrow Min$$

Constraints:

$$\sum_{j=1}^{n} Na_{ij} x_j \geq b_i \pm \delta_i \quad ; \; i = 1,2, \dots, m$$

$$x_j \geq 0$$

In both cases, we have $x_j \geq 0$, which are the decision variables, unknown values that we obtain after solving the linear model.
$Nc_j = c_j \pm \varepsilon_j$ and $Nb_i = b_i \pm \delta_i$ and $Na_{ij} = a_{ij} \pm \mu_{ij}$, where: ($j = 1,2, \dots, n$, $i = 1,2, \dots, m$) are neutrosophic values, which are undefined values with a margin of error that are determined by the characteristics of the situation as it is represented by the linear model.

7-1-3- Constructing neutrosophic dual linear models using tables:

The following procedures are followed in order to create a double table for the original and dual models in order to construct linear neutrosophic models utilizing tables:

1. The coefficients of the objective function in the original model are the constants column in the companion model, and the constants column in the original model are the coefficients of the objective function in the companion model.
2. We invert the signs of the inequalities of the constraints (if they were in the original model of type (=), they become in the dual model of type =).
3. We change the objective from maximizing in the original model to minimizing in the dual model.
4. We place each constraint (row) in the original model corresponding to a column in the dual model, meaning there is one variable for each constraint in the original model.
5. The variables in the original model and the dual model satisfy the non-negative constraints.

We explain the above using the following two cases:

First case:

The original model is symmetrical and of the maximization type:

First case: The original model is symmetrical and of the maximization type

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \text{N}a_{1n}x_n \leq Nb_1$$
$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \leq Nb_2$$
$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$
$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \leq Nb_m$$
$$x_1, x_2, \ldots, x_n \geq 0$$

The binary table for the original model and the dual model is as follows:

		Original model			
		objective function / **constants**	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$		***Max*** / **Constants column**
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	$\leq$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	$\leq$	Nb_2
	…	…	… … … … … … … … … … … … … … …	$\leq$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	$\leq$	Nb_m
		Non-negative constraints	$x_1, x_2, \ldots, x_n$	$\geq$	0
		Dual model			
		Objective function / **constraints**	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m$		***Min*** / **Constants column**
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	$\geq$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	$\geq$	Nc_2
		…	… … … … … … … … … … … … … … …	$\geq$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	$\geq$	Nc_n
		Non-negative constraints	$y_1, y_2, \ldots, y_m$	$\geq$	0

Table No. (1) Objective follower of the maximization type

The second case: The original model is symmetrical and of the reduction type:

Find:

$$NL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n \longrightarrow Min$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + \text{N}a_{1n}x_n \geq Nb_1$$

$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n \geq Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \ldots, x_n \geq 0$$

The binary table for the original model and the dual model is as follows:

		Original model			
		objective function / constants	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$		Min / **Constants column**
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n$	$\geq$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n$	$\geq$	Nb_2
	…	…	… … … … … … … … … … … … … …	$\geq$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n$	$\geq$	Nb_m
		Non-negative constraints	$x_1, x_2, \ldots, x_n$	$\geq$	0
		Dual model			
		objective function / constants	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m$		Max / **Constants column**
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m$	$\leq$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m$	$\leq$	Nc_2
		…	… … … … … … … … … … … … … …	$\leq$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m$	$\leq$	Nc_n
		Non-negative constraintS	$y_1, y_2, \ldots, y_m$	$\geq$	0

Table No. (2) objective follower in the original model of the reduce type

7-2- Formulation of the binary neutrosophic algorithm.

The binary simplex algorithm is neutrosophic (for both the original and dual models). This approach allows us to simultaneously identify the two optimal solutions for the dual and original models. The modified simplex algorithm that will be used inside each step of the binary algorithm must be mentioned before beginning the binary simplex algorithm.

Modified simplex algorithm:

In the modified Simplex algorithm, after converting the regular linear model to the basic form, we place the coefficients in a short table whose first column includes the basic variables and whose top row includes the non-basic variables only. We define the pivot column, which is the column corresponding to the largest positive value in the objective function row if the objective function is a maximization function (but if the objective function is a minimization function, it is the column corresponding to the most negative values). Let this column be the column of the variable x_s. We define the pivot row. The pivot row is determined. Through the following indicator:

$$N\theta = min\left[\frac{Nb_i}{Na_{is}}\right] = \frac{Nb_t}{Na_{ts}} > 0; \quad Na_{is} > 0, Nb_i > 0$$

Let this line be the line of the base variable y_t, then the anchor element is the element resulting from the intersection of the anchor column and the anchor line, i.e., Na_{ts}. Then we calculate the new elements corresponding to the anchor line and the anchor column as follows:

1. We put opposite the pivot element Na_{ts} the reciprocal of $\frac{1}{Na_{ts}}$
2. We calculate the elements of the row corresponding to the pivot row (except the pivot element) by dividing the elements of the pivot row by the pivot element Na_{ts}
3. We calculate all the elements of the column opposite the pivot (except the pivot element) by dividing the elements

of the pivot column by the pivot element Na_{ts} and then multiplying them by (-1)

4. We calculate the other elements from the following relation:

$$Nb_i' = Nb_i - Nb_t \frac{Na_{is}}{Na_{ts}} = \frac{Nb_i Na_{ts} - Nb_t Na_{is}}{Na_{ts}}$$

$$Na_{ij}' = Na_{ij} - Na_{tj} \frac{Na_{is}}{Na_{ts}} = \frac{Na_{ij} Na_{ts} - Na_{tj} Na_{is}}{Na_{ts}}$$

$$Nc_j' = Nc_j - Nc_s \frac{Na_{tj}}{Na_{ts}} = \frac{Nc_j Na_{ts} - Nc_s Na_{tj}}{Na_{ts}}$$

On the objective function row, we use the stopping criterion of the direct Simplex algorithm. If the objective function is of the maximum type, the objective function row in the table must not contain any positive value. However, if the objective function is of the minimization function, the objective function row in the new table must not have any negative values. If the condition is not fulfilled, we continue the process until the stopping criterion is met and the desired ideal solution is obtained.

7-2-1- Steps of the binary simplex algorithm:

a. We write the two models in basic form by adding or subtracting additional variables or using synthetic variables and isolating the non-restricting variables.

Basal form of the original model:

Find:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m \longrightarrow Max$$

Constraints:

$$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1 = Nb_1$$
$$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2 = Nb_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m = Nb_m$$

$$x_j \geq 0 \ ; \mathrm{j} = 1,2, \ldots, \mathrm{n}$$

$$u_i \geq 0 \ ; \mathrm{i} = 1,2, \ldots, \mathrm{m}$$

Here we do not require that $Nb_i \geq 0$.

Basic form of the dual model:

Find:

$$NL = Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n \longrightarrow Min$$

Constraints:

$$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1 = Nc_1$$

$$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2 = Nc_2$$

$$\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n = Nc_n$$

$$y_i \geq 0 \ ; i = 1,2, \ldots, m$$

$$v_j \geq 0 \ ; \mathrm{j} = 1,2, \ldots, \mathrm{n}$$

Here we do not require that $Nc_j \geq 0$.

The coefficients in both models are the same, and the matrix of instances in the dual model is the transpose of the matrix of instances in the original model. The two models are written in the binary table below:

		Original model			
		objective function / **constants**	$Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n + 0u_1 + 0u_2 + \cdots + 0u_m$		Max / **Constants column**
Dual vibrable	y_1	1	$Na_{11}x_1 + Na_{12}x_2 + \cdots + Na_{1n}x_n + u_1$	$=$	Nb_1
	y_2	2	$Na_{21}x_1 + Na_{22}x_2 + \cdots + Na_{2n}x_n + u_2$	$=$	Nb_2
	…	…	… … … … … … … … … … … … … …	$=$	…
	y_m	m	$Na_{m1}x_1 + Na_{m2}x_2 + \cdots + Na_{mn}x_n + u_m$	$=$	Nb_m
		Non-negative constraints	$x_1, x_2, \dots, x_n, u_1, u_2, \dots, u_m$	$\geq$	0
		Dual model			
		objective function / **constants**	$Nb_1y_1 + Nb_2y_2 + \cdots + Nb_iy_m + 0v_1 + 0v_2 + \cdots + 0v_n$		$\boldsymbol{Min}$ / **Constants column**
		1	$Na_{11}y_1 + Na_{21}y_2 + \cdots + Na_{m1}y_m - v_1$	$=$	Nc_1
		2	$Na_{12}y_1 + Na_{22}y_2 + \cdots + Na_{m2}y_m - v_2$	$=$	Nc_2
		…	… … … … … … … … … … … … … …	$=$	…
		n	$Na_{1n}y_1 + Na_{2n}y_2 + \cdots + Na_{mn}y_m - v_n$	$=$	Nc_n
		Non-negative constraints	$y_1, y_2, \dots, y_m, v_1, v_2, \dots, v_n$	$\geq$	0

