NEUTROSOPHIC TRANSPORT AND ASSIGNMENT ISSUES

The authors

Florentin Smarandache

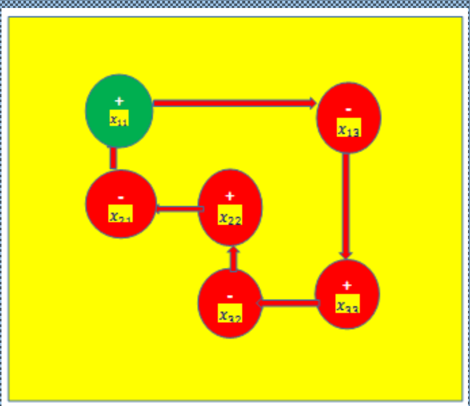
An Emeritus Professor of Mathematics

University of New Mexico, United States

Maissam Ahmad Jdid

Faculty member at the Faculty of Science

Damascus University, Syria.



$$NZ = \sum_{i=1}^{m}\sum_{j=1}^{n} Nc_{ij}x_{ij} \rightarrow Min$$

$$\sum_{j=1}^{n} Nx_{ij} = Na_i \ ; i = 1, 2, \dots, m$$

$$\sum_{i=1}^{m} Nx_{ij} = Nb_i \ ; j = 1, 2, \dots, n$$

$$Nx_{ij} \geq 0 \ ; i = 1, 2, \dots, m, j = 1, 2, \dots, n$$

2022 – 2023

NEUTROSOPHIC TRANSPORT AND ASSIGNMENT ISSUES

The authors

Florentin Smarandache
An Emeritus Professor of Mathematics
University of New Mexico, United States

Maissam Ahmad Jdid
Faculty member at the Faculty of Science
Damascus University, Syria.

ISBN 978-1-59973-769-0

2022-2023

Global Knowledge
Publishing House
Brickell Ave. Ste. 950 848
Miami, Florida 33131, United States
URL: https://egk.ccgecon.us

Florentin Smarandache

University of New Mexico, Mathematics, Physics and Natural Sciences Division,
705 Gurley Ave., Gallup, NM 87301, USA
smarand@unm.edu

Born: Balcesti, Valcea district, Romania**.**

Date of Birth: 12.10.1954

Academic degree:

- PhD, PostDocs, is an Emeritus Professor of Mathematics at the University of New Mexico, United States.
- He got his MSc in Mathematics and Computer Science from the University of Craiova, Romania, Ph.D in Mathematics from the State University of Kishinev,
- Postdoctoral in Applied Mathematics from Okayama University of Sciences, Japan,
  And The Guangdong University of Technology, Guangzhou, China.

His scientific achievements:

- He is the founder of neutrosophy (generalization of dialectics), neutrosophic set, logic, probability and statistics since 1995.
- He published hundreds of papers and books on neutrosophic physics, superluminal and instantaneous physics, unmatter, quantum paradoxes, absolute theory of relativity, redshift and blueshift due to the medium gradient and refraction index besides the Doppler effect, paradoxism, outerart, neutrosophy as a new branch of philosophy, Law of Included Multiple-Middle, Law of Included Infinitely-Many-Middles, multispace and multistructure, HyperSoft set, TreeSoft Set,

IndetermSoft Set and IndetermHyperSoft Set, SuperHyperGraph, SuperHyperTopology, SuperHyperAlgebra, SuperHyperFunction, Neutrosophic SuperHyperAlgebra, degree of dependence and independence between neutrosophic components, refined neutrosophic set, neutrosophic over-under-off-set (with degrees of membership/ indeterminacy/nonmembership less than 0 and bigger than1), plithogenic set / logic / probability / statistics, symbolic plithogenic algebraic structures, neutrosophic triplet and duplet structures, quadruple neutrosophic structures, extension of algebraic structures to NeutroAlgebra and AntiAlgebra, NeutroGeometry and AntiGeometry, NeutroTopology and AntiTopology, Refined Neutrosophic Topology, Refined Neutrosophic Crisp Topology, Dezert-Smarandache Theory.

- He reviewed many peers and many books in international journals.
- He presented papers and plenary lectures to tenths of international conferences and seminars around the world.
- He also published many books of poetry, dramas, children stories, literary translations, essays, a novel, folklore collections, traveling memories, and art albums.
- He wrote articles and books in four languages: English, Romanian, French, and Spanish.
- His books were translated to Arabic, Chinese, Russian, Spanish, Greek, Portuguese, Italian, German, Serbo-Croatian, and Turkish, see https://fs.unm.edu/LiteratureLibrary.htm and https://fs.unm.edu/ScienceLibrary.htm .
  [http://fs.unm.edu/FlorentinSmarandache.htm]

Maissam Ahmad Jdid

Faculty of Science, Damascus University, Damascus, Syria.
maissam.jdid66@damascusuniversity.edu.sy
Correspondence: jdidmaisam@gmail.com

Born: Damascus – Syria

Date of Birth: 27.04.1966

Origin: Tartous - Husayn al-Baher

Academic degree:

- **(**Ph.D) in mathematical modeling from Tver State University - Russia

Current place of work:

- Faculty member at Damascus University - Faculty of Science - Department of mathematics - since 2006

Previous place of work:

- lecturer at Al-Sham Private University - Faculty of Informatics Engineering from 2015-2023
- lecturer at Antioch Private Syrian University in 2022-2023

  College of Civil Engineering - College of Architecture

Honorary Membership

- Neutrosophic Science International Association –University of New Mexico, USA
- International Association of Paradoxism - University of New Mexico, USA

Editor-in-Chief

- Journal Prospects for Applied Mathematics and Data Analysis, (ASPG), USA

https://americaspg.com/articlesvoulume/34

https://orcid.org/my-orcid?orcid=0000-0003-4413-4783
https://www.scopus.com/authid/detail.uri?authorId=57353028500
https://www.researchgate.net/profile/Maissam-Jdid
https://scholar.google.com/citations?user=-5pTuFcAAAAJ&hl=ar
https://www.facebook.com/profile.php?id=61551427142439&mibextid=9R9pXO
https://digitalrepository.unm.edu/do/search/?q=Jdid&start=0&context=8211305&facet=

In the name of God, the most gracious, the most merciful

God Almighty says (And say, "My Lord, increase me in knowledge.")

Dear colleagues, we present to you this humble work, which is an episode of a series of works presented by a number of researchers and those interested in Neutrosophic science, entitled

(Neutrosophic transport and allocation issues)

We all know that problems of transportation and allocation appear frequently in practical life. We need to transfer materials from production centers to consumption centers to secure the areas' need for the transported material or allocate machines or people to do a specific job at the lowest cost, or in the shortest time. We know that the cost factors Time is one of the most important factors that decision-makers care about because it plays an "important" role in many of the practical and scientific issues that we face in our daily lives, and we need careful study to enable us to avoid losses. For this, the linear programming method was used, which is one of the research methods. Processes, where the problem data is converted into a linear mathematical model for which the optimal solution achieves the desired goal. Since these models are linear models, we can solve them using the direct simplex method and its modifications, but the specificity that these models enjoy has enabled scholars and researchers to find special methods that help us in obtaining the optimal solution. Whatever the method used, the goal is to determine the number of units transferred from any material from production centers to consumption centers, or to allocate a machine or person to do a job so that the cost or time is as short as possible. These issues were addressed according to classical logic, but the ideal solution was a specific value appropriate to the conditions in which the data was collected and does not take into account the changes that may occur in the work environment. In order to obtain results that are more accurate and enjoy a margin of freedom, we present in this book a study of transport issues and neutrosophic allocation issues and some methods for solving them. By neutrosophic issues we mean These are the problems in which the data are neutrosophic values, i.e. the required quantities and the available quantities. Neutrosophic values are of the form $a_i \in a_i + \varepsilon_i$, where ε_i is the indeterminacy of the produced quantities. It can be taken in the form $\varepsilon_i \in [\lambda_{i1}, \lambda_{i2}]$and the quantities required are also neutrosophic values of the form $Nb_j \in b_j + \delta_j$, where δ_j is the indeterminacy of the required quantities. One can take the form $\delta_j \in [\mu_{i1}, \mu_{i2}]$, as well as the costs or profit, that is, the cost (or the return profit) of transporting or uniting the material from the production center i to the consumption center j or allocating a machine Or a factor to accomplish some work, it is

$Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$, where $\varepsilon_{ij}$is indeterminacy and takes one of the forms $Nc_{ij} = \left[c_{ij} \pm \varepsilon_{ij}\right]$ or $\varepsilon_{ij} \in \left\{\lambda_{\mathrm{ij1}}, \lambda_{\mathrm{ij2}}\right\}$or otherwise, which is any neighborhood of the value c_{ij} that we get While collecting data on the issue, the cost (or profit) matrix becomes $Nc_{ij} = \left[c_{ij} \pm \varepsilon_{ij}\right]$. The need for a study that provides us with more results is what prompted us to prepare this book, which includes five chapters:

Chapter I: Neutrosophic transport models at the lowest cost.

Chapter II: Methods for finding the initial solution to neutrosophic transport Problems.

Chapter III: The optimal solution for neutrosophic transport models based on an initial solution.

Chapter IV: Neutrosophic transport models with the shortest time.

Chapter V: Optimal neutrosophic Allocation and the Hungarian Method.

We hope to God Almighty that I have succeeded in doing so.

The authors

Florentin Smarandache and Maissam Ahmad Jdid

Subject	Page

Subject	Page
	110

Chapter I
Neutrosophic transport models at the lowest cost

1.1. Introduction

1.2. Formulating neutrosophic transport issues at the lowest cost

1.3. Types of neutrosophic transport models at the lowest cost:

- 1.3.1. Equilibrated transportation models
- 1.3.2. Unbalanced transport models
  - 1.3.2.1. A state of surplus production.
  - 1.3.2.2. Production shortage.

1.4. Building neutrosophic transport models at the lowest cost

- 1.4.1. the cost of transportation is neutrosophic values
  - Formulating the issue
  - Building a mathematical model if the model is balanced
  - Building a mathematical model in case the model is unbalanced

    A state of surplus production

    A production deficit
- 1.4.2. Available quantities and required quantities are neutrosophic values
  - Formulating the issue
  - Building a mathematical model if the model is balanced
  - Building a mathematical model in case the model is unbalanced

A state of surplus production

A production deficit

1.4.3. The cost of transportation, the quantities supplied, and the quantities required are neutrosophic values

- Formulating the issue
- Building a mathematical model if the model is balanced
- Building a mathematical model if the model is unbalanced

A state of surplus production

A production deficit

1.1. Introduction:

In any production or service institution, the cost of transportation is considered one of the costs that affect the budget of this institution, so decision makers seek to make this cost as low as possible,. To obtain the lowest cost of transportation, scientific methods that help in making the optimal decision for the institution's workflow must be used. This is what prompted those interested in the science of operations research to present a study dealing with transportation issues using the linear programming method, and this was helped by the special nature of transportation issues. In this chapter, we present a study of transportation models at the lowest possible cost using the concepts of neutrosophy, the modern science that has brought about a major revolution in all fields. Science through the thought put forward by the founder of this science, the American scientist Florentin Smarandache, in 1995, through his belief that there is no absolute truth, which is in line with modern living conditions, which is the instability of the surrounding conditions of the work environment, and therefore the data collected about any system carries something. It is not clear which neutrosophic values should be taken to guarantee the institution a safe working environment, and therefore the transportation models with the lowest cost take the following formulation.

1.2. Formulating neutrosophic transport problems at the lowest cost:

These issues were found to help companies and institutions obtain the lowest cost for transporting materials from production centers or storage centers A_i, where $i = 1,2, \dots, m$, knowing that the quantity available in production center i is a_i, to consumption centers B_j, where $j = 1,2, \dots, n$ and the need of the j consumer center is b_j. These issues were addressed according to classical logic, and, below, we reformulate them using the concepts of neurosophic science.

The transport issue is neutrosophic if the quantities required, the quantities available, and the cost of transportation are neutrosophic values, one or all of them, i.e.:

The available quantities are $Na_i = a_i + \varepsilon_i$where the indeterminacy of the quantities is produced; it may be any neighbor of the true value a_i i.e. $\varepsilon_i \in [\lambda_{i1}, \lambda_{i2}]$ or $\varepsilon_i \in \{\lambda_{i1}, \lambda_{i2}\}$or otherwise.

The quantities required are $Nb_j = b_j + \delta_j$, where δ_j is the indeterminacy of the quantities produced, and it may be any neighborhood of the true value b_j, i.e. $\delta_j \in [\mu_{j1}, \mu_{j2}]$ or $\delta_j \in \{\mu_{j1}, \mu_{j2}\}$ or something else.

The cost of transporting one unit from production center i to consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$ where ε_{ij} is the indeterminacy of the production cost and it may be any neighborhood of the true value c_{ij}, i.e. $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$ or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$ Then the payments matrix becomes $Nc_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

1.3. Types of neutrosophic transport models at the lowest cost:

By comparing the quantities required and the quantities available, we distinguish the following types:

1.3.1. Balanced transport models:

The transportation model is balanced if the available quantities are equal to the required quantities, i.e.:

$$\sum_{i=1}^{m} Na_i = \sum_{j=1}^{n} Nb_j$$

1.3.2. Unbalanced transport models:

The transportation model is unbalanced if the quantities available do not equal the quantities demanded, i.e.:

$$\sum_{i=1}^{m} Na_i \neq \sum_{j=1}^{n} Nb_j$$

Among the unbalanced transport models, we have two cases:

1.3.2.1. A state of overproduction:

The sum of the quantities produced is greater than the sum of the required quantities, i.e.:

$$\sum_{i=1}^{m} Na_i > \sum_{j=1}^{n} Nb_j$$

We transform it into a balanced model by adding an artificial consumption center whose need is:

$$Nb_{n+1} = \sum_{i=1}^{m} Na_i - \sum_{j=1}^{n} Nb_j$$

1.3.2.2. Production shortage:

The total quantities produced are less than the total **available quantities**, i.e.:

$$\sum_{i=1}^{m} Na_i < \sum_{j=1}^{n} Nb_j$$

We transform it into a balanced model by adding an artificial production center with its production capacity:

$$Na_{m+1} = \sum_{j=1}^{n} Nb_i - \sum_{i=1}^{m} Na_i$$

In two cases (b & a), a surplus in production and a deficit in production, we obtain a balanced model.

Noting the necessity of determining the type of transportation model before starting to build the mathematical model, as we will see in the following paragraph:

1.4. Building neutrosophic transport models at the lowest cost:

When dealing with the issue of transportation according to neutrosophic logic, we encounter one of the following forms:

1.4.1. the cost of transportation, neutrosophic values:

That is, the cost of transporting one unit from production center i to consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$, where ε_{ij} is the indeterminacy of the production cost, and it may be $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$ or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$ Then the payments matrix becomes $Nc_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

Formulation of the issue:

Suppose that we want to transfer a material from production centers A_i, where $i = 1,2,\dots,m$, to consumption centers B_j, where $j = 1,2,\dots,n$, so that the transportation cost is as low as possible, bearing in mind that the quantities available in the production centers are $a_1, a_2, \dots, a_m$

The quantities required in the consumption centers are $b_1, b_2, \dots, b_n$, and the cost of transporting one unit from production center i to consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$ where ε_{ij} is indeterminacy and it may be $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$ or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$ then the payments matrix becomes $Nc_{ij} = [c_{ij} \pm \varepsilon_{ij}]$

To build the appropriate mathematical model, we symbolizethe quzntities transferred from te production center i to the consumption center j by x_{ij}. Then we can put the unknowns of the problem in the following matrix form: $X = [x_{ij}]$

We place the data in the problem in the table as follows:

Consumption centers / Production centers	B_1	B_2	B_3	...	B_n	Available quantities
A_1	Nc_{11} x_{11}	Nc_{12} x_{12}	Nc_{13} x_{13}	...	Nc_{1n} x_{1n}	a_1
A_2	Nc_{21} x_{21}	Nc_{22} x_{22}	Nc_{23} x_{23}	...	Nc_{2n} x_{2n}	a_2
A_3	Nc_{31} x_{31}	Nc_{32} x_{32}	Nc_{33} x_{33}	...	Nc_{3n} x_{3n}	a_3
...	...	...	...	...	...	...
A_m	Nc_{m1} x_{m1}	Nc_{m2} x_{m2}	Nc_{m3} x_{m3}	...	Nc_{mn} x_{mn}	a_m
Required quantities	b_1	b_2	b_3	...	b_n	

Table No. (1) Issue data cost neutrosophic values

Building the mathematical model in case the model is balanced, i.e.:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

The mathematical model is given by the following formulation:

Find

$$NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} Nc_{ij} x_{ij} \rightarrow Min$$

Conditions:

$$\sum_{j=1}^{n} x_{ij} = a_i \ ; i = 1,2, \dots, m$$

$$\sum_{i=1}^{m} x_{ij} = b_j \ ; j = 1,2, \dots, n$$

$$x_{ij} \geq 0 \ ; i = 1,2, \dots, m \, , j = 1,2, \dots, n$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / **Production centers**	B_1	B_2	B_3	B_4	**Available quantities**
A_1	$7 + \varepsilon$ x_{11}	$4 + \varepsilon$ x_{12}	$15 + \varepsilon$ x_{13}	$9 + \varepsilon$ x_{14}	120
A_2	$11 + \varepsilon$ x_{21}	$2 + \varepsilon$ x_{22}	$7 + \varepsilon$ x_{23}	$3 + \varepsilon$ x_{24}	80
A_3	$4 + \varepsilon$ x_{31}	$5 + \varepsilon$ x_{32}	$2 + \varepsilon$ x_{33}	$8 + \varepsilon$ x_{34}	100
Required quantities	85	65	90	60	

Table No. (2) Example data cost neutrosophic values

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$

From the data of the problem, we note that:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$$

That is, the model is balanced, and the mathematical model is written in the following formula:

Find

$$NZ \in \{[7,9]x_{11} + [4,6]x_{12} + [15,17]x_{13} + [9,11]x_{14} + [11,13]x_{21} + [2,4]x_{22} + [7,9]x_{23} + [3,5]x_{24} + [4,6]x_{31} + [5,7]x_{32} + [2,4]x_{33} + [8,10]x_{34}\} \to Min$$

Conditions:

$$\sum_{j=1}^{4} x_{ij} = 300 \ \ ; i = 1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = 300 \ \ ; j = 1,2,3,4$$

$$x_{ij} \geq 0 \ \ ; i = 1,2,3 \quad , j = 1,2,3,4$$

Building a mathematical model in case the model is unbalanced, i.e.:

$$\sum_{i=1}^{m} a_i \neq \sum_{j=1}^{n} b_j$$

A state of overproduction:

The total quantities produced are greater than the total available quantities, i.e.:

$$\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j$$

We transform the model into a balanced model by adding an artificial consumption center B_{n+1}whose need is:

$$b_{n+1} = \sum_{i=1}^{m} a_i - \sum_{j=1}^{n} b_j$$

The cost of transporting one unit from all production centers to this artificial consumer center is equal to zero, i.e., $c_{in+1} = 0$, where $i = 1,2, \dots, m$. To build the appropriate mathematical model, we symbolize x_{ij} to indicate the quantity transferred from the center. Production i to consumption center j Then we can put the unknowns of the problem in the following matrix form: $X = [x_{ij}]$

We place the information in the question in the table as follows:

Consumption centers / Production centers	B_1	B_2	B_3	…	B_n	B_{n+1}	Available quantities
A_1	Nc_{11} x_{11}	Nc_{12} x_{12}	Nc_{13} x_{13}	…	Nc_{1n} x_{1n}	Nc_{1n+1} x_{1n+1}	a_1
A_2	Nc_{21} x_{21}	Nc_{22} x_{22}	Nc_{23} x_{23}	…	Nc_{2n} x_{2n}	Nc_{2n+1} x_{2n+1}	a_2
A_3	Nc_{31} x_{31}	Nc_{32} x_{32}	Nc_{33} x_{33}	…	Nc_{3n} x_{3n}	Nc_{3n+1} x_{3n+1}	a_3
…	…	…	…	…	…	…	…
A_m	Nc_{m1} x_{m1}	Nc_{m2} x_{m2}	Nc_{m3} x_{m3}	…	Nc_{mn} x_{mn}	Nc_{mn+1} x_{mn+1}	a_m
Required quantities	b_1	b_2	b_3	…	b_n	b_{n+1}	

Table No. (3): Issue data cost neutrosophic values, surplus

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m} \sum_{j=1}^{n+1} Nc_{ij} x_{ij} \rightarrow Min$$

Conditions:

$$\sum_{i=1}^{m} x_{ij} = b_j \ ; j = 1,2, \dots, n+1$$

$$\sum_{j=1}^{n+1} x_{ij} = a_i \ ; i = 1,2, \dots, m$$

$$x_{ij} \geq 0 \;\; ; i = 1,2, \dots, m \, , j = 1,2, \dots, n$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / **Production centers**	B_1	B_2	B_3	B_4	**Available quantities**
A_1	$7 + \varepsilon$ x_{11}	$4 + \varepsilon$ x_{12}	$15 + \varepsilon$ x_{13}	$9 + \varepsilon$ x_{14}	120
A_2	$11 + \varepsilon$ x_{21}	$2 + \varepsilon$ x_{22}	$7 + \varepsilon$ x_{23}	$3 + \varepsilon$ x_{24}	95
A_3	$4 + \varepsilon$ x_{31}	$5 + \varepsilon$ x_{32}	$2 + \varepsilon$ x_{33}	$8 + \varepsilon$ x_{34}	100
Required quantities	85	65	90	60	

