Neutrosophic Soft Graphs

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Abstract

In this research article, we present a novel frame work for handling nutrosophic soft information by combining the theory of nutrosophic soft sets with graphs. We introduce the certain notions including neutrosophic soft graphs, strong neutrosophic soft graphs, complete neutrosophic soft graphs, and illustrate these notions by several examples. We then discuss various methods of their construction and investigate some of their related properties. We also present an application of neutrosophic soft graph in decision making.

Key-words: Neutrosophic soft sets, Neutrosophic soft graphs, Decision making, Algorithm.
Mathematics Subject Classification 2000: 03E72, 68R10; 68R05

1 Introduction

The concept of fuzzy set theory was introduced by Zadeh [31] to solve difficulties in dealing with uncertainties. Since then the theory of fuzzy sets and fuzzy logic have been examined by many researchers to solve many real life problems involving ambiguous and uncertain environment. Atanassov [7] introduced the concept of intuitionistic fuzzy sets as an extension of Zadeh's fuzzy set [31]. The concept of intuitionistic fuzzy set can be viewed as an alternative approach when available information is not sufficient to define the impreciseness by the conventional fuzzy set. In fuzzy sets the degree of acceptance is considered only but intuitionistic fuzzy set is characterized by a membership(truth-membership) function and a non-membership(falsity-membership) function, the only requirement is that the sum of both values is less than one. Smarandache [27] initiated the concept of neutrosophic set in 1995. A neutrosophic set is characterized by three components: truth-membership, indeterminacy-membership, and falsity-membership which are represented independently for dealing problems involving imprecise, indeterminacy and inconsistent data. Wang [29] introduced the concept of single valued neutrosophic set(SVNS) and defined the set theoretic operators on an instance of neutrosophic set called single valued neutrosophic set.
Molodtsov [23] introduced the concept of soft set theory as a new mathematical tool for dealing with uncertainties. Molodtsov's soft sets give us new technique for dealing with uncertainty from the viewpoint of parameters. It has been revealed that soft sets have potential applications in several fields. Some new operations on soft sets proposed in [6]. The algebraic structure of soft set theory and fuzzy soft set theory dealing with uncertainties has also been studied in more detail [4, 5, 34, 35, 36]. Cağman *et al* [10, 11, 12] presented applications of fuzzy soft set theory, soft matrix theory and intuitionistic fuzzy soft set theory in decision making. Maji *et al* [18, 19, 20] proposed fuzzy soft sets, intuitionistic fuzzy soft sets and neutrosophic soft sets. Said and Smarandache [26] proposed intuitionistic neutrosophic soft set and its application in decision making problem. Deli and Broumi [13, 14, 15] introduced several concepts including neutrosophic soft relations, neutrosophic soft matrices and neutrosophic soft multi-set theory.

Based on Zadeh's fuzzy relations [32] Kaufanm defined in [17] a fuzzy graph. Rosenfeld [25] described the structure of fuzzy graphs obtaining analogs of several graph theoretical concepts. Bhattacharya [8] gave some remarks on operations on fuzzy graphs introduced by Mordeson and Nair in [24]. Akram *et al.*[1-3] introduced many new concepts, including soft graphs, fuzzy soft graphs and operations on fuzzy soft graphs. In this research article, we present a novel frame work for handling nutrosophic soft information by combining the theory of nutrosophic soft sets with graphs. We introduce the certain notions including neutrosophic soft graphs, strong neutrosophic soft graphs, complete neutrosophic soft graphs, and illustrate these notions by several examples. We then discuss various methods of their construction and investigate some of their related properties. We also present an application of neutrosophic soft graph in decision making.

2 Preliminaries

In this section, we review some basic concepts that are necessary for fully benefit of this paper.

Definition 2.1. [27] Let X be a space of points (objects). A neutrosophic set A in X is characterized by a truth-membership function $T_A(x)$, an indeterminacy-membership function $I_A(x)$ and a falsity-membership function $F_A(x)$. The functions $T_A(x)$, $I_A(x)$, and $F_A(x)$ are real standard or non-standard subsets of $]0^-, 1^+[$. That is, $T_A(x) : X \to \;]0^-, 1^+[, I_A(x) : X \to \;]0^-, 1^+[$ and $F_A(x) : X \to \;]0^-, 1^+[$ and $0^- \leq T_A(x) + I_A(x) + F_A(x) \leq 3^+$. From philosophical point of view, the neutrosophic set takes the value from real standard or non-standard subsets of $]0^-, 1^+[$. In real life applications in scientific and engineering problems, it is difficult to use neutrosophic set with value from real standard or non-standard subset of $]0^-, 1^+[$, it is considered the neutrosophic set (single valued neutrosophic set) which takes the value from the subset of $[0, 1]$.

Definition 2.2. [27] A neutrosophic set set A is contained in another neutrosophic set B, i.e., $A \subseteq B$ if $\forall\; x \in X, T_A(x) \leq T_B(x), I_A \leq I_B(x)$ and $F_A(x) \geq F_B(x)$.

Wang et al. [29] introduced the the notion of single valued neutrosophic set(SVNS). Single valued neutrosophic set is an instance of neutrosophic set which can be used in real life application in scientific and engineering problems.

Definition 2.3. [29] Let X be a space of points (objects), with a generic element in X denoted by x. A *single valued neutrosophic set* (SVNS) A in X is characterized by truth-membership function $T_A(x)$, indeterminacy-membership function $I_A(x)$ and falsity-membership function $F_A(x)$. For each point x in X, $T_A(x), I_A(x), F_A(x) \in [0, 1]$, i.e.,

$$\begin{aligned} A = \{\langle x, T_A, I_A(x), F_A(x)\rangle : x \in X\} \;\; &and \\ 0 \leq TA(x) + IA(x) + FA(x) \leq 3& \end{aligned}$$

Definition 2.4. [29] The *complement* of a neutrosophic set A over X is denoted by A^c and is defined by

$$A^c = \{\langle x, F_A(x), 1 - I_A(x), T_A(x)\rangle : x \in X\}.$$

Definition 2.5. [29] The *union* of two single valued neutrosophic sets A and B is a single valued neutrosophic set $C = A \cup B$, truth-membership, indeterminacy-membership and falsity membership functions of C is defined by

$$\begin{aligned} T_C(x) &= \max\{T_A(x), T_B(x)\}, \\ I_C(x) &= \max\{I_A(x), I_B(x)\}, \\ F_C(x) &= \min\{F_A(x), F_B(x)\} \;\forall\; x \in X. \end{aligned}$$

Definition 2.6. [29] The *intersection* of two single valued neutrosophic sets A and B is a single valued neutrosophic set $C = A \cap B$, truth-membership, indeterminacy-membership and falsity membership functions of C is defined by

$$\begin{aligned} T_C(x) &= \min\{T_A(x), T_B(x)\}, \\ I_C(x) &= \min\{I_A(x), I_B(x)\}, \\ F_C(x) &= \max\{F_A(x), F_B(x)\} \ \forall\, x \in X. \end{aligned}$$

Definition 2.7. [16]A *neutrosophic graph* is defined as a pair $G^* = (V, E)$ where

(i) $V = \{v_1, v_2, v_3, \cdots, v_n\}$ such that $T_1 : V \to [0,1]$, $I_1 : V \to [0,1]$ and $F_1 : V \to [0,1]$ denote the degree of truth-membership function, indeterminacy-membership function and falsity-membership function, respectively, and

$$0 \leq T_1(v) + I_1(v) + F_1(v) \leq 3 \quad \forall\, v \in V.$$

(ii) $E \subset V \times V$ where $T_2 : E \to [0,1]$, $I_2 : E \to [0,1]$ and $F_2 : E \to [0,1]$ are such that

$$\begin{aligned} T_2(uv) &\leq \min\{T_1(u), T_1(v)\}, \\ I_2(uv) &\leq \min\{I_1(u), I_1(v)\}, \\ F_2(uv) &\leq \max\{F_1(u), F_1(v)\}, \end{aligned}$$

and $0 \leq T_2(uv) + I_2(uv) + F_2(uv) \leq 3 \quad \forall\, uv \in E$.

Soft set theory was proposed by Molodtsov [23] in 1999. This theory provides a parameterized point of view for uncertainty modelling and soft computing. Let U be the universe of discourse and P be the universe of all possible parameters related to the objects in U. Each parameter is a word or a sentence. In most cases, parameters are considered to be attributes, characteristics or properties of objects in U. The pair (U, P) is also known as a soft universe. The power set of U is denoted by $\mathscr{P}(U)$.

Definition 2.8. A pair (F, A) is called soft set over U, where $A \subseteq P$, F is a set-valued function $F : A \to \mathscr{P}(U)$. In other words, a soft set over U is a parameterized family of subsets of U.

By means of parametrization, a soft set produces a series of approximate descriptions of a complicated object being perceived from various points of view. It is apparent that a soft set $F_A = (F, A)$ over a universe U can be viewed as a parameterized family of subsets of U. For any parameter $\epsilon \in A$, the subset $F(\epsilon) \subseteq U$ may be interpreted as the set of ϵ-approximate elements.

Definition 2.9. [20] Let U be an intitial universe and P be a set of parameters. Consider $A \subset P$. Let $P(U)$ denotes the set of all neutrosophic sets of U. The collection (F, A) is termed to be the *neutrosophic soft set* over U, where F is a mapping given by $F : A \to P(U)$.

Definition 2.10. [20] Let (F, A) and (G, B) be two neutrosophic soft sets over the common universe U. (F, A) is said to be neutrosophic soft subset of (G, B) if $A \subset B$, and $T_{F(e)}(x) \leq T_{G(e)}(x), I_{F(e)}(x) \leq I_{G(e)}(x)$ and $F_{F(e)}(x) \geq F_{G(e)}(x)$ for all $e \in M, x \in U$.

Definition 2.11. [28] Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. The *union of two neutrosophic soft sets* (H, A) and (G, B) is neutrosophic soft set $(K, C) = (H, A) \cup (G, B)$, where $C = A \cup B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are defined by

$$T_{K(e)}(x) = \begin{cases} T_{H(e)}(x), & \text{if } e \in A - B, \\ T_{G(e)}(x), & \text{if } e \in B - A, \\ \max(T_{H(e)}(x), T_{G(e)}(x)) & \text{if } e \in A \cap B. \end{cases}$$

$$I_{K(e)}(x) = \begin{cases} I_{H(e)}(x), & \text{if } e \in A - B, \\ I_{G(e)}(x), & \text{if } e \in B - A, \\ \max(I_{H(e)}(x), I_{G(e)}(x)) & \text{if } e \in A \cap B. \end{cases}$$

$$F_{K(e)}(x) = \begin{cases} F_{H(e)}(x), & \text{if } e \in A - B, \\ F_{G(e)}(x), & \text{if } e \in B - A, \\ \min(F_{H(e)}(x), F_{G(e)}(x)) & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.12. [28] Let (H, A) and (G, B) be two neutrosophic soft sets over the common universe U. The *intersection of two neutrosophic soft sets* (H, A) and (G, B) is neutrosophic soft set $(K, C) = (H, A) \cup (G, B)$, where $C = A \cap B$ and the truth-membership, indeterminacy-membership and falsity-membership of (K, C) are defined by

$$T_{K(e)}(x) = \begin{cases} T_{H(e)}(x), & \text{if } e \in A - B, \\ T_{G(e)}(x), & \text{if } e \in B - A, \\ \min T_{H(e)}(x), T_{G(e)}(x)) & \text{if } e \in A \cap B. \end{cases}$$

$$I_{K(e)}(x) = \begin{cases} I_{H(e)}(x), & \text{if } e \in A - B, \\ I_{G(e)}(x), & \text{if } e \in B - A, \\ \min(I_{H(e)}(x), I_{G(e)}(x)) & \text{if } e \in A \cap B. \end{cases}$$

$$F_{K(e)}(x) = \begin{cases} F_{H(e)}(x), & \text{if } e \in A - B, \\ F_{G(e)}(x), & \text{if } e \in B - A, \\ \max(F_{H(e)}(x), F_{G(e)}(x)) & \text{if } e \in A \cap B. \end{cases}$$

Definition 2.13. Let (H, A) and G, B be two neutrosophic soft sets over the same universe U. The *Cartesian product* of (H, A) and (G, B) is denoted by $(H, A) \times (G, B) = (K, A \times B)$, the truth-membership, indeterminacy-membership and falsity-membership functions of $(K, A \times B)$ is defined by

$$\begin{aligned} T_{K(a,b)}(u) &= \min\{T_{H(a)}(u), T_{G(b)}(u)\}, \\ I_{K(a,b)}(u) &= \min\{I_{H(a)(u), I_{H(b)}(u)}\}, \\ F_{K(a,b)}(u) &= \max\{F_{H(a)}(u), F_{G(b)}(u)\}. \end{aligned}$$

Definition 2.14. Let (H, A) and G, B be two neutrosophic soft sets over the same universe U. A *neutrosophic soft relation* from (H, A) to (G, B) is of the form (R, C), where $C \subset A \times B$ and $R(x, y) \subset (H, A) \times (G, B)$ for all $(x, y) \in C$.

