On NeutroQuadrupleGroups

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NeutroAlgebraic structures and AntiAlgebraic structures

As generalizations and alternatives of classical algebraic structures Florentin Smarandache has introduced in 2019 the NeutroAlgebraic structures (or NeutroAlgebras) and AntiAlgebraic structures (or AntiAlgebras). Unlike the classical algebraic structures, where all operations are well-defined and all axioms are totally true, in NeutroAlgebras and AntiAlgebras the operations may be partially well-defined and the axioms partially true or respectively totally outer-defined and the axioms totally false.

NeutroAlgebraic structures and AntiAlgebraic structures

These NeutroAlgebras and AntiAlgebras form a new field of research, which is inspired from our real world. In this talk, we study neutrosophic quadruple algebraic structures and NeutroQuadrupleAlgebraicStructures. NeutroQuadrupleGroup is studied in particular and several examples are provided. It is shown that $(NQ(\mathbb{Z}), \div)$ is a NeutroQuadrupleGroup.

Operation, NeutroOperation, AntiOperation

When we define an operation on a given set, it does not automatically mean that the operation is well-defined. There are three possibilities:

- The operation is well-defined (or inner-defined) for all set's elements (as in classical algebraic structures this is classical Operation).
- The operation if well-defined for some elements, indeterminate for other elements, and outer-defined for others elements (this is NeutroOperation).
- The operation is outer-defined for all set's elements (this is AntiOperation).

Axiom, NeutroAxiom, AntiAxiom

Similarly for an axiom, defined on a given set, endowed with some operation(s). When we define an axiom on a given set, it does not automatically mean that the axiom is true for all set's elements. We have three possibilities again:

- The axiom is true for all set's elements (totally true) (as in classical algebraic structures; this is a classical Axiom).
- The axiom if true for some elements, indeterminate for other elements, and false for other elements (this is NeutroAxiom).
- The axiom is false for all set's elements (this is AntiAxiom).

Algebra, NeutroAlgebra, AntiAlgebra

- An algebraic structure who's all operations are well-defined and all axioms are totally true is called Classical Algebraic Structure (or Algebra).
- An algebraic structure that has at least one NeutroOperation or one NeutroAxiom (and no AntiOperation and no AntiAxiom) is called NeutroAlgebraic Structure (or NeutroAlgebra).
- An algebraic structure that has at least one AntiOperation or Anti Axiom is called AntiAlgebraic Structure (or AntiAlgebra).

$$< \mathbf{Algebra}, \mathbf{NeutroAlgebra}, \mathbf{AntiAlgebra} >.$$

The Neutrosophic Quadruple Numbers and the Absorbance Law were introduced by Smarandache in 2015 [1]; they have the general form:
$N = a + bT + cI + dF$, where a, b, c, d may be numbers of any type (natural, integer, rational, irrational, real, complex, etc.), where "a" is the known part of the neutrosophic quadruple number N, while "$bT + cI + dF$" is the unknown part of the neutrosophic quadruple number N; then the unknown part is split into three subparts: degree of confidence (T), degree of indeterminacy of confidence-nonconfidence (I), and degree of nonconfidence (F). N is a four-dimensional vector that can also be written as: $N = (a, b, c, d)$.

There are transcendental, irrational etc. numbers that are not well known, they are only partially known and partially unknown, they may have infinitely many decimals. Not even the most modern supercomputers can compute more than a few thousands decimals, but the infinitely many left decimals still remain unknown. Therefore, such numbers are very little known (because only a finite number of decimals are known), and infinitely unknown (because an infinite number of decimals are unknown). Take for example: $\sqrt{2} = 1.4142\ldots$.

Definition

A neutrosophic set of quadruple numbers denoted by $NQ(X)$ is a set defined by

$$NQ(X) = \{(a, bT, cI, dF) : a, b, c, d \in \mathbb{R} \text{ or } \mathbb{C}\}$$

where T, I, F have their usual neutrosophic logic meanings.

Multiplication of two neutrosophic quadruple numbers

Multiplication of two neutrosophic quadruple numbers cannot be carried out like multiplication of two real or complex numbers. In order to multiply two neutrosophic quadruple numbers the prevalence order of $\{T, I, F\}$ is required. Consider the following prevalence orders: Suppose in an optimistic way we consider the prevalence order $T \succ I \succ F$. Then we have:

$TI = IT = \max\{T, I\} = T,\quad TF = FT = \max\{T, F\} = T,$
$IF = FI = \max\{I, F\} = I,\quad TT = T^2 = T,$
$II = I^2 = I,\quad FF = F^2 = F.$

Or we consider the prevalence order $T \prec I \prec F$. Then we have:

$TI = IT = \max\{T, I\} = I,\quad TF = FT = \max\{T, F\} = F,$
$IF = FI = \max\{I, F\} = F,\quad TT = T^2 = T,$
$II = I^2 = I,\quad FF = F^2 = F.$

Division of two neutrosophic quadruple numbers

Two neutrosophic quadruple numbers $m = (a_1, b_1T, c_1I, d_1F)$ and $n = (a_2, b_2T, c_2I, d_2F)$ cannot be divided as we do for real and complex numbers. Since the literal neutrosophic components T, I and F are not invertible, the inversion of a neutrosophic quadruple number or the division of a neutrosophic quadruple number by another neutrosophic quadruple number must be carried out a systematic way. Suppose we are to evaluate m/n. Then we must look for a neutrosophic quadruple number $p = (x, yT, zI, wF)$ equivalent to m/n. In this way, we write

$$\begin{aligned} m/n &= p \\ \Rightarrow \frac{(a_1, b_1T, c_1I, d_1F)}{(a_2, b_2T, c_2I, d_2F)} &= (x, yT, zI, wF) \\ \Leftrightarrow \quad (a_2, b_2T, c_2I, d_2F)(x, yT, zI, wF) &\equiv (a_1, b_1T, c_1I, d_1F). \qquad (1) \end{aligned}$$

