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NEUTROSOPHIC METHODS IN GENERAL RELATIVITY

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Preface of the Editor

In this work the authors apply concepts of Neutrosophic Logic to the General Theory of Relativity to obtain a generalisation of Einstein's four-dimensional pseudo-Riemannian differentiable manifold in terms of Smarandache Geometry (Smarandache manifolds), by which new classes of relativistic particles and non-quantum teleportation are developed.

Fundamental features of Neutrosophic Logic are its denial of the Law of Excluded Middle, and open (or estimated) levels of truth, falsity and indeterminancy.

Both Neutrosophic Logic and Smarandache Geometry were invented some years ago by one of the authors (F. Smarandache). The application of these purely mathematical theories to General Relativity reveals hitherto unknown possibilities for Einstein's theory.

The issue of how closely the new theoretical possibilities account for physical phenomena, and indeed the viability of the concept of a four-dimensional space-time continuum itself as a fundamental model of Nature, must of course be explored by experiment.

December 2005 *Stephen J. Crothers*

Chapter 1

PROBLEM STATEMENT. THE BASICS OF NEUTROSOPHY

1.1 Problem statement

Neutrosophic method is a new method for scientific research.

This method is based on neutrosophy — "a theory developed by Florentin Smarandache in 1995 as a generalization of dialectics. Neutrosophy considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i.e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <NON-A>. The theory proves that every idea <A> tends to be neutralized and balanced by <Anti-A> and <Non-A> ideas — as a state of equilibrium", (see the Afterword of *Neutrosophic Dialogues* by Smarandache and Liu [1]).

"I coined the term "neutrosophy" in 1995 in my correspondence with Charlie Le. Neutrosophy actually resulted from paradoxism (which I initiated in the 1980's) from my effort to characterize a paradox, which did not work in fuzzy logic or in intuitionistic fuzzy logic because of the restriction that the sum of components had to be 1. Paradoxism is a literary and artistic vanguard movement, as an anti-totalitarian protest, based on excessive use of antitheses, antinomies, contradictions and paradoxes in creation. It was later extended to the sciences, philosophy, psychology, etc. The first thing published that mentioned neutrosophy, was my book: *Neutrosophy. Neutrosophic Probability, Set and Logic*, Am. Res. Press, 1998" [2].

"I then introduced the notion of neutrosophy: Etymologically Neutrosophy (from Latin "neuter" — neutral, Greek "sophia" — skill/wisdom) is a branch of philosophy which studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra" [4, 5, 6].

The main task of this study is to apply neutrosophic method to the General Theory of Relativity, aiming to discover new hidden effects. Here it's why we decided to employ neutrosophic method in this field.

Einstein said that the meaning of the General Theory of Relativity is that all properties of the world we observe are derived from the geometrical structure of our space-time. All the other postulates and laws of both the General Theory and its particular case, the Special Theory, may be considered as only consequences of the space-time geometry. This is a way of geometrizing physics. All postulates — "outer" laws introduced by Einstein before this thesis became historical; only geometry remained under consideration.

Einstein made a four-dimensional pseudo-Riemannian space with the signature $(+---)$ or $(-+++)$ the basic space-time of the General Theory of Relativity, reserving one dimension for time, while the other three were used for three-dimensional space. Experiments, starting with Eddington (1919) and until today, verify the main conclusions of the theory. It is therefore supposed that a four-dimensional pseudo-Riemannian space is the basic space-time of our world.

Of course, the theory does not explain all of today's problems in physics and astronomy (no theory can do that). For instance, when an experiment shows a deviation from the theory, we need to expand the General Theory of Relativity. But even in this case, we have to start this expansion from this basic four-dimensional pseudo-Riemannian space.

One of the main properties of such spaces is the *continuity property*, in contrast to discrete spaces. This property is derived from the fact that Riemannian spaces, being a generalization of Euclidean geometry spaces, are continued. So if we consider two infinitely closed points in a Riemannian space, we can put infinitely many points in between them. There are no omitted points, lines, surfaces or sub-spaces exist in Riemannian spaces. As a result there are no omitted points in intersection of lines, surfaces or sub-spaces in any Riemannian spaces. This phenomenon, derived from pure geometry, is like numbering houses in crossed streets: each corner-house has a double number, one number of which is related to one street, the second number is related to the other. If a city is referred to as a Riemannian space, no street intersection without its own double-numbered corner-house exists in the city. As a result, considering such an intersection, we see that the corner-house is labelled in two ways simultaneously: "13 Sheep Street / 21 Wolf Road", for instance. So the corner-house bears properties of both streets simultaneously, although each of its neighbouring houses "11 Sheep Street" and "23 Wolf Road" bear the property of only that street in which it is located.

Neutrosophic method "means to find common features to uncommon entities: i. e., <A> intersected with <Non-A> is different from the empty set, and even more: <A> intersected with <Anti-A> is different from the empty set" [4]. Therefore, considering the property of space continuity as a basis, we have wide possibilities for the application of the neutrosophic method in the General Theory of Relativity. The neutrosophic method will be employed here in the following directions of research.

Each particle located in space-time has its own four-dimensional trajectory (world-trajectory), which fully characterizes this particle. So each kind of world-trajectory defines a specific kind of particle. As many kinds of trajectories exist in space-time as kinds of particles inhabit space-time.

Following this notion we will study a set of trajectories (particles) which have previously been considered in the General Theory of Relativity. Neutrosophic method will display new trajectories (particles) of "mixed" kinds, bearing properties of trajectories (particles) of two uncommon kinds, which have never studied before. For instance, there will be putting together non-isotropic/isotropic trajectories corresponding to both mass-bearing particles (their rest-masses are non-zero) and massless light-like particles (which have no rest-mass) simultaneously. It will be further shown that such non-isotropic/isotropic trajectories lie outside the basic space-time of the General Theory of Relativity, but particles of such "mixed" kinds are accessible to observation — we can see them in different phenomena in Nature.

The basis of the General Theory of Relativity, a four-dimensional pseudo-Riemannian space, is that particular case of Riemannian metric spaces where the space metric is sign-alternating (denoted by the prefix "pseudo"). Because of the sign-alternating metric, the space is split into three-dimensional spatial sections "pinned up" by a time axis. All relativistic laws, like the Lorentz transformation etc., are derived only from the sign alternation of the space metric. There are 4 signature conditions in total, which define the space metric of such sign-alternating kind.

Neutrosophic method on the foundations of geometry leads to S-denying an axiom [7, 8, 9, 10], i. e. in the same space an "axiom is false in at least two different ways, or is false and also true. Such an axiom, not only in geometry but in any domain, is said to be Smarandachely denied, or S-denied for short" [11]. As a result, it is possible to introduce geometries, which have common points bearing mixed properties of Euclidean, Lobachevsky-Bolyai-Gauss, and Riemann geometry simultaneously. Such geometries have been called paradoxist geometries or Smarandache geometries. For instance, Iseri in his book *Smarandache Manifolds* [11] and his articles [12, 13] introduced manifolds that support particular cases of such geometries.

In this research we will S-deny each of 4 signature conditions in the four-dimensional pseudo-Riemannian space one after another, obtaining thereby four kinds of expanded space-time for the General Theory of Relativity. We will see that the expanded space-time of the 4th kind (where the 4th signature condition is S-denied) permits instant motion of photons — photon teleportation, well-established in recent experiments but which is inconsistent with the basic space-time. We will also find out that only the expanded space-time of the 4th kind permits virtual photons (predicted by Quantum Electrodynamics) — instant-moving massless mediators between entangled regular particles.

It is important to note that this research, using only the mathematical apparatus of Riemannian geometry, does not introduce any additional equations or additional physical requirements. For this reason all results herein are derived from only the geometrical structure of the four-dimensional pseudo-Riemannian spaces we are considering.

1.2 The basics of neutrosophy

Neutrosophy studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. It considers that every idea <A> tends to be neutralized, balanced by <Non-A> ideas; as a state of equilibrium.

Neutrosophy is the basis of *neutrosophic logic*, *neutrosophic set* which generalizes the fuzzy set, and of *neutrosphic probability* and *neutrosophic statistics*, which generalize the classical and imprecise probability and statistics respectively.

Neutrosophic Logic is a multiple-valued logic in which each proposition is estimated to have percentages of truth, indeterminacy, and falsity in T, I, and F respectively, where T, I, F are standard or non-standard subsets included in the non-standard unit interval $]^{-}0, 1^{+}[$. It is an extension of fuzzy, intuitionistic, paraconsistent logics.

Etymology

Neutro-sophy (French "neuter", Latin "neuter" — neutral, and Greek "sophia" — skill/wisdom) means knowledge of neutral thought.

Definition

Neutrosophy is a branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Characteristics

This mode of thinking:

- proposes new philosophical theses, principles, laws, methods, formulae, movements;
- reveals that the world is full of indeterminacy;
- interprets the uninterpretable, i. e. to deal with paradoxes (Le, 1996 [14, 15]) and paradoxism (Popescu, 2002 [16]);
- regards, from many different angles, old concepts and systems: showing that an idea, which is true in a given referential system, may be false in another one, and vice versa;
- attempts to make peace in the war of ideas, and to make war in the peaceful ideas;
- measures the stability of unstable systems, and instability of stable systems.

Methods of neutrosophic study

Mathematization (neutrosophic logic, neutrosophic probability and statistics, duality), generalization, complementarity, contradiction, paradox, tautology, analogy, reinterpretation, combination, interference, aphoristic, linguistic, transdisciplinarity.

Introduction to non-standard analysis

In the 1960's Abraham Robinson [17] developed *non-standard analysis*, a formalization of analysis and a branch of mathematical logic that rigorously defines the infinitesimals. Informally, an infinitesimal is an infinitely small number. Formally, x is said to be infinitesimal if and only if for all positive integers n one has $|x| < 1/n$. Let $\varepsilon > 0$ be such an infinitesimal number. The *hyper-real number set* is an extension of the real number set, which includes classes of infinite numbers and classes of infinitesimal numbers. Let us consider the non-standard finite numbers $1^+ = 1 + \varepsilon$, where 1 is its standard part and ε its non-standard part, and $-0 = 0 - \varepsilon$, where 0 is its standard part and ε its non-standard part.

We call $]^-a, b^+[$ a non-standard unit interval. Obviously, 0 and 1, and analogously non-standard numbers infinitely small but less than 0 or infinitely small but greater than 1, belong to the non-standard unit interval. Actually, by a one signifies a monad, i. e. a set of hyper-real numbers in non-standard analysis:

$$(^-a) = \{a - x : x \in R^*, x \text{ is infinitesimal}\}\,, \tag{1.1}$$

and similarly b^+ is a monad:

$$(b^+) = \{b + x : x \in R^*, x \text{ is infinitesimal}\}\,. \tag{1.2}$$

Generally, the left and right borders of a non-standard interval $]^-a, b^+[$ are vague, imprecise, themselves being non-standard (sub)sets (^-a) and (b^+) as defined above.

Combining the two aforementioned definitions one gets, what we would call, a binad of $^-c^+$: $(^-c^+) = \{c - x : x \in R^*, x \text{ is infinitesimal}\} \bigcup \{c + x : x \in R^*, x \text{ is infinitesimal}\}$, which is a collection of open punctured neighborhoods (balls) of c.

Of course, $^-a < a$, and $b^+ > b$. There is no order between $^-c^+$ and c.

Neutrosophic components

Let T, I, F be standard or non-standard real subsets of $]^-a, b^+[$. These T, I, F are not necessarily intervals, but may be any real sub-unitary subsets: discrete or continuous; single-element, finite, or (countable or uncountable) infinite; union or intersection of various subsets; etc. They may also overlap. The real subsets could represent the relative errors in determining t, i, f (in the case when the subsets T, I, F are reduced to points).

In this article, T, I, F, called *neutrosophic components*, will represent the truth value, indeterminacy value, and falsehood value respectively referring to neutrosophy, neutrosophic logic, neutrosophic set, neutrosophic probability, neutrosophic statistics.

This representation is closer to human reasoning. It characterizes or catches the *imprecision* of knowledge or linguistic inexactitude received by various observers (that is why T, I, F are subsets — not necessarily single-elements), *uncertainty* due to incomplete knowledge or acquisition errors or stochasticity (that is why the subset I exists), and *vagueness* due to lack of clear contours or limits (that is why T, I, F are subsets and I exists; in particular for the appurtenance to the neutrosophic sets).

Formalization

Let us note by <A> an idea, or proposition, theory, event, concept, entity, and by <Non-A> what is not <A>, and by <Anti-A> the opposite of <A>. Also, <Neut-A> means what is neither <A> nor <Anti-A>, i. e. neutrality in between the two extremes. And by <A′> a version of <A>.

> <Non-A> is different to <Anti-A>.
>
> For example: If <A> = white, then <Anti-A> = black (antonym), but <Non-A> = green, red, blue, yellow, black, etc. (any color, except white), while <Neut-A> = green, red, blue, yellow, etc. (any color, except white and black), and <A′> = dark white, etc. (any shade of white). <Neut-A>, <Neut-(Anti-A)>, neutralities of <A> are identical with neutralities of <Anti-A>. <Non-A> includes <Anti-A>, and <Non-A> includes <Neut-A> as well,

also:

> <A> intersection <Anti-A> is equal to the empty set,
>
> <A> intersection <Non-A> is equal to the empty set.
>
> <A>, <Neut-A>, and <Anti-A> are disjoint in pairs.
>
> <Non-A> is the completeness of <A> with respect to the universal set.

Main principle

Between an idea <A> and its opposite <Anti-A>, there is a continuum-power spectrum of neutralities <Neut-A>.

Fundamental thesis

Any idea <A> is T% true, I% indeterminate, and F% false, where the subsets T, I, F are included in the non-standard interval $]^-0, 1^+[$.

Main laws

Let <∀> be an attribute, and (T, I, F) in $]^-0, 1^+[$. Then:

- There is a proposition <P> and a referential system R, such that <P> is T% <∀>, I% indeterminate or <Neut-∀>, and F% <Anti-∀>.
- For any proposition <P>, there is a referential system R, such that <P> is T% <∀>, I% indeterminate or <Neut-∀>, and F% <Anti-∀>.
- <∀> is at some degree <Anti-∀>, while <Anti-∀> is at some degree <∀>.

Therefore:

For each proposition <P> there are referential systems R_1, R_2, ..., so that <P> looks different in each of them — having all possible states from <P> to <Non-P> until <Anti-P>.

And, as a consequence, for any two propositions <M> and <N>, there exist two referential systems R_M and R_N respectively, such that <M> and <N> look the same.

The referential systems are like mirrors of various curvatures reflecting the propositions.

Mottos

- All is possible, the impossible too!
- Nothing is perfect, not even the perfect!

Fundamental theory

Every idea <A> tends to be neutralized, diminished, balanced by <Non-A> ideas (which includes, besides Hegel's <Anti-A>, the <Neut-A> too) — as a state of equilibrium. In between <A> and <Anti-A> there are infinitely many <Neut-A> ideas, which may balance <A> without necessarily <Anti-A> versions.

To neutralise an idea one must discover all of its three sides: of sense (truth), of nonsense (falsity), and of undecidability (indeterminacy), then reverse/combine them. Afterwards, the idea will be classified as neutrality.

Delimitation from other philosophical concepts and theories

(a) Neutrosophy is based not only on analysis of oppositional propositions, as *dialectic* does, but on analysis of neutralities in between them as well.

(b) While *epistemology* studies the limits of knowledge and justification, neutrosophy passes these limits and places under the magnifying glass not only the defining features and substantive conditions of an entity <E>, but the whole <E′> derivative spectrum in connection with

<Neut-E>. Epistemology studies philosophical contraries, e. g. <E> versus <Anti-E>, neutrosophy studies <Neut-E> versus <E> and versus <Anti-E>, which means logic based on neutralities.

(c–d) *Neutral monism* asserts that ultimate reality is neither physical nor mental. Neutrosophy considers a more than pluralistic viewpoint: infinitely many separate and ultimate substances making up the world.

(e) *Hermeneutics* is the art or science of interpretation, while neutrosophy also creates new ideas and analyzes a widely ranging ideational field by balancing unstable systems and unbalancing stable systems.

(f) *Philosophia Perennis* expounds the common truth of contradictory viewpoints, neutrosophy combines with the truth of neutral ones as well.

(g) *Fallibilism* attributes uncertainty to every class of beliefs or propositions, while neutrosophy accepts 100% true assertions, and 100% false assertions as well and, moreover, checks in what referential systems the percent of uncertainty approaches zero or 100.

Evolution of an idea

<A> in the world is not cyclic (as Marx said), but discontinuous, knotted, boundless:

<Neut-A> = existing ideational background, before the arising of <A>;

<Pre-A> = a pre-idea, a forerunner of <A>;

<Pre-A′> = the spectrum of <Pre-A> versions;

<A> = the idea itself, which implicitly gives birth to <Non-A> = what is outside <A>;

<A′> = the spectrum of <A> versions after (mis)interpretations (mis)understanding by different people, schools, cultures;

<A/Neut-A> = spectrum of <A> derivatives/deviations, because <A> partially mixes/melts first with neutral ideas;

<Anti-A> = the exact opposite of <A>, developed inside of <Non-A>;

<Anti-A′> = the spectrum of <Anti-A> versions after (mis)interpretations (mis)understanding by different people, schools, cultures;

<Anti-A/Neut-A> = the spectrum of <Anti-A> derivatives/deviations, which means partial <Anti-A> and partial <Neut-A> combined in various percentages;

<A′/Anti-A′> = the spectrum of derivatives/deviations after mixing <A′> and <Anti-A′> spectra;

<Post-A> = after <A>, a post-idea, a conclusiveness;

<Post-A′> = the spectrum of <Post-A> versions;

<Neo-A> = <A> retaken in a new form, at a different level, in new conditions, as in a non-regular curve with inflection points, in evolute and involute periods, in a recurrent mode; the life of <A> restarts.

Marx's "spiral" of evolution is replaced by a more complex differential curve with ups-and-downs, with knots — because evolution means cycles of involution too.

This is *dynaphilosophy* = the study of the infinite path of an idea.

<Neo-A> has a larger sphere (including, besides parts of old <A>, parts of <Neut-A> resulting from previous combinations), more characteristics, is more heterogeneous (after combinations with various <Non-A> ideas). But, <Neo-A>, as a whole in itself, has the tendency to homogenize its content, and then to de-homogenize by mixture with other ideas.

And so on, until the previous <A> gets to a point where it paradoxically incorporates the entire <Non-A>, being indistinct from the whole. And this is the point where the idea dies, can not be distinguished from others. The Whole breaks down, because the motion is characteristic of it, in a plurality of new ideas (some of them containing grains of the original <A>), which begin their life in a similar way.

Thus, in time, <A> comes to mix with <Neut-A> and <Anti-A>.

1.3 Neutrosophic subjects

1. Neutrosophic topology including neutrosophic metric spaces and smooth topological spaces.
2. Neutrosophic numbers and arithmetical operations, including ranking procedures for neutrosophic numbers.
3. Rough sets, neutrosophic rough sets, rough neutrosophic sets.
4. Neutrosophic relational structures, including neutrosophic relational equations, neutrosophic similarity relations and neutrosophic orderings.
5. Neutrosophic geometry.
6. Uncertainty theories including possibility and necessity theory, plausibility and belief measures, imprecise probabilities.
7. Logical operations, including n-norms, n-conorms, neutrosophic implicators, neutrosophic quantifiers.
8. Measures of neutrosophication.
9. Deneutrosophication techniques.
10. Neutrosophic measures and neutrosophic integrals.
11. Neutrosophic multivalued mappings.
12. Neutrosophic differential calculus.
13. Neutrosophic mathematical morphology.

Applications

Neutrosophic relational databases, neutrosophic image processing, neutrosophic linguistic variables, neutrosophic decision making and preference structures, neutrosophic expert systems, neutrosophic reliability theory, neutrosophic soft computing techniques in e-commerce and e-learning

1.4 Neutrosophic logic. The origin of neutrosophy

As an alternative to the existing logics we propose a non-classical one, which represents a mathematical model of uncertainty, vagueness, ambiguity, imprecision, undefined, unknown, incompleteness, inconsistency, redundancy, contradiction.

Definition

A logic in which each proposition is estimated to have the percentage of truth in a subset T, the percentage of indeterminacy in a subset I, and the percentage of falsity in a subset F, where T, I, F are defined above, is called *Neutrosophic Logic*.

We use a subset of truth (or indeterminacy, or falsity), instead of a number only, because in many cases we are not able to exactly determine the percentages of truth and of falsity but only approximate them: for example a proposition is between 30–40% true and between 60–70% false, or even worse: between 30–40% or 45–50% true (according to various analyzers), and 60% or between 66–70% false.

The subsets are not necessarily intervals, but can be any sets (discrete, continuous, open or closed or half-open/half-closed intervals, intersections or unions of the previous sets, etc.) in accordance with the given proposition.

A subset may have one element only in special cases of this logic.