3 Neutrosophic soft graphs

Let U be an initial universe and P be the set of all parameters. $\mathcal{P}(U)$ denotes the set of all neutrosophic sets of U. Let A be a subset of P. A pair (F, A) is called a neutrosophic soft set over U. Let $\mathcal{P}(V)$ denotes the set of all neutrosophic sets of V and $\mathcal{P}(E)$ denotes the set of all neutrosophic sets of E.

Definition 3.1. A neutrosophic soft graph $G = (G^*, F, K, A)$ is an ordered four tuple if it satisfies the following conditions:

(i) $G^* = (V, E)$ is a simple graph,

(ii) A is a non-empty set of parameters,

(iii) (F, A) is a neutrosophic soft set over V,

(iv) (K, A) is a neutrosophic soft set over E,

(v) $(F(e), K(e))$ is a neutrosophic graph of G^*, then

$$T_{K(e)}(xy) \leq \min\{T_{F(e)}(x), T_{F(e)}(y)\},$$

$$I_{K(e)}(xy) \leq \min\{I_{F(e)}(x), I_{F(e)}(y)\},$$
$$F_{K(e)}(xy) \leq \max\{F_{F(e)}(x), F_{F(e)}(y)\}$$

such that

$$0 \leq T_{K(e)}(xy) + I_{K(e)}(xy) + F_{K(e)}(xy) \leq 3$$

$\forall\ e \in A,\ x, y \in V$.

The neutrosophic graph $(F(e), K(e))$ is denoted by $H(e)$ for convenience. A neutrosophic soft graph is a parameterized family of neutrosophic graphs. The class of all neutrosophic soft graphs is denoted by $\mathcal{NS}(G^*)$. Note that $T_{K(e)}(xy) = I_{K(e)}(xy) = 0$ and $F_{K(e)}(xy) = 1\ \forall xy \in V \times V - E, e \notin A$.

Definition 3.2. Let $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs of G^*. Then G_1 is neutrosophic soft subgraph of G_2 if

(i) $A \subseteq B$

(ii) $H_1(e)$ is a partial subgraph of $H_2(e)$ for all $e \in A$.

Example 3.1. Consider a simple graph $G^* = (V, E)$ such that $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{v_1v_2, v_1v_3, v_1v_4, v_2v_4, v_3v_4\}$. Let $A = \{e_1, e_2\}$ be a set of parameters and let (F, A) be a neutrosophic soft set over V with neutrosophic approximation function $F : A \to \mathcal{P}(V)$ defined by
$F(e_1) = \{(v_1, 0.5, 0.4, 0.6), (v_2, 0.2, 0.6, 0.7), (v_3, 0.2, 0.4, 0.5), (v_4, 0.1, 0.4, 0.3)\}$,
$F(e_2) = \{(v_1, 0.2, 0.3, 0.5), (v_2, 0.4, 0.7, 0.3), (v_3, 0.6, 0.7, 0.4), (v_4, 0.2, 0.4, 0.5)\}$.
Let (K, A) be a neutrosophic soft set over E with neutrosophic approximation function $K : A \to \mathcal{P}(E)$ defined by
$K(e_1) = \{(v_1v_2, 0.1, 0.3, 0.5), (v_1v_3, 0.2, 0.3, 0.3), (v_1v_4, 0.1, 0.2, 0.4)\}$,
$K(e_2) = \{(v_1v_3, 0.1, 0.2, 0.4), (v_2v_4, 0.1, 0.3, 0.4), (v_3v_4, 0.2, 0.3, 0.5)\}$.
Clearly, $H(e_1) = (F(e_1), K(e_1))$ and $H(e_2) = (F(e_2), K(e_2))$ are neutrosophic graphs corresponding to the parameters e_1 and e_2, respectively as shown in Figure. 3.1.

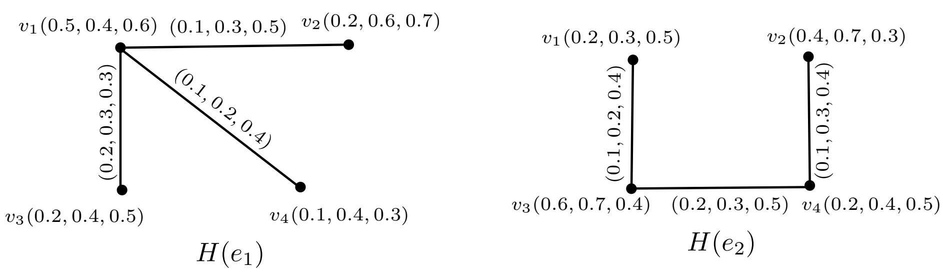


Figure 3.1: Neutrosophic soft graph $G = \{H(e_1), H(e_2)\}$.

Hence $G = \{H(e_1), H(e_2)\}$ is a neutrosophic soft graph of G^*. Tabular representation of a neutrosophic soft graph is given in Table. 1.

Table 1: Tabular representation of an intuitionistic fuzzy soft graph.

F	v_1	v_2	v_3	v_4
e_1	$(0.5, 0.4, 0.6)$	$(0.2, 0.6, 0.7)$	$(0.2, 0.4, 0.5)$	$(0.1, 0.4, 0.3)$
e_2	$(0.2, 0.3, 0.5)$	$(0.4, 0.7, 0.3)$	$(0.6, 0.7, 0.4)$	$(0.2, 0.4, 0.5)$

K	v_1v_2	v_2v_3	v_1v_3	v_1v_4	v_2v_4	v_3v_4
e_1	$(0.1, 0.3, 0.5)$	$(0.0, 0.0, 0.0)$	$(0.2, 0.3, 0.3)$	$(0.1, 0.2, 0.4)$	$(0.0, 0.0, 0.0)$	$(0.0, 0.0, 0.0)$
e_2	$(0.0, 0.0, 0.0)$	$(0.0, 0.0, 0.0)$	$(0.1, 0.2, 0.4)$	$(0.0, 0.0, 0.0)$	$(0.1, 0.3, 0.4)$	$(0.2, 0.3, 0.5)$

Definition 3.3. The neutrosophic soft graph $G_1 = (G^*, F_1, K_1, B)$ is called *spanning neutrosophic soft subgraph* of $G = (G^*, F, K, A)$ if

(i) $B \subseteq A$,

(ii) $T_{F_1(e)}(v) = T_{F(e)}(v)$,
$I_{F_1(e)}(v) = I_{F(e)}(v)$,
$F_{F_1(e)}(v) = F_{F(e)}(v)$ for all $e \in A, v \in V$.

Definition 3.4. Let $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. The *Cartesian product* of G_1 and G_2 is a neutrosophic soft graph $G = G_1 \times G_2 = (F, K, A \times B)$, where $(F = F_1 \times F_2, A \times B)$ is a neutrosophic soft set over $V = V_1 \times V_2$, $(K = K_1 \times K_2, A \times B)$ is a neutrosophic soft set over $E = \{((u, v_1), (u, v_2)) : u \in V_1, (v_1, v_2) \in E_2\} \cup \{((u_1, v), (u_2, v)) : v \in V_2, (u_1, u_2) \in E_1\}$ and $(F, K, A \times B)$ are neutrosophic soft graphs such that

(i) $T_{F(a,b)}(u, v) = T_{F_1(a)}(u) \wedge T_{F_2(b)}(v)$,
$I_{F(a,b)}(u, v) = I_{F_1(a)}(u) \wedge I_{F_2(b)}(v)$,
$F_{F(a,b)}(u, v) = F_{F_1(a)}(u) \vee F_{F_2(b)}(v) \ \forall \ (u, v) \in V, (a, b) \in A \times B$,

(ii) $T_{K(a,b)}\big((u, v_1), (u, v_2)\big) = T_{F_1(a)}(u) \wedge T_{K_2(b)}(v_1, v_2)$,
$I_{K(a,b)}\big((u, v_1), (u, v_2)\big) = I_{F_1(a)}(u) \wedge I_{K_2(b)}(v_1, v_2)$,
$F_{K(a,b)}\big((u, v_1), (u, v_2)\big) = F_{F_1(a)}(u) \vee F_{K_2(b)}(v_1, v_2) \ \forall \ u \in V_1, (v_1, v_2) \in E_2$,

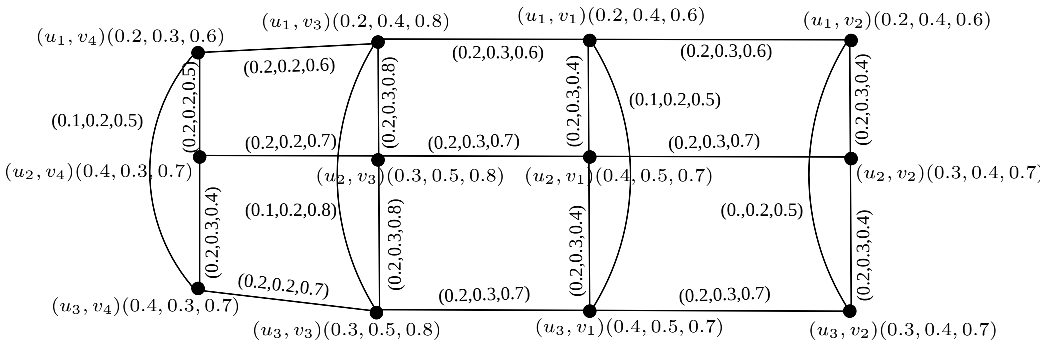
(iii) $T_{K(a,b)}\big((u_1, v), (u_2, v)\big) = T_{F_2(b)}(v) \wedge T_{K_1(a)}(u_1, u_2)$,
$I_{K(a,b)}\big((u_1, v), (u_2, v)\big) = I_{F_2(b)}(v) \wedge I_{K_1(a)}(u_1, u_2)$,
$F_{K(a,b)}\big((u_1, v), (u_2, v)\big) = F_{F_2(b)}(v) \vee F_{K_1(a)}(u_1, u_2) \ \forall \ v \in V_2, (u_1, u_2) \in E_1$.

$H(a, b) = H_1(a) \times H_2(b)$ for all $(a, b) \in A \times B$ are neutrosophic graphs of G.