Table No. (3) Standard format for the original and companion models

b. We place the variables and coefficients of the original model in the modified simplex table, and we place the variables of the dual model outside the table as follows:

			basic variables with a (-) sign in the dual model				
			$-v_1$	$-v_2$	…	$-v_n$	
		Non-basic vibrable / **basic vibrable**	x_1	x_2	…	x_n	**Follow the objective of the dual model** NB_i
Non-basic vibrable of the dual model	y_1	u_1	Na_{11}	Na_{12}	…	Na_{1n}	Nb_1
	y_2	u_2	Na_{21}	Na_{22}	…	Na_{2n}	Nb_2
	…	…	…	…	…	…	…
	y_m	u_m	Na_{m1}	Na_{m2}	…	Na_{mn}	Nb_m
		objective of the original model	Nc_1	Nc_2	…	Nc_n	$L - 0 \rightarrow Min$ / $Z - 0 \rightarrow Max$

Table No. (4): The binary table for the original and dual models according to the modified Simplex algorithm

7-2-2- Binary simplex algorithm for the original and dual models:

From the modified simplex algorithm of the original model, we obtain the optimal solution of the original model when all the elements are in the last row (the objective function row of the original model) $Nc_j \leq 0 \ ; \mathrm{j} = 1,2, \dots, \mathrm{n}$ and at the same time all the elements are in the last column (associated objective function column) $Nb_i \geq 0 \ ; i = 1,2, \dots, m$ and we get the optimal solution for the dual model when all elements in the last column (associated objective function column) are $Nb_i \geq 0 \ ; i = 1,2, \dots, m$ and at the same time the last row (the objective function row of the original model)

$Nc_j \leq 0 \ ; \mathrm{j} = 1,2, \dots, \mathrm{n}.$

(Because it will correspond to $Nc_j = -v_j$) which are the two conditions Same for both models. Therefore, when searching for the optimal solution for both models together, we must work to make all elements $Nb_i \geq 0\ ; i = 1,2, \dots, m$ and to make all elements $Nc_j \leq 0\ ; \text{j} = 1,2, \dots, \text{n}$, to achieve this we rely on one of the two models, put its variables and coefficients in a table, and place the dual model in an external frame of that table. In general, we find that the necessity of placing the two models in a short table does not allow us to eliminate the negative constants on the right side, and therefore the general case of the previous binary table can include negative constants $Nb_i < 0$, and the elements of the last row can include positive elements $Nc_j > 0$, so when searching for the optimal solution for the two models, we must work to address these elements based on one of the two models.

Depending on the original model, we do this in two stages:

First stage:

We make the constant Nb_i non-negative, which corresponds to obtaining a non-negative basic solution for the original model.

Second stage:

We make every element of the objective function row non-positive (maximization in the case of the objective function), which corresponds to finding the optimal solution required for the original model.

Based on the dual model, we do this in two stages:

First stage:

We must make the elements of the dual model objective function column

$Nb_i \geq 0\ ; \text{i} = 1,2, \dots, \text{m}$. The last row is non-negative.

The second stage:

We must make the free constants for the dual model $-Nc_j$ non-positive, and this corresponds to obtaining the optimal solution for the dual model. We explain the above through the following example:

Find the optimal solution for both the following neutrosophic linear model and its dual using the binary algorithm

Example 1:

Find:

$$[5,8]\ x_1 + [3,6]\ x_2 \longrightarrow Max$$

Constraints:

$$2x_1 + 3x_2 \leq [14,20]$$

$$2x_1 + x_2 \leq [10,16]$$

$$3x_2 \leq [12,18]$$
$$3x_1 \leq [15,21]$$

$$x_1 \geq 0, x_2$$

We form the binary table of the model and the dual model:

Dual vibrable		Original model			
		objective function / constants	$[5,8]\ x_1 + [3,6]\ x_2$		***Max*** / Constants column
	y_1	1	$2x_1 + 3x_2$	$\leq$	$[14,20]$
	y_2	2	$2x_1 + x_2$	$\leq$	$[10,16]$
	y_3	3	$3x_2$	$\leq$	$[12,18]$
	y_4	4	$3x_1$	$\leq$	$[15,21]$
		Non-negative constraints	x_1, x_2	$\geq$	0
		Dual model			
		objective function / constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4$		***Min*** / Constants column
		1	$2y_1 + 2y_2 + 3y_4$	$\geq$	$[5,8]$
		2	$3y_1 + y_2 + 3y_4$	$\geq$	$[3,6]$
		Non-negative constraints	y_1, y_2, y_3, y_4	$\geq$	0

Table No. (5) The original model and its dual model

We used standard form to write the two models in the table that follows:

		Original model			
		objective function / **constants**	$[5,8]\, x_1 + [3,6]\, x_2 + 0u_1 + 0u_2 + 0u_3 + 0u_4$		Max / **Constants column**
Dual vibrable	y_1	1	$2x_1 + 3x_2 + u_1$	$=$	$[14,20]$
	y_2	2	$2x_1 + x_2 + u_2$	$=$	$[10,16]$
	y_3	3	$3x_2 + u_3$	$=$	$[12,18]$
	y_4	4	$3x_1 + u_4$	$=$	$[15,21]$
		Non-negative constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$	0
		Dual model			
		objective function / **constants**	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$		Min / **Constants column**
		1	$2y_1 + 2y_2 + 3y_4 - v_1$	$=$	$[5,8]$
		2	$3y_1 + y_2 + 3y_4 - v_2$	$=$	$[3,6]$
		Non-negative constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0

Table No. (6) Standard format for the original model and the dual model

We notice from the table that the standard form of the original model includes a ready-made base of additional variables u_1, u_2, u_3, u_4, but for the dual model there is no ready-made base. Therefore, we multiply the two restrictions by (-1) and we obtain the basic form of the dual model.

The following table shows the basic form of the original and dual models:

		Original model			
		objective function constants	$[5,8]\,x_1 + [3,6]\,x_2 + 0u_1 + 0u_2 + 0u_3 + 0u_4$		***Max*** Constants column
Dual vibrable	y_1	1	$2x_1 + 3x_2 + u_1$	$=$	$[14,20]$
	y_2	2	$2x_1 + x_2 + u_2$	$=$	$[10,16]$
	y_3	3	$3x_2 + u_3$	$=$	$[12,18]$
	y_4	4	$3x_1 + u_4$	$=$	$[15,21]$
		Non-negative constraints	$x_1, x_2, u_1, u_2, u_3, u_4$	$\geq$	0
		Dual model			
		objective function constants	$[14,20]y_1 + [10,16]y_2 + [12,18]y_3 + [15,21]y_4 + 0v_1 + 0v_2$		***Min*** Constants column
		1	$-2y_1 - 2y_2 - 3y_4 + v_1$	$=$	$-[5,8]$
		2	$-3y_1 - y_2 - 3y_4 + v_2$	$=$	$-[3,6]$
		Non-negative constraints	$y_1, y_2, y_3, y_4, v_1, v_2$	$\geq$	0

Table No. (7): The basic shape of the original model and the dual model

We put the two models in the modified Simplex algorithm table and we get the following table:

			According to the original model		
			$-v_1$	$-v_2$	
		Non-basic vibrable / **basic vibrable**	x_1	x_2	**objective function** NB_i
Non-basic vibrable Dual model	y_1	u_1	2	3	$[14,20]$
	y_2	u_2	2	1	$[10,16]$
	y_3	u_3	0	3	$[12,18]$
	y_4	u_4	3	0	$[15,21]$
		objective function Nc_i	$[5,8]$	$[3,6]$	$L-0$ / $Z-0$

			According to the dual model				
			u_1	u_2	u_3	u_4	
		Non-basic vibrable / **basic vibrable**	y_1	y_2	y_3	y_4	**objective function Original model** Nc_i
Non-basic vibrable	x_1	v_1	-2	-2	0	-3	$-[5,8]$
	x_2	v_2	-3	-1	-3	0	$-[3,6]$
		objective function Dual model NB_i	$[14,20]$	$[10,16]$	$[12,18]$	$[15,21]$	$Z-0$ / $L-0$

Table No. (8): The binary table for the original and dual models according to the modified Simplex algorithm

First stage:

1- For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row and determine the largest positive value. We find:

$\max([5,8],[3,6]) = [5,8]$,which is an expression of the variable x_1. This means that it will enter the base. To determine the element that will exit from the base, we calculate the index $N\theta$, where:

$$N\theta \in min\left[\frac{[14,20]}{2},\frac{[10,16]}{2},\frac{[15,21]}{3}\right] = \frac{[15,21]}{3} = [5,7]$$

We find that the pivot column is the column of the non-base variable x_1, meaning that the variable x_1 will enter the base instead of the variable u_4, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (3)

We perform the switching between variables using a modified simplex algorithm.

