NEW TECHNIQUES TO ANALYSE THE PREDICTION OF FUZZY MODELS AND NEUTROSOPHIC MODELS
New Techniques to Analyse the Prediction of Fuzzy Models

W. B. Vasantha Kandasamy
Florentin Smarandache
Ilanthenral K

2014
CONTENTS

Preface 5

Chapter One
INTRODUCTION 7

Chapter Two
MERGED FCMS AND NCMS MODELS 9

Chapter Three
KOSKO-HAMMING DISTANCE IN FCMS AND NCMS 127

Chapter Four
NEW AVERAGE FCMS AND NEW AVERAGE NCMS 159
For the first time authors have ventured to study, analyse and investigate the properties of the fuzzy models, the experts opinion and so on. Here the concept of merged Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps are carried out, which are based on merged graphs and merged matrices. This concept is better than the usual combined Fuzzy Cognitive Maps. Further by this new technique we are able to give equal importance to all the experts who work with the problem.

Here the new concept of New Average Fuzzy Cognitive Maps and Neutrosophic Cognitive Maps is defined and described. This new tool helps in saving time and economy.

Another new tool called Kosko Hamming distance of FCMs and NCMs are defined which measures the closeness or otherwise of the experts. The node with maximum vertices is usually termed as a powerful node but here the influential node
in a FCMs (NCMs) is a node whose on state makes on the most number of nodes in the hidden pattern given by it.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

W.B.VASANTHA KANDASAMY
FLORENTIN SMARANDACHE
ILANTHENRAL K
Chapter One

INTRODUCTION

In this book we mainly analyse FCMs and NCMs. This analysis by authors will lead the expert to understand more about the problem. The main aim of the authors was a FCM or a NCM does not in general function on the opinion of a single expert but several experts. In [45, 79] the notion of combined FCMs and NCMs are given. However combined FCMs (or NCMs) has the disadvantage of canceling of the $-1$ with $+1$.

But one believes in the law of large numbers so we have to build a method which can cater to each and every experts opinion equally and also save time and economy. This has been done in chapter IV where the new average simple FCMs and NCMs are built and described. This newly modeled FCMs (NCMs model) not only treats every expert equally but also saves time and economy by working with a single dynamical system.

Also a study of distance between hidden patterns of the same initial state vectors analysed by two different experts by a new method is carried out. This is defined as Kosko-Hamming distance which measures whether two experts opinion are close or very much deviant for a given initial state vector.
This study is new and innovative for these Kosko Hamming distance is defined only if two experts work on the same problem with same number of attributes on the same initial state vector. Such study exhibits the distance between two experts on one specific initial state.

Now another important technique when the number of attributes involved for study is very large we use the newly defined concept of merged FCMs and NCMs. There are three types of merging and they are discussed with examples. The authors wish to keep on record that all the examples given in this book are just illustrations and they are not any real material worked with the real world problems.

To get the merged FCMs or NCMs the reader must be familiar with working of the directed merged neutrosophic graphs. For this concept please refer [100]. Now using this concept of merged graphs in the directed graphs given by the experts we can study merged FCMs. This is better than combined FCMs (or NCMs) for merging does not affect the entries of the connection matrix drastically. Such analysis and study is described and developed in chapter II of this book.

However in [100] the authors have already done a new concept on FCMs known as the special combined FCMs. In this case entries greater than 1 can also occur. This is not merged so we call them as overlapping FCMs.

For more about FCMs and NCMs please refer [79].
Chapter Two

**Merged FCMs and NCMs Models**

In this chapter we introduce the new notion of Merged Fuzzy Cognitive Maps model (MFCMs model) and Merged Neutrosophic Cognitive Maps model (MNCMs model).

Merged graphs and lattices got by merging the vertices or edges or both have been discussed in the book [100].

Here we study mainly pseudo lattice graphs of type II where we take two graphs and merge a vertex of one with other or take two graphs and merge a edge of one with other or both or several vertices or several edges or both are merged.

We will illustrate this situation by some examples.
Example 2.1: Let $G_1 =$

![Graph 1]

and $G_2 =$

![Graph 2]

be any two directed graphs. The pseudo lattice graph of type II is got by merging the vertex $C_1$ of $G_1$ with vertex $C_1$ of $G_2$ [100].

This is a special type of merging for only the node $C_1$ is common so merging of other types cannot take place in this case.
Now these graphs cannot be merged in any other way other than the one mentioned.

However merging of any two arbitrary graphs can be made in any number of ways.

**Example 2.2:** Let $G_1 =$

```
    v1
   /\  \
 v2   v3
```
be any two graphs. The vertices are different so the merging a single vertex of $G_1$ with $G_2$ or an edge of $G_1$ with $G_2$ or both or many vertices and many edges of $G_1$ with $G_2$ can be carried out.

It is in fact an open problem if $G_1$ has $n_1$ vertices and $m_1$ edges and if the graph $G_2$ has $n_2$ vertices and $m_2$ edges how many pseudo lattice graphs of type II can be got.

Now we give a few pseudo lattice graphs of type II.

$S_1 =$

is pseudo lattice graph of type II.

We can get $S_2$ by merging $v_1$ with $u_5$ which is as follows.
is again a graph.

Consider $S_3$ got by merging $v_1v_2$ of $G_1$ with $u_1u_4$ of $G_2$ which is as follows.

We merge $u_1$ with $v_1$, $v_2$ with $u_2$ and $u_4$ with $v_3$ and obtain the following pseudo lattice graph of type II.

We can have several such pseudo lattice graphs of type II.
Example 2.3: Let $G_1 =$

![Graph G1](image1)

$G_2 =$

![Graph G2](image2)

be any two graphs.

Find the number of pseudo lattice graphs of type II got using $G_1$ and $G_2$.

We give one or two examples of them.
We can merge $v_1$ with $v_2$ and $u_2$ with $v_5$.

We have several such pseudo lattice graphs of type II.

Now we proceed onto describe merging of vertices or edges or so of more than two graphs by some examples.

**Example 2.4:** Let $G_1$, $G_2$ and $G_3$ be three graphs given in the following.

We can merge $v_3$ with $u_4$ and $w_1$ and $u_5$ get the following pseudo lattice graph of type II.
We can also merge $w_2\ w_3$ with $u_3\ u_2$ and $v_1\ v_4$ and get the following pseudo lattice graph of type II.

Now we can merge the vertices $v_3$ and $u_4$ and $u_2$ with $w_1$ and get the pseudo lattice graph of type II which is as follows:
It is pertinent to keep on record that we need not always merge all the graphs together. We can merge them in a cycle say $G_1$ to $G_2$, $G_2$ to $G_3$ or $G_3$ to $G_1$ and $G_1$ to $G_2$ or $G_2$ to $G_3$ and $G_3$ to $G_1$.

It is still an open problem to find the number of pseudo lattice graphs of type II using 3 graphs whose number of vertices and edges are known.

Thus we can have pseudo lattice graphs of type II having more than 3 graphs also. All these new techniques are used in the problems of FCMs models and NCMs models. However in case of using in these models we have one and only one pseudo lattice graph of type II. For more about these concepts refer [100].

We will now describe the use of these in Fuzzy Cognitive Maps models.

In the first place to use the concept of merging of vertices or edges or both of the directed graphs associated with the FCMs model we mainly need all the related directed graphs pertain to the same problem and they are modeled or studied using only FCMs.

Only after ascertaining this we can proceed onto use the concept of merging of graphs. Further we also need the concept of merged matrices.

We will just define the notion of merged matrices.

Suppose $G_1$ and $G_2$ are two graphs given in the following:
G₁ =

\[
\begin{array}{cccccc}
\text{v}_1 & \text{v}_2 & \text{v}_3 & \text{v}_4 & \text{v}_5 & \text{v}_6 \\
\text{v}_1 & 0 & 0 & 1 & 0 & 1 \\
\text{v}_2 & 1 & 0 & 0 & 0 & 0 \\
\text{v}_3 & 0 & 0 & 0 & 0 & 0 \\
\text{v}_4 & 1 & 0 & 0 & 0 & 1 \\
\text{v}_5 & 0 & 0 & 0 & 0 & 0 \\
\text{v}_6 & 0 & 0 & 0 & 1 & 0
\end{array}
\]

G₂ =

\[
\begin{array}{cccccc}
\text{v}_1 & \text{v}_2 & \text{v}_3 & \text{v}_4 & \text{v}_5 & \text{v}_6 \\
\text{v}_1 & 0 & 1 & 0 & 1 & 0 \\
\text{v}_3 & 0 & 0 & 1 & 0 & 1 \\
\text{v}_7 & 1 & 0 & 0 & 0 & 0 \\
\text{v}_8 & 0 & 0 & 0 & 1 & 0 \\
\end{array}
\]

The matrices M₁ and M₂ associated with G₁ and G₂ are given in the following.

The merged graph of G₁ with G₂ is as follows:

The merged graph of G₁ with G₂ is as follows:
The merged matrix $M$ to $M_1$ and $M_2$ is as follows.

\[
M = \begin{bmatrix}
0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}.
\]

We see the presence of both $M_1$ and $M_2$ as submatrices exist. So we can merge two matrices $M_1$ and $M_2$ into a matrix $M$ if $M_1$ and $M_2$ are submatrices of $M$. (when the rows and columns of a matrix is deleted the resultant matrix is also defined as a submatrix).

Now in case of merged FCMs the merged matrix serves as the dynamical system of the merged FCMs. Further these merged matrices are the matrices associated with the directed graphs of the FCMs.

Finally we can merge two FCMs if and only if they have atleast a common node or edge or both. Further if they have an
edge in common they should also be in the same direction as that of the others. Only then merging can be done.

We cannot merge $u_1 \rightarrow u_2$ with $u_1 \rightarrow u_2$.

For we can in this case merge $u_2$ with $u_2$ or $u_1$ with $u_1$ and the edges cannot be merged as they are in opposite directions.

Suppose we have three experts working in the same problem with some concepts $c_1, c_2, c_3, \ldots, c_8$. They express their opinion in the graphs $G_1$, $G_2$ and $G_3$ which is given in the following:

$G_1 =$

$G_2 =$

$G_3 =$

The matrices related with the graphs are as follows:

$M_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}$
is the matrix of the graph $G_1$.

$$M_2 = \begin{bmatrix} c_1 & 0 & 1 & 1 \\ c_2 & 0 & 0 & 1 \\ c_7 & 0 & 0 & 0 \end{bmatrix}$$

is the matrix of the graph $G_2$.

$$M_3 = \begin{bmatrix} c_3 & 0 & 1 & 1 & 1 \\ c_5 & 0 & 0 & 0 & 1 \\ c_6 & 0 & 1 & 0 & 0 \\ c_8 & 0 & 0 & 0 & 0 \end{bmatrix}$$

is the matrix of $G_3$.

The merged graph $G$ of $G_1$, $G_2$ and $G_3$ is as follows:

The matrix related with the merged graph $G$ is as follows:
Now we can easily verify that $M$ is also the merged matrix of the matrices $M_1$, $M_2$ and $M_3$. This is the way we get merged matrices and pseudo lattice graphs of type II.

Merging of matrices must obey the following law. If $M_1$, $M_2$, $\ldots$, $M_n$ are $n$ matrices if $M$ is the merged matrix then $M_1$, $M_2$, $\ldots$, $M_n$ must be submatrices of $M$. Then only we call $M$ to be the merged matrix of $M_1$, $M_2$, $\ldots$, $M_n$.

We see for merging of matrices $M_i$ and $M_j$ they must have at least a $a_{i,j}$ to be common between $M_i$ and $M_j$ for some $i$ and $j$ in $1 \leq i, j \leq n$.

Similarly for merging graph at least a vertex or an edge must be common in case of graphs associated with fuzzy models like FCMs or NCMs or NRMs or FRMs.

Keeping these conditions in mind we illustrate the situation in case of FCMs.

**Example 2.5:** Let us consider the problem of passengers preference maximum utilization of a bus route in Madras city [92], the comfort, waiting time, congestion in the vehicle and so on. We first give the attributes suggested by the experts in the following.
C₁ - Frequency of the vehicle along the route.
C₂ - In-vehicle travel time along the route.
C₃ - Travel fare along the route.
C₄ - Speed of the vehicles along the route.
C₅ - Number of intermediate points in the route.
C₆ - Waiting time.
C₇ - Number of transfers in the route.
C₈ - Congestion in the vehicle.

Suppose the first expert wishes to work with the five attributes:

C₁, C₂, C₃, C₄, C₅ and C₆.

The second expert works with the nodes C₃, C₆, C₈ and C₇ and the directed graph given by him is as follows:
Now the graph can be merged by merging the vertex $C_3$ with $C_3$ and the vertex $C_6$ with $C_6$.

The merged graph is as follows:

![Merged graph](image)

The merged matrix of the merged graph $G$ is as follows:

$$
M = \begin{bmatrix}
0 & -1 & 0 & 1 & 0 & -1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -1 & 0 & 1 & -1 & 0 & 0 & -1 \\
0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Using $M$ we get the solution of the problem using the merged FCMs model.

Here merging of the common vertex $C_3$ of the directed graphs related with the FCMs is carried out.
We can also have a merging of an edge. This is illustrated in the following.

The directed graph given by expert I using $C_1$, $C_5$, $C_6$, $C_7$ and $C_8$.

Let $C_1$, $C_7$, $C_8$, $C_3$, $C_4$ and $C_2$ be the nodes taken by the second expert who has given the following directed graph.
Now the unique merged graph of the directed graphs associated with the FCM is as follows.

We merge vertices $C_1$, $C_7$ and $C_8$ and edges $C_1$, $C_7$ and $C_1$, $C_8$ in this merged graph.

Thus using this merged model we can find the merged connection matrix of the FCM using which we can analyse the problem.

Let $C_1$, $C_2$, $C_3$, $C_4$ and $C_6$ be used by the first expert who gives the following directed graph.
Using $C_1$, $C_2$, $C_5$, $C_7$ and $C_8$ the following directed graph is given by the second expert.

Now we can merge these two directed graphs associated with the FCMs in one edge $C_1$ to $C_2$ and two vertices $C_1$ and $C_2$ which is as follows:

This merged graph acts as the merged matrix for the merged FCMs.

Now it may also so happen that three or more number of experts work on the same problem with a collection of attributes with only one common node.
They all use of same model viz the fuzzy cognitive maps models.

Then we see the merged model, which have that concept / node to be common and all other nodes of these three persons get related indirectly and the merged FCM model is formed.

We will first illustrate this situation by an example.

Let three experts work on the same problem and give the following three directed graphs.

\[ G_1 = \]

\[
\begin{array}{c}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_7 \\
C_8 \\
\end{array}
\]

be the directed graph given by the first expert who wishes to work with the FCM model.

Let

\[ G_2 = \]
be the directed graph of the FCMs model given by the second expert.

\[ G_3 = \]

be the directed graph given by the third expert by using the FCMs model.

We can merge these three in only one way and obtain the merged graph \( G \) which is a pseudo lattice graph of type II.

\[ G = \]
Now thus we have 11 attributes which are got after merging the common vertex $C_1$ in all the three of the graph $G_1$, $G_2$ and $G_3$.

The merged connection matrix $M$ of the merged graph is as follows:

$$
M = \begin{bmatrix}
    c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} \\
    c_1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
    c_2 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_5 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
    c_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_8 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
    c_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{10} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    c_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0
\end{bmatrix}.
$$

Using the merged matrix $M$ we can study merged FCMs model for which $M$ is the merged dynamical system.

We can merge more number of vertices and get the FCMs which are merged.

We will still illustrate some other new type of merging.

Let $G_1$, $G_2$, $G_3$ and $G_4$ be the directed graphs given by four experts using FCM on the same problem.

Graph given by the first expert working with the nodes $C_1$, $C_2$, $C_5$ and $C_6$. 
Let $G_1 =$

\[
\begin{array}{c}
C_1 \\
\rightarrow \\
\rightarrow \\
\rightarrow \\
\rightarrow \\
C_5 \\
\end{array}
\]

Let $G_2$ be the directed graph given by the second expert working on the same problem with the nodes $C_2, C_3, C_4, C_9$ and $C_{10}$.

\[
\begin{array}{c}
C_2 \\
\rightarrow \\
\rightarrow \\
\rightarrow \\
C_4 \\
\end{array}
\]

Let $G_3$ be the graph given by the third expert who works with nodes $C_3, C_7, C_8$ and $C_{11}$ as follows:

\[
\begin{array}{c}
C_3 \\
\rightarrow \\
\rightarrow \\
\rightarrow \\
C_8 \\
\end{array}
\]
Finally the fourth expert works with the nodes $C_7$, $C_8$, $C_{12}$ and $C_{13}$ which is as follows:

$$G_4 =$$

We can merge the four graphs only in the following way.

This $G$ is finally the merged graphs of $G_1$, $G_2$, $G_3$ and $G_4$.

So using 13 nodes four expert work on the problem they felt as relevant. However merged FCM gives the working model of the experts on the 13 nodes which saves time and economy.
Further no expert will feel he was not preferred or his expertise was not given equal importance. Only this merged FCMs alone can serve the best.

Now suppose two experts work on the same problem and they both also have a same pair of common nodes

for the second and first expert respectively then we can merge \( C_i \) with \( C_i \) and \( C_j \) with \( C_j \) however the edge will be annulled.

Suppose on the other hand \( C_i \to C_j \) and \( C_i \leftarrow C_j \) then we merge as \( C_i \leftrightarrow C_j \).

We will give some more illustrations of these types of merging. Suppose we have say 15 concepts \( C_1, C_2, \ldots, C_{15} \) associated with the problem and all the experts wish to work with a selective number of nodes from these 15 nodes. We have four experts working on this problem and we see every expert has at least one among the other four experts with a common node or edge or both in their directed graphs. All of them use the FCMs model and we get using these four experts directed graph and the merged FCM model is obtained.

Let \( G_1 \) be the directed graph of the FCM given by the first expert.

\[
G_1 =
\]
Let $G_2$ be the directed graph of FCMs given by the second expert.

Let $G_3$ be the directed graph of the third expert.

Let $G_4$ be the directed graph of the FCM.
We can merge these four graphs appropriately and get at the final graph which is as follows:

Merging of FCMs paves way for integrated study of the experts opinion. However merged FCMs model are different from combined FCMs model.

It is left as an open problem for the reader to give a program for getting a merged FCMs merged graph and the merged matrix in the analysis of a problem by several experts.

We will give illustrations of this concept.
However we wish to keep on record that these examples are only illustrations and we have not worked on any real model.

**Example 2.6:** Let \(C_1, C_2, C_3, \ldots, C_{12}\) be some 12 concepts related with a social problem.

3 experts work on the same problem using some of the concepts from \(C_1, C_2, \ldots, C_{12}\) using the FCMs model.

Let \(G_1\) be the directed graph given by the first expert who uses the concepts \(C_1, C_2, C_5, C_7, C_8\) and \(C_9\).

Let \(M_1\) be the related connection matrix of the FCM given by the first expert.

\[
M_1 = \begin{bmatrix}
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Let $G_2$ be the graph given by the second expert. He works on the nodes $c_1$, $c_3$, $c_7$, $c_8$, and $c_{10}$.

\[ G_2 = \]

The connection matrix $M_2$ associated with the graph $G_2$ is as follows:

\[
\begin{bmatrix}
0 & 1 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

Let $G_3$ be the graph given by the third expert who works with the nodes $c_3$, $c_4$, $c_{10}$, $c_{11}$, and $c_{12}$.

\[ G_3 = \]
The connection matrix associated with this graph is as follows.

\[
\begin{pmatrix}
0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
\end{pmatrix}
\]

Now we get the unique merged graph of the three graphs \( G_1, G_2 \) and \( G_3 \).

Now using this merged directed graph we get the merged connection matrix \( M \) which is as follows:
Note: We use at times capital $C_i$’s and at times small $c_i$ but both mean the same, easily followed from the context.

This $M$ serves as the connection matrix or the dynamical system of the merged FCM.

The concept of merged FCMs play a vital role in the study of FCMs using multi experts opinion with certain conditions imposed on the concepts used by them.

If in the FCMs the concepts are so tailored (that is attributes / nodes without changing the notion they carry) we can always make the directed graph to have weights 0 and 1 only we assume $-1$ does not occur as a weight of the directed graph. This is so conditioned so as to make while merging or while adding the matrices they do not cancel out. Further they are different from the overlapping FCMs and NCMs developed by the authors in [76]. These are different and they give equal importance to each and every expert and the merged matrix also contains only entries from 0 and 1.

\[
\begin{bmatrix}
    c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} & c_{11} & c_{12} \\
    c_1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
    c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
    c_3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    c_4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_5 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    c_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_7 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    c_8 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_9 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{10} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
    c_{12} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
Another flexibility of the merged FCMs is if we have \( c_1, \ldots, c_n \) number of attributes and say some \( t \) expert work with them. One expert can work with \( r_1 \) attributes, another say \( r_2 \) attributes and so on and the researcher who works with problem can work with these \( t \) FCMs models if he is interested in getting \( t \)-bunch of opinions.

These \( r_1, r_2, \ldots, r_t \) sets of attributes are such that any set of \( r_i \) and \( r_j \) attributes have a common attribute or attributes for each \( 1 \leq i, j \leq t \). So if the expert wishes to work with only two set of attributes from two experts he can do so. Likewise one can choose to get the merged matrix with 2 experts or 3 experts to 4 experts or so on say upto \( s \) experts \( s \leq r \).

We will describe this with examples.

However the authors make it clear to the readers this illustration is not a product of working with any of the problems only an example to show how the merged FCMs functions and nothing more.

**Example 2.7:** Let \( c_1, c_2, c_3, \ldots, c_{12} \) be 12 attributes or nodes of a problem. Suppose 4 experts wish to work on it using only FCMs model. Further the experts do not work with all the 12 concepts only a few of the concepts from the 12 concepts. However each expert has a common concept with the other three experts. The directed graphs given by the four experts are as follows:

Suppose expert 1 works with the nodes \( c_1, c_2, c_7, c_{10} \) and \( c_{11} \). Expert 2 prefers to work with the nodes \( c_5, c_7, c_{10}, c_4, c_{12} \). Expert 3 works with the nodes \( c_1, c_2, c_5 \) and \( c_9 \) and expert four work with \( c_3, c_6, c_8, c_{10}, c_{12} \) and \( c_9 \). We see experts, 1 and 2 have the common nodes \( \{c_{10}, c_7\} \).

Experts 1 and 3 have the common nodes \( \{c_1, c_2\} \).
Experts 1 and 4 have \( \{c_{10}\} \) to be the common node.
Experts 2 and 3 have \( \{c_5\} \) to be the common node.
Experts 2 and 4 have \( \{c_{10}\} \) to be the common node.
Experts 3 and 4 have \( \{c_0\} \) to be the common node.

Thus the first criteria of having common attributes / nodes between any two experts are satisfied.

Now the researcher or problem solver may like to study the expert opinion in twos or threes or all the four. All these situations will be described.

Now the directed graph given by the four experts and the related connection matrices are given in the following.

The directed graph given by the first expert is

The connection matrix \( M_1 \) of graph I is as follows:

\[
M_1 = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]
The directed graph given by the second expert is as follows:

The connection matrix $M_2$ associated with the directed graph of second expert using graph II is as follows:

$$
M_2 = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 \\
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0
\end{bmatrix}.
$$

The directed graph given by the third expert is as follows:
The connection matrix $M_3$ associated with the directed graph III of the 3rd expert is as follows:

$$M_3 = \begin{bmatrix}
0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0
\end{bmatrix}.$$ 

The directed graph given by the fourth expert is as follows:

$$M_4 = \begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.$$ 

The related connection matrix of the graph IV is as follows:
Now we get the merged FCM of experts 1 and 2 is given in the following.

The merged graph of graph I and graph II of the FCMs of experts 1 and 2 is denoted by \( A \). The related connection matrix \( M_A \) is as follows.

\[
M_A = \begin{bmatrix}
    c_1 & c_2 & c_4 & c_5 & c_{10} & c_{11} & c_{12} \\
    c_1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_2 & 0 & 0 & 0 & 1 & 0 & 1 \\
    c_4 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_5 & 0 & 0 & 1 & 0 & 1 & 0 \\
    c_7 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_{10} & 0 & 1 & 0 & 0 & 0 & 0 \\
    c_{11} & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_{12} & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]
\( M_A \) serves as the merged dynamical system of the merged FCMs of the experts 1 and 2. Using \( M_A \) one can study the problem for the 8 attributes.

Suppose some wants to study the experts opinion of 1 and 3 alone. Then we merge the graph I and III of the FCMs given by the experts 1 and 3. Let \( B \) denote the merged graph of the two FCMs which is as follows:

Using the merged graph \( B \) we obtain the merged FCMs merged connection matrix which is denoted by \( M_B \).

\[
M_B = \begin{bmatrix}
    c_1 & c_2 & c_5 & c_7 & c_9 & c_{10} & c_{11} \\
    c_1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
    c_2 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
    c_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
    c_7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{10} & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{11} & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]
M_B serves as the merged dynamical system of the merged FCMs of experts 1 and 3. Now we find the merged FCM associated with the experts 1 and 4.

The merged directed graph C of the experts 1 and 4 is as follows.

The connection merged matrix of the merged graph C is as follows:

\[
\begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]
We get the merged opinion of the two experts 2 and 3. The merged directed graph of the experts 2 and 3 be denoted by $D$ which is as follows:

\[
\begin{align*}
\text{C}_1 & \quad \text{C}_2 \\
\text{C}_3 & \quad \text{C}_4 \\
\text{C}_5 & \quad \text{C}_6 \\
\text{C}_7 & \quad \text{C}_8 \\
\text{C}_9 & \quad \text{C}_{10} \\
\text{C}_{11} & \quad \text{C}_{12}
\end{align*}
\]

The merged matrix of the merged graph $D$ is as follows:

\[
M_D = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}
\]

Using the merged connection matrix $M_D$ we can use it as the merged FCMs dynamical system of the two experts 2 and 3.
Now we give the merged directed graph $E$ of the experts 2 and 4 which is as follows:

The merged connection matrix $M_E$ of the merged directed graph $E$ is as follows:

$$
M_E = 
\begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
$$
Using dynamical system $M_E$ of the FCMs we can get the hidden pattern of the any desired merged state vector.