Example 3.2. Let $A = \{e_1, e_2\}$ and $B = \{e_3, e_4\}$ be a set of parameters. Consider two neutrosophic soft graphs $G_1 = (H_1, A) = \{H_1(e_1), H_1(e_2)\}$ and $G_2 = (H_2, B) = \{H_2(e_2), H_2(e_3)\}$ such that

$H_1(e_1) = \big(\{(u_1, 0.2, 0.4, 0.6), (u_2, 0.4, 0.5, 0.7), (u_3, 0.4, 0.5, 0.7)\},$
$\{(u_1u_2, 0.2, 0.3, 0.4), (u_2u_3, 0.2, 0.3, 0.4), (u_1u_3, 0.1, 0.2, 0.5)\}\big),$
$H_1(e_2) = \big(\{(u_1, 0.3, 0.5, 0.7), (u_2, 0.4, 0.5, 0.6), (u_3, 0.5, 0.4, 0.3)\},$
$\{(u_1u_2, 0.2, 0.4, 0.5), (u_1u_3, 0.2, 0.3, 0.4)\}\big),$
$H_2(e_3) = \big(\{(v_1, 0.4, 0.5, 0.3), (v_2, 0.3, 0.4, 0.1), (v_3, 0.3, 0.5, 0.8), (v_4, 0.5, 0.3, 0.4)\},$
$\{(v_1v_2, 0.2, 0.3, 0.3), (v_1v_3, 0.2, 0.3, 0.5), (v_3v_4, 0.2, 0.2, 0.5)\}\big),$
$H_2(e_4) = \big(\{(v_1, 0.4, 0.5, 0.8), (v_2, 0.6, 0.3, 0.7), (v_3, 0.4, 0.4, 0.5), (v_4, 0.7, 0.2, 0.6)\},$
$\{(v_1v_2, 0.3, 0.4, 0.6), (v_1v_3, 0.2, 0.3, 0.5), (v_1v_4, 0.3, 0.2, 0.5)\}\big).$

The Cartesian product of G_1 and G_2 is $G_1 \times G_2 = G = (H, A \times B)$, where $A \times B = \{(e_1, e_3), (e_1, e_4), (e_2, e_3), (e_2, e_4)\}$, $H(e_1, e_3) = H_1(e_1) \times H_2(e_3)$, $H(e_1, e_4) = H_1(e_1) \times H_2(e_4)$, $H(e_2, e_3) = H_1(e_2) \times H_2(e_3)$ and $H(e_2, e_4) = H_1(e_2) \times H_2(e_4)$ are neutrosophic graphs of $G = G_1 \times G_2$. $H(e_1, e_3) = H_1(e_1) \times H_2(e_3)$ is shown in Figure. 3.2.



$H(e_1) \times H(e_3)$

Figure 3.2: Cartesian product: $H_1(e_1) \times H_2(e_3)$

In the similar way, Cartesian product of $H(e_1, e_4) = H_1(e_1) \times H_2(e_4)$, $H(e_2, e_3) = H_1(e_2) \times H_2(e_3)$, and $H(e_2, e_4) = H_1(e_2) \times H_2(e_4)$ can be drawn.
Hence $G = G_1 \times G_2 = \{H(e_1, e_3), H(e_1, e_4), H(e_2, e_3), H(e_2, e_4)\}$ is a neutrosophic soft graph.

Theorem 3.1. *The Cartesian product of two neutrosophic soft graph is a neutrosophic soft graph.*

Proof. Let $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Let $G = G_1 \times G_2 = (F, K, A \times B)$ be the Cartesian product of G_1 and G_2. We claim that $G = (F, K, A \times B)$ is a neutrosophic soft graph and $(H, A \times B) = \{F_1 \times F_2(a_i, b_j), K_1 \times K_2(a_i, b_j)\}$ $\forall$ $a_i \in A, b_j \in B$ for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$ are neutrosophic graphs of G.
Consider,
$T_{K_{(a_i,b_j)}}\big((u, v_1), (u, v_2)\big) = \min\{T_{F_1(a_i)}(u), T_{K_2(b_j)}(v_1, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n.$

$\leq \min\{T_{F_1(a_i)}(u), \min\{T_{F_2(b_j)}(v_1), T_{F_2(b_j)}(v_2)\}\}.$

$= \min\{\min\{T_{F_1(a_i)}(u), T_{F_2(b_j)}(v_1)\}, \min\{T_{F_1(a_i)}(u), T_{F_2(b_j)}(v_2)\}\}$

$T_{K_{(a_i,b_j)}}\big((u, v_1), (u, v_2)\big) \leq \min\{(T_{F_1(a_i)} \times T_{F_2(b_j)})(u, v_1), (T_{F_1(a_i)} \times T_{F_2(b_j)})(u, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n,$
$I_{K_{(a_i,b_j)}}\big((u, v_1), (u, v_2)\big) = \min\{I_{F_1(a_i)}(u), I_{K_2(b_j)}(v_1, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n.$

$\leq \min\{I_{F_1(a_i)}(u), \min\{I_{F_2(b_j)}(v_1), I_{F_2(b_j)}(v_2)\}\}$

$= \min\{\min\{I_{F_1(a_i)}(u), I_{F_2(b_j)}(v_1)\}, \min\{I_{F_1(a_i)}(u), I_{F_2(b_j)}(v_2)\}\}$

$I_{K_{(a_i,b_j)}}\big((u, v_1), (u, v_2)\big) \leq \min\{(I_{F_1(a_i)} \times I_{F_2(b_j)})(u, v_1), (I_{F_1(a_i)} \times I_{F_2(b_j)})(u, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$ and
$F_{K_{(a_i,b_j)}}\big((u, v_1), (u, v_2)\big) = \max\{F_{F_1(a_i)}(u), F_{K_2(b_j)}(v_1, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n.$

$\leq \max\{F_{F_1(a_i)}(u), \max\{F_{F_2(b_j)}(v_1), F_{F_2(b_j)}(v_2)\}\}$

$= \max\{\max\{F_{F_1(a_i)}(u), F_{F_2(b_j)}(v_1)\}, \max\{F_{F_1(a_i)}(u), F_{F_2(b_j)}(v_2)\}\}$

$F_{K_{(a_i,b_j)}}\big((u,v_1),(u,v_2)\big) \leq \max\{(F_{F_1(a_i)} \times F_{F_2(b_j)})(u,v_1), (F_{F_1(a_i)} \times F_{F_2(b_j)})(u,v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$.
Similarly,
$T_{K_{(a_i,b_j)}}((u_1,v),(u_2,v)) \leq \min\{(T_{F_1(a_i)} \times T_{F_2(b_j)})(u_1,v), (T_{F_1(a_i)} \times T_{F_2(b_j)})(u_2,v)\}$ for
$i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$,
$I_{K_{(a_i,b_j)}}((u_1,v),(u_2,v)) \leq \min\{(I_{F_1(a_i)} \times I_{F_2(b_j)})(u_1,v), (I_{F_1(a_i)} \times I_{F_2(b_j)})(u_2,v)\}$ for
$i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$ and
$F_{K_{(a_i,b_j)}}((u_1,v),(u_2,v)) \leq \max\{(F_{F_1(a_i)} \times F_{F_2(b_j)})(u_1,v), (F_{F_1(a_i)} \times F_{F_2(b_j)})(u_2,v)\}$ for
$i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$.
Hence $G = (F, K, A \times B)$ is a neutrosophic soft graph. □

Definition 3.5. The *cross product* of G_1 and G_2 is a neutrosophic soft graph $G = G_1 \circledcirc G_2 = (F, K, A \times B)$, where $(F, A \times B)$ is a neutrosophic soft set over $V = V_1 \times V_2$, $(K, A \times B)$ is a neutrosophic soft set over $E = \{((u_1,v_1),(u_2,v_2)) : (u_1,u_2) \in E_1, (v_1,v_2) \in E_2\}$ and $(F, K, A \times B)$ are neutrosophic soft graphs such that

(i) $T_{F(a,b)}(u,v) = T_{F_1(a)}(u) \wedge T_{F_2(b)}(v)$,
$I_{F(a,b)}(u,v) = I_{F_1(a)}(u) \wedge I_{F_2(b)}(v)$,
$F_{F(a,b)}(u,v) = F_{F_1(a)}(u) \vee F_{F_2(b)}(v)\ \forall\ (u,v) \in V, (a,b) \in A \times B$

(ii) $T_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = T_{K_1(a)}(u_1,u_2) \wedge T_{K_2(b)}(v_1,v_2)$,
$I_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = I_{K_1(a)}(u_1,u_2) \wedge I_{K_2(b)}(v_1,v_2)$,
$F_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = F_{K_1(a)}(u_1,u_2) \vee F_{K_2(b)}(v_1,v_2)\ \forall\ (u_1,u_2) \in E_1, (v_1,v_2) \in E_2$.

$H(a,b) = H_1(a) \circledcirc H_2(b)$ for all $(a,b) \in A \times B$ are neutrosophic graphs of G.

Theorem 3.2. *The cross product of two neutrosophic soft graph is a neutrosophic soft graph.*

Proof. Let $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Let $G = G_1 \circledcirc G_2 = (F, K, A \times B)$ be the cross product of G_1 and G_2. We claim that $G = (F, K, A \times B)$ is a neutrosophic soft graph and $(H, A \times B) = \{F_1 \circledcirc F_2(a_i,b_j), K_1 \circledcirc K_2(a_i,b_j)\}\ \forall\ a_i \in A,\ b_j \in B$ for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$ are intuitonistic fuzzy graphs of G.
Consider,
$T_{K(a_i,b_j)}\big((u_1,v_1),(u_2,v_2)\big) = \min\{T_{K_1(a_i)}(u_1,u_2), T_{K_2(b_j)}(v_1,v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$.

$\leq \min\{\min\{T_{F_1(a_i)}(u_1), T_{F_1(a_i)}(u_2)\}, \min\{T_{F_2(b_j)}(v_1), T_{F_2(b_j)}(v_2)\}\}$

$= \min\{\min\{T_{F_1(a_i)}(u_1), T_{F_2(b_j)})(v_1)\}, \min\{T_{F_1(a_i)}(u_2), T_{F_2(b_j)}(v_2)\}\}$

$T_{K(a_i,b_j)}\big((u_1,v_1),(u_2,v_2)\big) \leq \min\{T_{F_1(a_i)} \circledcirc T_{F_2(b_j)}(u_1,v_1), T_{F_1(a_i)} \circledcirc T_{F_2(b_j)}(u_2,v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$,
$I_{K(a_i,b_j)}\big((u_1,v_1),(u_2,v_2)\big) = \min\{I_{K_1(a_i)}(u_1,u_2), I_{K_2(b_j)}(v_1,v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$.

$\leq \min\{\min\{I_{F_1(a_i)}(u_1), I_{F_1(a_i)}(u_2)\}, \min\{I_{F_2(b_j)}(v_1), I_{F_2(b_j)}(v_2)\}\}$

$= \min\{\min\{I_{F_1(a_i)}(u_1), I_{F_2(b_j)})(v_1)\}, \min\{I_{F_1(a_i)}(u_2), I_{F_2(b_j)}(v_2)\}\}$

$I_{K(a_i,b_j)}\big((u_1,v_1),(u_2,v_2)\big) \leq \min\{I_{F_1(a_i)} \circledcirc I_{F_2(b_j)}(u_1,v_1), I_{F_1(a_i)} \circledcirc I_{F_2(b_j)}(u_2,v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$, and

$F_{K(a_i,b_j)}\big((u_1,v_1),(u_2,v_2)\big) = \min\{F_{K_1(a_i)}(u_1,u_2), F_{K_2(b_j)}(v_1,v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n.$

$\leq \min\{\min\{F_{F_1(a_i)}(u_1), F_{F_1(a_i)}(u_2)\}, \min\{F_{F_2(b_j)}(v_1), F_{F_2(b_j)}(v_2)\}\}$

$= \min\{\min\{F_{F_1(a_i)}(u_1), F_{F_2(b_j)})(v_1)\}, \min\{F_{F_1(a_i)}(u_2), F_{F_2(b_j)}(v_2)\}\}$

$F_{K(a_i,b_j)}\big((u_1,v_1),(u_2,v_2)\big) \leq \min\{F_{F_1(a_i)} \circledcirc F_{F_2(b_j)}(u_1,v_1), F_{F_1(a_i)} \circledcirc F_{F_2(b_j)}(u_2,v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n.$
Hence $G = (F, K, A \times B)$ is a neutrosophic soft graph. □

Definition 3.6. The *lexicographic product* of G_1 and G_2 is a neutrosophic soft graph $G = G_1 \odot G_2 = (F, K, A \times B)$, where $(F, A \times B)$ is a neutrosophic soft set over $V = V_1 \times V_2$, $(K, A \times B)$ is a neutrosophic soft set over $E = \{((u,v_1),(u,v_2)) : u \in V_1, (v_1,v_2) \in E_2\} \cup \{((u_1,v_1),(u_2,v_2)) : (u_1,u_2) \in E_1, (v_1,v_2) \in E_2\}$ and $(F, K, A \times B)$ are neutrosophic soft graphs such that

(i) $T_{F(a,b)}(u,v) = T_{F_1(a)}(u) \wedge T_{F_2(b)}(v)$,
$I_{F(a,b)}(u,v) = I_{F_1(a)}(u) \wedge I_{F_2(b)}(v)$,
$F_{F(a,b)}(u,v) = F_{F_1(a)}(u) \vee F_{F_2(b)}(v)\ \forall\ (u,v) \in V, (a,b) \in A \times B$,

(ii) $T_{K(a,b)}\big((u,v_1),(u,v_2)\big) = T_{F_1(a)}(u) \wedge T_{K_2(b)}(v_1,v_2)$,
$I_{K(a,b)}\big((u,v_1),(u,v_2)\big) = I_{F_1(a)}(u) \wedge I_{K_2(b)}(v_1,v_2)$,
$F_{K(a,b)}\big((u,v_1),(u,v_2)\big) = F_{F_1(a)}(u) \vee F_{K_2(b)}(v_1,v_2)\ \forall\ u \in V_1, (v_1,v_2) \in E_2$,

(iii) $T_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = T_{K_1(a)}(u_1,u_2) \wedge T_{K_2(b)}(v_1,v_2)$,
$I_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = I_{K_1(a)}(u_1,u_2) \wedge I_{K_2(b)}(v_1,v_2)$,
$F_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = F_{K_1(a)}(u_1,u_2) \vee F_{K_2(b)}(v_1,v_2)\ \forall\ (u_1,u_2) \in E_1, (v_1,v_2) \in E_2$.