Table No. (4) Example data cost neutrosophic values, surplus

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$

From the data of the issue, we note that:

$$\sum_{i=1}^{3} a_i \neq \sum_{j=1}^{4} b_j$$

Because$\sum_{i=1}^{3} a_i = \; 315$ and $\sum_{j=1}^{4} b_j \;\; = 300$, that is,

$$\sum_{i=1}^{3} a_i > \sum_{j=1}^{4} b_j$$

The model is an unbalanced state of surplus production, so we add an artificial consumer center B_5, its need**:**

$$b_5 = \sum_{i=1}^{3} a_i - \sum_{j=1}^{4} b_j \;\; = 315 - 300 = 15$$

We get the following table:

Production centers \ Consumption centers	B_1	B_2	B_3	B_4	B_5	**Available quantities**
A_1	$7+\varepsilon$ x_{11}	$4+\varepsilon$ x_{12}	$15+\varepsilon$ x_{13}	$9+\varepsilon$ x_{14}	0 x_{15}	120
A_2	$11+\varepsilon$ x_{21}	$2+\varepsilon$ x_{22}	$7+\varepsilon$ x_{23}	$3+\varepsilon$ x_{24}	0 x_{25}	95
A_3	$4+\varepsilon$ x_{31}	$5+\varepsilon$ x_{32}	$2+\varepsilon$ x_{33}	$8+\varepsilon$ x_{34}	0 x_{35}	100
Required quantities	85	65	90	60	15	

Table No. (5) Example data cost neutrosophic values with imaginary center

Mathematical model:

Find

$$\begin{aligned} NZ \in \{&[7,9]x_{11} + [4,6]x_{12} + [15,17]x_{13} + [9,11]x_{14} + 0.x_{15} \\ &+ [11,13]x_{21} + [2,4]x_{22} + [7,9]x_{23} + [3,5]x_{24} + 0.x_{25} \\ &+ [4,6]x_{31} + [5,7]x_{32} + [2,4]x_{33} + [8,10]x_{34} + 0.x_{35}\} \\ &\rightarrow Min \end{aligned}$$

Conditions:

$$\sum_{j=1}^{5} x_{ij} = 315 \ ; i = 1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = 315 \ ; j = 1,2,3,4,5$$

$$x_{ij} \geq 0 \ ; i = 1,2,3 \ , j = 1,2,3,4,5$$

Production shortfall:

If the total quantities produced, are less than the total quantities required, that is:

$$\sum_{i=1}^{m} a_i < \sum_{j=1}^{n} b_j$$

We transform the model into a balanced model by adding an artificial production center A_{m+1}whose needs are:

$$a_{m+1} = \sum_{j=1}^{n} b_j - \sum_{i=1}^{m} a_i$$

We get the following table and the cost of transporting one unit of it to all consumption centers is equal to zero, i.e., $c_{m+1j} = 0$ where $j = 1,2, \dots, n$. To build the appropriate mathematical model, we denote x_{ij} to denote the quantity transferred from the Production center i to the consumption center. Then we can put the unknowns of the problem in the following matrix form: $X = [x_{ij}]$

We place the information in question in a table as follows:

Consumption centers / Production centers	B_1	B_2	B_3	…	B_n	Available quantities
A_1	Nc_{11} x_{11}	Nc_{12} x_{12}	Nc_{13} x_{13}	…	Nc_{1n} x_{1n}	a_1
A_2	Nc_{21} x_{21}	Nc_{22} x_{22}	Nc_{23} x_{23}	…	Nc_{2n} x_{2n}	a_2
A_3	Nc_{31} x_{31}	Nc_{32} x_{32}	Nc_{33} x_{33}	…	Nc_{3n} x_{3n}	a_3
…	…	…	…	…	…	…
A_m	Nc_{m1} x_{m1}	Nc_{m2} x_{m2}	Nc_{m3} x_{m3}	…	Nc_{mn} x_{mn}	a_m
A_{m+1}	Nc_{m1} x_{m+11}	Nc_{m1} x_{m+12}	Nc_{m1} x_{m+13}	…	Nc_{m1} x_{m+1n}	a_{m+1}
Required quantities	b_1	b_2	b_3	…	b_n	

Table No. (6): Issue data, cost neutrosophic values, deficit

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m+1} \sum_{j=1}^{n} Nc_{ij} x_{ij} \rightarrow Min$$

Conditions:

$$\sum_{j=1}^{n} x_{ij} = a_i \ ; i = 1,2, \dots, m+1$$

$$\sum_{i=1}^{m+1} x_{ij} = b_j \ ; j = 1,2, \dots, n$$

$$x_{ij} \geq 0 \ ; i = 1,2, \dots, m+1 \, , j = 1,2, \dots, n$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7+\varepsilon$ x_{11}	$4+\varepsilon$ x_{12}	$15+\varepsilon$ x_{13}	$9+\varepsilon$ x_{14}	120
A_2	$11+\varepsilon$ x_{21}	$2+\varepsilon$ x_{22}	$7+\varepsilon$ x_{23}	$3+\varepsilon$ x_{24}	80
A_3	$4+\varepsilon$ x_{31}	$5+\varepsilon$ x_{32}	$2+\varepsilon$ x_{33}	$8+\varepsilon$ x_{34}	100
Required quantities	85	100	90	60	

Table No. (7) Example data cost neutrosophic values, deficit

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$

From the data of the issue, we note that:

$$\sum_{i=1}^{3} a_i \neq \sum_{j=1}^{4} b_j$$

Because $\sum_{i=1}^{3} a_i = 300$ and $\sum_{j=1}^{4} b_j = 335$, that is, the model is unbalanced because $\sum_{j=1}^{4} b_j > \sum_{i=1}^{3} a_i$

A production deficit, so we add a fictitious production center A_4 to its production capacity:

$$a_4 = \sum_{j=1}^{4} b_j - \sum_{i=1}^{3} a_i = 335 - 300 = 35$$

The cost of transporting one unit of it to any consumer center is equal to zero, i.e.: $c_{4j} = 0 \,; j = 1,2,3,4$, we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7+\varepsilon$ x_{11}	$4+\varepsilon$ x_{12}	$15+\varepsilon$ x_{13}	$9+\varepsilon$ x_{14}	120
A_2	$11+\varepsilon$ x_{21}	$2+\varepsilon$ x_{22}	$7+\varepsilon$ x_{23}	$3+\varepsilon$ x_{24}	80
A_3	$4+\varepsilon$ x_{31}	$5+\varepsilon$ x_{32}	$2+\varepsilon$ x_{33}	$8+\varepsilon$ x_{34}	100
A_4	0 x_{41}	0 x_{42}	0 x_{43}	0 x_{44}	35
Required quantities	85	100	90	60	

Table No. (8) Example data cost neutrosophic values, artificial center

Mathematical model:

Find

$$NZ \in \{[7,9]x_{11} + [4,6]x_{12} + [15,17]x_{13} + [9,11]x_{14} + 0x_{15} + [11,13]x_{21} + [2,4]x_{22} + [7,9]x_{23} + [3,5]x_{24} + 0x_{25} + [4,6]x_{31} + [5,7]x_{32} + [2,4]x_{33} + [8,10]x_{34} + 0x_{35}\} \rightarrow Min$$

Conditions:

$$\sum_{j=1}^{5} x_{ij} = 315 \ ; i = 1,2,3$$

$$\sum_{i=1}^{3} x_{ij} = 315 \ ; j = 1,2,3,4,5$$

$$x_{ij} \geq 0 \ ; i = 1,2,3 \ , j = 1,2,3,4,5$$

1.4.2. Available quantities and required quantities. neutrosophic values:

The available quantities are neutrosophic values, i.e. $Na_i = a_i \pm \varepsilon_i$, where $\varepsilon_i$is the indeterminacy related to the quantity available in production center i, and it may be $\varepsilon_i \in [\lambda_{i1}, \lambda_{i2}]$ or $\varepsilon_i \in \{\lambda_{i1}, \lambda_{i2}\}$ or otherwise. And the quantities required are neutrosophic values, i.e. $Nb_j = b_j \pm \delta_j$, where $\delta_j$is the indeterminacy related to the quantity required in the j the consumption center, and it may be $\delta_j \in \left[\mu_{j1}, \mu_{j2}\right]$or $\delta_j \in \left\{\mu_{j1}, \mu_{j2}\right\}$or something else.

Formulation of the issue:

Suppose that we want to transfer a material from production centers A_i, where $i = 1,2, \dots, m$, to consumption centers B_j, where $j = 1,2, \dots, n$, so that the transportation cost is as low as possible, bearing in mind that the quantities available in these centers are $a_1 \pm \varepsilon_1, a_2 \pm \varepsilon_2, \dots, a_m \pm \varepsilon_m$. And the quantities required in consumption centers $b_1 \pm \delta_1, b_2 \pm \delta_2, \dots, b_n \pm \delta_n$

The cost of moving one unit from the production center i to the consumption center j is c_{ij}, where ε_{ij} then the payments matrix becomes $c_{ij} = \left[c_{ij}\right]$

To build the appropriate mathematical model, we symbolize x_{ij} to indicate the quantity transferred from the production center i to the consumption center j. Then we can put the unknowns of the problem in the following matrix form: $X = [x_{ij}]$

We place the data in the problem in the table as follows:

Consumption centers / Production centers	B_1	B_2	B_3	…	B_n	**Available quantities**
A_1	Nc_{11} x_{11}	Nc_{12} x_{12}	Nc_{13} x_{13}	…	Nc_{1n} x_{1n}	$a_1 \pm \varepsilon_1$
A_2	Nc_{21} x_{21}	Nc_{22} x_{22}	Nc_{23} x_{23}	…	Nc_{2n} x_{2n}	$a_2 \pm \varepsilon_2$
A_3	Nc_{31} x_{31}	Nc_{32} x_{32}	Nc_{33} x_{33}	…	Nc_{3n} x_{3n}	$a_3 \pm \varepsilon_3$
…	…	…	…	…	…	…
A_m	Nc_{m1} x_{m1}	Nc_{m2} x_{m2}	Nc_{m3} x_{m3}	…	Nc_{mn} x_{mn}	$a_m \pm \varepsilon_m$
Required quantities	$b_1 \pm \delta_1$	$b_2 \pm \delta_2$	$b_3 \pm \delta_3$	…	$b_n \pm \delta_n$	

Table No. (9) Issue data: Available quantities and required quantities. eutrosophic values

Building the mathematical model in case the model is balanced, i.e.:

$$\sum_{i=1}^{m} Na_i = \sum_{j=1}^{n} Nb_j$$

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m}\sum_{j=1}^{n} Nc_{ij}x_{ij} \to Min$$

Conditions:

$$\sum_{j=1}^{n} Nx_{ij} = Na_i \ ; i = 1,2,\dots,m$$

$$\sum_{i=1}^{m} Nx_{ij} = Nb_i \ ; j = 1,2,\dots,n$$

$$Nx_{ij} \geq 0 \ ; i = 1,2,\dots,m \,, j = 1,2,\dots,n$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	$120 + \varepsilon_1$
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	$80 + \varepsilon_2$
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	$100 + \varepsilon_3$
Required quantities	$85 + \delta_1$	$65 + \delta_2$	$90 + \delta_3$	$60 + \delta_4$	

Table No. (10): Example data Available quantities and required quantities Neutrosophic values

Where ε_i is the indeterminacy of the available quantities and we take it as follows:

$$\varepsilon_1 \in [0,35]\,, \varepsilon_2 \in [0,10], \varepsilon_3 \in [0,15]$$

Where δ_j is the indeterminacy of the required quantities, we take it as follows:

$$\delta_1 \in [0,7]\,, \delta_2 \in [0,18], \delta_3 \in [0,25]\,, \delta_4 \in [0,10]$$

From the above we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	[120,155]
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	[80,90]
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	[100,115]
Required quantities	[85,92]	[65,83]	[90,115]	[60,70]	

Table No. (11): Example data: Available and required quantities neutrosophic values, Overproduction

From the data on the issue, we note:

$$\sum_{i=1}^{3} Na_i = [120,155] + [80,90] + [100,115] = [300,360]$$

$$\sum_{j=1}^{4} Nb_j = [85,92] + [65,83] + [90,115] + [60,70] = [300,360]$$

We note that $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,360]$, meaning that the model is balanced.

Mathematical model:

Find

$$NZ \in (7Nx_{11} + 4Nx_{12} + 15Nx_{13} + 9Nx_{14} + 11Nx_{21} + 2Nx_{22} + 7Nx_{23} + 3Nx_{24} + 4Nx_{31} + 5Nx_{32} + 2Nx_{33} + 8Nx_{34}) \to Min$$

Conditions:

$$\sum_{i=1}^{3} Nx_{ij} = Na_i \;\; ; j = 1,2,3,4$$

$$\sum_{j=1}^{4} Nx_{ij} = Nb_j \;\; ; i = 1,2,3$$

$$Nx_{ij} \geq 0 \;\; ; i = 1,2,3 \;\; , j = 1,2,3,4$$

Building a mathematical model in case the model is unbalanced, i.e.:

$$\sum_{i=1}^{m} Na_i \neq \sum_{j=1}^{n} Nb_j$$

A state of surplus production:

The total quantities produced are greater than the total quantities demanded, i.e.:

$$\sum_{i=1}^{m} Na_i > \sum_{j=1}^{n} Nb_j$$

We transform the model into a balanced model by adding an imaginary consumer center B_{n+1}whose need is:

$$Nb_{n+1} = \sum_{i=1}^{m} Na_i - \sum_{j=1}^{n} Nb_j$$

The cost of transporting one unit from all production centers to this artificial consumer center is equal to zero, i.e. $c_{in+1} = 0$, where $i = 1,2, \dots, m$. To build the appropriate mathematical model, we symbolize x_{ij} to indicate the quantity transferred from the Production center i to the consumption center j Then we can put the unknowns of the problem in the following matrix form: $X = [x_{ij}]$

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m} \sum_{j=1}^{n+1} Nc_{ij} x_{ij} \to Min$$

Conditions:

$$\sum_{j=1}^{n+1} Nx_{ij} = Na_i \ ; i = 1,2, \dots, m$$

$$\sum_{i=1}^{m} Nx_{ij} = Nb_i \ ; j = 1,2, \dots, n+1$$

$$Nx_{ij} \geq 0 \ ; i = 1,2, \dots, m \ , j = 1,2, \dots, n+1$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	$120 + \varepsilon_1$
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	$95 + \varepsilon_2$
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	$100 + \varepsilon_3$
Required quantities	$85 + \delta_1$	$65 + \delta_1$	$90 + \delta_1$	$60 + \delta_1$	

Table No. (12): Example data: Available and required quantities Neutrosophic values, Overproduction

Where ε_i is the indeterminacy of the available quantities and we take it as follows:

$$\varepsilon_1 \in [0,35] \ , \varepsilon_2 \in [0,10], \varepsilon_3 \in [0,15]$$

Where δ_j is the indeterminacy of the required quantities, we take it as follows:

$$\delta_1 \in [0,7]\,,\delta_2 \in [0,18], \delta_3 \in [0,25]\,, \delta_4 \in [0,10]$$

From the above we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	**Available quantities**
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	$[120,155]$
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	$[95,105]$
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	$[100,115]$
Required quantities	$[85,92]$	$[65,83]$	$[90,115]$	$[60,70]$	

Table No. (13): Example data: Available and required quantities Neutrosophic values, Overproduction

From the data on the issue, we note:

$$\sum_{i=1}^{3} Na_i = [120,155] + [95,105] + [100,115] = [315,375]$$

$$\sum_{j=1}^{4} Nb_j = [85,92] + [65,83] + [90,115] + [60,70] = [300,360]$$

We note that $\sum_{i=1}^{3} Na_i > \sum_{j=1}^{4} Nb_j$, state of surplus production.

We transform the model into a balanced model by adding an artificial consumer center B_{n+1}whose need is:

$$Nb_5 = \sum_{i=1}^{3} Na_i - \sum_{j=1}^{4} Nb_j = [315,375] - [300,360] = [15,15]$$

The transportation cost from all production centers to this imaginary consumer center is zero.

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	B_5	Available quantities
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	0 Nx_{15}	[120,155]
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	0 Nx_{25}	[95,105]
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	0 Nx_{35}	[100,115]
Required quantities	[85,92]	[65,83]	[90,115]	[60,70]	[15,15]	

Table No. (14): Example data: Available and required quantities. Neutrosophic values with artificial consumption center

Mathematical model:

Find

$$NZ \in (7Nx_{11} + 4Nx_{12} + 15Nx_{13} + 9Nx_{14} + 0.Nx_{15} + 11Nx_{21} + 2Nx_{22} + 7Nx_{23} + 3Nx_{24} + 0.Nx_{25} + 4Nx_{31} + 5Nx_{32} + 2Nx_{33} + 8Nx_{34} + 0.Nx_{35}) \to Min$$

Conditions:

$$\sum_{i=1}^{3} Nx_{ij} = Na_i \;\; ; j = 1,2,3,4,5$$

$$\sum_{j=1}^{5} Nx_{ij} = Nb_j \;\; ; i = 1,2,3$$

$$Nx_{ij} \geq 0 \;\; ; i = 1,2,3 \;\; , j = 1,2,3,4,5$$

Production shortage:

If the total quantities produced are less than the total quantities required, that is:

$$\sum_{i=1}^{m} Na_i < \sum_{j=1}^{n} Nb_j$$

We transform the model into a balanced model by adding an artificial production center A_{m+1}whose needs are:

$$Na_{m+1} = \sum_{j=1}^{n} Nb_j - \sum_{i=1}^{m} Na_i$$

The cost of transporting one unit to all consumption centers equals to zero, that is, $c_{m+1j} = 0$, where $j = 1,2, \dots, n$. For the table, we add a row A_{m+1} .

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m+1} \sum_{j=1}^{n} Nc_{ij} x_{ij} \rightarrow Min$$

Conditions:

$$\sum_{j=1}^{n} Nx_{ij} = Na_i \ ; i = 1,2, \dots, m+1$$

$$\sum_{i=1}^{m+1} Nx_{ij} = Nb_i \ ; j = 1,2, \dots, n$$

$$Nx_{ij} \geq 0 \ ; i = 1,2, \dots, m+1 \, , j = 1,2, \dots, n$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	$120 + \varepsilon_1$
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	$80 + \varepsilon_2$
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	$100 + \varepsilon_3$
Required quantities	$85 + \delta_1$	$100 + \delta_1$	$90 + \delta_1$	$60 + \delta_1$	

Table No. (15) Example data Available and required quantities neutrosophic values, production shortfall

Where ε_i is the indeterminacy of the available quantities and we take it as follows:

$$\varepsilon_1 \in [0,35]\, , \varepsilon_2 \in [0,10], \varepsilon_3 \in [0,15]$$

Where δ_j is the indeterminacy of the required quantities, we take it as follows:

$$\delta_1 \in [0,7]\, , \delta_2 \in [0,18], \delta_3 \in [0,25]\, , \delta_4 \in [0,10]$$

From the above we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	$[120,155]$
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	$[80,90]$
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	$[100,115]$
Required quantities	$[85,92]$	$[100,135]$	$[90,115]$	$[60,70]$	

Table No. (16) Example data Available and required quantities neutrosophic values, production shortfall

From the data on the issue, we note:

$$\sum_{i=1}^{3} Na_i = [120,155] + [80,90] + [100,115] = [300,360]$$

$$\sum_{j=1}^{4} Nb_j = [85,92] + [100,135] + [90,115] + [60,70] = [335,395]$$

We note that:

$$\sum_{i=1}^{3} Na_i \neq \sum_{j=1}^{4} Nb_j$$

Because $\sum_{i=1}^{3} Na_i = [300,360]$ and $\sum_{j=1}^{4} Nb_j = [335,395]$, that is, the model is unbalanced because $\sum_{j=1}^{4} Nb_j > \sum_{i=1}^{3} Na_i$

A production deficit, so we add an artificial production center A_4 to its production capacity:

$$Na_4 = \sum_{j=1}^{4} Nb_j - \sum_{i=1}^{3} Na_i = [335,395] - [300,360] = [35,35]$$

The cost of transporting one unit of it to any consumer center equals zero, i.e.:

$$c_{4j} = 0\,;\, j = 1,2,3,4$$

We get the following table:

Consumption centers / **Production centers**	B_1	B_2	B_3	B_4	**Available quantities**
A_1	7 Nx_{11}	4 Nx_{12}	15 Nx_{13}	9 Nx_{14}	[120,155]
A_2	11 Nx_{21}	2 Nx_{22}	7 Nx_{23}	3 Nx_{24}	[80,90]
A_3	4 Nx_{31}	5 Nx_{32}	2 Nx_{33}	8 Nx_{34}	[100,115]
A_4	0 Nx_{41}	0 Nx_{42}	0 Nx_{43}	0 Nx_{44}	[35,35]
Required quantities	[85,92]	[100,135]	[90,115]	[60,70]	

Table No. (17) Example data Available and required quantities neutrosophic values artificial production center

Mathematical model:

Find

$$NZ \in (7Nx_{11} + 4Nx_{12} + 15Nx_{13} + 9Nx_{14} + 11Nx_{21} + 2Nx_{22} + 7Nx_{23} + 3Nx_{24} + 4Nx_{31} + 5Nx_{32} + 2Nx_{33} + 8Nx_{34} + 0.Nx_{41} + 0.Nx_{42} + 0.Nx_{43} + 0.Nx_{44}) \rightarrow Min$$

Conditions:

$$\sum_{i=1}^{4} Nx_{ij} = Na_i \ \ ; j = 1,2,3,4$$

$$\sum_{j=1}^{4} Nx_{ij} = Nb_j \ \ ; i = 1,2,3,4$$

$$Nx_{ij} \geq 0 \ \ ; i = 1,2,3,4 \ \ , j = 1,2,3,4$$

1.4.3. The cost of transportation, available quantities, and required quantities. neutrosophic values:

The transportation cost is a neutrosophic value, meaning that the cost of transporting one unit transported from production center i to consumption center j isNc_{ij}, where $Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$, and ε_{ij} is indeterminacy, and it may be $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$ or other. Then the payments matrix becomes $Nc_{ij} = [c_{ij} \pm \varepsilon_{ij}]$

Available quantities are neutrosophic values $Na_i = a_i \pm \varepsilon_i$ where ε_i is the indeterminacy related to the available quantity in production center i. It may be $\varepsilon_{ij} \in [\lambda_{1i}, \lambda_{2i}]$ or $\varepsilon_{ij} \in \{\lambda_{1i}, \lambda_{2i}\}$or otherwise, the quantities demanded are neutrosophic values $Nb_j = b_j \pm \delta_j$ where δ_j is the indeterminacy related to the quantity available in the j consumer center, it may be $\delta_j \in [\mu_{1j}, \mu_{2j}]$ or $\delta_j \in \{\mu_{1j}, \mu_{2j}\}$ or otherwise.