Now we get the merged directed graph $F$ of the experts 3 and 4 which is as follows:

The merged connection matrix $M_F$ of the two experts is as follows:

$$
M_F = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$
Using $M_F$ as the merged FCMs dynamical system we can get the opinion.

Now we can merge the opinion of three of the experts 1, 2, and 3. We can get the merged graph $G$ of the three experts which is as follows:

Let $M_G$ be the connection matrix related with the merged graph $G$.

$$
M_G = \begin{bmatrix}
    c_1 & c_2 & c_4 & c_5 & c_7 & c_9 & c_{10} & c_{11} & c_{12} \\
    c_1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
    c_2 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 1 \\
    c_4 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    c_5 & 0 & 1 & 1 & 0 & 1 & 0 & 0 & 0 \\
    c_7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    c_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{10} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_{11} & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    c_{12} & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{bmatrix}
$$

Using $M_G$ we can get the opinion of the three experts at a time.
Now let us get the merged graph of the three experts 1, 2 and 4. Let \( H \) be the related merged graph.

Let \( M_H \) be the related connection matrix of the merged graph \( H \) of the three experts 1, 2 and 4. We have the following merged matrix \( M_H \).

\[
M_H = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}
\]
Using the merged matrix $M_H$ we can get the resultant of all the 12 attributes. Let $I$ denote the merged graph of the three experts 1, 3 and 4 which is as follows:

\[
I = \begin{array}{ccccccccccc}
C_1 & C_2 & C_5 \\
C_{11} & C_7 & C_3 \\
C_{10} & C_{12} & C_8 \\
\end{array}
\]

Let $M_I$ be the merged matrix of the directed merged graph $I$ which is as follows:

\[
M_I = \begin{bmatrix}
c_1 & c_2 & c_3 & c_5 & c_6 & c_7 & c_8 & c_{10} & c_{11} & c_{12} \\
c_1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
c_2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
c_3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
c_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
c_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
c_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_9 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
c_{10} & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
c_{11} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
c_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
$M_I$ is the merged matrix of the merged FCMs of the three experts 1, 3 and 4. Let $J$ be the merged graph of the three experts 2, 3 and 4 which is as follows.

Let $M_J$ be the merged connection matrix of the merged graph $J$ of the three experts 2, 3 and 4.

$M_J = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$
Using the merged dynamical system $M_J$ of FCMs we can get the resultant of all attributes other than $c_{11}$. Now we get the merged graph $K$ of all the four experts 1, 2, 3 and 4 which is as follows.

\[ K = \]

Using the merged directed graph $K$ of the experts we get the associated merged matrix $M_K$ of the graph which is as follows:

\[
M_K = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]
We see $M_K$ and $M_{III}$ are different graphs and the resultant also may not be the same for any initial state vector. For the merged graph $M_K$ contains all the merged graphs and the graphs I, II, III and IV.

This is the way the merged graphs of any $t$ experts who have non empty intersection of any two concepts associated with the experts works.

Another type of merging is as follows:

Let $C_1, C_2, \ldots, C_n$ be $n$ concepts associated with the problem. Suppose $t$ experts wishes to work with the problem taking a few of the attributes using the FCMs model.

Suppose the $t$ experts work with this problem we see as in the earlier case no two experts need to have the non empty intersection of the attribute set; what we demand is every expert has only with two other expert the non empty intersection of the attributes selected by them. Further two of the experts have only one expert who has a non empty intersection with the attributes.

For better understanding we give the following example. Further this type of choice of attributes can also occur for always one cannot demand every expert to choose the attributes in such a way that the attributes set intersect giving a non empty set.

**Example 2.8:** Let $C_1, C_2, C_3, C_4, \ldots, C_{11}$ be the 11 attributes associated with the problem where four experts choose to work with a selected set of attributes from $C_1, \ldots, C_{11}$ using the FCMs model.

Let $E_1, E_2, E_3$ and $E_4$ be the four experts who work with the problem using attributes from the set $\{C_1, C_2, \ldots, C_{11}\}$. The expert $E_1$ works with the attributes $\{C_1, C_2, C_7, C_8\}$. 
The expert E_2 works with the attributes \{C_7, C_8, C_4, C_5, C_{10}\}. The expert E_3 works with the attributes \{C_4, C_5, C_6, C_{11}\}. The expert E_4 works with the attributes \{C_6, C_{11}, C_9, C_3\}.

We see the experts E_1 and E_2 have
\[ \{C_7, C_8\} = \{C_1, C_2, C_7, C_8\} \cap \{C_7, C_8, C_5, C_4, C_{10}\}. \]

The experts E_1 \cap E_3 = \emptyset, E_1 \cap E_4 = \emptyset,
\[ E_2 \cap E_3 = \{C_4, C_5, C_7, C_8, C_{10}\} \cap \{C_4, C_5, C_6, C_{11}\} = \{C_4, C_5\}. \]
\[ E_2 \cap E_4 = \emptyset \quad \text{(Here we use } E_i \cap E_j \text{ to represent the intersection of the sets of attributes used by experts } E_i \text{ and } E_j). \]

\[ E_3 \cap E_4 = \{C_4, C_5, C_6, C_{11}\} \cap \{C_3, C_6, C_{11}, C_9\} \]
\[ = \{C_6, C_{11}\}. \]

This sort of choice of attributes by experts will be called as chain like merging and the resultant merged graphs will be known as chain like merged graph and the corresponding matrices as chain like merged matrix. Finally the merged FCMs will be known as chain like merged FCMs.

As in case of merged FCMs we cannot merge any of the two experts.

Here we can get the merged graphs of experts E_1 with E_2, E_2 with E_3 and E_3 with E_4 and merged graphs of E_1, E_2 and E_3 or E_2, E_3 and E_4 and E_1, E_2, E_3 and E_4.

Let us exhibit the directed graph given by the first expert E_1 and denote it by A.
The connection matrix $A$ associated with the directed graph $A$ is as follows:

$$
\begin{bmatrix}
C_1 & C_2 & C_7 & C_8 \\
C_1 & 0 & 1 & 0 & 0 \\
C_2 & 0 & 0 & 1 & 1 \\
C_7 & 0 & 1 & 0 & 0 \\
C_8 & 0 & 0 & 1 & 0 \\
\end{bmatrix}
$$

Let $B$ be the directed graph given by the expert $E_2$.

$$
\begin{array}{c}
C_4 \\
\downarrow \\
C_5 \\
\downarrow \\
C_8 \\
\downarrow \\
C_{10}
\end{array}
$$

Let $M_B$ be the connection matrix of the directed graph.

$$
\begin{bmatrix}
C_4 & C_5 & C_7 & C_8 & C_{10} \\
C_4 & 0 & 1 & 0 & 0 & 0 \\
C_5 & 0 & 0 & 1 & 1 & 1 \\
C_7 & 0 & 0 & 0 & 1 & 0 \\
C_8 & 0 & 0 & 0 & 0 & 1 \\
C_{10} & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
$$

Let $C$ be the directed graph given by the third expert $E_3$. 
The connection matrix $M_C$ associated with the directed graph $C$ is as follows:

$$
M_C = \begin{bmatrix}
C_4 & C_5 & C_6 & C_{11} \\
C_4 & 0 & 1 & 0 & 1 \\
C_5 & 0 & 0 & 1 & 1 \\
C_6 & 0 & 1 & 0 & 0 \\
C_{11} & 0 & 0 & 1 & 0
\end{bmatrix}.
$$

Let $D$ be the directed graph given by the fourth expert.

The connection matrix $M_D$ associated with graph $D$ is as follows.
Merged FCMs and NCMs Models

We have no option of merging $E_3$ and $E_4$.

The only option is $E_1$ can be merged with $E_2$. The merged graph of $A$ and $B$ of the experts $E_1$ and $E_2$ is as follows. Let $E$ be the directed merged graph of $A$ and $B$.

\[
E = 
\begin{array}{c}
C_1 \\
C_2 \\
C_7 \\
C_8 \\
C_{11}
\end{array}
\begin{array}{c}
C_2 \\
C_4 \\
C_5 \\
C_8 \\
C_{11}
\end{array}
\]

The merged connection matrix of the merged directed graph $E$ is denoted by $M_E$ which is as follows:

\[
M_E = 
\begin{bmatrix}
c_1 & c_2 & c_4 & c_7 & c_8 & c_{11} \\
c_1 & 0 & 1 & 0 & 0 & 0 & 0 \\
c_2 & 0 & 0 & 0 & 0 & 1 & 1 \\
c_4 & 0 & 0 & 0 & 1 & 0 & 0 \\
c_7 & 0 & 0 & 0 & 0 & 1 & 1 \\
c_8 & 0 & 1 & 0 & 0 & 0 & 1 \\
c_9 & 0 & 0 & 0 & 1 & 0 & 1 \\
c_{11} & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]
Now we merge the directed graphs $B$ and $C$ of experts $E_2$ and $E_3$ which is as follows:

Let us denote the merged graph by $F$.

The connection merged matrix of $M_F$ of $F$ is as follows:

\[
M_F = \begin{bmatrix}
  c_4 & c_5 & c_6 & c_7 & c_8 & c_{11} \\
  c_4 & 0 & 1 & 0 & 0 & 0 \\
  c_5 & 0 & 0 & 1 & 1 & 1 \\
  c_6 & 0 & 1 & 0 & 0 & 0 \\
  c_7 & 0 & 0 & 0 & 1 & 0 \\
  c_8 & 0 & 0 & 0 & 0 & 1 \\
  c_{11} & 0 & 1 & 1 & 0 & 0 \\
\end{bmatrix}
\]

Now we can merge the directed graph $C$ with that of $D$ and get the merged directed graph $G$ which is as follows:
The connection merged matrix $M_G$ of the merged graph $G$ is as follows:

$$
M_G = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}
$$

Thus using $M_E$, $M_F$ and $M_G$ we can study the problem using each of the two experts opinion.

Now we can find also 3 experts opinion only in two ways. Taking the experts $E_1$, $E_2$ and $E_3$ or $E_2$, $E_3$ and $E_4$.

To find the opinion of the experts $E_1$, $E_2$ and $E_3$ we should get the merged graph of the three directed graph $A$, $B$ and $C$. Let $H$ denote the merged graph of the graphs $A$, $B$ and $C$. 

\[ \text{The connection merged matrix } M_H \text{ of the merged graph } H \text{ is as follows:} \]

\[ M_H = \begin{bmatrix}
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix} \]
Let $M_H$ denote the connection matrix of the merged graph $M_H$ which is as follows.

$$
\begin{array}{cccccccc}
    & c_1 & c_2 & c_4 & c_5 & c_6 & c_7 & c_8 & c_{11} \\
    c_1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_2 & 0 & 0 & 1 & 0 & 0 & 1 & 1 & 0 \\
    c_4 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    c_5 & 0 & 0 & 1 & 0 & 1 & 1 & 1 & 0 \\
    c_6 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    c_7 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_8 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 \\
    c_{11} & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
\end{array}
$$

Using $M_H$ we can get the resultant of all attributes except $C_3$, $C_9$ and $C_{10}$.

Now we find the merged graph $I$ of the three directed graphs $B$, $C$ and $D$ of the experts $E_2$, $E_3$ and $E_4$.

The merged graph $I$ of the directed graphs $B$, $C$ and $D$ are as follows.
The connection matrix $M_I$ of the merged graph $I$ as follows.

$$
\begin{bmatrix}
  c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{11} \\
  c\_1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
  c\_4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
  c\_5 & 0 & 0 & 0 & 1 & 1 & 1 & 0 & 1 \\
  c\_6 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  c\_7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
  c\_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
  c\_9 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
  c_{11} & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

Using the connection matrix $M_I$ we can study the effect of the 8 attributes.

We cannot find the effect of $C_1$, $C_2$ and $C_{10}$. 
Finally we get the merged directed graph $J$ of the four graphs $A$, $B$, $C$ and $D$ which is as follows.

Now we find the connection merged matrix of the merged directed graph $J$ which is denoted by $M_J$.

Thus we see all the four experts opinion is merged and we get the merged connection matrix $M_J$. 
Now we proceed onto describe the model abstractly.

Let $c_1, c_2, \ldots, c_n$ be $n$ attributes with which $t$ experts say $E_1, E_2, \ldots, E_t$ work on some problem using FCMs. However these experts choose only a subset from the set of concepts $c_1, c_2, \ldots, c_n$.

Without loss of generality let us assume the expert $E_1$ and $E_2$ have common attributes and the expert $E_1$ does not share any common attribute with any of the other experts $E_3, E_4, \ldots, E_t$. Consider the expert $E_2$, $E_2$ has common attributes with $E_1$ and $E_3$ and none others $E_4, E_5, \ldots, E_t$. Likewise expert $E_3$ has common attributes only with $E_4$ and $E_2$ and none others. Similarly expert $E_4$ has common attributes only with $E_5$ and $E_3$ and none others and so on. Thus the expert $E_i$ has common attributes with the expert $E_{i-1}$ and $E_{i+1}$ for $i = 1, 2, \ldots, t-1$. We see the expert $E_i$ has only common attributes with $E_{i-1}$ only we see thus $E_1 \cap E_2 \neq \emptyset$. $E_1 \cap E_i = \emptyset$ for all $3 \leq i \leq t$. That is $E_i \cap E_{i+1} \neq \emptyset$ for $i = 1, 2, \ldots, t-1$.

When experts distribute the nodes and concepts in this way among themselves we can work with the merged FCMs which we call as specially linked merged FCMs.

Study of this concept is described and developed in an example. Now using this specially linked merged FCMs we can study the problem.

Now we introduce yet another new type of merged FCMs which is little different from the other two merged FCMs models. Let us suppose we have say $C = \{C_1, C_2, \ldots, C_n\}$ to be $n$-attributes associated a problem. Suppose $E_1, E_2, \ldots, E_t$ be $t$ experts who works with some attributes from the subset of $C$. Suppose $r$ of the experts from the $t$ experts $r < t$ happen to contribute to the merged FCMs in such a way that these $r$ experts say $E_1, \ldots, E_r$ cover $C$ with $E_i \cap E_j \neq \emptyset$ for $1 \leq i, j \leq r$ (or they cover $C$ with $E_i \cap E_{i+1} \neq \emptyset$, $1 \leq i \leq r-1$) then we get the merged FCMs model to study the problem.
Further we can choose some other $s$ experts from the $t$ experts ($s < t$) to cover $C$ such that some from these $s$-experts are also in the $r$-experts mentioned earlier. Thus we can have say $m$ such groups and these $m$-group of experts have non empty intersection. While studying these merged FCMs we clearly see some experts are vital that is they appear in many of the groups, so that unintentionally these experts play a major role in every merged FCMs.

Some experts may appear only in one group of experts and some experts may appear in two groups and so on.

If an expert finds place in every $m$-group we call that expert to be a strongly influencing vital expert. It may so happen we can have more than one expert to be a strongly influencing vital expert. The expert who finds in one and only group will be known as the non vital or non influencing expert. No expert need to feel their expertise is lost for grouping is not going to bias as no role is played by the humans.

We will illustrate this model by an example.

The example is only artificial as this example is not based on any real world problem. Further adopting this to any real world problem is at a risk of bias as this example is only a mere illustration and nothing more.

However the techniques of this model are vital and this model is described for experts / researchers to understand the situation.

**Example 2.9:** Let $C = \{c_1, c_2, ..., c_{13}\}$ be the 13 concepts related with the problem. Suppose 5 experts work with some attributes from the 13 attributes set $C$.

Let $E_1$, $E_2$, $E_3$, $E_4$ and $E_5$ be the five experts who work on the problem.
Let the expert E₁ work with the nodes \{c₁, c₂, c₃, c₁₀, c₄\}. The expert E₂ works with the following nodes \{c₁, c₄, c₁, c₆, c₉\}. The third expert E₃ works with the nodes \{c₈, c₅, c₇, c₁₂, c₁₃\}, expert E₄ works with the nodes \{c₁, c₃, c₅, c₁₀, c₁₁\} and the expert E₅ works with attributes \{c₂, c₅, c₁₀, c₁₂\}.

We give the directed graph associated with each of the five experts in the following.

The directed graph I given by the first expert E₁ is as follows.

```
\begin{center}
\begin{tikzpicture}
\node (C1) at (0,0) {C₁};
\node (C2) at (2,1) {C₂};
\node (C3) at (0,-1) {C₃};
\node (C4) at (2,-2) {C₄};
\node (C10) at (4,0) {C₁₀};
\draw (C1) -- (C2);
\draw (C1) -- (C3);
\draw (C2) -- (C3);
\draw (C2) -- (C10);
\draw (C3) -- (C4);
\end{tikzpicture}
\end{center}
```

The directed graph II given by the second expert E₂ is as follows.

```
\begin{center}
\begin{tikzpicture}
\node (C4) at (0,0) {C₄};
\node (C3) at (2,1) {C₃};
\node (C1) at (-2,-1) {C₁};
\node (C6) at (2,-2) {C₆};
\node (C₉) at (0,-2) {C₉};
\draw (C4) -- (C3);
\draw (C1) -- (C₉);
\draw (C₉) -- (C₆);
\end{tikzpicture}
\end{center}
```
The directed graph III of the third expert is given by the following.

![Diagram of graph III]

The directed graph IV of the expert four E₄ is given in the following.

![Diagram of graph IV]

The directed graph V given by the fifth expert is as follows.
We get the following connection matrices for these directed graphs I, II, III, IV and V respectively. They are denoted by \( M_I, M_{II}, M_{III}, M_{IV}, \) and \( M_V \) respectively.

The connection matrix \( M_I \) of the directed graph I given by the expert I is as follows.

\[
M_I = \begin{bmatrix}
  0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 1 & 1 & 1 \\
  0 & 0 & 0 & 0 & 1 \\
  0 & 0 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 0
\end{bmatrix}
\]

The connection matrix \( M_{II} \) of the directed graph II is as follows:

\[
M_{II} = \begin{bmatrix}
  0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 0 & 0 & 1 \\
  0 & 0 & 1 & 0 & 0 \\
  0 & 1 & 0 & 0 & 0 \\
  0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}
\]
The connection matrix $M_{III}$ of the directed graph III is as follows.

$$M_{III} = \begin{bmatrix}
c_5 & c_7 & c_8 & c_{12} & c_{13} \\
c_5 & 0 & 1 & 0 & 1 & 1 \\
c_7 & 0 & 0 & 1 & 0 & 1 \\
c_8 & 0 & 0 & 0 & 0 & 1 \\
c_{12} & 0 & 0 & 0 & 0 & 0 \\
c_{13} & 0 & 0 & 1 & 0 & 0
\end{bmatrix}$$

Let $M_{IV}$ denote the connection matrix of the directed graph IV of expert four which is as follows.

$$M_{IV} = \begin{bmatrix}
c_1 & c_3 & c_4 & c_{10} & c_{11} \\
c_1 & 0 & 1 & 0 & 1 & 0 \\
c_3 & 0 & 0 & 1 & 0 & 0 \\
c_5 & 0 & 0 & 0 & 0 & 1 \\
c_{10} & 0 & 1 & 0 & 0 & 0 \\
c_{11} & 0 & 0 & 0 & 1 & 0
\end{bmatrix}$$

Let $M_V$ denote the connection matrix of the directed graph V of the expert which is as follows:

$$M_V = \begin{bmatrix}
c_2 & c_5 & c_{10} & c_{12} \\
c_2 & 0 & 1 & 1 & 1 \\
c_5 & 0 & 0 & 1 & 0 \\
c_{10} & 0 & 1 & 0 & 0 \\
c_{12} & 0 & 0 & 0 & 0
\end{bmatrix}$$

Now see by merging the experts $E_2$, $E_4$, $E_3$ and $E_5$ or experts $E_1$, $E_2$, $E_3$ and $E_4$ we get covered all the 13 attributes or nodes.
However to get all the nodes we need the three experts set E₂, E₃ and E₄. For the expert E₂ alone has used the nodes C₆ and C₉ and no other expert has used them. Expert E₃ cannot be overlooked for the expert E₃ alone has worked with the nodes C₈ and C₁₃. Expert E₄ cannot be overlooked for he alone has used the node C₁₁. Thus working with the merged or integrated FCMs the three experts are very essential we can choose expert E₁ or expert E₂ as per the wishes of the experts or the researcher who works in the problem.

We merge the four experts E₁, E₂, E₃ and E₄ by merging the directed graphs given by them which is as follows.

Let A denote the merged directed graph of the four experts E₁, E₂, E₃ and E₄.

The merged directed graph A of the directed graphs E₁, E₂, E₃ and E₄ and the merged connection matrix of A be denoted by M_A, which is as follows.

\[
\begin{array}{cccccccccccccc}
C₁ & C₂ & C₄ & C₅ & C₆ & C₇ & C₈ & C₉ & C₁₀ & C₁₁ & C₁₂ & C₁₃
\end{array}
\]
Suppose we merge the opinion of the experts $E_2$, $E_3$, $E_4$ and $E_5$ by merging the directed graph II, III, IV and V. Let $B$ denote the merged directed graph which is as follows.
Using the merged directed graph B we obtained the merged connection matrix of B which will serve as the dynamical system of the merged FCMs. Let \( \mathbf{M}_B \) denote the connection matrix of B.

\[
\begin{bmatrix}
\mathbf{c}_1 & \mathbf{c}_2 & \mathbf{c}_3 & \mathbf{c}_4 & \mathbf{c}_5 & \mathbf{c}_6 & \mathbf{c}_7 & \mathbf{c}_8 & \mathbf{c}_9 & \mathbf{c}_{10} & \mathbf{c}_{11} & \mathbf{c}_{12} & \mathbf{c}_{13} \\
\mathbf{c}_1 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\mathbf{c}_2 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
\mathbf{c}_3 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_4 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\mathbf{c}_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_6 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_8 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_{10} & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_{11} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\mathbf{c}_{12} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\mathbf{c}_{13} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

We see both the connection matrices are different and they give different resultants.

We see \( E_1 \) and \( E_5 \) are experts who are not that strong only partly strong one of them is sufficient to give a merged or integrated FCMs model.

However both models can be used as they are different. In the further following chapter a new average technique will be used and also in another chapter the concept of Kosko-Hamming distance will be introduced and that can applied to the resultants given by the two models for the same initial state vector. We have throughout used only FCMs whose related matrices take values from the set \{0, 1\}.
Now when we have a subset from the set of experts set \( E = \{E_1, E_2, \ldots, E_t\} \) which can give the merged FCMs taking all the \( n \) concepts we take all such subsets of \( E \) and find the merged FCMs.

Using new average simple FCMs which will be defined in chapter four of this book we find a new single of integrated model which can predict the solutions of the problem.

Further we proceed onto describe the notion of merged NCMs.

The basic notion of NCMs and the concept of neutrosophic graphs have been introduced in chapter I.

We now give a few types of merged neutrosophic NCMs and mixed merged NCMs and FCMs. In case of NCMs we have two types of merging and both merging pave way to only NCMs.

We will define, develop and describe these situations by some examples.

We know a neutrosophic directed graph associated with NCMs.

We just show how merging takes place among neutrosophic graphs.

Suppose we have two neutrosophic graphs \( G_1 \) and \( G_2 \) which has some common vertices.
We see both the graphs have only $C_4$ to be the common vertex so merging can be done without any difficulty for only one common vertex is $C_4$.

Further we assume in NCMs in general the vertices are not neutrosophic only the edges are neutrosophic. Further merging of an neutrosophic edge with the real edge cannot be accepted how to overcome or redefine the edge.
We redefine the edge in a very flexible way.

If the expert feels neutrosophy that is indeterminacy over usual edge let them opt for indeterminacy if they feel contrary let them take the real.

But however if one chooses to take indeterminacy till the end of the problem that is while forming each and every merged graph the same should be adopted. Only under these conditions we can get NCMs merged model.

We will first illustrate this situation by some examples.

**Example 2.10:** Let $G_1$ and $G_2$ be any two neutrosophic graphs which has some neutrosophic edges and vertices in common

![Graph](image-url)
be the two neutrosophic graphs.

We can merge the two graphs in one and only one way.

The merging is carried out in a direct way as there is no conflicts about the edges.
This is the way merging is carried out without any difficulty.

Next we find merging of the two neutrosophic graphs $G_1$ and $G_2$. 

\[ G_1 = \begin{array}{c}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_7 \\
\end{array} \]

\[ G_2 = \begin{array}{c}
C_1 \\
C_2 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_7 \\
\end{array} \]
The merged graph of $G_1$ with $G_2$ is as follows.

This is the way neutrosophic graphs are merged. We see the merging is done not under any assumption.

Suppose we have two neutrosophic graphs $G_1$ and $G_2$ which is as follows.
We can merge the graph $G_1$ and $G_2$ which is as follows.

We see by merging the neutrosophic edge and real edge we get the neutrosophic edge and so on. Here the experts opts to take $C_3$ to $C_2$ as a neutrosophic edge only.
This is not the usual merging.

Now using these techniques we work for the merging NCMs.