$H(a,b) = H_1(a) \odot H_2(b)$ for all $(a,b) \in A \times B$ are neutrosophic graphs of G.

Theorem 3.3. *The lexicographic product of two neutrosophic soft graph is a neutrosophic soft graph.*

Definition 3.7. The *strong product* of G_1 and G_2 is a neutrosophic soft graph $G = G_1 \otimes G_2 = (F, K, A \times B)$, where $(F, A \times B)$ is a neutrosophic soft set over $V = V_1 \times V_2$, $(K, A \times B)$ is a neutrosophic soft set over $E = \{((u,v_1),(u,v_2)) : u \in V_1, (v_1,v_2) \in E_2\} \cup \{((u_1,v),(u_2,v)) : v \in V_2, (u_1,u_2) \in E_1\} \cup \{((u_1,v_1),(u_2,v_2)) : (u_1,u_2) \in E_1, (v_1,v_2) \in E_2\}$ and $(F, K, A \times B)$ are neutrosophic soft graphs such that

(i) $T_{F(a,b)}(u,v) = T_{F_1(a)}(u) \wedge T_{F_2(b)}(v)$,
$I_{F(a,b)}(u,v) = I_{F_1(a)}(u) \wedge I_{F_2(b)}(v)$,
$F_{F(a,b)}(u,v) = F_{F_1(a)}(u) \vee F_{F_2(b)}(v)\ \forall\ (u,v) \in V, (a,b) \in A \times B$,

(ii) $T_{K(a,b)}\big((u,v_1),(u,v_2)\big) = T_{F_1(a)}(u) \wedge T_{K_2(b)}(v_1,v_2)$,
$I_{K(a,b)}\big((u,v_1),(u,v_2)\big) = I_{F_1(a)}(u) \wedge I_{K_2(b)}(v_1,v_2)$,
$F_{K(a,b)}\big((u,v_1),(u,v_2)\big) = F_{F_1(a)}(u) \vee F_{K_2(b)}(v_1,v_2)\ \forall\ u \in V_1, (v_1,v_2) \in E_2$,

(iii) $T_{K(a,b)}\big((u_1,v),(u_2,v)\big) = T_{F_2(b)}(v) \wedge T_{K_1(a)}(u_1,u_2)$,
$I_{K(a,b)}\big((u_1,v),(u_2,v)\big) = I_{F_2(b)}(v) \wedge I_{K_1(a)}(u_1,u_2)$,
$F_{K(a,b)}\big((u_1,v),(u_2,v)\big) = F_{F_2(b)}(v) \vee F_{K_1(a)}(u_1,u_2)\ \forall\ v \in V_2, (u_1,u_2) \in E_1$,

(iv) $T_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = T_{K_1(a)}(u_1,u_2) \wedge T_{K_2(b)}(v_1,v_2)$,
$I_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = I_{K_1(a)}(u_1,u_2) \wedge I_{K_2(b)}(v_1,v_2)$,
$F_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = F_{K_1(a)}(u_1,u_2) \vee F_{K_2(b)}(v_1,v_2)\ \forall\ (u_1,u_2) \in E_1, (v_1,v_2) \in E_2$.

$H(a,b) = H_1(a) \otimes H_2(b)$ for all $(a,b) \in A \times B$ are neutrosophic graphs of G.

Theorem 3.4. *The strong product of two neutrosophic soft graph is a neutrosophic soft graph.*

Definition 3.8. The *composition* of G_1 and G_2 is a neutrosophic soft graph $G = G_1[G_2] = (F, K, A\times B)$, where $(F, A\times B)$ is a neutrosophic soft set over $V = V_1 \times V_2$, $(K, A\times B)$ is a neutrosophic soft set over $E = \{((u,v_1),(u,v_2)) : u \in V_1, (v_1,v_2) \in E_2\} \cup \{((u_1,v),(u_2,v)) : v \in V_2, (u_1,u_2) \in E_1\} \cup \{((u_1,v_1),(u_2,v_2)) : (u_1,u_2) \in E_1, v_1 \neq v_2\}$ and $(F, K, A \times B)$ are neutrosophic soft graphs such that

(i) $T_{F(a,b)}(u,v) = T_{F_1(a)}(u) \wedge T_{F_2(b)}(v)$,
$I_{F(a,b)}(u,v) = I_{F_1(a)}(u) \wedge I_{F_2(b)}(v)$,
$F_{F(a,b)}(u,v) = F_{F_1(a)}(u) \vee F_{F_2(b)}(v) \ \forall \ (u,v) \in V, (a,b) \in A \times B$,

(ii) $T_{K(a,b)}((u,v_1),(u,v_2)) = T_{F_1(a)}(u) \wedge T_{K_2(b)}(v_1,v_2)$,
$I_{K(a,b)}((u,v_1),(u,v_2)) = I_{F_1(a)}(u) \wedge I_{K_2(b)}(v_1,v_2)$,
$F_{K(a,b)}((u,v_1),(u,v_2)) = F_{F_1(a)}(u) \vee F_{K_2(b)}(v_1,v_2) \ \forall \ u \in V_1, (v_1,v_2) \in E_2$,

(iii) $T_{K(a,b)}\big((u_1,v),(u_2,v)\big) = T_{F_2(b)}(v) \wedge T_{K_1(a)}(u_1,u_2)$,
$I_{K(a,b)}\big((u_1,v),(u_2,v)\big) = I_{F_2(b)}(v) \wedge I_{K_1(a)}(u_1,u_2)$,
$F_{K(a,b)}\big((u_1,v),(u_2,v)\big) = F_{F_2(b)}(v) \vee F_{K_1(a)}(u_1,u_2) \ \forall \ v \in V_2, (u_1,u_2) \in E_1$,

(iv) $T_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = T_{F_1(a)}(u_1,u_2) \wedge T_{F_2(b)}(v_1) \wedge T_{F_2(b)}(v_2)$,
$I_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = I_{F_1(a)}(u_1,u_2) \wedge I_{F_2(b)}(v_1) \wedge I_{F_2(b)}(v_2)$,
$F_{K(a,b)}\big((u_1,v_1),(u_2,v_2)\big) = F_{F_1(a)}(u_1,u_2) \vee F_{F_2(b)}(v_1) \vee F_{F_2(b)}(v_2) \ \forall \ (u_1,u_2) \in E_1$, where $v_1 \neq v_2$.

$H(a,b) = H_1(a)[H_2(b)]$ for all $(a,b) \in A \times B$ are neutrosophic graphs of G.

Example 3.3. Let $A = \{e_1\}$ and $B = \{e_2, e_3\}$ be the parameter sets. Let G_1 and G_2 be the two neutrosophic soft graphs defined as follows:
$G_1 = \{H_1(e_1)\} = \{(\{(u_1, 0.3, 0.4, 0.6), (u_2, 0.4, 0.5, 0.7)\}, \{(u_1u_2, 0.3, 0.4, 0.6)\})\}$,
$G_2 = \{H_2(e_2), H(e_3)\} = \{(\{(v_1, 0.4, 0.5, 0.3), (v_2, 0.7, 0.2, 0.4), (v_3, 0.5, 0.6, 0.3)\}, \{(v_1v_3, 0.4, 0.5, 0.2), (v_2v_3, 0.5, 0.2, 0.4)\}), (\{(v_1, 0.3, 0.4, 0.4), (v_2, 0.2, 0.4, 0.8), (v_3, 0.6, 0.5, 0.7)\}, \{(v_1v_2, 0.2, 0.3, 0.7), (v_1v_3, 0.1, 0.3, 0.6)\})\}$.
The composition of G_1 and G_2 is $G = G_1[G_2] = (H, A\times B)$, where $A\times B = \{(e_1,e_2),(e_1,e_3)\}$, $H(e_1,e_2) = H_1(e_1)[H_2(e_2)]$ and $H(e_1,e_3) = H_1(e_1)[H_2(e_3)]$ are neutrosophic graphs of $G_1[G_2]$. $H_1(e_1)[H_2(e_2)]$ is shown in Figure. 3.3

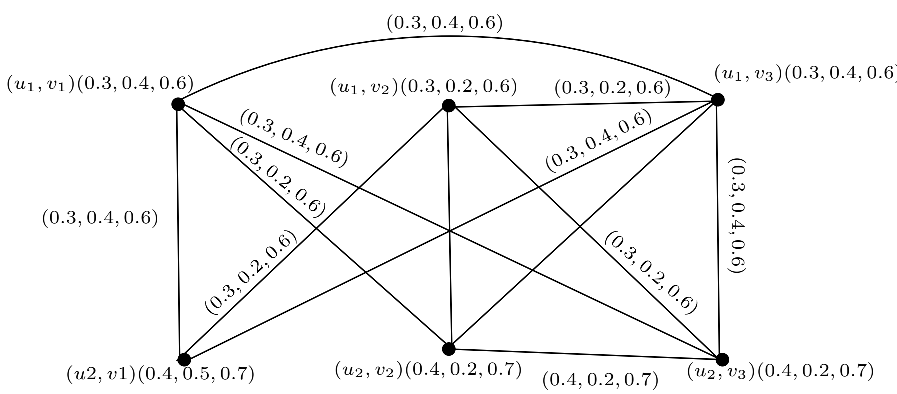


Figure 3.3: Composition: $H_1(e_1)[H_2(e_2)]$

Similarly, composition of neutrosophic graphs $H_1(e_1)$ and $H_2(e_3)$ of G_1 and G_2, respectively can be drawn.
Hence $G = G_1[G_2] = \{H_1(e_1)[H_2(e_2)], H_1(e_1)[H_2(e_3)]\}$ is a neutrosophic soft graph.