Formulation of the issue:

Suppose that we want to transfer a material from production centers A_i, where $i = 1,2, \dots, m$, to consumption centers B_j, where $j = 1,2, \dots, n$, so that the transportation cost is as low as possible, knowing that the quantities available in these centers are $a_1 \pm \varepsilon_1, a_2 \pm \varepsilon_2, \dots, a_m \pm \varepsilon_m$ and the quantities required in the consumption centers are:

$$b_1 \pm \delta_1, b_2 \pm \delta_2, \dots, b_n \pm \delta_n$$

To build the appropriate mathematical model, we symbolize the quantity transferred from the production center i to the consumption center j by Nx_{ij}. Then, we can put the unknowns of the problem in the following matrix form: $NX = [Nx_{ij}]$

We place the information in the question in the table as follows:

Consumption centers / **Production centers**	B_1	B_2	B_3	...	B_n	**Available quantities**
A_1	Nc_{11} Nx_{11}	Nc_{12} Nx_{12}	Nc_{13} Nx_{13}	...	Nc_{1n} Nx_{1n}	$a_1 \pm \varepsilon_1$
A_2	Nc_{21} Nx_{21}	Nc_{22} Nx_{22}	Nc_{23} Nx_{23}	...	Nc_{2n} Nx_{2n}	$a_2 \pm \varepsilon_2$
A_3	Nc_{31} Nx_{31}	Nc_{32} Nx_{32}	Nc_{33} Nx_{33}	$\dots$	Nc_{3n} Nx_{3n}	$a_3 \pm \varepsilon_3$
...	...	...	...	...	...	...
A_m	Nc_{m1} Nx_{m1}	Nc_{m2} Nx_{m2}	Nc_{m3} Nx_{m3}	$\dots$	Nc_{mn} Nx_{mn}	$a_m \pm \varepsilon_m$
Required quantities	$b_1 \pm \delta_1$	$b_2 \pm \delta_2$	$b_3 \pm \delta_3$	$\dots$	$b_n \pm \delta_n$	

Table No. (18) Issue data: transportation cost, available quantities, and required quantities. Neutrosophic values

Building the mathematical model in case the model is balanced, i.e.:

$$\sum_{i=1}^{m} Na_i = \sum_{j=1}^{n} Nb_j$$

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m} \sum_{j=1}^{n} Nc_{ij} x_{ij} \rightarrow Min$$

Conditions:

$$\sum_{j=1}^{n} Nx_{ij} = Na_i \ ; i = 1,2, \dots, m$$

$$\sum_{i=1}^{m} Nx_{ij} = Nb_j \ ; j = 1,2, \dots, n$$

$$Nx_{ij} \geq 0 \ ; i = 1,2, \dots, m \, , j = 1,2, \dots, n$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7 + \varepsilon$ Nx_{11}	$4 + \varepsilon$ Nx_{12}	$15 + \varepsilon$ Nx_{13}	$9 + \varepsilon$ Nx_{14}	$120 + \varepsilon_1$
A_2	$11 + \varepsilon$ Nx_{21}	$2 + \varepsilon$ Nx_{22}	$7 + \varepsilon$ Nx_{23}	$3 + \varepsilon$ Nx_{24}	$80 + \varepsilon_2$
A_3	$4 + \varepsilon$ Nx_{31}	$5 + \varepsilon$ Nx_{32}	$2 + \varepsilon$ Nx_{33}	$8 + \varepsilon$ Nx_{34}	$100 + \varepsilon_3$
Required quantities	$85 + \delta_1$	$65 + \delta_2$	$90 + \delta_3$	$60 + \delta_4$	

Table No. (19): Example data: transportation cost, available and required quantities. Neutrosophic values

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$

Where ε_i is the indeterminacy of the available quantities and we take it as follows:

$$\varepsilon_1 \in [0,35] \, , \varepsilon_2 \in [0,10], \varepsilon_3 \in [0,15]$$

Where δ_j is the indeterminacy of the required quantities, we take it as follows:

$$\delta_1 \in [0,7] \, , \delta_2 \in [0,18], \delta_3 \in [0,25] \, , \delta_4 \in [0,10]$$

From the above we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$[7,9]$ Nx_{11}	$[4,6]$ Nx_{12}	$[15,17]$ Nx_{13}	$[9,11]$ Nx_{14}	$[120,155]$
A_2	$[11,13]$ Nx_{21}	$[2,4]$ Nx_{22}	$[7,9]$ Nx_{23}	$[3,5]$ Nx_{24}	$[80,90]$
A_3	$[4,6]$ Nx_{31}	$[5,7]$ Nx_{32}	$[2,4]$ Nx_{33}	$[8,10]$ Nx_{34}	$[100,115]$
Required quantities	$[85,92]$	$[65,83]$	$[90,115]$	$[60,70]$	

Table No. (20): Example data: transportation cost, available quantities, and required quantities. Neutrosophic values

From the data of the issue, we note that:

$$\sum_{i=1}^{3} Na_i = [120,155] + [80,90] + [100,115] = [300,360]$$

$$\sum_{j=1}^{4} Nb_j = [85,92] + [65,83] + [90,115] + [60,70] = [300,360]$$

We note that:

$$\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,360]$$

Mathematical model:

Find

$$NZ \in \{[7,9]x_{11} + [4,6]x_{12} + [15,17]x_{13} + [9,11]x_{14} + [11,13]x_{21} + [2,4]x_{22} + [7,9]x_{23} + [3,5]x_{24} + [4,6]x_{31} + [5,7]x_{32} + [2,4]x_{33} + [8,10]x_{34}\} \rightarrow Min$$

Conditions:

$$\sum_{i=1}^{3} Nx_{ij} = Na_i \;\; ; j = 1,2,3,4$$

$$\sum_{j=1}^{4} Nx_{ij} = Nb_j \ ; i = 1,2,3$$

$$Nx_{ij} \geq 0 \ ; i = 1,2,3 \ , j = 1,2,3,4$$

Building a mathematical model in case the model is unbalanced, i.e.:

$$\sum_{i=1}^{m} Na_i \neq \sum_{j=1}^{n} Nb_j$$

A state of overproduction:

The total quantities produced are greater than the total available quantities, i.e.:

$$\sum_{i=1}^{m} Na_i > \sum_{j=1}^{n} Nb_j$$

We transform the model into a balanced model by adding an artificial consumer center B_{n+1} whose need is:

$$Nb_{n+1} = \sum_{i=1}^{m} Na_i - \sum_{j=1}^{n} Nb_j$$

The cost of transporting one unit from all production centers to this imaginary consumer center is equal to zero, i.e., $c_{in+1} = 0$, where $i = 1,2, \dots , m$. To build the appropriate mathematical model, we symbolize x_{ij} to indicate the quantity transferred from the center. Production i to consumption center j. Then, we can put the unknowns of the problem in the following matrix form: $X = [x_{ij}]$

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m} \sum_{j=1}^{n+1} Nc_{ij} x_{ij} \rightarrow Min$$

Conditions:

$$\sum_{j=1}^{n+1} Nx_{ij} = Na_i \ ; i = 1,2, \dots, m$$

$$\sum_{i=1}^{m} Nx_{ij} = Nb_i \ ; j = 1,2, \dots, n+1$$

$$Nx_{ij} \geq 0 \ ; i = 1,2, \dots, m \, , j = 1,2, \dots, n+1$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	available quantities
A_1	$7+\varepsilon$ Nx_{11}	$4+\varepsilon$ Nx_{12}	$15+\varepsilon$ Nx_{13}	$9+\varepsilon$ Nx_{14}	$120+\varepsilon_1$
A_2	$11+\varepsilon$ Nx_{21}	$2+\varepsilon$ Nx_{22}	$7+\varepsilon$ Nx_{23}	$3+\varepsilon$ Nx_{24}	$95+\varepsilon_2$
A_3	$4+\varepsilon$ Nx_{31}	$5+\varepsilon$ Nx_{32}	$2+\varepsilon$ Nx_{33}	$8+\varepsilon$ Nx_{34}	$100+\varepsilon_3$
Required quantities	$85+\delta_1$	$65+\delta_2$	$90+\delta_3$	$60+\delta_4$	

Table No. (21): Example data: transportation cost, available, and required quantities Neutrosophic values, overproduction

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$

Where ε_i is the indeterminacy of the available quantities and we take it as follows:

$$\varepsilon_1 \in [0,35]\,, \varepsilon_2 \in [0,10], \varepsilon_3 \in [0,15]$$

Where δ_j is the indeterminacy of the required quantities, we take it as follows:

$$\delta_1 \in [0,7]\,, \delta_2 \in [0,18], \delta_3 \in [0,25]\,, \delta_4 \in [0,10]$$

From the above we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	available quantities
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[95,105]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
Required quantities	[85,92]	[65,83]	[90,115]	[60,70]	

Table No. (22): Example data: transportation cost, available, and required quantities Neutrosophic values, overproduction

From the data on the issue, we note:

$$\sum_{i=1}^{3} Na_i = [120,155] + [95,105] + [100,115] = [315,375]$$

$$\sum_{j=1}^{4} Nb_j = [85,92] + [65,83] + [90,115] + [60,70] = [300,360]$$

We note that $\sum_{i=1}^{3} Na_i > \sum_{j=1}^{4} Nb_j$, state of surplus production.

We transform the model into a balanced model by adding an artificial consumer center B_5 whose need is:

$$Nb_5 = \sum_{i=1}^{3} Na_i - \sum_{j=1}^{4} Nb_j = [315,375] - [300,360] = [15,15]$$

we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	B_5	Available quantities
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	0 x_{15}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	0 x_{25}	[95,105]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	0 x_{35}	[100,115]
Required quantities	[85,92]	[65,83]	[90,115]	[60,70]	15	

Table No. (23) Example data Transportation cost, available and required quantities, neutrosophic values, artificial consumption center

Mathematical model:

Find

$$\begin{aligned} NZ \in \{&[7,9]x_{11} + [4,6]x_{12} + [15,17]x_{13} + [9,11]x_{14} + 0.x_{15} \\ &+ [11,13]x_{21} + [2,4]x_{22} + [7,9]x_{23} + [3,5]x_{24} + 0.x_{25} \\ &+ [4,6]x_{31} + [5,7]x_{32} + [2,4]x_{33} + [8,10]x_{34} + 0.x_{35}\} \\ &\rightarrow Min \end{aligned}$$

Conditions:

$$\sum_{i=1}^{3} Nx_{ij} = Na_i \ ; j = 1,2,3,4,5$$

$$\sum_{j=1}^{5} Nx_{ij} = Nb_j \ ; i = 1,2,3$$

$$Nx_{ij} \geq 0 \ ; i = 1,2,3 \ , j = 1,2,3,4,5$$

Production shortfall:

If the total quantities produced are less than the total quantities required, that is:

$$\sum_{i=1}^{m} Na_i < \sum_{j=1}^{n} Nb_j$$

We transform the model into a balanced model by adding an artificial production center A_{m+1} whose needs are:

$$Na_{m+1} = \sum_{j=1}^{n} Nb_j - \sum_{i=1}^{m} Na_i$$

The cost of transporting one unit of it to all consumption centers is equal to zero, that is, $c_{m+1j} = 0$, where $j = 1,2, \dots, n$. For the table, we add a row A_{m+1}

Mathematical model:

Find

$$NZ = \sum_{i=1}^{m+1} \sum_{j=1}^{n} Nc_{ij} x_{ij} \to Min$$

Conditions:

$$\sum_{j=1}^{n} Nx_{ij} = Na_i \ ; i = 1,2, \dots, m+1$$

$$\sum_{i=1}^{m+1} Nx_{ij} = Nb_i \ ; j = 1,2, \dots, n$$

$$Nx_{ij} \geq 0 \ ; i = 1,2, \dots, m+1 \,, j = 1,2, \dots, n$$

Example:

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the transportation cost are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7+\varepsilon$ Nx_{11}	$4+\varepsilon$ Nx_{12}	$15+\varepsilon$ Nx_{13}	$9+\varepsilon$ Nx_{14}	$120+\varepsilon_1$
A_2	$11+\varepsilon$ Nx_{21}	$2+\varepsilon$ Nx_{22}	$7+\varepsilon$ Nx_{23}	$3+\varepsilon$ Nx_{24}	$80+\varepsilon_2$
A_3	$4+\varepsilon$ Nx_{31}	$5+\varepsilon$ Nx_{32}	$2+\varepsilon$ Nx_{33}	$8+\varepsilon$ Nx_{34}	$100+\varepsilon_3$
Required quantities	$85+\delta_1$	$100+\delta_2$	$90+\delta_3$	$60+\delta_4$	

Table No. (24) Example data Transportation cost, available and required quantities Neutrosophic values, production shortfall

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$

Where ε_i is the indeterminacy of the available quantities and we take it as follows:

$$\varepsilon_1 \in [0,35]\,, \varepsilon_2 \in [0,10], \varepsilon_3 \in [0,15]$$

Where δ_j is the indeterminacy of the required quantities, we take it as follows:

$$\delta_1 \in [0,7]\,, \delta_2 \in [0,18], \delta_3 \in [0,25]\,, \delta_4 \in [0,10]$$

From the above we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[80,90]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
Required quantities	[85,92]	[100,118]	[90,115]	[60,70]	

Table No. (25) Example data Transportation cost, available, and required quantities Neutrosophic values, production shortfall

From the data on the issue, we note:

$$\sum_{i=1}^{3} Na_i = [120,155] + [80,90] + [100,115] = [300,360]$$

$$\sum_{j=1}^{4} Nb_j = [85,92] + [100,118] + [90,115] + [60,70] = [335,395]$$

We note that:

$$\sum_{i=1}^{3} Na_i \neq \sum_{j=1}^{4} Nb_j$$

Because$\sum_{i=1}^{3} Na_i = [300,360]$ and $\sum_{j=1}^{4} Nb_j = [335,395]$, that is, the model is unbalanced because $\sum_{j=1}^{4} Nb_j > \sum_{i=1}^{3} Na_i$

A production deficit, so we add an artificial production center A_4 to its production capacity:

$$Na_4 = \sum_{j=1}^{4} Nb_j - \sum_{i=1}^{3} Na_i = [335,395] - [300,360] = [35,35]$$

The cost of transporting one unit of it to any consumer center is equal to zero, i.e.:

$$c_{4j} = 0 \,;\, j = 1,2,3,4$$

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	[7,9] Nx_{11}	[4,6] Nx_{12}	[15,17] Nx_{13}	[9,11] Nx_{14}	[120,155]
A_2	[11,13] Nx_{21}	[2,4] Nx_{22}	[7,9] Nx_{23}	[3,5] Nx_{24}	[80,90]
A_3	[4,6] Nx_{31}	[5,7] Nx_{32}	[2,4] Nx_{33}	[8,10] Nx_{34}	[100,115]
A_4	0 Nx_{41}	0 Nx_{42}	0 Nx_{43}	0 x_{34}	[35,35]
Required quantities	[85,92]	[100,118	[90,115]	[60,70]	

Table No. (26) Example data Transportation cost, available and required quantities Neutrosophic values, artificial production center

Mathematical model:

Find

$$
\begin{aligned}
NZ \in \{&[7,9]x_{11} + [4,6]x_{12} + [15,17]x_{13} + [9,11]x_{14} + [11,13]x_{21} \\
&+ [2,4]x_{22} + [7,9]x_{23} + [3,5]x_{24} + [4,6]x_{31} + [5,7]x_{32} \\
&+ [2,4]x_{33} + [8,10]x_{34} + 0.x_{41} + 0.x_{42} + 0.x_{43} \\
&+ 0.x_{44}\} \to Min
\end{aligned}
$$

Conditions:

$$\sum_{i=1}^{4} Nx_{ij} = Na_i \ \ ; j = 1,2,3,4$$

$$\sum_{j=1}^{4} Nx_{ij} = Nb_j \ \ ; i = 1,2,3,4$$

$$Nx_{ij} \geq 0 \ \ ; i = 1,2,3,4 \ \ , j = 1,2,3,4$$

Chapter II

Methods for finding the principled solution to Neutrosophic transport problems

2.1. Introduction

2.2. North-west corner method

2.2.1. the cost of transportation is Neutrosophic values

2.2.2. The quantities available in production centers and the quantities required in consumption centers are neutrosophic values

2.2.3. The cost of transportation, the quantities available in production centers, and the quantities required in consumption centers are neutrosophic values

2.3. The least - cost method

2.3.1. the cost of transportation is Neutrosophic values

2.3.2. The quantities available in production centers and the quantities required in consumption centers are neutrosophic values

2.3.3. The cost of transportation, the quantities available in production centers, and the quantities required in consumption centers are neutrosophic values

2.4. Vogel's Approximation Method

2.4.1. the cost of transportation is neutrosophic values

2.4.2. The quantities available in production centers and the quantities required in consumption centers are neutrosophic values

2.4.3. The cost of transportation, the quantities available in production centers, and the quantities required in consumption centers are neutrosophic values

2.1. Introduction:

In the first chapter, we looked at the neutrosophic transport models at the lowest cost, and, to find the optimal solution for them, we must obtain an initial solution. We present in this chapter some methods through which it is possible to obtain an initial solution for any transport model, noting that the initial solution must consider the balance between what is required and what is available. The number of unknowns of the rule in this initial solution must be the number of linear conditions, i.e., $n + m - 1$.

2.2. The North-West Corner Method:

In this method, we start from the North-west Corner (upper left), which is located at the intersection of the first row and the first column, and we place in it the largest possible quantity. This quantity is the smaller of the two quantities, the quantity available in the first production center and the quantity required in the first consumption center, if the quantity is available in the first production center is greater than the quantity required in the first consumer center. We move to the right and place in the adjacent cell the smaller of the two quantities, the quantity remaining in the first production center and the quantity required in the second consumer center. However, if the quantity required in the first consumer center is greater than the quantity available in the first production center, we descend to the bottom cell next to it, and we put in it the smaller of the two quantities, the quantity available to satisfy the need of the first consumption center and the quantity available in the second production center, and so we continue in the same way until we satisfy the needs of all consumption centers and empty all production centers. We obtain what resembles a staircase that descends from left to right, and thus we have arrived. To the first initial solution, the number of cells we occupy must equal $n + m - 1$ and then we calculate the total cost corresponding to this initial solution.

We explain the above through examples of neutrosophic transport models according to the three forms presented in the first chapter:

2.2.1. the transportation cost is neutrosophic values:

The cost of transporting one unit from the production center i to the consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$ where ε_{ij} is the indeterminacy of the production cost and it may be any neighborhood of the true valuec_{ij}, i.e. $\varepsilon_{ij} \in \left[\lambda_{1ij}, \lambda_{2ij}\right]$ or $\varepsilon_{ij} \in \left\{\lambda_{1ij}, \lambda_{2ij}\right\}$ Then the payments matrix becomes$Nc_{ij} = \left[c_{ij} \pm \varepsilon_{ij}\right]$.