2- For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that they are all negative values. We choose the most negative of them, which is $(-[5,8])$ which is the row of the base variable v_1, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$where:

$$N\theta' \in Max\left[\frac{[14,20]}{-2},\frac{[10,16]}{-2},\frac{[15,21]}{-3}\right] = \frac{[15,21]}{-3}$$

So, the column of the non-basic variable u_4 is the pivot column, meaning that the variable u_4 will enter the base instead of the variable v_1, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2) We get the following double table:

			According to the original model		
			$-y_4$	$-v_2$	
		Non-basic vibrable / **basic vibrable**	u_4	x_2	**objective function Original model** NB_i
Non-basic vibrable	y_1	u_1	$\frac{-2}{3}$	3	[4,6]
	y_2	u_2	$\frac{-2}{3}$	1	[0,4]
	y_3	u_3	0	3	[12,18]
	v_1	x_1	$\frac{1}{3}$	0	[5,7]
		objective function Original model Nc_i	$\left[\frac{-8}{3},\frac{-5}{3}\right]$	[3,6]	$L-[25,56]$ $Z-[25,56]$

			According to the dual model				
			u_1	u_2	u_3	x_1	
		Non-basic vibrable / **basic vibrable**	y_1	y_2	y_3	v_1	**objective function Original model** Nc_i
Non-basic vibrable	u_4	y_4	$\frac{2}{3}$	$\frac{2}{3}$	0	$\frac{-1}{3}$	$\left[\frac{8}{3},\frac{5}{3}\right]$
	x_2	v_2	-3	-1	-3	0	$-[3,6]$
		objective function Original model Nc_i	[4,6]	[0,4]	[12,18]	[5,7]	$Z-[25,56]$ $L-[25,56]$

Table No. (9): The binary table for the first stage, the solution according to the original and dual models

Second phase:

We apply the stopping criterion of the algorithm

For the original model:

Since the values in the constant's column are all positive, we study the values in the objective function row. We notice that there is a positive value, which is [3,6], meaning that we have not yet reached the optimal solution. Therefore, we specify the pivot column, which is the column of the variable x_2 corresponding to the only positive value in the objective function row. [3,6] to determine the pivot row and the pivot element, we calculate the index $N\theta$, where:

$$N\theta \in \min\left[\frac{[4,6]}{3}, \frac{[0,4]}{1}, \frac{[12,18]}{3}\right] = \frac{[4,6]}{3}$$

It corresponds to the base element u_1, so its row is the pivot row and the pivot element is $(\mathbf{3})$. We swap between the variables using the modified simplex algorithm.

For the dual model:

We study the elements of the objective function row. We notice that all the values are positive. Therefore, we study the elements of the constant's column. We find that there is a single negative value, which is $(-[3,6])$, which is the line of the base variable v_2, so its row is the pivot row. To determine the pivot column and the pivot element, we calculate the index $N\theta'$ where:

$$N\theta' \in Max\left[\frac{[4,6]}{-3}, \frac{[0,4]}{-1}, \frac{[12,18]}{-3}\right] = \frac{[4,6]}{-3}$$

So, the column of the non-base variable y_1 is the pivot column, meaning that the variable y_1 will enter the base instead of the variable v_2, and the pivot element is the element resulting from the intersection of the pivot row and the pivot column, which is (-3). We perform the switching between the variables using the modified simplex algorithm, from (1) and (2). We get the following double table:

			According to the original model		
			$-y_4$	$-y_1$	
		Non-basic vibrable / **basic vibrable**	u_4	u_1	**objective function Dual model** NB_i
Non-basic vibrable	v_2	x_2	$\frac{-2}{9}$	$\frac{1}{3}$	$\left[\frac{4}{3},2\right]$
	y_2	u_2	$\frac{-4}{9}$	$\frac{-1}{3}$	$\left[\frac{4}{3},2\right]$
	y_3	u_3	$\frac{2}{3}$	-1	$[8,12]$
	v_1	x_1	1	0	$[5,7]$
		objective function Original model Nc_i	$[-6,-1]$	$[-2,-1]$	$L-[29,68]$ / $Z-[29,68]$

			According to the dual model				
			x_2	u_2	u_3	x_1	
		Non-basic vibrable / **basic vibrable**	v_2	y_2	y_3	v_1	objective function Original model Nc_i
Non-basic vibrable	u_4	y_4	$\frac{2}{9}$	$\frac{4}{9}$	$\frac{-2}{3}$	-1	$[-6,-1]$
	u_1	y_1	$\frac{-1}{3}$	$\frac{1}{3}$	1	0	$[-2,-1]$
		objective function Dual model NB_i	$\left[\frac{4}{3},2\right]$	$\left[\frac{4}{3},2\right]$	$[8,12]$	$[5,7]$	$Z-[29,68]$ / $L-[29,68]$

Table No. (10): The binary algorithm table for the second stage

We apply the stopping criterion of the algorithm:

1- In the original model, we investigate the elements of the objective function row until the requirement for ending the procedure, which is the lack of any positive element, is fulfilled.
2- We also investigate the elements of the constants column for the dual model until the requirement for ending the process, which is the lack of any negative element, is fulfilled.
3- We determine that the condition has been satisfied, and hence we have arrived at the optimal solution.

The optimal solution of the original model is:

$$x_2^* \in \left[\frac{4}{3}, 2\right] \ , u_2^* \in \left[\frac{4}{3}, 2\right] \ , u_3^* \in [8,12] \, , x_1^* \in [5,7] \ , u_1^* = u_4^* = 0$$

The value of the objective function corresponds to:

$$NZ^* = Max\, NZ \in [29,68]$$

The optimal solution of the dual model is:

$$y_1^* \in [1,2] \ , y_4^* \in [1,6] \ , v_2^* = y_2^* = y_3^* = v_1^* = 0$$

The value of the objective function corresponds to:

$$NL^* = MinNL \in [29,166]$$

We note that:

$$NZ^* = Max\, NZ \in [29,68] \leq NL^* = MinNL \in [29,166]$$

This solution is acceptable according to the following theory:

If $(x_1, x_2, \dots, x_n)$ is an acceptable solution to the original model of type Max and $(y_1, y, \dots, y_m)$ was an acceptable solution for the dual model of type Min, so the value of the objective function of the original model does not exceed the value of the objective function of the dual model for these two solutions, that is, it is

$$\sum_{j=1}^{n} Nc_j x_j \leq \sum_{i=1}^{m} Nb_i y_i$$

This applies to all appropriate solutions for both models (including the optimal solution).

7-3- Economic interpretation of the dual models:

The following example illustrates the economic interpretation of the dual model:

Example2:

A factory wants to move its products as cheaply as possible from two warehouses to three retail locations. The data supplied by the factory official is shown in the table below:

Sales centers / Stores	B_1	B_2	B_3	Available quantities
A_1	[1,3]	[2,4]	[0,3]	300
A_2	[4,6]	[1,4]	[1,5]	600
Quantities required	200	300	400	900 / 900

The plant manager demanded a low-cost transportation strategy so that the distribution centres' requests could be filled from the available amounts.

The previous issue is a balanced transfer issue because

$$\sum_{i=1}^{2} a_i = \sum_{j=1}^{3} b_j = 900$$

To formulate the mathematical model

We assume x_{ij} the quantity transported from store i where $i = 1,2$, to distribution center j, where $j = 1,2,3$. Thus, we obtain the following linear model:

Find:

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constraints:

$$x_{11} + x_{12} + x_{13} \leq 300$$

$$x_{11} + x_{21} + x_{23} \leq 600$$

$$x_{11} + x_{21} \geq 200$$

$$x_{12} + x_{22} \geq 300$$

$$x_{13} + x_{23} \geq 400$$

$$x_{\mathrm{ij}} \geq 0\ ;\ \ \mathrm{i} = 1,2\ \ , \mathrm{j} = 1,2,3$$

We write the model in the following symmetrical form:

Because the objective function is a minimization function, all constraints must be larger than or equal to, hence the model takes the symmetric form:

Find:

$$L \in [1,3]x_{11} + [2,4]x_{12} + [0,3]x_{13} + [4,6]x_{21} + [1,4]x_{22} + [1,5]x_{23} \longrightarrow Min$$

Constraints:

$$-x_{11} - x_{12} - x_{13} \geq -300$$

$$-x_{11} - x_{21} - x_{23} \geq -600$$

$$x_{11} + x_{21} \geq 200$$

$$x_{12} + x_{22} \geq 300$$

$$x_{13} + x_{23} \geq 400$$

$$x_{ij} \geq 0 \;;\; i = 1,2 \;, j = 1,2,3$$

Forming the dual model, we obtain the following linear model:

Find:

$$Z = -300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5 \longrightarrow Max$$

Constraints:

$$-y_1 + y_3 \leq [1,3]$$

$$-y_1 + y_4 \leq [2,4]$$

$$-y_1 + y_5 \leq [0,3]$$

$$-y_2 + y_3 \leq [4,6]$$

$$-y_2 + y_4 \leq [1,4]$$

$$-y_2 + y_5 \leq [1,5]$$

$$y_1, y_2, y_3, y_4, y_5 \geq 0$$

Based on the content of the original problem, we build an adequate text for the accompanying model:

It is clear from the original model that the factory's goal is to transport all of its products at the lowest possible cost:

Text of the issue dual to the attached form:

A transport company submitted to a factory an offer that it would transport the entire quantity in the first warehouse, i.e., 300 units, at a price of y_1 monetary unit per unit, and transfer the entire quantity available in the second warehouse, 600, at a price of y_2 monetary units per unit. The business promised to supply the three retail outlets with 200, 300, and 400 units, respectively. These units are sold in these centers at a price of (y_3, y_4, y_5) monetary units, respectively. So, you can convince the factory official that if he accepts your offer, the transportation cost in his factory will be less than the cost. Using the constraints in the dual model as follows:

You pay the cost of transporting one unit from the first factory to the first sales center, an amount whose value belongs to the range [1,3], but if you use the transport company, the cost is $(y_3 - y_1)$, and we have from the first entry in the accompanying model

$$y_3 - y_1 \leq [1,3]$$

Here the official in the laboratory will notice that the transportation company's offer is an appropriate offer.