Theorem 3.5. *If G_1 and G_2 are neutrosophic soft graphs, then $G_1[G_2]$ is a neutrosophic soft graph.*

Proof. $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs of $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$, respectively. Let $G_1[G_2] = G = (F, K, A \times B)$, be the composition of G_1 and G_2. We claim that $G_1[G_2] = G = (F, K, A \times B)$ is a neutrosophic soft graph and $(H, A \times B) = \{F_1(a_i)[F_2(b_j)], K_1(a_i)[K_2(b_j)]\}$ $\forall a_i \in A, b_j \in B$ for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$ are neutrosophic graphs of G.
Let $u \in V_1$ and $(v_1, v_2) \in E_2$, we have
$T_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) = \min\{T_{F_1(a_i)}(u), T_{K_2(b_j)}(v_1, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$.
$T_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) \leq \min\{T_{F_1(a_i)}(u), \min\{T_{F_2(b_j)}(v_1), T_{F_2(b_j)}(v_2)\}\}$

$= \min\{\min\{T_{F_1(a_i)}(u), T_{F_2(b_j)}(v_1)\} \min\{T_{F_1(a_i)}(u), T_{F_2(b_j)}(v_2)\}\}$

$= \min\{(T_{F_1(a_i)} \times T_{F_2(b_j)})(u, v_1), (T_{F_1(a_i)} \times T_{F_2(b_j)})(u, v_2)\},$

$T_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) \leq \min\{T_{F(a_i,b_j)}(u, v_1), T_{F(a_i,b_j)}(u, v_2)\},$

$I_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) = \min\{I_{F_1(a_i)}(u), I_{K_2(b_j)}(v_1, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$.
$I_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) \leq \min\{I_{F_1(a_i)}(u), \min\{I_{F_2(b_j)}(v_1), I_{F_2(b_j)}(v_2)\}\}$

$= \min\{\min\{I_{F_1(a_i)}(u), I_{F_2(b_j)}(v_1)\}, \min\{I_{F_1(a_i)}(u), I_{F_2(b_j)}(v_2)\}\}$

$= \min\{(I_{F_1(a_i)} \times I_{F_2(b_j)})(u, v_1), (I_{F_1(a_i)} \times I_{F_2(b_j)})(u, v_2)\}$

$I_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) \leq \min\{I_{F(a_i,b_j)}(u, v_1), I_{F(a_i,b_j)}(u, v_2)\},$

$F_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) = \max\{F_{F_1(a_i)}(u), F_{K_2(b_j)}(v_1, v_2)\}$
for $i = 1, 2, \cdots, m,\ j = 1, 2, \cdots, n$.
$F_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) \leq \max\{F_{F_1(a_i)}(u), \max\{F_{F_2(b_j)}(v_1), F_{F_2(b_j)}(v_2)\}\}$

$= \max\{\max\{F_{F_1(a_i)}(u), F_{F_2(b_j)}(v_1)\}, \max\{F_{F_1(a_i)}(u), F_{F_2(b_j)}(v_2)\}\}$

$= \max\{(F_{F_1(a_i)} \times F_{F_2(b_j)})(u, v_1), (F_{F_1(a_i)} \times F_{F_2(b_j)})(u, v_2)\},$

$F_{K(a_i,b_j)}\big((u, v_1), (u, v_2)\big) \leq \max\{F_{F(a_i,b_j)}(u, v_1), F_{F(a_i,b_j)}(u, v_2)\}.$

Similarly, for any $v \in V_2$ and $(u_1, u_2) \in E_1$, we have

$T_{K(a_i,b_j)}\big((u_1, v), (u_2, v)\big) \leq \min\{T_{F(a_i,b_j)}(u_1, v), T_{F(a_i,b_j)}(u_2, v)\},$

$I_{K(a_i,b_j)}\big((u_1, v), (u_2, v)\big) \leq \min\{I_{F(a_i,b_j)}(u_1, v), I_{F(a_i,b_j)}(u_2, v)\},$

$F_{K(a_i,b_j)}\big((u_1, v), (u_2, v)\big) \leq \max\{F_{F(a_i,b_j)}(u_1, v), F_{F(a_i,b_j)}(u_2, v)\}.$

Let $(u_1, v_1)(u_2, v_2) \in E^*$, $(u_1, u_2) \in E_1$, and $v_1 \neq v_2$. Then we have

$T_{K(a_i,b_j)}\big((u_1, v_1), (u_2, v_2)\big) = \min\{T_{K_1(a_i)}(u_1, u_2), T_{F_2(b_j)}(v_1), T_{F_2(b_j)}(v_2)\}$

$\leq \min\{\min\{T_{F_1(a_i)}(u_1), T_{F_1(a_i)}(u_2)\}, T_{F_2(b_j)}(v_1), T_{F_2(b_j)}(v_2)\}$

$= \min\{\min\{T_{F_1(a_i)}(u_1), T_{F_2(b_j)}(v_1)\}, \min\{T_{F_1(a_i)}(u_2), T_{F_2(b_j)}(v_2)\}\}$

$T_{K(a_i,b_j)}\big((u_1, v_1), (u_2, v_2)\big) \leq \min\{T_{F(a_i,b_j)}(u_1, v_1), T_{F(a_i,b_j)}(u_2, v_2)\}$,

$I_{K(a_i,b_j)}\big((u_1, v_1), (u_2, v_2)\big) = \min\{I_{K_1(a_i)}(u_1, u_2), I_{F_2(b_j)}(v_1), I_{F_2(b_j)}(v_2)\}$

$\leq \min\{\min\{I_{F_1(a_i)}(u_1), I_{F_1(a_i)}(u_2)\}, I_{F_2(b_j)}(v_1), I_{F_2(b_j)}(v_2)\}$

$= \min\{\min\{I_{F_1(a_i)}(u_1), I_{F_2(b_j)}(v_1)\}, \min\{I_{F_1(a_i)}(u_2), I_{F_2(b_j)}(v_2)\}\}$

$I_{K(a_i,b_j)}\big((u_1, v_1), (u_2, v_2)\big) \leq \min\{I_{F(a_i,b_j)}(u_1, v_1), I_{F(a_i,b_j)}(u_2, v_2)\}$,

$F_{K(a_i,b_j)}\big((u_1, v_1), (u_2, v_2)\big) = \max\{F_{K_1(a_i)}(u_1, u_2), F_{F_2(b_j)}(v_1), F_{F_2(b_j)}(v_2)\}$

$\leq \max\{\max\{F_{F_1(a_i)}(u_1), F_{F_1(a_i)}(u_2)\}, F_{F_2(b_j)}(v_1), F_{F_2(b_j)}(v_2)\}$

$= \max\{\max\{F_{F_1(a_i)}(u_1), F_{F_2(b_j)}(v_1)\}, \max\{F_{F_1(a_i)}(u_2), F_{F_2(b_j)}(v_2)\}\}$

$F_{K(a_i,b_j)}\big((u_1, v_1), (u_2, v_2)\big) \leq \max\{F_{F(a_i,b_j)}(u_1, v_1), F_{F(a_i,b_j)}(u_2, v_2)\}$.
Hence $G = (F, K, A \times B)$ is a neutrosophic soft graph. □

Definition 3.9. Let $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs. The *intersection* of G_1 and G_2 is a neutrosophic soft graph denoted by $G = G_1 \cap G_2 = (F, K, A \cup B)$, where $(F, A \cup B)$ is a neutrosophic soft set over $V = V_1 \cap V_2$, $(K, A \cup B)$ is a neutrosophic soft set over $E = E_1 \cap E_1$, the truth-membership, indeterminacy-membership, and falsity-membership functions of G for all $u, v \in V$ defined by,

(i) $T_{F(e)}(v) = \begin{cases} T_{F_1(e)}(v) & \text{if } e \in A - B; \\ T_{F_2(e)}(v) & \text{if } e \in B - A; \\ T_{F_1(e)}(v) \wedge T_{F_2(e)}(v), & \text{if } e \in A \cap B. \end{cases}$

$$I_{F(e)}(v) = \begin{cases} I_{F_1(e)}(v) & \text{if } e \in A - B; \\ I_{F_2(e)}(v) & \text{if } e \in B - A; \\ I_{F_1(e)}(v) \wedge I_{F_2(e)}(v), & \text{if } e \in A \cap B. \end{cases}$$

$$F_{F(e)}(v) = \begin{cases} F_{F_1(e)}(v) & \text{if } e \in A - B; \\ F_{F_2(e)}(v) & \text{if } e \in B - A; \\ F_{F_1(e)}(v) \vee F_{F_2(e)}(v), & \text{if } e \in A \cap B. \end{cases}$$

(ii) $T_{K(e)}(uv) = \begin{cases} T_{K_1(e)}(uv) & \text{if } e \in A - B; \\ T_{K_2(e)}(uv) & \text{if } e \in B - A; \\ T_{K_1(e)}(uv) \wedge T_{K_2(e)}(uv), & \text{if } e \in A \cap B. \end{cases}$

$$I_{K(e)}(uv) = \begin{cases} I_{K_1(e)}(uv) & \text{if } e \in A - B; \\ I_{K_2(e)}(uv) & \text{if } e \in B - A; \\ I_{K_1(e)}(uv) \wedge I_{K_2(e)}(uv), & \text{if } e \in A \cap B. \end{cases}$$

$$F_{K(e)}(uv) = \begin{cases} F_{K_1(e)}(uv) & \text{if } e \in A - B; \\ F_{K_2(e)}(uv) & \text{if } e \in B - A; \\ F_{K_1(e)}(uv) \vee F_{K_2(e)}(uv), & \text{if } e \in A \cap B. \end{cases}$$

Example 3.4. Let $A = \{e_1, e_2\}$ and $B = \{e_1, e_3\}$ be two parameters sets. Let G_1 and G_2 be two neutrosophic soft graphs defined by
$G_1 = \{H_1(e_1), H_1(e_2)\}$, where

$H_1(e_1) = (\{(u_1, 0.3, 0.4, 0.6), (u_2, 0.4, 0.5, 0.3), (u_3, 0.6, 0.4, 0.5), (u_4, 0.5, 0.4, 0.7)\},$
$\{(u_1u_2, 0.2, 0.1, 0.3), (u_1u_4, 0.2, 0.3, 0.5), (u_2u_4, 0.3, 0.2, 0.5), (u_2u_3, 0.1, 0.4, 0.5)\}),$
$H_1(e_2) = (\{(u_1, 0.4, 0.5, 0.6), (u_2, 0.3, 0.5, 0.4), (u_3, 0.5, 0.3, 0.4), (u_4, 0.6, 0.3, 0.4)\}, \{(u_1u_2,$
$0.2, 0.3, 0.3), (u_2u_3, 0.1, 0.2, 0.3), (u_2u_4, 0.2, 0.1, 0.4)\})$ and
$G_2 = \{e_1, e_3\}$, where
$H_2(e_1) = (\{(u_1, 0.3, 0.4, 0.5), (u_2, 0.5, 0.3, 0.2), (u_3, 0.5, 0.3, 0.6), (v_1, 0.5, 0.6, 0.4)\}, \{(u_1v_1,$
$0.2, 0.2, 0.3), (u_1u_2, 0.2, 0.3, 0.3)(u_2u_3, 0.2, 0.2, 0.4)\}),$
$H_2(e_3) = (\{(u_1, 0.5, 0.7, 0.3), (u_2, 0.7, 0.6, 0.5), (u_3, 0.5, 0.5, 0.6), (v_1, 0.3, 0.4, 0.6)\}, \{(u_1v_1,$
$0.2, 0.3, 0.2), (u_2v_1, 0.3, 0.3, 0.6), (u_2u_3, 0.4, 0.2, 0.6)\}).$

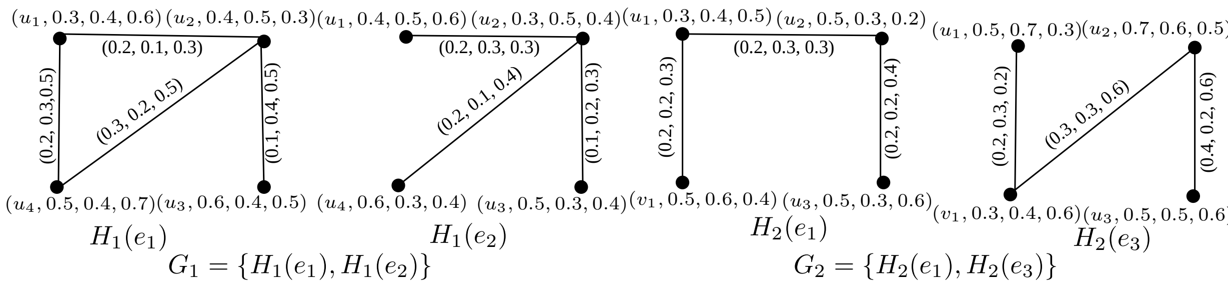


Figure 3.4: Neutrosophic soft graphs G_1 and G_2.

Intersection of G_1 and G_2 is a neutrosophic soft graph $G = G_1 \cap G_2 = (H, A \cup B)$, where $A \cup B = \{e_1, e_2, e_3\}$, $H(e_1) = H_1(e_1) \cap H_2(e_1)$, $H(e_2)$ and $H(e_3)$ are neutrosophic graphs of G corresponding to the parameters e_1, e_2 and e_3, respectively are shown in Figure.3.5.