Example (1):

It is intended to ship a quantity of oil from three stations A_1, A_2, A_3 to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7+\varepsilon$ x_{11}	$4+\varepsilon$ x_{12}	$15+\varepsilon$ x_{13}	$9+\varepsilon$ x_{14}	120
A_2	$11+\varepsilon$ x_{21}	$0+\varepsilon$ x_{22}	$7+\varepsilon$ x_{23}	$3+\varepsilon$ x_{24}	80
A_3	$4+\varepsilon$ x_{31}	$5+\varepsilon$ x_{32}	$2+\varepsilon$ x_{33}	$8+\varepsilon$ x_{34}	100
Required quantities	85	65	90	60	

Table No. (1) Data for example (1)

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$.

From the data of the problem, we note that:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \ = 300$$

That is, the model is balanced. Substituting $\in [0,2]$, we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$[7,9]$	$[4,6]$	$[15,17]$	$[9,11]$	120
A_2	$[11,13]$	$[0,2]$	$[7,9]$	$[3,5]$	80
A_3	$[4,6]$	$[5,7]$	$[2,4]$	$[8,10]$	100
Required quantities	85	65	90	70	300 / 300

Table No. (2) Data for example (1)

We use the north-west corner method to find an initial solution:

We start from the North-west corner that is, opposite the first production center and the first consumption center and put $Min\{85,120\} = 85$. Thus, we have fulfilled the need of the first consumption center from the first production center, and the quantity remaining in the first production center is $120 - 85 = 35$. We move to the right cell. If the first row intersects with the second column, and we set $Min\{65,35\} = 35$, the quantities available in the first production center become equal to zero, and the quantities required in the first consumption center equal to zero, and the second consumption center needs $65 - 35 = 30$. We go down to the cell located at the intersection of the second row and the second column, and we put $Min\{30,80\} = 30$, then we have met the need of the second consumption center, and what remains in the second production center is $80 - 30 = 50$. We continue in the same way until we empty all the production centers and satisfy the needs of all the consumption centers. We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	[7,9] 85	[4,6] 35	[15,17]	[9,11]	120
A_2	[11,13]	[0,2] 30	[7,9] 50	[3,5]	80
A_3	[4,6]	[5,7]	[2,4] 40	[8,10] 60	100
Required quantities	85	65	90	60	300 / 300

Table No. (3): Initial solution for example (1)

From the previous table we find:

$$x_{11} = 85 \ , x_{12} = 35 \, , x_{22} = 30 \ ,$$
$$x_{23} = 50 \ , x_{33} = 40 \, , x_{34} = 60$$
$$x_{13} = \ x_{14} = \ x_{21} = x_{24} = \ x_{31} = \ x_{32} = 0$$

We have $m = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution.

$$NL \in \{[7,9].85 + [4,6].35 + [15,17].0 + [9,11].0 + [11,13].0 + [0,2].30 + [7,9].50 + [3,5].0 + [4,6].0 + [5,7].0 + [2,4].40 + [8,10].60\} = [1645,2245]$$

It is the cost corresponding to the initial solution.

2.2.2. The quantities available in production centers and the quantities required in consumption centers are neutrosophic values.

Example (2):

It is intended to ship a quantity of oil from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7	4	15	9	$120+\varepsilon_1$
A_2	11	0	7	3	$80+\varepsilon_2$
A_3	4	5	2	8	$100+\varepsilon_3$
Required quantities	$85+\delta_1$	$65+\delta_1$	$90+\delta_1$	$60+\delta_1$	

Table No. (4) Data for example (2)

Where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ is the indeterminacy over the quantities available at the stations, ε_i is the indeterminacy over the quantities produced, i.e. $\varepsilon_i \in [\lambda_{1i}, \lambda_{2i}]$ or $\varepsilon_i \in \{\lambda_{1i}, \lambda_{2i}\}$, in this example we will take

$\varepsilon_1 \in [0,11]$,$\varepsilon_2 \in [0,9]$, and $\varepsilon_3 \in [0,15]\delta_1, \delta_2, \delta_3, \delta_4$. Is the indeterminacy over the quantities available at the stations, δ_j is the indeterminacy over the quantities produced, i.e., $\delta_j \in \left[\mu_{1j}, \mu_{2j}\right]$ or $\delta_j \in \left\{\mu_{1j}, \mu_{2j}\right\}$, in this example we will take

$$\delta_1 \in [0,8], \delta_2 \in [0,12], \ \delta_3 \in [0,9], \text{ and } \delta_4 \in [0,6]$$

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7	4	15	9	$[120,131]$
A_2	11	0	7	3	$[80,89]$
A_3	4	5	2	8	$[100,115]$
Required quantities	$[85,93]$	$[65,77]$	$[90,99]$	$[60,66]$	$[300,335]$ / $[300,335]$

Table No. (5) Data for example (2)

We note that $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j \ = [300,335]$ and therefore the model is balanced.

We use the northwest corner method to find an initial solution:

We start from the northwest corner that is opposite the first production center and the first consumption center, and put

$$Min\{[85,93],[120,131]\} = [85,93]$$

Thus, we have fulfilled the need of the first consumption center from the first production center and it remains in the first production center. Quantity $[120,131] - [85,93] = [35,38]$, We move to the right of the cell located at the intersection of the first row and the second column and put $Min\{[65,77],[35,38]\} = [35,38]$ in it, it becomes the quantities available in the first production center equal zero, and the quantities required in the first consumption center equal zero, and the second consumption center needs $[65,77] - [35,38] = [30,39]$ .We go down to the cell located at the intersection of the second row and the second column. We put $in\{[30,39],[80,89]\} = [30,39]$, then we have met the need of the second consumer center, and the second production center remains $[80,89] - [30,39] = [50,50]$ .We continue in the same way until we empty all production centers and satisfy the needs of all consumption centers. We obtain the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7 [85,93]	4 [35,38]	15 0	9 0	[120,131]
A_2	11 0	0 [30,39]	7 [50,50]	3 0	[80,89]
A_3	4 0	5 0	2 [40,49]	8 [60,66]	[100,115]
Required quantities	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

Table No. (6): Initial solution for example (2)

From the previous table we find that:

$$Nx_{11} \in [85{,}93],\ Nx_{12} \in [35{,}38],\ Nx_{22} \in [30{,}39],$$
$$Nx_{23} \in [50{,}50],\ Nx_{33} \in [40{,}49],\ Nx_{34} \in [60{,}66],$$
$$Nx_{13} = Nx_{14} = Nx_{21} = Nx_{24} = Nx_{31} = Nx_{32} = 0$$

We have $m = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{7.[85{,}93] + 4.[35{,}38] + 15.0 + 9.0 + 11.0 + 0.[30{,}39] + 7.[50{,}50] + 3.0 + 4.0 + 5.0 + 2.[40{,}49] + 8.[60{,}66]\} = [1645{,}1779]$$

It is the cost corresponding to the initial solution.

2.2.3. The cost of transportation, the quantities available in production centers, and the quantities required in consumption centers. neutrosophic values:

Example (3):

It is intended to ship a quantity of oil from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7+\varepsilon$	$4+\varepsilon$	$15+\varepsilon$	$9+\varepsilon$	$120+\varepsilon_1$
A_2	$11+\varepsilon$	$0+\varepsilon$	$7+\varepsilon$	$3+\varepsilon$	$80+\varepsilon_2$
A_3	$4+\varepsilon$	$5+\varepsilon$	$2+\varepsilon$	$8+\varepsilon$	$100+\varepsilon_3$
Required quantities	$85+\delta_1$	$65+\delta_2$	$90+\delta_3$	$60+\delta_4$	

Table No. (7) Data for example (3)

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$.

Where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ is the indeterminacy over the quantities available at the stations, ε_i is the indeterminacy over the quantities produced, i.e., $\varepsilon_i \in [\lambda_{1i}, \lambda_{2i}]$ or $\varepsilon_i \in \{\lambda_{1i}, \lambda_{2i}\}$, in this example we will take

$\varepsilon_1 \in [0,11]$,$\varepsilon_2 \in [0,9]$, and $\varepsilon_3 \in [0,15]\delta_1, \delta_2, \delta_3, \delta_4$. Is the indeterminacy over the quantities available at the stations, δ_j is the indeterminacy over the quantities produced, i.e., $\delta_j \in [\mu_{1j}, \mu_{2j}]$ or $\delta_j \in \{\mu_{1j}, \mu_{2j}\}$, in this example we will take

$$\delta_1 \in [0,8], \delta_2 \in [0,12],\ \delta_3 \in [0,9], \text{ and } \delta_4 \in [0,6]$$

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$[7,9]$	$[4,6]$	$[15,17]$	$[9,11]$	$[120,131]$
A_2	$[11,13]$	$[0,2]$	$[7,9]$	$[3,5]$	$[80,89]$
A_3	$[4,6]$	$[5,7]$	$[2,4]$	$[8,10]$	$[100,115]$
Required quantities	$[85,93]$	$[65,77]$	$[90,99]$	$[60,66]$	$[300,335]$ / $[300,335]$

Table No. (8) Data for example (3)

We use the northwest corner method to find an initial solution:

We start from the North-west corner, that is, opposite the first production center and the first consumption center and put

$Min\{[85,93],[120,131]\} = [85,93]$ thus, we have fulfilled the need of the first consumption center from the first production center and it remains in the first production center.

Quantity $[120,131] - [85,93] = [35,38]$.We move to the right of the cell located at the intersection of the first row and the second column and put $[65,77] - [35,38] = [30,39]$ in it. The quantities available in the first production center are equal to zero, and the quantities required in the first consumption center are equal to zero, and the second consumption center needs $[65,77] - [35,38] = [30,39]$. We go down to the cell located at the intersection of the second row and the second column. We put in it $Min\{[30,39],[80,89]\} = [30,39]$,then we have met the need of the second consumer center and the second production center remains $[80,89] - [30,39] = [50,50]$. We continue in the same way until we empty all production centers and satisfy the needs of all consumption centers. We obtain the following table**:**

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	[7,9] [85,93]	[4,6] [35,38]	[15,17] 0	[9,11] 0	[120,131]
A_2	[11,13] 0	[0,2] [30,39]	[7,9] [50,50]	[3,5] 0	[80,89]
A_3	[4,6] 0	[5,7] 0	[2,4] [40,49]	[8,10] [60,66]	[100,115]
Required quantities	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

Table No. (9): Initial solution for example (3)

From the previous table we find that:

$$Nx_{11} \in [85,93], Nx_{12} \in [35,38], Nx_{22} \in [30,39],$$
$$Nx_{23} \in [50,50], Nx_{33} \in [40,49], \ Nx_{34} \in [60,66],$$
$$Nx_{13} = Nx_{14} = Nx_{21} = Nx_{24} = Nx_{31} = Nx_{32} = 0$$

We have $\mathrm{m} = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{[7,9].[85,93] + [4,6].[35,38] + [15,17].0 + [9,11].0 + [11,13].0 + [0,2].[30,39] + [7,9].[50,50] + [3,5].0 + [4,6].0 + [5,7].0 + [2,4].[40,49] + [8,10].[60,66]\} = [1645,2449]$$

It is the cost corresponding to the initial solution.

2.3. The Least- Cost Method:

This method depends on satisfying the cells with the least cost first, so it is better than the northwest corner method. We begin by supplying the square with the least cost in the problem as a whole and provide it with the order it needs from the inventory corresponding to this cell. We continue to fill the cells with the least cost sequentially until we supply all centers. Consumption of quantities available in production centers

2.3.1. the cost of transportation is neutrosophic values:

The cost of transporting one unit from production center $i$to consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$ where ε_{ij} is the indeterminacy of the production cost and it may be any neighborhood of the true value c_{ij}, i.e.,

$\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$. Then the payments matrix becomes $Nc_{ij} = [c_{ij} \pm \varepsilon_{ij}]$

Example (4):

It is intended to ship a quantity of oil from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4.The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7 + \varepsilon$ x_{11}	$4 + \varepsilon$ x_{12}	$15 + \varepsilon$ x_{13}	$9 + \varepsilon$ x_{14}	120
A_2	$11 + \varepsilon$ x_{21}	$0 + \varepsilon$ x_{22}	$7 + \varepsilon$ x_{23}	$3 + \varepsilon$ x_{24}	80
A_3	$4 + \varepsilon$ x_{31}	$5 + \varepsilon$ x_{32}	$2 + \varepsilon$ x_{33}	$8 + \varepsilon$ x_{34}	100
Required quantities	85	65	90	60	

Table No. (10): Data for example (4)

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$.

From the data of the problem we note that:

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j \ = 300$$

That is, the model is balanced. Substituting $\varepsilon \in [0,2]$,we get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	[7,9]	[4,6]	[15,17]	[9,11]	120
A_2	[11,13]	[0,2]	[7,9]	[3,5]	80
A_3	[4,6]	[5,7]	[2,4]	[8,10]	100
Required quantities	85	65	90	70	300 / 300

Table No. (11): Data for example (4)

We use the least cost method to find an initial solution. We notice that the least cost is [0,2], which is located in the cell resulting from the intersection of the second row and the second column. Therefore, we put $Min\{65,80\} = 65$ in it. Thus, we have met the need of the second consumer center from the production center. The second, and the remaining quantity in the second production center is $80 - 65 = 15$. We move to the least cost among the remaining costs, which is [2,4]. We find it located in the cell resulting from the intersection of the third row and the third column, in which we put $Min\{90,100\} = 90$. Thus, we have met the need. The third consumption center comes from the third production center, and the remaining quantity in the third production center is

$100 - 90 = 10$. We continue in the same way until we satisfy all consumption centers and empty all production centers. We obtain the following table:

Production centers \ Consumption centers	B_1	B_2	B_3	B_4	Available quantities
A_1	[7,9] 75	[4,6]	[15,17]	[9,11] 45	120
A_2	[11,13]	[0,2] 65	[7,9]	[3,5] 15	80
A_3	[4,6] 10	[5,7]	[2,4] 90	[8,10]	100
Required quantities	85	65	90	70	300 / 300

Table No. (12): Initial solution for example (4)

From the previous table we find:

$$x_{11} = 75 \ , x_{14} = 45 \, , x_{22} = 65$$
$$x_{24} = 15 \ , x_{31} = 10 \, , x_{33} = 90$$
$$x_{12} = x_{13} = x_{21} = x_{23} = x_{32} = x_{34} = 0$$

We have $m = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{[7,9].75 + [4,6].0 + [15,17].0 + [9,11].45 + [11,13].0 + [0,2].65 + [7,9].0 + [3,5].15 + [4,6].10 + [5,7].0 + [2,4].90 + [8,10].0\} = [1195,1795]$$

It is the cost corresponding to the initial solution.

2.3.2. The quantities available in production centers and the quantities required in consumption centers are neutrosophic values.

Example (5):

It is intended to ship a quantity of oil from three stations A_1, A_2, A_3 to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table :

Consumption centers / Production centers	B_1	B_2	B_3	B_4	**Available quantities**
A_1	7	4	15	9	$120 + \varepsilon_1$
A_2	11	0	7	3	$80 + \varepsilon_2$
A_3	4	5	2	8	$100 + \varepsilon_3$
Required quantities	$85 + \delta_1$	$65 + \delta_1$	$90 + \delta_1$	$60 + \delta_1$	

Table (13) data for example (5)

Where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ is the indeterminacy over the quantities available at the stations, ε_i is the indeterminacy over the quantities produced, i.e., $\varepsilon_i \in [\lambda_{1i}, \lambda_{2i}]$ or $\varepsilon_i \in \{\lambda_{1i}, \lambda_{2i}\}$, in this example we will take

$\varepsilon_1 \in [0,11]$,$\varepsilon_2 \in [0,9]$, and $\varepsilon_3 \in [0,15]\delta_1, \delta_2, \delta_3, \delta_4$. Is the indeterminacy over the quantities available at the stations, δ_j is the indeterminacy over the quantities produced, i.e., $\delta_j \in [\mu_{1j}, \mu_{2j}]$ or $\delta_j \in \{\mu_{1j}, \mu_{2j}\}$, in this example we will take

$$\delta_1 \in [0,8], \delta_2 \in [0,12],\ \delta_3 \in [0,9], \text{ and } \delta_4 \in [0,6].$$

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	**Available quantities**
A_1	7	4	15	9	$[120,131]$
A_2	11	0	7	3	$[80,89]$
A_3	4	5	2	8	$[100,115]$
Required quantities	$[85,93]$	$[65,77]$	$[90,99]$	$[60,66]$	$[300,335]$ / $[300,335]$

Table (14) data for example (5)

We note that $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,335]$ and therefore the model is balanced.

We use the least cost method to find an initial solution. We note that the least cost is zero, and it is located in the cell resulting from the intersection of the second row and the second column. Therefore, we put $Min\{[65,77],[80,89]\} = [65,77]$ in it, thus we have demonstrated the need of the first consumption center from the first production center and the quantity $[80,89] - [65,77] = [12,15]$ remains in the first production center. We move to the least cost among the remaining costs, which is 2. We find it located in the cell resulting from the intersection of the line. The third and third column, we put $Min\{[90,99],[100,115]\} = [90,99]$ then we have met the need of the second consumer center and the second production center remains $[80,89] - [30,39] = [50,50]$,we continue in the same way until we satisfy all consumption centers and empty all production centers. We obtain the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7 [75,77]	4 0	15 0	9 [45,54]	[120,131]
A_2	11 0	0 [65,77]	7 0	3 [15,12]	[80,89]
A_3	4 [10,16]	5 0	2 [90,99]	8 0	[100,115]
Required quantities	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

Table (15) Initial solution for example (5)

From the previous table we find:

$$Nx_{11} \in [75,93], Nx_{12} \in [45,54], Nx_{22} \in [65,77],$$
$$Nx_{24} \in [15,12], Nx_{31} \in [10,16], \; Nx_{33} \in [90,99],$$
$$Nx_{12} = Nx_{13} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$$

We have $\mathrm{m} = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{7.[75,77] + 4.0 + 15.0 + 9.[45,54] + 0.[65,77] + 7.0 + 3.[12,15] + 4.[10,16] + 5.0 + 2.[90,99] + 8.0\} = [1259,1323]$$

It is the cost corresponding to the initial solution.

2.3.3. The cost of transportation, the quantities available in production centers, and the quantities required in consumption centers are neutrosophic values.

Example (6):

It is intended to ship a quantity of oil from three stations $A_1, A_2, A_3$to four citiesB_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$7+\varepsilon$	$4+\varepsilon$	$15+\varepsilon$	$9+\varepsilon$	$120+\varepsilon_1$
A_2	$11+\varepsilon$	$0+\varepsilon$	$7+\varepsilon$	$3+\varepsilon$	$80+\varepsilon_2$
A_3	$4+\varepsilon$	$5+\varepsilon$	$2+\varepsilon$	$8+\varepsilon$	$100+\varepsilon_3$
Required quantities	$85+\delta_1$	$65+\delta_2$	$90+\delta_3$	$60+\delta_4$	

Table (16) data for example (6)

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$.

Where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ is the indeterminacy over the quantities available at the stations, ε_i is the indeterminacy over the quantities produced, i.e., $\varepsilon_i \in [\lambda_{1i}, \lambda_{2i}]$ or $\varepsilon_i \in \{\lambda_{1i}, \lambda_{2i}\}$, in this example we will take

$\varepsilon_1 \in [0,11]$,$\varepsilon_2 \in [0,9]$, and $\varepsilon_3 \in [0,15]\delta_1, \delta_2, \delta_3, \delta_4$. Is the indeterminacy over the quantities available at the stations, δ_j is the indeterminacy over the quantities produced, i.e., $\delta_j \in [\mu_{1j}, \mu_{2j}]$ or $\delta_j \in \{\mu_{1j}, \mu_{2j}\}$, in this example we will take

$$\delta_1 \in [0,8], \delta_2 \in [0,12],\ \delta_3 \in [0,9], \text{ and } \delta_4 \in [0,6].$$

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$[7,9]$	$[4,6]$	$[15,17]$	$[9,11]$	$[120,131]$
A_2	$[11,13]$	$[0,2]$	$[7,9]$	$[3,5]$	$[80,89]$
A_3	$[4,6]$	$[5,7]$	$[2,4]$	$[8,10]$	$[100,115]$
Required quantities	$[85,93]$	$[65,77]$	$[90,99]$	$[60,66]$	$[300,335]$ / $[300,335]$

Table (17) data for example (6)

We use the least cost method to find an initial solution:

We use the least cost method to find an initial solution. We note that the least cost is $[0,2]$ and it is located in the cell resulting from the intersection of the second row and the second column, so we put $Min\{[80,89],[65,77]\} = [65,77]$ thus, we have met the need of the second consumption center from the second production center, and the remaining quantity in the second production center is

$[80,89] - [65,77] = [12,15]$. We move to the least cost among the remaining costs, which is $[2,4]$. We find it located in the cell resulting from the intersection of the third row and the third column, and we put $Min\{[90,99],[100,115]\} = [90,99]$in it. Thus, we have met the need of the third consumption center from the third production center, and the remaining quantity is in the third production center.