In the same way we discuss all the limitations of the dual model, the conclusion that the factory official will reach is that the cost of transportation on any route if the transportation company's offer is accepted is less than or equal to the cost that he would pay if he himself carried out the transportation process.

The transport company will adopt the values $(y_1, y_2, y_3, y_4, y_5)$, because it will achieve maximum profit through them, as the transport company's profit is calculated from the relation:

$$-300y_1 - 600y_2 + 200y_3 + 300y_4 + 400y_5$$

It is the same as the objective function of the dual model, meaning that the dual model represents the transportation company that is trying to maximize its profits

The best values of the dual model and the model itself are always equivalent, according to the fundamental theorem of association. Although it saves the manufacturer the trouble of solving the original model to determine the minimum cost of transportation, and because it guarantees the transportation company a deal to transport the goods with the maximum profit, the manufacturer does not save any money because he will pay the transportation company the minimum cost of transportation.

Conclusion:

The interpretation of the optimal solution for the original model is that it gives us the best production plan that makes the value of that production as large as possible, within available capabilities. Based on the previous study, we arrived at a solution for the original and utility models simultaneously, which are neutrosophic values from which we know the minimum and maximum profit that we can obtain. The optimum values for raw material prices are provided by the optimal solution for the dual model. If these prices are employed efficiently, they also yield the best production plan, which maximizes profit.

Chapter VIII: Some applications to neutrosophic linear models

Introduction.

8-1- Problem of the composition of mixtures.

8-2- Problem of product mixture.

Conclusion.

Chapter VIII

Some applications to neutrosophic linear models

Introduction:

In the majority of useful domains, one of the most popular operations research techniques is linear programming. This approach is predicated on turning the problem at hand into a linear mathematical model. From there, we use specialized algorithms designed for solving linear models to determine the best solution. This approach facilitates the process of making well-founded, scientifically-based judgments for the decision makers in charge of overseeing the system that follows this modelThe creation of the linear model, or representing the issue under investigation in mathematical relations, is the most crucial step in linear programming. To create the linear model, the following fundamental components must be present.

1. Determine the goal quantitatively, and it is expressed by the goal function, which is the function for which the maximum or minimum value is required. That is, we must be able to express the goal quantitatively, such as if the goal is to achieve the greatest profit or achieve the lowest cost.
2. Determine the constraints: The constraints that express the available resources must be specific, that is, the resources must be measurable, and expressed in a mathematical formula in the form of inequalities or equals.
3. Identifying the different alternatives: This element indicates that the problem should have more than one

solution so that linear programming can be applied, because if the problem had one solution, there would be no need to use linear programming, whose benefit is focused on helping to choose the best solution from among the acceptable solutions.

In this chapter, we present some problems that lead to neutrosophic linear models, that is, we will take some or all of the problem data as neutrosophic values.

8-1- Problem of the composition of mixtures:

By mixtures, we mean anything that is installed from a number of materials such as diets - medicine - any metal mixture- and here the stretch loop is to choose the materials that enter the composition of this mixture so that the cost of production is as little as possible, the goal of putting forward this model in the field of education is that the student can link between neutrosophic equations and linear inequations as well as the neutrosophic function and problems from real life.

General text of the problem:

We want to install a mixture of raw materials $A_1, A_2, \dots, A_n$and the price of one unit of each of them is equal to

$NC_1, NC_2, \dots, NC_n$ respectively, and the meal must include an amount of important elements

$B_1, B_2, \dots, B_m$ that the quantity of each element shall not be less than $B_1, B_2, \dots, B_m$

$Nb_1, Nb_2, .., Nb_m$ unit in the order required to find the necessary amounts of each of the materials

$A_1, A_2, \dots, A_n$ which must be included in the mixture so that its cost is as low as possible, knowing that the content of each of the materials $A_1, A_2, .., A_n$ of each of the elements $B_1, B_2, \dots, B_m$, is shown in the following table:

Materials / Elements	A_1	A_2	………	A_n	NB
B_1	a_{11}	a_{12}	……….	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	……….	a_{2n}	Nb_2
……………….	………..	………….	………..	………….	
B_m	a_{m1}	a_{m2}	……….	a_{mn}	Nb_m

Table No. (1) Raw materials and elements for the problem of composition of mixtures

If and $Nc_j = c_j \pm \varepsilon_j \quad j = 1,2,\dots,n$ where ε_j is indefinite and can be $\varepsilon_j = [\lambda_{1j}, \lambda_{2j}]$ or $\varepsilon_j \epsilon \{\lambda_{1j}, \lambda_{2j}\}$

Also values that express the quantities of elements that must be available in the mixture and $Nb_i = b_i \pm \delta_i \quad i = 1,2,\dots,m$ where δ_i is indefinite and can be, $\delta_i = [\mu_{1i}, \mu_{2i}]$ or $\delta_i \epsilon \{\mu_{1i}, \mu_{2i}\}$

Building the Mathematical Model:

We represent the necessary quantities of every material $A_1, A_2, \dots, A_n$ by $x_1, x_2, \dots, x_n$ and put all the information in the following table:

Materials / Elements	A_1	A_2	…	A_n	Minimum amounts
B_1	a_{11}	a_{12}	…	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	…	a_{2n}	Nb_2
………	………	……	…	………	………
B_m	a_{m1}	a_{m2}	…	a_{mn}	Nb_m
Profit per unit	Nc_1	Nc_2	…	Nc_n	
Required amounts	x_1	x_2	…	x_n	

Table No. (2) General data on the issue of composition of mixtures

What is required in the problem is to determine a value for each of the variables $x_1, x_2, \dots, x_n$ so that, given the constraints, the objective function opts for the lowest value.

Based on the data of the problem, the objective function is written in the following form:

$$NL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

We express the terms mathematically and offer the following explanation:

Each unit of the material A_1 gives us a_{11} unit of the element B_1 , and thus we find that x_1 unit gives us $a_{11}x_1$ unit of the element B_1, and so we find for the rest of the materials and elements, and therefore the condition related to the element B_1 is as follows:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq Nb_1$$

We proceed in the same manner to obtain the following mathematical model for all materials and elements:

$$MinNL = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \geq Nb_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \geq Nb_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \geq Nb_m$$

$$x_1, x_2, \dots, x_n \geq 0$$

We write in the following abbreviated form:

$$\text{Min}\, NL = \sum_{j=1}^{n} (c_j \pm \varepsilon_j)x_j$$

Constraints:

$$\sum_{j=1}^{n} a_{ij}x_j \geq b_i \pm \delta_i \ ; i = 1,2, \dots, m$$

$$x_j \geq 0$$

where $c_j \pm \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2, \dots, n$, $i = 1,2, \dots, m$

Example 1:

A school wishes to serve four different types of food for breakfast to its students: A_1, A_2, A_3, and A_4. The cost of one unit of each of these would be Nc_1, Nc_2, Nc_3, Nc_4. Additionally, let's assume that the meal must include a specific quantity of vital nutrients: proteins B_1, starch B_2, carbohydrates B_3. In order to ensure that the meal contains at least the minimum amount of nutrients required to be provided and that the amount of protein in it is not less than Nb_1 unit, and the amount of carbohydrates is Nb_2 unit, and the amount of carbohydrates is Nb_3 unit, it is necessary to determine the necessary amounts of substances that must be included into the meal. The following table outlines the requirements for each element and the essential nutrients they contain, where:

$Nc_j = c_j \pm \varepsilon_j$ and $j = 1,2,3,4$ where ε_j is indefinite and can be $\varepsilon_j = [\lambda_{1j} , \lambda_{2j}]$or $\varepsilon_j \epsilon \{\lambda_{1j} , \lambda_{2j}\}$

Also values that express the amounts of nutrients that must be available in the meal $Nb_i = b_i \pm \delta_i$ and $i = 1,2,3$ where δ_i it is indefinite and can be $\delta_i = [\mu_{1i} , \mu_{2i}]$ or $\delta_i \epsilon \{\mu_{1i} , \mu_{2i}\}$

We denote the required amounts of each of the materials A_1, A_2, A_3, A_4 with symbols x_1, x_2, x_3, x_4 respectively put the information contained in the text of the problem's table as follows:

Materials / Elements	A_1	A_2	A_3	A_4	Minimum amounts
B_1	a_{11}	a_{12}	a_{13}	a_{14}	Nb_1
B_2	a_{21}	a_{22}	a_{23}	a_{24}	Nb_2
B_3	a_{31}	a_{32}	a_{33}	a_{34}	Nb_3
Profit	Nc_1	Nc_2	Nc_3	Nc_4	
Required amounts	x_1	x_2	x_3	x_4	

Table No. (3): Basic data for building the linear model for example 1

Follow the objective function:

We find:

$$NL = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4x_4$$

Nutrient conditions:

Nutrient protein requirement B_1

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \geq Nb_1$$

Requirement of starch nutrient B_2

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \geq Nb_2$$

Requirement of carbohydrate nutrient B_3

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \geq Nb_3$$

Non-negative condition:

$$x_1, x_2, x_3, x_4 \geq 0$$

The appropriate mathematical model emerges.