Suppose we wish to merge two NCMs whose directed graphs $G_1$ and $G_2$ are as follows:

$$
G_1 = \begin{array}{c}
C_1 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_2
\end{array}
$$

$$
G_2 = \begin{array}{c}
C_1 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_2 \\
C_7 \\
C_8
\end{array}
$$

The merged graph of $G_1$ and $G_2$ is as follows:

$$
\begin{array}{c}
C_1 \\
C_3 \\
C_4 \\
C_5 \\
C_6 \\
C_7 \\
C_8
\end{array}
$$
G is the merged neutrosophic graph of G₁ and G₂.

As in case of merged FCMs we can get merged NCMs of the three types apart from those mixed merged NCMs and FCMs.

Let \{C₁, C₂, ..., Cₙ\} be n attributes or nodes. Suppose t experts E₁, E₂, ..., Eₜ work on the problem working will some attributes from the set C.

Suppose all of them work with only Neutrosophic Cognitive Maps (NCMs) model we can find the merged NCMs as in case of FCMs.

Let us suppose c₁, c₂, ..., c₁₀ are the 10 nodes or attributes related with the some problem. Suppose only three experts E₁, E₂ and E₃ work with the problem.

Let the expert E₁ work with the nodes \{C₁, C₃, C₅, C₇ and C₉\}. Let the expert E₂ work with the nodes \{C₁, C₂, C₄, C₇, C₆, C₈\} and the expert E₃ works with the nodes \{C₂, C₃, C₁₀, C₇, C₈, C₉\}. We can find the merged graph of the NCMs using the neutrosophic graphs G₁, G₂ and G₃ of the experts E₁, E₂ and E₃ respectively.

\[ G₁ = \]

The neutrosophic connection matrix \( M_{G₁} \) of the graph G₁ is as follows.
The neutrosophic directed graph $G_2$ given by the expert $E_2$ is as follows:

\[
G_2 =
\begin{array}{c}
\text{C}_1 \\
\text{C}_4 \\
\text{C}_7 \\
\text{C}_8 \\
\text{C}_6 \\
\text{C}_2
\end{array}
\]

The connection matrix $M_{G_2}$ is given by $G_2$ is as follows.

\[
M_{G_1} = \begin{bmatrix}
c_1 & c_3 & c_5 & c_7 & c_9 \\
c_1 & 0 & 1 & 0 & 0 & 1 \\
c_3 & 0 & 0 & 1 & 1 & 0 \\
c_5 & 0 & 1 & 0 & 1 & 0 \\
c_7 & 0 & 0 & 0 & 0 & 0 \\
c_9 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}.
\]

\[
M_{G_2} = \begin{bmatrix}
c_1 & c_2 & c_4 & c_6 & c_7 & c_8 \\
c_4 & 0 & 0 & 0 & 1 & 0 \\
c_6 & 0 & 0 & 0 & 0 & 0 \\
c_7 & 0 & 0 & 0 & 1 & 0 \\
c_8 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}.
\]
The directed neutrosophic graph $G_3$ given by expert 3 is as follows.

![Diagram of $G_3$.]

The neutrosophic connection matrix associated with $G_3$ is as follows:

$$
\begin{bmatrix}
    c_2 & c_3 & c_7 & c_8 & c_9 & c_{10} \\
    c_2 & 0 & 0 & 0 & 1 & 0 \\
    c_3 & 0 & 0 & 0 & 0 & 0 \\
    c_7 & 0 & 0 & 0 & 1 & 0 \\
    c_8 & 0 & 0 & 0 & 1 & 1 \\
    c_9 & 0 & 0 & 0 & 0 & 0 \\
    c_{10} & 0 & 1 & 0 & 0 & 0 
\end{bmatrix}
$$

Now we can merge the graphs $G_1$ and $G_2$.

Let $H$ be the merged graph of $G_1$ and $G_2$ which is as follows:
The neutrosophic merged connection matrix $M_H$ of $H$ is as follows:

$$M_H = \begin{bmatrix}
    c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \\
    c_1 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 1 \\
    c_2 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    c_3 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
    c_4 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
    c_5 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    c_7 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
    c_8 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
    c_9 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
\end{bmatrix}.$$

We can also merge the neutrosophic graph $G_1$ with $G_5$.
Let $I$ denote the merged neutrosophic graph which is as follows.

Let $M_I$ denote the neutrosophic merged connection matrix of the neutrosophic graph $I$.

\[
\begin{bmatrix}
    c_1 & c_2 & c_3 & c_5 & c_7 & c_8 & c_9 & c_{10} \\
    c_1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
    c_2 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
    c_3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
    c_5 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
    c_7 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_8 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
    c_9 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
    c_{10} & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Now we get the merged graph $J$ of the graphs $G_2$ with $G_3$ which is as follows.

Now we can get the merged neutrosophic graph $K$ of all the three experts $E_1$, $E_2$ and $E_3$ which is as follows.
The merged connection matrix $M_K$ of the three experts gives by the merged graph which is as follows.

$$
M_K = \begin{bmatrix}
    0 & 1 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
    0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 1 \\
    0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$M_K$ gives the total or integrated dynamical system of the merged model. Working with this given the every node of the problem.
By looking at the merged graph one will think $C_7$ is the vital or the most influential node. Such study about the graphs associated with the models is carried out in chapter III of this book.

Once such study is done the researcher will have more knowledge about the problem and its outcome.

Next we give the merged NCM such that the $t$ experts say $E_1, E_2, \ldots, E_t$ work with $n$ nodes of the problem using NCM by selecting some of the nodes from the $n$ nodes such that the node of the expert $E_i \cap E_{i+1} \neq \emptyset$ for $i = 1, 2, \ldots, t-1$ and $E_i \cap E_j = \emptyset$ if $j \neq i+1$.

Such a type of merged FCM was discussed earlier. Here we discuss the same type of problem using NCMs.

We will illustrate this situation by some examples.

Let $C_1, C_2, \ldots, C_{10}$ be the concepts associated with a problem. Let four experts work with the problem using NCMs taking some nodes from the ten nodes.

Let the expert $E_1$ work with the nodes $\{C_1, C_2, C_3, C_5\}$. Let the expert $E_2$ work with the nodes $\{C_3, C_2, C_6, C_7\}$. Let the expert $E_3$ work with the nodes $\{C_6, C_7, C_8, C_9\}$ and the expert $E_4$ works with the nodes $\{C_8, C_4, C_9, C_{10}\}$.

We see the common nodes between $E_1$ and $E_2$ is $\{C_3, C_2\}$, the common node between $E_2$ and $E_3$ is $\{C_6, C_7\}$ and the common node between $E_3$ and $E_4$ $\{C_8, C_9\}$.

However $E_i \cap E_j = \emptyset$ if $j = i+1; 1 \leq i \leq 4; 2 \leq j \leq 3$.

Now the directed neutrosophic graph given by the expert $E_1$ is as follows.
The connection neutrosophic matrix $M_I$ associated with the directed graph $I$ is as follows:

$$
M_I = \begin{bmatrix}
0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 \\
\end{bmatrix}
$$

The directed neutrosophic graph $II$ given by the second expert $E_2$ is as follows:
The connection neutrosophic matrix $M_{II}$ given by the expert $E_2$ is as follows:

\[
M_{II} = \begin{bmatrix}
0 & 1 & 1 & 0 \\
1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0
\end{bmatrix}.
\]

The directed graph $III$ given by the third expert $E_3$ is as follows:

The neutrosophic connection matrix $M_{III}$ of the graph $III$ is as follows:

\[
M_{III} = \begin{bmatrix}
0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 \\
1 & 1 & 0 & 0 \\
0 & 1 & 0 & 0
\end{bmatrix}.
\]
The directed graph IV given by the forth expert $E_4$ is as follows:

The neutrosophic connection matrix $M_{IV}$ associated with graph IV is as follows:

$$M_{IV} = \begin{bmatrix}
    c_4 & c_8 & c_9 & c_{10} \\
    c_4 & 0 & 1 & 1 \\
    c_8 & 0 & 0 & 1 \\
    c_9 & 0 & 0 & 1 \\
    c_{10} & 1 & 0 & 0 \\
\end{bmatrix}$$

We cannot merge the graph I with III or IV.

Likewise graph II cannot be merged with graph IV. Further graph III cannot be merged with graph I. Finally graph IV cannot be merged with graphs I and II. We now get the merged neutrosophic graphs.

First let us get the merged NCMs of experts $E_1$ and $E_2$ by merging the neutrosophic graphs I and II.

Let A denote the merged graph of graphs I and II which is as follows.
Let $M_A$ denote the neutrosophic merged connection matrix of $A$.

$$
M_A = \begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 
\end{bmatrix}.
$$

Let $B$ denote the merged graphs of the graphs II and III which is as follows:
The merged neutrosophic connection matrix $M_B$ of the graph $B$ is as follows:

$$M_B = \begin{bmatrix}
    c_2 & c_3 & c_6 & c_7 & c_8 & c_9 \\
    c_2 & 0 & 1 & I & 0 & 0 \\
    c_3 & 1 & 0 & 0 & I & 0 \\
    c_6 & 0 & 0 & 0 & 1 & I \\
    c_7 & 0 & 0 & 1 & 0 & 0 \\
    c_8 & 0 & 0 & 1 & I & 0 \\
    c_9 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}.$$

Let $C$ denote the merged neutrosophic graphs III and IV which is as follows:

Let $M_C$ denote the merged neutrosophic connection matrix of the merged neutrosophic graph $C$.

$$M_C = \begin{bmatrix}
    c_4 & c_6 & c_7 & c_8 & c_9 & c_{10} \\
    c_4 & 0 & 0 & 0 & I & 1 \\
    c_6 & 0 & 0 & 1 & 0 & I \\
    c_7 & 0 & 0 & 0 & 0 & 1 \\
    c_8 & 0 & 1 & I & 0 & 0 \\
    c_9 & 0 & 0 & 1 & 0 & 0 \\
    c_{10} & I & 0 & 0 & 0 & 0 \\
\end{bmatrix}.$$
Let $D$ denote the merged graph of all the four neutrosophic graphs I, II, III and IV which is as follows:

Let $M_D$ denote the merged neutrosophic connection matrix of the merged neutrosophic directed graph $D$ which is as follows:

\[
M_D = \begin{bmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
This is the way specially linked merged NCMs function.

Now we proceed onto describe the notion of specially linked (merged) NCMs. Suppose we have some n concepts say \( \{c_1, c_2, \ldots, c_n\} \) and say t experts work on it and all of them use only NCMs.

Now we proceed onto describe the specially merged NCMs model using groups of experts from the t experts.

Suppose say \( r_1 \) of the experts from the t experts give the total or integrated model. If we have another set of \( r_2 \) experts from t experts give the total or integrated model so on say some \( r_i \) of the t experts give the integrated model then to find the most influential expert and the least influential expert.

Most influential expert is one who cannot be compensated or replaced by some other expert.

A passive or a weak expert is one who can be replaced by one or more experts. This has been described developed in the case of FCMs.

Now we will describe the situation by an example.

Let \( C = \{C_1, C_2, \ldots, C_{10}\} \) be the ten concepts associated with a problem. Let \( E_1, E_2, E_3, E_4, E_5 \) be 5 experts working on this problem using NCMs by selecting some attributes from the set \( C \). Suppose the expert \( E_1 \) works with the attributes \( \{C_1, C_3, C_4, C_5\} \) and the expert \( E_2 \) works with the attributes \( \{C_5, C_4, C_2, C_7\} \). The third expert \( E_3 \) works with the attributes \( \{C_6, C_9, C_8, C_{10}\} \).

The forth expert works with \( \{C_1, C_7, C_6\} \) and the fifth expert works with \( \{C_2, C_5, C_1, C_4\} \).

Now we see \( \{C_1, C_3, C_4, C_5\} \cup \{C_5, C_4, C_2, C_7\} \cup \{C_6, C_9, C_8, C_{10}\} = C \).
However the set \( \{C_5, C_4, C_2, C_7\} \) is replaced by \( \{C_1, C_2, C_7, C_6\} \) and still we get C.

However the sets \( \{C_1, C_3, C_4, C_5\} \) and \( \{C_6, C_8, C_9, C_{10}\} \) cannot be replaced by any other set. We can as in case of FCMs get the merged connection neutrosophic matrix of the merged neutrosophic cognitive maps model.

It is pertinent to keep on record that all the five experts work only with the NCMs.

Finally we describe the mixed FCMs and NCMs model. Suppose we have a problem which is associated with \( n \) attributes say \( C = \{c_1, \ldots, c_n\} \). Some s experts agree to work on the problem using some attributes from the set \( C \) using only the FCMs model. Some t expert wish to work on the problem using some attributes from the set \( C \) using the NCMs model only. Thus these \( t + s \) number of experts alone can contribute for the integrated merged mixed FCMs and NCMs model.

That is all the s-experts do not cover the set of \( n \) attributes neither the set of t-experts cover the set of all \( n \)-attributes only a subcollection from the s-experts and the t-experts alone are in a positive to cover all \( n \) concepts in \( C \).

Thus we are forced to merge a directed graph with a directed neutrosophic graph to arrive at a solution. The resultant model will be defined as the mixed merged FCMs and NCMs model.

This will be illustrated by the following example.

Let \( C = \{C_1, C_2, C_3, \ldots, C_{10}\} \) be the set of 10 attributes. Suppose 3 experts choose to work with the problem with some nodes from the set \( C \) using only NCMs model. Some 2 experts work the problem with some nodes from \( C \) using only the FCMs model. Thus 5 experts \( E_1, E_2, \ldots, E_5 \) work on the problem.
Let the three experts $E_1$, $E_2$ and $E_3$ work with the NCMs and the experts $E_4$ and $E_5$ work with the FCMs model.

Let the first expert $E_1$ work with the concepts $\{C_1, C_2, C_4, C_5\}$ and the second expert $E_2$ work with the nodes $\{C_6, C_3, C_7, C_9\}$. The third expert $E_3$ works with the nodes $\{C_4, C_5, C_6, C_9\}$. Thus all the three experts work only with the NCMs model.

Let the forth experts $E_4$ work with the nodes $\{C_1, C_2, C_3, C_8, C_9\}$ and the expert $E_5$ work with the nodes $\{C_4, C_5, C_{10}, C_7, C_2\}$.

In the first place we observe even when all the three experts $E_1$, $E_2$ and $E_3$ join together to get the merged NCMs still the nodes $C_8$ and $C_{10}$ are left out from the set $C$.

Now it is also observed that two experts $E_4$ and $E_5$ cannot give in the merged FCMs model accounting for all the nodes from $C$.

They also cannot account for $C_6$, for $C_6$ is missing. Thus we to get whole of $C$ define the merged mixed FCMs and NCMs model.

Now let $A$ be the neutrosophic graph associated with the first expert $E_1$ which is as follows.

![Neutrosophic Graph](image-url)
Let B be the directed neutrosophic graph given by second expert E_2.

\[ B = \]

Let C be the directed neutrosophic graph given by the third expert E_3 which is as follows:

\[ C = \]

Let D be the directed graph given by the fourth expert E_4 which is as follows:

\[ D = \]
Let $E$ be the directed graph of the FCMs model given by the fifth expert $E_5$.

Now the connection matrix of the neutrosophic graph $A$ be $M_A$ which is as follows:

$$M_A = \begin{bmatrix}
c_1 & 1 & 1 & 0 \\
c_2 & 0 & 1 & 1 \\
c_3 & 0 & 0 & 1 \\
c_4 & 0 & 1 & 0 \\
c_5 & 0 & 0 & 0
\end{bmatrix}.$$

Let $M_B$ be the connection matrix associated with the neutrosophic directed graph given by the second expert which is as follows:

$$M_B = \begin{bmatrix}
c_3 & 0 & 0 & 1 & 0 \\
c_6 & 0 & 0 & 1 & 1 \\
c_7 & 1 & 1 & 0 & 0 \\
c_8 & 0 & 0 & 1 & 0
\end{bmatrix}.$$
Let $M_c$ denote the connection neutrosophic matrix associated with the neutrosophic graph $C$.

\[
M_c = \begin{bmatrix}
    c_{4} & c_{5} & c_{6} & c_{9} \\
    c_{4} & 0 & 1 & 1 & 0 \\
    c_{5} & 0 & 0 & 0 & 1 \\
    c_{6} & 0 & 1 & 0 & 1 \\
    c_{9} & 1 & 0 & 0 & 0
\end{bmatrix}.
\]

Let $M_D$ denote the connection matrix of the directed graph $D$ given by the fourth expert $E_4$ which is as follows:

\[
M_D = \begin{bmatrix}
    c_{1} & c_{2} & c_{3} & c_{8} & c_{9} \\
    c_{1} & 0 & 0 & 0 & 1 & 0 \\
    c_{2} & 0 & 0 & 0 & 0 & 0 \\
    c_{3} & 1 & 0 & 0 & 1 & 0 \\
    c_{8} & 0 & 0 & 0 & 1 & 0 \\
    c_{9} & 0 & 1 & 0 & 0 & 0
\end{bmatrix}.
\]

Let $M_E$ denote the connection matrix of the directed graph $E$ given by the fifth expert $E_5$.

\[
M_E = \begin{bmatrix}
    c_{2} & c_{4} & c_{5} & c_{7} & c_{10} \\
    c_{2} & 0 & 1 & 1 & 0 & 0 \\
    c_{4} & 1 & 0 & 0 & 1 & 1 \\
    c_{5} & 0 & 0 & 0 & 0 & 0 \\
    c_{7} & 0 & 0 & 0 & 0 & 1 \\
    c_{10} & 0 & 0 & 1 & 0 & 0
\end{bmatrix}.
\]
Now let $F$ give the merged graph of the first two experts $E_1$ and $E_2$ which is as follows:

\[ F = \]

We see the graphs $A$ and $B$ cannot be merged to a graph so $F$ does not exist as the graphs have no common vertex or edge.

Let $G$ denote the merged graph of the experts 1 and 3 which is as follows:

The merged graph $G$ is a neutrosophic graph.

Let $M_G$ be the connection matrix of $G$. 
The merged graph $H$ of experts 1 and 4 is as follows:

$H = \begin{pmatrix}
C_1 & C_2 & C_3 & C_4 & C_5 & C_6 & C_7 & C_8 & C_9 \\
C_1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
C_2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
C_3 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
C_4 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
C_5 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\
C_6 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
C_7 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
C_8 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
C_9 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{pmatrix}$

Let $M_H$ be the connection matrix of the merged graph $H$. 

Let $M_G = c_1 \begin{pmatrix} 0 & 1 & 1 & 0 & 0 & 0 \end{pmatrix}, c_2 \begin{pmatrix} 0 & 0 & 1 & 1 & 0 & 0 \end{pmatrix}, c_3 \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}, c_4 \begin{pmatrix} 0 & 0 & 0 & 1 & 1 & 0 \end{pmatrix}, c_5 \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 1 \end{pmatrix}, c_6 \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}, c_7 \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, c_8 \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}, c_9 \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$
Let $I$ denote the merged graph of the experts (1) and (5) which is as follows:

Let $M_I$ denote the connection matrix of the merged graph $I$ which is as follows:

$$
M_I = 
\begin{bmatrix}
0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 \\
0 & 1 & 0 & 1 & 1 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 
\end{bmatrix}
$$
The merged graph $J$ of experts 2 and 3 are as follows:

![Graph representation]

The merged connection matrix $M_J$ of the merged graph $J$ is as follows:

$$
M_J = \begin{bmatrix}
0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1
\end{bmatrix}
$$

Now we get the merged graph $K$ given by the experts 2 and 4 which is as follows:
The merged connection matrix $M_K$ of the directed graph $K$ is as follows:

$$
M_K = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 
\end{bmatrix}.
$$

Let $L$ denote the merged graph given by the experts 2 and 5 which is as follows:
The merged connection matrix $M_L$ of the neutrosophic merged graph $L$ is as follows:

$$M_L = \begin{bmatrix}
    c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_9 & c_{10} \\
    c_2 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
    c_3 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
    c_4 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
    c_6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_7 & 0 & 1 & 1 & 0 & 1 & 0 & 0 \\
    c_9 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_{10} & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}$$

Let $M$ denote the merged graph of the experts 3 and 4 which is as follows:
Let $M_M$ be the merged neutrosophic connection matrix of the merged neutrosophic graph $M$ which is as follows:

$$
M_M = \begin{bmatrix}
c_{i1} & c_{i2} & c_{i3} & c_{i4} & c_{i5} & c_{i6} & c_{i7} & c_{i8} & c_{i9} \\
c_{1} & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
c_{2} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_{3} & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
c_{4} & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
c_{5} & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
c_{6} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
c_{7} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_{8} & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\
c_{9} & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 
\end{bmatrix}
$$

Let $N$ denoted the merged neutrosophic graphs of the experts 3 and 5.
Let $M_N$ be the merged neutrosophic connection matrix of the merged neutrosophic graph $N$ which is as follows:

\[
M_N = \begin{bmatrix}
    c_2 & c_4 & c_5 & c_6 & c_7 & c_9 & c_{10} \\
    c_2 & 0 & 1 & 1 & 0 & 0 & 0 \\
    c_4 & 1 & 0 & 1 & 1 & 1 & 0 \\
    c_5 & 0 & 0 & 0 & 0 & 1 & 0 \\
    c_6 & 0 & 0 & 1 & 0 & 0 & 1 \\
    c_7 & 0 & 0 & 0 & 0 & 0 & 1 \\
    c_9 & 0 & 1 & 0 & 0 & 0 & 0 \\
    c_{10} & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}
\]

Let $O$ denote the merged graph given by the experts 4 and 5 which is as follows:
We see the merged graph is not a neutrosophic graph. Infact a usual graph.

Hence the related connection merged matrix $M_o$ of $O$ will not be a neutrosophic graph.

Hence the merged graph of these two expert also work with the FCM and not a NCM.

Only when one of them works with a neutrosophic graph and other the usual graph we will get a merged neutrosophic graph hence the merged connection matrix is also a neutrosophic matrix forcing the dynamical system associated with it to be a NCM and not a FCM.

We now give the connection matrix $M_o$ of the merged graph $O$ which as follows:
Merged FCMs and NCMs Models

We see this merged model is only a merged FCMs model. All nodes except $C_6$ is present. We can also merge the directed graphs of the experts 1, 2 and 4.

Let the merged graph of the experts 1, 2 and 3 be denoted by $P$ which is as follows:

The merged connection matrix $M_P$ of $P$ is as follows:
\begin{align*}
M_P &= 
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}.
\end{align*}

Let Q denote the merged graph of the expert 1, 2 and 4 which is as follows:

Let $M_Q$ denote the merged connection matrix of the merged graph Q.
Let $R$ denote the merged graph of the experts 1, 2 and 5 which is as follows:

Let $R$ denote the merged graph of the experts 1, 2 and 5 which is as follows:

% Let $R$ denote the merged graph of the experts 1, 2 and 5 which is as follows:

\[
M_Q = \begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \\
  c_1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
  c_2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
  c_3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
  c_4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
  c_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
  c_7 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
  c_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
  c_9 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 
\end{bmatrix}.
\]

% Let $R$ denote the merged graph of the experts 1, 2 and 5 which is as follows:

Let $R$ denote the merged graph of the experts 1, 2 and 5 which is as follows:

* Let $M_R$ denote the merged connection matrix of the graph $R$ which is as follows:

% Let $M_R$ denote the merged connection matrix of the graph $R$ which is as follows:

% Let $M_R$ denote the merged connection matrix of the graph $R$ which is as follows:

\[
M_R = \begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 \\
  c_1 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
  c_2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
  c_3 & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
  c_4 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
  c_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_6 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\
  c_7 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
  c_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
  c_9 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 
\end{bmatrix}.
\]
Let \( S \) denote the merged graph of the experts 1, 3 and 4 which as follows:

![Graph](image_url)

Let \( M_S \) denote the merged connection matrix of the merged graph \( S \) which is as follows:

\[
M_S = \begin{bmatrix}
c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} \\
c_1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
c_2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
c_3 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
c_4 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
c_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_6 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
c_7 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 \\
c_8 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
c_9 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
c_{10} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Let $T$ denote the merged graph of the experts 1, 3 and 5 which is as follows:

Let $M_T$ denote the merged connection matrix of the merged graph $T$ which is as follows:

\[
M_S = \begin{bmatrix}
c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} & c_{17} & c_{18} & c_{19} \\
c_{21} & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
c_{22} & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
c_{23} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
c_{31} & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\
c_{32} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_{33} & 1 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
c_{34} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
c_{35} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
c_{36} & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]
Let $U$ denote the merged graph of the graphs given by the experts 1, 4 and 5 which is as follows:

Let $M_U$ denote the merged neutrosophic connection matrix of the merged graph $U$ which is as follows:
\[
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 
\end{bmatrix}
\]

However \( M_0 \) and \( M_U \) are distinct one is a FCM model and other is a NCM model. Let \( V \) denote the merged graph of the experts 2, 3 and 4 which is as follows:

The merged connection matrix \( M_V \) associated with neutrosophic directed graph \( V \) is as follows:
Next let $W$ denote the merged graph given by the experts 2, 3 and 5 which is as follows:

Let $M_W$ denote the merged connection neutrosophic matrix of the merged graph $W$ which is as follows:
Next let $X$ denote the merged graph given by the experts 3, 4, 5 which is as follows:

Let $M_X$ denote the merged connection matrix of the merged graph $X$ which is as follows:
This merged graph $X$ gives the complete merged NCM model of all the 10 attributes. So we see three experts are sufficient to make up for the integrated NCMs model.

We now find the merged graph $Y$ given by the experts 1, 2, 3 and 4 which is as follows:

$$
M_X = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
\end{bmatrix}
$$
Let $M_Y$ denote the merged connection matrix of the merged graph $Y$ which is as follows:

$$
M_Y = 
\begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 1 & 0 & 0 & 0 & 0 
\end{bmatrix}.
$$

Now we get the merged graph $Z$ of the experts 1, 2, 3 and 5.
Let $M_Z$ be the merged connection matrix of the merged graph $Z$.

\[
M_Z = \begin{bmatrix}
0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Now we get the merged graph $\alpha$ of the four experts 2, 3, 4 and 5.