Constants: (T, I, F) truth-values, where T, I, F are standard or non-standard subsets of the non-standard interval $]^{-}0, 1^{+}[$, where $n_{inf} = \inf \mathrm{T} + \inf \mathrm{I} + \inf \mathrm{F} \geqslant 0$, and $n_{sup} = \sup \mathrm{T} + \sup \mathrm{I} + \sup \mathrm{F} \leqslant 3^{+}$.

Atomic formulae: a, b, c, ...

Arbitrary formulae: A, B, C, ...

Neutrosophic logic is a formal frame endeavouring to qunatify truth, indeterminacy, and falsehood. There are many neutrosophic rules of inference (Dezert, 2002 [18]).

History

Classical Logic, also called Bivalent Logic for its taking only two the values 0, 1, or Boolean Logic after the British mathematician George Boole (1815–64), was called by the philosopher Quine in 1981 [19] “sweet simplicity”.

Peirce, before 1910, developed a semantic for three-valued logic in an unpublished note, but Emil Post's dissertation (1920's) is cited for originating the three-valued logic. Here 1 is used for truth, 1/2 for indeterminacy, and 0 for falsehood. Reichenbach, the leader of logical empiricism, also studied it.

Three-valued logic was employed by Halldán in 1949 [20], Körner in 1960 [21], and Tye in 1994 [22] to solve Sorites Paradoxes. They used truth tables, such as Kleene's, but everything depended on the definition of validity.

A three-valued paraconsistent system (LP) has the values: "true", "false", and "both true and false". The ancient Indian metaphysics considered four possible values of a statement: "true (only)", "false (only)", "both true and false", and "neither true nor false". J. M. Dunn in 1976 [23] formalized this in a four-valued paraconsistent system in his First Degree Entailment semantics;

Buddhist logic added a fifth value to the previous ones, "none of these" (called *catushkoti*).

The 0, $a_1, \ldots, a_n$, 1 Multi-Valued, or Plurivalent Logic, was developed by Lukasiewicz, while Post originated the m-valued calculus.

The many-valued logic was replaced by Goguen in 1969 [24] and Zadeh in 1975 [25, 26] with an Infinite-Valued Logic (of continuum power, as in classical mathematical analysis and classical probability) called Fuzzy Logic, where the truth-value can be any number in the closed unit interval [0, 1]. The Fuzzy Set was introduced by Zadeh in 1975.

Therefore, we finally generalize the fuzzy logic to a transcendental logic, called "neutrosophic logic": where the interval [0, 1] is exceeded, i. e., the percentages of truth, indeterminacy, and falsity are approximated by non-standard subsets, not by single numbers, and these subsets may overlap and exceed the unit interval in the sense of non-standard analysis. Furthermore, the superior sum, $n_{sup} = \sup \mathrm{T} + \sup \mathrm{I} + \sup \mathrm{F} \in \,]^{-}0, 3^{+}[$, may be as high as 3 or 3^{+}, while the inferior sum $n_{inf} = \inf \mathrm{T} + \inf \mathrm{I} + \inf \mathrm{F} \in \,]^{-}0, 3^{+}[$, may be as low as 0 or -0.

The idea of tripartition (truth, falsehood, indeterminacy) appeared in 1764 when J. H. Lambert investigated the credibility of one witness affected by the contrary testimony of another. He generalized Hooper's rule of combination of evidence (1680's), which was a Non-Bayesian approach to find a probabilistic model. In the 1940's Koopman introduced the notions of lower and upper probability, followed by Good, and in 1967 Dempster [27] gave a rule for combining two arguments. Shafer in 1976 [28] extended it to the Dempster-Shafer Theory of Belief Functions by defining the Belief and Plausibility functions and using the rule of inference of Dempster for combining two pieces of evidence obtained from two different sources. A Belief Function is a connection between fuzzy reasoning and probability. The Dempster-Shafer Theory of Belief Functions is a generalization of Bayesian Probability (Bayes 1760's, Laplace 1780's). It uses mathematical probability in a more general way, and is based on probabilistic combination of evidence in artificial intelligence.

In Lambert "there is a chance p that the witness will be faithful and accurate, a chance q that he will be mendacious, and a chance $1-p-q$ that he will simply be careless", according to Shafer [29]. Thus there are three components: accuracy, mendacity, and carelessness, which add up to 1.

Van Fraassen [30] introduced the supervaluation semantics in his attempt to solve the Sorites paradoxes, followed by Dummett in 1975 [31] and Fine in 1975 [32]. They all tripartitioned, considering a vague predicate which, having border cases, is undefined for these border cases. Van Fraassen took the vague predicate "heap" and extended it positively to those objects to which the predicate definitively applies and negatively to those objects to which it definitively does not apply. The remaining objects border was called the penumbra. A sharp boundary between these two extensions does not exist for a soritical predicate. Inductive reasoning is no longer valid either; if S is a Sorites predicate, the proposition $\exists\,\mathrm{n}\,(\mathrm{Sa}_n - - \mathrm{Sa}_{n+1})$ is false. Thus, the predicate Heap (positive extension) = true, Heap (negative extension) = false, Heap (penumbra) = indeterminate.

Narinyani in 1980 [33] used the tripartition to define what he called the "indefinite set", and Atanassov in 1982 [34] extended the tripartition and gave five generalizations of the fuzzy set, and studied their properties and applications to neural networks in medicine:

(a) **Intuitionistic Fuzzy Set (IFS):** Given a universe E, an IFS A over E is a set of ordered triples <universe-element, degree-of-membership-to-A(M), degree-of-non-membership-to-A(N)> such that $\mathrm{M}+\mathrm{N} \leqslant 1$ and M, $\mathrm{N} \in [0,1]$. When $\mathrm{M}+\mathrm{N} = 1$ one obtains the fuzzy set, and if $\mathrm{M}+\mathrm{N} < 1$ there is an indeterminacy $\mathrm{I} = 1 - \mathrm{M} - \mathrm{N}$.

(b) **Intuitionistic L-Fuzzy Set (ILFS):** Is similar to IFS, but M and N belong to a fixed lattice L.

(c) **Interval-Valued Intuitionistic Fuzzy Set (IVIFS):** Is similar to IFS, but M and N are subsets of [0, 1] and $\sup \mathrm{M} + \sup \mathrm{N} \leqslant 1$.

(d) **Intuitionistic Fuzzy Set of Second Type (IFS2):** Is similar to IFS, but $\mathrm{M}^2 + \mathrm{N}^2 \leqslant 1$. M and N are inside the upper right quadrant of the unit circle.

(e) **Temporal IFS:** Is similar to IFS, but M and N are in addition, functions of the time-moment.

Neutrosophic logic is an attempt to unify many logics in a single field. Yet, too large a generalization may sometimes have no practical impact. Such attempts at unification have been throughout the history of science.

1.5 Definitions of neutrosophics

Neutrosophic Logic is a general framework for unification of many existing logics. The main idea of NL is to characterize each logical statement in a 3D Neutrosophic Space, where each dimension of the space represents respectively the truth (T), the falsehood (F), and the indeterminacy (I) of the statement under consideration, where T, I, F are standard or non-standard real subsets of $]^{-}0, 1^{+}[$. For software engineering proposals the classical unit interval $[0, 1]$ can be used. T, I, F are independent components, leaving room for incomplete information (when their superior sum < 1), paraconsistent and contradictory information (when the superior sum > 1), or complete information (sum of components $= 1$). By way of example, a statement can be between $[0.4, 0.6]$ in truth, $\{0.1\}$ or between $(0.15, 0.25)$ indeterminant, and either 0.4 or 0.6 false.

Neutrosophic Set. Let U be a universe of discourse, and M a set included in U. An element x from U is denoted with respect to the set M as x (T, I, F) and belongs to M in the following way: it is t% true in the set, i% indeterminate (unknown if it is) in the set, and f% false, where t varies in T, i varies in I, f varies in F. Statically T, I, F are subsets, but dynamically T, I, F are functions/operators depending on many known or unknown parameters.

Neutrosophic Probability is a generalization of the classical probability and imprecise probability in which the chance that an event A occurs is t% true — where t varies in the subset T, i% indeterminate — where i varies in the subset I, and f% false — where f varies in the subset F. In classical probability n_sup $[1$, while in neutrosophic probability n_sup $[3^{+}$. In imprecise probability, the probability of an event is a subset T_$[0, 1]$, not a number $p\ \chi\ [0, 1]$, and what is left is supposed to be the opposite, a subset F (also from the unit interval $[0, 1]$). There is no indeterminate subset I in imprecise probability.

Neutrosophic Statistics is the analysis of events described by the neutrosophic probability. The function that models the neutrosophic probability of a random variable x is called the neutrosophic distribution: $\mathrm{NP}(x) = \big(\mathrm{T}(x), \mathrm{I}(x), \mathrm{F}(x)\big)$, where $\mathrm{T}(x)$ represents the probability that value x occurs, $\mathrm{F}(x)$ represents the probability that value x does not occur, and $\mathrm{I}(x)$ represents the indeterminant / unknown probability of the variable x.

Neutrosophy is a new branch of philosophy that studies the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra. The neutrosophics were introduced by Dr. F. Smarandache in 1995. This theory considers every notion or idea <A> together with its opposite or negation <Anti-A> and the spectrum of "neutralities" <Neut-A> (i. e. notions or ideas located between the two extremes, supporting neither <A> nor <Anti-A>). The <Neut-A> and <Anti-A> ideas together are referred to as <Non-A>. According to this theory every idea <A> tends to be neutralized and balanced by <Anti-A> and <Non-A> ideas — as a state of equilibrium.

Neutrosophy is the basis of neutrosophic logic, neutrosophic set, neutrosophic probability and statistics used in engineering applications (especially for software and information fusion), medicine, military, cybernetics,

Chapter 2

TRAJECTORIES AND PARTICLES

2.1 Einstein's basic space-time

What is a four-dimensional pseudo-Riemannian space, which is the basic space-time of the General Theory of Relativity?

As it is well-known, Euclidean geometry is set forth by Euclid's five axioms:

1. Given two points there is an interval that joins them;
2. An interval can be prolonged indefinitely;
3. A circle can be constructed when its centre, and a point on it, are given;
4. All right angles are equal;
5. If a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which the angles are less than the two right angles.

Non-Euclidean geometries are derived from making assumptions which deny some of the Euclidean axioms. Three main kinds of non-Euclidean geometries are conceivable — Lobachevsky-Bolyai-Gauss geometry, Riemann geometry, and Smarandache geometries. They can be illustrated by the following example: let's consider two rays connected altogether by the common perpendicular. In a space of Euclidean geometry the rays are infinitely parallel (i. e. they never intersect). In the Lobachevsky-Bolyai-Gauss geometry space the rays diverge. Such a geometric space is known as hyperbolic (from the Greek *hyperballein* — "to throw beyond"). In a space of Riemann geometry the rays converge and ultimately intersect. Such a geometric space is known as elliptic (from the Greek *elleipein* — "to fall short").

In a Smarandache geometric space we may have altogether two or three of the previous geometric cases, i. e. either rays which are infinitely parallel in a subspace, or other rays which diverge in another subspace, or again other rays which converge in a different subspace of the same space.

The second Euclidean axiom asserts that an interval can be prolonged indefinitely. The fifth axiom says that, if a line meets two other lines so that the two angles the crossed lines make on one side of it are together less than two right angles, the other lines, if prolonged indefinitely, will meet on this side.

In Lobachevsky-Bolyai-Gauss (hyperbolic) geometry the fifth axiom is denied. In Riemann (elliptic) geometry, the fifth Euclidean axiom stated above is satisfied formally, because there are no lines parallel to the given line. But if we state the fifth axiom in a broader form, such as "through a point not on a given line there is only one line parallel to the given line", the fifth axiom is also denied in Riemann geometry. Besides the fifth axiom, the second axiom is also denied in Riemann geometry, because herein the straight lines are closed: a circle or, alternatively, an infinitely long straight line is possible but then all other straight lines are of the same infinite length.

"Because it is impossible in practice to measure how far apart the rays will be when extended millions of miles, it is quite conceivable that man is living in a non-Euclidean universe. Because intuition is developed from relatively limited observations, it is not to be trusted in this regard. In such a world, railroad tracks can still be equidistant, but not necessarily perfectly straight" [35].

To illustrate non-Riemannian geometries in the best way we have to look at the sum of the angles of a triangle. So, it is 180° in Euclidean geometry, while it less than 180° in hyperbolic geometry and more than 180° in elliptic geometry. In Smarandache geometries the sum of the angles of a triangle can be either 180° in a subspace and less or bigger than 180° in another subspace because these geometries can be partially Euclidean and partially non-Euclidean, and so, different from 180°. Actually, Euclidean geometry is a geometry in the plane. Lobachevsky-Bolyai-Gauss geometry is a geometry on a hyperbolic surface. Riemann geometry is a geometry on the surface of a sphere. Smarandache geometries are geometries whose space includes as subspaces, combinations of these.

Riemannian geometry is the generalization of Riemann geometry, so that in a space of Riemannian geometry (we consider a space of n dimensions):

(1) The differentiable field of a non-degenerate symmetric tensor of the 2nd rank $g_{\alpha\beta}$ is given by

$$g_{\alpha\beta} = \begin{pmatrix} g_{00} & g_{01} & \cdots & g_{0n} \\ g_{10} & g_{11} & \cdots & g_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n0} & g_{n1} & \cdots & g_{nn} \end{pmatrix}, \tag{2.1}$$

$$g_{\alpha\beta} = g_{\beta\alpha}\,, \tag{2.2}$$

$$g = \det \|g_{\alpha\beta}\| \neq 0\,, \tag{2.3}$$

so that the distance ds between any two infinitely close points in the space are given by the non-degenerate quadratic differential form

$$ds^2 = \sum_{1 \leqslant \alpha, \beta \leqslant n} g_{\alpha\beta}(x)\, dx^\alpha dx^\beta = g_{\alpha\beta}\, dx^\alpha dx^\beta, \tag{2.4}$$

known as the Riemann metric. According to this definition, the tensor $g_{\alpha\beta}$ is called the fundamental metric tensor, and its components define the geometrical structure of the space;

(2) The space curvature may take different numerical values at different points in the space.

Actually, a Riemann geometry space is the space of the Riemannian geometry family, where the curvature is constant and has positive numerical value [36].

In the particular case where the fundamental metric tensor $g_{\alpha\beta}$ takes strictly diagonal form

$$g_{\alpha\beta} = \begin{pmatrix} 1 & 0 & \dots & 0 \\ 0 & 1 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \dots & 1 \end{pmatrix}, \tag{2.5}$$

the Riemannian space becomes Euclidean.

Pseudo-Riemannian spaces are specific kinds of Riemannian spaces, where the fundamental metric tensor $g_{\alpha\beta}$ (and also the Riemannian metric ds^2) has sign-alternating form so that the diagonal components of the metric tensor bear numerical values of opposite sign, for instance

$$g_{\alpha\beta} = \begin{pmatrix} 1 & g_{01} & \dots & g_{0n} \\ g_{10} & -1 & \dots & g_{1n} \\ \vdots & \vdots & \ddots & \vdots \\ g_{n0} & g_{n1} & \dots & -1 \end{pmatrix}. \tag{2.6}$$

Here the prefix "pseudo" is used to distinguish Riemannian spaces of the sign-alternating metric from Riemannian spaces whose metric is definite-signed.

Einstein's basic space-time of the General Theory of Relativity is a four-dimensional pseudo-Riemannian space having the sign-alternating signature $(+---)$ or $(-+++)$, which reserves one dimension for time $x^0 = ct$ whilst the other three dimensions $x^1 = x$, $x^2 = y$, $x^3 = z$ are reserved for three-dimensional space, so that the space metric is

$$ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta = g_{00}\, c^2 dt^2 + 2g_{0i}\, c\, dt\, dx^i + g_{ik}\, dx^i dx^k. \tag{2.7}$$

In general, there is no real difference between the signatures used — $(+---)$ or $(-+++)$. Each of them has its own advantages and drawbacks. For instance, Landau and Lifshitz in their *The Classical Theory of Fields* [37] use the signature $(-+++)$, where time is imaginary while spatial coordinates are real so that the three-dimensional *coordinate impulse* (the spatial part of the four-dimensional impulse vector) is real. We, following Eddington [38], use the signature $(+---)$, where time is real while spatial coordinates are imaginary, because in this case the three-dimensional *observable impulse*, being the projection of the four-dimensional impulse vector on an observer's spatial section, is real. But all these are only pure mathematical tricks for getting some profit in calculations and representing the results in a more conceivable form.

In the particular case where the fundamental metric tensor $g_{\alpha\beta}$ of the four-dimensional pseudo-Riemannian space takes the strictly diagonal form

$$g_{\alpha\beta}=\begin{pmatrix}1&0&0&0\\0&-1&0&0\\0&0&-1&0\\0&0&0&-1\end{pmatrix}, \tag{2.8}$$

the space becomes four-dimensional pseudo-Euclidean. Its metric is

$$ds^2=g_{\alpha\beta}\,dx^\alpha dx^\beta=c^2dt^2-dx^2-dy^2-dz^2, \tag{2.9}$$

and such a four-dimensional pseudo-Euclidean space is known as Minkowski's space, because he had introduced it first. It is the basic space-time of the Special Theory of Relativity.

In the general case a four-dimensional pseudo-Riemannian space is curved, inhomogeneous, gravitating, rotating, and deforming (any or all of these properties may be anisotropic).

The first property implies that the space may be of a non-zero curvature. Here there are four cases: (1) the four-dimensional curvature $K\neq 0$ and the three-dimensional curvature $C\neq 0$; (2) the four-dimensional curvature $K\neq 0$, while the three-dimensional $C=0$; (3) the four-dimensional curvature $K=0$, while the three-dimensional curvature $C\neq 0$; (4) the four-dimensional curvature $K=0$ and the three-dimensional curvature $C=0$.

The space inhomogenity implies that Christoffel's symbols (the space coherence coefficients) are non-zero.

The presence of gravitational potential* implies inhomogeneity of time references — values of the zero component g_{00} of the fundamental metric tensor $g_{\alpha\beta}$ of the space are different at each point of the space.

If the space rotates, the space-time (mixed) components g_{0i} of the fundamental metric tensor are non-zero. From the geometrical viewpoint this implies that time lines become locally non-orthogonal to the three-dimensional spatial section. Such spaces are known as *non-holonomic*, in contrast to *holonomic* (free of rotation) spaces.

*It should be noted that the presence of gravitational potential does not necessarily imply hat forces of gravity are also present. For instance, in a homogeneous gravitational field the potential can be very strong, but no forces of gravity (the potential gradient) exit there because the field is homogeneous.

If the space produces deformations, its metric is non-stationary so the derivative of the metric tensor with respect to time is not zero (in different directions).

All space properties are linked to one another by different equations, proceeding from Riemannian geometry.

2.2 Standard set of trajectories and particles. A way to expand the set

Each particle, located in a four-dimensional pseudo-Riemannian space, has its own four-dimensional trajectory (world-trajectory). No two different particles having the same world-trajectory exist. So its own trajectory characterizes all specific properties of the particle moving in it, distinguishing this particle from other particles located in the space-time. Hence, as many specific kinds of trajectories exist in the space-time as specific kinds of particles inhabit the space-time.

From the purely mathematical viewpoint each world-trajectory is characterized by two four-dimensional vectors (world-vectors):

(1) A vector Q^α tangential to it at each of its points;

(2) And also by a vector N^α orthogonal to it at each of its points.

The first of them is the derivative of the world-coordinate increment along the trajectory with respect to a parameter ρ which could be monotone and non-zero along the trajectory

$$Q^\alpha = \varepsilon \frac{dx^\alpha}{d\rho}\,, \qquad \alpha,\mu,\nu = 0,1,2,3 \tag{2.10}$$

where ε is a parameter making each point of the trajectory a particle moving on it, so the parameter ε could be a scalar which is a world-invariant like rest-mass, etc. The second vector, which is orthogonal to the trajectory, is the absolute derivative of the previous

$$N^\alpha = \frac{\mathrm{D}Q^\alpha}{d\rho} = \frac{dQ^\alpha}{d\rho} + \Gamma^\alpha_{\mu\nu} Q^\mu \frac{dx^\nu}{d\rho}\,, \tag{2.11}$$

which is different to the regular differential dQ^α owing to the presence of Christoffel's symbols of the 2nd kind $\Gamma^\alpha_{\mu\nu}$ — the coherence coefficients of the given Riemannian space. The Christoffel symbols of the 2nd kind are calculated through the Christoffel symbols (the coherence coefficients) of the 1st kind $\Gamma_{\mu\nu,\rho}$ and they are functions of the first derivatives of the fundamental metric tensor $g_{\alpha\beta}$, namely

$$\Gamma^\alpha_{\mu\nu} = g^{\alpha\rho}\,\Gamma_{\mu\nu,\rho}\,, \quad \Gamma_{\mu\nu,\rho} = \frac{1}{2}\left(\frac{\partial g_{\mu\rho}}{\partial x^\nu} + \frac{\partial g_{\nu\rho}}{\partial x^\mu} - \frac{\partial g_{\mu\nu}}{\partial x^\rho}\right). \tag{2.12}$$

Motion of a particle in a Riemannian space, in particular in a four-dimensional pseudo-Riemannian space, is a parallel transfer of the particle's own four-dimensional vector Q^α tangential to its trajectory. In this process the vector N^α, which is normal to the particle's trajectory, also undergoes parallel transfer. Parallel transfer in Riemannian spaces is of the Levi-Civita's kind, where the square of any transferred vector remains unchanged along the entire trajectory

$$Q_\alpha Q^\alpha = \text{const}, \tag{2.13}$$

$$N_\alpha N^\alpha = \text{const}. \tag{2.14}$$

Paths of each family may be *geodesic* or *non-geodesic*. A geodesic trajectory is the shortest path between any two points in the space. The laws of mechanics (both Classical Mechanics and relativistic) require that a particle affected by gravitational fields moves along shortest (geodesic) line. Such motion is known as *geodesic motion*. If the particle is affected by additional forces of non-gravitational origin, the latter causes the particle to diverge from its geodesic trajectory, so the motion becomes non-geodesic.