Find:

$$MinNL = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4x_4$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \geq Nb_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \geq Nb_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \geq Nb_3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

We apply the neutrosophic simplex approach outlined in the research to get the best solution.

8-2- Problem of product mixture:

When a product is said to be a mixture of products, it refers to one that is produced in all production facilities using a variety of raw materials in order to achieve optimal workflow and maximum profit. This process is guided by a scientific analysis that determines the quantities required to produce each product using the best available resources in order to meet market demand and turn a profit. This model can be used to demonstrate to students the application of linear models. As a result, students will learn that the pens, notebooks, benches, tables, transportation, and other items they use on a daily basis are produced using process research methods that rely on developing mathematical models. The optimal solution for solving a model is what the institution must undertake.

General text of the problem:

A production institution that can produce products $A_1, A_2, \dots, A_n$ and includes in its composition of raw materials, the quantities used of each $B_1, B_2, \dots, B_m$ of the raw materials in each of the products are shown in the following table:

Materials / Elements	A_1	A_2	...	A_n	NB
B_1	a_{11}	a_{12}	...	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	...	a_{2n}	Nb_2
...	...	...	...	...	
B_m	a_{m1}	a_{m2}	...	a_{mn}	Nb_m

Table No. (4) Raw materials and elements for the product mix issue

The quantities available to the institution of these raw materials are $Nb_1, Nb_2, \ldots, Nb_m$ where:

$Nb_i = b_i \pm \delta_i$ and δ_i is indefinite and can be

$\delta_i = [\mu_{1i}, \mu_{2i}]$ or $\delta_i \epsilon \{\mu_{1i}, \mu_{2i}\}$, $i = 1,2, \ldots, m$

$NC_1, NC_2, \ldots, NC_n$, is required to find the amount of what must be produced from each of the products, knowing that the profit returned from one unit of each of the products is respectively

$Nc_j = c_j \pm \varepsilon_j$ where ε_j is indeterminate and can be

$\varepsilon_j = [\lambda_{1j}, \lambda_{2j}]$ $\varepsilon_j \epsilon \{\lambda_{1j}, \lambda_{2j}\}$; $j = 1,2, \ldots, n$

Building the Mathematical Model:

We code the quantities produced from each of the products $A_1, A_2, \ldots, A_n$ be $x_1, x_2, \ldots, x_n$ and put all the information in the following table:

Products / Materials	A_1	A_2	…	A_n	Available Quantities
B_1	a_{11}	a_{12}	…	a_{1n}	Nb_1
B_2	a_{21}	a_{22}	…	a_{2n}	Nb_2
…	…	…	…	…	…
B_m	a_{m1}	a_{m2}	…	a_{mn}	Nb_m
Profit	Nc_1	Nc_2	…	Nc_n	
Quantities produced	x_1	x_2	…	x_n	

Table No. (5) Neutrosophic data for the model

What is required in the problem is to determine a value for each of the variables $x_1, x_2, \dots, x_n$ so that the objective function takes the greatest value, within the imposed conditions.

Based on the data of the problem, the objective function is written in the following form:

$$NZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

We give the following explanation by mathematically formulating the terms:

To produce one unit of the product A_1, we need a_{11} unit of the material B_1, and thus we find that x_1 unit of the product A_1 needs A_1a_{11} unit of the material B_1, and so we find for the rest of the products and materials, and therefore the condition related to the material is as follows: $B_1x_1A_1a_{11}x_1B_1B_1$

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \leq Nb_1$$

Following the same procedure for all goods and materials, we obtain the following mathematical model:

Find the maximum value of the function

$$MaxNZ = Nc_1x_1 + Nc_2x_2 + \cdots + Nc_nx_n$$

Constraints:

$$a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n \le Nb_1$$

$$a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n \le Nb_2$$

… … … … … … … … … … … … ….

$$a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n \le Nb_m$$

$$x_1, x_2, \dots, x_n \ge 0$$

It shall be written in the following abbreviated form:

$$\mathrm{M}axNZ = \sum_{j=1}^{n} (c_j \pm \varepsilon_j)x_j$$

Constraints:

$$\sum_{j=1}^{n} a_{ij}x_j \le b_i \pm \delta_i \quad ; \quad i = 1,2,3, \dots, m$$

$$x_j \ge 0$$

where $c_j \pm \varepsilon_j$, $b_i \pm \delta_i$, a_{ij}, $j = 1,2, \dots, n$, $i = 1,2, \dots, m$ are constants having set or interval values according to the nature of the given problem, x_j are decision variables.

Example2:

A factory for manufacturing pens produces four types S_4 , S_3 , S_2 , S_1 and uses the following raw materials M_3 , M_2, M_1 for this. The factory management wants to study the optimal organization of production during a period of time (for example, a month) and determine the monthly production of each product to achieve a maximum profit, knowing that the profit is directly proportional to the number of units sold of products. We explain the available quantities of raw materials required for each product and the profit returned in the following table:

Materials / Elemants	S_1	S_2	S_3	S_4	Available Quantities
M_1	a_{11}	a_{12}	a_{13}	a_{14}	Nm_1
M_2	a_{21}	a_{22}	a_{23}	a_{24}	Nm_2
M_3	a_{31}	a_{32}	a_{33}	a_{34}	Nm_3

Table No. (6): Data in Example 2

To construct the mathematical model, we assume x $_1$ counting units produced from S_1

x_2 number of units produced fromS_2

x_3 number of units produced fromS_3

x_4 Number of units produced from S_4

During the productive period (for example, a month) we put the information in the following table:

Matereals / Elements	S_1	S_2	S_3	S_4	Available Quantities
M_1	a_{11}	a_{12}	a_{13}	a_{14}	Nm_1
M_2	a_{21}	a_{22}	a_{23}	a_{24}	Nm_2
M_3	a_{31}	a_{32}	a_{33}	a_{34}	Nm_3
Profit	Nc_1	Nc_2	Nc_3	Nc_4	
Quantities produced	x_1	x_2	x_3	x_4	

Table No. (7): Basic information for building the mathematical model

From the table we can see that the primary material condition M_1:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \leq Nm_1$$

Initial material requirement M_2:

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \leq Nm_2$$

Initial material requirement M_3:

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \leq Nm_3$$

In addition, the quantities produced must be non-negative, i.e.:

$$x_1, x_2, x_3, x_4 \geq 0$$

We now define the objective function. If units of the same type are produced x_4 , x_3 , x_2 , x_1 respectively, the profit during the production period will be:

$$NZ = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4x_4$$

Therefore, the mathematical model of the problem is:

Find the maximum value of the function

$$MaxNZ = Nc_1x_1 + Nc_2x_2 + Nc_3x_3 + Nc_4$$

Within the conditions

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + a_{14}x_4 \leq Nm_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + a_{24}x_4 \leq Nm_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + a_{34}x_4 \leq Nm_3$$

$$x_1, x_2, x_3, x_4 \geq 0$$

Using neutrosophic science principles, we shall apply the above to a model of optimal agricultural land use. We will be employing data that is influenced by the environment, i.e., neutrosophic values.

Text of the issue:

Let us assume that we have n agricultural areas (plain or cultivated), each of which has an area equal to $A_1, A_2, \cdots\cdots, A_n$. We want to plant it with m types of agricultural crops to secure the community's requirements for it. Knowing that we need of crop i the amount b_i , if the average productivity of one area in plain j of crop i is equal to

Na_{ij} tons/ha. where $j = 1,2, \dots \dots, n$ and $i = 1,2, \dots \dots, m$, and the profit returned from one unit of crop i equal to Np_i, whereNp_i is a neutrosophic value, an undefined non-specific value that designates a perfect and can be any neighbor of the value a_{ij}, also Np_iwhich can be any neighbor of p_i.

Requirement:

Ascertain the acreage required for cultivation of each crop in each region in order to maximize profit and satisfy societal demands.