The connection merged matrix $M_\alpha$ of the merged graph $\alpha$ is as follows.
The matrix is a neutrosophic matrix with all the 10 concepts $C_1, C_2, \ldots, C_{10}$.

\[
\begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} \\
  c_1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
  c_2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
  c_3 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
  c_4 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
  c_5 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 \\
  c_6 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_{10} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{bmatrix}
\]

Now we give the merged directed graph $\beta$ given by 1, 3, 4 and 5.
Let $M_\beta$ denote the merged connection matrix of the merged graph $\beta$ which is as follows:

$$
M_\beta =
\begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 & c_8 & c_9 & c_{10} \\
  c_1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\
  c_2 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\
  c_3 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
  c_4 & 0 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 1 \\
  c_5 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
  c_6 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 1 & 0 \\
  c_7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
  c_8 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
  c_9 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
  c_{10} & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

We see this gives the integrated merged NCMs of the problem.

It is important to note that even without the inclusion of all the experts a few of the experts can contribute to the integrated merged NCMs.

It may so happen that we may get some integrated merged NCMs leading to the complete problems. They may also be distinct.

So in this case to overcome the problem of selecting which set of experts define a new NCMs model called the average NCMs model.

We can adopt the method new average NCMs model for these merged NCMs which is introduced in the last chapter of this book.
Further we have defined in chapter IV the notion of Kosko Hamming distance for NCMs and FCMs. We can use this distance to study how far two systems vary from each other and if the distance / deviation is small we accept the solution.

If on the other hand the deviation is large we investigate the cause for it. Thus by these methods applied to the merged models we can analyse the problem in a sensitive and in a productive way which will help. The experts draw appropriate conclusions.

We suggest a few problems for this chapter.

**Problem:**

1. Obtain some special features enjoyed by merged graphs.

2. Show merged FCMs are in general better than the combined FCMs.

3. Use merged FCMs to study some special real world problem.

4. Find the special features enjoyed by merged FCMs.

5. Give a real world problem illustration by using merged FCMs.

6. Describe with an example the merged NCMs.

7. Give an illustration in the real world problem the notion of the linked merged FCMs.

8. What are the advantages of using merged FCMs and linked merged FCMs?

9. Study question 8 for NCMs.
10. Describe with an example the notion of specially merged FCMs (NCMs).

11. Describe with an example the notion of mixed merged FCMs and NCMs.

12. Prove these new techniques of merged FCMs (NCMs) make the analysis more sensitive with a better solution.
Chapter Three

**Kosko - Hamming Distance in FCMs and NCMs**

In this chapter authors for the first time introduce the new notion of Kosko - Hamming distance (K-H) distance in vectors related to FCMs and NCMs. However K-H distance is nothing but a Hamming distance defined for a special type of vectors \( x, y \in V^n \) which enjoy same initial properties.

So K-H distance in general cannot be defined for any two arbitrary \( x, y \in V^n = \{(x_1, \ldots, x_n) \mid x_i \in Z_2 = \{0, 1\}, 1 \leq i \leq n\} \), they are resultant state vectors related with a FCMs or a NCMs model. Such study is innovative and show how far two experts agree or defer over an issue.

Such study will help to get more information on the problem their by making the solution more closer to truth and feasibility.
As said at the outset Kosko-Hamming distance is a distance function depending on the NCMs or FCMs. So for one to define a K-H distance they basically need the resultants and properties of the FCMs (or NCMs).

Let $C = \{c_1, \ldots, c_n\}$ be the $n$ concepts / attributes related with the problem. Suppose some experts work with the same problem using all the $n$ attributes using a FCM (or NCM) model.

However the researcher is interested in comparing the results of the $t$ experts and wants to know how much they differ over the predicted results.

To this end we denote by the on state of the node $C_1$ alone by $X_1 = (1, 0, 0, \ldots, 0)$ the on state of the node $C_2$ by $X_2 = (0, 1, 0, \ldots, 0)$ and so on and the on state of the node $C_i$ by $X_i = (0, 0, \ldots, 0, i, 0, \ldots, 0)$ and finally the state on the node $C_n$ by $X_n = (0, 0, \ldots, 0, 1)$.

Let us denote the $t$ experts by $E_1, E_2, \ldots, E_t$. Let $M_1$ denote the connection matrix of the FCM (or NCM) which serves as the dynamical system of the first expert.

Let $M_2$ be the connection matrix associated with the directed graph given by the second expert. $M_2$ acts as the dynamical system of the FCMs for the second expert and so on.

Thus $M_i$ denotes the connection matrix associated with the directed graph given by the $i^{th}$ expert and $M_i$ serves as the dynamical system of the FCMs for $E_i$ the $i^{th}$ expert. Finally $M_i$ denotes the connection matrix associated with the directed graph given by the $i^{th}$ expert $E_i$ and $M_i$ serves as the dynamical system of the FCMs.
Now to find Kosko Hamming distance denoted $d_k$ we need the following conditions to be satisfied by the two vectors in $\mathbb{Z}_2^n = \{(a_1, \ldots, a_n) | a_i \in \{0, 1\}, 1 \leq i \leq n\}$ for which the Kosko-Hamming distance can be found.

(i) For any initial state vector $A = (a_1, \ldots, a_n)$ we find using each of the $t$ experts the hidden pattern by finding $A_{Mi}$, $i = 1, 2, \ldots, t$.

The resultant state vector that is the hidden pattern may be a fixed point or a limit cycle. Let $Y_1^A, Y_2^A, \ldots, Y_t^A$ be the hidden patterns of the initial state vector $A$ of the $t$ experts. We can define the Kosko-Hamming distance only on the set of vectors $Y = (Y_1^A, Y_2^A, \ldots, Y_t^A)$ clearly

$Y_i^A \in \{(a_1, a_2, \ldots, a_n) | a_j \in \{0, 1\}, 1 \leq j \leq n\}$ and $i = 1, 2, \ldots, t$.

$d_k(Y_r^A, Y_s^A) = \{\text{the Hamming distance between the vectors } Y_r^A \text{ and } Y_s^A \text{ in the set } Y\}$

$= d_k(E_r, E_s)$ because $E_r$ is the $r^{th}$ expert and $E_s$ is the $s^{th}$ expert and $Y_r^A$ and $Y_s^A$ are their the resultant vectors given by the experts on the initial state vector $A$.

Now clearly (1) $d_k(x, y) \geq 0$ or (2) $d_k(x, y) \leq n$.

If $d_k(E_r, E_s) = 0$ we say both the experts $E_r$ and $E_s$ agree on the outcome of the initial state vector $A$.

If $d_k(E_r, E_s) = m$ has a bigger value $0 \leq m \leq n$ we say the two experts do not agree over the outcome of the initial state vector $A$. So the experts have a different opinion hence this sort of comparison can make the study more sensitive and very closer to the solution or suggestions in
the investigation of the problem. In the final chapter of this book we have given about averages of the FCMs and NCMs and certainly this notion will be used there.

Now we will technically describe the working of the problem. Suppose we have n concepts or attributes related with the problem. We have say t experts E_1, E_2, ..., E_t working with the problem using all the n concepts / attributes using either FCM or NCM. Suppose M_1, M_2, ..., M_t are the t connection matrices given by the t experts respectively.

Let X_1 = (1, 0, 0, ..., 0), X_2 = (0, 1, 0, ..., 0), X_3 = (0, 0, 1, ..., 0) so on X_n = (0, 0, ..., 0, 1) be the initial state vectors with which we work.

X_1M_1 is calculated and the hidden pattern of X_1 is a fixed point or a limit cycle given by

Y^1_i = {(a_1, a_2, ..., a_n) | a_i \in \{0, 1\}, 2 \leq i \leq n}.

Let the hidden pattern for X_2 = {0, 1, 0, ..., 0) given by X_2M_1 be denoted by

Y^2_i = {(a_1, a_3, ..., a_n) | a_i \in \{0, 1\}, i \neq 2, i = 1, 3, ..., n}.

The hidden pattern for the initial state vector X_3 = (0 0 1 0 ... 0) is as follows.

The hidden pattern X_3M of the initial state vector X_3 be

Y^3_i = {(a_1, a_2, a_4, ..., a_n) | a_i \in \{0, 1\} i = 1, 2, 4, ..., n}.

Finally for X_i = (0, 0, ..., 0, i, 0, ..., 0) the initial vector.
Let $X_i M_1$ give the resultant vector that is the hidden pattern of $X_i M_i$ as

$$Y_i = \{(a_1, a_2, \ldots, a_{i-1}, 1, a_{i+1}, a_{i+2}, \ldots, a_n) \mid a_j \in \{0, 1\}; j = 1, 2, \ldots, i-1, i+1, \ldots, n\} \text{ and so on.}$$

Finally for $X_n = (0, 0, \ldots, 0, 1)$ we get $X_n M_1$’s resultant state vector of $X_n$’s to be

$$Y_n = \{(a_1, \ldots, a_{n-1}, 1) \mid a_i \in \{0, 1\} \text{ and } 1 \leq i \leq n-1\}.$$ 

On similar lines for each of the initial state vectors $X_1 = (1 0 0 \ldots 0), X_2 = (0 \ldots 1 0)$ and so on $X_n = (0, \ldots, 1, 0)$ we get using the dynamical system given by the second expert $E_2$ the hidden pattern to be $Y_2^1 = (1, a_2, \ldots, a_n), Y_2^2 = (a_1, 1, a_3, \ldots, a_n)$ and so on.

$$Y_i^1 = (a_1, \ldots, a_{i-1}, 1, a_{i+1}, \ldots, a_n), \ldots, Y_n^1 = (a_1, a_2, \ldots, a_{n-1}, 1).$$

Similarly for expert 3 and so on.

For expert $i$ we get the resultant of $X_1, \ldots, X_n$ to be $Y_i^1 = (1, a_2, \ldots, a_n), Y_i^2 = (a_1, 1, a_3, \ldots, a_n)$ and so on.

$$Y_i^1 = (a_1, a_2, \ldots, a_{i-1}, 1, a_{i+1}, \ldots, a_n), \ldots, Y_n^1 = (a_1, a_2, \ldots, a_{n-1}, 1).$$

Thus for the $t^{th}$ expert $E_t$ using the connection matrix $M_t$ of the FCMs or NCMs we for the initial state vectors $X_1, X_2, \ldots, X_n$ get the resultant state vectors or hidden patterns to be $Y_t^1 = (1, a_2, \ldots, a_n), Y_t^2 = (a_1, 1, a_3, \ldots, a_n)$ and so on $Y_n^1 = (a_1, a_2, \ldots, a_{n-1}, 1).$

Now we give the sample representation of how the Kosko-Hamming distance is calculated this distance can be found provided at that time of comparison both the
experts work only on the same initial state vector otherwise the Kosko-Hamming distance cannot be found or it is meaningless. So if we compare the jth expert with a pth expert \(1 \leq j, p \leq t\), we denote it by \(d_k(Y_j^i, Y_p^p)\) and find that value.

That value can be found for \(i = 1, 2, \ldots, n\). So we can say for the ith initial state vector how close or how far the two experts \(j\) and \(p\) are in their predictions, however keeping in mind that the experts opinions show his ignorance or capabilities in tackling the problem and it also varies from expert to expert and problem to problem.

This is tabulated in the following form.

Hidden pattern of the initial state vectors given by the experts and K-H distance

<table>
<thead>
<tr>
<th>Initial state vector</th>
<th>(E_1)</th>
<th>(\ldots)</th>
<th>(E_i)</th>
<th>(\ldots)</th>
<th>(E_t)</th>
<th>(d_k(E_1,E_2))</th>
<th>(\ldots)</th>
<th>(d_k(E_{t-1}, E_t))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_1)</td>
<td>(Y_1^i)</td>
<td>(\ldots)</td>
<td>(Y_1^i)</td>
<td>(\ldots)</td>
<td>(Y_1^i)</td>
<td>(n_1^i)</td>
<td>(\ldots)</td>
<td>(n_1^i)</td>
</tr>
<tr>
<td>(X_2)</td>
<td>(Y_2^i)</td>
<td>(\ldots)</td>
<td>(Y_2^i)</td>
<td>(\ldots)</td>
<td>(Y_2^i)</td>
<td>(n_2^i)</td>
<td>(\ldots)</td>
<td>(n_2^i)</td>
</tr>
<tr>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\ldots)</td>
<td>(\vdots)</td>
<td>(\ldots)</td>
<td>(\vdots)</td>
<td>(\vdots)</td>
<td>(\ldots)</td>
<td>(\vdots)</td>
</tr>
<tr>
<td>(X_n)</td>
<td>(Y_n^i)</td>
<td>(\ldots)</td>
<td>(Y_n^i)</td>
<td>(\ldots)</td>
<td>(Y_n^i)</td>
<td>(n_n^i)</td>
<td>(\ldots)</td>
<td>(n_n^i)</td>
</tr>
</tbody>
</table>

This table is called one initial on state vectors K–H distance comparison table.

Note that \(Y_j^i\) gives the resultant state vector of the initial state vector \(X_j\) for the \(i^{th}\) expert in the above table; \(1 \leq i \leq t\) and \(1 \leq j \leq n\).
However it is pertinent to keep on record that these initial vectors $X_1, X_2, \ldots, X_n$ are by no means given any form of even sample representations of the resultant by the resultant of the initial state vector $X_1, \ldots, X_n$. That is we want to keep on record that if $X_1 + X_2 = (1 \ 1 \ 0 \ 0 \ \ldots \ 0) = X_{1,2}$ then

$$X_1 M_1 + X_2 M_1 \neq (X_1 + X_2) M_1$$

$$\neq (Y_1^i + Y_2^i) \text{ in general.}$$

This is true for every $X_i$, $i = 1, 2, \ldots, n$ and for every one of the connection matrices $M_1, \ldots, M_t$, so we are forced to work with $2^n$ vectors with entries from $n$-vectors in $\{(a_1, a_2, \ldots, a_n) | a_i \in \{0, 1\}, 1 \leq i \leq n\}$.

Now we can work with initial state vectors

$$X_{1,2} = (1 \ 1 \ 0 \ 0 \ \ldots \ 0),$$
$$X_{1,3} = (1 \ 0 \ 1 \ 0 \ \ldots \ 0),$$
$$X_{1,4} = (1 \ 0 \ 0 \ 1, 0 \ \ldots \ 0) \text{ and}$$
so on $X_{1,n} = (1, 0, \ldots, 0, 1)$.

Likewise

$$X_{2,3} = (0, 1, 1, 0, \ldots, 0),$$
$$X_{2,4} = (0, 1, 0, 1, 0, \ldots, 0)$$

and so on

$$X_{2,n} = (0, 1, 0, \ldots, 0, 1).$$
$$X_{t, n} = (0, \ldots, 0, 1, 0, \ldots, 1),$$
$$X_{t+1, n} = (0, 0, \ldots, 0, 1, 0, \ldots, 1)$$
and so on.

$$X_{n-1, n} = (0, 0, \ldots, n-1, n).$$
We also work with three on state which gives \( \text{nC}_3 \) number of such states.

Four number on state of nodes gives \( \text{nC}_4 \) number of such initial state vectors and so on.

Finally we get \( \text{nC}_{n-1} \) number of such state vectors.

However we see if \( X_i \) has resultant \( Y_i \) and \( X_j \) has resultant \( Y_j \) then

\[
X_iM + X_jM \neq Y_i + Y_j
\]

because of this only we are forced to work in a different way to arrive at a solution.

We will illustrate this situation by an example.

**Example 3.1:** Let

\[
M = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & -1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & -1 & -1 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & -1 & 0 \\
\end{bmatrix}
\]

be a matrix associated with a problem involving 10 concepts \( c_1, c_2, \ldots, c_{10} \).
Let $X_1 = (1 0 0 0 0 0 0 0 0) \) be the given initial state vector where only the $C_1$ node is in the on state and all other nodes are in the off state.

We find $X_1M = (0 0 0 0 1 1 -1 1) \) (after updating and thresholding we get $Y_1 = (1 0 0 0 0 1 1 0 1) \) ($\rightarrow$ denotes the vector has been updated and thresholded).

We find $Y_1M = (0 0 0 0 2 3 0 3) \rightarrow Y_2 = (1 0 0 0 0 0 1 0 1) \) = $Y_1$ \) … I

Thus the hidden pattern is a fixed point.

Next let us take $X_2 = (0 1 0 0 0 0 0 0 0) \) we see only the node $C_2$ is in the on state and all other nodes are in the off state. We find the resultant of the state vector $X_2$ on the dynamical system $M$.

$X_2 \rightarrow (0 0 0 0 0 1 1 1 0) \rightarrow (0 1 0 0 0 1 1 1 0) \) \) = $T_1$ (say)

$T_2M \rightarrow (0 0 0 0 0 3 2 3 0) \rightarrow (0 1 0 0 0 1 1 1) \) = $T_2$ \) (say) \) \) = $T_1$ \) \) \) … II

Using equations I and II we get

$X_1M + X_2M = Y_1 + T_1$

$= (1 0 0 0 0 1 1 0 1) + (0 1 0 0 0 1 1 1 0) \)

$= (1 1 0 0 0 0 0 1 1)$ \) \) \) III

Now consider $X_1 + X_2 = (1 0 0 0 0 0 0 0 0) + (0 1 0 0 0 0 0 0 0)$ \)

$= (1 1 0 0 0 0 0 0 0)$.

We find the effect of $X_1 + X_2$ on the dynamical system $M$. 
\[(X_1 + X_2) M = (0 \ 0 \ 0 \ 0 \ 0 \ 2 \ 2 \ 0 \ 1) \rightarrow (1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1) = S_1 \text{ (say)}\]

\[S_1M = (0 \ 0 \ 0 \ 0 \ 0 \ 4 \ 3 \ 0 \ 3) \rightarrow (1 \ 1 \ 0 \ 0 \ 0 \ 1 \ 1 \ 0 \ 1) = S_2 (= S_1) \quad \text{IV}\]

Hence this is also a fixed point.

However III and IV are distinct so in general while working with FCM or NCM we see for the dynamical system \(M: (X_1 + X_2) M \neq X_1M + X_2M\).

This forces us to find the Kosko-Hamming distance for every \(2^n\) elements in the set \(A = \{(a_1, \ldots, a_n) | a_i \in \{0, 1\}, 1 \leq i \leq n\}\).

Hence we have to find \(d_k\) for two vectors in \(A\) provided they are resultant of the same basic initial state vector used by both experts but only the dynamical system used by them viz the connection matrices used by the concerned two experts are different.

Further we cannot as in case of vector spaces think if we work for the state vectors which forms a basis set we will get the resultant for other vectors. This is clearly proved in the example.

So it is not very difficult to write a program to find using the matrix which serves as the dynamical system for the FCM (or NCM) and find the appropriate Kosko-Hamming distance of the resultant state vectors which are the hidden patterns of the state vectors.

So for the Kosko-Hamming distance function we in the first place should have two distinct dynamical systems given by two different experts working on the same
problem with same number of nodes. Secondly for $d_k$ to be defined at that time both experts should have worked only with the same initial state vector.

Only under these conditions we will be in a position to define $d_k$ and compare them.

We will illustrate the situation by an example or two.

**Example 3.2:** Let $E_1$ and $E_2$ be any two experts working on the same problem with same number of nodes. Let them work with seven nodes $c_1, c_2, \ldots, c_7$.

Let $G_1$ be the directed graph given by the expert $E_1$.

Let $M_1$ be the connection matrix associated with the graph $G_1$ which is as follows:

$$
C_1 \quad 1 \quad 1 \\
C_6 \quad 1 \quad 1 \\
C_7 \quad 1 \\
C_3 \\
C_5 \\
C_4 \\
C_2
$$
Let $G_2$ be the directed graph given by the second expert $E_2$ using the same set of concepts $C_1, C_2, \ldots, C_7$. 

Let $M_2$ be the connection matrix associated with the graph $G_2$ which is as follows:
Kosko Hamming distance in FCMs and NCMs

\[ M_2 = \begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \\
  c_1 & 0 & 1 & -1 & 1 & 0 & 0 & -1 \\
  c_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_3 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_4 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
  c_5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
  c_6 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\
  c_7 & 0 & 1 & 0 & 0 & 0 & -1 & 0 
\end{bmatrix}. \]

Now using these two matrices \( M_1 \) and \( M_2 \) of the FCM we work for the Kosko-Hamming distance between them.

Let \( X_1 = (1\ 0\ 0\ 0\ 0\ 0\ 0) \) be the initial state vector. To find the effect of \( X_1 \) on the dynamical system \( M_1 \) and \( M_2 \) respectively.

\[
X_1 M_1 = (0\ 0\ 0\ 1\ 1\ 1\ -1) \rightarrow (1\ 0\ 0\ 1\ 1\ 1\ 0) = Y_1
\]

\[
Y_1 M_1 = (2\ 0\ 0\ 1\ 1\ 1\ -1) \rightarrow (1\ 0\ 0\ 1\ 1\ 1\ 0) = Y_2 (= Y_1).
\]

\[
X_1 M_2 = (0\ 1\ -1\ 1\ 0\ 0\ -1) \rightarrow (1\ 1\ 0\ 1\ 0\ 0\ 0) = Z_1
\]

\[
Z_1 M_2 = (1\ 1\ -1\ 1\ 0\ 1\ -1) \rightarrow (1\ 1\ 0\ 1\ 0\ 1\ 0) = Z_2
\]

\[
Z_2 M_2 \rightarrow (1\ 1\ 0\ 1\ 0\ 1\ 0) = Z_3 (= Z_2).
\]

\( X_1 M_1 \) gives the hidden pattern as \((1\ 0\ 0\ 1\ 1\ 1\ 0)\) and \( X_1 M_2 \) gives the hidden pattern as \((1\ 1\ 0\ 1\ 0\ 1\ 0)\).

We see \( d_k((1\ 0\ 0\ 1\ 1\ 1\ 0)\ (1\ 1\ 0\ 1\ 0\ 1\ 0)) = 2. \)
So from the Kosko-Hamming distance we see for the state vector \( X_1 = (1 0 0 0 0 0 0) \) both the experts do agree or approximately close in their predictions.

Now let us consider the on state of the vector \( S_1 = (0 0 1 0 0 0) \).

To find the effect of \( S_1 \) on \( M_1 \) and \( M_2 \).

\[
S_1M_1 = (1 0 0 0 0 0 0) \mapsto (1 0 0 1 0 0 0) = (P_1 \text{ say}) \\
P_1M_1 = (1 0 0 1 1 1 -1) \mapsto (1 0 0 1 1 1 0) = (P_2 \text{ say}) \\
P_2M_1 = (2 0 0 1 1 1 0) \mapsto (1 0 0 1 1 1 0) = P_3 (= P_2). \\
S_1M_2 = (1 0 0 0 0 1 0) \mapsto (1 0 0 1 0 1 0) = R_1 \\
R_1M_2 = (1 1 -1 2 0 1 -1) \mapsto (1 1 0 1 0 1 0) = R_2 \\
R_2M_2 = (1 1 -1 2 0 1 -2) \mapsto (1 1 0 1 0 1 0) = R_3 (= P_2).
\]

We find \( d_k(E_1, E_2) \)
\[
= d_k((1 0 0 1 1 1 0), (1 1 0 1 0 1 0)) \\
= 2.
\]

Now we will find the Kosko - Hamming distance between the two experts opinion for the state vector \( S_1 = (1 0 0 1 0 0 0) \). We find both \( S_1M_1 \) and \( S_1M_2 \) in the following.

Consider
\[
S_1M_1 = (1 0 0 1 1 1 -1) \mapsto (1 0 0 1 1 1 0) = T_1 \\
T_1M_1 = (1 0 0 1 1 1 0) = T_2 (=T_1) \quad \ldots \quad I
\]

\[
S_1M_2 = (1 1 -1 1 0 1 -1) \mapsto (1 1 0 1 0 1 0) = P_1 \text{ (say)} \\
P_1M_2 = (1 1 -1 1 0 1 -1) \mapsto (1 1 0 1 0 1 0) = P_2
\]

Now \( d_k((1 0 0 1 1 1 0), (1 1 0 1 0 1 0)) = 2. \)
We see the effect or resultant of the initial state vectors \((1 0 0 0 0 0), (0 0 0 1 0 0)\) and \((1 0 0 1 0 0)\) using \(M_1\). The hidden pattern of all the three vectors are \((1 0 0 1 1 0), (1 0 0 1 1 1)\) and \((1 0 0 1 1 0)\) respectively. That is the hidden pattern of all the three initial state vectors are the same in case of the dynamical system \(M_1\) given by the first expert.

The hidden pattern associated with the initial state vector \((1 0 0 0 0 0), (0 0 0 1 0 0)\) and \((1 0 0 1 0 0)\) using the \(M_2\) are \((1 1 0 1 0 1 0), (1 1 0 1 0 1 0), (1 1 0 1 0 1 0)\) respectively. In case of the matrix \(M_2\) or the second expert \(E_2\) also we see the hidden patterns of the three vectors are the same.

Consider

\[ A_1 = (0 0 0 0 0 1 0) \]

to be the initial state vector for which we wish to find the hidden pattern using \(M_1\).

\[ A_1M_1 = (0 0 0 0 0 0 0) \rightarrow (0 0 0 0 0 1 0) \]

a fixed point with no change.