Equations of motion along geodesic world-trajectories, known as *geodesic equations*, are actually given by $N^\alpha = 0$, i. e.

$$\frac{dQ^\alpha}{d\rho} + \Gamma^\alpha_{\mu\nu} Q^\mu \frac{dx^\nu}{d\rho} = 0\,, \tag{2.15}$$

while equations of motion along non-geodesic world-lines $N^\alpha \neq 0$ include a deviating "non-geodesic" force on the right side.

In general, all world-trajectories can be split into different kinds by numerical values of the space-time interval ds along each of them: $ds^2 > 0$, $ds^2 = 0$, or $ds^2 < 0$. So, considering the possible trajectories we arrive at a standard set of three kinds of known trajectories. Actually these are three different regions of the basic space-time, each of which has world-trajectories and particles of its own kind, specific for only this region. So we have trajectories as follows:

Non-isotropic real trajectories, which lay "within" the light hypercone in the well-known Minkowski diagram. Along such trajectories the square of the space-time interval $ds^2 > 0$, so the interval ds itself is real. These are trajectories of regular mass-bearing particles which, having non-zero rest-masses $m_0 > 0$, move at sub-light velocities $\mathrm{v} < c$ so that their relativistic masses $m = \frac{m_0}{\sqrt{1-\mathrm{v}^2/c^2}}$ are real. Each particle moving along such a trajectory is characterized by its own four-dimensional impulse vector

$$P^\alpha = m_0 \frac{dx^\alpha}{ds} = \frac{m}{c}\frac{dx^\alpha}{d\tau}\,, \tag{2.16}$$

which is tangential to the trajectory at any of its points. The square of the vector is constant like that of any transferred vector in Riemannian spaces, and its length is equal to the rest-mass of the particle

$$P_\alpha P^\alpha = m_0^2 g_{\alpha\beta} \frac{dx^\alpha}{ds}\frac{dx^\beta}{ds} = m_0^2 = \text{const}\,. \tag{2.17}$$

Isotropic trajectories, which lie on the surface of the light hyper-cone and are trajectories of particles with zero rest-mass (massless light-like particles), which travel at the light velocity $\mathrm{v} = c$. The relativistic mass $m = \frac{m_0}{\sqrt{1-\mathrm{v}^2/c^2}}$ and energy $E = mc^2$ of any massless particle, being indeterminate like $\frac{0}{0}$, are non-zero. Photons, having zero rest-mass, bear non-zero relativistic masses and energies. Along isotropic trajectories the space-time interval is zero $ds = 0$, but the time interval as well as the three-dimensional interval are non-zero. Because $ds^2 = 0$ there, the space-time interval ds cannot be used as a parameter for differentiation along isotropic trajectories. For this reason one of two other parameters may be used there [39, 40, 41], which are an interval of physical observable time $d\tau$ and an observable interval of three-dimensional length $d\sigma$. The quantities $d\tau$ and $d\sigma$ are defined in the reference frame of an observer who accompanies his references, as projections of the increment of four-dimensional coordinates dx^α on the observer's time line and the spatial section

$$d\tau = \frac{1}{c}\, b_\alpha dx^\alpha = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}} dx^i, \tag{2.18}$$

$$d\sigma^2 = \left(-g_{\alpha\beta} + b_\alpha b_\beta\right) dx^\alpha dx^\beta = \left(-g_{ik} + \frac{g_{0i}\, g_{0k}}{g_{00}}\right) dx^i dx^k, \tag{2.19}$$

where b^α is the projection operator on the observer's time line, $h_{\alpha\beta} = -g_{\alpha\beta} + b_\alpha b_\beta$ is the projection operator on his spatial section [39, 40, 41]. The world-vector b^α is actually the four-dimensional velocity of the observer with respect to his references; $b^i = 0$ in the accompanying reference frame considered. As a result the space-time interval is

$$ds^2 = b_\alpha b_\beta\, dx^\alpha dx^\beta - h_{\alpha\beta}\, dx^\alpha dx^\beta = c^2 d\tau^2 - d\sigma^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right). \tag{2.20}$$

As soon as $\mathrm{v} = c$, $ds^2 = c^2 d\tau^2 - d\sigma^2 = 0$. Thus a massless particle moving along a isotropic trajectory has the four-dimensional impulse vector P^α, taken by the non-zero parameter $cd\tau = d\sigma$, so the vector takes the non-zero form

$$P^\alpha = m_0 \frac{dx^\alpha}{ds} = \frac{m_0}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} \frac{dx^\alpha}{cd\tau} = m \frac{dx^\alpha}{d\sigma}\,, \tag{2.21}$$

while its square as well as the square of any isotropic vector is zero

$$P_\alpha P^\alpha = m^2 g_{\alpha\beta} \frac{dx^\alpha}{d\sigma} \frac{dx^\beta}{d\sigma} = m^2 \frac{ds^2}{d\sigma^2} = 0\,. \tag{2.22}$$

Non-isotropic imaginary trajectories lie "outside" the light hyper-cone. Along such trajectories the square of the space-time interval is $ds^2 < 0$, so the ds is imaginary. These are trajectories of super-light tachyon particles (from the Greek *tachus* — speedy), and they have imaginary relativistic masses $m = \dfrac{im_0}{\sqrt{\mathrm{v}^2/c^2 - 1}}$ [42, 43]. Each tachyon is characterized by its own four-dimensional impulse vector P^{α}, which having the same form as that of the impulse vector $P^{\alpha} = m_0 \dfrac{dx^{\alpha}}{ds}$ of a regular sub-light particle, takes imaginary numerical values. Its square is also $P_{\alpha} P^{\alpha} = m_0^2 = \text{const}$.

We collect the main characteristics of each kind of trajectory and the associated particles altogether into Table 1.

From these three kinds of mass-bearing particles those moving at sub-light velocities and the massless light-like particles (photons) moving at the velocity of light form everyday reality. Super-light moving particles (tachyons) have never been observed.

The theory of physical observable quantities [39, 40] section of the General Theory of Relativity, shows that super-light moving tachyons are unobservable from the viewpoint of the regular observer located in sub-light regions.

Numerous researchers, beginning with Paul Dirac, have predicted that particles bearing masses and energies inhabit a mirror Universe, as the antipode to our Universe. That is because relativistic masses of particles in our Universe are positive, whereas particles in the mirror Universe must evidently be negative.

Joseph Weber [44] wrote that neither Newton's law of gravitation nor the relativistic theory of gravitation rule out the existence of negative masses, but our empirical experience has not found them. Both the Newtonian theory of gravitation and Einstein's General Theory of Relativity predict the behavior of negative masses as totally different from what electrodynamics prescribes for negative charges. For two bodies, one of which bears positive mass and the other bearing a negative one, but equal to the first one in absolute value, it would be expected that positive mass will attract the negative one, while the negative mass will repel the positive one, so that one will chase the other! If motion occurs along a line which links the centers of the two bodies, such a system will move with a constant acceleration. This problem had been studied by Bondi [45]. Assuming the gravitational mass of the positron to be negative (observations show that its inertial mass is positive) and using the methods of Quantum Electrodynamics, Schiff found that there is a difference between the inertial mass of the positron and its gravitational mass. The difference proved to be much greater than the margin of error in well-known Eötvös' experiment, which showed equality of gravitational and inertial masses [46]. Consequently, Schiff concluded that a negative gravitational mass for the positron cannot exist (see Chapter 1 of Weber's book [44]).

Besides, "co-habitation" of positive and negative masses in the same space-time region would cause ongoing annihilation. Possible consequences of particles of a "mixed" kind, which bear both positive and negative masses, were also studied by Terletski [47, 48].

Anyway, from the purely mathematical viewpoint, all three standard

Table 1: Standard set of world-trajectories and particles located in Einstein's basic space-time.

Kind	Trajectories/particles	Area	Tangential vector Q^α	Normal vector N^α
I	Non-isotropic sub-light trajectories — sub-light mass-bearing particles	$ds^2 > 0$	$Q_\alpha Q^\alpha = m_0^2$	$N_\alpha N^\alpha = 0$, $N_\alpha N^\alpha \neq 0$ $\left\{ \begin{array}{l} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{array} \right.$
II	Isotropic trajectories — massless light-like speed particles	$ds^2 = 0$	$Q_\alpha Q^\alpha = 0$	$N_\alpha N^\alpha = 0$, $N_\alpha N^\alpha \neq 0$ $\left\{ \begin{array}{l} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{array} \right.$
III	Non-isotropic super-light trajectories — super-light mass-bearing tachyons	$ds^2 < 0$	$Q_\alpha Q^\alpha = m_0^2$	$N_\alpha N^\alpha = 0$, $N_\alpha N^\alpha \neq 0$ $\left\{ \begin{array}{l} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{array} \right.$

Table 2: Additional kinds of world-trajectories located in Einstein's basic space-time.

Kind	Trajectory kind	Region	Particle kind
I/II	Non-isotropic sub-light/isotropic light-like trajectories	$ds^2 = \text{undef}$	Real mass-bearing/massless light-like particles
II/III	Isotropic light-like/non-isotropic super-light trajectories	$ds^2 = \text{undef}$	Massless light-like/imaginary mass-bearing particles

kinds of trajectories/particles given in Table 1 are surely present in Einstein's basic space-time — a four-dimensional pseudo-Riemannian space. So in looking at Table 1 we can ask the question:

Question 1 Is this list of trajectory/particle types complete for Einstein's basic space-time, or not?

In answering this question we should take into account that Riemannian spaces have the property of continuity by definition. For this reason any pseudo-Riemannian space, being a specific case of Riemannian spaces, is continuous.

Answer The Neutrosophic method, considering the Einstein basic space-time continuous, answers the question thus — no, this list is incomplete. Besides the standard three kinds of trajectories/particles two additional "intermediate" kinds should exist: "non-isotropic/isotropic" trajectories of the I/II kind, common to sub-light mass-bearing particles and light-like massless particles (photons), and also "isotropic/non-isotropic" trajectories of the II/III kind common to light-like massless photons and super-light mass-bearing tachyons.

Non-isotropic/isotropic trajectories of the I/II kind, in which the square of the space-time interval may take numerical values $ds^2 \geqslant 0$. Such trajectories, having common properties for sub-light and light-like trajectories, are partially non-isotropic and isotropic. So particles moving along such trajectories should be of a mixed "real-mass/light-like" kind possessing properties partially of sub-light (real) mass-bearing particles and also of light-like massless particles (photons).

Isotropic/non-isotropic trajectories of the II/III kind, in which the square of the space-time interval may take numerical values $ds^2 \leqslant 0$. Such trajectories, having common properties for light-like and super-light trajectories, are partially isotropic and non-isotropic. So particles moving along such trajectories should be of a mixed "light-like/tachyon" kind possessing properties partially of light-like massless particles (photons) and of super-light mass-bearing tachyons.

Surely trajectories of such "mixed" kinds should exist, because any four-dimensional pseudo-Riemannian space is continuous everywhere. From this two questions arise:

Question 2 What are particles of the mixed I/II kind, possessing common properties of sub-light mass-bearing particles and light-like particles like photons?

Question 3 What are particles of the mixed II/III kind, possessing common properties of light-like particles (like photons) and super-light mass-bearing tachyons?

In relation to the Question 2 and Question 3 let us recall that each particle moving along a world-trajectory is characterized by its own four-dimensional vector, which actually is a vector Q^{α} tangential to the trajectory. Its absolute derivative is the vector N^{α} normal to the trajectory. Setting $N^{\alpha}=0$ gives the equations of geodesic motion; if $N^{\alpha}\neq 0$ the motion is non-geodesic (the right side contains a deviating "non-geodesic" force). So the vectors Q^{α} and N^{α} together define all the properties of the particle and its motion in the space-time in which it is located.

Therefore, looking at Questions 2 and 3 from purely mathematical perspective, we can reformulate them together in the another way:

Question 4 What are the tangential vector Q^{α} and the normal vector N^{α} to trajectories of "mixed" non-isotropic/isotropic kinds?

Such trajectories, which we have herein predicted through a neutrosophic method, were unknown until now. Answering Question 4 in the next paragraph, we will see that trajectories and particles of the mixed kind I/II surely exist in theory. They were merely unconsidered heretofore. Moreover, as we will see in final paragraphs of this Chapter, particles moving along trajectories of these kinds can be observable in experiments.

2.3 Itroducing trajectories of mixed isotropic/non-isotropic kind

In this paragraph we are going to answer Question 4: what are the tangential vector Q^{α} and the normal vector N^{α} to trajectories of "mixed" non-isotropic/isotropic kinds located in Einstein basic space-time? To answer this question it is actually required to find:

(1) The mathematical definition of such "mixed" trajectories;

(2) Specific properties of particles moving in such "mixed" trajectories.

Here we will study only one kind of "mixed" trajectory. We are going to study trajectories of the mixed I/II kind (see Table 2 on page 27) — non-isotropic/isotropic trajectories, which are common for mass-bearing particles moving at sub-light velocities and also light-like massless particles (photons).

It is first required to introduce such mixed trajectories. We will do this one by one. So let us get started. Here is the first question:

Question 5 Can a non-isotropic trajectory and an isotropic trajectory intersect, so that they can have a common point?

To answer this question, let us refer to §6 in Chapter I of Synge's well-known book *Relativity: the General Theory*. Therein Synge wrote, "Isotropic geodesics play a very important part in the General Theory of Relativity, because most astronomical data are obtained using optical observations, so they are obtained by received photons while photon is moving along isotropic geodesic lines in the space-time. . .

Let C_1 and C_2 be time-like (sub-light-speed) arcs in the space-time, not necessarily geodesics although they could be geodesics. Suppose the arcs show motions of an observer and of a source of light. Let P_1 be a point on the arc C_1. The total sum of isotropic geodesics transecting the arc in the point P_1 is an *isotropic cone*. There are two areas: one of them is known as the *past area*, the other — the *future area*. Here we consider only the past area. The other arc C_2 transects this area in a point P_2, so we can say that the isotropic cone maps the P_1 into the P_2. Thus the whole arc C_1 is mapped into the curve C_2 meaning that every point located in C_1 is mapped into a point located in C_2, and vice versa. . . The total sum of those isotropic geodesics builds a two-dimensional space, defined by the arcs C_1 and C_2" [49].

So here is the answer:

Answer Yes, a non-isotropic trajectory can be intersected by an isotropic trajectory, so they can have a common point.

We will also take into account a "theorem on a geodesic trajectory passing through a point in a given direction", see §6 in Petrov's book *Einstein Spaces* [50]:

Theorem For any given point there is only one geodesic trajectory passing through this point in a given direction.

From this we now formulate a set of theorems that we will call "theorems on intersections between non-isotropic and isotropic trajectories in a pseudo-Riemannian space". Here and below, just as it was considered by Synge, non-isotropic and isotropic trajectories are not necessarily geodesics, although they could be geodesics.

A Set of Theorems about Intersections between Non-Isotropic and Isotropic Trajectories in a Pseudo-Riemannian Space

Theorem 1 Given a non-isotropic trajectory, each point located on it is passed by at least one isotropic trajectory. So this point of intersection is common for the non-isotropic and isotropic trajectories.

Theorem 2 An isotropic trajectory meets infinitely many non-isotropic trajectories, having at least one common point of intersection with each of them.

Theorem 3 Given a surface, whose elements are non-isotropic trajectories, each point located on it is passed by at least one isotropic trajectory. So this point of intersection is common for the non-isotropic surface and the isotropic trajectory.

Theorem 4 An isotropic trajectory meets infinitely many non-isotropic surfaces, having at least one common point of intersection with each of them.

Theorem 5 Given a sub-space, whose elements are non-isotropic trajectories and surfaces, each point located in it is passed by at least one isotropic trajectory. So this point of intersection is common for the non-isotropic sub-space and the isotropic trajectory.

Theorem 6 An isotropic trajectory meets infinitely many non-isotropic sub-spaces, having at least one common point of intersection with each of them.

If only one of these theorems was false, the space would have at least one omitted point, so that the continuity property would be broken, and therefore the space would not be continuous.

Thus, with Theorems 1–6 as a basis, we continue this set of theorems with:

Theorem 7 Given a non-isotropic surface, whose elements are non-isotropic trajectories, each line located on it is passed by at least one isotropic surface, whose elements are isotropic trajectories. So this line of intersection is common for the non-isotropic surface and the isotropic surface.

Theorem 8 An isotropic surface (two-dimensional isotropic space) meets infinitely many non-isotropic surfaces (two-dimensional non-isotropic spaces), having at least one common line of intersection with each of them.

Taking the previous theorems into account, we finish our set of theorems with:

Theorem 9 Given any point in a pseudo-Riemannian space there is a common trajectory for the non-isotropic sub-space and an isotropic sub-space that can be chosen.

Theorem 10 Given any point in a pseudo-Riemannian space there are infinitely many common trajectories for the non-isotropic and isotropic sub-spaces, passing through this point.

Thus trajectories of the mixed isotropic/non-isotropic kind are introduced by the set of Theorems 1–10. If no such trajectories existed, the pseudo-Riemannian space would not be continuous.

It should be noted that the foregoing is specific for only pseudo-Riemannian spaces, owing to their sign-alternating metrics. In a Riemannian space whose metric is sign-definite, no isotropic lines, surfaces, or subspaces exist, and so the set of theorems would be senseless.

However they are all true in pseudo-Riemannian spaces. In particular, it is true in the four-dimensional pseudo-Riemannian space at the base of the General Theory of Relativity. Therefore we shall continue to study trajectories of the isotropic/non-isotropic kind.

2.4 Particles moving along mixed isotropic/non-isotropic trajectories

We are going to find physical characteristics of particles moving along mixed non-isotropic/isotropic trajectories, so we will study the tangential vector and the normal vector to the trajectory of such particles.

Non-isotropic trajectories are occupied by mass-bearing particles; their rest-masses $m_0 \neq 0$ and relativistic masses $m \neq 0$. Isotropic trajectories are occupied by massless light-like particles — photons, their rest-masses $m_0 = 0$ while their relativistic masses $m \neq 0$.

Question 6 What properties could be attributed to particles moving along mixed non-isotropic/isotropic trajectories? What common properties of mass-bearing particles and massless light-like particles could there be from the purely geometrical viewpoint?

To answer this question let us consider the vector tangential to the trajectory of a particle.

As mentioned in paragraph 2.3, according to today's physical concepts a particle located in a four-dimensional pseudo-Riemannian space (the basic space-time of the General Theory of Relativity) is characterized by it own four-dimensional vector impulse P^α, which is tangential to the particle's trajectory at every point. This vector for a mass-bearing particle ($m_0 \neq 0$, $m \neq 0$) is

$$P^\alpha = m_0 \frac{dx^\alpha}{ds}\,, \tag{2.23}$$

while for a massless light-like particle ($m_0 = 0$, $m \neq 0$) this vector takes the form

$$P^\alpha = m \frac{dx^\alpha}{d\sigma}\,. \tag{2.24}$$

Here, along non-isotropic trajectories (mass-bearing particles), the space-time interval $ds \neq 0$ is used as a differential parameter, while along isotropic trajectories (massless particles) $ds = 0$ so the differential parameter is the three-dimensional observable interval $d\sigma \neq 0$. For a mass-bearing particle moving along both geodesic (shortest) and non-geodesic trajectories the square of the particle impulse vector P^α is non-zero

$$P_\alpha P^\alpha = g_{\alpha\beta} P^\alpha P^\beta = m_0^2 = \text{const} \neq 0\,, \tag{2.25}$$

so in this case P^α is non-isotropic vector. This fact is independent of the particle's trajectory being geodesic (the normal vector is zero $N^\alpha = 0$) or non-geodesic ($N^\alpha \neq 0$). The square of the impulse vector of a massless light-like particle is zero, so its impulse vector P^{α} is isotropic

$$P_{\alpha}P^{\alpha} = g_{\alpha\beta}\,P^{\alpha}P^{\beta} = m\frac{g_{\alpha\beta}\,dx^{\alpha}dx^{\beta}}{d\sigma^2} = m\frac{ds^2}{d\sigma^2} = 0 \tag{2.26}$$

independent of the light-like particle's motion being along a geodesic trajectory ($N^{\alpha} = 0$) or a non-isotropic one ($N^{\alpha} \neq 0$).