Formulation of the mathematical model:

We represent by $\boldsymbol{x_{ij}}$ the amount of area in area j that must be cultivated with crop, and we place the data for the problem in the following table:

Regions / **Crops**	1	2	$\cdots$	n	**Order amount** $\boldsymbol{b_i}$	**profit amount** $\boldsymbol{Np_i}$
1	Na_{11} x_{11}	Na_{12} x_{12}	$\cdots$	Na_{1n} x_{1n}	b_1	Np_1
2	Na_{21} x_{21}	Na_{22} x_{22}	$\cdots$	Na_{2n} x_{2n}	b_2	Np_2
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots\cdots$	$\cdots$	$\cdots$
m	Na_{m1} x_{m1}	Na_{m2} x_{m2}	$\cdots$	Na_{mn} x_{mn}	b_m	Np_m
Available space $\boldsymbol{a_i}$	a_1	a_2	$\cdots$	a_n		

Table No. (8) Issue data

Then we find that the conditions imposed on the variables x_{ij} are:

1- Space restrictions:

The total area allocated to various crops in area **j** must be equal to a_j , that is, it must be:

$$x_{11} + x_{12} + \cdots \cdots + x_{m1} = a_1$$
$$x_{12} + x_{22} + \cdots \cdots + x_{m2} = a_2$$

$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$

$$x_{1n} + x_{2n} + \cdots\cdots + x_{mn} = a_n$$

2- Conditions for meeting community requirements:

The total production of crop i in all regions must not be less than the amount b_i , that is, it must be:

$$Na_{11}x_{11} + Na_{12}x_{12} + \cdots\cdots + Na_{1n}x_{1n} \geq b_1$$
$$Na_{21}x_{21} + Na_{22}x_{22} + \cdots\cdots + Na_{2n}x_{2n} \geq b_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$Na_{m1}x_{m1} + Na_{m2}x_{m2} + \cdots\cdots + Na_{mn}x_{mn} \geq b_m$$

Find the objective function:

We note that the profit resulting from the production of crop i only and from all regions is equal to the product of the profit times the quantity, which is:

$$Np_i(Na_{i1}x_{i1} + Na_{i2}x_{i2} + \cdots\cdots + Na_{in}x_{in})$$

Thus, we find that the objective function, which expresses the total profit resulting from all crops, is equal to:

$$Z = Np_1\left(\sum_{j=1}^{n} Na_{1j}\ x_{1j}\right) + Np_2\left(\sum_{j=1}^{n} Na_{2j}\ x_{2j}\right) + \cdots\cdots + Np_m\left(\sum_{j=1}^{n} Na_{mj}\ x_{mj}\right) \to Max$$

From the above we get the following mathematical model:

Find the maximum value of

$$Z = Np_1\left(\sum_{j=1}^{n} Na_{1j}\ x_{1j}\right) + Np_2\left(\sum_{j=1}^{n} Na_{2j}\ x_{2j}\right) + \cdots\cdots + Np_m\left(\sum_{j=1}^{n} Na_{mj}\ x_{mj}\right) \to Max$$

Constraints:

$$x_{11} + x_{12} + \cdots\cdots + x_{m1} = a_1$$
$$x_{12} + x_{22} + \cdots\cdots + x_{m2} = a_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$x_{1n} + x_{2n} + \cdots\cdots + x_{mn} = a_n$$
$$a_{11}x_{11} + a_{12}x_{12} + \cdots\cdots + a_{1n}x_{1n} \geq b_1$$
$$a_{21}x_{21} + a_{22}x_{22} + \cdots\cdots + a_{2n}x_{2n} \geq b_2$$
$$\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots\dots$$
$$a_{m1}x_{m1} + a_{m2}x_{m2} + \cdots\cdots + a_{mn}x_{mn} \geq b_m$$

$$x_{ij} \geq 0 \quad ; i = 1,2,\cdots\cdots,m \,, j = 1,2,\cdots\cdots,n$$

Example3:

Let us assume that we want to exploit four agricultural areas A_1, A_2, A_3, A_4, and the area of each of them, respectively, is $60,150,20,10$, by planting them with the following crops: wheat, barley, cotton, tobacco, and beet, from which we need the following: $800, 200,600,1000,2500$. Let us assume that the regions' productivity of these crops and their prices are given in the following table:

Regions / Crops	A_1	A_2	A_3	A_4	order	Price per ton
wheat	$\{4,6\}$	4	3	6	2500	$\{1400,1600\}$
barley	7	5	4	$\{3,5\}$	1000	$\{900,1100\}$
cotton	4	$\{9,11\}$	8	5	600	$\{4500,6000\}$
tobacco	6	$\{2,4\}$	0	0	200	$\{4000,5000\}$
beet	3	$\{10,14\}$	10	6	800	$\{400,700\}$
Space	60	150	20	10		

Table No. (9) Example data

Requirement:

Formulate the mathematical model for this problem in a way that maximizes the production value.
To formulate the mathematical model, we extract the following linear conditions:

Space restrictions:

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 150$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 20$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 10$$

Order restrictions:

$$\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14} \geq 2500$$

$$7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24} \geq 1000$$

$$4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34} \geq 600$$

$$6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44} \geq 200$$

$$3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54} \geq 800$$

Non-Negative restrictions:

$$x_{ij} \geq 0 \;; i = 1,2,3,4,5 \quad and \quad j = 1,2,3,4$$

Objective function that expresses the value of production is:

$$\begin{aligned} Z = \{1400,1600\}(\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14}) \\ + \{900,1100\}(7x_{21} + 5x_{22} + 4x_{23} \\ + \{3,5\}x_{24}) \\ + \{4500,6000\}(4x_{31} + \{9,11\}x_{32} + 8x_{33} \\ + 5x_{34}) \\ + \{4000,5000\}(6x_{41} + \{2,4\}x_{42} + 0x_{43} \\ + 0x_{44}) + \{400,700\}(3x_{51} + \{10,14\}x_{52} \\ + 10x_{53} + 6x_{54}) \rightarrow Max \end{aligned}$$

Mathematical model:

Find the maximum value of

$$Z = \{1400,1600\}(\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14}) + \{900,1100\}(7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24}) + \{4500,6000\}(4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34}) + \{4000,5000\}(6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44}) + \{400,700\}(3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54}) \rightarrow Max$$

Constraints:

$$x_{11} + x_{21} + x_{31} + x_{41} + x_{51} = 60$$

$$x_{12} + x_{22} + x_{32} + x_{42} + x_{52} = 150$$

$$x_{13} + x_{23} + x_{33} + x_{43} + x_{53} = 20$$

$$x_{14} + x_{24} + x_{34} + x_{44} + x_{54} = 10$$

$$\{4,6\}x_{11} + 4x_{12} + 3x_{13} + 6x_{14} \geq 2500$$

$$7x_{21} + 5x_{22} + 4x_{23} + \{3,5\}x_{24} \geq 1000$$

$$4x_{31} + \{9,11\}x_{32} + 8x_{33} + 5x_{34} \geq 600$$

$$6x_{41} + \{2,4\}x_{42} + 0x_{43} + 0x_{44} \geq 200$$

$$3x_{51} + \{10,14\}x_{52} + 10x_{53} + 6x_{54} \geq 800$$

$$x_{ij} \geq 0 \ ; i = 1,2,3,4,5 \quad \text{and} \quad j = 1,2,3,4$$

The first issue:

An operations research expert was consulted by an executive in one of the companies to help him find the best way to operate warehouses at the lowest possible cost and minimize transportation costs. The executive gave the expert information that allowed the expert to formulate the following problem:

The text of the problem according to the concepts of neutrosophic science: A retail company plans to expand its activities in a specific area by establishing two new warehouses. The following table shows the potential locations, the number of customers and the possibility of meeting the demand for the sites where ($*$) has been placed in the event that the site can meet the customer's request and put ($\times$) the opposite and code Nc_{ij} The transfer of one unit from location i to customer j is shown in the following table:

site \ Customer	B_1	B_2	B_3	B_4
A_1	$*$ Nc_{11}	$*$ Nc_{12}	$\times$	$*$ Nc_{14}
A_2	$*$ Nc_{21}	$*$ Nc_{22}	$*$ Nc_{23}	$*$ Nc_{24}
A_3	$\times$	$*$ Nc_{32}	$*$ Nc_{33}	$*$ Nc_{34}
Customer orders	D_1	D_2	D_3	D_4

Table (10) Transportation cost in case of location selection

We have the following information available for each of the candidate locations for warehouses

site \ information	Operating cost per unit (monetary unit)	Initial Invested Capital (Monetary Unit)	Site Capacity
first	Np_1	k_1	A_1
second	Np_2	k_2	A_2
third	Np_3	k_3	A_3

Table (11) operation information

It is required to choose suitable locations for warehouses that make the total costs of investment, operation and transportation as small as possible.

Building the mathematical model:

The total cost of setting up and running the warehouse is therefore a non-linear function of the stored quantity. The problem of locating the warehouse can be formulated using binary integer variables in a program with integers, where we assume that the binary integer variable $\boldsymbol{\delta_i}$ represents the decision to choose the site or not. Each site has a fixed capital cost independent of the quantity stored in the warehouse referred to that site and also has a variable cost proportional to the quantity transported:

$$\delta_i = \begin{cases} 1 & \text{if we chose the site } i \\ 0 & \text{otherwise} \end{cases}$$

Suppose that $\boldsymbol{x_{ij}}$ is the quantity transferred from site $\boldsymbol{i}$ to customer $\boldsymbol{j}$, so the constraint expressing the ability of the first site to meet the requests is as follows:

$$x_{11} + x_{12} + x_{14} \leq A_1\delta_1$$

When $\delta_1 = 1$, the first location with capacity A_1 is chosen. The quantity transported from the first site cannot exceed the capacity of that site A_1 when $\delta_1 = 0$ the non-negative variables

$x_{11}, x_{12}, x_{14} = 0$ directly, indicating that it is not possible to ship from the first location

In a similar way, we obtain the following two constraints for the second and third signatories.

$$\mathrm{x}_{21} + \mathrm{x}_{22} + \mathrm{x}_{23} + \mathrm{x}_{24} \leq \mathrm{A}_2\delta_2$$

$$\mathrm{x}_{31} + \mathrm{x}_{33} + \mathrm{x}_{34} \leq \mathrm{A}_3\delta_3$$

To choose exactly two locations, we need the following restriction:

$$\delta_1 + \delta_2 + \delta_3 = 2$$

As δ_1 can take one of the values of 0 or 1 only, the new constraint will force two variables from among the three variables, δ_i to be equal to one.