Now we find \(A_1M_2 = (0 0 0 1 0 0 -1) \rightarrow (0 0 0 1 0 1 0) = A_2\)

\[ A_2M_2 = (1 0 0 1 0 1 0) \rightarrow (1 0 0 1 0 1 0) = A_3 \]

\[ A_3M_2 = (1 1 -1 2 0 1 -1) \rightarrow (1 1 0 1 0 1 0) = A_4 \]

\[ A_4M_2 = (1 1 -1 1 0 1 -1) \rightarrow (1 1 0 1 0 1 0) = A_4 \]

We see

\[ d_k(E_1, E_2) = d_k((0 0 0 0 0 1 0), (1 1 0 1 0 1 0)) = 3. \]

Now we find the effect of \(B_1 = (0 0 0 0 0 0 1)\) on the dynamical systems \(M_1\) and \(M_2\).
\[ B_1 M_1 = (-1 \ 0 \ 0 \ 0 \ 0 \ 0) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 1) \] no change.

Now \[ B_1 M_2 = (-1 \ 0 \ 0 \ 0 \ 0 \ -1 \ 0) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1) \] no change. Thus \[ d_k(E_1, E_2) = d_k((0 \ 0 \ 0 \ 0 \ 0 \ 1), (0 \ 0 \ 0 \ 0 \ 0 \ 1)) = 0. \]

That is the distance between these two resultant vectors is zero. That is for the initial state vector \( B_1 \) we see both the experts agree upon the effect that is why the Kosko-Hamming distance is zero.

Thus Kosko-Hamming distance measures how far two experts agree on the effect of a initial state vector or how much they disagree upon it.

Such type of study is new and for the first time authors study this as it would throw light on the deviations from one expert to another while analyzing the problem using FCMs or NCMs.

Let us now study the same situation for NCMs for the same problem by the experts \( E_3 \) and \( E_4 \). Now let \( G_3 \) be the neutrosophic directed graph given by the expert \( E_3 \).
Let $M_3$ be the neutrosophic connection matrix associated with the neutrosophic directed graph which is as follows:

$$
M_3 = \begin{bmatrix}
0 & 1 & -1 & 1 & 1 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 0 & 0 \\
-1 & 1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & -1 \\
-1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

Let $G_4$ be the directed neutrosophic graph given by the forth expert on the same problem.

Let $M_4$ denote the neutrosophic connection matrix associated with the neutrosophic directed graph $G_4$. 
144 New Techniques to Analyse the Prediction of Fuzzy Models

\[
M_4 = \begin{bmatrix}
    c_1 & c_2 & c_3 & c_4 & c_5 & c_6 & c_7 \\
    c_1 & 0 & 1 & 0 & 0 & -1 & 0 \\
    c_2 & 0 & 0 & 0 & 1 & 0 & 1 \\
    c_3 & -1 & 0 & 0 & 0 & 0 & 0 \\
    c_4 & 1 & 0 & 0 & 0 & 0 & 0 \\
    c_5 & 0 & 0 & 0 & 0 & -1 & 0 \\
    c_6 & 1 & 0 & 0 & 0 & 0 & -1 \\
    c_7 & -1 & 1 & 0 & 0 & 0 & -1 \\
\end{bmatrix}
\]

Now we will find the on state of the node $C_1$ alone. That is let $X_1 = (1 0 0 0 0 0 0)$. To find the effect of $X_1$ on $M_3$ and $M_4$.

\[
X_1M_3 = (0 1 1 1 0 0) \rightarrow (1 1 0 1 1 0 0) = Y_1 \text{ (say)}
\]

\[
Y_1M_3 = (0 1 1 1 0 0) \rightarrow (1 1 0 1 1 0 0) = Y_1 \text{ (say)}
\]

\[
Y_2M_3 = (1 1 1 1 1 0 0) \rightarrow (1 1 1 1 1 0 0) = Y_3 \text{ (say)}
\]

\[
Y_3M_3 = (1 1 1 1 1 0 0) \rightarrow Y_4 \text{ (}=Y_3 \text{ say).}
\]

Thus the hidden pattern is a fixed point.

Now consider

\[
X_1M_4 = (0 1 0 1 1 0 -1) \rightarrow (1 1 0 1 1 0 0) = Z_1 \text{ say}
\]

\[
Z_1M_4 = (1, 1+1, 0, 1, I+1, 0, -1 + 1) \rightarrow (1 1 0 1 1 0 1) = Z_2 \text{ (say)}
\]

\[
Z_2M_4 = (1 1 0 1 1 0 1) = Z_3 \text{ (}=Z_2 \text{ say is a fixed point.}
\]

Now we have not yet defined Kosko-Hamming distance or for that matter Hamming distance in case of
neutrosophic n-tuples of the form \((a_1, a_2, \ldots, a_n)\) where 
\(a_i \in \{\langle Z_2 \cup I \rangle, \langle Z \cup I \rangle \text{ or } \langle R \cup I \rangle \text{ and so on}\} \ 1 \leq i \leq n.\)

We define this now in the following.

Let \(x = (x_1, \ldots, x_n)\) and \(y = (y_1, \ldots, y_n) \in \{\langle Z_2 \cup I \rangle, \langle Z \cup I \rangle \text{ or } \langle Q \cup I \rangle \text{ or } \langle R \cup I \rangle\}\) we define the Hamming distance \(d(x, y) = d((x_1, \ldots, x_n), (y_1, \ldots, y_n)) = t\) where \(t\) a positive integer denotes the number of places in which \(x\) differs from \(y\), \(0 \leq t \leq n.\)

We will illustrate this situation.

Let \(x = (I, 0, 1, I+1, I, 1, 0)\) and \(y = (1, 0, I, 1 + I, I, I, 0)\) then the Hamming distance \(d(x, y) = 3.\) This is the way Hamming distance is defined.

Let \(x = (1, 1, 1, 1, I, I, I, 0, 1 + I)\) and \(y = (2, 3, 4, 1, I, 1+I, 3+5I, 7I, 8).\)

Now \(d(x, y) = 7.\) Thus the Hamming distance between \(x\) and \(y\) is 7 that is \(x\) differs from \(y\) in seven places.

Now to define Kosko-Hamming distance we need two state vectors associated with two NCMs working on the same initial state vector over the same problem using the same number of concepts. This is a basic need for one to define the Kosko-Hamming distance for two resultant vectors. Such study helps one to compare how far two experts agree or disagree over the same nodes influence on their respective dynamical systems. Thus in this case from the example we see
\[d_k(E_3, E_4) = d_k((1, I, I, 1, 0, 0), (1, I, 0, 1, 0, 1)) = 2,\]
which shows they agree on majority of the nodes and disagree only on the two nodes.
Nothing prevents us from comparing the experts $E_i$ and $E_j$ where one of them works on the NCMs and other works using FCMs but both work on the same problem with the same set of nodes.

We now find for the initial state vector $X_3 = (0 \ 0 \ 1 \ 0 \ 0 \ 0 \ 0)$ the effect of $X_3$ on $M_3$ and $M_4$ respectively.

$$X_3M_3 = (-1, 1, 0, 0, 1, 0, 0) \rightarrow (0, 1, 0, 1, 0, 0) = Y_1$$ (say)

$$Y_1M_3 = (2I-1, I, 0, I, 0, 0) \rightarrow (I, I, 0, I, 0, 0) = Y_2$$ (say)

$$Y_2M_3 = (2I-1, 2I, 0, I, 0, 0) \rightarrow (I, I, I, I, 0, 0) = Y_3$$ (say)

$$Y_3M_3 = (I, 2I, 0, I, 0, 0) \rightarrow (I, I, I, I, 0, 0) = Y_4$$ (= $Y_3$) (say); clearly the hidden pattern is a fixed point.

Now we find

$$X_3M_4 = (-1, 0, 1, 0, 0, 0) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 0) = X_3$$

So as far as $X_3$ on the dynamical system $M_3$ is concerned we see it is fixed point.

Now we find

$$d_k (E_3, E_4)$$

$$= d_k((I, I, 1, I, 1, 0, 0), (0, 0, 1, 0, 0, 0)) = 4.$$  

Thus we see both the experts have different opinion as far as the node $C_3$ is concerned.
Consider the on state of the node $X_7 = (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)$ to find the effect of $X_7$ of the dynamical systems $M_3$ and $M_4$.

$$X_1M_3 = (-1 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1) \rightarrow (0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 1)$$

is a fixed point.

$$X_7M_4 = (-1 \ 1 \ 0 \ 0 \ 0 \ -1 \ 0) \rightarrow (0 \ 1 \ 0 \ 0 \ 0 \ 0 \ 1) = P_1 \text{(say)}$$

$$P_1M_4 = (-1 \ 1 \ 0 \ 0 \ I \ -1 \ I) \rightarrow (0 \ 1 \ 0 \ 1 \ 0 \ 1) = P_2 \text{ say}$$

$$P_2M_4 = (-1+I, 2I, 0, 0, I, -I, I)$$
$$\rightarrow (I, I, 0, 0, I, 0, 1)$$
$$= P_3 \text{ (say)}$$

$$P_3M_4 = (I-1, 3I, 0, 0, I, 0, 0)$$
$$\rightarrow (I, I, 0, 0, I, 0, 1)$$
$$= P_4 \text{ (say)} = P_3.$$ 

Thus

$$d_k(E_3, E_4)$$
$$= d_k((0, 0, 0, 0, 0, 0, 1), (I, I, 0, 0, I, 0, 1))$$
$$= 3.$$

This is the way comparisons are performed between $M_3$ and $M_4$.

We will tabulate in a table of the four experts two of them working using FCMs and two using NCMs on the same problem.
### Table 1: New Techniques to Analyse the Prediction of Fuzzy Models

<table>
<thead>
<tr>
<th>C₁</th>
<th>E₁</th>
<th>E₂</th>
<th>E₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1000000)</td>
<td>(1001110)</td>
<td>(1101010)</td>
<td>(111100)</td>
</tr>
<tr>
<td>(0100000)</td>
<td>(0100000)</td>
<td>(0100000)</td>
<td>((111100) or (110100))</td>
</tr>
<tr>
<td>(0010000)</td>
<td>(0010000)</td>
<td>(0010000)</td>
<td>(110100)</td>
</tr>
<tr>
<td>(0001000)</td>
<td>(1001110)</td>
<td>(1101010)</td>
<td>(1101100)</td>
</tr>
<tr>
<td>(0000100)</td>
<td>(1001110)</td>
<td>(0000100)</td>
<td>(1000100)</td>
</tr>
<tr>
<td>(0000010)</td>
<td>(1001110)</td>
<td>(1101010)</td>
<td>(110110)</td>
</tr>
<tr>
<td>(0000001)</td>
<td>(0000001)</td>
<td>(0000001)</td>
<td>(0000001)</td>
</tr>
<tr>
<td>(1100000)</td>
<td>(1101110)</td>
<td>(1101010)</td>
<td>(111100)</td>
</tr>
<tr>
<td>(1000100)</td>
<td>(1001110)</td>
<td>(1101010)</td>
<td>(1101100)</td>
</tr>
<tr>
<td>(0110010)</td>
<td>(1111110)</td>
<td>(1101010)</td>
<td>(111100)</td>
</tr>
<tr>
<td>(1111000)</td>
<td>(1111110)</td>
<td>(111010)</td>
<td>(111100)</td>
</tr>
</tbody>
</table>

### Table 2: New Techniques to Analyse the Prediction of Fuzzy Models

<table>
<thead>
<tr>
<th>E₄</th>
<th>dₙ(E₁, E₂)</th>
<th>dₙ(E₁, E₃)</th>
<th>dₙ(E₁, E₄)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1101101)</td>
<td>2</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>(1101101)</td>
<td>0</td>
<td>4 or 3</td>
<td>4</td>
</tr>
<tr>
<td>(0010000)</td>
<td>0</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>(1101101)</td>
<td>2</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>(1100101)</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>(1101101)</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>(0100101)</td>
<td>0</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>(1101101)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>(1101101)</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>(1111111)</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>(1111101)</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>
However one has to work with $\gamma C_1 + \gamma C_2 + \gamma C_3 + \ldots + \gamma C_5 + \gamma C_6$ number of possible state vectors and find the relative 6 columns to compare them.

For the 11 initial vectors state vectors tabulated above we see experts one and two agree very rarely for the maximum Kosko-Hamming distance in 3. So also experts 1 and 3 agree in this manner. However experts E_1 and E_4 does not agree on the initial state vector (1 0 0 0 0 0 0).

Expert E_2 and E_3, E_2 and E_4, and E_3 and E_4 disagree on certain initial state vectors as the Kosko-Hamming distance is four.

So we can study and analyse these in a special way to arrive at a result. It is pertinent to define a norm for acceptance or rejection or reanalysis of a node. This is carried out in the following way.

Suppose $(c_1, \ldots, c_n)$ are the $n$ concepts under study and $E_1, E_2, \ldots, E_t$ as the $t$ experts working with it.
If $d_k(E_i, E_j) < \frac{\|n\|}{2}$ we do not re analyse.

If $d_k(E_i, E_j) \geq \frac{\|n\|}{2}$ we analyse them and see why the underlying node is giving a result of this form $1 \leq i, j \leq t$. A special mention about that node will be made in the study and conclusions of the problem.

Now this technique can be made for merged FCMs and merged NCMs also. For we take each experts opinion and when we have a common node we study the Kosko-Hamming distance between them.

In case we get several integrated complete FCMs and NCMs we study the Kosko-Hamming distance between them.

Such study is illustrated here from the examples given in chapter II of this book. Consider the merged graphs of the four experts working with the concepts \{C_1, C_2, ..., C_{12}\}.

We only use the graphs of the I, II, III and IV of the four experts.

![Diagram of merged graphs](image_url)
Let $M_1$ be the directed graph given by the first expert.

The connection matrix associated with the graph $I$ is as follows:

$$
M_1 = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}.
$$

Let the graph given by the second expert be denoted by $II$ which is as follows:

Using the second experts graph $II$ we have the following connection matrix of the second expert.
Now we work with the on state of the node $C_7$ alone using $M_I$ in the initial state vector and all other nodes are in the off state.

Consider $X_7 = (00100)$

\[
X_7M_I = (00010) \rightarrow (00110) = Y_1 \text{ (say)}
\]
\[
Y_1M_I = (10010) \rightarrow (10110) = Y_2 \text{ (say)}
\]
\[
Y_2M_I = (11010) \rightarrow (11110) = Y_3 \text{ (say)}
\]
\[
Y_3M_I = (11111) \rightarrow (11111) = Y_4 \text{ (say)}
\]
\[
Y_4M_I \rightarrow (11111) = P_1.
\]

The hidden pattern is a fixed point given by $P_1$.

Now we find

\[
X_7M_{II} = (00010) \rightarrow (00110) = Y_1 \text{ (say)}
\]
\[
Y_1M_{II} = (00010) = Y_2 \text{ (say)} \rightarrow (00110) = Y_3 (=Y_1) = Q_1.
\]

Hence the fixed point is the hidden pattern given by $Q_1$. Obvious we cannot find the Kosko-Hamming between the two experts.

So we expand the state vectors to the final form. Authors by the term expand mean the missing nodes in all
the 12 nodes will be put with zeros as the value. Now $P_1$ is expanded as $P_1^c$ follows.

$$P_1 = (c_1, c_2, c_7, c_{10}, c_{11})$$
$$= (c_1, c_2, 0, 0, 0, c_7, 0, 0, c_{10}, c_{11}, 0)$$
$$P_1^c = (1\ 1\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 1\ 0).$$

Now $Q_1$ is also expanded as $Q_1^c$ which is as follows:

$$Q_1 = (c_4, c_5, c_7, c_{10}, c_{12})$$

$$Q_1^c = (0\ 0\ 0\ c_4\ 0\ c_5\ 0\ 0\ c_7\ 0\ c_{10}\ 0\ c_{12})$$
$$= (0\ 0\ 0\ 0\ 0\ 1\ 0\ 0\ 1\ 0\ 0)$$
$$d_k(P_1^c, Q_1^c) = 3.$$

This is the way we find in case of several experts for which we wish to work with merged FCM or NCMs find the Kosko-Hamming distance for expanded resultant vectors.

It is pertinent to keep on record in this case not all the nodes are comparable we may have one or two which happens to be common are comparable.

Let us consider the experts II and IV the connection matrices given by them are

$$M_{II} = \begin{bmatrix}
    c_4 & c_5 & c_7 & c_{10} & c_{12} \\
    c_4 & 0 & 0 & 1 & 0 \\
    c_5 & 1 & 0 & 1 & 0 \\
    c_7 & 0 & 0 & 1 & 0 \\
    c_{10} & 0 & 0 & 0 & 0 \\
    c_{12} & 0 & 1 & 0 & 1
\end{bmatrix}$$

and
The nodes $c_{10}$ and $c_{12}$ are common for both the experts II and IV. So we can find the resultant for the initial state vectors.

\[ A_1 = (00010), \quad A_2 = (00001) \text{ and } A_3 = (00011) \text{ in case of expert II and } B_1 = (000010), \quad B_2 = (000001) \text{ and } B_3 = (000011) \text{ in case of expert IV}
\]

we calculate the resultant in the following.

\[ A_1 M_{II} \rightarrow (00010) \]
\[ A_2 M_{II} = (01010) \rightarrow (01011) = X_1 \text{ say} \]
\[ X_1 M_{II} = (11110) \rightarrow (11111) = X_2 \text{ say} \]
\[ X_2 M_{II} = (11111) = X_3 (=X_2). \]

Now
\[ A_3 M_{II} = (01010) \rightarrow (01011) = Y_1 \text{ say} \]
\[ Y_1 M_{II} = (11110) \rightarrow (11111) = Y_2 \text{ (say)} \]
\[ Y_2 M_{II} \rightarrow 3 (=Y_2). \]

Thus we see in case of both $A_2$ and $A_3$ the hidden pattern of the dynamical system $M_2$ is a fixed point in fact the same resultant (11111).

Now we find the hidden $B_1 = (000010)$ using $M_{IV}$ which is as follows:
\( B_1 M_{IV} = (011000) \rightarrow (011010) = Z_1 \)
\( Z_1 M_{IV} = (011011) = Z_2 \)
\( Z_2 M_{IV} \rightarrow (011111) = Z_3 (=Z_2). \)
Thus the hidden pattern is a fixed point.

Now we expand the resultant vectors of the dynamical system \( M_{II} \) and \( M_{IV} \).

\[
\begin{align*}
X^c_2 & = (000110100101) \\
Y^c_2 & = (000110100101) \\
Z^c_2 & = (000001011101)
\end{align*}
\]

\[d_k(X^c_2, Z^c_2) = ((000110100101) (000001011101)) = 6.\]
The extended Kosko-Hamming distance is 6.

Now let \( B_2 = (000001) \)
\( B_2 M_{IV} \rightarrow (000001) = T_1. \)
\( T^c_1 = (000000000001) \) is the extended state vector of \( T_1 \)

Now \( d_k(Y^c_2, T^c_1) \)
\[= ((000110100101) (000000000001))\]
\[= 4. \) Thus the extended Kosko-Hamming distance is 4.

Now we proceed on to use the Kosko-Hamming distance for two merged FCMs where the merging of different set of experts is carried out.

Consider the two dynamical system of the merged FCMs given by the merged connection matrices \( M_D \) and \( M_K \) given in chapter two of this book.

Let \( \{C_1, C_2, \ldots, C_{10}\} \) be the 10 attributes we find the 10 attributes we find the Kosko-Hamming distance using a
few of the initial state vectors. Let $d_k(M_D, M_K)$ denote the Kosko-Hamming distance is not on two experts but on their merged FCMs.

We tabulate as follows:

<table>
<thead>
<tr>
<th>$C_i$</th>
<th>Hidden patterns given by $M_K$</th>
<th>Hidden pattern given by $M_D$</th>
<th>$d_k(M_D, M_K)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(100...0)</td>
<td>(1110111110)</td>
<td>(1110111000)</td>
<td>3</td>
</tr>
<tr>
<td>(010...0)</td>
<td>(0110111110)</td>
<td>(0111111111)</td>
<td>3</td>
</tr>
<tr>
<td>(0010...0)</td>
<td>(0010111111)</td>
<td>(0110111111)</td>
<td>1</td>
</tr>
<tr>
<td>(00010...0)</td>
<td>(0010111111)</td>
<td>(0001111111)</td>
<td>3</td>
</tr>
<tr>
<td>(000010...0)</td>
<td>(0010111111)</td>
<td>(0000100000)</td>
<td>7</td>
</tr>
<tr>
<td>(0000010...0)</td>
<td>(0000100000)</td>
<td>(0000011010)</td>
<td>2</td>
</tr>
<tr>
<td>(0000001000)</td>
<td>(0110111111)</td>
<td>(0000011010)</td>
<td>5</td>
</tr>
<tr>
<td>(0000000100)</td>
<td>(0110111111)</td>
<td>(0001011111)</td>
<td>5</td>
</tr>
<tr>
<td>(0000000010)</td>
<td>(0110111111)</td>
<td>(000001010)</td>
<td>6</td>
</tr>
<tr>
<td>(0000000001)</td>
<td>(0110111111)</td>
<td>(0001001111)</td>
<td>8</td>
</tr>
<tr>
<td>(1100...0)</td>
<td>(1110111110)</td>
<td>(1110111010)</td>
<td>2</td>
</tr>
<tr>
<td>(10010...0)</td>
<td>(1111111111)</td>
<td>(1101111111)</td>
<td>4</td>
</tr>
<tr>
<td>(1000110000)</td>
<td>(1111111111)</td>
<td>(110011110)</td>
<td>5</td>
</tr>
<tr>
<td>(0100000001)</td>
<td>(0110111111)</td>
<td>(0111111111)</td>
<td>3</td>
</tr>
<tr>
<td>(0000110000)</td>
<td>(0010111111)</td>
<td>(0001111010)</td>
<td>4</td>
</tr>
<tr>
<td>(0000001001)</td>
<td>(0110111111)</td>
<td>(0001011111)</td>
<td>6</td>
</tr>
<tr>
<td>(0000000110)</td>
<td>(0110111111)</td>
<td>(0001011111)</td>
<td>4</td>
</tr>
<tr>
<td>(0000000101)</td>
<td>(0110111111)</td>
<td>(0001011111)</td>
<td>4</td>
</tr>
<tr>
<td>(1101101100)</td>
<td>(1110111111)</td>
<td>(1111111111)</td>
<td>3</td>
</tr>
<tr>
<td>(0001110001)</td>
<td>(0110111111)</td>
<td>(0001011111)</td>
<td>7</td>
</tr>
<tr>
<td>(0000000111)</td>
<td>(0110111111)</td>
<td>(0001011111)</td>
<td>5</td>
</tr>
<tr>
<td>(0000011110)</td>
<td>(0110111111)</td>
<td>(0001011111)</td>
<td>3</td>
</tr>
<tr>
<td>(0011101110)</td>
<td>(0111111111)</td>
<td>(0111111111)</td>
<td>0</td>
</tr>
</tbody>
</table>

It is pertinent to mention that the above example is in no way connected with any of the problems. It is just
constructed for the example sake. Only this is an illustrate example.

Now one more quality of this notion is it can be used in finding the most influential / vital node. So the $d_h(c_i, c_j)$ can also predict the influential node describe in chapter III of this book.

Now finally we can also use this concept in New Average FCMs and New Average NCMs which will be developed in chapter V of this book. Thus this notion will also be used in finding the Kosko-Hamming distance between every expert and the average FCMs value given by all the experts.

If the Kosko-Hamming distance is small or negligible we need not find the hidden pattern for each of the experts but only FCM or New Average NCM which will save both time and economy.

Thus we will be using this concept in the last chapter of this book.

We suggest the few problems.

**Problems:**

1. Find some special features enjoyed by Kosko-Hamming distance on NCMs and FCMs.

2. Construct a real world problem model using FCMs with 10 experts, and use the concept of Kosko-Hamming distance on the experts opinion.
3. Study the same model using NCMs with same number of experts mentioned in problem 2.


5. Prove Kosko-Hamming distance aids in finding the influential nodes.

6. Illustrate problem 5 by some examples.

7. Construct a real world problem model and obtain the Kosko-Hamming distance table.

8. Develop any other property related with Hamming-Kosko distance.

9. Use Kosko-Hamming distance to study the merged FCMs model.

10. Use Kosko-Hamming distance to study the mixed merged FCMs and NCMs models.

11. Obtain some FCMs and NCMs models and use Kosko-Hamming distance to study a few properties associated with it.

12. Prove some interesting results on Kosko-Hamming distance for extended (expanded) hidden patterns.

13. Study the problem 12 on a real world model.

14. Using Kosko-Hamming distance predict a problem on two experts $E_1$ and $E_2$ who work with FCM and NCMs respectively.
Chapter Four

**NEW AVERAGE FCMs AND NEW AVERAGE NCMs**

In this chapter authors for the first time introduce two types of New Average FCMs and NCMs, one is a New Simple Average FCMs (NSAFCMs) and another is New Average FCMs. The analogue for NCMs is also carried out in this book. We make some simple assumptions before we define the new average FCMs and new average NCMs.

We redefine or rename the concepts in such a way that all these FCMs or NCMs take values from the set \{0, 1\} or \{0, 1, I\} respectively. Only such study will nullify the draw back of canceling of two opinions of the two expert if one is \(-1\) and other is \(+1\). So this is over come and we use this concept of new average FCMs and NCMs only under this basic assumption.

We now describe define and develop first the notion of Average Simple FCMs.
Let us suppose \( n \) experts work on a problem with the same set of attributes \( c_1, c_2, \ldots, c_t \).

All the \( n \) experts choose to work with FCMs using only these \( t \) attributes.

Let \( M_1, M_2, \ldots, M_n \) be the \( n \) connection matrices where
\[
M = \frac{1}{n} \sum_{i=1}^{n} M_i
\]
given by the \( n \)-experts who use only 0 or 1 as edge weights.

We see \( M = (a_{ij}); a_{ij} \in \{0, 1\}; 1 \leq i, j \leq t \).

We call \( M \) the new average dynamical system of the new average simple FCM as the entries are 0 or 1.