$[100,115] - [90,99] = [10,16]$We continue in the same way until we satisfy all consumption centers and empty all production centers. We obtain the following table**:**

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	[7,9] [75,77]	[4,6] 0	[15,17] 0	[9,11] [45,54]	[120,131]
A_2	[11,13] 0	[0,2] [65,77]	[7,9] 0	[3,5] [12,15]	[80,89]
A_3	[4,6] [10,16]	[5,7] 0	[2,4] [90,99]	[8,10] 0	[100,115]
Required quantities	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

Table No. (18): Initial solution for example (6)

From the previous table we find:

$$Nx_{11} \in [75,77], Nx_{13} \in [45,54], Nx_{22} \in [65,77],$$
$$Nx_{24} \in [12,15], Nx_{13} \in [10,16], \ Nx_{33} \in [90,99],$$
$$Nx_{12} = Nx_{22} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$$

We have $m = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{[7,9].[75,77] + [4,6].0 + [15,17].0 + [9,11].[45,54] + [11,13].0 + [2,4].[65,77] + [7,9].0 + [3,5].[12,15] + [4,6].[10,16] + [5,7].0 + [2,4].[90,99] + [8,10].0\} = [1195,1993]$$

It is the cost corresponding to the initial solution.

2.4. Vogel's Approximation Method:

This method often leads to the optimal solution or to a solution close to it, and it is better than the previous two methods. To reach the initial solution in this way, we follow the following steps:

We calculate the difference between the two least unequal costs in each row and each column. We take the row and column with the largest difference and choose the least cost cell in the chosen row or column and work to satisfy the order of the consumption center in which this cell is located from the production center it corresponds to. We recalculate the difference again. For all columns and rows, we repeat the previous process until we meet all the requests of the consumption centers from the quantities available in the production centers.

2.4.1. the cost of transportation using neutrosophic values:

The cost of transporting one unit from production center i to consumption center j is $Nc_{ij} = c_{ij} \pm \varepsilon_{ij}$where ε_{ij} is the indeterminacy of the production cost and it may be any neighborhood of the true value c_{ij}, i.e. $\varepsilon_{ij} \in [\lambda_{1ij}, \lambda_{2ij}]$ or $\varepsilon_{ij} \in \{\lambda_{1ij}, \lambda_{2ij}\}$ Then the payments matrix becomes $Nc_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

Example (7):

It is intended to ship a quantity of oil from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / **Production centers**	B_1	B_2	B_3	B_4	**Available quantities**
A_1	$7 + \varepsilon$	$4 + \varepsilon$	$15 + \varepsilon$	$9 + \varepsilon$	120
A_2	$11 + \varepsilon$	$0 + \varepsilon$	$7 + \varepsilon$	$3 + \varepsilon$	80
A_3	$4 + \varepsilon$	$5 + \varepsilon$	$2 + \varepsilon$	$8 + \varepsilon$	100
Required quantities	85	65	90	60	

Table No. (19): Data for example (7)

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$.

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	$[7,9]$	$[4,6]$	$[15,17]$	$[9,11]$	120
A_2	$[11,13]$	$[0,2]$	$[7,9]$	$[3,5]$	80
A_3	$[4,6]$	$[5,7]$	$[2,4]$	$[8,10]$	100
Required quantities	85	65	90	60	$[300,335]$ / $[300,335]$

Table No. (20): Data for example (7)

We calculate the difference between the two least unequal costs in each row and each column, and here it is not correct for the difference to be zero. We take the row and column with the largest difference and choose the least cost cell in the chosen row or column and work to meet the order of the consumption center in which this cell is located from the production center that corresponds to it. We take the largest differences and find them to be 5 and they are present in two cells. We choose one of them, let it be column B_4. Then we choose the cell with the least cost, which is opposite the production center A_2. We saturate this cell and put $Min\{60,80\} = 60$ in it. Thus, we have met the need of the fourth consumer center from the center. In the second productive position, we cross out the fourth column, and the remaining quantity in the second productive position is $80 - 60 = 20$, as in the following table:

Consumption centers / Production centers		B_1	B_2	B_3	B_4	Available quantities	row difference Δ_1
A_1		$[7,9]$	$[4,6]$	$[15,17]$	$[9,11]$	120	3
A_2		$[11,13]$	$[0,2]$	$[7,9]$	$[3,5]$ $[60,66]$	80	3
A_3		$[4,6]$	$[5,7]$	$[2,4]$	$[8,10]$	100	2
Required quantities		85	65	90	60	$[300,335]$ $[300,335]$	
Column teams	Δ'_1	3	4	5	5		

Table No. (21) First differences for example (7)

We recalculate the difference again for all columns and rows except the fourth column. We repeat the previous process until we meet all the demands of the consumption centers from the quantities available in the production centers, empty all the production centers, and satisfy the needs of all the consumption centers. We obtain the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities	difference Δ_1	Δ_2	Δ_3
A_1	[7,9] [75,77]	[4,6] [45,54]	[15,17]	[9,11]	120	3	3	3
A_2	[11,13]	[0,2] [20,23]	[7,9]	[3,5] [60,66]	80	3	7	$-$
A_3	[4,6] [10,16]	[5,7]	[2,4] [90,99]	[8,10]	100	2	2	2
الكميات المطلوبة	85	65	90	60	[300,335] [300,335]			
Difference Δ'_1	3	4	5	5				
Difference Δ'_2	3	4	5	$-$				
Difference Δ'_3	3	1	13	$-$				

Table No. (22): Initial solution for example (7)

From the previous table we find:

$$Nx_{11} = 75, Nx_{14} = 45, Nx_{22} = 65,$$
$$Nx_{24} = 15, Nx_{31} = 10, \ Nx_{33} = 90,$$
$$Nx_{12} = Nx_{13} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$$

We have $m = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{[7,9].75 + [4,6].45 + [15,17].0 + [9,11].0 + [11,13].0 + [0,2].20 + [7,9].0 + [3,5].60 + [4,6].10 + [5,7].0 + [2,4].90 + [8,10].0\} = [1105,1705]$$

It is the cost corresponding to the initial solution.

2.4.2. The quantities available in production centers and the quantities required in consumption centers are neutrosophic values:

Example (8):

A quantity of oil is to be shipped from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7	4	15	9	$120 + \varepsilon_1$
A_2	11	0	7	3	$80 + \varepsilon_2$
A_3	4	5	2	8	$100 + \varepsilon_3$
Required quantities	$85 + \delta_1$	$65 + \delta_1$	$90 + \delta_1$	$60 + \delta_1$	

Table No. (23) Data for example (8)

Where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ is the indeterminacy over the quantities available at the stations, ε_i is the indeterminacy over the quantities produced, i.e., $\varepsilon_i \in [\lambda_{1i}, \lambda_{2i}]$ or $\varepsilon_i \in \{\lambda_{1i}, \lambda_{2i}\}$, in this example we will take

$\varepsilon_1 \in [0,11]$,$\varepsilon_2 \in [0,9]$, and $\varepsilon_3 \in [0,15]\delta_1, \delta_2, \delta_3, \delta_4$. Is the indeterminacy over the quantities available at the stations, δ_j is the indeterminacy over the quantities produced, i.e., $\delta_j \in \left[\mu_{1j}, \mu_{2j}\right]$ or $\delta_j \in \left\{\mu_{1j}, \mu_{2j}\right\}$, in this example we will take

$$\delta_1 \in [0,8], \delta_2 \in [0,12],\ \delta_3 \in [0,9], \text{ and } \delta_4 \in [0,6].$$

We get the following table:

Consumption centers / Production centers	B_1	B_2	B_3	B_4	Available quantities
A_1	7	4	15	9	[120,131]
A_2	11	0	7	3	[80,89]
A_3	4	5	2	8	[100,115]
Required quantities	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]

Table No. (24) Data for example (8)

We note that $\sum_{i=1}^{3} Na_i = \sum_{j=1}^{4} Nb_j = [300,335]$ and therefore the model is balanced.

According to the difference between the two least unequal costs in each row and each column, then we take the row and column with the largest difference, choose the cell with the least cost in the chosen line or column, and work to meet the order of the consumption center in which this cell of the production center is located.

We take the largest differences and find them 5, which are located in two cells. We choose one of them, let it be column B_4. Then we choose the cell with the least cost, which is opposite the production center A_2. We saturate this cell and put it in it.$Min\{[60,66],[80,89]\} = [60,66]$, thus, we have met the need of the fourth consumption center from the second production center. We cross out the fourth column and the remaining quantity is in the second production center $[80,89] - [60,66] = [20,23]$.

As in the following table:

Consumption centers / Production centers		B_1	B_2	B_3	B_4	Available quantities	difference Δ_1
A_1		7	4	15	9	[120,131]	3
A_2		11	0	7	3 [60,66]	[80,89] [20,23]	3
A_3		4	5	2	8	[100,115]	2
Required quantities		[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]	
difference	Δ'_1	3	4	5	5		

Table No. (26) First differences for example (8)

We recalculate the difference again for all columns and rows except the fourth column. We repeat the previous process until we meet all the demands of the consumption centers from the quantities available in the production centers, empty all the production centers, and satisfy the needs of all the consumption centers. We obtain the following table:

Consumption centers / Production centers		B_1	B_2	B_3	B_4	Available quantities	difference Δ_1	Δ_2	Δ_3
A_1		7 [75,77]	4 [45,54]	15	9	[120,131]	3	3	3
A_2		11	0 [20,23]	7	3 [60,66]	[80,89]	3	7	–
A_3		4 [10,16]	7	2 [90,99]	8	[100,115]	2	2	2
Required quantities		[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]			
difference	Δ'_1	3	4	5	5				
	Δ'_2	3	4	5	–				
	Δ'_3	3	1	13	–				

Table No. (27): Initial solution for example (8)

From the previous table we find:

$$Nx_{11} \in [75,77], Nx_{12} \in [45,54], Nx_{22} \in [20,23],$$
$$Nx_{24} \in [60,66], Nx_{31} \in [10,16],\ Nx_{33} \in [90,99],$$
$$Nx_{13} = Nx_{14} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$$

We have $m = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{7.[75,77] + 4.[45,54] + 15.0 + 9.0 + 11.0 + 0.[20,23] + 7.0 + 3.[60,66] + 4.[10,16] + 5.0 + 2.[90,99] + 8.0\} = [1105,1215]$$

It is the cost corresponding to the initial solution.

2.4.3. The cost of transportation, the quantities available in production centers and the quantities required in consumption centers are neutrosophic values:

Example (9):

It is intended to ship a quantity of oil from three stations A_1, A_2, A_3 to four cities B_1, B_2, B_3, B_4. The quantities available at each station, the quantities required in each city, and the cost of transportation in each direction are shown in the following table:

Consumption centers / **Production centers**	B_1	B_2	B_3	B_4	**Available quantities**
A_1	$7 + \varepsilon$	$4 + \varepsilon$	$15 + \varepsilon$	$9 + \varepsilon$	$120 + \varepsilon_1$
A_2	$11 + \varepsilon$	$0 + \varepsilon$	$7 + \varepsilon$	$3 + \varepsilon$	$80 + \varepsilon_2$
A_3	$4 + \varepsilon$	$5 + \varepsilon$	$2 + \varepsilon$	$8 + \varepsilon$	$100 + \varepsilon_3$
Required quantities	$85 + \delta_1$	$65 + \delta_2$	$90 + \delta_3$	$60 + \delta_4$	

Table No. (28): Data for Example (9)

Where ε is the indeterminacy and we take it as $\varepsilon \in [0,2]$.

Where $\varepsilon_1, \varepsilon_2, \varepsilon_3$ is the indeterminacy over the quantities available at the stations, ε_i is the indeterminacy over the quantities produced, i.e., $\varepsilon_i \in [\lambda_{1i}, \lambda_{2i}]$ or $\varepsilon_i \in \{\lambda_{1i}, \lambda_{2i}\}$, in this example we will take

$\varepsilon_1 \in [0,11]$,$\varepsilon_2 \in [0,9]$, and $\varepsilon_3 \in [0,15]\delta_1, \delta_2, \delta_3, \delta_4$. Is the indeterminacy over the quantities available at the stations, δ_j is the indeterminacy over the quantities produced, i.e., $\delta_j \in \left[\mu_{1j}, \mu_{2j}\right]$ or $\delta_j \in \left\{\mu_{1j}, \mu_{2j}\right\}$, in this example we will take

$$\delta_1 \in [0,8], \delta_2 \in [0,12],\ \delta_3 \in [0,9], \text{ and } \delta_4 \in [0,6].$$

We get the following table:

Consumption centers / **Production centers**	B_1	B_2	B_3	B_4	**Available quantities**
A_1	[7,9]	[4,6]	[15,17]	[9,11]	[120,131]
A_2	[11,13]	[0,2]	[7,9]	[3,5]	[80,89]
A_3	[4,6]	[5,7]	[2,4]	[8,10]	[100,115]
Required quantities	[85,93]	[65,77]	[90,99]	[60,66]	[300,335] / [300,335]

Table No. (29): Data for example (9)

We calculate the difference between the two least unequal costs in each row and each column, and here it is not correct for the difference to be zero. We take the row and column with the largest difference and choose the least cost cell in the chosen row or column and work to meet the order of the consumption center in which this cell is located from the production center that corresponds to it. We take the largest differences, we find them 5, and they are present in two cells. We choose one of them, let it be column B_4, then we choose the cell with the least cost, which is opposite the production center A_2. We saturate this cell and put $Min\{[60,66],[80,89]\} = [60,66]$, we have thus fulfilled the need of the fourth consumer center from the second production center. We delete the fourth column and the remaining quantity in the second production center is $[80,89] - [60,66] = [20,23]$ as in the following table:

Consumption centers / Production centers		B_1	B_2	B_3	B_4	Available quantities	difference Δ_1
A_1		[7,9]	[4,6]	[15,17]	[9,11]	[120,131]	3
A_2		[11,13]	[0,2]	[7,9]	[3,5] [60,66]	[80,89]	3
A_3		[4,6]	[5,7]	[2,4]	[8,10]	[100,115]	2
Required quantities		[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]	
difference	Δ_1'	3	4	5	5		

Table No. (30): First differences table for example (9)

We recalculate the difference again for all columns and rows except the fourth column. We repeat the previous process until we meet all the demands of the consumption centers from the quantities available in the production centers, empty all the production centers, and satisfy the needs of all the consumption centers. We obtain the following table.

Consumption centers / Production centers		B_1	B_2	B_3	B_4	Available quantities	difference		
							Δ_1	Δ_2	Δ_3
A_1		[7,9] [75,77]	[4,6] [45,54]	[15,17]	[9,11]	[120,131]	3	3	3
A_2		[11,13]	[0,2] [20,23]	[7,9]	[3,5] [60,66]	[80,89]	3	7	–
A_3		[4,6] [10,16]	[5,7]	[2,4] [90,99]	[8,10]	[100,115]	2	2	2
Required quantities		[85,93]	[65,77]	[90,99]	[60,66]	[300,335] [300,335]			
difference	Δ'_1	3	4	5	5				
	Δ'_2	3	4	5	–				
	Δ'_3	3	1	13	–				

Table No. (31): Initial solution for example (9)

From the previous table we find:

$$Nx_{11} \in [75,77], Nx_{12} \in [45,54], Nx_{22} \in [20,23],$$
$$Nx_{24} \in [60,66], Nx_{31} \in [10,16], \; Nx_{33} \in [90,99],$$
$$Nx_{13} = Nx_{14} = Nx_{21} = Nx_{23} = Nx_{32} = Nx_{34} = 0$$

We have $m = 3, n = 4$, Therefore $n + m - 1 = 6$. This means that the initial solution satisfies the condition.

We calculate the cost corresponding to this initial solution and replace it with the following cost function:

$$L = c_{11}x_{11} + c_{12}x_{12} + c_{13}x_{13} + c_{14}x_{14} + c_{21}x_{21} + c_{22}x_{22} + c_{23}x_{23} + c_{24}x_{24} + c_{31}x_{31} + c_{32}x_{32} + c_{33}x_{33} + c_{34}x_{34}$$

We get the cost corresponding to the initial solution:

$$NL \in \{[7,9].[75,77] + [4,6].[45,54] + [15,17].0 + [9,11].0 + [11,13].0 + [0,2].[20,23] + [7,9].0 + [3,5].[60,66] + [4,6].[10,16] + [5,7].0 + [2,4].[90,99] + [8,10].0\} = [11055,1885]$$

It is the cost corresponding to the initial solution.

Chapter III

The optimal solution for neutrospheric transport models based on an initial solution

3.1. Introduction.

3.2. The Stepping-Stone Method.

3.3. Modified Distribution Method.

3.1. Introduction:

After obtaining an initial neutrosophic solution using the methods presented in the second chapter, we must test this solution whether it is the optimal solution or not. If it is the optimal solution, then we have reached what is required, but if the solution is other than that, we must search for the optimal solution. Based on this, there are several methods from which we study in this book the following two methods:

- **The Stepping-Stone Method.**
- **Modified Distribution Method.**

3.2. The Stepping - Stone Method:

a. We find the initial solution using one of the methods mentioned in Chapter Two, then we calculate the total cost according to the initial solution**.**

b. We identify the basic variables from the non-basic variables from the preliminary solution table.

c. We determine the indirect cost by finding closed paths, as each closed path has its beginning and end as a non-basic variable and consists of horizontal and vertical lines whose pillars are basic variables, as it happens that there are two basic variables in the way of the path, so we deviate from the basic non-basic variable and in general the closed path represents in the following form:

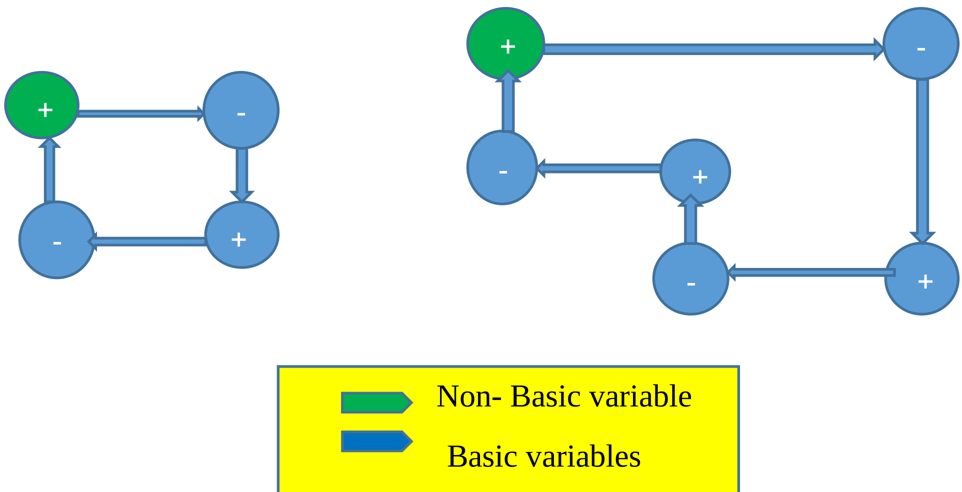


Figure NO. (1): A representation of c path ways

We calculate the indirect cost of each non-basic variable by giving the cost of the non-basic variable a positive sign, and the cost of the basic variables we give it alternating negative and then positive signs, and so on. Of the basic variables are positive or zero, this means that the solution that we got is an optimal solution and we stop. But if at least one of the indirect costs is negative, then we must develop the solution by choosing one of the non-basic variables to become basic and the exit of one of the basic variables.

Note:

To determine the basic internal variable, we take the non-basic variable that achieved the most negative in the indirect cost, and in order to make the solution the best possible, we try to pass in it the largest possible amount, we explain the above through the following example:

Example:

The following table represents the cost of transporting goods from sources A_i ; $i = 1,2,3,4$ to distribution centers B_j ; $j = 1,2,3,4$ it is required to use the mobile quarantine method to improve the solution and obtain the ideal solution**:**

consumption center / production centers	B_1	B_2	B_3	Available quantities
A_1	2	4	0	150
A_2	{3,4.5}	{1,2}	{5,8}	200
A_3	6	2	4	325
A_4	1	7	9	25
required quantities	180	320	200	700 / 700

Table No. (1) Issue data

In this example the cost of transportation the product at the production center A_2 has neutrosophic values we take it form $c_{2j} \in \left\{\alpha_{1_{2j}} \alpha_{2_{2j}}\right\}$

The solution:

We find the initial solution using the least cost method; we get the following preliminary solution table:

consumption centers / production centers	B_1	B_2	B_3	Available quantities
A_1	2	4	0 150	150
A_2	{3,4.5}	{1,2} 200	{5,8}	200
A_3	6 155	2 120	4 50	325
A_4	1 25	7	9	25
required quantities	180	320	200	700 / 700

Table No (2) preliminary solution

We note that the number of occupied squares is equal to $m + n - 1 = 6$

The total transportation cost according to the preliminary solution is:

$$NZ_1 \in \{0 \times 150 + \{1,2\} \times 200 + 6 \times 155 + 2 \times 120 + 4 \times 50 + 1 \times 25\}$$

For $c_{22} = 1 \Rightarrow Z_1 = 1595$

For $c_{22} = 2 \Rightarrow Z_1 = 1795$

That is, against this preliminary solution, we have a neutrosophic value for the total transportation cost:

$$NZ_1 \in \{1595,1795\}$$

Now we see whether this solution is an optimal solution or not?