Calculation of contravariant (upper-index) components of the particle's impulse vector P^{α} gives

$$P^0 = m\frac{dt}{d\tau}\,, \tag{2.27}$$

$$P^i = \frac{m}{c}\frac{dx^i}{d\tau} = \frac{1}{c}\,m\mathrm{v}^i\,. \tag{2.28}$$

The formula $\frac{dt}{d\tau}$ can be obtained from the square of the four-dimensional velocity vector of a particle U^{α}, which for sub-light speed, light speed and super-light speed is, respectively

$$g_{\alpha\beta}U^{\alpha}U^{\beta} = +1\,, \qquad U^{\alpha} = \frac{dx^{\alpha}}{ds} \qquad ds = c\,d\tau\sqrt{1-\frac{\mathrm{v}^2}{c^2}}\,, \tag{2.29}$$

$$g_{\alpha\beta}U^{\alpha}U^{\beta} = 0\,, \qquad U^{\alpha} = \frac{dx^{\alpha}}{d\sigma} \qquad ds = 0\,, \qquad d\sigma = c\,d\tau\,, \tag{2.30}$$

$$g_{\alpha\beta}U^{\alpha}U^{\beta} = -1\,, \qquad U^{\alpha} = \frac{dx^{\alpha}}{|ds|} \qquad |ds| = c\,d\tau\sqrt{\frac{\mathrm{v}^2}{c^2}-1}\,. \tag{2.31}$$

Let us substitute into these formulae the space self-rotation three-dimensional velocity v_i and the particle's three-dimensional velocity v^i, defined in the reference frame of an observer who accompanies his references [39, 40, 41] as follows

$$v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}\,, \qquad \mathrm{v}^i = \frac{dx^i}{d\tau} \tag{2.32}$$

and also the formula for the observable metric tensor, defined in the same observer's reference frame,

$$h_{ik} = -g_{ik} + \frac{1}{c^2}\,v_i v_k\,. \tag{2.33}$$

Using these definitions in each formula for $g_{\alpha\beta}\,U^{\alpha}U^{\beta}$ we obtain three quadratic equations with respect to $\frac{dt}{d\tau}$. They are the same for sub-light, light-like and super-light velocities

$$\left(\frac{dt}{d\tau}\right)^2 - \frac{2v_i\mathrm{v}^i}{c^2\left(1-\frac{\mathrm{w}}{c^2}\right)}\frac{dt}{d\tau} + \frac{1}{\left(1-\frac{\mathrm{w}}{c^2}\right)^2}\left(\frac{1}{c^4}\,v_i v_k \mathrm{v}^i\mathrm{v}^k - 1\right) = 0\,. \tag{2.34}$$

This quadratic equation has the two solutions

$$\left(\frac{dt}{d\tau}\right)_{1,2} = \frac{1}{1-\frac{\mathrm{w}}{c^2}}\left(\frac{1}{c^2}\,v_i\mathrm{v}^i \pm 1\right). \tag{2.35}$$

The function $\frac{dt}{d\tau}$ allows us to define what direction in time the particle takes. If $\frac{dt}{d\tau} > 0$ then the temporal coordinate t increases, i. e. the particle moves from the past into the future (direct flow of time). If $\frac{dt}{d\tau} < 0$ then temporal coordinate decreases, i. e. the particle moves from the future into the past (reverse flow of time).

The quantity $1-\frac{\mathrm{w}}{c^2} = \sqrt{g_{00}} > 0$, because the other cases $\sqrt{g_{00}} = 0$ and $\sqrt{g_{00}} < 0$ contradict the signature condition $(+---)$. Therefore the coordinate time t stops, $\frac{dt}{d\tau} = 0$, provided

$$v_i\mathrm{v}^i = -c^2, \qquad v_i\mathrm{v}^i = +c^2. \tag{2.36}$$

The coordinate time t has direct flow, $\frac{dt}{d\tau} > 0$, if in the first and in the second solutions, respectively

$$\frac{1}{c^2}\,v_i\mathrm{v}^i + 1 > 0\,, \qquad \frac{1}{c^2}\,v_i\mathrm{v}^i - 1 > 0\,. \tag{2.37}$$

The coordinate time t has reverse flow, $\frac{dt}{d\tau} < 0$, when

$$\frac{1}{c^2}\,v_i\mathrm{v}^i + 1 < 0\,, \qquad \frac{1}{c^2}\,v_i\mathrm{v}^i - 1 < 0\,. \tag{2.38}$$

For sub-light speed particles $v_i\mathrm{v}^i < c^2$ is always true. Hence direct flow of time for regularly observed mass-bearing particles takes place under the first condition (2.37) while reverse flow of time takes place under the second condition (2.38).

It is to be noted that we looked at the problem of the direction of coordinate time t assuming that physical observable time is $d\tau > 0$ always.

Now, using above formulae, we calculate the covariant (lower-index) component P_i as well as the projection of the four-dimensional impulse vector P^α on the time line

$$P_i = -\frac{m}{c}\left(\mathrm{v}_i \pm v_i\right), \tag{2.39}$$

$$\frac{P_0}{\sqrt{g_{00}}} = \pm m\,, \tag{2.40}$$

where the relativistic mass $+m$ takes a place in observation of a particle that moves into the future (the direct flow of time), while the value $-m$ takes a place in observation of a particle moving into the past (the reverse flow of time).

We can show the same in representing a particle as a wave of inherent frequence ω (see [37], for instance). In such an approach each massless particle could be represented by its own four-dimensional wave vector, tangential to the trajectory.

$$K^{\alpha} = \frac{\omega}{c}\frac{dx^{\alpha}}{d\sigma}\,. \tag{2.41}$$

Its square is zero, because it is an isotropic vector, tangential to an isotropic trajectory

$$K_{\alpha}K^{\alpha} = g_{\alpha\beta}K^{\alpha}K^{\beta} = \frac{\omega^2}{c^2}\frac{g_{\alpha\beta}\,dx^{\alpha}dx^{\beta}}{d\sigma^2} = \frac{\omega^2}{c^2}\frac{ds^2}{d\sigma^2} = 0\,. \tag{2.42}$$

We can write K^{α} down as

$$K^{\alpha} = \frac{\omega}{c}\frac{dx^{\alpha}}{d\sigma} = \frac{k}{c}\frac{dx^{\alpha}}{d\tau}\,, \tag{2.43}$$

where $k = \frac{\omega}{c}$ is the wave number. From this formula we can see that physical observable time τ can be used instead of the three-dimensional observable interval σ as the differential parameter.

Calculating the contravariant component of the wave vector K^{α} gives

$$K^0 = k\frac{dt}{d\tau}\,, \qquad K^i = \frac{k}{c}\frac{dx^i}{d\tau} = \frac{1}{c}\,k\mathrm{v}^i\,, \tag{2.44}$$

where $k\mathrm{v}^i$ is the three-dimensional wave vector of the massless particle.

Substituting the formula obtained earlier for $\frac{dt}{d\tau}$, we obtain the component K_i and the projection of the four-dimensional wave vector K^{α} on time

$$K_i = -\frac{k}{c}\left(\mathrm{v}_i \pm v_i\right), \tag{2.45}$$

$$\frac{K_0}{\sqrt{g_{00}}} = \pm k\,, \tag{2.46}$$

where $+k$ takes a place in observation of a light-like particle moving into the future (the direct flow of time), while $-k$ would be observed when a light-like particle moves into the past (the reverse flow of time).

As it easy to see, this gives the same effect as that resulting from the four-dimensional impulse vector of both mass-bearing and massless light-like particles.

Therefore, physical observable quantities, in terms of the observed four-dimensional impulse vector, are: relativistic mass $\pm m$ and the three-dimensional value $\frac{1}{c}m\mathrm{v}^i$, where $p^i = m\mathrm{v}^i$ is the three-dimensional vector of observable impulse (for massless light-like particles $p^i = mc^i$, where c^i is the three-dimensional vector of the light velocity).

Conclusion Particles having properties "between" mass-bearing particles and massless particles should be of zero relativistic mass $m = 0$, because only in the particular case $m = 0$ can mass-bearing particles and massless particles have a common nature. Given that, whether the motion is geodesic or non-geodesic does not effect this peculiarity.

We have thus obtained the answer to Question 6, as follows:

Answer A particle moving along trajectories of the common isotropic/non-isotropic kind could be of zero rest-mass $m_0 = 0$ and also zero relativistic mass $m = 0$ while its four-dimensional impulse vector, tangential to the trajectory, should be strictly non-zero $P^\alpha \neq 0$. The vector N^α normal to the trajectory of a such particle could be both zero (geodesic motion) and non-zero (non-geodesic motion).

For this reason properties of particles moving along common isotropic/non-isotropic trajectories in comparison to properties of regular mass-bearing and massless particles should be as follows — see Table 3.

2.5 S-denying the signature conditions. Classification of the expanded spaces

In a four-dimensional pseudo-Riemannian space of signature $(+---)$ or $(-+++)$ there are *four signature conditions* which define this space as pseudo-Riemannian. The higher the dimension of a space the more signature conditions there are. So the basic space-time of the General Theory of Relativity, being such a space, is defined by the aforementioned four signature conditions.

Here is a question:

Question 7 What happens if we S-deny one of the four signature conditions in the basic space-time of the General Theory of Relativity? What happens if we postulate that one of the signature conditions is to be denied in two ways, or, alternatively, to be true and false?

We will S-deny every signature condition one by one. We consider first the space where the 1st signature condition is S-denied; then the space where the 2nd signature condition is S-denied, and so on. So we study four cases; in each of them one of the four signature conditions will be S-denied.

Looking at the foundations of Smarandache geometries where S-denying* was introduced (see [7]–[13] for references), here is the solution:

Answer If we S-deny one of the four signature conditions in the basic space-time of the General Theory of Relativity, we obtain a new basic space-time of an expanded kind. Such a space-time will be partially Riemannian and partially not. There are four main kinds of such expanded spaces, due to four possible cases where one of the signature conditions is S-denied. The other kinds of such expanded spaces are "mixtures" between the four main ones.

*Smarandachely denying.

Table 3: Properties of different kind particles.

Kind	Particles	Rest-mass	Rel. mass	Tangen. vector	Normal vector
I	Sub-light mass-baring particles	$m_0 \neq 0$	$m \neq 0$	$\begin{cases} P^\alpha \neq 0 \\ P_\alpha P^\alpha = m_0^2 \end{cases}$	$\begin{array}{l} N_\alpha N^\alpha = 0 \\ N_\alpha N^\alpha \neq 0 \end{array} \begin{cases} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{cases}$
I/II	Particles of mixed kind	$m_0 = 0$	$m = 0$	$\begin{cases} P^\alpha \neq 0 \\ P_\alpha P^\alpha \geqslant 0 \end{cases}$	$\begin{array}{l} N_\alpha N^\alpha = 0 \\ N_\alpha N^\alpha \neq 0 \end{array} \begin{cases} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{cases}$
II	Massless light-like particles	$m_0 = 0$	$m \neq 0$	$\begin{cases} P^\alpha \neq 0 \\ P_\alpha P^\alpha = 0 \end{cases}$	$\begin{array}{l} N_\alpha N^\alpha = 0 \\ N_\alpha N^\alpha \neq 0 \end{array} \begin{cases} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{cases}$
II/III	Particles of mixed kind	$m_0 = 0$	$m = 0$	$\begin{cases} P^\alpha \neq 0 \\ P_\alpha P^\alpha \geqslant 0 \end{cases}$	$\begin{array}{l} N_\alpha N^\alpha = 0 \\ N_\alpha N^\alpha \neq 0 \end{array} \begin{cases} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{cases}$
III	Super-light mass-bearing tachyons	$m_0 \neq 0$	$m \neq 0$	$\begin{cases} P^\alpha \neq 0 \\ P_\alpha P^\alpha = m_0^2 \end{cases}$	$\begin{array}{l} N_\alpha N^\alpha = 0 \\ N_\alpha N^\alpha \neq 0 \end{array} \begin{cases} N^\alpha = 0 \text{ (geodesics)} \\ N^\alpha \neq 0 \text{ (non-geodesics)} \end{cases}$

Following this logical procedure, resulting from a purely mathematical viewpoint of the basic space-time, more queries arise:

Question 8 What happens if we S-deny all four signature conditions in the basic space-time of the General Theory of Relativity?

Answer We obtain the fifth kind of an expanded space-time for the General Theory of Relativity. Such a space-time will also be partially Riemannian and partially not.

Here we are going to consider each of the five kinds of expanded spaces.

Starting from a pure mathematical viewpoint, the signature conditions in a four-dimensional pseudo-Riemannian space are derived from the fundamental metric tensor $g_{\alpha\beta}$ of this space, namely from the sign-alternation in its diagonal terms g_{00}, g_{11}, g_{22}, g_{33} in the matrix

$$g_{\alpha\beta} = \begin{pmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{pmatrix}. \tag{2.47}$$

From a physical perspective, the signature conditions are derived from the fact that the three-dimensional (spatial) observable interval

$$d\sigma^2 = h_{ik}\, dx^i dx^k \tag{2.48}$$

must be strictly positive. Hence the three-dimensional observable metric tensor

$$h_{ik} = -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}\,, \tag{2.49}$$

being the three-dimensional matrix

$$h_{ik} = \begin{pmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{pmatrix} \tag{2.50}$$

defined in an observer's reference system accompanying its physical references (reference body), must satisfy three evident conditions

$$\det \| h_{11} \| = h_{11} > 0\,, \tag{2.51}$$

$$\det \begin{Vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{Vmatrix} = h_{11}\, h_{22} - h_{12}^2 > 0\,, \tag{2.52}$$

$$\det \begin{Vmatrix} h_{11} & h_{12} & h_{13} \\ h_{21} & h_{22} & h_{23} \\ h_{31} & h_{32} & h_{33} \end{Vmatrix} > 0\,, \tag{2.53}$$

see §84 in *The Classical Theory of Fields* [37] for details.

From these conditions we obtain the signature conditions in the fundamental metric tensor's matrix (2.47). Therefore, in a space of signature $(+---)$, the *first signature condition* is

$$\det \| g_{00} \| = g_{00} > 0\,, \tag{2.54}$$

the *second signature condition* is

$$\det \begin{Vmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{Vmatrix} = g_{00}\, g_{11} - g_{01}^2 < 0\,, \tag{2.55}$$

the *third signature condition* is

$$\det \begin{Vmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & g_{22} \end{Vmatrix} > 0\,, \tag{2.56}$$

and, at last, the *fourth signature condition* is

$$g = \det \| g_{\alpha\beta} \| = \det \begin{Vmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{Vmatrix} < 0\,. \tag{2.57}$$

An expanded basic space-time of kind I

In such a space-time the first signature condition $g_{00} > 0$ (2.54) is S-denied, while the other signature conditions (2.55, 2.56, 2.57) remain unchanged. Namely, given the expanded space-time of kind I, the first signature condition is S-denied in the following form

$$\det \| g_{00} \| = g_{00} \geqslant 0\,, \tag{2.58}$$

which includes two particular cases

$$g_{00} > 0\,, \qquad g_{00} = 0\,, \tag{2.59}$$

or the initial first signature condition $g_{00} > 0$ (2.54) is partially true and partially not at any point of such space, in other words the first signature condition is true for some points ($g_{00} > 0$) and false for others ($g_{00} = 0$).

What is the space-time from a physical viewpoint? Landau and Lifshitz in their *The Classical Theory of Fields* wrote, "nonfulfilling of the condition $g_{00} > 0$ would only mean that the corresponding system of reference cannot be accomplished with real bodies; if the condition on the principal values is fulfilled, then a suitable transformation of the coordinates can make g_{00} positive (an example of such a system is given by the rotating system of coordinates)" [37].

The General Theory of Relativity defines gravitational potential as follows [39, 40, 41]:

$$\mathrm{w} = c^2 \left(1 - \sqrt{g_{00}}\right). \tag{2.60}$$

Let us begin with this well-known definition. Then the first signature condition in its S-denied form $g_{00} \geqslant 0$ (2.58) has the physical sense that in such a space-time two different physical states occur

$$1 - \frac{\mathrm{w}}{c^2} > 0\,, \qquad 1 - \frac{\mathrm{w}}{c^2} = 0\,, \tag{2.61}$$

or, in other words

$$\mathrm{w} < c^2\,, \qquad \mathrm{w} = c^2\,. \tag{2.62}$$

The first one corresponds to the regular space-time state, where the first signature condition is $g_{00} > 0$. The second corresponds to a special state of the space-time, where the first signature condition is simply denied $g_{00} = 0$. Because we have both conditions together in the same expanded space-time of kind I, we call equation

$$\mathrm{w} \leqslant c^2 \tag{2.63}$$

the *physical condition of S-denying the first signature condition*.

From this we get that, when in an expanded space of kind I the first signature condition $g_{00} > 0$ is broken, i. e. $g_{00} = 0$, the space-time is in a state where the condition

$$\mathrm{w} = c^2 \tag{2.64}$$

is true. The equality $\mathrm{w} = c^2$ is the well-known condition under which gravitational collapse occurs (called the *collapse condition*). So we come to the following conclusion for expanded spaces of kind I:

Conclusion on expanded spaces of kind I: An expanded space-time of kind I ($g_{00} \geqslant 0$) is merely the generalization of the basic space-time of the General Theory of Relativity ($g_{00} > 0$), including regions where this space-time is in with the collapse state ($g_{00} = 0$).

An expanded basic space-time of kind II

In such a space-time the second signature condition (2.55) is S-denied, the other signature conditions (2.54, 2.56, 2.57) remain unchanged. Thus, given the expanded space-time of kind II, the second signature condition is S-denied in the following form

$$\det \begin{Vmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{Vmatrix} = g_{00}\, g_{11} - g_{01}^2 \leqslant 0\,, \tag{2.65}$$

which includes two different cases

$$g_{00}\, g_{11} - g_{01}^2 < 0\,, \qquad g_{00}\, g_{11} - g_{01}^2 = 0\,, \tag{2.66}$$

whence the second initial signature condition $g_{00}\, g_{11} - g_{01}^2 < 0$ (2.55) is partially true and partially not at each point of the space.

Let's consider the second signature condition S-denied in the form $g_{00}\, g_{11} - g_{01}^2 \leqslant 0$ (2.65) from a physical viewpoint.

The component g_{00} is defined by gravitational potential $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$. The component g_{0i} is defined by the space rotation linear velocity (see [39, 40, 41] for details)

$$v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}\,, \qquad v^i = -c\,g^{0i}\sqrt{g_{00}}\,, \qquad v_i = h_{ik}\, v^k. \tag{2.67}$$

The component g_{ik} can be obtained from the representation of the fundamental metric tensor in basis vectors in the fashion described below.

We are going to use a *local geodesic frame of reference*. The fundamental metric tensor within infinitesimal vicinities of any point of such a frame is

$$\tilde{g}_{\mu\nu} = g_{\mu\nu} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\mu\nu}}{\partial \tilde{x}^\rho \partial \tilde{x}^\sigma}\right)(\tilde{x}^\rho - x^\rho)\,(\tilde{x}^\sigma - x^\sigma) + \dots\ , \tag{2.68}$$

i. e. the values of its components in the vicinities of a point are different from those of this point itself, which are only different by factors of 2nd order of smallness, which can be neglected. Therefore at any point of the local geodesic frame of reference the fundamental metric tensor (up to 2nd order) is a constant, while the first derivatives of the metric, i. e. the Christoffel symbols, are zero [39].

Certainly, within infinitesimal vicinities of any point of a Riemannian space, a local geodesic frame of reference can be defined. Subsequently, at any point of the local geodesic frame of reference a tangential flat space can be defined so the local geodesic frame of reference of the Riemannian space is a global geodesic one for that flat space. Because the metric tensor is constant in a flat space, in the vicinities of a point of the Riemannian space the values $\tilde{g}_{\mu\nu}$ converge to values of that tensor $g_{\mu\nu}$ in the tangential flat space. That means that in the tangential flat space we can build a system of basic vectors $\vec{e}_{(\alpha)}$ tangential to the curved coordinate lines of the Riemannian space. Because coordinate lines of any Riemannian space can be generally curved and in a non-holonomic space are not even orthogonal to each other, the lengths of the basis vectors are sometimes substantially different from unity.

Let $d\vec{r}$ be a four-dimensional infinitesimal displacement vector $d\vec{r} = (dx^0, dx^1, dx^2, dx^3)$. Then $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, where the components are

$$\begin{aligned} \vec{e}_{(0)} &= (e^0_{(0)}, 0, 0, 0), \qquad & \vec{e}_{(1)} &= (0, e^1_{(1)}, 0, 0), \\ \vec{e}_{(2)} &= (0, 0, e^2_{(2)}, 0), \qquad & \vec{e}_{(3)} &= (0, 0, 0, e^3_{(3)}). \end{aligned} \tag{2.69}$$

The scalar product of the vector $d\vec{r}$ with itself gives $d\vec{r}d\vec{r}=ds^2$, i. e. the square of the four-dimensional interval. On the other hand $ds^2=g_{\alpha\beta}\,dx^\alpha dx^\beta$. Hence

$$g_{\alpha\beta}=\vec{e}_{(\alpha)}\vec{e}_{(\beta)}=e_{(\alpha)}e_{(\beta)}\cos\left(x^\alpha;x^\beta\right), \tag{2.70}$$

which facilitates a better understanding of the geometric structure of different regions within the Riemannian space and even beyond. Since according to the formula

$$g_{00}=e^2_{(0)}\,, \tag{2.71}$$

and on the other hand $\sqrt{g_{00}}=1-\frac{\mathrm{w}}{c^2}$, the length of the time basis vector $\vec{e}_{(0)}$ tangential to the time coordinate line $x^0=ct$ is

$$e_{(0)}=\sqrt{g_{00}}=1-\frac{\mathrm{w}}{c^2} \tag{2.72}$$

and is smaller than unity as the greater is the gravitational potential w. In the case of collapse ($\mathrm{w}=c^2$) the length of the time basis vector $\vec{e}_{(0)}$ becomes zero.