This FCM associated with the connection matrix \( M \) is defined as the New Average Simple FCMs (NASFCMs).

Now instead of FCMs all the \( m \) experts work on the problem using NCMs with values from the set \( \{0, 1, I\} \) using the same \( t \) attributes \( \{c_1, c_2, \ldots, c_t\} \) and if \( N_1, N_2, \ldots, N_m \) are the neutrosophic matrices then we find
\[
N = \frac{1}{m} \sum_{i=1}^{m} N_i = (a_{ij})
\]
New Average FCMs and New Average NCMs

$\alpha_{ij} = \begin{cases} 
1 & \text{if } a_{ij} = x + y \text{ and } y \geq \frac{m}{2} \text{ with } y < x \\
1 & \text{if } a_{ij} = x + y \text{ and } y \geq \frac{m}{2} \text{ with } x < y \\
0 & \text{if } a_{ij} = x + y \text{ and } x < \frac{m}{2} \text{ with } y < \frac{m}{2} \\
a + b & \text{if } a_{ij} = x + y \text{ and } x \geq \frac{m}{2} \text{ and } y \geq \frac{m}{2} 
\end{cases}$

Now $N = (a_{ij})$ is called the New Simple Average dynamical system of the new simple average neutrosophic matrices $N_1, N_2, \ldots, N_m$.

Now we can also use the mixed new average simple FCM and NCM whose dynamical system is defined as follows:

Let $T = \frac{(N + M)}{2}$

$= \left( \frac{n_{ij} + m_{ij}}{2} \right)$

where
\[ t_{ij} = \begin{cases} 
1 & \text{if } \frac{n_{ij} + m_{ij}}{2} \geq 0.5 \\
0 & \text{if } \frac{n_{ij} + m_{ij}}{2} < 0.5 \\
1 + I & \text{if both neutrosophic part and real part is } \geq 0.5 \\
(I \leq i, j \leq t) 
\end{cases} \]

Now T has the values from the set \{0, 1, I\} so T is associated with the New Average simple mixed FCMs and NCMs.

We will first illustrate this by an example before we proceed onto define New Average FCMs, New Average NCMs and New average mixed FCMs and NCMs.

However we keep on record, this example is just an illustration and is not a resultant of working with any real world problem.

**Example 4.1:** Let us consider some 6 attributes \{c_1, c_2, c_3, c_4, c_5, c_6\} related to one problem. Four experts agree to work with the problem using FCMs.

All of them agree to work on these 6 attributes using the weights of the graphs to be either 0 or 1 only.

Let \(G_1\) be the directed graph given by the first expert on the problem.
Let $M_1$ be the connection matrix associated with the direct graph $G_1$ which is as follows:

$$
M_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

Let $G_2$ be the directed graph given by the second expert using the nodes $c_1, c_2, \ldots, c_6$. 

\begin{center}
\includegraphics[width=.5\textwidth]{diagram}
\end{center}
The connection matrix of the graph $G_2$ be $M_2$ which is as follows:

$$
M_2 = \begin{bmatrix}
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0
\end{bmatrix}.
$$

Let $G_3$ be the directed graph given by the third expert which is as follows:

![Graph G3](image-url)
Let $M_3$ be the connection matrix associated with the graph $G_3$ which is as follows:

$$M_3 = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 1 & 1 & 0 \\
\end{bmatrix}.$$ 

Let $G_4$ be the directed graph

$G_4 =$

![Diagram of the directed graph $G_4$ with nodes $C_1$ to $C_6$.]

Let $M_4$ be the connection matrix related with $G_4$ which is as follows:

$$M_4 = \begin{bmatrix}
c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\
c_1 & 0 & 1 & 0 & 0 & 0 \\
c_2 & 0 & 0 & 1 & 0 & 0 \\
c_3 & 0 & 0 & 0 & 1 & 0 \\
c_4 & 0 & 0 & 0 & 0 & 1 \\
c_5 & 0 & 0 & 0 & 0 & 0 \\
c_6 & 1 & 0 & 0 & 1 & 1 \\
\end{bmatrix}.$$
Now we find the average of the FCMs which is defined as the New Simple Average FCMs (NSAFCMs) by finding

\[
\frac{1}{4}(M_1 + M_2 + M_3 + M_4)
\]

\[
\begin{bmatrix}
0 & 1 & 1 & 0 & 0 & 2 \\
1 & 0 & 2 & 3 & 0 & 0 \\
2 & 1 & 0 & 2 & 0 & 0 \\
0 & 1 & 0 & 0 & 3 & 1 \\
0 & 0 & 0 & 3 & 0 & 2 \\
1 & 0 & 0 & 1 & 3 & 0
\end{bmatrix}
\]

\[
\begin{bmatrix}
0 & 0.25 & 0.25 & 0 & 0 & 0.5 \\
0.25 & 0 & 0.5 & 0.75 & 0 & 0 \\
0.5 & 0.25 & 0 & 0.5 & 0 & 0 \\
0 & 0.25 & 0 & 0 & 0.75 & 0.25 \\
0 & 0 & 0 & 0.75 & 0 & 0.5 \\
0.25 & 0 & 0 & 0.25 & 0.75 & 0
\end{bmatrix}
= (m_{ij}).
\]
We see if \( m_{ij} \geq 0.25 \) put 1

if \( m_{ij} < 0.25 \) put 0.

\[
\begin{bmatrix}
  c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \\
  c_1 & 0 & 1 & 1 & 0 & 0 & 1 \\
  c_2 & 1 & 0 & 1 & 1 & 0 & 0 \\
\end{bmatrix}
\]

We see \( M = \begin{bmatrix}
  c_3 & 1 & 1 & 0 & 1 & 0 & 0 \\
  c_4 & 0 & 1 & 0 & 0 & 1 & 1 \\
  c_5 & 0 & 0 & 1 & 0 & 1 \\
  c_6 & 1 & 0 & 0 & 1 & 1 & 0
\end{bmatrix} \).

\( M \) is defined as the new average simple dynamical system associated with the new average simple FCMs.

We work with a few attributes in the on state using all the 5 dynamical systems in the following.

Let us now work with the dynamical system given by the first expert with the following initial state vectors.

\[
X_1 = (1 \ 0 \ 0 \ 0 \ 0 \ 0) \\
X_2 = (0 \ 1 \ 0 \ 0 \ 0 \ 0) \\
X_3 = (0 \ 0 \ 1 \ 0 \ 0 \ 0) \\
X_4 = (0 \ 0 \ 0 \ 1 \ 0 \ 0) \\
X_5 = (0 \ 0 \ 0 \ 0 \ 1 \ 0) \\
X_6 = (0 \ 0 \ 0 \ 0 \ 0 \ 1) \\
X_{1,2} = (1 \ 1 \ 0 \ 0 \ 0 \ 0)
\]
\[ X_{3,6} = (0 \ 0 \ 1 \ 0 \ 0 \ 1) \]
\[ X_{4,5} = (0 \ 0 \ 0 \ 1 \ 1 \ 0) \]
\[ X_{1,3,5} = (1 \ 0 \ 1 \ 0 \ 1 \ 0) \]
\[ X_{2,4,6} = (0 \ 1 \ 0 \ 1 \ 0 \ 1). \]

We will be working only with these 11 initial state vectors in case of all the experts as well as M the dynamical system of NASFCMs.

Let table 1 denote the initial set of state vectors in the first column and the second column the hidden pattern using the dynamical system \( M_1 \).

| \( X_1 \) = (1 0 0 0 0 0) | (1 0 0 0 1) |
| \( X_2 \) = (0 1 0 0 0 0) | (1 1 1 1 1 1) |
| \( X_3 \) = (0 0 1 0 0 0) | (0 0 1 0 0 0) |
| \( X_4 \) = (0 0 0 1 0 0) | (1 1 1 1 1 1) |
| \( X_5 \) = (0 0 0 1 0) | (1 1 1 1 1 1) |
| \( X_6 \) = (0 0 0 0 1) | (0 0 0 0 1) |
| \( X_{1,2} \) = (1 1 0 0 0 0) | (1 1 1 1 1 1) |
| \( X_{3,6} \) = (0 0 1 0 0 1) | (0 0 1 0 0 1) |
| \( X_{4,5} \) = (0 0 0 1 1 0) | (1 1 1 1 1 1) |
| \( X_{1,3,5} \) = (1 0 1 0 1 0) | (1 1 1 1 1 1) |
| \( X_{2,4,6} \) = (0 1 0 1 0 1) | (1 1 1 1 1 1) |

Now we work with same set of initial vectors and obtain the hidden pattern using \( M_2 \) which is given in table 2 in the following.
### Table 2

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$(1\ 0\ 0\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_2$</td>
<td>$(0\ 1\ 0\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_3$</td>
<td>$(0\ 0\ 1\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_4$</td>
<td>$(0\ 0\ 0\ 1\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_5$</td>
<td>$(0\ 0\ 0\ 0\ 1\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_6$</td>
<td>$(0\ 0\ 0\ 0\ 0\ 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,2}$</td>
<td>$(1\ 1\ 0\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{3,6}$</td>
<td>$(0\ 0\ 1\ 0\ 0\ 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{4,5}$</td>
<td>$(0\ 0\ 0\ 1\ 1\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,3,5}$</td>
<td>$(1\ 0\ 1\ 0\ 1\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{2,4,6}$</td>
<td>$(0\ 1\ 0\ 1\ 0\ 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now using the dynamical system $M_3$ given by the $3^{rd}$ expert we use the 11 initial values $X_1, X_2, \ldots, X_{1,3,5}$ and $X_{2,4,6}$, find the hidden pattern and tabulate in Table 3 in the following.

### Table 3

<p>| | | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>$(1\ 0\ 0\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_2$</td>
<td>$(0\ 1\ 0\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_3$</td>
<td>$(0\ 0\ 1\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_4$</td>
<td>$(0\ 0\ 0\ 1\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_5$</td>
<td>$(0\ 0\ 0\ 0\ 1\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_6$</td>
<td>$(0\ 0\ 0\ 0\ 0\ 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,2}$</td>
<td>$(1\ 1\ 0\ 0\ 0\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{3,6}$</td>
<td>$(0\ 0\ 1\ 0\ 0\ 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{4,5}$</td>
<td>$(0\ 0\ 0\ 1\ 1\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{1,3,5}$</td>
<td>$(1\ 0\ 1\ 0\ 1\ 0)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_{2,4,6}$</td>
<td>$(0\ 1\ 0\ 1\ 0\ 1)$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Now we use the dynamical system given by the fourth expert $M_4$ and find the hidden pattern for all the 11 initial
state vectors and tabulate in the table 4 which is as follows.

Table 4

| X_1  = (1 0 0 0 0 0) | (1 1 0 1 1 1) |
| X_2  = (0 1 0 0 0 0) | (0 1 0 1 0 0) |
| X_3  = (0 0 1 0 0 0) | (1 1 1 1 1 1) |
| X_4  = (0 0 0 1 0 0) | (0 1 0 1 0 0) |
| X_5  = (0 0 0 0 1 0) | (1 1 1 1 1 1) |
| X_6  = (0 0 0 0 0 1) | (0 1 0 1 1 1) |
| X_{1,2} = (1 1 0 0 0 0) | (1 1 0 1 1 1) |
| X_{3,6} = (0 0 1 0 0 1) | (1 1 1 1 1 1) |
| X_{4,5} = (0 0 0 1 1 0) | (0 1 0 1 1 1) |
| X_{1,3,5} = (1 0 1 0 1 0) | (1 1 1 1 1 1) |
| X_{2,4,6} = (0 1 0 1 0 1) | (0 1 0 1 1 1) |

Now we find the hidden pattern using the new average simple FCM for the 11 initial state vectors and give them in the following table 5.

Table 5

| X_1  = (1 0 0 0 0 0) | (1 1 1 1 1 1) |
| X_2  = (0 1 0 0 0 0) | (1 1 1 1 1 1) |
| X_3  = (0 0 1 0 0 0) | (1 1 1 1 1 1) |
| X_4  = (0 0 0 1 0 0) | (1 1 1 1 1 1) |
| X_5  = (0 0 0 0 1 0) | (1 1 1 1 1 1) |
| X_6  = (0 0 0 0 0 1) | (1 1 1 1 1 1) |
| X_{1,2} = (1 1 0 0 0 0) | (1 1 1 1 1 1) |
| X_{3,6} = (0 0 1 0 0 1) | (1 1 1 1 1 1) |
| X_{4,5} = (0 0 0 1 1 0) | (1 1 1 1 1 1) |
| X_{1,3,5} = (1 0 1 0 1 0) | (1 1 1 1 1 1) |
| X_{2,4,6} = (0 1 0 1 0 1) | (1 1 1 1 1 1) |
New Average FCMs and New Average NCMs

From the table 5 we see the thresholding value 0.25 ought to be changed.

So now we in the average matrix put \( m_{ij} = 1 \) if \( m_{ij} \geq 0.5 \) and \( m_{ij} = 0 \) if \( m_{ij} < 0.5 \).

So the modified \( M \) is denoted by \( M' \).

\[
\begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 1 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1
\end{bmatrix}
\]

Now using this \( M' \) we find the hidden pattern of the 11 initial state vectors and tabulate them in the following table 6 which is as follows:

<table>
<thead>
<tr>
<th>( X_1 ) = (1 0 0 0 0 0)</th>
<th>( X_1,2 ) = (1 1 0 0 0 0)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_2 ) = (0 1 0 0 0 0)</td>
<td>( X_3 ) = (0 0 1 0 0 0)</td>
</tr>
<tr>
<td>( X_4 ) = (0 0 0 1 0 0)</td>
<td>( X_5 ) = (0 0 0 1 0 0)</td>
</tr>
<tr>
<td>( X_6 ) = (0 0 0 0 1 0)</td>
<td>( X_1,3,5 ) = (1 0 1 0 1 0)</td>
</tr>
<tr>
<td>( X_2,4,6 ) = (0 1 0 1 0 1)</td>
<td>( X_2 ) = (0 1 0 0 0 0)</td>
</tr>
<tr>
<td>( X_1,2 ) = (1 1 0 0 0 0)</td>
<td>( X_3 ) = (0 0 1 0 0 0)</td>
</tr>
<tr>
<td>( X_4 ) = (0 0 0 1 0 0)</td>
<td>( X_5 ) = (0 0 0 1 0 0)</td>
</tr>
<tr>
<td>( X_6 ) = (0 0 0 0 1 0)</td>
<td>( X_1,3,5 ) = (1 0 1 0 1 0)</td>
</tr>
<tr>
<td>( X_2,4,6 ) = (0 1 0 1 0 1)</td>
<td>( X_2 ) = (0 1 0 0 0 0)</td>
</tr>
</tbody>
</table>
Now we use the concept of Kosko-Hamming distance to study the closeness or distance of each of the experts in the predictions. We write under the columns $d_k(E_i, E_j)$ the Kosko-Hamming distance of the 11 hidden patterns of the experts $E_i$ and $E_j$, $1 \leq i, j \leq 4$ ($E_i, A$) means the Kosko-Hamming distance between the 11 hidden patterns of the expert $E_i$ and the NASFCMs, $1 \leq i \leq 4$.

This is given in the following table 7. However table 5 is not useful as the hidden pattern of all the 11 initial state vectors is $(1 \ 1 \ 1 \ 1 \ 1 \ 1)$.

**Table 7**

<table>
<thead>
<tr>
<th>$X_1 = (1 \ 0 \ 0 \ 0 \ 0 \ 0)$</th>
<th>$d_k(E_1,E_2)$</th>
<th>$d_k(E_1,E_3)$</th>
<th>$d_k(E_1,E_4)$</th>
<th>$d_k(E_1,A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_2 = (0 \ 1 \ 0 \ 0 \ 0 \ 0)$</td>
<td>4</td>
<td>4</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (0 \ 0 \ 1 \ 0 \ 0 \ 0)$</td>
<td>2</td>
<td>2</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>$X_4 = (0 \ 0 \ 0 \ 1 \ 0 \ 0)$</td>
<td>5</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>$X_5 = (0 \ 0 \ 0 \ 0 \ 1 \ 0)$</td>
<td>3</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$X_6 = (0 \ 0 \ 0 \ 0 \ 0 \ 1)$</td>
<td>3</td>
<td>4</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>$X_{1,2} = (1 \ 1 \ 0 \ 0 \ 0 \ 0)$</td>
<td>2</td>
<td>5</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$X_{3,6} = (0 \ 0 \ 1 \ 0 \ 0 \ 1)$</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>3</td>
</tr>
<tr>
<td>$X_{4,5} = (0 \ 0 \ 0 \ 1 \ 1 \ 0)$</td>
<td>3</td>
<td>4</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$X_{1,3,5} = (1 \ 0 \ 1 \ 0 \ 1 \ 0)$</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>$X_{2,4,6} = (0 \ 1 \ 0 \ 1 \ 0 \ 1)$</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$d_k(E_2,E_3)$</th>
<th>$d_k(E_2,E_4)$</th>
<th>$d_k(E_2,A)$</th>
<th>$d_k(E_3,E_4)$</th>
<th>$d_k(E_3,A)$</th>
<th>$d_k(E_4,A)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>4</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>1</td>
<td>3</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
<td>3</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
<td>0</td>
<td>4</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>0</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>0</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>
We see from the table 7 the deviation of each of the experts from the average $\bar{A}$ is very less. Now we find $\bar{A}$ the average of all the four experts for all the 11 hidden pattern and compare it $A$ in the following table.

Table 8

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>$\bar{A}$</th>
<th>$A$</th>
<th>$d_k(A, \bar{A})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1\ 0\ 0\ 0\ 0)$</td>
<td>$A = (1\ 0\ 0\ 1\ 1\ 1\ 1)$</td>
<td>$A = (1\ 0\ 0\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 2$</td>
</tr>
<tr>
<td>$X_2 = (0\ 1\ 0\ 0\ 0)$</td>
<td>$A = (0\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 1$</td>
</tr>
<tr>
<td>$X_3 = (0\ 0\ 1\ 0\ 0)$</td>
<td>$A = (1\ 0\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (1\ 0\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 1$</td>
</tr>
<tr>
<td>$X_4 = (0\ 0\ 0\ 1\ 0)$</td>
<td>$A = (0\ 0\ 0\ 1\ 1\ 1\ 1)$</td>
<td>$A = (0\ 0\ 0\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 1$</td>
</tr>
<tr>
<td>$X_5 = (0\ 0\ 0\ 0\ 1)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (0\ 0\ 0\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 3$</td>
</tr>
<tr>
<td>$X_6 = (0\ 0\ 0\ 0\ 0)$</td>
<td>$A = (0\ 0\ 0\ 0\ 1\ 1\ 1)$</td>
<td>$A = (0\ 0\ 0\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 1$</td>
</tr>
<tr>
<td>$X_{1,2} = (1\ 1\ 0\ 0\ 0\ 0)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 0$</td>
</tr>
<tr>
<td>$X_{3,6} = (0\ 0\ 0\ 1\ 0\ 0)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (1\ 0\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 1$</td>
</tr>
<tr>
<td>$X_{4,5} = (0\ 0\ 0\ 1\ 1\ 0)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (0\ 0\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 1$</td>
</tr>
<tr>
<td>$X_{1,3,5} = (1\ 0\ 1\ 0\ 1\ 0)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (1\ 0\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 1$</td>
</tr>
<tr>
<td>$X_{2,4,6} = (0\ 1\ 0\ 1\ 0\ 1)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$A = (1\ 1\ 1\ 1\ 1\ 1\ 1)$</td>
<td>$d_k(A, \bar{A}) = 0$</td>
</tr>
</tbody>
</table>

We see except for the initial state vector $X_5$ all the deviations or 0, 1 and only one two. Thus there is not much of deviation as per the Kosko-Hamming distance between the hidden patterns of the average $\bar{A}$ and that of the hidden pattern got from the NASFCMs.

Thus we by using NASFCMs can save time and economy. Now in the following table we give the value of $d_k(E_1, \bar{A})$, $d_k(E_2, \bar{A})$, $d_k(E_3, \bar{A})$ and $d_k(E_4, \bar{A})$ in the following.

Table 9

<table>
<thead>
<tr>
<th>$X_i$</th>
<th>$d_k(E_1, \bar{A})$</th>
<th>$d_k(E_2, \bar{A})$</th>
<th>$d_k(E_3, \bar{A})$</th>
<th>$d_k(E_4, \bar{A})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1\ 0\ 0\ 0\ 0)$</td>
<td>4</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$X_2 = (0\ 1\ 0\ 0\ 0)$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$X_3 = (0\ 0\ 1\ 0\ 0)$</td>
<td>5</td>
<td>1</td>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>$X_4 = (0\ 0\ 0\ 1\ 0)$</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$X_5 = (0\ 0\ 0\ 0\ 1)$</td>
<td>0</td>
<td>0</td>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>$X_6 = (0\ 0\ 0\ 0\ 0)$</td>
<td>3</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$X_{1,2} = (1\ 1\ 0\ 0\ 0)$</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$X_{3,6}$ = (0 0 1 0 0 1)</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$X_{4,5}$ = (0 0 0 1 1 0)</td>
<td>2</td>
<td>0</td>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>$X_{1,3,5}$ = (1 0 1 0 1 0)</td>
<td>3</td>
<td>1</td>
<td>1</td>
<td>0</td>
</tr>
<tr>
<td>$X_{2,4,6}$ = (0 1 0 1 0 1)</td>
<td>3</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
</tbody>
</table>

Except for the first expert as in case of $d_k(E,A)$ we see the deviation is not very high.

Thus we have defined NASFCMs and shown using the new technique of Kosko-Hamming distance we see the deviations are considerably small so we have no problem of using NASFCMs instead of working with several experts. This gives equal importance to each and every expert. Further this saves time and economy so this new model will serve as a better one.

Next we proceed onto define the notion of New Average Simple NCMs. We assume NCMs take values only from the set $\{0, 1, I\}$.

Suppose we have $n$ experts who wish to work only with the NCMs, then to find the New Average Simple NCMs (NASNCMs).

Let $\{c_1, c_2, \ldots, c_t\}$ be the t-concepts associated with the problem and $m$-experts work with the problem using only the NCMs model. To find the new average simple NCMs. Let $N_1, N_2, \ldots, N_m$ be the $m$-connection neutrosophic matrices given by the $m$-experts.

Let $N = \frac{1}{m} \sum_{i=1}^{m} N_i$

$= (n_{ij})$ we see if $n_{ij} \geq \left\lfloor \frac{n}{2} \right\rfloor$;
then put $n_{ij} = 1$ (n_{ij} real)

if $n_{ij} = t + s$ where $t, s$ are real then
put $n_{ij} = 1$ if $t \geq s$
$= I$ if $t < s$
if $n_{ij}$ is real and $n_{ij} < \left\lfloor \frac{\pi}{2} \right\rfloor$ put 0.

Thus $N$ takes values from the set \{0, 1, I\}.

Using $N$ we can find the hidden pattern and $N$ is defined as the New Average Simple dynamical NCM system.

We will illustrate this situation by an example.

**Example 4.2:** Let $C = \{C_1, C_2, C_3, C_4, C_5\}$ be the five attributes associated with the problem.

Let $E_1$, $E_2$ and $E_3$ be the three experts who work with the problem using the NCMs model and using the five attributes.

Let $G_1$ be the neutrosophic graph given by the first expert which is as follows.

Let $N_1$ be the neutrosophic connection matrix associated with the neutrosophic graph $G_1$. 

![Neutrosophic Graph G1](image-url)
Let $G_2$ be the neutrosophic directed graph associated with the NCM given by the second expert which is as follows:

\[
N_1 = \begin{bmatrix}
0 & 1 & 0 & 1 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}.
\]

Let $G_2$ be the neutrosophic directed graph associated with the NCM given by the second expert which is as follows:

Let $N_2$ be the connection matrix associated with neutrosophic directed graph $G_2$:

\[
N_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}.
\]
Let $G_3$ be the graph associated with third expert $E_3$ which is as follows:

$$G_3$$

The neutrosophic connection matrix associated with the graph $G_3$ is as follows:

$$N_3 = \begin{pmatrix}
0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 1 & 0 & 1 & 0
\end{pmatrix}.$$ 

Let us find the hidden pattern of 12 initial state vectors and tabulate them in the following for the NCM given by the expert $E_1$.

**Table 1**

<table>
<thead>
<tr>
<th>Initial state vectors</th>
<th>Hidden pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1 \ 0 \ 0 \ 0 \ 0)$</td>
<td>$(1 \ 1 \ 1 \ 1 \ 1)$</td>
</tr>
<tr>
<td>$X_2 = (0 \ 1 \ 0 \ 0 \ 0)$</td>
<td>$(0 \ 1 \ 1 \ 0 \ 1)$</td>
</tr>
<tr>
<td>$X_3 = (0 \ 0 \ 1 \ 0 \ 0)$</td>
<td>$(0 \ 1 \ 1 \ 0 \ 1)$</td>
</tr>
</tbody>
</table>
Now in table 2 we give for the 12 initial state vectors the hidden pattern given by the second expert using the dynamical system N₂ which is as follows:

<table>
<thead>
<tr>
<th>Initial state vectors</th>
<th>Hidden pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁ = (1 0 0 0 0)</td>
<td>(1 0 0 0 0)</td>
</tr>
<tr>
<td>X₂ = (0 1 0 0 0)</td>
<td>(1 1 0 1 1)</td>
</tr>
<tr>
<td>X₃ = (0 0 1 0 0)</td>
<td>(1 0 1 1 1)</td>
</tr>
<tr>
<td>X₄ = (0 0 0 1 0)</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>X₅ = (0 0 0 0 1)</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>X₁₂ = (1 1 0 0 0)</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>X₁₃ = (1 0 1 0 0)</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>X₁₄ = (1 0 0 1 0)</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>X₁₅ = (1 0 0 0 1)</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>X₄₅ = (0 0 0 1 1)</td>
<td>(0 0 1 1 1)</td>
</tr>
<tr>
<td>X₃₄ = (0 0 1 1 0)</td>
<td>(0 1 1 1 1)</td>
</tr>
<tr>
<td>X₂₄ = (0 1 0 1 0)</td>
<td>(0 1 1 1 1)</td>
</tr>
</tbody>
</table>

We now find the 3rd expert opinion the neutrosophic matrix N₃ and tabulate the hidden pattern of the 12 initial state vectors in the following table 3.
Next we find the average of the three NCMs \( N_1, N_2, N_3. \)

\[
N = \frac{1}{3} (N_1 + N_2 + N_3)
\]

\[
\begin{bmatrix}
0 & 1 + I & 0 & 0.33 & 0 \\
0.33 & 0 & 0.66 & 0 & I \\
0 & 0.33 & 0 & 0.33 & 0.33 \\
0 & 1 + I & 0 & 0 & 1 + I \\
0.33 & 0.33 & 0 & I & 0
\end{bmatrix}
\]

We after thresholding put 1 if \( \alpha \geq 0.33 \)

I if \( \alpha = 1 \)

0 if \( \alpha < 0.33 \) finally 1 if \( \alpha = 1 + I. \)
Now \( N \) is known as the New Average Simple NCMs dynamical system.