For this we define basic variables and non-basic variables, it is clear that the basic variables are:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

and the non-basic variables are:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

We have six basic variables and six non-basic variables, so we get six closed paths:

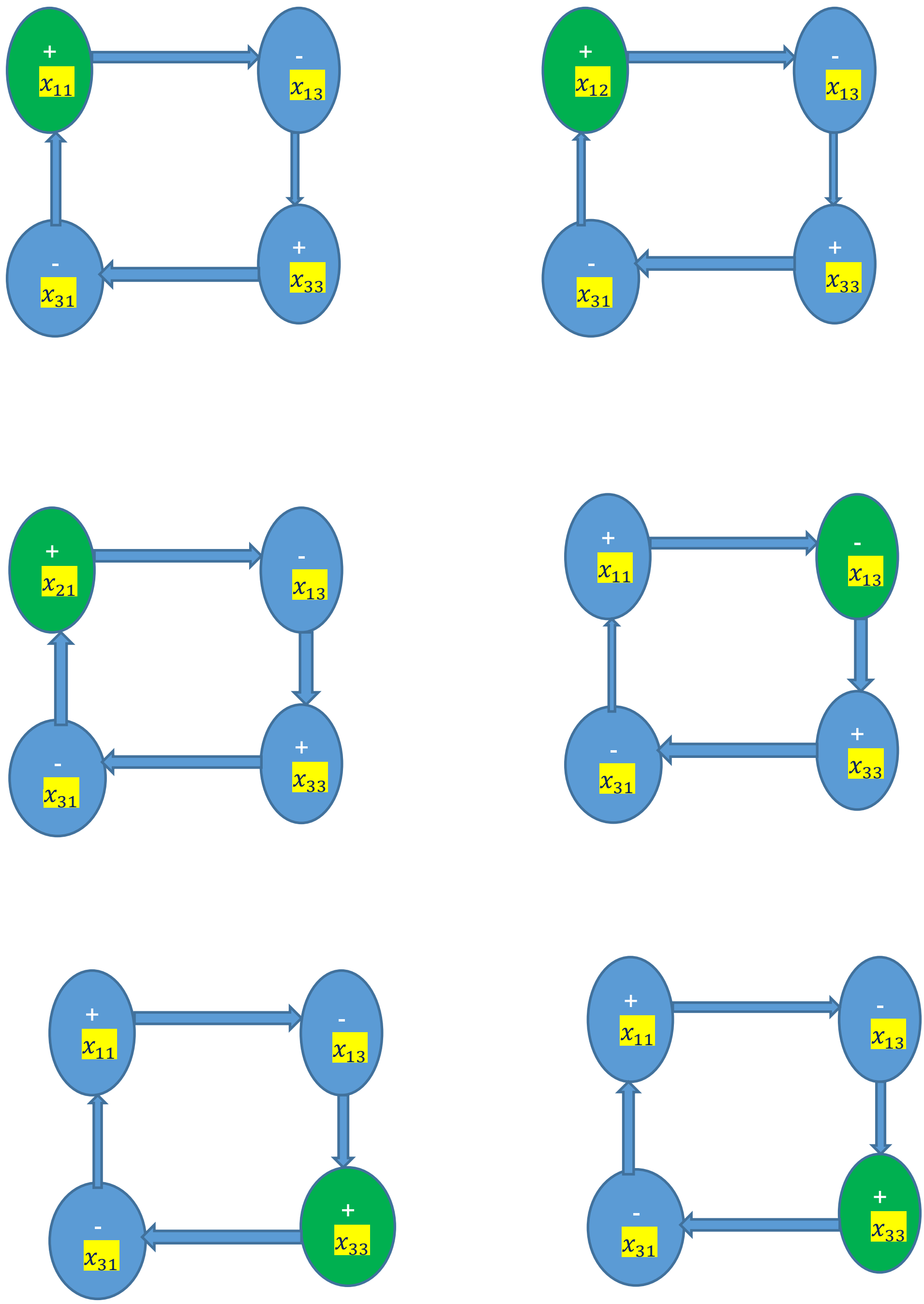


Figure No. (2) Possible closed paths after finding the initial solution
The non- basic variables green

The following table shows how the path is formed:

consumption center / production centers	B_1	B_2	B_3	Available quantities
A_1	2	4	0 150	150
A_2	{3,4.5}	{1,2} 200	{5,8}	200
A_3	6 155	2 120	4 50	325
A_4	1 25	7	9	25
required quantities	180	320	200	700 / 700

Table No (3) Closed path identification

We calculate the indirect cost, we find:

From the previous table and according to the drawn path, we explain how to calculate the indirect cost:

- We start from the cell (A_1B_1) we have the cost $c_{11} = 2$, we take it with a positive sign because it is the cell of the non-basic variable.
- In the cell (A_1B_3) we have the cost $c_{13} = 0$ we take here the minus sign and the variable in this cell is definitely a basic variable
- Then to the cell (A_3B_3)we have the cost $c_{33} = 4$, we take here the sign is positive, and the variable in this cell is definitely a basic variable.
- Then to the cell (A_3B_1) we have the cost $c_{31} = 6$ we take here the minus sign, and the variable in this cell is definitely a basic variable,

therefore, the indirect cost of the non- basic variable x_{11} is:

$$x_{11}: 2 - 0 + 4 - 6 = 0$$

in the same way, we calculate the cost for all non-essential variables we find:

$$x_{12} : 4 - 0 + 4 - 2 = 6$$

$$x_{21} : \{3,4.5\} - \{1,2\} + 2 - 6 = \{-2, -3, -0.5, -1,5\}$$

$$x_{23} : \{5,8\} - 4 + 2 - \{1,2\} = \{2,1,5,4\}$$

$$x_{42} : 7 - 1 + 6 - 2 = 10$$

$$x_{43} : 9 - 1 + 6 - 4 = 10$$

We note that the indirect cost corresponding to the basic variable x_{21} is a negative amount and it is the only one, so we enter this variable, and it becomes one of the basic variables and we exit instead of x_{31} .

We notice that we can pass the quantity $x_{21} = 155$, then it becomes:

$$x_{31} = 0,\ x_{32} = 275\ , x_{22} = 45$$

We get the following table:

consumption center / production centers	B_1	B_2	B_3	Available quantities
A_1	2	4	0 150	150
A_2	{3,4.5} 155	{1,2} 45	{5,8}	200
A_3	6	2 275	4 50	325
A_4	1 25	7	9	25
required quantities	180	320	200	700 / 700

Table No (4) the first improvement

We note that the transportation cost is according to the previous solution:

$$NZ_2 \in (0 \times 150 + \{3,4.5\} \times 155 + \{1,2\} \times 45 + 2 \times 275 + 4 \times 50 + 1 \times 25)$$

For $c_{21} = 3 \; and \; c_{22} = 1 \Rightarrow Z_2 = 1285$

For $c_{21} = 3 \; and \; c_{22} = 2 \Rightarrow Z_2 = 1330$

For $c_{21} = 4.5 \; and \; c_{22} = 1 \Rightarrow Z_2 = 1517.5$

For $c_{21} = 4.5 \; and \; c_{22} = 2 \Rightarrow Z_2 = 1562.5$

Therefore:

$$NZ_2 \in \{1285 , 1330 , 1517.5 ,1562.5 \}$$

$$\forall \, NZ_2 \in \{1285 , 1330 , 1517.5 ,1562.5 \}$$
$$\Longrightarrow NZ_2 < NZ_1 \in \{1595,1795\}$$

This solution is better than the previous one, the question now is whether this solution is the optimal solution, for this we define the basic variables and the non-basic variables we find:

The basic variables are:

$$x_{41} , x_{33} , x_{32} , x_{22} , x_{21} , x_{13}$$

The non- basic variables are:

$$x_{43} , x_{42} , x_{31} , x_{23} , x_{12} , x_{11}$$

We have six basic variables and six non-basic variables, so we get six closed paths; we form the closed paths for the six non- basic variables:

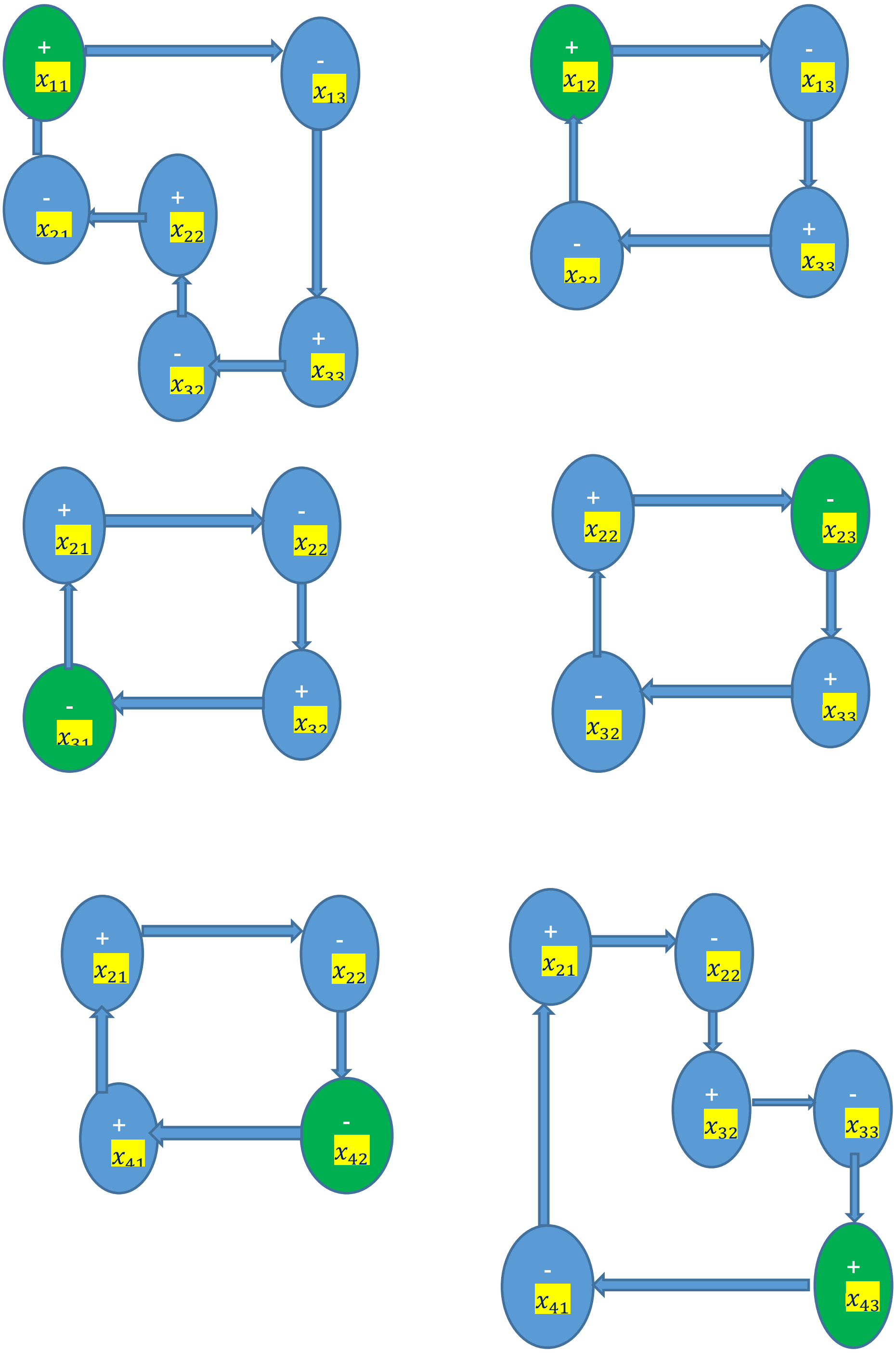


Figure No. (3) Possible closed paths after the first optimization
The non- basic variables green

We calculate the indirect cost:

$$x_{11} : 2 - 0 + 4 - \{1,2\} + 1 - 3 = \{3,2\}$$
$$x_{12} : 4 - 0 + 4 - 2 = 8$$
$$x_{32} : 6 - 2 + \{1,2\} - \{3,4.5\} = \{2,0.5,3,1.5\}$$
$$x_{42} : 7 - \{1,2\} + \{3,4.5\} - 1 = \{8,9.5,7,8.5\}$$
$$x_{43} : 9 - 4 + 2 - \{1,2\} + \{3,4.5\} - 1 = \{8,9.5,7,8.5\}$$

We note that the indirect cost for each non- basic variable is positive, and therefore we cannot introduce any non- basic variable to the basic rule, i.e., the solution that we obtained in the first improvement is an optimal vinegar, and the minimum transportation cost is the one that we obtained previously:

Therefor the optimal solution is:

$x_{13} = 150$, $x_{21} = 155$, $x_{22} = 45$, $x_{32} = 275$, $x_{33} = 50$, $x_{41} = 25$

The minimum cost of transportation is:

$$NZ_2 \in \{1285 , 1330 , 1517.5 ,1562.5 \}$$

It is a neutrosophic value that can be any element of the set,

$$\{1285 , 1330 , 1517.5 ,1562.5 \}$$

The second method:

3.3. Modified Distribution Method:

This method is another method of finding the optimal solution for transportation issues, and it is similar to the previous method (the mobile stone method). Conjunction.

To find the optimal solution to the transportation issue according to this method, we use the following steps:

1. We find the initial solution in one of the previously mentioned ways.
2. We define the essential variables and non-basic variables for the solution.
3. We associate with each row i multiplied by u_i, and with each column j multiplied, we call it v_j, so it is:

   For each basic variable, x_{ij} we have:

$$u_i + v_j = c_{ij} \qquad (*)$$

Where c_{ij} is the cost from A_i to B_j:

Since the number of basic variables is $m + n - 1$, we get the $m + n - 1$ equation from the previous figure $(*)$ and by solving these equations we must find the values of u_i, v_j which have $m + n$, so we must give one of these multipliers an optional value, Then we solve these equations according to this value.

After we find the values u_i, v_j, for each non-basic variable x_{ij} we calculate the quantities:

$$c_q = c_{ij} - u_i - v_j$$

In a similar way to the moving stone method, but if one of these quantities is negative, then we must introduce a non-basic variable to the set of basic variables and output instead a basic variable, and the primary variable entered is chosen in the same previous way:

Example:

Let's take the previous example, where we found the preliminary solution according to the cost method:

consumption center / production centers		v_1	v_2	v_3	Available quantities
		B_1	B_2	B_3	
u_1	A_1	2	4	0 / 150	150
u_2	A_2	{3,4.5}	{1,2} / 200	{5,8}	200
u_3	A_3	6 / 155	2 / 120	4 / 50	325
u_4	A_4	1 / 25	7	9	25
required quantities		180	320	200	700 / 700

Table No (5) the preliminary solution

Transportation cost is: $Z_1 = 1595$

Basic variables:

$$x_{13}, x_{22}, x_{31}, x_{32}, x_{33}, x_{41}$$

Non- basic variables:

$$x_{11}, x_{12}, x_{21}, x_{23}, x_{42}, x_{43}$$

Multipliers is u_i ; i = 1,2,3,4 and v_j ; $j = 1,2,3$.

For basic variables, we have:

For x_{13}, we have $u_1 + v_3 = 0$

For x_{22}, we have $u_2 + v_2 \in \{1,2\}$

For x_{31}, we have $u_3 + v_1 = 6$

For x_{32}, we have $u_3 + v_2 = 2$

For x_{33}, we have $u_3 + v_3 = 4$

For x_{41}, we have $u_4 + v_1 = 1$

It is six equations with seven unknowns. To solve them, we impose $u_1 = 0$, so we find the rest of the variables:

$$u_1 = 0\,, u_2 = 3\,, u_3 = 4\,, \qquad u_4 = -1$$
$$v_1 = 2\,, v_2 = -2\,, v_3 = 0$$

For non-basic variables, we have:

$$x_{11}\,, x_{12}\,, x_{21}\,, x_{23}\,, x_{42}\,, x_{43}$$

For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 2 = 0$

For x_{12} it is: $\bar{c}_{12} = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$

For x_{21} it is: $\overline{Nc}_{21} \in (c_{21} - u_2 - v_1) = \{3{,}4.5\} - 3 - 2 = \{-2\,, -3.5\}$

For x_{23} it is: $\overline{Nc}_{23} \in (c_{23} - u_2 - v_3) = \{5{,}8\} - 3 - 0 = \{2\,, 5\}$

For x_{42} it is: $\bar{c}_{42} = c_{42} - u_4 - v_2 = 7 + 1 + 2 = 10$

For x_{43} it is: $\bar{c}_{43} = c_{43} - u_4 - v_3 = 9 + 1 - 0 = 10$

We note that the quantity $\overline{Nc}_{21} \in \{-2, -3.5\}$ is a negative value, and therefore the initial solution that we got is not an optimal solution, and we must develop this solution, and for that, we form the closed path for the non -basic variable x_{21} , so we find it from the shape:

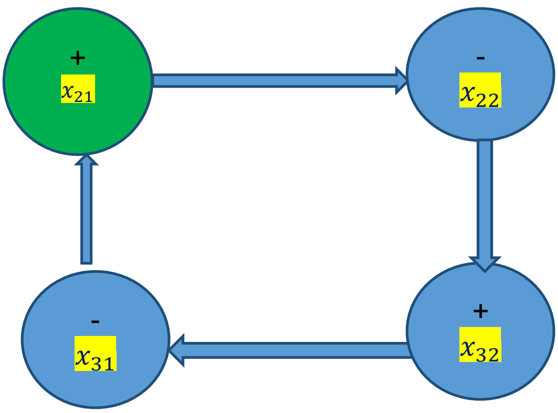


Figure 4: Possible closed pathways for the non-basic variable x_{21}

We enter x_{21} into the set of basic variables, by giving it the value $x_{21} = 155$, and we remove the variable x_{31} so it becomes a non-basic variable, and then it becomes $x_{22} = 45$ and $x_{32} = 275$, so we get the following table:

consumption center / production centers		v_1	v_2	v_3	Available quantities
		B_1	B_2	B_3	
u_1	A_1	2	4	0 150	150
u_2	A_2	{3,4.5} 155	{1,2} 45	{5,8}	200
u_3	A_3	6	2 275	4 50	325
u_4	A_4	1 25	7	9	25
required quantities		180	320	200	700 700

Table No (6) the first improvement

The new transfer cost is:

$$NZ_2 \in \{1285\,, 1330\,, 1517.5\,, 1562.5\,\}$$

$$\forall\, NZ_2 \in \{1285\,, 1330\,, 1517.5\,, 1562.5\,\} \Longrightarrow NZ_2 < NZ_1 \in \{1595, 1795\}$$

This solution is better than the previous solution, but is it the optimal solution?

For basic variables, we have:

For x_{13} we have $u_1 + v_3 = 0$

For x_{21} we have $u_2 + v_1 = 3$

For x_{22} we have $u_2 + v_2 = 1$

For x_{32} we have $u_3 + v_2 = 2$

For x_{33} we have $u_3 + v_3 = 4$

For x_{41} we have $u_4 + \ v_1 = 1$

We assume $u_1 = 0$ and solve the system of equations we find:

$$u_1 = 0 \ , u_2 = 3 \ \ , u_3 = 4 \ , u_4 = 1$$
$$v_3 = 0 \ \ , v_2 = -2 \ \ , \ \ , v_1 = 0 \ ,$$

For non-basic variables:

For x_{11} it is: $\bar{c}_{11} = c_{11} - u_1 - v_1 = 2 - 0 - 0 = 2$
For x_{12} it is: $\bar{c}_{12} \ = c_{12} - u_1 - v_2 = 4 - 0 + 2 = 6$
For x_{31} it is: $\bar{c}_{31} = c_{31} - u_3 - v_1 = 6 - 4 - 0 = 2$
For x_{23} it is: $\overline{Nc}_{23} \ \in (c_{23} - u_2 - v_3) = \{5,8\} - 3 - 0 = \{2,5\}$
For x_{42} then: $\bar{c}_{42} \ = c_{42} - u_4 - v_2 = 7 - 1 + 2 = 8$
For x_{43} it is: $\bar{c}_{43} \ = c_{43} - u_4 - v_3 = 9 - 1 - 0 = 8$

We note that all quantities $\bar{c}_{ij}$ are positive quantities, so the solution that we got is an optimal solution, and the minimum cost of transportation is:

$$NZ_2 \in \{1285 \ , 1330 \ , 1517.5 \ ,1562.5 \ \}$$

$$\forall \ NZ_2 \in \{1285 \ , 1330 \ , 1517.5 \ ,1562.5 \ \} \Longrightarrow NZ_2 < NZ_1 \in \{1595,1795\}$$

Therefor the optimal solution is:

$x_{13} = 150$, $x_{21} = 155$, $x_{22} = 45$, $x_{32} = 275$, $x_{33} = 50$, $x_{41} = 25$

The minimum cost of transportation is:

$$NZ_2 \in \{1285 , 1330 , 1517.5 ,1562.5 \}$$

It is a neutrosophic value that can be any element of the set,

$$\{1285 , 1330 , 1517.5 ,1562.5 \}$$

Chapter IV

The fourth chapter shortest time neutrosophic transport models

4.1. Introduction:

The time factor plays an important role in many issues, and the most important of these issues is the issue of transportation. When we need to transport perishable materials such as milk, medicines, blood... or develop military plans to secure battle necessities of ammunition, food, and soldiers... At maximum speed, we need an accurate scientific study to be able to avoid losses, so the researchers studied transportation models in the shortest possible time using classical values, and the ideal solution for such models is a specific value that is subject to increase or decrease because nothing is certain in real reality. The results of the studies are related to the circumstances surrounding the system under study. Given the sensitivity of these issues, they had to be reformulated according to science that accounts for all the situations that the system may go through so that we can take all possible precautions that help us reduce losses and secure what is required in the shortest possible time.