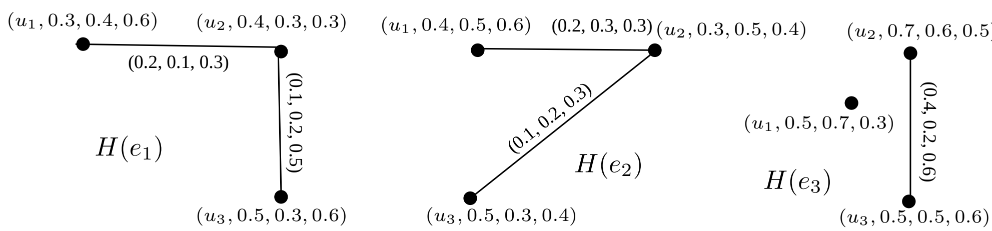


Figure 3.5: Neutrosophic graph $G = G_1 \cap G_2$.

Definition 3.10. Let $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ be two neutrosophic soft graphs. The *union* of G_1 and G_2 is a neutrosophic soft graph denoted by $G = G_1 \cup G_2 = (F, K, A \cup B)$, where $(F, A \cup B)$ is a neutrosophic soft set over $V = V_1 \cup V_2$, $(K, A \cup B)$ is a neutrosophic soft set over $E = E_1 \cup E_1$, the truth-membership, indeterminacy-membership, and falsity-membership functions of G for all $u, v \in V$ defined by,

(i) $$T_{F(e)}(v) = \begin{cases} T_{F_1(e)}(v) & \text{if } e \in A - B; \\ T_{F_2(e)}(v) & \text{if } e \in B - A; \\ T_{F_1(e)}(v) \vee T_{F_2(e)}(v), & \text{if } e \in A \cap B. \end{cases}$$

$$I_{F(e)}(v) = \begin{cases} I_{F_1(e)}(v) & \text{if } e \in A - B; \\ I_{F_2(e)}(v) & \text{if } e \in B - A; \\ I_{F_1(e)}(v) \vee I_{F_2(e)}(v), & \text{if } e \in A \cap B. \end{cases}$$

$$F_{F(e)}(v) = \begin{cases} F_{F_1(e)}(v) & \text{if } e \in A - B; \\ F_{F_2(e)}(v) & \text{if } e \in B - A; \\ F_{F_1(e)}(v) \wedge F_{F_2(e)}(v), & \text{if } e \in A \cap B. \end{cases}$$

(ii) $$T_{K(e)}(uv) = \begin{cases} T_{K_1(e)}(uv) & \text{if } e \in A - B; \\ T_{K_2(e)}(uv) & \text{if } e \in B - A; \\ T_{K_1(e)}(uv) \vee T_{K_2(e)}(uv), & \text{if } e \in A \cap B. \end{cases}$$

$$I_{K(e)}(uv) = \begin{cases} I_{K_1(e)}(uv) & \text{if } e \in A - B; \\ I_{K_2(e)}(uv) & \text{if } e \in B - A; \\ I_{K_1(e)}(uv) \vee I_{K_2(e)}(uv), & \text{if } e \in A \cap B. \end{cases}$$

$$F_{K(e)}(uv) = \begin{cases} F_{K_1(e)}(uv) & \text{if } e \in A - B; \\ F_{K_2(e)}(uv) & \text{if } e \in B - A; \\ F_{K_1(e)}(uv) \wedge F_{K_2(e)}(uv), & \text{if } e \in A \cap B. \end{cases}$$

Theorem 3.6. *Let G_1 and G_2 be two neutrosophic soft graph over G^* such that $A \cap B \neq \emptyset$, then $G_1 \cup G_2$ is a neutrosophic soft graph.*

Proof. The union of $G_1 = (F_1, K_1, A)$ and $G_2 = (F_2, K_2, B)$ defined by $G_1 \cup G_2 = (H, A \cup B)$, where

$$H(e) = \begin{cases} H_1(e) & \text{if } e \in A - B; \\ H_2(e) & \text{if } e \in B - A; \\ H_1(e) \cup H_2(e), & \text{if } e \in A \cap B. \end{cases}$$

Since $G_1 \in \mathcal{NS}(G_1^*)$ and $G_2 \in \mathcal{NS}(G_2^*)$, then $H_1(e)$ and $H_2(e)$ are neutrosophic graphs for all $e \in A \cup B$. The union of two intuitionistic fuzzy graphs $H_1(e) \Cup H_2(e)$ is a neutrosophic graph for all $e \in A \cap B$. Therefore, $H(e)$ are neutrosophic graph of G for all $e \in A \cup B$. Hence $G = (H, A \cup B)$ is a neutrosophic soft graph over G^*.

□

Definition 3.11. Let G_1 and G_2 be two neutrosophic soft graphs. The *join* of G_1 and G_2 is a neutrosophic soft graph denoted by $G_1 + G_2 = (F_1 + F_2, K_1 + K_2, A \cup B)$, where $(F_1 + F_2, A \cup B)$ is a neutrosophic soft set over $V_1 \cup V_2$, $(K_1 + K_2, A \cup B)$ is a neutrosophic soft set over $E_1 \cup E_2 \cup \acute{E}$ defined by

(i) $(F_1 + F_2, A \cup B) = (F_1, A) \cup (F_2, B)$,

(ii) $(K_1 + K_2, A \cup B) = (K_1, A) \cup (K_2, B)$ if $uv \in E_1 \cup E_2$,
when $e \in A \cap B, uv \in \acute{E}$, where $\acute{E}$ is the set of all edges joining the vertices of V_1 and V_2, the truth-membership, indeterminacy-membership, and falsity-membership functions are defined by

$$\begin{aligned} T_{K_1+K_2(e)}(uv) &= \min\{T_{F_1(e)}(u), T_{F_2(e)}(v)\}, \\ I_{K_1+K_2(e)}(uv) &= \min\{I_{F_1(e)}(u), I_{F_2(e)}(v)\}, \\ F_{K_1+K_2(e)}(uv) &= \max\{F_{F_1(e)}(u), F_{F_2(e)}(v)\} \ \forall uv \in \acute{E}. \end{aligned}$$

Example 3.5. Let $A = \{e_1, e_2\}$ and $B = \{e_1, e_3\}$ be parameter sets. Let G_1 and G_2 be two neutrosophic soft graphs defined as follows:
$G_1 = \{H_1(e_1), H_1(e_2)\}$, where
$H_1(e_1) = \big(\{(v_1, 0.5, 0.5, 0.6), (v_2, 0.6, 0.5, 0.3), (v_3, 0.8, 0.5, 0.2)\}, \{(v_1v_2, 0.3, 0.4, 0.3), (v_1v_3, 0.3, 0.5, 0.5)\}\big)$,
$H_1(e_2) = \big(\{(v_1, 0.5, 0.6, 0.7), (v_2, 0.4, 0.4, 0.3), (v_3, 0.7, 0.9, 0.7), (v_4, 0.4, 0.3, 0.5)\}, \{(v_1v_2, 0.3, 0.4, 0.6), (v_1v_3, 0.4, 0.5, 0.6), (v_1v_4, 0.2, 0.1, 0.4)\}\big)$ as shown in Figure.3.6

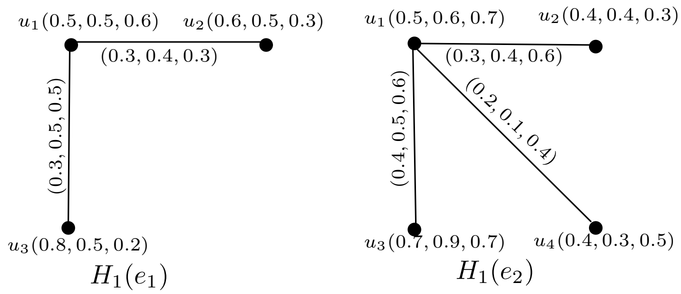


Figure 3.6: Neutrosophic soft graph $G_1 = \{H_1(e_1, H_1(e_2))\}$.

$G_2 = \{H_2(e_1)\}$, where
$H_2(e_2) = \big(\{(v_1, 0.5, 0.6, 0.4), (v_2, 0.5, 0.4, 0.7)\}, \{(v_1v_2, 0.4, 0.3, 0.6), \}\big)$,
$H_2(e_3) = (\{(v_1, 0.6, 0.5, 0.7), (v_2, 0.5, 0.6, 0.4), (v_3, 0.5, 0.7, 0.5), \{(v_1v_2, 0.5, 0.4, 0.2), (v_1v_3, 0.3, 0.2, 0.6), (v_2v_3, 0.4, 0.3, 0.2)\}\}$
as shown in Figure.3.5.

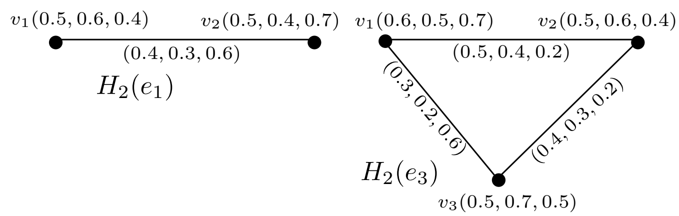


Figure 3.7: Neutrosophic soft graph $G_2 = \{H_2(e_1), H_2(e_3)\}$

Join of G_1 and G_2 is a neutrosophic soft graph $G_1 + G_2 = (H, A \cup B)$, where
$A \cup B = \{e_1, e_2, e_3\}$, $H(e_1) = H_1(e_1) + H_2(e_1)$, $H(e_2)$ and $H(e_3)$ are neutrosophic graphs corresponding to the parameters e_1, e_1 and e_3, respectively are shown in Figure.3.8.

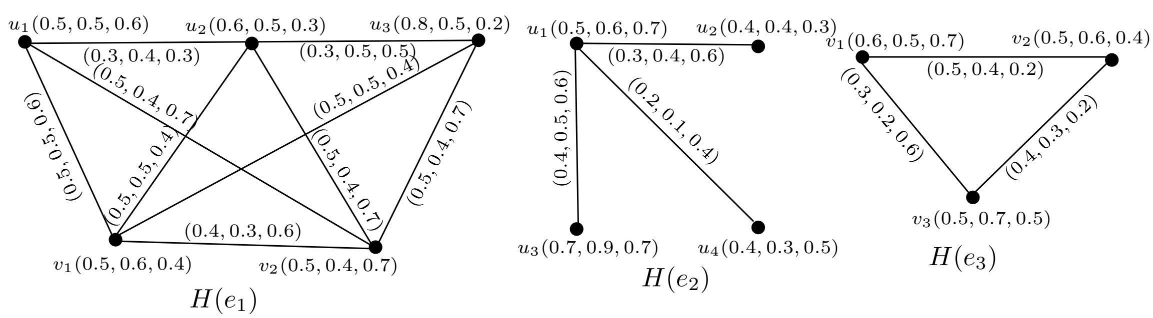


Figure 3.8: Join: $G_1 + G_2 = \{H(e_1), H(e_2), H(e_3)\}$.

Proposition 3.1. *If G_1 and G_2 are two neutrosophic soft graphs then their join $G_1 + G_2$ is also a neutrosophic soft graph.*

Definition 3.12. The *complement* of a neutrosophic soft graph $G = (F, K, A)$ denoted by $G^c = (F^c, K^c, A^c)$ is defined as follows:

(i) $A^c = A$,

(ii) $F^c(e) = F(e)$,

(iii) $T_{K^c_\mu(e)}(u,v) = T_{F(e)}(u) \wedge T_{F(e)}(v) - T_{K(e)(u,v)}$,

iv $I_{K^c_\mu(e)}(u,v) = I_{F(e)}(u) \wedge I_{F(e)}(v) - I_{K(e)(u,v)}$, and

(v) $F_{K^c_\mu(e)}(u,v) = F_{F(e)}(u) \vee F_{F(e)}(v) - F_{K(e)(u,v)}$, for all $u, v \in V, e \in A$.