Then according to general formula (2.70) we have finally

$$g_{ik}=e_{(i)}e_{(k)}\cos\left(x^i;x^k\right), \tag{2.73}$$

that gives the required formula for g_{11}

$$g_{11}=e_{(1)}e_{(1)}\cos\left(x^1;x^1\right)=e^2_{(1)}\,. \tag{2.74}$$

Looking back at the second signature condition in its S-denied form (2.65) we see that the condition can be written as follows

$$g_{00}\left(g_{11}-\frac{1}{c^2}\,v_1^2\right)\leqslant 0\,. \tag{2.75}$$

If the first signature condition is not denied, so $g_{00}>0$ is true, the second signature condition in its S-denied form is

$$g_{11}-\frac{1}{c^2}\,v_1^2\leqslant 0\,, \tag{2.76}$$

having two particular cases

$$g_{11}-\frac{1}{c^2}\,v_1^2<0\,,\qquad g_{11}-\frac{1}{c^2}\,v_1^2=0\,. \tag{2.77}$$

To better see the physical sense of the condition (2.76), we take a case where $g_{11}=e^2_{(1)}$ is close to -1.* Then, denoting $v^1=v$, we obtain

$$-1-\frac{1}{c^2}\,v^2<0\,,\qquad -1-\frac{1}{c^2}\,v^2=0\,, \tag{2.78}$$

*Because if we consider the signature $(+---)$ we have $g_{11}=-1$.

which actually means

$$v^2 > -c^2\,, \qquad v^2 = -c^2\,. \tag{2.79}$$

The first condition $v^2 > -c^2$ is true in the regular basic space-time. Because the velocities v and c take positive numerical values, this condition uses the well-known fact that positive numbers are greater than negative ones.

The second condition $v^2 = -c^2$ has no place in the basic space-time; it is true as a particular case of the common condition $v^2 \geqslant -c^2$ in the expanded spaces of kind II. This condition means that as soon as the linear velocity of the space rotation reaches light velocity, the space signature changes its own kind from $(+---)$ to $(-+++)$. That is, given an expanded space-time of kind II, the transit from a non-isotropic sub-light region into an isotropic light-like region implies change of signs in the space signature — the time axis and the three-dimensional space inerchange. So we conclude for expanded spaces of kind II:

Conclusion on expanded spaces of kind II: An expanded space-time of kind II ($v^2 \geqslant -c^2$) is the generalization of the basic space-time of the General Theory of Relativity ($v^2 > -c^2$) which permits the peculiarity that the space-time changes signs in its own signature as soon as we, reaching the light velocity of the space rotation, encounter a light-like isotropic region.

An expanded basic space-time of kind III

In this space-time the third signature condition (2.56) is S-denied, the other signature conditions (2.54, 2.55, 2.57) remain unchanged. So, given the expanded space-time of kind III, the third signature condition is S-denied in the following form

$$\det \begin{Vmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & g_{22} \end{Vmatrix} \geqslant 0\,, \tag{2.80}$$

which, taking the other form of the third signature condition (2.52) into account, can be transformed into the formula

$$\det \begin{Vmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{Vmatrix} = h_{11}\,h_{22} - h_{12}^2 \geqslant 0\,, \tag{2.81}$$

that includes two different cases

$$h_{11}\,h_{22} - h_{12}^2 > 0\,, \qquad h_{11}\,h_{22} - h_{12}^2 = 0\,. \tag{2.82}$$

Thus the third initial signature condition (2.56) is partially true and partially not at any point of such a space.

This condition is not clear, because many free parameters are present. Unfortunately, we cannot conclude anything definite about specific peculiarities of expanded spaces of kind III. Future research is required.

An expanded basic space-time of kind IV

In this space-time the fourth signature condition (2.57) is S-denied, the other signature conditions (2.54, 2.55, 2.56) remain unchanged. So, given the expanded space-time of kind IV, the fourth signature condition is S-denied in the following form

$$g = \det \| g_{\alpha\beta} \| = \det \begin{Vmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{Vmatrix} \leqslant 0 \,, \qquad (2.83)$$

that includes two different cases

$$g = \det \| g_{\alpha\beta} \| < 0 \,, \qquad g = \det \| g_{\alpha\beta} \| = 0 \,. \qquad (2.84)$$

Thus the fourth initial signature condition $g < 0$ (2.57) is partially true and partially not at any point of such a space. The initial condition $g < 0$ is true in the basic space-time. The second condition $g = 0$, being the particular case of the common condition $g \leqslant 0$, could only be true in the expanded spaces of kind IV.

Because the determinant g of the fundamental metric tensor $g_{\alpha\beta}$, being taken in the reference frame of an observer who accompanies his references, depends on the determinant of the observable metric tensor h_{ik} as follows [39, 40, 41]

$$h = -\frac{g}{g_{00}} \,, \qquad (2.85)$$

the equality $g = 0$, as a degeneration of the fundamental metric tensor, implies degeneration of the observable metric tensor $h = 0$. So an expanded space-time of kind IV includes regions where the space-time metric is fully degenerate. Such a region will be referred to as a *degenerate space-time*.

In such fully degenerate areas the space-time interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, the spatial observable interval $d\sigma^2 = h_{ik}\, dx^i dx^k$ and the observable time interval become zero*

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = 0 \,, \qquad c^2 d\tau^2 = d\sigma^2 = 0 \,. \qquad (2.86)$$

The condition $d\tau^2 = 0$ means that physical observable time τ has the same value along the entire trajectory. The condition $d\sigma^2 = 0$ means that

*It should be noted that $ds^2 = 0$ is true not only at $c^2 d\tau^2 = d\sigma^2 = 0$, but also when $c^2 d\tau^2 = d\sigma^2 \neq 0$ is true in the basic space-time in light-like (isotropic) region, where light propagates.

all three-dimensional trajectories have zero length. Taking into account the definitions of $d\tau$ and $d\sigma^2$ (2.18, 2.19) in the accompanying observer's reference frame

$$d\tau = \sqrt{g_{00}}\, dt + \frac{g_{0i}}{c\sqrt{g_{00}}} dx^i, \tag{2.87}$$

$$d\sigma^2 = \left(-g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}\right) dx^i dx^k, \tag{2.88}$$

and also the fact that in the accompanying reference frame we have $h_{00} = 0$, $h_{0i} = 0$, we write the conditions $d\tau^2 = 0$ and $d\sigma^2 = 0$ as

$$cd\tau = \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] cdt = 0\,, \qquad dt \neq 0\,, \tag{2.89}$$

$$d\sigma^2 = h_{ik}\, dx^i dx^k = 0\,, \tag{2.90}$$

where $u^i = \frac{dx^i}{dt}$ is the three-dimensional coordinate velocity of a particle, which is different from its physical observable velocity $\mathrm{v}^i = \frac{dx^i}{d\tau}$.

Substituting $h_{ik} = -g_{ik} + \frac{1}{c^2} v_i v_k$ into (2.89) and dividing it by dt^2 we obtain the *physical conditions of degeneration* (2.89) and (2.90) in the final form

$$\mathrm{w} + v_i u^i = c^2, \tag{2.91}$$

$$g_{ik} u^i u^k = c^2 \left(1 - \frac{\mathrm{w}}{c^2}\right)^2, \tag{2.92}$$

where $v_i u^i$ is the scalar product of the space rotation linear velocity v_i and the coordinate velocity of the particle u^i.

Finally, we come to a conclusion on expanded spaces of kind IV:

Conclusion on expanded spaces of kind IV: An expanded space-time of kind IV ($g \leqslant 0$) is the generalization of the basic space-time of General Relativity ($g < 0$) including regions where this space-time is in a fully degenerate state ($g = 0$). Looking at such fully degenerate regions from the viewpoint of a regular observer we see that time intervals between any events inside the area are zero, and spatial intervals are zero. Thus, all the regions are actually a point.

An expanded basic space-time of kind V

In this space-time all four signature conditions (2.54, 2.55, 2.56, 2.57) are S-denied, therefore given the expanded space-time of kind V all signature conditions are S-denied as follows:

$$\det \| g_{00} \| = g_{00} \geqslant 0\,, \tag{2.93}$$

$$\det \begin{Vmatrix} g_{00} & g_{01} \\ g_{10} & g_{11} \end{Vmatrix} = g_{00}\, g_{11} - g_{01}^2 \leqslant 0\,, \tag{2.94}$$

$$\det \begin{Vmatrix} g_{00} & g_{01} & g_{02} \\ g_{10} & g_{11} & g_{12} \\ g_{20} & g_{21} & g_{22} \end{Vmatrix} \geqslant 0\,, \tag{2.95}$$

$$g = \det \| g_{\alpha\beta} \| = \det \begin{Vmatrix} g_{00} & g_{01} & g_{02} & g_{03} \\ g_{10} & g_{11} & g_{12} & g_{13} \\ g_{20} & g_{21} & g_{22} & g_{23} \\ g_{30} & g_{31} & g_{32} & g_{33} \end{Vmatrix} \leqslant 0\,, \tag{2.96}$$

so all four signature conditions are partially true and partially not at each point of the expanded space-time.

It is obvious that an expanded space of kind V contains expanded spaces of kind I, II, III, and IV as particular cases, being a common space for all of them. Taking their properties into account, we come to a conclusion on expanded spaces of kind V:

Conclusion on expanded spaces of kind V: An expanded space-time of kind V, being the common space for expanded spaces of kinds I, II, III, and IV, is the generalization of the basic space-time of General Relativity that: (1) permits its collapse, (2) has the peculiarity that its signature changes signs as soon as we, reaching the light velocity of the space rotation, encounter a light-like isotropic region, (3) permits full degeneration of the metric, when all degenerate regions become points, where all motions are instantaneous, and (4) has some other peculiarities, linked to the third signature condition (the meaning of which is not yet clear).

Negative S-denying expanded spaces and the mixed kinds

We could also S-deny the signatures with the possibility that say $g_{00} > 0$ for kind I, but this means that the gravitational potential would be imaginary, etc., or, even take into account the "mixed" cases of kind I/II, etc. But most of them are senseless from the geometrical viewpoint. Hence we have only included five main kinds in our discussion.

2.6 More on an expanded space-time of kind IV

Here we are going to consider the detailed structure of an expanded space-time of kind IV, which, being the generalization of the basic space-time of General Relativity (where the metric tensor $g_{\alpha\beta}$ is strictly non-degenerate $g < 0$), includes regions where this space-time is in a fully degenerate state ($g = 0$, a zero-space). Let us consider the conditions, which are true there

$$ds^2 = c^2 d\tau^2 - d\sigma^2 = 0\,, \qquad c^2 d\tau^2 = d\sigma^2 = 0\,. \tag{2.97}$$

The condition $d\tau^2 = 0$ implies that, from the viewpoint of a regular observer, the observable time τ has the same value along the entire trajectory inside the zero-space. The condition $d\sigma^2 = 0$ implies that all the observable spatial trajectories inside the zero-space have zero length. Taking into account the definitions of $d\tau$ (2.18) and of $d\sigma^2 = h_{ik}\,dx^i dx^k$ (2.19), we can write down the conditions $d\tau^2 = 0$ and $d\sigma^2 = 0$ in the form

$$d\tau = \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] dt = 0\,, \qquad dt \neq 0\,, \tag{2.98}$$

$$d\sigma^2 = h_{ik}\,dx^i dx^k = 0\,, \tag{2.99}$$

where the three-dimensional coordinate velocity $u^i = dx^i/dt$ of a particle is not the same as its physical observable velocity $\mathrm{v}^i = dx^i/d\tau$.

As is known, the necessary and sufficient condition of full degeneration of a quadratic metric form is equality to zero of the determinant of its metric tensor. For degeneration of the three-dimensional observable metric form $d\sigma^2 = h_{ik}\,dx^i dx^k$ this condition is $h = \det\|h_{ik}\| = 0$. The determinant of the observable metric tensor h_{ik} has the form [39, 40]

$$h = -\frac{g}{g_{00}}\,, \qquad g = \det\|\,g_{\alpha\beta}\|\,, \tag{2.100}$$

so if the three-dimensional form $d\sigma^2$ is degenerate, $h = 0$, then the four-dimensional form ds^2 is also degenerate, $g = 0$. Hence a four-dimensional space-time, where conditions (2.98) and (2.99) are true, is *a fully degenerate space-time*.

Taking into account formula (2.98), the observable metric tensor in an accompanying frame of reference is

$$h_{ik} = -g_{ik} + b_i b_k = -g_{ik} + \frac{1}{c^2}\,v_i v_k\,, \tag{2.101}$$

and we arrive at the conditions (2.98) and (2.99) in the final form

$$\mathrm{w} + v_i u^i = c^2, \qquad g_{ik}\,u^i u^k = c^2\left(1 - \frac{\mathrm{w}}{c^2}\right)^2, \tag{2.102}$$

where $v_i u^i$ is the scalar product of the velocity of rotation of space v_i and the coordinate velocity u^i of a particle located in it. We will refer to the conditions (2.102) as *the physical conditions of degeneration* of space.

If a space under the conditions of degeneration does not rotates $v_i = 0$, then the 1st condition becomes $\mathrm{w} = c^2$, so $\sqrt{g_{00}} = 0$. This implies that the gravitational potential w of the body of reference w is strong enough to make the reference space twice degenerate in the zero-space.

Using the 1st condition of degeneration in the form (2.98) we obtain the relationship between the observable velocity v^i of a particle in the zero-space and its coordinate velocity u^i there

$$\mathrm{v}^i = \frac{u^i}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)}\,, \tag{2.103}$$

so we can express the space-time interval ds^2 in a form to have the conditions of degeneration presented

$$\begin{aligned} ds^2 = c^2 d\tau^2 - h_{ik}\,dx^i dx^k = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right) = \\ = c^2 dt^2 \left\{ \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_k u^k\right)\right]^2 - \frac{u^2}{c^2} \right\}. \end{aligned} \tag{2.104}$$

In a four-dimensional pseudo-Riemannian space ($g<0$) this metric becomes the regular space-time of General Relativity. Under the conditions of degeneration, the metric becomes the zero-space metric, which is fully degenerate ($g=0$). For these reasons we can accept metric (2.104) as the metric of a four-dimensional generalized space-time ($g \leqslant 0$), consisting of the pseudo-Riemannian space and the zero-space as well.

Let us turn to the geometrical interpretation of the conditions of degeneration we have obtained. Substituting the formula for h_{ik} (2.101) into $d\sigma^2=0$, we obtain the four-dimensional metric inside the zero-space

$$ds^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\,dx^i dx^k = 0\,, \tag{2.105}$$

so in the zero-space the three-dimensional space does not rotate, while the rotation of the zero-space as a whole is present with the time term of its metric, where $\mathrm{w} = c^2 - v_i u^i$ in accordance with the 1st condition of degeneration (2.102).

Under gravitational collapse ($\mathrm{w}=c^2$) the metric inside the zero-space (2.105) becomes

$$ds^2 = g_{ik}\,dx^i dx^k = 0\,, \qquad \det\|\,g_{ik}\| = 0\,, \tag{2.106}$$

i. e. it becomes purely spatial. The fact that the quadratic form $g_{ik}dx^i dx^k$ is sign-definite leads to the fact that $g_{ik}dx^i dx^k$ can only become zero provided the determinant of the metric tensor g_{ik} becomes zero. Therefore under collapse in the zero-space its three-dimensional space becomes degenerate.

Because in the 1st condition of degeneration $\mathrm{w} + v_i u^i = c^2$ the value $v_i u^i = v\,u\cos(v_i; u^i)$ is the scalar product of the velocity of space rotation and the coordinate velocity of a particle, we see the three particular cases of gravitational fields allowable in the zero-space:

- If we have $v_i u^i > 0$, then the angle between v_i and u^i is within $\frac{3\pi}{2} < \alpha < \frac{\pi}{2}$. Because the 2nd condition of degeneration implies that $u = c\sqrt{g_{00}}$, then gravitational potential is $\mathrm{w} < c^2$ (a regular gravitational field);
- If we have $v_i u^i < 0$, then α is within $\frac{\pi}{2} < \alpha < \frac{3\pi}{2}$ and $\mathrm{w} > c^2$ (a super-strong gravitational field);
- The condition $v_i u^i = 0$ is only true when $\alpha = \frac{\pi}{2}$ or $\frac{3\pi}{2}$, or under the condition $\mathrm{w} = c^2$ (gravitational collapse).

Let us express the conditions of degeneration with the basis vectors $\vec{e}_{(\alpha)}$ taken in a flat space tangential to the given space at a given point. The basis vectors $\vec{e}_{(\alpha)}$ are tangential to the curved coordinate lines of the given space. It is evident that the tangential flat space could be placed at each point of the given space [39, 40]. Then, because $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ is the scalar product of two vectors of an infinitesimal displacement $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, we have

$$g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)}e_{(\beta)} \cos\left(x^\alpha; x^\beta\right), \tag{2.107}$$

$$\mathrm{w} = c^2\left(1 - e_{(0)}\right), \qquad v_i = -c\, e_{(i)} \cos\left(x^0; x^i\right), \tag{2.108}$$

and the 1st condition of degeneration $\mathrm{w} + v_i u^i = c^2$ becomes

$$c\, e_{(0)} = -e_{(i)} u^i \cos\left(x^0; x^i\right). \tag{2.109}$$

In this formula the time basis vector $\vec{e}_{(0)}$ is linearly dependent on from all the spatial basis vectors $\vec{e}_{(i)}$. This means degeneration of the space-time, so formula (2.109) is *the geometrical condition of degeneration*. Under gravitational collapse ($\mathrm{w} = c^2$) the length $e_{(0)} = \sqrt{g_{00}}$ of the time basis vector $\vec{e}_{(0)}$ becomes zero $e_{(0)} = 0$. In the absence of gravitational fields the length is $e_{(0)} = 1$. In intermediate cases the value $e_{(0)}$ becomes shorter as the acting gravitational field becomes stronger.

As is known, at any point of a four-dimensional pseudo-Riemannian space there exists a hyper-surface, the equation of which is $g_{\alpha\beta}\, dx^\alpha dx^\beta = 0$. This is a space-time region that hosts light-like particles. Because in this region $ds^2 = 0$, all those directions located inside it are equivalent, so the directions are isotropic. Therefore the region is commonly referred to as the isotropic cone or the light cone.

Because the metric inside the zero-space is actually zero (2.105), an isotropic cone can be set at any of its points. However, although having the equation

$$-\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k = 0\,, \tag{2.110}$$

it is not a light cone. The difference between this isotropic cone and the light cone is that the first term here, derived from the condition of degeneration $\mathrm{w} + v_i u^i = c^2$, is only typical for the zero-space. Therefore we will call it *the degenerate isotropic cone*. Note that because the specific term is the direct function of rotation of space, the degenerate isotropic cone is a cone of rotation. Under gravitational collapse the term is zero, while the remaining equation

$$g_{ik}\, dx^i dx^k = 0 \tag{2.111}$$

describes the three-dimensional degenerate hyper-surface. If we have $\mathrm{w} = 0$, then $v_i u^i = c^2$ and the equation of the degenerate isotropic cone (2.111) becomes

$$-c^2 dt^2 + g_{ik}\, dx^i dx^k = 0\,, \tag{2.112}$$

i. e. coordinate time flows evenly. The greater the gravitational potential w the "closer" the degenerate cone is to the spatial section. In the ultimate case, when $\mathrm{w}=c^2$, the degenerate cone becomes flattened over the three-dimensional space (collapses). The degenerate cone in the absence of gravitational fields ($\mathrm{w}=0$) is the most "distant" one from the spatial section.

As it is easy to see, being represented by the ratio w and c^2 the metric in a regular pseudo-Riemannian space

$$ds^2 = \left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 - 2\left(1-\frac{\mathrm{w}}{c^2}\right) v_i dx^i dt + g_{ik}\, dx^i dx^k, \qquad (2.113)$$

under gravitational collapse takes the form $g_{ik}\, dx^i dx^k = 0$, analogous to the collapsed metric of the zero-space (2.111). However it is not the same, because the additional condition $\mathrm{w}+v_i u^i = c^2$ typical for the zero-space takes no place in a pseudo-Riemannian space. The only conclusion is that the zero-space observed by a regular observer is the same as that of a doubly degenerate regular space from the viewpoint of a hypothetical observer located in the zero-space.