4.2. Formulate the neutrosophic transport problem in the shortest time:

Text of the issue:

Model of transporting materials in the shortest possible time using neutrosophic values:

Based on the study contained in the research [19], we can formulate the issue as follows:

The text of the issue in general form:

Suppose we want to move a material from production centers A_i where $i = 1,2,\dots,m$,whose production capacities are respectively

$Na_1, Na_2, \dots, Na_m$, to consumption centers B_j where $j = 1,2,\dots,n$ whose needs are $Nb_1, Nb_2, \dots, Nb_n$ It is in order, and let the matrix of times necessary to transfer the appropriate quantity from the center i to the center j be known and equal $NT = [Nt_{ij}]$, it is required to formulate the appropriate mathematical model to transfer all the quantities ,available in the production centers and meet the needs of all consumption centers in the shortest time. To build the appropriate mathematical model, we denote Nx_{ij} for the quantity transferred from the production center i where $i = 1,2,3, \dots, m$ to the consumption center j where $= 1,2,3, \dots, n$, then we can put the problem unknowns in matrix form. $NX = [Nx_{ij}]$, and we put the information in the question in a table as follow:

consumption / production	B_1	B_2	B_3	…	B_n	quantities
A_1	Nt_{11} Nx_{11}	Nt_{12} Nx_{12}	Nt_{13} Nx_{13}	…	Nt_{1n} Nx_{1n}	Na_1
A_2	Nt_{21} Nx_{21}	Nt_{22} Nx_{22}	Nt_{23} Nx_{23}	…	Nt_{2n} Nx_{2n}	Na_2
A_3	Nt_{31} Nx_{31}	Nt_{32} Nx_{32}	Nt_{33} Nx_{33}	…	Nt_{3n} Nx_{3n}	Na_3
…	…	…	…	…	…	…
A_m	Nt_{m1} Nx_{m1}	Nt_{m2} Nx_{m2}	Nt_{m3} Nx_{m3}	…	Nt_{mn} Nx_{mn}	Na_m
Required quant	Nb_1	Nb_2	Nb_3	…	Nb_n	

Table No. (1) Data of the issue of transport in the shortest time

4.3. Building the mathematical model:

To build the appropriate mathematical model, we distinguish two cases:

4.3.1. the model is balanced:

The model is balanced if:

$$\sum_{i=1}^{m} Na_i = \sum_{j=1}^{n} Nb_j$$

4.3.2. Unbalanced Model:

The model is unbalanced if:

$$\sum_{i=1}^{m} Na_i \neq \sum_{j=1}^{n} Nb_j$$

From this case, two cases result:

4.3.2.1. Overproduction:

$$\sum_{i=1}^{m} Na_i \geq \sum_{j=1}^{n} Nb_j$$

This model is returned to a balanced model by adding an imaginary consumer center that needs it:

$$Nb_{n+1} = \sum_{i=1}^{m} Na_i - \sum_{j=1}^{n} Nb_j$$

4.3.2.2. Deficit in production:

This model is returned to a balanced model by adding a fictitious production center with a production capacity:

$$Na_{m+1} = \sum_{j=1}^{n} Nb_i - \sum_{i=1}^{m} Na_i$$

In both cases$(b\ \&\ a)$, a case of surplus production and a deficit in production, we get a balanced model.

4.4. Formulation of the mathematical model of transport models in the shortest possible time:

Our symbol for the quantity transferred from the center i to the center j with the symbol Nx_{ij} then these variables must meet the following conditions:

$$\sum_{j=1}^{n} Nx_{ij} = \mathrm{N}a_i \qquad (i = 1,2,3,\ldots,m)$$

$$\sum_{i=1}^{m} Nx_{ij} = Nb_j \qquad (j = 1,2,3,\ldots,n)$$

$$Nx_{ij} \geq 0 \quad (i = 1,2,3,\ldots,m)\,,(j = 1,2,3,\ldots,n)$$

In these models, the objective function cannot be formulated with a mathematical follower, so we extract its most important qualities and properties through the following discussion:

To find the optimal solution for any transport model, we must find the values of unknowns:

$$Nx_{ij}\ ;\ (i = 1,2,3, \dots, m)\ , (j = 1,2,3, \dots, n)$$

In the second chapter, we presented methods for finding the initial solution to transportation issues, we know that any optimal solution is includes n+m-1 basic variable that are not equal to zero and against this solution there is a set of times that we will symbolize as $[Nt_{ij}]_X$.

It represents the time required to transport all materials available in all production centers and meet the needs of all consumption centers.

The time required to finish the transfer process, which we will symbolize as corresponding to the largest element of the matrix $[Nt_{ij}]_X$ must achieve the following relationship:

$$Nt_x = Max_{i,j}[Nt_{ij}]_X$$

Since we have many acceptable solutions, the optimal solution is given by the following relationship:

$$Nt_x^* = MinNt_x = Min(Max_{i,j}[Nt_{ij}]_X)$$

This means that we solve the model without a target function we get a base solution and then we determine the number set from the matrix $[Nt_{ij}]_X$ corresponding to this base solution.

Note 1:

If the issue is unbalanced and when adding an imaginary production center or an imaginary consumer center, we determine the time according to the following:

Since the time required to finish the transfer process achieves the following relationship:

$$Nt_x = Max_{i,j}[Nt_{ij}]_X$$

So we take the time required to transfer the quantities available in this imaginary production center to all consumption centers equal to zero.

And we take the time required to transport quantities from all production centers to the imaginary consumer center is equal to zero.

Note 2:
For the transport model to be a neutrosophic transport model, at least one of the data in Table 1 must be a neutrosophic value.

4.5. A special method to find the optimal solution for neutrosophic transport models in the shortest time:

The general method used to obtain the smallest transfer time is to move from one neutrosophic base solution to another base solution using the simplex method.

and the goal is to make the largest elements Nt_x in the matrix $NT = \left[Nt_{ij}\right]_X$ as small as possible.

In this chapter, we will use a special method to solve neutrosophic transport models according to time, which we explain through the following example:

Example:

Four pharmaceutical plants distribute their production of one type to three pharmacies the available quantities, the quantities required, and the times required to transport them are shown in the following table:

Consumption center / **Production centers**	B_1	B_2	B_3	**Required quantities**
A_1	[1 ,1.5] Nx_{11}	[2 ,2.4] Nx_{12}	6 Nx_{13}	11
A_2	[3 ,3.2] Nx_{21}	8 Nx_{22}	[1 ,1.5] Nx_{23}	9
A_3	[7 ,7.5] Nx_{31}	10 Nx_{32}	[4 ,4.6] Nx_{33}	13
A_4	12 Nx_{41}	8 Nx_{42}	[5 ,5.1] Nx_{43}	17
Available quantity	18	10	12	40 / 40

Table No. (2) Example data

We look for the smallest time in the cells (i,j), we find that it is found in two cells $(1,1)$ and $(3,2)$:

$$Min(Nt_{ij)} = Nt_{11} = Nt_{32} \in [1,1.5]$$

We denote b.i Ω_1 for all table stone except the two cells $(1,1)$ and $(2,3)$

We saturate the two chambers opposite them $(1,1)$ and $(2,3)$ then we put in the other stone $(*)$ we get the following table:

consumption / **production**				**Required quantities**
A_1	11	*	*	11
A_2	*	*	9	9
A_3	*	*	*	13
A_4	*	*	*	7
Available quantities	18	10	12	40 / 40

Table No. (3) First Step

The first solution is $x_{11}^{(1)} = 11\,, x_{23}^{(1)} = 9$ which expresses the total quantities transferred and is equal to $x_{ij} = 11 + 9 = 20$ it crosses a quantity less than the quantity required for that time solution from which we started and the author of $Nt_{11} = Nt_{23} \in [1,1.5]$ is an imperfect solution to the problem at hand.

From the elements of the sat Ω_1, where Ω_1 equals:

$$\Omega_1 = \{\, [3\,,3.2], [7\,,7.5], 12, [2\,,2.4], 8,10,8,6, [4\,,4.6], [5\,,5.1] \,\}$$

We look for the smallest time we find:

$$\min_{\Omega_1}\left(Nt_{ij}\right) = Min\{[3\,,3.2], [7\,,7.5], 12, [2\,,2.4], 8,10,8,6, [4\,,4.6], [5\,,5.1]\} \\ \in [2\,,2.4]$$

Any smallest time is this $Nt_{12} \in [2\ ,2.4]$ means that we must transfer the quantities available in the production center A_1 to the consumption center B_2 and this is not possible because the center A_1 no longer contains any quantity and therefore this step is not useful.

We form the sat:

$$\Omega_2 = \{\ [3\ ,3.2], [7\ ,7.5], 12{,}8{,}10{,}8{,}6, [4\ ,4.6], [5\ ,5.1]\ \}$$

Is the resulting Ω_1 after deleting $Nt_{12} \in [2\ ,2.4]$ and we choose from Ω_2 the smallest time we find:

$$\min_{\Omega_2}\left(Nt_{ij}\right) = Min\{[3\ ,3.2], [7\ ,7.5], 12{,}8{,}10{,}8{,}6, [4\ ,4.6], [5\ ,5.1]\} \in [3\ ,3.2]$$

Any smaller time is this $Nt_{21} \in [3\ ,3.2]$,means that we have to transfer the quantities available in the production center A_2 to the consumption center B_1 and this is not possible because the center A_2 no longer contains any quantity and therefore this step is also not useful.

We form:

$$\Omega_3 = \{[7\ ,7.5], 12{,}8{,}10{,}8{,}6, [4\ ,4.6], [5\ ,5.1]\ \}$$

The sat, which is the resulting, sat Ω_2 after deleting $Nt_{21} \in [3\ ,3.2]$ and choosing from the Ω_3 smallest time we find:

$$\min_{\Omega_3}\left(Nt_{ij}\right) = Min\{[7\ ,7.5], 12{,}8{,}10{,}8{,}6, [4\ ,4.6], [5\ ,5.1]\} \in [4\ ,4.6]$$

Any smallest time is $Nt_{33} \in [4\ ,4.6]$ this means that we must transfer the quantities available in the production center A_3 to the consumption center B_3 i.e. we put $x_{33} = 12$ and therefore we must shift the amount in the cell (2,3) with the same time [1 ,1.5] to a cell with a time immediately followed and located in the same row i.e. to the cell (2,1) with the same time [3 ,3.2] and we put $x_{21} = 9$.

Then we make a balance process for the quantities that have been distributed and here we must reduce the quantity in the cell (1,1) becomes $x_{11} = 9$ and we put the quantity that has been reduced in the cell with the lowest time immediately following the time in that cell and in the same row, i.e in the cell (1,2) we get $x_{12} = 2$ from this step we get the following distribution:

$$x_{11} = 9 ، \ x_{12} = 2 , x_{21} = 9 , x_{33} = 12$$

But this solution is not ideal because the sum of the quantities that were distributed is not equal to the 40 total quantities available, that is, the time $Nt_{33} \in [4 ,4.6]$ Not the shortest time, we proceed in the same way in the eighth time to reach the distribution shown in the following table:

consumption / production	$\boldsymbol{B_1}$	$\boldsymbol{B_2}$	$\boldsymbol{B_3}$	**Required amounts**
A_1	1	10	$*$	11
A_2	9	$*$	$*$	9
A_3	8	$*$	5	13
A_4	$*$	$*$	7	7
Available quantities	18	10	12	40 / 40

Table No. (4) Optimal Solution

In return for $Min\left(Nt_{ij}\right) = Nt_{31} \in [7 ,7.5]$ this time, the entire quantities available in the production centers have been transferred and the needs of all consumer centers have been met, the ideal solution is:

$$x_{11} = 1 , \ x_{21} = 9 , \ x_{31} = 8 , \ x_{12} = 10 , \ x_{33} = 5 , x_{43} = 7$$

The rest of the variables is equal to zero, and the shortest time is:

$$Nt^{*} = Nt_{31} \in [7 ,7.5]$$

It should be noted that the same example was presented and solved according to classical logic, and the data of the problem were as in the following table:

The available quantities, the quantities required, and the times required for their transportation are shown in the following table:

production \ consumption	B_1	B_2	B_3	Required amounts
A_1	1 Nx_{11}	2 Nx_{12}	6 Nx_{13}	11
A_2	3 Nx_{21}	8 Nx_{22}	1 Nx_{23}	9
A_3	7 Nx_{31}	10 Nx_{32}	4 Nx_{33}	13
A_4	12 Nx_{41}	8 Nx_{42}	5 Nx_{43}	7
Available amounts	18	10	12	40 \ 40

Table No. (5) Example data according to classical logic

The optimal solution was as follows:

Time $Min(t_{ij}) = t_{31} = 7$ and in exchange for this time, the entire quantities available in the production centers have been transferred and the needs of all consumer centers have been met, the best solution is:

$$x_{11} = 1 \ , x_{21} = 9 \ , \ x_{31} = 8 \ , \ x_{12} = 10 \ , x_{33} = 5 \ , x_{43} = 7$$

The rest of the variables is equal to zero, and the shortest time is

$t^* = t_{31} = 7$

Chapter V
Optimal allocation Neutrosophic and Hungarian method

5.1. Introduction.

5.2. Standard allocation issues:

5.3. Formulation of the standard Neutrosophic assignment problem of the minimization type.

5.4. Building a mathematical model for the standard allocation problem (data are classical values).

5.5. Building a mathematical model for the standard allocation problem (data are neutrosophic values).

5.6. Hungarian neutrosophic Method.

5.7. Standard allocation problem and cost neutrosophic values.

5.8. Steps of the Hungarian method.

5.9. Important notes.

5.1. Introduction:

Allocation issues are a special case of linear programming issues concerned with the optimal allocation of various economic, productive, and human resources to the various work to be accomplished, and we encounter them frequently in practical life in educational institutions - hospitals - construction projects... etc. To obtain an optimal allocation that achieves the greatest profit and the least loss in all conditions that the working environment can pass through. In this chapter, we present the issue of optimal neutrosophic allocation and the Hungarian neutrosophic method.

5.2. Standard assignment issues:

In these issues, the number of machines or people equals the number of works, which we will address in this research.

5.3. Text of the minimum cost type neutrosophic normative assignment problem:

If we have n machines, we denote them by $M_1, M_2, \dots\dots, M_n$ and we have a set of works consisting of n different work we denote them by $N_1, N_2, \dots\dots, N_n$ we want to designate the machines to do these jobs, cost of doing any work j on the device i, it is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$. Assuming any machine can do only one job, it is required to find the optimal assignment so that the cost is as small as possible.

Formulation of the mathematical model:

To formulate the linear mathematical model, we assume:

$$x_{ij=} \begin{cases} 1 & if\ job\ j\ was\ given\ to\ machine\ i \\ 0 & otherwise \end{cases}$$

Then write the target function as follows:

$$NZ = \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} \pm \varepsilon_{ij}) x_{ij}$$

Conditions for machines:

Since each machine accepts only one action, we find:

$$\sum_{j=1}^{n} x_{ij} = 1 \quad ; i = 1,2, \dots \dots, n$$

Business terms:

Since each work is assigned to only one machine, we find:

$$\sum_{i=1}^{n} x_{ij} = 1 \quad ; j = 1,2, \dots \dots, n$$

Accordingly, the neutrosophic mathematical model is written as follows:

Find the minimum value:

$$NZ = \sum_{i=1}^{n} \sum_{j=1}^{n} (c_{ij} \pm \varepsilon_{ij}) x_{ij}$$

Machine terms:

$$\sum_{j=1}^{n} x_{ij} = 1 \quad ; i = 1,2, \dots \dots, n$$

Business terms:

$$\sum_{i=1}^{n} x_{ij} = 1 \quad ; j = 1,2, \dots \dots, n$$

5.4. Formulation of the mathematical model for the problem of standard assignment of minimum cost:

Example 1: (The data are classic values).

We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table:

Business / **The machines**	N_1	N_2	N_3	N_4
M_1	10	9	8	7
M_2	3	4	5	6
M_3	2	1	1	2
M_4	4	3	5	6

Table No. (1) Table of Distribution cost table and classic values

To formulate the linear mathematical model:

We impose:

$$x_{ij} = \begin{cases} 1 & if\ job\ j\ was\ given\ to\ machine\ i \\ 0 & otherwise \end{cases} \quad ;\ i,j = 1,2,3,4$$

Using the problem data, we get the following objective function:

$$Z = 10x_{11} + 9x_{12} + 8x_{13} + 7x_{14} + 3x_{21} + 4x_{22} + 5x_{23} + 6x_{24} + 2x_{31} + x_{32} + x_{33} + 2x_{34} + 4x_{41} + 3x_{42} + 5x_{43} + 6x_{44}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:

Find the minimum value of the function:

$$Z = 10x_{11} + 9x_{12} + 8x_{13} + 7x_{14} + 3x_{21} + 4x_{22} + 5x_{23} + 6x_{24} + 2x_{31} + x_{32} + x_{33} + 2x_{34} + 4x_{41} + 3x_{42} + 5x_{43} + 6x_{44}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

In the previous model, there is some indeterminacy in the assignment process, as we do not know which machine will perform a certain work. In addition to that, we will also use neutrosophic data. We will take the cost of neutrosophic values, i.e. the cost of assignment machine i to perform work j is $Nc_{ij} \in c_{ij} \pm \varepsilon_{ij}$, where ε_{ij} is the indeterminacy and $\varepsilon_{ij} \in [\lambda_{\mathrm{ij1}}, \lambda_{\mathrm{ij2}}]$, which is any neighborhood to the value c_{ij} then the cost matrix becomes $NC_{ij} = [c_{ij} \pm \varepsilon_{ij}]$.

5.5. Building a mathematical model for the standard allocation problem (data are neutrosophic values):

Example 2: (Cost is neutrosophic values):

We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table**:**

Business / The machines	N_1	N_2	N_3	N_4
M_1	$10+\varepsilon_{11}$	$9+\varepsilon_{12}$	$8+\varepsilon_{13}$	$7+\varepsilon_{14}$
M_2	$3+\varepsilon_{21}$	$4+\varepsilon_{22}$	$5+\varepsilon_{23}$	$6+\varepsilon_{24}$
M_3	$2+\varepsilon_{31}$	$1+\varepsilon_{32}$	$1+\varepsilon_{33}$	$2+\varepsilon_{34}$
M_4	$4+\varepsilon_{41}$	$3+\varepsilon_{42}$	$5+\varepsilon_{43}$	$6+\varepsilon_{44}$

Table No. (2) Table of allocation cost of neutrosophic values

Where ε_{ij} is the limitation on the costs of assignment and it can be any neighborhood of the values contained in Table No. (1)

To formulate the linear mathematical model, we assume:

$$x_{ij} = \begin{cases} 1 & if\ job\ j\ was\ given\ to\ machine\ i \\ 0 & otherwise \end{cases} \qquad ; i,j = 1,2,3,4$$

Using the problem data, we get the following objective function:

$$\begin{aligned} NZ \in \{&(10+\varepsilon_{11})x_{11} + (9+\varepsilon_{12})x_{12} + (8+\varepsilon_{13})x_{13} + (7+\varepsilon_{14})x_{14} \\ &+ (3+\varepsilon_{21})x_{21} + (4+\varepsilon_{22})x_{22} + (5+\varepsilon_{23})x_{23} + (6 \\ &+ \varepsilon_{24})x_{24} + (2+\varepsilon_{31})x_{31} + (1+\varepsilon_{32})x_{32} + (1+\varepsilon_{33})x_{33} \\ &+ (2+\varepsilon_{34})x_{34} + (4+\varepsilon_{41})x_{41} + (3+\varepsilon_{42})x_{42} + (5 \\ &+ \varepsilon_{43})x_{43} + (6+\varepsilon_{44})x_{44}\} \end{aligned}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:

Find the minimum value of the function:

$$\begin{aligned} NZ \in \{&(10 + \varepsilon_{11})x_{11} + (9 + \varepsilon_{12})x_{12} + (8 + \varepsilon_{13})x_{13} + (7 + \varepsilon_{14})x_{14} \\ &+ (3 + \varepsilon_{21})x_{21} + (4 + \varepsilon_{22})x_{22} + (\ 5 + \varepsilon_{23})x_{23} + (6 \\ &+ \varepsilon_{24})x_{24} + (2 + \varepsilon_{31})x_{31} + (1 + \varepsilon_{32})x_{32} + (1 + \varepsilon_{33})x_{33} \\ &+ (2 + \varepsilon_{34})x_{34} + (4 + \varepsilon_{41})x_{41} + (3 + \varepsilon_{42})x_{42} + (5 \\ &+ \varepsilon_{43})x_{43} + (6 + \varepsilon_{44})x_{44}\} \end{aligned}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

Since the number of works is equal to the number of machines, the issue is a standard assignment issue, and the optimal solution can be obtained using several methods, including the Hungarian method in this research.