The restrictions for customer requests can be written as follows:

First customer $x_{11} + x_{21} = D_1$

Second customer $x_{12} + x_{22} + x_{32} = D_2$

Third customer $x_{23} + x_{33} = D_3$

Forth customer $x_{14} + x_{24} + x_{34} = D_4$

To write the objective function, we note that the total cost of investment, operation and transportation for the first site is as follows:

$$k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14}$$

When we do not choose the first site, variable $\delta_1 = 0$ And that forces the variables

x_{11}، x_{12}، $x_{14} = 0$

In a similar way, the cost functions of the second and third sites can be written, and thus the full formulation of the issue of assigning the location of the warehouse is reduced to the following correct mixed program: Z is meant to be made minimal

$Z = k_1\delta_1 + Np_1(x_{11} + x_{12} + x_{14}) + Nc_{11}x_{11} + Nc_{12}x_{12} + Nc_{14}x_{14} + k_2\delta_2 + Np_2(x_{21} + x_{22} + x_{23} + x_{24}) + Nc_{21}x_{21} + Nc_{22}x_{22} + Nc_{23}x_{23} + Nc_{24}x_{24} + k_3\delta_3 + Np_3(x_{32} + x_{33} + x_{34}) + Nc_{32}x_{32} + Nc_{33}x_{33} + Nc_{34}x_{34}$

considering the following restrictions:

$$x_{11} + x_{12} + x_{14} \leq A_1\delta_1$$

$$x_{21} + x_{22} + x_{23} + x_{24} \leq A_2\delta_2$$

$$x_{31} + x_{33} + x_{34} \leq A_3\delta_3$$

$$\delta_1 + \delta_2 + \delta_3 = 2$$

$$x_{11} + x_{21} = D_1$$

$$x_{12} + x_{22} + x_{32} = D_2$$

$$x_{23} + x_{33} = D_3$$

δ_i true variable for $i = 1,2,3$

$$x_{ij} \geq 0 \ ; \ i = 1,2,3 \ \ and \ j = 1,2,3,4$$

The second problem:

The executive's second request concerned how to select the best projects to carry out the limited capital that the company has available among the various projects that have been presented. Using the data supplied by the official responsible for overseeing the business, the expert developed the following problem:

The issue of the capital budget: A company plans to disburse its capital during the T_j periods. Where:

$j = 1,2, \dots ,$ n, and there isA_iA proposed project where:

$i = 1,2, - - -,$ m versus a limited capital B_j Available for investment in period j and when choosing any project i

becomes in need of a certain capital in each period j we denote it Na_{ij}. It is a neutrosophic value, the value of each project is measured in terms of the liquidity flow corresponding to the project in each period minus the value of inflation, and this is called net present value (NPV), we denote it Nv_i Accordingly, the following table can be organized:

period / project	T_1	T_2	…	T_n
A_1	Na_{11}	Na_{12}	…	Na_{11}
A_2	Na_{21}	Na_{22}	…	Na_{2n}
…	…	…	…	…
A_m	Na_{m1}	Na_{m2}	…	Na_{mn}
Limited capital	B_1	B_2	…	B_n

Table (12) Return on Investment during Periods

What is required in this problem is to select the right projects that maximize the total value (NPV) of all selected projects. Formulation of the mathematical model:

Here we assume a binary integer variable x_jIt takes the value one if the project j is selected and takes the value zero if the project ***j*** is not selected

$$x_i = \begin{cases} 1 & \text{if we chose project } i \\ 0 & \text{otherwise} \end{cases}$$

Then the objective function is given by the following relation:

$$Z = \sum_{i=1}^{m} Nv_i \, x_i$$

Then the objective function is given by the following relation:

$$\sum_{i=1}^{m} Na_{ij} \, x_i \le B_j \quad ; j = 1, \dots, n$$

Accordingly, we get the following mathematical model:

Find the maximum value of the function:

$$Z = \sum_{i=1}^{m} Nv_i \, x_i$$

considering the following restrictions:

$$\sum_{i=1}^{m} Na_{ij} \, x_i \leq B_j \quad ; j = 1, \dots, n$$

x_i A binary variable takes one of the values 0 or 1 for all values of $i = 1, \dots, m$ in the previous two issues, we got models with integers that have special methods of solution. This research cannot be presented and we will present them in later research using the concepts of neutrosophic science

1- **Formulation of the problem and the construction of mathematical model according to neutrosophic values:**

The study concluded in the research [12] shows us how to construct neutrosophic linear models, (the linear model is a neutrosophic model if at least one of the likes of variables in the objective function or neutrosophic value constraints)

The text of the issue:

The company has n rank for inspectors and wants to assign the task of quality control to them, and K pieces should be audited daily during an S hour of work per day, in the following table we explain the full information about the inspectors and for all ranks:

About the Inspector / Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	NM_1	ND_1	G_1	A_1	R
2	NM_2	ND_2	G_2	A_2	R
...	...	...	...	...	...
n	NM_n	ND_n	G_n	A_n	R

Table (13) Information on inspectors using neutrosophic values

The number of pieces is a neutrosophic value $NM_j = M_j + \varepsilon_j$ where ε_j is the indeterminacy on the number of pieces, it can take one of the shapes $[\lambda_{j1}, \lambda_{j2}]$ or $\{\lambda_{j1}, \lambda_{j2}\}$ or any value close to M_j as well as the precision, neutrosophic values

$ND_j = D_j + \delta_j$

where δ_j is the indeterminacy on the precision that can take one of the shapes $[\mu_{j1}, \mu_{j2}]$ or $\{\mu_{j1}, \mu_{j2}\}$ or any value close to D_j.

Requirement:

Create a suitable mathematical model that will allow us to allocate the inspectors with the optimal support, resulting in the lowest possible inspection cost.

Building the neutrosophic mathematical model:

To build the mathematical model, we impose

$x_1, x_2, ----, x_n$ the number of inspectors of each rank on the order assigned to the inspection task, then the following inequality must be met:

$$x_j \leq A_j \quad ; \quad j = 1,2, --, n$$

Since the company needs to audit K piece daily within S working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} S(NM_j)x_j \geq K$$

In order to derive the objective function, we first notice that the corporation is responsible for paying the inspector's fee as well as the fine for each mistake the inspector makes. Based on this information, the target follower writes as follows:

$$NZ = S\sum_{j=1}^{n} G_j + (NM_j)R_j\left[\frac{100 - ND_j}{100}\right]\ x_j$$

Then the mathematical model is written as follows:

$$NZ = S\sum_{j=1}^{n} G_j + (NM_j)R_j\left[\frac{100 - ND_j}{100}\right]\ x_j\ \rightarrow \text{Min}$$

Constraints:

$$x_j \leq A_j \quad ; \quad j = 1,2,--,n$$

$$\sum_{j=1}^{n} S(NM_j)x_j \geq K$$

$$x_j \geq 0\ ;\ j = 1,2,----n$$

Example4:

The following table explains all the information about the inspectors and for all ranks. In this example, we will use the number of pieces checked by the inspectors from each rank as neutrosophic values. A company wants to assign the task of quality control to its inspectors, who have three ranks. The inspectors should audit 1500 pieces every day during eight working hours.

About the Inspector / Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (Monetary Unit per Hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	{15,16}	95	4	10	2
2	{10,11}	90	3	6	2
3	{25,26}	98	5	8	2

Table (14) Information on inspectors using neutrosophic values

Requirement:

Create a suitable mathematical model that will allow us to assign the best inspection assignments to the inspectors while keeping the inspection cost as low as feasible.

To build the mathematical model, we impose $x_{1,}x_2, x_3$ as the number of inspectors from the three ranks in the order assigned to the inspection task, then the following inequality must be met:

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

Since the company needs to audit *K* pieces daily within *S* working hour per day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} 8M_j x_j \geq 1500$$

That is:

$$8(M_1x_1 + M_2x_2 + M_3x_3) \geq 1500$$

From it we get the following restriction:

$$8\{15,16\}x_1 + 8\{10,11\}x_2 + 8\{25,26\}x_3 \geq 1500$$

To establish the objective function, we observe that the company pays two types of expenses throughout the inspection process, the inspector's fee and the fine corresponding to the inspector's fault committed for each piece, and then the target follower writes as follows:

Then the cost of the inspector is calculated from j the hourly rank through the following relation:

$$NC_j = G_j + (NM_j)R_j\left(\frac{100 - D_j}{100}\right) \quad ; \quad j = 1,2,---,n$$

We get:

$$NC_1 = 4 + \{15,16\} \times 2 \times \left(\frac{100 - 95}{100}\right) = \{5.5,5.6\}$$

$$NC_2 = 3 + \{10,11\} \times 2 \times \left(\frac{100 - 90}{100}\right) = \{5,5.2\}$$

$$NC_3 = 5 + \{25,26\} \times 2 \times \left(\frac{100 - 98}{100}\right) = \{6,6.04\}$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relation:

$$NTC_j = \sum_{j=1}^{n}\left[G_j + (NM_j)R_j\left(\frac{100 - D_j}{100}\right)\right] x_j$$

$$NTC_j = \{5.5,5.6\}x_1 + \{5,5.2\}x_2 + \{6,6.04\}x_3$$

substituting the following target phrase:

$$NZ = S\sum_{j=1}^{n}\left[G_j + (NM_j)R_j\left(\frac{100 - D_j}{100}\right)\right] x_j$$

We get:

$$NZ = \{44,44.8\}x_1 + \{40,41,6\}x_2 + \{48,48.32\}x_3$$

From the above, we can develop the following mathematical model:

We want to find:

$$Min(NZ) = \{44,44.8\}x_1 + \{40,41,6\}x_2 + \{48,48.32\}x_3$$

Constraints:

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

$$8\{15,16\}x_1 + 8\{10,11\}x_2 + 8\{25,26\}x_3 \geq 1500$$

$$x_j \geq 0 \ ; \ j = 1,2,3$$

Example 5:

The company has three ranks for inspectors and wants to assign the task of quality control to them. 1500 pieces should be checked daily during eight working hours per day. For the purposes of this example, we will use each inspector's accuracy of inspection as a neutrosophic value, with the lowest range representing the inspector's level of accuracy and the highest range representing the inspector's level of accuracy by rank. The following table provides comprehensive information about inspectors and all ranks.