Assuming these we can conclude that the isotropic light cone $g_{\alpha\beta}\, dx^\alpha dx^\beta = 0$ contains the degenerate isotropic cone (2.110) as the ultimate case, which in its turn contains the collapsed degenerated isotropic cone (2.111) filled with degenerate matter inside the zero-space. This is an illustration of the fractal structure of the world presented here as a system of isotropic cones found inside each other.

As is well known, any particle in a space-time corresponds to its own world line, which sets spatial coordinates of the particle at any given moment of time. With a single mass-bearing particle of a rest-mass m_0 there is present its own four-dimensional impulse vector P^α, while a massless particle of frequency ω is present with its own four-dimensional wave vector K^α

$$P^\alpha = m_0 \frac{dx^\alpha}{ds}\,, \qquad K^\alpha = \frac{\omega}{c}\frac{dx^\alpha}{d\sigma}\,, \qquad (2.114)$$

where a differential parameter along an isotropic world line of the massless particle is the non-zero observable spatial interval $d\sigma^2 \neq 0$, because $ds^2 = 0$ there. However the vectors cannot describe a particle located in the zero-space, because both the differential parameters become zero.

To characterize a particle located in a generalized space-time ($g \leqslant 0$), consisting of a pseudo-Riemannian space and a fully degenerate space-time (the zero-space), let us express the impulse vector P^α in a form whereby the condition of degeneration is represented by

$$P^\alpha = m_0 \frac{dx^\alpha}{ds} = \frac{M}{c}\frac{dx^\alpha}{dt}\,, \qquad (2.115)$$

$$M = \frac{m_0}{\sqrt{\left[1-\frac{1}{c^2}\left(\mathrm{w}+v_k u^k\right)\right]^2 - \frac{u^2}{c^2}}}\,, \qquad (2.116)$$

where the mass M containing the 1st condition of degeneration depends not only upon the three-dimensional velocity of the particle, but upon gravitational potential w and upon the velocity of rotation v_i of space. The formula so obtained shows that a differential parameter along world lines in the generalized space-time is coordinate time t.

We can also write down the mass M (2.116) in the form

$$M = \frac{m}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)}\,, \tag{2.117}$$

which is a ratio between two values, each one equal to zero under the conditions of degeneration, however the ratio itself is not zero $M \neq 0$. This fact is not a surprise. The same is true for a relativistic mass m at the light velocity, when the mass is $m \neq 0$ being a ratio between the zero rest-mass $m_0 = 0$ and the zero relativistic root. Therefore light-like (massless) particles are the ultimate case of mass-bearing ones at $\mathrm{v} = c$, while zero-particles can be regarded the ultimate case of light-like ones at the conditions of degeneration. As a result two ultimate transitions are possible in the generalized space-time:

- *the light barrier*, to overcome which a particle should exceed the speed of light;
- *the zero-transition* for which a particle should be in a state defined by the conditions of degeneration.

Chronometrically invariant projections of the vector P^α (2.115) in the frame of reference of a regular observer are

$$\frac{P_0}{\sqrt{g_{00}}} = M\left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] = m\,, \tag{2.118}$$

$$P^i = \frac{M}{c}\,u^i = \frac{m}{c}\,\mathrm{v}^i, \tag{2.119}$$

while the remaining components are

$$P^0 = M = \frac{m}{1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)}\,, \tag{2.120}$$

$$P_i = -\frac{M}{c}\left[u_i + v_i - \frac{1}{c^2}\,v_i\left(\mathrm{w} + v_k u^k\right)\right]. \tag{2.121}$$

In the zero-space, where the conditions of degeneration (2.102) prevail, the components become

$$\frac{P_0}{\sqrt{g_{00}}} = m = 0\,, \qquad P^i = \frac{M}{c}\,u^i, \tag{2.122}$$

$$P^0 = M\,, \qquad P_i = -\frac{M}{c}\,u_i\,, \tag{2.123}$$

i. e. zero-particles bearing zero rest-mass and zero relativistic mass, bear the non-zero mass M (2.117).

Now let us consider a single particle in the generalized space-time within the wave-particle duality. In other words, we will consider the particle as a wave in terms of the geometrical optics approximation. Because the four-dimensional wave vector of a massless particle in the geometrical optics approximation is [37]

$$K_\alpha = \frac{\partial\psi}{\partial x^\alpha}\,, \tag{2.124}$$

where ψ is the wave phase (eikonal), we write the four-dimensional impulse vector P^α in a similar way

$$P_\alpha = \frac{\hbar}{c}\frac{\partial\psi}{\partial x^\alpha}\,, \tag{2.125}$$

where $\hbar$ is Planck's constant. Its chronometrically invariant projections in the generalized space-time are

$$\frac{P_0}{\sqrt{g_{00}}} = \frac{\hbar}{c^2}\frac{{}^*\partial\psi}{\partial t}\,, \qquad P^i = -\frac{\hbar}{c}\,h^{ik}\frac{{}^*\partial\psi}{\partial x^k} \tag{2.126}$$

while the other components are

$$P_i = \frac{\hbar}{c}\left(\frac{{}^*\partial\psi}{\partial x^i} - \frac{1}{c^2}\,v_i\frac{{}^*\partial\psi}{\partial t}\right), \tag{2.127}$$

$$P^0 = \frac{\hbar}{c^2\sqrt{g_{00}}}\left(\frac{{}^*\partial\psi}{\partial t} - v^i\frac{{}^*\partial\psi}{\partial x^i}\right). \tag{2.128}$$

From these the following two formulae can be obtained. The first one (2.129) links the mass M to its corresponding total energy E. The second one (2.130) links the spatial generalized impulse Mu^i to change of the wave phase ψ

$$Mc^2 = \frac{1}{1-\frac{1}{c^2}\left(\mathrm{w}+v_iu^i\right)}\,\hbar\frac{{}^*\partial\psi}{\partial t} = \hbar\,\Omega = E\,, \tag{2.129}$$

$$Mu^i = -\,\hbar\,h^{ik}\frac{{}^*\partial\psi}{\partial x^k}\,, \tag{2.130}$$

where Ω is the generalized frequency and ω is the regular frequency

$$\Omega = \frac{\omega}{1-\frac{1}{c^2}\left(\mathrm{w}+v_iu^i\right)}\,, \qquad \omega = \frac{{}^*\partial\psi}{\partial t}\,. \tag{2.131}$$

The condition $P_\alpha P^\alpha = \text{const}$ in the geometrical optics approximation is known as the eikonal equation. For the impulse vector located in the generalized space-time (2.125) the eikonal equation becomes

$$\frac{E^2}{c^2} - M^2u^2 = \frac{E_0^2}{c^2}\,, \tag{2.132}$$

where $E = mc^2$ and $E_0 = m_0 c^2$. Using the relationship (2.131) we write the impulse vector P^α (2.125) in the wave-like form

$$P^\alpha = \frac{\hbar\,\Omega}{c^3}\frac{dx^\alpha}{dt} = \frac{\hbar\,\dfrac{^*\partial\psi}{\partial t}}{c^3\left[1-\dfrac{1}{c^2}\left(\mathrm{w}+v_i u^i\right)\right]}\frac{dx^\alpha}{\partial t}\,, \tag{2.133}$$

$$P_\alpha P^\alpha = \frac{\hbar^2\Omega^2}{c^4}\left\{\left[1-\frac{1}{c^2}\left(\mathrm{w}+v_i u^i\right)\right]^2 - \frac{u^2}{c^2}\right\}, \tag{2.134}$$

which includes the conditions of degeneration. For a particle located in zero-space the condition $P_\alpha P^\alpha = 0$ is true. So, from the viewpoint of a regular observer, the square of the four-dimensional impulse vector presenting a zero-particle remains unchanged.

Taking the condition $P_\alpha P^\alpha = 0$ for the wave form of the four-dimensional impulse vector, after substituting $\omega = 0$ that is typical in the zero-space, we conclude, from the viewpoint of a regular observer, that the eikonal equation for a zero-particle is a standing wave equation

$$h^{ik}\frac{^*\partial\psi}{\partial x^i}\frac{^*\partial\psi}{\partial x^k} = 0\,, \tag{2.135}$$

In other words, a regular observer looking at zero-particles sees them as *light-like standing waves*; that is, as light-like holograms.

2.7 An expanded space-time of kind IV: a home space for virtual photons

We are now going to consider one of the advantages obtained from considering rather than a four-dimensional pseudo-Riemannian space, but its generalization, that is, an expanded space-time of kind IV. Recall that an expanded space-time of kind IV, in contrast to the basic pseudo-Riemannian space-time which is strictly non-degenerate (the determinant of the fundamental metric tensor is strictly less that zero $g < 0$), permits the space-time metric to be fully degenerate ($g = 0$). So the metric of an expanded space-time of kind IV can be non-degenerate and degenerate so that the condition $g \leqslant 0$ is true therein.

It is known that Feynman diagrams clearly show that the actual carriers of interactions between elementary particles are virtual particles — almost all the processes rely upon emission and absorption of virtual particles. Quantum Electrodynamics considers virtual particles also as particles for which, contrary to regular ones the energy/impulse relationship

$$E^2 - c^2p^2 = E_0^2\,, \tag{2.136}$$

where $E = mc^2$, $p^2 = m^2\mathrm{v}^2$, $E_0 = m_0c^2$, is not true. In other words, for virtual particles $E^2 - c^2p^2 \neq E_0^2$. In pseudo-Riemannian space the relationship

(2.136), derived from the condition $P_\alpha P^\alpha = \text{const}$ along world line

$$P_\alpha P^\alpha = g_{\alpha\beta} P^\alpha P^\beta = m_0^2 g_{\alpha\beta} \frac{dx^\alpha}{ds} \frac{dx^\beta}{ds} = m_0^2\,, \tag{2.137}$$

has the same representation as (2.136), where $p^i = m\mathrm{v}^i$ is the observable impulse vector of the particle and its square is $p^2 = h_{ik}\, p^i p^k$ (see [39, 40]). So $E^2 - c^2p^2 \neq E_0^2$ implies that the square of the four-dimensional impulse of a virtual particle does not conserve $P_\alpha P^\alpha$; $P_\alpha P^\alpha \neq \text{const}$ in parallel transfer.

As previously mentioned, for a particle located in zero-space the condition $P_\alpha P^\alpha = 0$ is true. Thus, from the viewpoint of a regular observer the square of the four-dimensional impulse vector representing a zero-particle remains unchanged. However from the viewpoint of an inner observer located inside the zero-space this is not true. The reason is that the metric of the zero-space observed by him, $d\mu^2 = g_{ik}\, dx^i dx^k$, is not invariant, because formula (2.110) of the four-dimensional metric of the zero-space gives

$$d\mu^2 = g_{ik}\, dx^i dx^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 \neq \text{inv}\,. \tag{2.138}$$

Thus, from the viewpoint of the inner observer, the square of the four-dimensional velocity of a zero-particle, becoming the spatial one,

$$U_\alpha U^\alpha = g_{ik} u^i u^k = \left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 \neq \text{const}, \tag{2.139}$$

is not conserved. This fact leads us to the conclusion that virtual particles can be equated to zero-particles in generalized space-time ($g \leqslant 0$), which permits degeneration of its own metric.

Now we are going to see what kinds of particles inhabit the zero-space. First we look at the conditions of degeneration (2.102) in the absence of gravitational fields ($\mathrm{w} = 0$). These are

$$v_i u^i = c^2, \qquad g_{ik} u^i u^k = c^2, \tag{2.140}$$

so in the absence of gravitation a zero-particle travels at the coordinate velocity, the value of which is equal to the speed of the light

$$u = \sqrt{g_{ik} u^i u^k} = c\,. \tag{2.141}$$

The first condition of degeneration

$$v_i u^i = v u \cos\left(v_i; u^i\right) = c^2, \tag{2.142}$$

including $u = c$, is true if the vectors v_i and u^i are co-directed. Hence in the absence of gravitational fields a zero-particle moves with a forward velocity equal to the speed of the light, and at the same time rotates at light speed as well. We will refer to such particles as *virtual photons*. From the viewpoint of an inner observer located inside the zero-space, the zero-space metric along their trajectories is

$$d\mu^2 = g_{ik}\,dx^i dx^k = c^2 dt^2 \neq 0\,, \tag{2.143}$$

similar to the metric $d\sigma^2 = c^2 d\tau^2 \neq 0$ along the trajectories of regular photons in a pseudo-Riemannian space.

In general, when gravitational fields are present ($\mathrm{w} \neq 0$), the conditions of degeneration (2.102) become

$$v_i u_*^i = c^2\,, \quad u_*^i = \frac{dx^i}{dt_*}\,, \quad t_* = \left(1 - \frac{\mathrm{w}}{c^2}\right) t\,, \tag{2.144}$$

$$u_*^2 = g_{ik}\,u_*^i u_*^k = g_{ik}\frac{dx^i}{dt_*}\frac{dx^k}{dt_*} = c^2, \tag{2.145}$$

i. e. the zero-particles rotate and move inside zero-space with a velocity equal to the velocity of light. Hence, they are *virtual photons* as well.

Note that considering virtual mass-bearing particles is senseless, because all particles in the zero-space possess zero rest-mass by definition. Therefore only virtual photons and their varieties are virtual particles.

Virtual particles in the state of collapse ($\mathrm{w} = c^2$) will be referred as *virtual collapsars*. For them the conditions of degeneration (2.102) become

$$v_i u^i = 0\,, \qquad g_{ik}\,dx^i dx^k = 0\,, \tag{2.146}$$

which could manifest in two particular cases: (1) zero-collapsars either at rest with respect to the observer located in the zero-space, so that the world around him collapses into a point — all the $dx^i = 0$; (2) the three-dimensional metric inside the zero-space degenerates, $\det\|g_{ik}\| = 0$.

However from perspective of an outside observer, who is a regular observer located in an Earth-bound laboratory, the observable velocity $\mathrm{v}^i = dx^i/d\tau$ of a zero-particle both a virtual photon and a virtual collapsar is infinite. This is true because the 1st condition of degeneration $\mathrm{w} + v_i u^i = c^2$ turn the observable time interval $d\tau$ between any two events inside the zero-space into zero

$$d\tau = \left[1 - \frac{1}{c^2}\left(\mathrm{w} + v_i u^i\right)\right] dt = 0\,, \qquad dt \neq 0\,. \tag{2.147}$$

Assuming all the above results, we can conclude that from the viewpoint of a regular observer that the motion of a zero-particle is perceived as instant displacement.

Moreover, zero-particles are actually virtual particles transferring interaction instantly between elementary particles of our world. This implies that the space for virtual particles and virtual interactions supposed by Quantum Electrodynamics is actually zero-space in General Relativity.

Quantum Electrodynamics contends that all interactions between elementary particles, including their birth and destruction, rely upon emission and absorbtion of virtual particles. Therefore zero-particles can possess the birth and death processes for particles of our world. Thus, their instant displacement can realize teleportation. Because zero-particles can be considered in our world as standing light-like waves (light-like hologram), the possiblility of teleportation should be linked with the physical conditions to realize the halting of light.

We have found the place for virtual particles in the General Theory of Relativity. Actually, this is the way to join the General Theory of Relativity with Quantum Electrodynamics.

2.8 An expanded space-time of kind IV: non-quantum teleportation of photons

A second advantage arising from considering instead of a four-dimensional pseudo-Riemannian space, its generalization in an expanded space-time of kind IV, is the fact that the velocity of any motions in fully degenerate regions of the expanded space appears infinite.

As is well known, the basic space-time of the General Theory of Relativity is a four-dimensional pseudo-Riemannian space, which is, in general, inhomogeneous, curved, rotating, and deformed. Therein the square of the space-time interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, being expressed in the terms of physical observable quantities — chronometric invariants [39, 40], takes the form

$$ds^2 = c^2 d\tau^2 - d\sigma^2. \tag{2.148}$$

Here the quantity

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i, \tag{2.149}$$

is an interval of physical observable time, $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ is the gravitational potential, $v_i = -c\dfrac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation, $d\sigma^2 = h_{ik}\, dx^i dx^k$ is the square of a observable spatial interval, $h_{ik} = -g_{ik} + \dfrac{1}{c^2}\, v_i v_k$ is the observable metric tensor, g_{ik} are spatial components of the fundamental metric tensor $g_{\alpha\beta}$ (space-time indices are Greek $\alpha, \beta = 0, 1, 2, 3$, while spatial indices — Roman $i, k = 1, 2, 3$).

In these terms we consider a particle displaced by ds in the space-time. We write ds^2 as follows

$$ds^2 = c^2 d\tau^2 \left(1 - \frac{\mathrm{v}^2}{c^2}\right), \tag{2.150}$$

where $\mathrm{v}^2 = h_{ik}\, \mathrm{v}^i \mathrm{v}^k$, and $\mathrm{v}^i = \dfrac{dx^i}{d\tau}$ is the three-dimensional observable velocity of the particle. So ds is: (1) a substantial quantity under $\mathrm{v} < c$; (2) a zero quantity under $\mathrm{v} = c$; (3) an imaginary quantity under $\mathrm{v} > c$.

Particles of non-zero rest-masses $m_0 \neq 0$ (substance) can be moved: (1) along real world-trajectories $c d\tau > d\sigma$, having real relativistic masses

$m = \dfrac{m_0}{\sqrt{1-\mathrm{v}^2/c^2}}$; (2) along imaginary world-trajectories $cd\tau < d\sigma$, having imaginary relativistic masses $m = \dfrac{im_0}{\sqrt{\mathrm{v}^2/c^2-1}}$ (tachyons). World-lines of both kinds are known as *non-isotropic trajectories*.

Particles of zero rest-masses $m_0 = 0$ (massless particles), having non-zero relativistic masses $m \neq 0$, move along world-trajectories of zero four-dimensional lengths $cd\tau = d\sigma$ at light velocity. They are known as *isotropic trajectories*. To massless particles are related light-like particles — quanta of electromagnetic fields (photons).

A condition under which a particle may realize an instant displacement (*teleportation*) is equality to zero of the observable time interval $d\tau = 0$ so that the *teleportation condition* is

$$\mathrm{w} + v_i u^i = c^2, \tag{2.151}$$

where $u^i = \dfrac{dx^i}{dt}$ is its three-dimensional coordinate velocity. From this the square of that space-time interval this particle instantly traverses takes the form

$$ds^2 = -d\sigma^2 = -\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2dt^2 + g_{ik}\,dx^i dx^k, \tag{2.152}$$

where in this case $1-\frac{\mathrm{w}}{c^2} = \frac{v_i u^i}{c^2}$, because $d\tau = 0$.

Actually, with the signature $(+---)$ in the space-time region of a regular observer, the signature becomes $(-+++)$ in that space-time region where particles may be teleported. So the terms "time" and "three-dimensional space" interchange in that region. "Time" of teleporting particles is "space" of the regular observer, and vice versa "space" of teleporting particles is "time" of the regular observer.

Let us first consider substantial particles. It easy to see that instant displacements (teleportation) of such particles manifests along world-trajectories in which $ds^2 = -d\sigma^2 \neq 0$ is true. So the trajectories represented in the terms of observable quantities are pure spatial lines of imaginary three-dimensional lengths $d\sigma$, although being expressed in ideal world-coordinates t and x^i the trajectories are four-dimensional. In a particular case, where the space is free of rotation ($v_i = 0$) or its rotation velocity v_i is orthogonal to the particle's coordinate velocity u^i (their scalar product is $v_i u^i = |v_i||u^i|\cos(v_i; u^i) = 0$), substantial particles may be teleported only if gravitational collapse occurs ($\mathrm{w} = c^2$). In this case world-trajectories of teleportation taken in ideal world-coordinates become purely spatial $ds^2 = g_{ik}\,dx^i dx^k$.

Second, massless light-like particles (photons) may be teleported along world-trajectories located in a space of the metric

$$ds^2 = -d\sigma^2 = -\left(1-\frac{\mathrm{w}}{c^2}\right)^2 c^2dt^2 + g_{ik}\,dx^i dx^k = 0\,, \tag{2.153}$$

because for photons $ds^2 = 0$ by definition. So the space of photon teleportation characterizes itself by the conditions $ds^2 = 0$ and $d\sigma^2 = c^2 d\tau^2 = 0$.

The equation obtained is like the "light cone" equation $c^2 d\tau^2 - d\sigma^2 = 0$ ($d\sigma \neq 0$, $d\tau \neq 0$), elements of which are world-trajectories of light-like particles. But, in contrast to the light cone equation the obtained equation is built by ideal world-coordinates t and x^i — having no representation in the terms of observable quantities. So teleporting photons move along trajectories which are elements of the world-cone (like the light cone) in that space-time region where substantial particles may be teleported (the metric inside that region has been obtained above).