Example 3.6. Consider an undirected graph $G^* = (V, E)$, where $V = \{v_1, u_2, u_3, u_4\}$ and $E = \{u_1u_2, u_2u_4, u_3u_4\}$ Let $A = \{e_1, e_2\}$ and let (F, A) be a neutrosophic soft set over V with its approximate function $F : A \to \mathcal{P}(V)$ giuen by
$F(e_1) = \{(u_1, 0.5, 0.6, 0.7), (u_2, 0.4, 0.5, 0.3), (u_3, 0.7, 0.5, 0.8), (u_4, 0.4, 0.9, 0.5)\}$,
$F(e_2) = \{(u_1, 0.4, 0.5, 0.2), (u_2, 0.3, 0.6, 0.8), (u_3, 0.3, 0.4, 0.5), (u_4, 0.7, 0.8, 0.5)\}$.
Let (K, A) be a neutrosophic soft set over E with its approximate function $K : A \to \mathcal{P}(E)$ giuen by
$K(e_1) = \{(u_1u_2, 0.3, 0.4, 0.5), (u_2u_4, 0.3, 0.4, 0.4), (u_1u_3, 0.4, 0.3, 0.6)\}$,
$K(e_2) = \{(u_1u_2, 0.2, 0.3, 0.5), (u_2u_3, 0.1, 0.3, 0.4), (u_3u_4, 0.2, 0.2, 0.4)\}$.
By routine calculations, it is easy to see that $H(e_1)$ and $H(e_2)$ are neutrosophic graphs corresponding to the parameters e_1 and e_2, respectively as shown in Figure.3.9

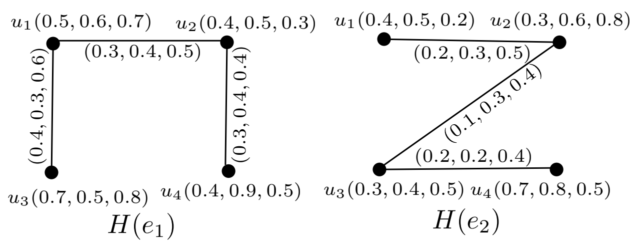


Figure 3.9: $G = \{H(e_1) = (F(e_1), K(e_1)), H(e_2) = (F(e_2), K(e_2))\}$

By the complement of neutrosophic soft graph G is the complement of neutrosophic graphs $H(e_1)$ and $H(e_2)$ which are shown in Figure. 3.10.

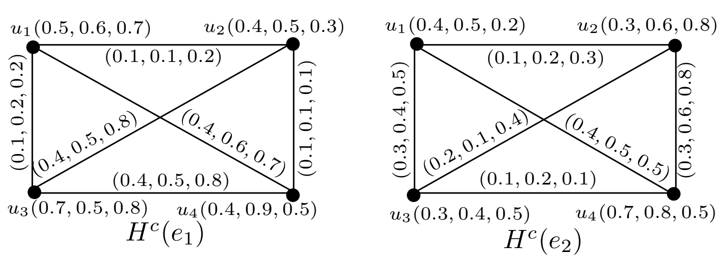


Figure 3.10: $G^c = \{H^c(e_1) = (F^c(e_1), K^c(e_1)), H^c(e_2) = (F^c(e_2), K^c(e_2))\}$

Definition 3.13. A neutrosophic soft graph G is *self complementary* if $G \approx G^c$.

Definition 3.14. A neutrosophic soft graph G is a *complete neutrosophic soft graph* if $H(e)$ is a complete neutrosophic graph of G for all $e \in A$, i.e.,

$T_{K(e)}(uv) = \min\{T_{F(e)}(u), T_{F(e)}(v)\}$

$I_{K(e)}(uv) = \min\{I_{F(e)}(u), I_{F(e)}(v)\}$ and

$F_{K(e)}(uv) = \max\{F_{F(e)}(u), F_{F(e)}(v)\} \quad \forall\, u, v \in V, e \in A.$

Example 3.7. Consider the simple graph $G^* = (V, E)$ where $V = \{u_1, u_2, u_3, u_4\}$ and $E = \{u_1u_2, u_2u_3, u_3u_4, u_1u_3, u_1u_4, u_2u_4$
Let $A = \{e_1, e_2, e_3\}$. Let (F, A) be a neutrosophic soft set over V with its approximation function $F : A \to \mathcal{P}(V)$ defined by
$F(e_1) = \{(u_1, 0.5, 0.7, 0.7), (u_2, 0.3, 0.4, 0.6), (u_3, 0.5, 0.4, 0.6)\}$,
$F(e_2) = \{(u_1, 0.8, 0.5, 0.4), (u_2, 0.4, 0.6, 0.8), (u_3, 0.4, 0.5, 0.6), (u_4, 0.7, 0.8, 0.3)\}$,
$F(e_3) = \{(u_1, 0.6, 0.7, 0.4), (u_2, 0.7, 0.4, 0.9), (u_3, 0.8, 0.5, 0.9), (u_4, 0.5, 0.7, 0.7)\}$.
Let (K, A) be a neutrosophic soft set over E with its approximation function $K : A \to \mathcal{P}(E)$ defined by
$K(e_1) = \{(u_1u_2, 0.3, 0.4, 0.7), (u_1u_3, 0.5, 0.4, 0.7), (u_2u_3, 0.3, 0.4, 0.6)\}$,
$K(e_2) = \{(u_1u_2, 0.4, 0.5, 0.8), (u_2u_3, 0.4, 0.5, 0.8), (u_3u_4, 0.4, 0.5, 0.6), (u_1u_3, 0.4, 0.5, 0.6),$
$(u_1u_4, 0.7, 0.5, 0.4), (u_2u_4, 0.4, 0.6, 0.8)\}$,
$K(e_3) = \{(u_1u_2, 0.6, 0.4, 0.9), (u_2u_3, 0.7, 0.4, 0.9), (u_3u_4, 0.5, 0.5, 0.9), (u_1u_3, 0.6, 0.5, 0.9),$
$(u_1u_4, 0.5, 0.7, 0.7), (u_2u_4, 0.5, 0.4, 0.9)\}$.
It is easy to see that $H(e_1)$, $H(e_2)$ and $H(e_3)$ are complete neutrosophic graphs of G corresponding to the parameters e_1 e_2 and e_3, respectively as shown in Figure 3.11.

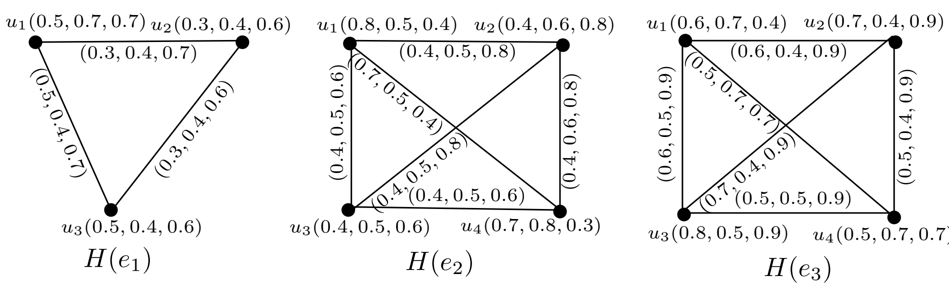


Figure 3.11: Complete neutrosophic soft graph $G = \{H(e_1), H(e_2), H(e_3)\}$.

Definition 3.15. A neutrosophic soft graph G is a *strong neutrosophic soft graph* if $H(e)$ is a strong neutrosophic graph for all $e \in A$.

Example 3.8. Consider the simple graph $G^* = (V, E)$ where $V = \{u_1, u_2, u_3, u_4\}$ and $E = \{u_1u_2, u_2u_3, u_3u_4, u_1u_3, u_1u_4, u_2u_4$
Let $A = \{e_1, e_2, e_3\}$. Let (F, A) be a neutrosophic soft set over V with its approximation function $F : A \to \mathcal{P}(V)$ defined by
$F(e_1) = \{(u_1, 0.5, 0.7, 0.7), (u_2, 0.3, 0.4, 0.6), (u_3, 0.5, 0.4, 0.6)\}$,
$F(e_2) = \{(u_1, 0.8, 0.5, 0.4), (u_2, 0.4, 0.6, 0.8), (u_3, 0.4, 0.5, 0.6), (u_4, 0.7, 0.8, 0.3)\}$,
$F(e_3) = \{(u_1, 0.6, 0.7, 0.4), (u_2, 0.7, 0.4, 0.9), (u_3, 0.8, 0.5, 0.9), (u_4, 0.5, 0.7, 0.7)\}$.
Let (K, A) be a neutrosophic soft set over E with its approximation function $K : A \to \mathcal{P}(E)$ defined by
$K(e_1) = \{(u_1u_2, 0.3, 0.4, 0.7), (u_1u_3, 0.5, 0.4, 0.7), (u_2u_3, 0.3, 0.4, 0.6)\}$,
$K(e_2) = \{(u_2u_3, 0.4, 0.5, 0.8), (u_1u_4, 0.7, 0.5, 0.4)\}$,
$K(e_3) = \{(u_1u_2, 0.6, 0.4, 0.9), (u_1u_3, 0.6, 0.5, 0.9), (u_2u_4, 0.5, 0.4, 0.9)\}$.
$H(e_1) = (F(e_1), K(e_1))$, $H(e_2) = (F(e_2), K(e_2))$ and $H(e_3) = (F(e_3), K(e_3))$ are strong neutrosophic graphs of G corresponding to the parameters e_1, e_2 and e_3, respectively as shown in Figure 3.12.

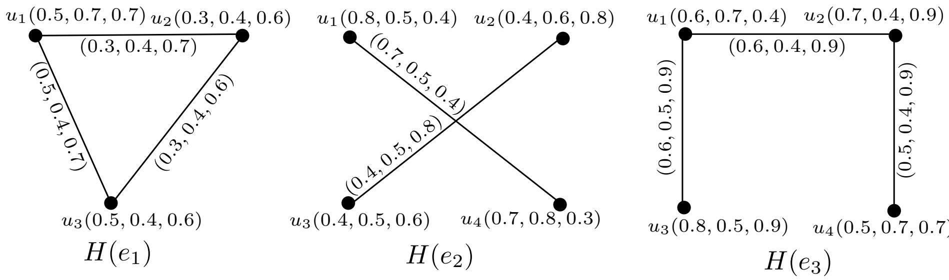


Figure 3.12: Strong neutrosophic soft graph $G = \{H(e_1), H(e_2), H(e_3)\}$.

Hence $G = \{H(e_1), H(e_2), H(e_3)\}$ is a strong neutrosophic soft graph of G^*.

Proposition 3.2. *If G_1 and G_2 are strong neutrosophic soft graphs, then $G_1 \times G_2$, $G_1[G_2]$ and $G_1 \tilde{+} G_2$ are strong neutrosophic soft graphs.*

Definition 3.16. The complement of a strong neutrosophic soft graph $G = (F, K, A)$ is a neutrosophic soft graph $G^c = (F^c, K^c, A^c)$ defined by

(i) $A^c = A$,

(ii) $F^c(e)(u) = F(e)(u)$ for all $e \in A$ and $u \in V$,

(iii) $T_{K^c(e)}(u, v) = \begin{cases} 0 & \text{if } T_{K(e)}(u, v) > 0, \\ \min\{T_{F(e)}(u), T_{F(e)}(v)\}, & \text{if } T_{K(e)}(u, v) = 0, \end{cases}$

$$I_{K^c(e)}(u, v) = \begin{cases} 0 & \text{if } I_{K(e)}(u, v) > 0, \\ \min\{I_{F(e)}(u), I_{F(e)}(v)\}, & \text{if } I_{K(e)}(u, v) = 0, \end{cases}$$

$$F_{K^c(e)}(u, v) = \begin{cases} 0 & \text{if } F_{K(e)}(u, v) > 0, \\ \max\{F_{F(e)}(u), F_{F(e)}(v)\}, & \text{if } F_{K(e)}(u, v) = 0, \end{cases}$$

We state the following propositions without their proofs.

Proposition 3.3. *If G is a strong neutrosophic soft graph over G^*, then G^c is also a strong neutrosophic soft graph.*

Proposition 3.4. *If G and G^c are strong neutrosophic soft graphs of G^*. Then $G \cup G^c$ is a complete neutrosophic soft graph.*

4 Application

Neutrosophic soft set has several applications in decision making problems and used to deal with uncertainties from our different daily life problems. In this section we apply the concept of neutrosophic soft sets in a decision making problem and then give an algorithm for the selection of optimal object based upon given set of information. Suppose that $V = \{h_1, h_2, h_3, h_4, h_5, h_6\}$ be the set of five houses under consideration which Mr. X is going to buy a house on the basis of wishing parameters or attributes set $A = \{e_1 = large, e_2 = beautiful, e_3 = greensurrounding\}$. (F, A) is the neutrosophic soft set on V which describe the value of the houses based upon the given parameters $e_1 = large, e_2 = beautiful, e_3 = greensurrounding$, respectively.