Now we find the hidden pattern of all the 12 initial state vectors and tabulate them in table 4 in the following.

**Table 4**

<table>
<thead>
<tr>
<th>Initial state vectors</th>
<th>Hidden pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 = (1 0 0 0 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_2 = (0 1 0 0 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_3 = (0 0 1 0 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_4 = (0 0 0 1 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_5 = (0 0 0 0 1) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_{1,2} = (1 1 0 0 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_{1,3} = (1 0 1 0 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_{1,4} = (1 0 0 1 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_{1,5} = (1 0 0 0 1) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_{4,5} = (0 0 0 1 1) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_{34} = (0 0 1 1 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
<tr>
<td>( X_{2,4} = (0 1 0 1 0) )</td>
<td>(1 1 1 1 1)</td>
</tr>
</tbody>
</table>

From the hidden pattern we see all the nodes come to on state for every initial vector. This is not any form of good prediction.

So review of the results forces us to change the thresholding function from \( \alpha \geq 0.33 \) to “1 if \( \alpha \geq 0.5 \).
Now we redo the matrix $N$ and denote it by $N'$ which is as follows:

$$
N' = \begin{bmatrix}
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}.
$$

Now we find the hidden pattern for all the 12 initial vectors.

<table>
<thead>
<tr>
<th>Initial state vectors</th>
<th>Hidden pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(11111)</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(01111)</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(00100)</td>
</tr>
<tr>
<td>$X_4 = (00010)$</td>
<td>(01111)</td>
</tr>
<tr>
<td>$X_5 = (00001)$</td>
<td>(01111)</td>
</tr>
<tr>
<td>$X_{1,2} = (11000)$</td>
<td>(11111)</td>
</tr>
<tr>
<td>$X_{1,3} = (10100)$</td>
<td>(11111)</td>
</tr>
<tr>
<td>$X_{1,4} = (10010)$</td>
<td>(11111)</td>
</tr>
<tr>
<td>$X_{1,5} = (10001)$</td>
<td>(11111)</td>
</tr>
<tr>
<td>$X_{4,5} = (00011)$</td>
<td>(01111)</td>
</tr>
<tr>
<td>$X_{3,4} = (00110)$</td>
<td>(01111)</td>
</tr>
<tr>
<td>$X_{2,4} = (01010)$</td>
<td>(01111)</td>
</tr>
</tbody>
</table>

Now we will use the Kosko-Hamming distance defined in chapter III to find the distance or how far two experts agree or disagree also how far they agree or disagree from the New Average Simple Neutrosophic Cognitive Maps model and the average of the three experts 12 hidden patterns in the following. Let $\bar{A}$ denote the average and $A$ denote the hidden of the NASNCMs.
Now we find $d_k(E_i, E_j)$, $d_k(A, E_i)$, $d_k(A, E_i)$ and $d_k(A, \bar{A})$ in the following.

<table>
<thead>
<tr>
<th>$X_1 = (1 0 0 0 0)$</th>
<th>$d_k(E_1, E_2)$</th>
<th>$d_k(E_1, E_3)$</th>
<th>$d_k(E_2, E_3)$</th>
<th>$d_k(A, E_1)$</th>
<th>$d_k(A, E_2)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_2 = (0 1 0 0 0)$</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$X_3 = (0 0 1 0 0)$</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (0 0 0 1 0)$</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$X_5 = (0 0 0 0 1)$</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>$X_{1,2} = (1 1 0 0 0)$</td>
<td>2</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>$X_{1,3} = (1 0 1 0 0)$</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>$X_{1,4} = (1 0 0 1 0)$</td>
<td>0</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>$X_{1,5} = (1 0 0 0 1)$</td>
<td>3</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>$X_{4,5} = (0 0 0 1 1)$</td>
<td>3</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>$X_{3,4} = (0 0 1 1 0)$</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>$X_{2,4} = (0 1 0 1 0)$</td>
<td>3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$d_k(E_3, A)$</th>
<th>$\bar{A}$</th>
<th>$d_k(E, \bar{A})$</th>
<th>$d_k(E_2, \bar{A})$</th>
<th>$d_k(E_3, \bar{A})$</th>
<th>$d_k(A, \bar{A})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>(11000)</td>
<td>4</td>
<td>1</td>
<td>0</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>(11100)</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>(11101)</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>(10011)</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>(10001)</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>(11100)</td>
<td>2</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>(11101)</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(11111)</td>
<td>0</td>
<td>0</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>(10101)</td>
<td>2</td>
<td>1</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>2</td>
<td>(10011)</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>3</td>
</tr>
<tr>
<td>3</td>
<td>(01111)</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>(01111)</td>
<td>0</td>
<td>3</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

This is only an illustrative example and is not a real world model in which we have worked. This is more to explain the working so the answers may be little deviant. Any interested reader can work with the real world model.

We on similar lines work with some problem which has say $C = \{c_1, \ldots, c_n\}$ concepts with $s + t$ experts work using the set $C$.  


with $s$ of them work using the FCMs and $t$ of them work on the problem with NCMs. We find the mixed new average FCMs and NCMs where we add all the $s + t$, $n \times n$ matrices and divide it by $s + t$ we use some suitable parameter $\alpha$ so that using $\alpha$ the Mixed New average NCM and FCM is obtained such that the entries are from the set $\{0, 1, I\}$ only. This working is similar to that of NASFCMs and NASNCMs.

Here we keep on record that we need not work with values in between the interval $[0,1]$ for the dynamical system is ultimately only going to give hidden pattern as 0 or 1 the off or on state of nodes for otherwise the dynamical system will not function. So under these conditions it is deem fit we can work only with NASFCMs NASNCMs and mixed NASFCMs and mixed NASNCMs using elements from the set $\{0, 1, I\}$.

Interested reader can work with them using them in the real world model. Here we suggest some problems for the reader.

**Problems:**

1. Obtain some special features enjoyed by Average New Simple FCMs.

2. Show by a real world problem the working of NASFCMs.

3. Compare NASFCMs with combined FCMs.

4. Which of the models is better NASFCMs or combined FCMs?

5. What are the special features enjoyed by NASNCMs?

6. Exhibit by a real world model the functioning of NASNCMs.

7. Compare NASNCMs with combined NCMs.
8. Which model is better combined NCMs or NASNCMs.

9. Give a real world model and describe the functioning of the mixed new average simple FCMs and NCMs.

10. Compare NASFCMs with the overlapping FCMs.

11. Distinguish both mentioned in problem 10 by applying it in the real world problem.

12. Compare FTCMs with NASTCMs.

13. Compare both the model NTCMs with NASNTCMs by using it in a real world model.

14. Can NASFCMs be used in predicting the users web behavior?

15. Illustrate the working of NASNCMs in the users web behavior.

16. Use NASFCMs to study the bonded labour problem.

17. Can the study given problem 16 be done using NASNCMs?

18. Using problems (16) and (17) make the mixed NASNCMs and NCMs model to study the bonded labour problem.

19. Prove these new average models saves time and money.

20. Prove the advantage of using new average models eradicates the bias in taking the opinion of only few experts.
Chapter Five

In this chapter we study the vital or the most influential nodes of FCMs and NCMs. We know when we have a graph a vertex which has the maximum number of vertices adjacent with it is usually considered as the vital node or the most influential node.

In this book we study the nodes in case of FCMs and NCMs. As FCMs and NCMs mainly function on the directed graph of the dynamical system we ventured to study such nodes. We saw these graphs are not like usual graphs for a node with the maximum number of edges incident to it need not in general to be vital node. This was proved by real valued problems using FCMs / NCMs model and their associated directed graphs given by the experts.

Further we for these directed graphs of the FCMs define most influential node, more influential node,
influential node, less influential node, least influential node or a passive node in a very different way.

Let \( \{C_1, C_2, \ldots, C_n\} = C \) be the set of nodes / attributes with which an expert works with the problem using FCMs or NCMs. The expert will give the experts opinion in the form of a directed graph say with \( C_1, C_2, \ldots, C_n \) as its nodes. Suppose \( C_i \) is a node with maximum number of edges adjacent with it then in general \( C_i \) is not defined as the most influential node by us; on the contrary we define a node \( C_i \) to be the most influential node if the on state of the state vector \( C_i \) alone say \( X_i = (0, 0, \ldots, 0, 1, 0, \ldots, 0) \) that is the \( i^{th} \) coordinate alone is in the on state and all other nodes are in the off state then we find the effect of \( X_i \) on the dynamical system and if \( X_i \) gives the maximum number of on states of the node in the hidden pattern of the model which may be a fixed point or a limit cycle then we define that node to be the most influential node. The \( C_i \) which when on and rest of the nodes are in the off states gives maximum on states in the result vector is defined as the most influential node. However for a given FCMs model we can have more than one most influential node. Suppose the most influential node \( C_i \) whose initial state vector is \( X_i \) gives \( r \) number of on states of \( r \) node including the \( i^{th} \) node \( C_i \) (\( r < n \)).

We say the most influential node of the dynamical system of the FCMs makes \( (r-1) \) nodes on. Now when we study the related graph of the FCMs it may not be the node of the graph which has the maximum number of edges adjacent to it.

So by studying the role of the node we can derive several important properties about the problem at hand. We can have more than one node for a graph of an FCMs to be a most influential node. Now a more influential node
of a FCMs will be a node $C_j$ say $X_j = (0, \ldots, 0, 1, 0, \ldots, 0)$ only the $j$th coordinate is in the state and all other coordinates are in the off state. Suppose the hidden pattern of $X_j$ using the dynamical system makes $s$ of the coordinate to be in on state and $r > s$ then we have no other state vector which can give on state of more than $s$ state vectors then we call $C_j$ to be the more influential node of the FCMs or NCMs we may have more than one node in $C$ to be such more influential nodes. However the vertices of these nodes may not in general contain the maximum number of edges incident to it.

Thus we have now defined the notion of more influential node and the most influential node. Now we can define on similar lines the influential node, less influent node and so on.

A node is said to be a more influential node if the on state of the node gives the on state of several nodes but the number of nodes it makes on is less than that of the most influential node. Next we can go for the just influential node and an influential node and so on.

Hence $(\text{number of on state of most influential node}) > (\text{number of on state of more influential node}) > (\text{number of on state of just influential node}) > (\text{number of on state of influential node}) > (\text{number of less influential node}) > (\text{number of least influential node}) > (\text{number of non influential node})$ for a given initial state vector.

This is the way the concept of influential node is studied. Here the authors keep on record that there influential nodes of a FCMs or NCMs are not the popular nodes called hubs or influential nodes. These are entirely a different concept varying with the problem in hand. Further it is proved beyond doubt a node with the
maximum number of edges adjacent to it in a directed graph given by an expert of the FCMs or NCMs in general is not the most influential node but it is the capacity of the node after working with the dynamical system gives in the hidden pattern with the maximum number of on state of the nodes which is addressed in this book as the most influential node.

Thus it may so happen a node with only one edge adjacent to it may be the most influential node in that problem. Thus in this study it is very clearly established that in the directed graphs of the FCMs or NCMs that is in the net working of the problem as given by an expert the concept of the most influential node is not the maximum number of edges adjacent to it. So these special class of directed graphs (or networks) do not agree with the usual concept of influential node; on the other hand a most influential node will be node on which or around which the problem spins. So such study is very vital for any one who uses FCMs or NCMs. We are the first one to make such a study. This also answers the long standing question of the graph theorists who had doubts about the influential nodes of a graph in general.

Now we have also classified the nodes as “most influential”, “more influential”, “just influential”, “influential less influential”, least influential” and “not influential” etc.,. Such study throws of new way of analysis of the problem which uses FCMs or NCMs whatever is said for the directed graphs associated with FCMs are also true in case of neutrosophic directed graphs associated with the NCMs. Here we need to study only the on state of one and only one node. For on state of two nodes simultaneously etc does not come under the purview of this study. We will illustrate these situations before we describe them more technically.
From table 1 in chapter IV we have

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (100000)$</td>
<td>(100001)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (010000)$</td>
<td>(111111)</td>
<td>4</td>
</tr>
<tr>
<td>$X_3 = (001000)$</td>
<td>(001000)</td>
<td>1</td>
</tr>
<tr>
<td>$X_4 = (000100)$</td>
<td>(111111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_5 = (000010)$</td>
<td>(111111)</td>
<td>3</td>
</tr>
<tr>
<td>$X_6 = (000001)$</td>
<td>(000001)</td>
<td>2</td>
</tr>
</tbody>
</table>

We see $X_4$ that is the node $C_4$ has only two edges incident to it yet it is also a most influential node. So the nodes $C_2, C_4$ and $C_5$ are the most influential nodes.

Refer graph $G_1$

Now we remove the node $C_2$ and find the most influential node.
The connection matrix

\[ c_1 \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = S_1. \]

We find for \( X_1 = (1 \ 0 \ 0 \ 0 \ 0) \) the hidden pattern using \( S_1 \).

\( X_1 = (1 \ 0 \ 0 \ 0 \ 0) \) gives the hidden pattern as \((1 \ 0 \ 0 \ 0 \ 1)\). For \( X_3 = (0 \ 1 \ 0 \ 0 \ 0) \) we find the hidden pattern using \( S_1 \). The hidden pattern is \((0 \ 1 \ 0 \ 0 \ 0)\).

Using \( X_4 = (0 \ 0 \ 1 \ 0 \ 0) \) we find the hidden pattern using \( S_1 \) which is \((0 \ 0 \ 1 \ 1 \ 1)\).

For \( X_5 = (0 \ 0 \ 0 \ 1 \ 0) \) we find the hidden pattern using \( S_1 \) to be \((00111)\).

For \( X_6 = (0 \ 0 \ 0 \ 0 \ 1) \) we find the hidden pattern using \( S_1 \) to be \((0 \ 0 \ 0 \ 0 \ 1)\).
Thus we see the most influential node of the graph $G_1 \setminus \{C_2\}$ is still $X_4$ and $X_5$.

So the removal of $C_2$ has not collapsed the system. Now we find the graph $G_1 \setminus \{C_4\}$.

Let $S_2$ be the associated connection matrix which is as follows.

$$
S_2 = \begin{bmatrix}
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}.
$$

Now using $X_1 = (1 \ 0 \ 0 \ 0 \ 0)$ we find the hidden pattern using $S_2$ which is $(1 \ 0 \ 0 \ 0 \ 1)$.

Now for $X_2 = (0 \ 1 \ 0 \ 0 \ 0)$ we find the hidden pattern using $S_2$ which is $(1 \ 1 \ 1 \ 0 \ 1)$. 
For $X_3 = (0 \ 0 \ 1 \ 0 \ 0)$ we using $S_2$ find the hidden pattern which is $(0 \ 0 \ 1 \ 0 \ 0)$.

For $X_5 = (0 \ 0 \ 0 \ 1 \ 0)$ we find the hidden pattern using $S_2$ which is $(1 \ 1 \ 1 \ 1 \ 1)$.

Now for $X_6 = (0 \ 0 \ 0 \ 0 \ 1)$ we find the hidden pattern using $S_2$ which is as follows $(0 \ 0 \ 0 \ 1)$.

The most influential node is $X_5 = (0 \ 0 \ 0 \ 1 \ 0)$; $C_5$ is the most influence node. Though $C_4$ is very influential its removal has weakened only the most influential node $C_2$ but has no impact on the other most influential node $C_5$.

Now $G_1 \setminus \{C_5\}$ gives the following graph.

Let $S_3$ be the connection matrix of the graph $G_1 \setminus \{C_5\}$.

$$S_3 = \begin{bmatrix}
c_1 & c_2 & c_3 & c_4 & c_6 \\
0 & 0 & 0 & 0 & 1 \\
1 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}.$$
Now $X_1 = (1 0 0 0 0)$ on the dynamical system $S_3$ yields $(1 0 0 0 1)$.

$X_2 = (0 1 0 0 0)$ on $S_3$ yields $(1 1 1 1 1)$.

$X_3 = (0 0 1 0 0)$ on $S_3$ yields $(0 0 1 0 0)$.

$X_4 = (0 0 0 1 0)$ on $S_3$ yields $(0 0 0 1 0)$.

$X_6 = (0 0 0 0 1)$ on $S_3$ yields $(0 0 0 0 1) = X_6$.

Thus the most influential node $X_4$ becomes a least influential node. However $X_2$ remains as the most influential node.

Thus $C_5$ is most influential vital node for it can also affect the most influential node to become a least influential node.

Now we study the graph $G_2$ given in chapter IV.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>Number of edges incident to the vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (100000)$</td>
<td>$(111111)$</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (010000)$</td>
<td>$(010111)$</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (001000)$</td>
<td>$(111111)$</td>
<td>3</td>
</tr>
<tr>
<td>$X_4 = (000100)$</td>
<td>$(000111)$</td>
<td>4</td>
</tr>
<tr>
<td>$X_5 = (000010)$</td>
<td>$(000111)$</td>
<td>3</td>
</tr>
<tr>
<td>$X_6 = (000001)$</td>
<td>$(000111)$</td>
<td>2</td>
</tr>
</tbody>
</table>
Only the nodes $X_1$ and $X_3$ are the most influential node and $X_2$ is more influential node.

However the nodes $X_4$, $X_5$ and $X_6$ are just influential node. There is no influential node or less influential node or least influential node for this particular system.

Now $G_2 \setminus C_3$ gives the following graph.

The related connection matrix $R_1$ of the graph $G_2 \setminus C_3$ is as follows:

$$R_1 = \begin{bmatrix}
1 & 2 & 4 & 5 & 6 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}.$$ 

Now using $R_1$ we find the hidden pattern of $X_1 = (1 0 0 0 0 0)$ which is $(1 0 0 0 0 0)$.

The hidden pattern of $X_2 = (01000)$ using the dynamical system $R_1$ is $(0 1 1 1 1 1)$.
The hidden pattern of the node $X_4 = (0 \ 0 \ 1 \ 0 \ 0)$ is as follows: $(0 \ 0 \ 1 \ 1 \ 1)$.

Now the hidden pattern of $X_5 = (0 \ 0 \ 0 \ 1 \ 0)$ is given by $(0 \ 0 \ 1 \ 1 \ 1)$. Finally the hidden pattern of $X_6 = (0 \ 0 \ 0 \ 0 \ 1)$ is $(0 \ 0 \ 1 \ 1 \ 1)$.

Thus more influential node becomes the most influential node and all the other three nodes are unaffected the removal of the most influential node. Now we see the most influential node viz $X_1$ becomes the least influential node.

Now we remove the other most influential node $C_1$ from the graph $G_2$. $G_2 \ {\{C_1\}}$ gives the following graph.

![Graph Diagram]

$$R_2 = \begin{bmatrix}
c_2 & c_3 & c_4 & c_5 & c_6 \\
c_2 & 0 & 0 & 1 & 0 & 0 \\
c_3 & 1 & 0 & 0 & 0 & 0 \\
c_4 & 0 & 0 & 0 & 1 & 1 \\
c_5 & 0 & 0 & 1 & 0 & 0 \\
c_6 & 0 & 0 & 0 & 1 & 0 \\
\end{bmatrix}.$$
We find the hidden pattern of the on state of the nodes \( C_4, C_2, C_3, C_5 \) and \( C_6 \).

Let \( X_2 = (1\ 0\ 0\ 0\ 0) \), to find the hidden pattern of \( X_2 \) using \( R_2 \). The hidden pattern of \( X_2 \) is \( (1\ 0\ 1\ 1\ 1) \).

The hidden pattern of the node initial state vector \( X_3 = (0\ 1\ 0\ 0\ 0) \) is \( (1\ 1\ 1\ 1\ 1) \). For the initial state of \( X_4 = (0\ 0\ 1\ 0\ 0) \) the hidden pattern is \( (0\ 0\ 1\ 1\ 1) \).

The hidden pattern for the state vector \( X_5 = (0\ 0\ 0\ 1\ 0) \) is \( (0\ 0\ 1\ 1\ 1) \). Finally the hidden pattern for the initial state vector \( X_6 = (0\ 0\ 0\ 1) \) is \( (0\ 0\ 1\ 1\ 1) \). Thus \( X_3 \) continues to be the most influential node even if the most influential node \( C_1 \) is removed.

So \( C_3 \) happens to be the better of the two influential nodes \( C_1 \) and \( C_3 \).

Now we analyse the graph III of the problem.

![Graph III](image)

Now the influential nodes are tabled using the connection matrix \( M_3 \) of the graph \( G_3 \) in the following.
The most influential node is $X_6$, the more influential node is $X_1$. However the vertex $C_4$ which has maximum number of edges adjacent to it is the least influential node of the system. Now $G_3 \setminus C_6$ gives the following graph.

$$S_1 = \begin{pmatrix}
   c_1 & c_2 & c_3 & c_4 & c_5 \\
   c_1 & 0 & 1 & 0 & 0 \\
   c_2 & 0 & 0 & 1 & 0 \\
   c_3 & 0 & 0 & 0 & 1 \\
   c_4 & 0 & 0 & 0 & 1 \\
   c_5 & 0 & 0 & 1 & 0 
\end{pmatrix}.$$
Now using $S_1$ we find the most influential node from the table calculated.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(11111)</td>
<td>1</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(01111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(00111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (00010)$</td>
<td>(00011)</td>
<td>3</td>
</tr>
<tr>
<td>$X_5 = (00001)$</td>
<td>(00011)</td>
<td>1</td>
</tr>
</tbody>
</table>

The most influential node now is $X_1$ and the more influential node is $X_2$.

So the removal of the most influential node $X_6$ makes the more influential node $X_1$ to be the most influential node and so on.

Now we remove the node $X_1$ from $G_3$.

We get the following graph $G_3 \setminus C_1 =$

![Graph G3 \setminus C1](image)

The connection matrix of the graph $G_3 \setminus C_1$ is as follows:
Now we tabulate the hidden patterns;

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_2 = (10000)</td>
<td>(11110)</td>
<td>1</td>
</tr>
<tr>
<td>X_3 = (01000)</td>
<td>(01110)</td>
<td>2</td>
</tr>
<tr>
<td>X_4 = (00100)</td>
<td>(00110)</td>
<td>4</td>
</tr>
<tr>
<td>X_5 = (00010)</td>
<td>(00110)</td>
<td>3</td>
</tr>
<tr>
<td>X_6 = (00001)</td>
<td>(00111)</td>
<td>2</td>
</tr>
</tbody>
</table>

X_2 happens to be the most influential node X_3 and X_6 are the more influential node. Removal of the more influential node C_1 makes the most influential node X_6 into a more influential node.

Thus we will now remove X_5 from the graph G_3.

\[
\text{G}_3 \setminus \text{C}_5 =
\]
The associated connection matrix of $G_3 \setminus C_5$ is as follows:

$$
T_1 = \begin{bmatrix}
    c_1 & c_2 & c_3 & c_4 & c_6 \\
    c_1 & 0 & 1 & 0 & 0 \\
    c_2 & 0 & 0 & 1 & 0 \\
    c_3 & 0 & 0 & 0 & 1 \\
    c_4 & 0 & 0 & 0 & 0 \\
    c_6 & 0 & 0 & 1 & 0
\end{bmatrix}.
$$

The table of influential nodes is given in the following:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(11111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(01110)</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(00110)</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (00010)$</td>
<td>(00010)</td>
<td>2</td>
</tr>
<tr>
<td>$X_6 = (00001)$</td>
<td>(00001)</td>
<td>2</td>
</tr>
</tbody>
</table>

Most influential node is $X_1$, the non influential nodes are $X_4$ and $X_6$.

However all the fives nodes have the same number of edges incident to it. So the removal of the least influential node into a non influential node. We call such nodes as the most powerful nodes of the dynamical system.

Powerful nodes in general need not be the most influential node. Likewise the most influential node need not be a powerful node. Thus a most powerful node is a node whose deletion makes the most influential node into a non influential node.
The more powerful node is that node whose deletion makes the more influential node or most influential node into a least influential node. Likewise the other types of powerful nodes are defined.

Now we study the graph \( G_4 \).

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_1 = (100000) )</td>
<td>(110111)</td>
<td>3</td>
</tr>
<tr>
<td>( X_2 = (010000) )</td>
<td>(010100)</td>
<td>2</td>
</tr>
<tr>
<td>( X_3 = (001000) )</td>
<td>(111111)</td>
<td>2</td>
</tr>
<tr>
<td>( X_4 = (000100) )</td>
<td>(010100)</td>
<td>5</td>
</tr>
<tr>
<td>( X_5 = (000010) )</td>
<td>(010111)</td>
<td>3</td>
</tr>
<tr>
<td>( X_6 = (000001) )</td>
<td>(010111)</td>
<td>3</td>
</tr>
</tbody>
</table>

The most influential node is \( C_3 \) and the least influential nodes are \( X_4 \) and \( X_2 \).