Considering the photon teleportation cone equation from the viewpoint of a regular observer, we can see that the spatial observable metric $d\sigma^2 = h_{ik}\,dx^i dx^k$ becomes degenerate, $h = \det\|h_{ik}\| = 0$, in the space-time region of that cone. Taking the relationship $g = -h\,g_{00}$ [39, 40] into account, we conclude that the four-dimensional metric $ds^2 = g_{\alpha\beta}\,dx^\alpha dx^\beta$ degenerates there as well: $g = \det\|g_{\alpha\beta}\| = 0$. The last fact implies that signature conditions defining pseudo-Riemannian spaces are broken, so that photon teleportation is realized outside the basic space-time of the General Theory of Relativity. Such fully degenerate space was considered in [41, 51], where it was referred to as *zero-space* because, from viewpoint of a regular observer, all spatial intervals and time intervals are zero there.

At $d\tau = 0$ and $d\sigma = 0$ observable relativistic mass m and the frequency ω become zero. So from viewpoint of a regular observer all particles located in zero-space (in particular, teleporting photons) having zero rest-masses $m_0 = 0$ appear as zero relativistic masses $m = 0$ and the frequencies $\omega = 0$. Therefore particles of such a kind may be assumed the ultimate case of massless light-like particles.

We will refer to all particles located in zero-space as *zero-particles*.

Within the framework of the particle-wave concept each particle is described by its own wave world-vector $K_\alpha = \dfrac{\partial\psi}{\partial x^\alpha}$, where ψ is the wave phase (eikonal). the eikonal equation $K_\alpha K^\alpha = 0$ [37], sets forth that the length of the wave vector K^α remains unchanged*, and for regular massless light-like particles (regular photons) becomes the travelling wave equation

$$\frac{1}{c^2}\left(\frac{^*\partial\psi}{\partial t}\right)^2 - h^{ik}\,\frac{^*\partial\psi}{\partial x^i}\,\frac{^*\partial\psi}{\partial x^k} = 0\,, \tag{2.154}$$

that is obtained after taking $K_\alpha K^\alpha = g^{\alpha\beta}\,\dfrac{\partial\psi}{\partial x^\alpha}\,\dfrac{\partial\psi}{\partial x^\beta} = 0$ in terms of physical observable quantities [39, 40], where we formulate regular derivatives through chronometrically invariant (physical observable) derivatives

*According to Levi-Civita's rule, in a Riemannian space of n dimensions the length of any n-dimensional vector Q^α remains unchanged in parallel transport, so $Q_\alpha Q^\alpha = \text{const}$, so it is therefore true for the four-dimensional wave vector K^α in a four-dimensional pseudo-Riemannian space — the basic space-time of the General Theory of Relativity. It is well-known that since along isotropic trajectories $ds = 0$ is true (because $c\,d\tau = d\sigma$), the length of any isotropic vector is zero so that we have $K_\alpha K^\alpha = 0$.

$\frac{{}^*\partial}{\partial t} = \frac{1}{\sqrt{g_{00}}}\frac{\partial}{\partial t}$ and $\frac{{}^*\partial}{\partial x^i} = \frac{\partial}{\partial x^i} + \frac{1}{c^2}v_i\frac{{}^*\partial}{\partial t}$ and we use $g^{00} = \frac{1}{g_{00}}\Big(1 - \frac{1}{c^2}v_i v^i\Big)$, $v_k = h_{ik}v^i$, $v^i = -cg^{0i}\sqrt{g_{00}}$, $g^{ik} = -h^{ik}$.

The eikonal equation in zero-space takes the form

$$h^{ik}\frac{{}^*\partial\psi}{\partial x^i}\frac{{}^*\partial\psi}{\partial x^k} = 0 \tag{2.155}$$

because therein $\omega = \frac{{}^*\partial\psi}{\partial t} = 0$, making the equation's time term zero. It is a standing wave equation. From viewpoint of a regular observer, in the framework of the particle-wave concept, all particles located in zero-space appear as *standing light-like waves*, so that all zero-space appears filled with a system of light-like standing waves — a light-like hologram. This implies that an experiment discovering non-quantum teleportation of photons should be linked to the halting of light.

There is no problem in photon teleportation being realized along fully degenerate world-trajectories ($g=0$) outside the basic pseudo-Riemannian space ($g<0$), while teleportation trajectories of substantial particles are strictly non-degenerate ($g<0$) so the trajectories are located in the pseudo-Riemannian space*. This is not a problem because at any point of the pseudo-Riemannian space we can place a tangential space of $g\leqslant 0$ consisting of the regular pseudo-Riemannian space ($g<0$) and zero-space ($g=0$) as two different regions of the same manifold. Such spaces of $g\leqslant 0$ will be a natural generalization of the basic space-time of the General Theory of Relativity, permitting teleportation of both substantial particles (without experiment verification as yet) and photons, which has been realized in experiments.

The only difference is that from the perspective of a regular observer the square of any parallely transported vector remains unchanged. It is also an "observable truth" for vectors in zero-space, because the observer reasons standards of his pseudo-Riemannian space anyway. So that eikonal equation in zero-space, expressed in his observable world-coordinates, is $K_\alpha K^\alpha = 0$. But being taken in ideal world-coordinates t and x^i, the metric inside zero-space $ds^2 = -\Big(1-\frac{\mathrm{w}}{c^2}\Big)^2 c^2dt^2 + g_{ik}\,dx^i dx^k = 0$, degenerates into a three-dimensional $d\mu^2$ which, depending on the gravitational potential w being uncompensated by something else, is not invariant $d\mu^2 = g_{ik}\,dx^i dx^k = \Big(1-\frac{\mathrm{w}}{c^2}\Big)^2 c^2dt^2 \neq \text{inv}$. As a result, within zero-space the square of a transferred vector, a four-dimensional coordinate velocity vector U^α for instance, being degenerated into a three-dimensional U^i, does not remain unchanged

$$U_iU^i = g_{ik}U^iU^k = \Big(1-\frac{\mathrm{w}}{c^2}\Big)^2 c^2 \neq \text{const}, \tag{2.156}$$

*Any space of Riemannian geometry has the strictly non-degenerate metric of $g\neq 0$ by definition of such metric spaces. Pseudo-Riemannian spaces are a particular case of Riemannian spaces, where the metric is sign-alternating. So a four-dimensional pseudo-Riemannian space of the signature $(+{-}{-}{-})$ or $(-{+}{+}{+})$, Einstein laid at the base of the General Theory of Relativity, is also a strictly non-degenerate metric ($g<0$).

so that corresponding to the Riemannian geometry for a regular observer, the real geometry of zero-space within the space itself is of non-Riemannian shape.

We conclude that instant displacements of particles are naturally permitted in the space-time of the General Theory of Relativity. As shown herein, teleportation of substantial particles and photons realizes itself in different space-time regions. But it would be a mistake to think that teleportation requires acceleration of a substantial particle to super-light speeds (the tachyon region), while a photon needs to be accelerated to infinite speed. No — as it is easy to see from the teleportation condition $\mathrm{w} + v_i u^i = c^2$, if gravitational potential is essential and the space rotates at a speed close to light velocity, substantial particles may be teleported at regular sub-light speeds. Photons can reach the teleportation condition easier, because they move at light velocity. From the viewpoint of a regular observer, as soon as the teleportation condition is realized in the neighbourhood around a moving particle, such a particle "disappears" although it continues its motion at a sub-light coordinate velocity u^i (or at the velocity of light) in another space-time region invisible for us. Then, having its velocity reduced, or if something else disrupts the teleportation condition (reduction of gravitational potential or the space rotation speed), it "appears" at the same observable moment at another point of our observable space at that distance and in that direction of its u^i there.

These results, derived from purely geometrical considerations, verify the old proposition of the 1970's that "there is no speed barrier for a wave phase nor for entangled particles" as given by F. Smarandache, who worked from a basis in phenomenological analysis and General Relativity space-time geometry (Einstein-Podolsky-Rosen paradox) taken from the viewpoint of linear logic (see [52, 53, 54] for details).

2.9 Conclusions

In closing this Chapter we would like to repeat, in brief, the main results we have obtained.

A four-dimensional pseudo-Riemannian space, the basic space-time of the General Theory of Relativity, is continuous by definition, so there are no omitted points, lines, or surfaces therein. Proposing the real space-time to be continuous, the neutrosophic method predicted that the General Theory of Relativity should permit trajectories in common for regular mass-bearing particles that move at sub-light velocities and massless light-like particles moving at the velocity of light. Particles moving along such "mixed" trajectories should have properties of both regular mass-bearing particles and massless light-like particles.

Detailed analysis of this proposition showed that trajectories of such a 'mixed" kind can exist in a four-dimensional pseudo-Riemannian space, but particles that move along such trajectories can even move outside the pseudo-Riemannian space, outside the basic space-time of the General Theory of Relativity.

Such trajectories have been found employing S-denying signature conditions in the basic four-dimensional pseudo-Riemannian space (there are four signature conditions), when a signature condition is partially true and partially not in the same space. S-denying each of the signature conditions (or even all the conditions at once) gave an expanded space for the General Theory of Relativity, which, being an instance of the family of Smarandache spaces, include the pseudo-Riemannian space as a particular case. S-denying the 4th signature condition gave an expanded space of kind IV, which permits full degeneration of its metric.

Particles of the "mixed" mass-bearing/massless kind move along fully degenerate trajectories in an expanded space of kind IV and are moved instantly, from the viewpoint of a regular observer located in an Earth-bound laboratory. But their true motions realise themselves at finite sub-light velocities up to the light velocity. Such particles were called "virtual photons", because the energy-impulse relationship is not valid, in similar fashion for any virtual photons as predicted by Quantum Electrodynamics. Such particles were also called "zero-particles", because their own masses and frequencies are zero according to a regular observer.

This research currently expounds the sole explanation of virtual particles and virtual interaction given by the purely geometrical methods of Einstein's theory. It is possible that this method will form a link between Quantum Electrodynamics and the General Theory of Relativity.

Moreover, this research currently gives the sole theoretical explanation of the well-known phenomenon of photon teleportation in terms of the General Theory of Relativity.

◇

Chapter 3

ENTANGLED STATES AND QUANTUM CAUSALITY THRESHOLD

3.1 Disentangled and entangled particles in General Relativity. Problem statement

In his article of in 2000, dedicated to the 100th anniversary of the discovery of quanta, Belavkin [55] generalizes definitions assumed *de facto* in Quantum Mechanics for entangled and disentangled particles. He writes:

> "The only distinction of the classical theory from quantum is that the prior mixed states cannot be dynamically achieved from pure initial states without a procedure of either statistical or chaotic mixing. In quantum theory, however, the mixed, or decoherent states can be dynamically induced on a subsystem from the initial pure disentangled states of a composed system simply by a unitary transformation.
>
> Motivated by hte Einstein-Podolsky-Rosen paper, in 1935 Schrödinger published a three part essay* on *The Present Situation in Quantum Mechanics*. He turns to the EPR paradox and analyses completeness of the description by the wave function for the entangled parts of the system. (The word *entangled* was introduced by Schrödinger for the description of nonseparable states.) He notes that if one has pure states $\psi(\sigma)$ and $\chi(\upsilon)$ for each of two completely separated bodies, one has maximal knowledge, $\psi_1(\sigma,\upsilon) = \psi(\sigma)\chi(\upsilon)$, for two taken together. But the converse is not true for the entangled bodies, described by a non-separable wave function $\psi_1(\sigma,\upsilon) \neq \psi(\sigma)\chi(\upsilon)$: Maximal knowledge of a total system does not necessary imply maximal knowledge of all its parts, not even when these are completely separated from one another, and at the time cannot influence one another at all."

In other words, because Quantum Mechanics considers particles as stochastic clouds, there can be entangled particles — particles whose states are entangled, and they build a whole system so that if the state of one particle changes the state of the other particles changes immediately although they are located far from one another.

In particular, because of the admission of entangled states, Quantum Mechanics permits quantum teleportation — an experimentally verified phenomenon. The term "quantum teleportation" had been introduced into theory in 1993 [56]. The first experiment teleporting massless particles

*Schrödinger E. *Naturwissenschaften*, 1935, Band 23, 807–812, 823–828, 844–849.

(quantum teleportation of photons) was performed five years later, in 1998 [57]. Experiments teleporting mass-bearing particles (atoms as a whole) were done in 2004 by two independent groups of scientists: quantum teleportation of the an ion of the Calcium atom [58] and of an ion of the Beryllium atom [59]. There are many followers who continue experiments with quantum teleportation, see [60]–[70] for instance.

It should be noted that the experimental statement on quantum teleportation has two channels in which information (the quantum state) transfers between two entangled particles: the "teleportation channel" where information is transferred instantly, and the "synchronization channel" — the classical channel where information is transferred in the regular way, at or less than the the speed of light (the classical channel is targeted to inform the receiving particle about the initial state of the first one). After teleportation the state of the first particle is destroyed, so there is data transfer (not data copying).

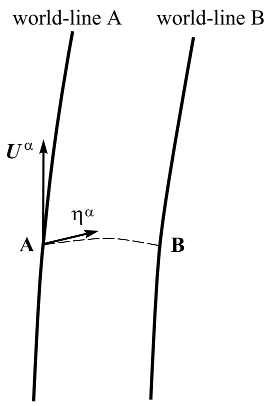
General Relativity draws another picture of data transfer: the particles are considered as point-masses or waves, not stochastic clouds. This statement is true for both mass-bearing particles and massless ones (photons). Data transfer between any two particles is realized as well by point-mass particles, so in General Relativity this process is not of stochastic origin.

In the classical problem statement accepted in General Relativity [71, 38, 37], two mass-bearing particles are considered which are moved along neighbouring world-lines, and a signal is transferred between them by a photon. One of the particles radiates the photon to the other, where the photon is absorbed, realizing data transfer between the particles. Of course, the signal can as well be carried by a mass-bearing particle.

If there are two free mass-bering particles, they fall freely along neighbouring geodesic lines in a gravitational field. This classical problem has been developed in Synge's book [49] where he has deduced the geodesic lines deviation equation (Synge's equation, 1950's). If these are two particles connected by a non-gravitational force (for instance, by a spring), they are moved along neighbouring non-geodesic world-lines. This classical statement was developed a few years later by Weber [44], who obtained the world-lines deviation equation (Synge-Weber's equation).

Anyway in this classical problem of General Relativity two interacting particles moved along both neighbouring geodesic and non-geodesic world-lines are *disentangled*. This happens for two reasons:

1. In this problem statement a signal moves between two interacting particles at a velocity no faster than light, so their states are absolutely separated — these are *disentangled states*;
2. Any particle, being considered in General Relativity's space-time, has its own four-dimensional trajectory (world-line) which is the set of the particle's states from its birth to decay. Two different particles cannot occupy the same world-line, so they are in absolutely separated states — they are *disentangled* particles.



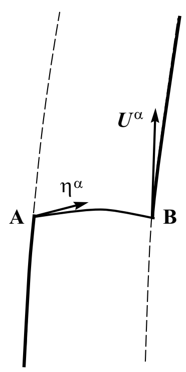


Fig. 1: Synge-Weber's statement. Fig. 2: The advanced statement.

The second reason is much stronger than the first. In particular, the second reason leads to the fact that, in General Relativity, *entangled states* are only neighbouring states of the same particle along its own world-line — its own states separated in time, not in three-dimensional space. No two different particles could be entangled. Any two different particles, both mass-bearing and massless ones, are *disentangled* in General Relativity.

On the other hand, experiments on teleportation indicate that *entanglement* is really an existing state that occurs with particles if they reach specific physical conditions. This is the fact that should be taken into account by General Relativity.

Therefore our task in this research is to introduce entangled states into General Relativity. Of course, for of the reasons above, two particles cannot be in an entangled state if they are located in the basic space-time of General Relativity — that four-dimensional pseudo-Riemannian space with sign-alternating elements $(+---)$ or $(-+++)$. Its metric is strictly non-degenerate as for any space of Riemannian space family, namely, there the determinant $g = \det \| g_{\alpha\beta} \|$ of the fundamental metric tensor $g_{\alpha\beta}$ is strictly negative $g < 0$. We expand the Synge-Weber problem statement, considering it in a *generalized space-time* whose metric can become degenerate $g = 0$ under specific physical conditions. This space is one of the Smarandache family of geometric spaces [7, 8, 9, 10, 11, 12, 13], because its geometry is partially Riemannian, partially not.

It was shown in [51, 41] (Borissova and Rabounski, 2001), when General Relativity's basic space-time degenerates, physical conditions can imply *observable teleportation* of both a mass-bearing and massless particle — its instant displacement from one point of the space to another, although it moves no faster than light in the degenerate space-time region, outside the basic space-time. In the generalized space-time the Synge-Weber problem statement about two particles interacting by a signal (see Fig. 1) can be reduced to the case where the same particle is located at two different points A and B of the basic space-time at the same moment of time, so the states A and B are entangled (see Fig. 2). This particle, being actually two particles in the entangled states A and B, can interact with itself by radiating a photon (signal) at the point A and absorbing it at the point B. That is our goal: to incorporate entangled states into General Relativity.

Moreover, as we will see, under specific physical conditions the entangled particles in General Relativity can reach a state where neither particle A nor particle B can be the cause of future events. We call this specific state *Quantum Causality Threshold*.

3.2 Incorporating entangled states into General Relativity

In the classical problem statement, Synge [49] considered two free-particles (Fig. 1) moving along neighbouring geodesic world-lines $\Gamma(v)$ and $\Gamma(v+dv)$, where v is a parameter along the direction orthogonal to the geodesics (it is taken in the plane normal to the geodesics). There $v=\text{const}$ along each geodesic line. Motion of the particles is determined by the well-known geodesic equation

$$\frac{dU^\alpha}{ds}+\Gamma^\alpha_{\mu\nu}\,U^\mu\frac{dx^\nu}{ds}=0\,, \tag{3.1}$$

which is the actual fact that the absolute differential $\mathrm{D}U^\alpha=dU^\alpha+\Gamma^\alpha_{\mu\nu}\,U^\mu dx^\nu$ of a tangential vector U^α (the velocity world-vector $U^\alpha=\frac{dx^\alpha}{ds}$, in this case), transferred along that geodesic line to where it is tangential, is zero. Here s is an invariant parameter along the geodesic (we assume it the space-time interval), and $\Gamma^\alpha_{\mu\nu}$ are Christoffel's symbols of the 2nd kind. Greek $\alpha=0,1,2,3$ denote four-dimensional (space-time) indices.

The parameter v is different for the neighbouring geodesics, and the difference is dv. Therefore, in order to study relative displacements of two geodesics $\Gamma(v)$ and $\Gamma(v+dv)$, we shall study the vector of their infinitesimal relative displacement

$$\eta^\alpha=\frac{\partial x^\alpha}{\partial v}\,dv\,, \tag{3.2}$$

As Synge had deduced, a deviation of the geodesic line $\Gamma(v+dv)$ from the geodesic line $\Gamma(v)$ can be found as the solution to his equation

$$\frac{\mathrm{D}^2\eta^\alpha}{ds^2}+R^{\alpha\cdots}_{\cdot\beta\gamma\delta}\,U^\beta U^\delta\eta^\gamma=0\,, \tag{3.3}$$

that describes relative accelerations of two neighbouring free-particles ($R^{\alpha\cdots}_{\cdot\beta\gamma\delta}$ is the Riemann-Chrostoffel curvature tensor). This formula is known as the geodesic lines deviation equation or the *Synge equation*.

In Weber's statement [44] the difference is that he considers two particles connected by a non-gravitational force Φ^α, a spring for instance. So their world-trajectories are non-geodesic, and they are determined by the equation

$$\frac{dU^{\alpha}}{ds} + \Gamma^{\alpha}_{\mu\nu}\,U^{\mu}\,\frac{dx^{\nu}}{ds} = \frac{\Phi^{\alpha}}{m_0c^2}\,, \tag{3.4}$$

which is different to the geodesic equation in that the right side is not zero. His improved equation of the world lines deviation

$$\frac{\mathrm{D}^2\eta^{\alpha}}{ds^2} + R^{\alpha\cdot\cdot\cdot}_{\cdot\beta\gamma\delta}\,U^{\beta}U^{\delta}\eta^{\gamma} = \frac{1}{m_0c^2}\,\frac{\mathrm{D}\Phi^{\alpha}}{dv}\,dv\,, \tag{3.5}$$

describes relative accelerations of two particles (of the same rest-mass m_0), connected by a spring. His deviation equation is that of Synge, except for that non-gravitational force Φ^{α} on the right side. This formula is known as the *Synge-Weber equation*. In this case the angle between the vectors U^{α} and η^{α} does not remain unchanged along the trajectories

$$\frac{\partial}{\partial s}(U_{\alpha}\eta^{\alpha}) = \frac{1}{m_0c^2}\,\Phi_{\alpha}\eta^{\alpha}. \tag{3.6}$$

Now, proceeding from this problem statement, we are going to introduce entangled states into General Relativity. At first we determine such states in the space-time of General Relativity, then we find specific physical conditions under which two particles reach a state to be entangled.

Definition Two particles A and B, located in the same spatial section* at the distance $dx^i \neq 0$ from each other, are in non-separable states if the observable time interval $d\tau$ between linked events in the particles† is zero $d\tau = 0$. If only $d\tau = 0$, the states become non-separated from one another, so the particles A and B become entangled.

So we will refer to $d\tau = 0$ as the *entanglement condition* in General Relativity.

Let us consider the entanglement condition $d\tau = 0$ in connection with the world-lines deviation equations.