About the Inspector / Inspector rank	Number of pieces checked (hour)	Accuracy (percent)	Inspector's remuneration (monetary unit per hour)	Number of inspectors	The fine paid by the company for each fault to the inspector
1	15	[95,97]	4	10	2
2	10	[90,92]	3	6	2
3	25	[98,99.5]	5	8	2

Table (15) Information on inspectors using neutrosophic values

Requirement: Formulate a suitable mathematical model that will allow to allocate the inspectors in the best feasible way, minimizing the inspection cost.

To build the mathematical model, we impose $x_{1,}x_2, x_3$ the number of inspectors from the three ranks in the order assigned to the inspection task, then the following inequality must be met:

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

Since the company needs to audit K pieces daily during S hours of work each day, the following set of restrictions must be met:

$$\sum_{j=1}^{n} 8M_j x_j \geq 1500$$

That is:

$$8(M_1x_1 + M_2x_2 + M_3x_3) \geq 1500$$

We get the following entry:

$$120x_1 + 80x_2 + 200x_3 \geq 1500$$

In order to derive the objective function, we first notice that the corporation is responsible for paying the inspector's fee as well as the fine for each mistake the inspector makes. Based on this information, the target follower writes as follows:

Then the cost of the inspector is calculated from j the hourly rank through the following relation:

$$NC_j = G_j + M_jR_j\left(\frac{100 - ND_j}{100}\right) \;\; ; \;\; j = 1,2,---,n$$

We get

$$NC_1 = 4 + 15 \times 2 \times \left(\frac{100 - [95,97]}{100}\right) = [4.9,5.5]$$

$$NC_2 = 3 + 10 \times 2 \times \left(\frac{100 - [90,92]}{100}\right) = [4.6,5]$$

$$NC_3 = 5 + 25 \times 2 \times \left(\frac{100 - [98,99.5]}{100}\right) = [5.25,6]$$

The total costs for all inspectors assigned to the task of quality control per hour shall be given by the following relation:

$$NTC_j = \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - \mathrm{N}D_j}{100}\right)\right] x_j$$

$$NTC_j = [4.9,5.5]x_1 + [4.6,5]x_2 + [5.25,6]x_3$$

substituting the following target phrase:

$$NZ = S \sum_{j=1}^{n} \left[G_j + M_j R_j \left(\frac{100 - ND_j}{100}\right)\right] x_j$$

We get:

$$NZ = [39.2,44]x_1 + [36.8,40]x_2 + [42,48]x_3$$

From the above, we can develop the following mathematical model:

We want to find:

$$Min(NZ) = [39.2,44]x_1 + [36.8,40]x_2 + [42,48]x_3$$

Constraints:

$$x_1 \leq 10$$

$$x_2 \leq 6$$

$$x_3 \leq 8$$

$$120x_1 + 80x_2 + 200x_3 \geq 1500$$

$$x_j \geq 0 \; ; \; j = 1,2,3$$

In the two examples! and 2 for the optimal solution we use the neutrosophic simplex method.

From the previous model, we notice that x_j takes a positive value only when $\delta_j = 1$, and in this case, the production of the product j is limited by the quantity d_j and the fixed production cost K_j is included in the goal function

The idea of indeterminacy is the basis of neutrosophic science represented here through the use of the binary integer variable because the optimal solution depends on the decision to produce a product or not to produce it.

However, given the significant changes in the labor market due to price strikes, resource availability or non-availability, and other factors, we cannot guarantee the company a safe working environment.

So it was necessary to reformulate this problem using neutrosophic values for the sales opportunity d_j and the cost of producing one unit of each product C_j and selling price P_j so that the sales opportunity becomes $d_j + \varepsilon_j$, production cost $C_j + \mu_j$ and selling price $P_j + \varphi_j$ where ε_j and μ_j and φ_j are the indeterminacy the change in the sales opportunity, cost and selling price respectively depending on the conditions of the work environment and takes one of the following forms:

$\varepsilon_j \in [\lambda_{j1}, \lambda_{j2}]$ Or $\varepsilon_j \in \{\lambda_{j1}, \lambda_{j2}\}$ … and $\mu_j \in [\nu_{j1}, \nu_{j2}]$ or

$\mu_j \in \{\nu_{j1}, \nu_{j2}\}$… and $\varphi_j \in [\theta_{j1}, \theta_{j2}]$ or $\varphi_j \in \{\theta_{j1}, \theta_{j2}\}$

which are values close to the values d_j and C_j and can be any neighborhood to them.

Then the text of the problem becomes as follows:

The text of the problem according to neutrosophic science:

A company is planning to produce N product where the product j needs a fixed preparation cost or a fixed production cost K_j independent of the quantity produced, and needs a variable cost $C_j + \mu_j$ per production unit commensurate with the quantity produced, we suppose that each unit of the product j needs a_{ij} a unit of the supplier i where there is M supplier. Assuming that the product j that has a sales opportunity $d_j + \varepsilon_j$ is sold at the price of $P_j + \varphi_j$ monetary unit per unit and that only b_j unit of the supplier i is available where $i = 1,2, \dots, M$ the goal of the problem becomes to determine the optimal product mix that makes the net profit as great as possible.

Formulation of the mathematical model:

Determination of the cost:

The problem's text indicates that the variable cost, which is a nonlinear function of the quantity produced, and the fixed cost make up the overall cost of production.

However, the issue may be represented as a linear model with integers using binary integer variables δ_j.

It is assumed that the binary integer variable δ_j represents the choice of whether or not to generate the product j.

$$\delta_j = \begin{cases} 1 & j \text{ if production decision was taken} \\ 0 & \text{otherwise} \end{cases}$$

Then the cost of producing one unit of the product becomes as follows $K_j\delta_j + (C_j + \mu_j)x_j$, where $\delta_j = 1$ if $x_j > 0$ and $\delta_j = 0$ if $x_j = 0$ and therefore the goal function becomes as follows:

$$Z = \sum_{j=1}^{N} (P_j + \varphi_j)\, x_j - \sum_{j=1}^{N} \big(K_j\delta_j + (C_j + \mu_j)x_j\big)$$

Restrictions of the problem:

A restriction on the supplier i is given in the following relation:

$$\sum_{j=1}^{N} a_{ij}\, x_j \le b_j \ \ ; i = 1,2, \dots , M$$

The restriction of the demand for the product j is given by the following relation:

$$x_j \le \big(d_j + \varepsilon_j\big)\delta_j \ \ ; j = 1,2, \dots , N$$

Mathematical model: Find the maximum value of the function:

$$Z = \sum_{j=1}^{N} (P_j + \varphi_j)\, x_j - \sum_{j=1}^{N} \big(K_j\delta_j + (C_j + \mu_j)x_j\big)$$

Within Restrictions

$$\sum_{j=1}^{N} a_{ij}\, x_j \le b_j \ \ ; i = 1,2, \dots , M$$

$$x_j \le \big(d_j + \varepsilon_j\big)\delta_j \ \ ; j = 1,2, \dots , N$$

$x_j \ge 0$ and $\delta_j = 1$ or $\delta_j = 0$

And or for all values

$$j = 1,2, \dots , N$$

From the previous model, we note that x_j takes a positive value only when $\delta_j = 1$ and in this case the production of the product j is limited by the quantity $d_j + \varepsilon_j$ and the fixed production cost K_j included in the goal function, by solving this model we get an optimal neutrosophic value for the goal function NZ^* through which we know the profit that the company can achieve in the best and worst conditions and enable the company to develop appropriate plans for the workflow in it.

Conclusion:

In our study, we aim to provide the optimal solution to most of the problems that production companies can face by formulating the situation under treatment with a problem that can be converted into a linear model, the optimal solution of which helps decision-makers make optimal decisions for the workflow so that the greatest profit is achieved. To find solutions with a margin of freedom, we can employ data, neutrosophic values, values that account for all of the situations that the system represented by the linear model may encounter.

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