This method was named after the scientist who created it, a mathematician D. Konig. Its principle depends on finding the total opportunity-cost matrix, references .

5.6. Hungarian neutrosophic Method:

Explanation of the method based on what was stated in the reference:

This method is based on a mathematical property discovered by the scientist D. Konig.

If the cost is non-negative values, then subtracting or adding a fixed number of elements of any row or any column in the standard allocation cost matrix does not affect the optimal assignment, and specifically does not affect the optimal values x_{ij} .

The algorithm begins by identifying the smallest element in each row and subtracting it from all the elements of the row, or by selecting the smallest element in each column and subtracting it from all the elements of that column, we get a new cost matrix that includes at least one element equal to zero in each row or column. We do the assignment process using cells with a cost equal to zero. If possible, we have obtained the optimal allocation. For this assignment, the cost elements (c_{ij}) are non-negative, so the minimum value of the objective function cannot be

$\sum_{i=1}^{n} \sum_{j=1}^{n} c_{ij}\, x_{ij}$ is less than zero.

We will use the above to find the optimal assignment for the problem in Example 2 based on the following information:

Taking the indeterminacy $\varepsilon_{ij} = \varepsilon \in [0\,, 5]$, the problem becomes:

5.7. Standard allocation problem and cost neutrosophic values:

Example 3:

We want to find the optimal assignment for four jobs on four machines. The cost of assignment is given in the following table:

Business / The machines	N_1	N_2	N_3	N_4
M_1	[10,15]	[9 ,14]	[8 ,13]	[7 ,12]
M_2	[3 , 8]	[4 , 9]	[5 ,10]	[6 ,11]
M_3	[2 , 7]	[1,6]	[1,6]	[2 , 7]
M_4	[4 , 9]	[3 , 8]	[5 ,10]	[6 ,11]

Table No. (3) Table of Example data

To formulate the linear mathematical model:

We assume:

$$x_{ij} = \begin{cases} 1 & if\ job\ j\ was\ given\ to\ machine\ i \\ 0 & otherwise \end{cases} \quad ; i,j = 1,2,3,4$$

Using the problem data, we get the following objective function:

$$\begin{aligned} NZ \in \{&[10,15]x_{11} + [9\ ,14]x_{12} + [8\ ,13]x_{13} + [7\ ,12]x_{14} + [3\ ,8]x_{21} \\ &+ [4\ ,9]x_{22} + [5\ ,10]x_{23} + [6\ ,11]x_{24} + [2\ ,7]x_{31} \\ &+ [1\ ,6]x_{32} + [1\ ,6]x_{33} + [2\ ,7]x_{34} + [4\ ,9]x_{41} \\ &+ [3\ ,8]x_{42} + [5\ ,10]x_{43} + [6\ ,11]x_{44}\} \end{aligned}$$

Machine terms:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$

Business terms:

$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Therefore, the mathematical model is written as follows:

Find the minimum value of the function:

$$NZ \in \{[10,15]x_{11} + [9 ,14]x_{12} + [8 ,13]x_{13} + [7 ,12]x_{14} + [3 ,8]x_{21} + [4 ,9]x_{22} + [5 ,10]x_{23} + [6 ,11]x_{24} + [2 ,7]x_{31} + [1 ,6]x_{32} + [1 ,6]x_{33} + [2 ,7]x_{34} + [4 ,9]x_{41} + [3 ,8]x_{42} + [5 ,10]x_{43} + [6 ,11]x_{44}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} + x_{14} = 1$$
$$x_{21} + x_{22} + x_{23} + x_{24} = 1$$
$$x_{31} + x_{32} + x_{33} + x_{34} = 1$$
$$x_{41} + x_{42} + x_{43} + x_{44} = 1$$
$$x_{11} + x_{21} + x_{31} + x_{41} = 1$$
$$x_{12} + x_{22} + x_{32} + x_{42} = 1$$
$$x_{13} + x_{23} + x_{33} + x_{43} = 1$$
$$x_{14} + +x_{24} + x_{34} + x_{44} = 1$$

Where x_{ij} it is either equal to zero or one.

Solution using the Hungarian method:
From the table Number 3 we find the following:

Business / The machines	N_1	N_2	N_3	N_4
M_1	[10,15]	[9 ,14]	[8 ,13]	[7 ,12]
M_2	[3 , 8]	[4 , 9]	[5 ,10]	[6 ,11]
M_3	[2 , 7]	[1,6]	[1,6]	[2 , 7]
M_4	[4 , 9]	[3 , 8]	[5 ,10]	[6 ,11]

Table No. (4)

To form the opportunity cost matrix for the rows, we do the following:

We form the opportunity cost matrix for the rows as follows:

In the first row, the lowest cost is $[7\ ,12]$ we subtract it from all the elements of the first row.

In the second row, the lowest cost is$[3\ ,8]$, we subtract from all elements of the second row.

In the third row, the lowest cost is$[1,6]$, we subtract it from all elements of the third row.

In the fourth row, the lowest cost is$[3\ ,8]$, we subtract from all elements of the fourth row.

We get the opportunity cost matrix for the following rows:

Business / The machines	N_1	N_2	N_3	N_4
M_1	3	2	1	0
M_2	0	1	2	3
M_3	1	0	0	1
M_4	1	0	2	3

Table No. (5) Table of Total opportunity cost matrix table

We try to make the assignment using cells with cost equal to zero we find:

Dedicate the machine M_1 to get the job N_4done.
Dedicate the machine M_2 to get the job N_1done.
Dedicate the machine M_3 to get the job N_3 done.
Dedicate the machine M_4 to get the job N_2 done.

Thus, we have obtained the optimal assignment and the minimum cost:

$$NZ \in \{[10,15] \times 0 + [9\,,14] \times 0 + [8\,,13] \times 0 + [7\,,12] \times 1 + [3\,,8] \times 1 + [4\,,9] \times 0 + [5\,,10] \times 0 + [6\,,11] \times 0 + [2\,,7] \times 0 + [1,6] \times 0 + [1\,,6] \times 1 + [2\,,7] \times 0 + [4\,,9] \times 0 + [3\,,8] \times 1 + [5\,,10] \times 0 + [6\,,11] \times 0\}$$

$$NZ \in [7\,,12] + [3\,,8] + [1\,,6] + [3\,,8] = [14\,,34]$$

That is, the optimal allocation is:

Dedicate the machine M_1 to get the job N_4 done.
Dedicate the machine $M_2$to get the job N_1done.
Dedicate the machine M_3 to get the job N_3 done.
Dedicate the machine M_4 to get the job N_2 done.

The cost:

$$NZ \in [14\,,34]$$

5.8. Steps of the Hungarian method:

The Hungarian method is summarized based on what was stated in the reference:

1- We determine the smallest element in each row and subtract it from the rest of the elements of that row.
 Thus, we get a new matrix that is the opportunity cost matrix for the rows.
2- We determine the smallest element in each column of the opportunity cost matrix for the rows and
 Subtract it from the elements of that column. Thus, we get the total opportunity cost matrix.
3- We draw as few horizontal and vertical straight lines as possible to pass through all zero elements of the total opportunity cost matrix.
4- If the number of the straight lines drawn passing through the zero elements is equal to the number of rows (columns). Then we say that we have reached the optimal assignment**.**
5- If the number of straight lines passing through the zero elements is less than the number of rows
 (Columns). Then we move on to the next step.

6- We choose the lesser element from the elements that no straight line passed through and subtract it from all the elements that no straight line. Then we add it to all the elements that lie at the intersection of two lines. The elements that the straight lines passed through remain the same without any change.

We get a new matrix that we call it the modified total opportunity cost matrix.

7- We draw vertical and horizontal straight lines passing through all the zero elements in the modified total opportunity cost matrix. If the number of straight lines drawn passing through the zero elements is equal to the number of rows (columns). Then we have reached the optimal assignment solution.

8- If the number of the lines is not equal to the number of rows (columns). We go back to step (1), we
Repeat the previous steps until reaching the optimal assignment that makes the total opportunity cost equal to zero.

Example 4:

We have three machines M_1 , M_2, M_3 and three works N_1, N_2, N_3 and each work is done completely using any of the three machines and in return each machine can perform any of the three works as well. What is required is to allocate these mechanisms to the existing works so that we get the optimal assignment , i.e. the assignment that gives us here the minimum total cost, bearing in mind that the costs of completing these works vary according to the different mechanisms implemented for these works, and this cost is related to the performance of each work and is shown in the following table:

Business / The machines	N_1	N_2	N_3
M_1	[20,23]	[27,30]	[30,33]
M_2	[10,13]	[18,21]	[16,19]
M_3	[14,17]	[16,19]	[12,15]

Table No. (6) Table of allocation cost neutrosophic values example data

Mathematical model:

Find the minimum value of the function:

$$NZ \in \{[20,23]x_{11} + [27,30]x_{12} + [30,33]x_{13} + [10,13]x_{21} + [18,21]x_{22} + [16,19]x_{23} + [14,17]x_{31} + [16,19]x_{32} + [12,15]x_{33}\}$$

Within the conditions:

$$x_{11} + x_{12} + x_{13} = 1$$
$$x_{21} + x_{22} + x_{23} = 1$$
$$x_{31} + x_{32} + x_{33} = 1$$
$$x_{11} + x_{21} + x_{31} = 1$$
$$x_{12} + x_{22} + x_{32} = 1$$
$$x_{13} + x_{23} + x_{33} = 1$$

Where $\boldsymbol{x_{ij}}$ it is either equal to zero or one.

Finding the optimal assignment using the Hungarian ethod:

We take Table No. (5)

Business / The machines	N_1	N_2	N_3
M_1	[20,23]	[27,30]	[30,33]
M_2	[10,13]	[18,21]	[16,19]
M_3	[14,17]	[16,19]	[12,15]

Table No. (7)

1.

In the first row, the lowest cost is [20,23], which we subtract from all the elements of the first row.

In the second row, the least cost is [10,13] and we subtract it from all the elements of the second row.

In the third row, the lowest cost is $[12,15]$, which we subtract from all the elements of the third row.

We get the opportunity cost table for the following rows:

Business / The machines	N_1	N_2	N_3
M_1	0	7	10
M_2	0	8	6
M_3	2	4	0

Table No. (8) Table of opportunity cost matrix for lines

2.

In the first column, the lowest cost is 0 we subtract it from all the items in the first column.

In the second column, the lowest cost is 4 we subtract it from all the items in the second column.

In the third column, the lowest cost is 4 we subtract it from all the items in the third column.

We get the table:

Business / The machines	N_1	N_2	N_3
M_1	0	3	10
M_2	0	4	6
M_3	2	0	0

Table No. (9) Table of total opportunity cost matrix

3. We draw as few horizontal and vertical straight lines as possible to pass through all zero elements of the total opportunity cost matrix.
4. If the number of straight lines drawn passing through the zero elements is equal to the number of rows (columns), then we say that we have reached the optimal assignment.

5. If the number of straight lines passing through the zero elements is less than the number of rows or columns, then we move on to the third step.

Business / The machines	N_1	N_2	N_3
M_1	0	3	10
M_2	0	4	6
M_3	2	0	0

Table No. (10) Total Opportunity Cost Matrix

We Note that the number of lines is less than the number of rows (columns).

Therefore, we go to (6).

6.
   a. We choose the lowest element through which no straight line has passed. Smallest element is (3).
   b. We subtract it from the rest of the elements through which none of the lines drawn are passed.
   c. We add it to all the elements at the intersection of two straight lines drawn.
   d. Elements through which straight lines pass remain unchanged.
   e. We draw vertical and horizontal straight lines passing through all zero elements of the adjusted total opportunity cost matrix, and we get:

Business / The machines	N_1	N_2	N_3
M_1	**0**	**0**	**7**
M_2	**0**	**1**	**3**
M_3	**5**	**0**	**0**

Table No. (11) Modified Total Opportunity Cost Matrix

7. If the number of drawn lines is equal to the number of rows (columns), then we have reached the optimal assignment.
   From Table No. (8), we note that the number of drawn lines is equal to the number of rows, meaning that we have obtained the optimal assignment, which is as follows:
   In the third column, we have zero only in the cell that is M_3N_3, so the third machine is used to perform the third work. $M_3 \rightarrow N_3$
   We delete the third row and the third column, and we get the following table:

Business / The machines	N_1	N_2
M_1	0	0
M_2	0	1

Table No. (12)

In the same way. The first machine is used to perform the second work, $M_1 \rightarrow N_2$.

The second machine used to perform the first work, $M_2 \rightarrow N_1$.

The minimum total cost is:

$$NZ \in \{[20,23] \times 0 + [27,30] \times 1 + [30,33] \times 0 + [10,13] \times 1 + [18,21] \times 0 + [16,19] \times 0 + [14,17] \times 0 + [16,19] \times 0 + [12,15] \times 1\}$$

$$NZ \in [27,30] + [10,13] + [12,15] = [49,58]$$

That is, the optimal assignment is:

Machine M_1 is assigned to do N_2work.
Machine M_2 is assigned to perform N_1work.
Machine M_3 is assigned to perform N_3work.
The cost:

$$NZ \in [49\,,58]$$

5.9. Important notes:

When we study the issues of optimal assignment, we come across the following:

1. **There is two types of assignment issues according to the objective function:**

The first type:

It is required to obtain a minimum value for the objective function, knowing that the cost of completing any work by a machine is a known value, and therefore the total cost is as small as possible.

The second type:

What is required is to obtain a maximum value for the objective function, and here it is known that we have the profit accruing from the completion of any work, by a machine, and therefore the total cost is the greatest possible.

In this type, we transform matter to the first type according to the following steps:

   a. Multiply the elements of the cost matrix by the value (-1).
   b. If some elements of the matrix are negative, we add enough positive numbers to the corresponding rows and columns so that all elements become non-negative.
   c. Then the issue becomes a matter of assignment and we want to make the objective function smaller, and all elements of the cost matrix are non-negative, so we can apply the Hungarian method.

2. **There are two types of customization issues according to the number of businesses and the number of machines**:

In this research, we studied the standard assignment issues. It should be noted that there are non-standard assignment issues. In these issues, the number of works is not equal to the number of machines, and here we convert them into standard issues by adding a fictitious work or a fictitious machine and make the cost equal to zero So that the objective function does not change, then we build the mathematical model as it is in the standard models.

References

1- Alali. Ibrahim Muhammad, Operations Research. Tishreen University Publications, 2004. (Arabic version).

2- Bugaha J.S, Mualla.W , Nayfeh.M , Murad.H , Al-Awar.M.N - Operations Research Translator into Arabic,The Arab Center for Arabization, Translation, Authoring and Publishing, Damascus,1998.(Arabic version).

3- Al Hamid .M, Mathematical programming, Aleppo University, Syria, 2010. (Arabic version).

4- Maissam Jdid, Operations Research, Faculty of Informatics Engineering, Al-Sham Private University Publications, 2021. (Arabic version).

5- Wael Khansa- Ola Abu Amsha. Operations Research (1), Faculty of Informatics Engineering - Damascus University Publications, 2005.

6- DavidG. Luenbrgrr.YinyuYe, Linear and Nonlinear Programming, Springer Science + Business Media-2015.

7- Zadeh, L. A., Fuzzy Sets. Inform. Control 8 (1965).

8- Smarandache, F., Introduction to Neutrosophic statistics, Sitech & Education Publishing, 2014.

9- Maissam Jdid, AA Salama, Huda E Khalid ,Neutrosophic Handling of the Simplex Direct Algorithm to Define the Optimal Solution in Linear Programming ,International Journal of Neutrosophic Science, Vol.18,No. 1, 2022

10- Maissam Jdid, Huda E Khalid, Mysterious Neutrosophic Linear Models, International Journal of Neutrosophic Science, Vol.18,No. 2, 2022

11- Maissam Jdid, Huda E Khalid, An Investigation in the Initial Solution for Neutrosophic Transportation Problems (NTP),Neutrosophic sets and Systems NSS, Vol.50,2022

12- Maissam Jdid, Huda E. Khalid, Neutrosophic Mathematical formulas of Transportation Problems, Neutrosophic sets and Systems ,NSS , Vol .51,2022

13- Maissam Jdid, The Use of Neutrosophic linear Programming Method in the Field of Education, Handbook of Research on the Applications of Neutrosophic Sets Theory and Their Extensions in Education, Chapter 15, IGI-Global,2023

14- Maissam Jdid, Florentin Smarandache, Optimal Neutrosophic Assignment and the Hungarian Method, Neutrosophic Sets and Systems, NSS, Vol.57,2023

15- Maissam Jdid, Studying Transport Models with the Shortest Time According to the Neutrosophic Logic, Journal Neutrosophic Sets and Systems, NSS Vol. 58, 2023.

Scientific Reviewer

Dr. Mountajab Al-Hasan

Faculty of Science, Al-Baath University , Homs, Syria

Email: alhasanmountajab@gmail.com

Date of Birthday: 28 November 1966

Scientific Certificate: PHD in Mathematical Science (Mechanics of Continuous Media))From Wroclaw University of Science and Technology-Poland

Current Work: Professor in Department of Mathematics -Faculty of Science –Al-Baath University

Previous Works:

Professor in Al-Hwash Private University From 2020 Until 2023 (Part Time)

Professor in Hama University From 2021 Until 2023 (Part Time)

Professor in Tartous University From 2016 Until 2022 (Part Time)

Assistant Professor in Al-Baath University- Faculty of Economy From 2009 Until 2011 (Part Time)

Assistant Professor in Aleppo University-Faculty of Electrical Engineering From 2004 Until 2005 (Part Time)

Scientific Experience :

Published 43 Scientific Papers

Supervised: 21 Master Degree and PHD Students.

Khalifa Zayid Al Shaqsi

Department of Mathematical and Physical Sciences, College of Arts and Sciences University of Nizwa, Nizwa, Oman

Khalifa.alshaqsi@unizwa.edu.com

Born: Bahla - Oman

Date of Birth: 16.12.1976

Academic degree: (Ph.D) in complex analysis from the National University of Malaysia.

Current place of work:

Faculty member at the University of Nizwa - College of Arts and Sciences - Department of Mathematical and Physical Sciences since 2020.

Previous place of work:

Faculty member - University of Technology and Applied Sciences - Nizwa Technical College 2013-2020.

Mathematics teacher and educational supervisor - Ministry of Education 1999 - 2013.

Dr Nada Adel Nabeeh

Assistant Professor Information Systems Department

Faculty of Computers and Information Sciences

Mansoura University Egypt nadaadel@mans.edu.eg

B.S. from Mansoura University (Excellent with honor the first) in 2011.

M.Sc., in information systems received from Mansoura University in 2015.

Ph.D., in information Systems received from Mansoura University in 2019.

More than ten years of teaching and research experience. The current working is as Assistant Professor at Faculty of Computer and Information Sciences Mansoura University Egypt.

Research Interests

Neutrosophic sets; machine Learning; decision support systems; multi-criteria decision making; industry 4.0; big data; smart city; internet of things; neural networks, artificial intelligence; evolutionary algorithms.

List of Publications

- An Electric Vehicle Analysis Model for Sustainable Environment in Devoicing Nationals. 2023.
- Optimal selection of battery recycling plant location: strategies, challenges, perspectives, and sustainability. 2023.
- Neutrosophic model for vehicular malfunction detection. 2023.
- A Neutrosophic Proposed Model for Evaluation Blockchain Technology in Secure Enterprise Distributed Applications. 2023.
- A Neutrosophic Model for Blockchain Platform Selection based on SWARA and WSM. Neutrosophic and Information Fusion. 2023
- Evaluation of Production of Digital Twins Based on Blockchain Technology. Electronics.; 11(8):1268.

https://doi.org/10.3390/electronics11081268.2022.

- A Comparative Analysis for a Novel Hybrid Methodology using Neutrosophic theory with MCDM for Manufacture Selection. 2022.
- A Neutrosophic Evaluation Model for Blockchain Technology in Supply Chain Management. 2022.
- A model for evaluating green credit rating and its impact on sustainability performance. 2021.
- An intelligent framework using disruptive technologies for COVID-19 analysis. 2020
- A Hybrid Neutrosophic Approach of DEMATEL with AR-DEA in Technology Selection. 2020.
- A Hybrid Approach of Neutrosophic with MULTIMOORA in Application of Personnel selection. 2020.

NEUTROSOPHIC TRANSPORT AND ASSIENMENT

ISBN 978-1-59973-769-0

9 781599 737690 >

2022 – 2023