\( X_4 \) has the most number of edges incident to it \( C_1 \) is the more influential node. There is no node which is non influential.

Let us study the graph \( G_4 \) with node \( C_3 \) removed which is as follows:
The connection matrix of $G_4 \setminus C_3$ is as follows:

$$
\begin{bmatrix}
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0 \\
\end{bmatrix} = L_1.
$$

Using $L_1$ we tabulate the hidden pattern of the on state of the nodes $C_1, C_2, C_4, C_5$ and $C_6$ in the following table.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(11111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(01100)</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (00100)$</td>
<td>(01100)</td>
<td>4</td>
</tr>
<tr>
<td>$X_5 = (00010)$</td>
<td>(01111)</td>
<td>1</td>
</tr>
<tr>
<td>$X_6 = (00001)$</td>
<td>(01111)</td>
<td>3</td>
</tr>
</tbody>
</table>

The most influential node is $X_1$, however it has only 2 edges adjacent to it.

The more influential nodes are $X_5$ and $X_6$ however the number of edges adjacent to $X_5$ is only one the least number of edges but it is the more influential node.

However there is no non influential nodes in this case.

Now we find the graph $G_4 \setminus C_4$ which is as follows:
The connection matrix $D_1$ of the graph $G_4 \setminus C_4$ is as follows:

$$D_1 = \begin{pmatrix}
1 & 0 & 0 & 0 & 1 \\
0 & 1 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0
\end{pmatrix}.$$ 

The table of hidden patterns to find the influential node is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(10011)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(01000)</td>
<td>0</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(10111)</td>
<td>1</td>
</tr>
<tr>
<td>$X_5 = (00010)$</td>
<td>(00011)</td>
<td>2</td>
</tr>
<tr>
<td>$X_6 = (00001)$</td>
<td>(00011)</td>
<td>3</td>
</tr>
</tbody>
</table>
From the table it is clear that the non influential node is $C_2$ as there is only zero number of edges incident to it.

Most influential node is $X_3$ and more influential node is $X_1$.

Just influential nodes are $C_5$ and $C_6$.

However $C_6$ has the maximum number of edges incident to it and $C_3$ has the least number of edges adjacent to it viz. one edge but it is the most influential node.

However the edge $C_2$ is non influential as it has no edge adjacent towards it.

Consider the graph $G_4 \setminus C_6$ which is as follows:

$$G_4 \setminus C_6 =$$

Now we give the connection matrix of the graph $G_4 \setminus C_6$ and is as follows:
The table of hidden patterns using the dynamical system $V_1$ is given by the following table.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(11010)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(01010)</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(11110)</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (00010)$</td>
<td>(01010)</td>
<td>5</td>
</tr>
<tr>
<td>$X_5 = (00001)$</td>
<td>(01011)</td>
<td>1</td>
</tr>
</tbody>
</table>

The most influential node is $C_3$ but the number of edges incident to it is 2.

However the node $C_4$ has the highest number of nodes adjacent to it however it is not even the most influential node only a just influential node.

Now we study $G_4 \setminus C_2$ directed graph and FCMs associated with it.
The related connection matrix of the graph $G_4 \setminus C_2$ is as follows:

$$W_1 = \begin{bmatrix}
0 & 0 & 1 & 0 & 1 \\
1 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 1 & 0
\end{bmatrix}.$$

Now we get the table of hidden patterns:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(10111)</td>
<td>3</td>
</tr>
<tr>
<td>$X_3 = (01000)$</td>
<td>(11111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (00100)$</td>
<td>(00100)</td>
<td>3</td>
</tr>
<tr>
<td>$X_5 = (00010)$</td>
<td>(00111)</td>
<td>3</td>
</tr>
<tr>
<td>$X_6 = (00001)$</td>
<td>(00111)</td>
<td>3</td>
</tr>
</tbody>
</table>
The most influential node is $C_3$ which has the least number of edges adjacent to it. However $C_4$ which has 3 edges incident to it however it is a non influential node. The more influential node is $C_1$ and the just influential nodes are $C_5$ and $C_6$.

Now we study the case for the NCMs given in example 4.2.

Consider the neutrosophic graph $G_1 \setminus \{C_2\}$ which is as follows:

The table of the graph $G_1$ is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(11111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(01101)</td>
<td>4</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(01101)</td>
<td>1</td>
</tr>
<tr>
<td>$X_4 = (00010)$</td>
<td>(00011)</td>
<td>2</td>
</tr>
<tr>
<td>$X_5 = (00001)$</td>
<td>(00001)</td>
<td>2</td>
</tr>
</tbody>
</table>

$C_1$ is the most influential node. $C_2$ and $C_3$ are more influential nodes but $C_3$ has only one edge incident to it.
but $C_2$ has four edges incident to it. However $X_5$ is a non influential node.

Now the connection matrix of $G_1 \setminus C_2$ is as follows;

$$
S_1 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}.
$$

The table of comparison of influential nodes using matrix $S_1$ is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1000)$</td>
<td>(1011)</td>
<td>1</td>
</tr>
<tr>
<td>$X_3 = (0100)$</td>
<td>(0100)</td>
<td>0</td>
</tr>
<tr>
<td>$X_4 = (0010)$</td>
<td>(0011)</td>
<td>2</td>
</tr>
<tr>
<td>$X_5 = (0001)$</td>
<td>(0001)</td>
<td>1</td>
</tr>
</tbody>
</table>

$X_2$ is a special node for $C_3$ which is a more influential node is made into a non influential node.

Now we remove the node $C_1$ from the graph $G_1$.

![Diagram](image-url)
The neutrosophic connection matrix of the graph $G_1 \setminus C_1$ is as follows:

$$
M_1 = \begin{bmatrix}
0 & 1 & 0 & 1 \\
1 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0
\end{bmatrix}.
$$

The table of hidden pattern using the matrix $M_1$ is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_2 = (1000)$</td>
<td>$(110I)$</td>
<td>3</td>
</tr>
<tr>
<td>$X_3 = (0100)$</td>
<td>$(110I)$</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (0010)$</td>
<td>$(0011)$</td>
<td>1</td>
</tr>
<tr>
<td>$X_5 = (0001)$</td>
<td>$(0001)$</td>
<td>1</td>
</tr>
</tbody>
</table>

The most influential nodes are $C_2$ and $C_3$. $C_3$ and $C_5$ non influential nodes.

More influential node $C_3$ is made into a most influential node and so on.

This type of study can be made to study the influential nodes as well as powerful nodes of the problem.

Consider the graph $G_2$ of the example 4.2.

The table of hidden patterns is as follows.
<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(10000)</td>
<td>1</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(11101)</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(10101)</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (00010)$</td>
<td>(11111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_5 = (00001)$</td>
<td>(10001)</td>
<td>3</td>
</tr>
</tbody>
</table>

The most influential node is $C_4$ and more influential node is $C_2$. $C_1$ is a non influential node.

Now we study the system with $C_1$ removed. Consider the graph $G_2 \setminus C_1$.

![Graph](image)

The connection matrix of $G_2 \setminus C_1$ is as follows:

$$
P_1 = \begin{bmatrix}
    c_2 & c_3 & c_4 & c_5 \\
    c_2 & 0 & 1 & 0 & 0 \\
    c_3 & 0 & 0 & 0 & 1 \\
    c_4 & 1 & 0 & 0 & 1 \\
    c_5 & 0 & 0 & 0 & 0
\end{bmatrix}.$$


Now the table of hidden pattern using $P_1$ is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_2 = (1000)$</td>
<td>(1101)</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (0100)$</td>
<td>(0101)</td>
<td>2</td>
</tr>
<tr>
<td>$X_4 = (0010)$</td>
<td>(1111)</td>
<td>2</td>
</tr>
<tr>
<td>$X_5 = (0001)$</td>
<td>(0001)</td>
<td>2</td>
</tr>
</tbody>
</table>

Removal the node $C_1$ does not alter the most influential node. However $C_5$ happens to be a non influential node. Consider $G_2 \setminus C_2$ the graph which is as follows:

The connection matrix of $G_2 \setminus C_2$ is as follows:

$$W_1 = \begin{bmatrix}
  c_1 & c_3 & c_4 & c_4 \\
  c_1 & 0 & 0 & 0 \\
  c_3 & 0 & 0 & 1 \\
  c_4 & 0 & 0 & 1 \\
  c_5 & 1 & 0 & 0 \end{bmatrix}.$$
The table of hidden patterns is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1000)$</td>
<td>(1000)</td>
<td>1</td>
</tr>
<tr>
<td>$X_3 = (0100)$</td>
<td>(1101)</td>
<td>1</td>
</tr>
<tr>
<td>$X_4 = (0010)$</td>
<td>(1011)</td>
<td>1</td>
</tr>
<tr>
<td>$X_5 = (0001)$</td>
<td>(1001)</td>
<td>3</td>
</tr>
</tbody>
</table>

Clearly $C_5$ is the node which has maximum number of edges incident to it but it is not the most influential node.

$C_3$ and $C_4$ which has only one edge incident towards it happens to be the most influential node.

However $C_1$ happens to be a non influential node.

The node $C_2$ is not a powerful node for the change made by it on the NCM is negligible.

Consider the graph $G_2 \setminus C_3$ which is follows:

$$G_2 \setminus C_3 =$$

The connection matrix $W_1$ of the graph $G_2 \setminus C_3$ is as follows:
W_1 = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 \\
1 & 0 & 0 & 0 & 0
\end{bmatrix}.

Now using W_1 we find the hidden pattern is as follows.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_1 = (1000)</td>
<td>(1000)</td>
<td>1</td>
</tr>
<tr>
<td>X_2 = (0100)</td>
<td>(0100)</td>
<td>1</td>
</tr>
<tr>
<td>X_4 = (0010)</td>
<td>(1111)</td>
<td>2</td>
</tr>
<tr>
<td>X_5 = (0001)</td>
<td>(1001)</td>
<td>2</td>
</tr>
</tbody>
</table>

X_4 is the most influential node and two nodes X_1 and X_2 are non influential.

Now we study the graph G_2 \setminus C_4 which is as follows:

The connection matrix of G_2 \setminus C_4 is as follows:
The table of hidden pattern is as follows.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>X₁ = (1000)</td>
<td>(1000)</td>
<td>1</td>
</tr>
<tr>
<td>X₂ = (0100)</td>
<td>(1111)</td>
<td>1</td>
</tr>
<tr>
<td>X₃ = (0010)</td>
<td>(1011)</td>
<td>2</td>
</tr>
<tr>
<td>X₅ = (0001)</td>
<td>(1001)</td>
<td>2</td>
</tr>
</tbody>
</table>

C₂ is the most influential node. Thus the removal of the most influential node. C₄ makes the more influential node C₂ to be the most influential node and nothing more.

Now consider the graph G₂ \ C₅ which is as follows.
The connection matrix associated with $G_2 \setminus C_5$ is as follows:

$$P_1 = \begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}.$$ 

The hidden pattern of the nodes is given by the following table.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1 \ 0 \ 0 \ 0)$</td>
<td>(1 0 0 0)</td>
<td>0</td>
</tr>
<tr>
<td>$X_2 = (0 \ 1 \ 0 \ 0)$</td>
<td>(0 1 1 0)</td>
<td>2</td>
</tr>
<tr>
<td>$X_3 = (0 \ 0 \ 1 \ 0)$</td>
<td>(0 0 1 0)</td>
<td>1</td>
</tr>
<tr>
<td>$X_4 = (0 \ 0 \ 0 \ 1)$</td>
<td>(0 1 1 1)</td>
<td>1</td>
</tr>
</tbody>
</table>

This removal of node $C_5$ makes $X_4$ the most influential nodes which has only one edge adjacent to it.

The nodes $C_1$ and $C_3$ are the non influential nodes.

Now we study the graph $G_3$. The table of hidden patterns is as follows.

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (10000)$</td>
<td>(11000)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (01000)$</td>
<td>(11000)</td>
<td>4</td>
</tr>
<tr>
<td>$X_3 = (00100)$</td>
<td>(11111)</td>
<td>1</td>
</tr>
</tbody>
</table>
The least number of edges are incident towards $C_3$ and $C_3$ is the most influential node $C_4$ and $C_5$ are more influential nodes.

There is no non influential node.

We now study the graph $G_3 \setminus C_1$ which is as follows:

![Graph G3 \setminus C1](image)

The connection matrix of $G_3 \setminus C_1$ is as follows:

$$
B_3 = \begin{bmatrix}
0 & 0 & 1 & 0 \\
0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 \\
1 & 0 & 1 & 0
\end{bmatrix}
$$

The table of hidden pattern is as follows:

<table>
<thead>
<tr>
<th>$X_4$</th>
<th>$X_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(00010)</td>
<td>(11011)</td>
</tr>
<tr>
<td>(00001)</td>
<td>(11011)</td>
</tr>
</tbody>
</table>
The least number of edges is incident to the node $C_3$ and it is the most influential node and $C_2$ is the non influential node though it has two edges incident towards it. $X_4$ and $X_5$ are more influential nodes.

Consider the graph $G_3 \setminus C_2$ is as follows.

The connection matrix associated with $G_3 \setminus C_2$ is as follows:

\[
E_1 = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 1 \\
0 & 0 & 1 & 0 & 0
\end{bmatrix}
\]
The table of hidden patterns is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1000)$</td>
<td>(1000)</td>
<td>0</td>
</tr>
<tr>
<td>$X_3 = (0100)$</td>
<td>(0111)</td>
<td>1</td>
</tr>
<tr>
<td>$X_4 = (0010)$</td>
<td>(0011)</td>
<td>3</td>
</tr>
<tr>
<td>$X_5 = (0001)$</td>
<td>(0011)</td>
<td>2</td>
</tr>
</tbody>
</table>

$X_3$ is the most influential node and $X_4$ and $X_5$ are more influential nodes.

$X_1$ is non influential node consider the graph $G_3 \setminus C_3$ which is as follows.

The connection matrix associated with $G_3 \setminus C_3$ is as follows:

$$Y_1 = \begin{bmatrix} c_1 & c_2 & c_4 & c_5 \\ c_1 & 0 & 1 & 0 & 0 \\ c_2 & 1 & 0 & 0 & 0 \\ c_4 & 0 & 1 & 0 & 1 \\ c_5 & 0 & 1 & 1 & 0 \end{bmatrix}.$$
We give the following table of hidden patterns using the connection matrix $Y_1$

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1000)$</td>
<td>$(1100)$</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (0100)$</td>
<td>$(1100)$</td>
<td>4</td>
</tr>
<tr>
<td>$X_4 = (0010)$</td>
<td>$(1111)$</td>
<td>3</td>
</tr>
<tr>
<td>$X_5 = (0001)$</td>
<td>$(1111)$</td>
<td>3</td>
</tr>
</tbody>
</table>

The most influential nodes are $X_4$ and $X_5$.

Every node is influential.

This is a unique one in which both the most influential nodes have the maximum number of edges incident to it.

Now consider the graph $G_3 \setminus C_5$ which is given in the following.

![Graph G3 \setminus C5](image)

The connection matrix of $G_3 \setminus C_5$ is as follows.
The table of comparison of the NCMs associated with the graph $G_3 \setminus C_5$ which is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No.of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1 = (1000)$</td>
<td>(1100)</td>
<td>2</td>
</tr>
<tr>
<td>$X_2 = (0100)$</td>
<td>(1100)</td>
<td>3</td>
</tr>
<tr>
<td>$X_3 = (0010)$</td>
<td>(0010)</td>
<td>1</td>
</tr>
<tr>
<td>$X_4 = (0001)$</td>
<td>(1101)</td>
<td>2</td>
</tr>
</tbody>
</table>

The most influential node is $X_4$.

$X_3$ is the non influential node.

Consider the graph $G_3 \setminus C_4$ which is as follows:

The connection matrix associated with $G_3 \setminus C_4$ is as follows:

$$
\begin{bmatrix}
 c_1 & c_2 & c_3 & c_4 \\
 c_1 & 0 & 1 & 0 & 0 \\
 c_2 & 1 & 0 & 0 & 0 \\
 c_3 & 0 & 0 & 1 & 0 \\
 c_4 & 0 & 1 & 0 & 0
\end{bmatrix}
$$
Influential or Vital nodes of FCMs and NCMs

\[ F_1 = \begin{bmatrix} c_1 & c_2 & c_3 & c_4 \\ c_1 & 0 & 1 & 0 & 0 \\ c_2 & 1 & 0 & 0 & 0 \\ c_3 & 0 & 0 & 0 \\ c_4 & 0 & 0 & 0 \end{bmatrix}. \]

The table of hidden pattern is as follows:

<table>
<thead>
<tr>
<th>Initial State vectors</th>
<th>Hidden patterns</th>
<th>No. of edges incident to vertices</th>
</tr>
</thead>
<tbody>
<tr>
<td>X_1 = (1000)</td>
<td>(1100)</td>
<td>2</td>
</tr>
<tr>
<td>X_2 = (0100)</td>
<td>(1100)</td>
<td>3</td>
</tr>
<tr>
<td>X_3 = (0010)</td>
<td>(0010)</td>
<td>0</td>
</tr>
<tr>
<td>X_4 = (0001)</td>
<td>(1101)</td>
<td>1</td>
</tr>
</tbody>
</table>

The most influential node is X_5 (C_5) which has the least number of edges adjacent to it. However C_3 is the non influential node. Now having studied about the most influential node, more influential node and so on.

From the study of influential node we can also study the powerful node the removal of which will collapse the system.

We suggest a few problems.

**Problems:**

1. Obtain the special contributing feature of the ‘most influential node’, ‘more influential node’ and so on.

2. Does there exist a directed graph of a FCM or NCM which is such that every node has equal number of edges incident to it?
3. Show using real world problems that a node with one edge incident to it can also be a most influential node.

4. Prove that in any NCM or FCM removal of a most influential node need not result in the collapsing of the dynamical system.

5. Prove for real world problem in which FCMs or NCMs are used a most influential node in general need not be the most powerful node.

6. Find any interesting relation that exist between the most influential node and the more powerful node.

7. Can there be a NCM or FCM associated with a real world problem in which every node is the most influential node?

8. Can there be a NCM or FCM in which every node is a most powerful node?

9. Let G be the graph associated with a FCM

![Graph Diagram]

- $C_1$ to $C_2$
- $C_6$ to $C_1$
- $C_5$ to $C_6$
- $C_4$ to $C_5$
- $C_3$ to $C_5$
- $C_4$ to $C_3$
- $C_7$ to $C_4$
- $C_2$ to $C_7$
(i) Find the most influential node of the FCMs associated with graph G.

(ii) Find the most influential nodes of $G \setminus C_i$; $1 \leq i \leq 7$.

(iii) Does the FCMs contain most powerful nodes?

10. Let $G$ be the directed neutrosophic graph associated with the NCM.

11. Prove or disprove a most influential node is not the node with maximum number of edges incident to that node.
12. Study the powerful nodes of FCMs and NCMs.

13. Can one say the most powerful node is a node with the maximum number of edges incident to it?


15. Is question (13) true in case of social networking?

16. Can you prove or disprove the fact that in networking graph does not function like the directed graphs of FCMs or NCMs?

17. Show the concept of influential node in an FCM or NCM can help the expert to analyse the problem in different angles.

18. Connect the notion of most influential node and the Kosko-Hamming distance of the same node given by two experts.

19. Prove or disprove the influential nodes of a NCMs or FCMs varies from expert to experts.

20. Prove or disprove the notion of most powerful node of an NCMs or FCMs is dependent on the experts.

21. Prove or disprove the notion of most powerful node varies from experts and experts.

22. Prove or disprove the notion of most influential node does not depend on NCM or FCMs.

23. Can merging of two FCMs affect the influential node?
24. Let $G_1 =$

![Graph G1](image1.png)

and $G_2 =$

![Graph G2](image2.png)

be two directed graphs of FCMs.
Now merging of $G_1$ with $G_2$ gives a graph with 7 vertices. $G_1$ is a graph with 6 vertices $G_2$ is also a graph with 7 vertices.
Now merging of $G_1$ with $G_2$ gives a graph $G$ of 7 vertices.
(i) Find \( G \setminus \{C_7, C_6\} \), \( G_1 \setminus \{C_6\} \) and \( G_2 \setminus \{C_7\} \).

(ii) Does the merged nodes act different from usual non merged nodes?

(iii) Does the merged graphs act differently on powerful nodes?

25. Study the influential nodes in case of New Average FCMs and New average NCMs.

26. Can one say if \( G_1, G_2, \ldots, G_t \) are directed graphs of the t-FCMs working on n nodes.

(i) Can we say the influence nodes of the t FCMs and NAFCMs are different?

(ii) Can we say the powerful nodes of the t-FCMs and the NAFCMs are different?

27. Study using 5 experts a real world problem using same number of nodes.

(i) For this find average and compare the influential nodes.

28. Can we establish that NAFCMs nodes which are most influential need not be most influential?

29. Study problem 27 in case of NANCMS.

30. Study the status of powerful node in case of NAFCMs.

31. Study the status of powerful nodes in case of NANCMS.
FURTHER READING


35. Hafner, V.V. Cognitive Maps for Navigation in Open Environments, http://citeseer.nj.nec.com/hafner00cognitive.html


Further Reading  233


60. Praseetha, V.R., A New Class of Fuzzy Relation Equation and its Application to a Transportation Problem, Masters


W.B. Vasantha Kandasamy, Department of Mathematics, Indian Institute of Technology, Chennai, March 1997.


92. Vasantha Kandasamy, W.B., and V. Indra. Applications of Fuzzy Cognitive Maps to Determine the Maximum Utility


100. Vasantha Kandasamy, W.B., Smarandache, F., and Ilanthenral, K., Pseudo Lattice Graphs and their Applications to Fuzzy and Neutrosophic Models, Educational Publisher Inc., Ohio, 2014.


INDEX

D
Directed neutrosophic graphs, 70-89

I
Influential node, 185-200

K
Kosko-Hamming distance (K-H distance), 127-140

L
Least influential node, 185-200
Less influential node, 185-200

M
Merged Fuzzy Cognitive Maps (MFCMs) model, 9-108
Merged Graphs, 9-35
Merged matrices, 16-40
Merged Neutrosophic Cognitive Maps (MNCMs) model, 9-108
Merged neutrosophic graphs, 72-95
More influential node of FCMs and NCMs, 185-200
Most influential node of FCMs and NCMs, 185-200
N

Neutrosophic merged connection matrix, 72-89
New Average simple FCMs (NASFCMs), 159-170
New Average simple NCMs (NASNCMs), 159-171
New Simple Average dynamical system, 159-170
New simple average neutrosophic matrices, 159-170
Non influential node, 185-200

P

Passive node, 185-200
Pseudo lattice graphs of type II, 9-35

V

Vital node, 185-200
ABOUT THE AUTHORS

Dr. W. B. Vasantha Kandasamy is a Professor in the Department of Mathematics, Indian Institute of Technology Madras, Chennai. In the past decade she has guided 13 Ph.D. scholars in the different fields of non-associative algebras, algebraic coding theory, transportation theory, fuzzy groups, and applications of fuzzy theory of the problems faced in chemical industries and cement industries. She has to her credit 653 research papers. She has guided over 100 M.Sc. and M.Tech. projects. She has worked in collaboration projects with the Indian Space Research Organization and with the Tamil Nadu State AIDS Control Society. She is presently working on a research project funded by the Board of Research in Nuclear Sciences, Government of India. This is her 98th book.

On India’s 60th Independence Day, Dr. Vasantha was conferred the Kalpana Chawla Award for Courage and Daring Enterprise by the State Government of Tamil Nadu in recognition of her sustained fight for social justice in the Indian Institute of Technology (IIT) Madras and for her contribution to mathematics. The award, instituted in the memory of Indian-American astronaut Kalpana Chawla who died aboard Space Shuttle Columbia, carried a cash prize of five lakh rupees (the highest prize-money for any Indian award) and a gold medal.

She can be contacted at vasanthakandasamy@gmail.com
Web Site: http://mat.iitm.ac.in/home/wbv/public_html/ or http://www.vasantha.in

Dr. Florentin Smarandache is a Professor of Mathematics at the University of New Mexico in USA. He published over 75 books and 200 articles and notes in mathematics, physics, philosophy, psychology, rebus, literature. In mathematics his research is in number theory, non-Euclidean geometry, synthetic geometry, algebraic structures, statistics, neutrosophic logic and set (generalizations of fuzzy logic and set respectively), neutrosophic probability (generalization of classical and imprecise probability). Also, small contributions to nuclear and particle physics, information fusion, neutrosophy (a generalization of dialectics), law of sensations and stimuli, etc. He got the 2010 Telesio-Galilei Academy of Science Gold Medal, Adjunct Professor (equivalent to Doctor Honoris Causa) of Beijing Jiaotong University in 2011, and 2011 Romanian Academy Award for Technical Science (the highest in the country). Dr. W. B. Vasantha Kandasamy and Dr. Florentin Smarandache got the 2012 New Mexico-Arizona and 2011 New Mexico Book Award for Algebraic Structures. He can be contacted at smarand@unm.edu

K. Ilanthenral is the editor of The Maths Tiger, Quarterly Journal of Maths. She can be contacted at ilanthenral@gmail.com
In this book for the first time authors have ventured to study, analyse and investigate fuzzy and neutrosophic models and the experts opinion. To make such a study, innovative techniques are defined and developed. Several important conclusions about these models can be derived using these new techniques. Open problems are suggested in this book.