In General Relativity, the interval of physical observable time $d\tau$ between two events separated by the distance dx^i is determined through components of the fundamental metric tensor as

$$d\tau = \sqrt{g_{00}}\,dt + \frac{g_{0i}}{c\sqrt{g_{00}}}\,dx^i, \tag{3.7}$$

(see §84 in the well-known *The Classical Theory of Fields* by Landau and Lifshitz [37]). The mathematical apparatus of physical observable quantities

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*A three-dimensional section of the four-dimensional space-time, placed in a given point in the time line. In the space-time there are infinitely many spatial sections, one of which is our three-dimensional space.

†Such linked events in the particles A and B can be radiation of a signal in one and its absorbtion in the other, for instance.

(Zelmanov's theory of chronometric invariants [39, 40] — see also the brief account in [41, 51]) transforms this formula to

$$d\tau = \left(1 - \frac{\mathrm{w}}{c^2}\right) dt - \frac{1}{c^2}\, v_i dx^i, \tag{3.8}$$

where $\mathrm{w} = c^2(1 - \sqrt{g_{00}})$ is the gravitational potential of an acting gravitational field, and $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$ is the linear velocity of the space rotation.

So, following the theory of physical observable quantities, in real observations where the observer accompanies his references the space-time interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ is

$$ds^2 = c^2 d\tau^2 - d\sigma^2, \tag{3.9}$$

where $d\sigma^2 = \left(-g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}\right) dx^i dx^k$ is a three-dimensional (spatial) invariant, built on the three-dimensional observable metric tensor $h_{ik} = -g_{ik} + \frac{g_{0i}g_{0k}}{g_{00}}$. This observable metric tensor, in real observations where the observer accompanies his references, is the same as the analogously constructed general covariant tensor $h_{\alpha\beta}$. So, $d\sigma^2 = h_{ik}\, dx^i dx^k$ is the spatial observable interval for any observer who accompanies his references.

As it is easy to see from (3.9), there are two possible cases where the entanglement condition $d\tau = 0$ occurs:

(1) $ds = 0$ and $d\sigma = 0$,

(2) $ds^2 = -d\sigma^2 \neq 0$, so $d\sigma$ becomes imaginary,

and we will refer to them as the *auxiliary entanglement conditions of the 1st kind and 2nd kind* respectively.

Let us get back to the Synge equation and the Synge-Weber equation.

According to Zelmanov's theory of physical observable quantities [39, 40], if an observer accompanies his references the projection of a general covariant quantity on the observer's spatial section is its spatial observable projection.

In this fashion Borissova has deduced (see eqs. 7.16–7.28 in [72]) that the spatial observable projection of the Synge equation is*

$$\frac{d^2\eta^i}{d\tau^2} + 2\left(D^i_k + A^{\cdot i}_{k\cdot}\right)\frac{d\eta^k}{d\tau} = 0\,, \tag{3.10}$$

*In this formula, according to Zelmanov's mathematical apparatus of physical observable quantities [39, 40], $D_{ik} = \frac{1}{2}\frac{{}^*\partial h_{ik}}{\partial t} = \frac{1}{2\sqrt{g_{00}}}\frac{\partial h_{ik}}{\partial t}$ is the three-dimensional symmetric tensor of the space deformation observable rate while $A_{ik} = \frac{1}{2}\left(\frac{\partial v_k}{\partial x^i} - \frac{\partial v_i}{\partial x^k}\right) + \frac{1}{2c^2}\left(F_i v_k - F_k v_i\right)$ is the three-dimensional antisymmetric tensor of the space rotation observable angular velocities, the indices of which can be raised and lowered by the observable metric tensor so that $D^i_k = h^{im} D_{km}$ and $A^{\cdot i}_{k\cdot} = h^{im} A_{km}$. See a brief account of Zelmanov's mathematical apparatus in [41, 72, 73, 74].

which she called the *Synge equation in chronometrically invariant form*. The Weber equation is different in its right side, containing there the non-gravitational force that connects the particles (of course, the force should be in the spatially projected form). For this reason, conclusions obtained for the Synge equation will be the same for the Weber equation.

In order to make the results of General Relativity applicable in practice, we should consider tensor quantities and equations in chronometrically invariant form, because in this way they contain only chronometrically invariant quantities — physical quantities and geometrical properties of space, measurable in real experiments [39, 40].

Let us consider our problem from this viewpoint.

It easy to see that the Synge equation in its chronometrically invariant form (3.10) under the entanglement condition $d\tau = 0$ becomes nonsense. The Weber equation becomes nonsense as well. So the classical problem statement becomes senseless as soon as particles reach entangled states.

At the same time, in their recent theoretical research [51], two authors of this paper (Borissova and Rabounski, 2005) have found two groups of physical conditions under which particles can be teleported in a non-quantum way. They have been called the *teleportation conditions*:

(1) $d\tau = 0$ $\{ds = 0\,,\ d\sigma = 0\}$, the conditions of photon teleportation;

(2) $d\tau = 0$ $\{ds^2 = -d\sigma^2 \neq 0\}$, the conditions of substantial (mass-bearing) particles teleportation.

There were also theoretically deduced physical conditions* which should be achieved in a laboratory in order to teleport particles in the non-quantum way [51].

It is easy to see the non-quantum teleportation conditions are identical to those introduced herein as the main entanglement condition $d\tau = 0$ in conjunction with the auxiliary entanglement conditions of the 1st and 2nd kind!

Taking this one into account, we transform the classical Synge and Weber problem statement into another. In our statement the world-line of a particle, being entangled with itself by definition, splits into two different world-lines under teleportation conditions. In other words, as soon as the teleportation conditions occur in a research laboratory, the world-line of a teleported particle breaks in one world-point A and immediately starts in the other world-point B (Fig. 2). Both particles A and B, being actually two different states of the same teleported particle at a remote distance from one another, are in *entangled states*. So, in this statement, the particles A and B themselves are *entangled*.

Of course, this entanglement exists only at the moment of the teleportation when the particle exists in two different states simultaneously. As soon as the teleportation process has finished, only one particle remains, so the entanglement disappears.

*A specific correlation between the gravitational potential w, the space rotation linear velocity v_i and the teleported particle's velocity u^i.

It should be noted that it follows from the entanglement conditions that only substantial particles can reach entangled states in the basic space-time of General Relativity — the four-dimensional pseudo-Riemannian space, not photons. Here is why.

It is known that the interval $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$ cannot be fully degenerate in a Riemannian space*: the condition is that the determinant of the fundamental metric tensor $g_{\alpha\beta}$ must be strictly negative $g = \det \| g_{\alpha\beta} \| < 0$ by definition of Riemannian spaces. In other words, in the basic space-time of General Relativity the fundamental metric tensor must be strictly non-degenerate as $g < 0$.

The observable three-dimensional (spatial) interval $d\sigma^2 = h_{ik}\, dx^i dx^k$ is positive definite [39, 40], proceeding from the physical sense. It fully degenerates ($d\sigma^2 = 0$) only if the space compresses into a point (a senseless case) or the determinant of the observable metric tensor becomes zero $h = \det \| h_{ik} \| = 0$.

It was shown by Zelmanov [39, 40] that, in real observations where an observer accompanies his references, the determinant of the observable metric tensor is connected with the determinant of the fundamental one by the relationship $h = -\frac{g}{g_{00}}$. From this we see that if the three-dimensional observable metric fully degenerates $h = 0$, the four-dimensional metric degenerates as well $g = 0$.

We have obtained that states of two substantial particles can be entangled, if $d\tau = 0$ $\{ds^2 = -d\sigma^2 \neq 0\}$ in the space neighbourhood. So $h > 0$ and $g < 0$ in the neighbourhood, and hence the four-dimensional pseudo-Riemannian space is not degenerate.

Conclusion Substantial particles can reach entangled states in the basic space-time of General Relativity (a four-dimensional pseudo-Riemannian space) under specific conditions in the neighbourhood.

Although $ds^2 = -d\sigma^2$ in the neighbourhood ($d\sigma$ should be imaginary), the substantial particles remain in a regular sub-light region, and they do not become super-light tachyons. It is easy to see, from the definition of physical observable time (8), the entanglement condition $d\tau = 0$ occurs only if the specific relationship

$$\mathrm{w} + v_i u^i = c^2 \tag{3.11}$$

holds between the gravitational potential w, the space rotation linear velocity v_i and the particles' true velocity $u^i = dx^i/dt$ in the observer's laboratory. For this reason, in the neighbourhood, the space-time metric is

$$ds^2 = -d\sigma^2 = -\left(1 - \frac{\mathrm{w}}{c^2}\right)^2 c^2 dt^2 + g_{ik}\, dx^i dx^k, \tag{3.12}$$

so the substantial particles can become entangled if the space initial signature $(+---)$ becomes inverted $(-+++)$ in the neighbourhood, while the

*It can only be partially degenerate. For instance, a four-dimensional Riemannian space can degenerate into a three-dimensional one.

particles' velocities u^i remain no faster than light.

Another case — massless particles (photons). States of two photons can be entangled in the space neighbourhood only if $d\tau = 0$ $\{ds = 0\,,\ d\sigma = 0\}$. In this case the determinant of the observable metric tensor becomes $h = 0$, so the space-time metric as well degenerates $g = -g_{00}\,h = 0$. This is not the four-dimensional pseudo-Riemannian space.

Where is that region? In previous works (Borissova and Rabounski, 2001 [41, 51]) a generalization to the basic space-time of General Relativity was introduced — that four-dimensional space which, having General Relativity's sign-alternating label $(+---)$, permits the space-time metric to be fully degenerate, so that $g \leqslant 0$ there.

As it was shown in those works, as soon as the specific condition $\mathrm{w} + v_i u^i = c^2$ occurs, the space-time metric becomes fully degenerate: there are $ds = 0$, $d\sigma = 0$, $d\tau = 0$ (it can be easy derived from the above definition for the quantities) and, hence $h = 0$ and $g = 0$. Therefore, in a space-time where the *degeneration condition* $\mathrm{w} + v_i u^i = c^2$ is permitted the determinant of the fundamental metric tensor is $g \leqslant 0$. This case includes both the Riemannian geometry case $g < 0$ and the non-Riemannian, fully degenerated one $g = 0$. For this reason such a space is one of Smarandache geometric spaces (because its geometry is partially Riemannian, partially not*). In the such generalized space-time the entanglement conditions of the 1st kind $d\tau = 0$ $\{ds = 0\,,\ d\sigma = 0\}$ (the entanglement conditions for photons) are permitted in that region where the space metric fully degenerates (there $h = 0$ and, hence $g = 0$).

Conclusion Massless particles (photons) can reach entangled states only if the basic space-time fully degenerates $g = \det \| g_{\alpha\beta} \| = 0$ in the neighbourhood. It is permitted in the generalized four-dimensional space-time whose metric can be fully degenerated $g \leqslant 0$ in that region where the degeneration conditions occur. The generalized space-time is attributed to Smarandache geometry spaces, because its geometry is partially Riemannian, partially not.

Thus, entangled states have been introduced into General Relativity for both substantial particles and photons.

3.3 Quantum Causality Threshold in General Relativity

This term was introduced by one of the authors two years ago (Smarandache, 2003) in our common correspondence [75] on the theme:

*In foundations of geometry it is known the *S-denying* of an axiom [7, 8, 9, 10], i. e. in the same space an "axiom is false in at least two different ways, or is false and also true. Such axiom is said to be Smarandachely denied, or S-denied for short" [11]. As a result, it is possible to introduce geometries, which have common points bearing mixed properties of Euclidean, Lobachevsky-Bolyai-Gauss, and Riemann geometry at the same time. Such geometries have been called paradoxist geometries or Smarandache geometries. For instance, Iseri in his book *Smarandache Manifolds* [11] and articles [12, 13] introduced manifolds that support particular cases of such geometries.

Definition Considering two particles A and B located in the same spatial section, *Quantum Causality Threshold* was introduced as a special state in which neither A nor B can be the cause of events located "over" the spatial section on the Minkowski diagram.

The term *Quantum* has been added to the *Causality Threshold*, because in this problem statement an interaction is considered between two infinitely separated particles (in infinitesimal vicinities of each particle) so this statement is applicable only to quantum scale interactions that occur in the scale of elementary particles.

Now, we are going to find physical conditions under which particles can reach the threshold in the space-time of General Relativity.

Because in this problem statement we look at causal relations in General Relativity's space-time from "outside", it is required to use an "outer viewpoint" — a point of view located outside the space-time.

We introduce such a point of outlook in a Euclidean flat space, which is tangential to our's at that world-point, where the observer is located. In this problem statement we have a possibility of comparing the absolute cause relations in that tangential flat space with those in ours. As a matter of fact, a tangential Euclidean flat space can be introduced at any point of the pseudo-Riemannian space.

At the same time, according to Zelmanov [39, 40], within infinitesimal vicinities of any point located in the pseudo-Riemannian space a *locally geodesic reference frame* can be introduced. In such a reference frame, within infinitesimal vicinities of the point, components of the fundamental metric tensor (denoted by tilde)

$$\tilde{g}_{\alpha\beta} = g_{\alpha\beta} + \frac{1}{2}\left(\frac{\partial^2 \tilde{g}_{\alpha\beta}}{\partial \tilde{x}^\mu \partial \tilde{x}^\nu}\right)(\tilde{x}^\mu - x^\mu)(\tilde{x}^\nu - x^\nu) + \dots \tag{3.13}$$

are different from those $g_{\alpha\beta}$ at the point of reflection to within only the higher order terms, which can be neglected. So, in a locally geodesic reference frame the fundamental metric tensor can be taken as constant, while its first derivatives (Christoffel's symbols) are zero. The fundamental metric tensor of a Euclidean space is also a constant, so values of $\tilde{g}_{\mu\nu}$, taken in the vicinity of a point of the pseudo-Riemannian space, converge to values of $g_{\mu\nu}$ in the flat space tangential at this point. Actually, we have a system of the flat space's basis vectors $\vec{e}_{(\alpha)}$ tangential to curved coordinate lines of the pseudo-Riemannian space. Coordinate lines in Riemannian spaces are curved, inhomogeneous, and are not orthogonal to each other (the latter is true if the space rotates). Therefore the lengths of the basis vectors may be very different from unity.

Writing the world-vector of an infinitesimal displacement as $d\vec{r} = (dx^0, dx^1, dx^2, dx^3)$, we obtain $d\vec{r} = \vec{e}_{(\alpha)} dx^\alpha$, where the components of the basis vectors $\vec{e}_{(\alpha)}$ tangential to the coordinate lines are $\vec{e}_{(0)} = \{e^0_{(0)}, 0, 0, 0\}$, $\vec{e}_{(1)} = \{0, e^1_{(1)}, 0, 0\}$, $\vec{e}_{(2)} = \{0, 0, e^2_{(2)}, 0\}$, $\vec{e}_{(3)} = \{0, 0, 0, e^2_{(3)}\}$. The scalar product of $d\vec{r}$ with itself is $d\vec{r}d\vec{r} = ds^2$ or, in another form, $ds^2 = g_{\alpha\beta}\, dx^\alpha dx^\beta$, so

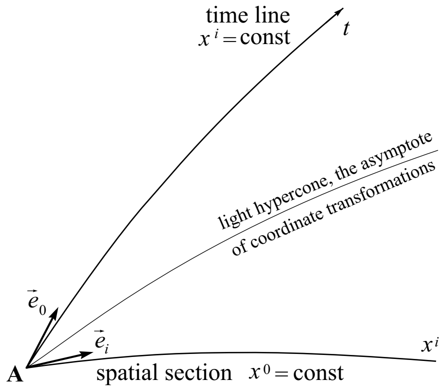

Fig. 3: The spatial section becomes non-orthogonal to time lines as soon as the space starts rotating.

$g_{\alpha\beta} = \vec{e}_{(\alpha)}\vec{e}_{(\beta)} = e_{(\alpha)}e_{(\beta)}\cos\,(x^\alpha; x^\beta)$. We obtain

$$g_{00} = e^2_{(0)}\,, \qquad g_{0i} = e_{(0)}e_{(i)}\cos\,(x^0; x^i)\,, \tag{3.14}$$

$$g_{ik} = e_{(i)}e_{(k)}\cos\,(x^i; x^k)\,, \quad i,k = 1,2,3\,. \tag{3.15}$$

Then, substituting g_{00} and g_{0i} from the formulae that determine the gravitational potential $\mathrm{w} = c^2(1-\sqrt{g_{00}})$ and the space rotation linear velocity $v_i = -c\frac{g_{0i}}{\sqrt{g_{00}}}$, we obtain

$$v_i = -c\,e_{(i)}\cos\,(x^0; x^i)\,, \tag{3.16}$$

$$h_{ik} = e_{(i)}e_{(k)}\Big[\cos(x^0; x^i)\cos(x^0; x^k) - \cos(x^i; x^k)\Big]\,. \tag{3.17}$$

From this we see that if the pseudo-Riemannian space is free of rotation, $\cos\,(x^0; x^i) = 0$, the observer's spatial section is strictly orthogonal to time lines. As soon as the space starts to rotate the cosine becomes different to zero, so the spatial section becomes non-orthogonal to the time lines (Fig. 3). By this process the light hypercone inclines with the time line to the spatial section. In this inclination the light hypercone does not remain unchanged; it "compresses" because of hyperbolic transformations in pseudo-Riemannian space. The more the light hypercone inclines, the more it symmetrically "compresses" because the space-time's geometrical structure changes according to the inclination.

In the ultimate case, where the cosine reaches the ultimate value $\cos\,(x^0; x^i) = 1$, time lines coincide with the spatial section: time "has fallen" into the three-dimensional space. Of course, in this case the light hypercone overflows time lines and the spatial section: the light hypercone "has also fallen" into the three-dimensional space.

It is easy to see from formula (3.16) that this ultimate case occurs as soon as the space rotation velocity v_i reaches light velocity. If particles A and B are located in the space entirely in this ultimate state, neither A nor B can be the cause of events located "over" the spatial section in the Minkowski diagrams we use in the illustrations. So in this ultimate case the entire space-time is in a special state called the Quantum Causality Threshold.

Conclusion Particles located in General Relativity's space-time reach the Quantum Causality Threshold as soon as the space rotation reaches light velocity. Quantum Causality Threshold is impossible if the space does not rotate (holonomic space), or if it rotates at a sub-light speed.

Thus, the Quantum Causality Threshold has been introduced into General Relativity.

3.4 Conclusions

We have shown that the Synge-Weber classical problem statement about two particles interacting by a signal can be reduced to the case where the same particle is located at two different points A and B of the basic space-time at the same moment of time (the states A and B are entangled). This particle, being actually two particles in the entangled states A and B, can interact with itself by radiating a photon (signal) at the point A and absorbing it at the point B. That is our goal: to incorporate entangled states into General Relativity. Under specific physical conditions the entangled particles in General Relativity can reach a state where neither particle A nor particle B can be the cause of future events. We call this specific state Quantum Causality Threshold.

We have found entangled states and Quantum Causality Threshold in General Relativity, which is standard non-quantum theory. This implies that entangled particles and long-lang action (teleportation) can be explained by regular physics, not only by quantum theory.

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Florentin Smarandache is a mathematician of international renown. He is the founder of a class of “paradoxist geometries”, where one or more axioms can be denied in two different ways or can be true and false at the same time (Smarandache geometries). In collaboration with Jean Dezert, a French mathematician, he constructed a new theory of plausible and paradoxical reasoning (DSmT) in information fusion. He is the inventor of a new class of logic (neutrosophic logic) upon which he built a new philosophy — neutrosophy — which extends the current dialectics by the inclusion of neutralities. He has worked in algebraic structures with W. B. Vasantha Kandasamy (see Smarandache n-structures) and in applications of neutrosophics to social sciences and psychology. In recent years he has also been working on fundamental problems in mathematical physics and their applications. He is the author of many books and articles on mathematics, logic, philosophy, and poetry.

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Neutrosophy is a theory developed by Florentin Smarandache in 1995 as a generalization of dialectics, which studies the origin, nature and properties of neutralities. This book applies neutrosophic method to the General Theory of Relativity, revealing thereby, previously unsuspected new effects.

Neutrosophic method applied to Einstein's basic space-time predicts the existence of new trajectories and particles never considered before. These trajectories and associated particles are of two "mixed" kinds, which are (1) common for sub-light mass-bearing particles and massless photons (non-isotropic/isotropic trajectories) and (2) common for massless photons and super-light mass-bearing tachyons (isotropic/non-isotropic trajectories). In is shown herein that mass-bearing/light-like particles are, theoretically, accessible to observation, manifest in various phenomena in Nature.

The fundamental feature of Smarandache geometries is the notion of S-denying an axiom, i. e. within a given space an axiom is false in at least two different ways, or is both true and false. In S-denying each of the four signature conditions of Einstein's basic space-time, four kinds of expanded space-time result for the General Theory of Relativity. The 4th type of expanded space-time permits photon teleportation, a phenomenon which has been well established in recent experiments, but which has had no explanation in basic space-time. In addition, expanded space-time of the 4th kind permits virtual photons — particles predicted by Quantum Electrodynamics as instantaneous mediators between entangled regular particles.

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