$F(e_1) = \{(h_1, 0.3, 0.5, 0.8), (h_2, 0.2, 0.8, 0.5), (h_3, 0.4, 0.5, 0.2), (h_4, 0.5, 0.2, 0.7),$
$(h_5, 0.4, 0.7, 0.6), (h_6, 0.2, 0.5, 0.8)\},$
$F(e_2) = \{(h_1, 0.6, 0.7, 0.4), (h_2, 0.6, 0.2, 0.9), (h_3, 0.2, 0.6, 0.3), (h_4, 0.7, 0.4, 0.2),$
$(h_5, 0.0, 0.0, 0.0), (h_6, 0.6, 0.2, 0.6)\},$
$F(e_3) = \{(h_1, 0.6, 0.3, 0.5), (h_2, 0.5, 0.2, 0.8), (h_3, 0.4, 0.4, 0.8), (h_4, 0.5, 0.6, 0.4),$
$(h_5, 0.6, 0.4, 0.2), (h_6, 0.4, 0.7, 0.8)\}.$

(K, A) is the neutrosophic soft set on $E = \{h_1h_2, h_1h_3, h_1h_5, h_1h_6, h_2h_4, h_2h_6, h_2h_3, h_2h_5, h_3h_4, h_3h_5, h_4h_5, h_4h_6, h_5h_6\}$ which describe the value of two houses corresponding to the given parameters $e_1 = large, e_2 = beautiful, e_3 = greensurrounding$, respectively.

$K(e_1) = \{(h_1h_2, 0.1, 0.3, 0.6), (h_1h_4, 0.2, 0.1, 0.4), (h_2h_3, 0.2, 0.4, 0.3), (h_2h_4, 0.1, 0.1, 0.6),$
$(h_2h_5, 0.2, 0.2, 0.4), (h_3h_5, 0.3, 0.4, 0.5), (h_3h_6, 0.1, 0.3, 0.6), (h_4h_5, 0.3, 0.1, 0.2),$
$(h_5h_6, 0.2, 0.4, 0.7)\},$
$K(e_2) = \{(h_1h_2, 0.5, 0.1, 0.6), (h_1h_3, 0.1, 0.5, 0.3), (h_1h_4, 0.4, 0.3, 0.3), (h_2h_4, 0.5, 0.1, 0.7),$
$(h_2h_6, 0.4, 0.1, 0.7), (h_3h_4, 0.1, 0.3, 0.3), (h_3h_6, 0.2, 0.1, 0.4)\},$
$K(e_3) = \{(h_1h_2, 0.4, 0.1, 0.7), (h_1h_5, 0.4, 0.2, 0.3), (h_2h_3, 0.3, 0.1, 0.6), (h_2h_4, 0.3, 0.1, 0.5),$
$(h_3h_5, 0.3, 0.2, 0.7), (h_3h_6, 0.3, 0.2, 0.6), (h_4h_5, 0.4, 0.3, 0.1), (h_5h_6, 0.2, 0.3, 0.5),$
$(h_4h_5, 0.3, 0.1, 0.2), (h_5h_6, 0.2, 0.4, 0.7)\}.$

The neutrosophic graphs $H(e_i)$ of neutrosophic soft graph $G = (F, K, A)$ corresponding to the parameters e_i for $i = 1, 2, 3$ are shown in Figure. 4.1.

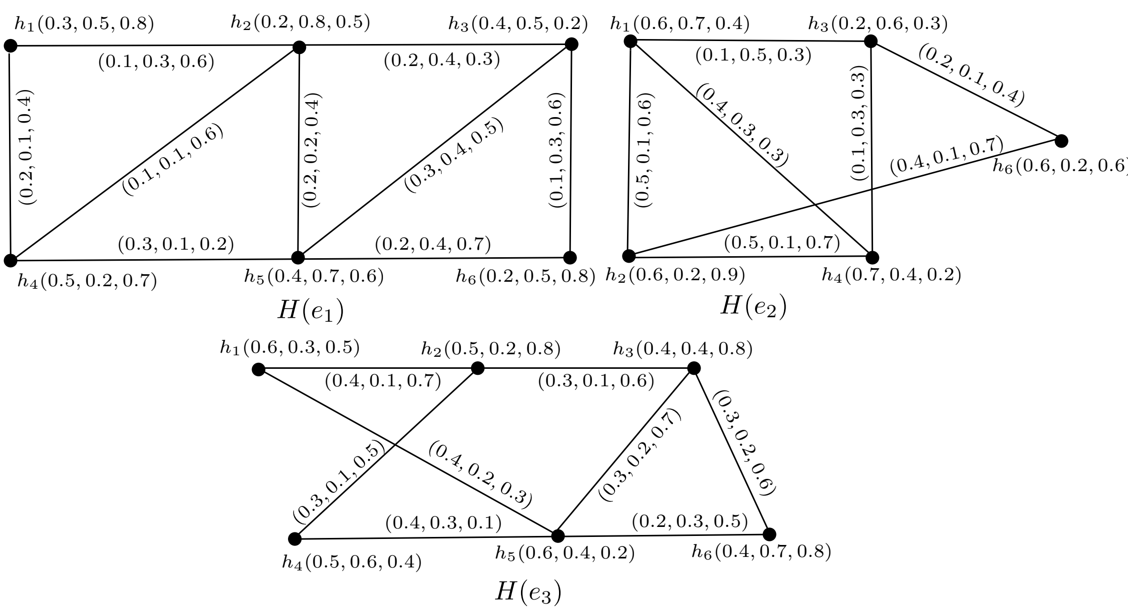


Figure 4.1: Neutrosophic soft graph $G = \{H(e_1), H(e_2), H(e_3)\}$.

The neutrosophic graphs $H(e_1)$, $H(e_2)$ and $H(e_3)$ corresponding to the parameters "large", "beautiful" and "green surrounding", respectively are represented by the following incidence matrices

$$H(e_1) = \begin{pmatrix} (0,0,0) & (0.1,0.3,0.6) & (0,0,0) & (0.2,0.1,0.4) & (0,0,0) & (0,0,0) \\ (0.1,0.3,0.6) & (0,0,0) & (0.2,0.4,0.3) & (0.1,0.1,0.6) & (0.2,0.2,0.4) & (0,0,0) \\ (0,0,0) & (0.2,0.4,0.3) & (0,0,0) & (0,0,0) & (0.3,0.4,0.5) & (0.1,0.3,0.6) \\ (0.2,0.1,0.4) & (0.1,0.1,0.6) & (0,0,0) & (0,0,0) & (0.3,0.1,0.2) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0.1,0.3,0.6) & (0,0,0) & (0.2,0.4,0.7) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0.1,0.3,0.6) & (0,0,0) & (0.2,0.4,0.7) & (0,0,0) \end{pmatrix},$$

$$H(e_2) = \begin{pmatrix} (0,0,0) & (0.5,0.1,0.6) & (0.1,0.5,0.3) & (0.4,0.3,0.3) & (0,0,0) & (0,0,0) \\ (0.5,0.1,0.6) & (0,0,0) & (0,0,0) & (0.5,0.1,0.7) & (0,0,0) & (0.4,0.1,0.7) \\ (0.1,0.5,0.3) & (0,0,0) & (0,0,0) & (0.1,0.3,0.3) & (0,0,0) & (0.2,0.1,0.4) \\ (0.4,0.3,0.3) & (0.5,0.1,0.7) & (0.1,0.3,0.3) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0.4,0.1,0.7) & (0.2,0.1,0.4) & (0,0,0) & (0,0,0) & (0,0,0) \end{pmatrix},$$

and

$$H(e_3) = \begin{pmatrix} (0,0,0) & (0.4,0.1,0.7) & (0,0,0) & (0,0,0) & (0.4,0.2,0.3) & (0,0,0) \\ (0.4,0.1,0.7) & (0,0,0) & (0.3,0.1,0.6) & (0.3,0.1,0.5) & (0,0,0) & (0,0,0) \\ (0,0,0) & (0.3,0.1,0.6) & (0,0,0) & (0,0,0) & (0.3,0.2,0.7) & (0.3,0.2,0.6) \\ (0,0,0) & (0.3,0.1,0.5) & (0,0,0) & (0,0,0) & (0.4,0.3,0.1) & (0,0,0) \\ (0.4,0.2,0.3) & (0,0,0) & (0.3,0.2,0.7) & (0.4,0.3,0.1) & (0,0,0) & (0.2,0.3,0.5) \\ (0,0,0) & (0,0,0) & (0.3,0.2,0.6) & (0,0,0) & (0.2,0.3,0.5) & (0,0,0) \end{pmatrix}.$$

After performing some operations(AND or OR); we obtain the resultant neutrosophic graph $H(e)$, where $e = e_1 \wedge e_2 \wedge e_3$. The incidence matrix of resultant neutrosophic graph is

$$H(e) = \begin{pmatrix} (0,0,0) & (0.1,0.1,0.7) & (0,0,0.3) & (0,0,0.4) & (0,0,0.3) & (0,0,0) \\ (0.1,0.1,0.7) & (0,0,0) & (0,0,0.6) & (0.1,0.1,0.7) & (0,0,0.4) & (0,0,0.7) \\ (0,0,0.3) & (0,0,0.6) & (0,0,0) & (0,0,0.3) & (0,0,0.7) & (0.1,0.1,0.6) \\ (0,0,0.4) & (0.1,0.1,0.7) & (0,0,0.3) & (0,0,0) & (0,0,0.2) & (0,0,0) \\ (0,0,0.3) & (0,0,0.4) & (0,0,0.7) & (0,0,0.2) & (0,0,0) & (0,0,0.7) \\ (0,0,0) & (0,0,0.7) & (0.1,0.2,0.6) & (0,0,0) & (0,0,0.7) & (0,0,0) \end{pmatrix}.$$

Tabular representation of score values of incidence matrix of resultant neutrosophic graph $H(e)$ with average score function $S_k = \frac{T_k+I_k+1-F_k}{3}$ and choice value for each house h_k for $k = 1, 2, 3, 4, 5$.

Table 2: Tabular representation of score values with choice values.

	h_1	h_2	h_3	h_4	h_5	h_6	$\acute{h}_k$
h_1	0.334	0.167	0.234	0.2	0.234	0.334	1.503
h_2	0.167	0.334	0.133	0.334	0.2	0.334	1.502
h_3	0.234	0.133	0.334	0.234	0.1	0.2	1.235
h_4	0.2	0.167	0.234	0.334	0.267	0.334	1.536
h_5	0.234	0.2	0.1	0.267	0.334	0.1	1.235
h_6	0.334	0.1	0.234	0.334	0.1	0.334	1.436

Clearly, the maximum score value is 1.536, scored by the h_4. Mr. X will buy the house h_4. We present our method as an algorithm that is used in our application.

Algorithm

1. Input the set P of choice parameters of Mr. X, A is a subset of P.
2. Input the neutrosophic soft sets (F, A) and (K, A).
3. Construct the neutrosophic soft graph $G = (F, K, A)$.
4. Compute the resultant neutrosophic soft graph $H(e) = \bigcap\limits_k H(e_k)$ for $e = \bigwedge\limits_k e_k \ \forall\ k$.

5. Consider the neutrosophic graph $H(e)$ and its incidence matrix form.
6. Compute the score S_k of $h_k \ \forall \ k$.
7. The decision is h_k if $\acute{h}_k = \max_i \acute{h}_i$.
8. If k has more than one value then any one of h_k may be chosen.

5 Conclusion and future work

Fuzzy graph theory is finding an increasing number of applications in modeling real time systems where the level of information inherent in the system varies with different levels of precision. Fuzzy models are becoming useful because of their aim of reducing the differences between the traditional numerical models used in engineering and sciences and the symbolic models used in expert systems. A neutrosophic set introduced by Smarandache is a powerful general formal framework, which generalizes the concept of the classic set, fuzzy set, interval valued fuzzy set and intuitionistic fuzzy set. A neutrosophic soft set is a generalization of fuzzy soft set. We have applied the concept of neutrosophic soft sets to graphs in this paper. We have discussed various methods of construction of neutrosophic soft graphs. We are extending our research of fuzzification to (1) Neutrosophic soft hypergraphs, (2) Application of neutrosophic multisets to graphs and (3) Intuitionistic neutrosophic soft graphs.

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