Division of two neutrosophic quadruple numbers

Assuming the prevalence order $T \succ I \succ F$ and from the equality of two neutrosophic quadruple numbers, we obtain from Equation (1)

$$a_2x = a_1 \quad (2)$$

$$b_2x + (a_2 + b_2 + c_2 + d_2)y + b_2z + b_2w = b_1 \quad (3)$$

$$c_2x + (a_2 + c_2 + d_2)z + c_2w = c_1 \quad (4)$$

$$d_2x + (a_2 + d_2)w = d_1 \quad (5)$$

a system of linear equations in unknowns x, y, z and w.

Division of two neutrosophic quadruple numbers

By similarly assuming the prevalence order $T \prec I \prec F$, we obtain from Equation (1)

$$
\begin{aligned}
a_2x &= a_1 && (6)\\
b_2x + (a_2+b_2)y &= b_1 && (7)\\
c_2x + c_2y + (a_2+b_2+c_2)z &= c_1 && (8)\\
d_2x + d_2y + d_2z + (a_2+b_2+c_2+d_2)w &= d_1 && (9)
\end{aligned}
$$

a system of linear equations in unknowns x, y, z and w.

Example

Let $a = (2, -T, I, 2F)$ and $b = (1, 2T, -I, F)$ be two neutrosophic quadruple numbers in $NQ(\mathbb{R})$.

(i) For the prevalence order $T \succ I \succ F$, we obtain

$$\frac{(2, -T, I, 2F)}{(1, 2T, -I, F)} = \left(2, -\frac{11}{3}T, 3I, 0F\right).$$

(ii) For the prevalence order $T \prec I \prec F$, we obtain

$$\frac{(2, -T, I, 2F)}{(1, 2T, -I, F)} = \left(2, -\frac{5}{3}T, \frac{2}{3}I, \frac{1}{3}F\right).$$

Neutrosophic quadruple set

Let $NQ(X)$ be a neutrosophic quadruple set and let $* : NQ(X) \times NQ(X) \to NQ(X)$ be a classical binary operation on $NQ(X)$. The couple $(NQ(X), *)$ is called a neutrosophic quadruple algebraic structure. The structure $(NQ(X), *)$ is named according to the classical laws and axioms satisfied or obeyed by $*$.

Neutrosophic quadruple hyper set

If $* : NQ(X) \times NQ(X) \to \mathbb{P}(NQ(X))$ is the classical hyper operation on $NQ(X)$. Then the couple $(NQ(X), *)$ is called a neutrosophic quadruple hyper algebraic structure; and the hyper structure $(NQ(X), *)$ is named according to the classical laws and axioms satisfied by $*$.

Neutrosophic quadruple homomorphism

If $(NQ(X), *)$ and $(NQ(Y), \circ)$ are two neutrosophic quadruple algebraic structures. The mapping $\phi : (NQ(X), *) \to (NQ(Y), \circ)$ is called a neotrosophic quadruple homomorphism if ϕ preserves $*$, $\circ$ and literal neutrosophic components T, I and F that is if:

(i) $\phi(x * y) = \phi(x) \circ \phi(y) \quad \forall\, x, y \in NQ(X)$.

(ii) $\phi(T) = T$.

(iii) $\phi(I) = I$.

(iv) $\phi(F) = F$.

Theorem

(i) $(NQ(\mathbb{Z}),+)$, $(NQ(\mathbb{Q}),+)$, $(NQ(\mathbb{R}),+)$ *and* $(NQ(\mathbb{C}),+)$ *are abelian groups.*

(ii) $(NQ(\mathbb{Z}),+,\times)$, $(NQ(\mathbb{Q}),+,\times)$, $(NQ(\mathbb{R}),+,\times)$ *and* $(NQ(\mathbb{C}),+,\times)$ *are commutative rings.*

(iii) $(NQ(\mathbb{Z}),\times)$ *is a commutative monoid.*

(iv) $(NQ(\mathbb{Z}),\times)$ *is not a group.*

(v) $(NQ(\mathbb{Z}),\div)$ *is not a group.*

Neutrosophic quadruple groups

Let $NQ(G)$ be a nonempty set and let $* : NQ(G) \times NQ(G) \rightarrow NQ(G)$ be a binary operation on $NQ(G)$. The couple $(NQ(G), *)$ is called a neutrosophic quadruple group if the following conditions hold:

(QG1)
$x * y \in G \; \forall x, y \in NQ(G)$ [closure law].

(QG2)
$x * (y * z) = (x * y) * z \; \forall x, y, z \in G$ [axiom of associativity].

(QG3)
There exists $e \in NQ(G)$ such that $x * e = e * x = x \; \forall x \in NQ(G)$ [axiom of existence of neutral element].
(QG4)
There exists $y \in NQ(G)$ such that $x * y = y * x = e \; \forall x \in NQ(G)$ [axiom of existence of inverse element] where e is the neutral element of $NQ(G)$.
If in addition $\forall x, y \in NQ(G)$, we have
(QG5)
$x * y = y * x$, then $(NQ(G), *)$ is called a commutative neutrosophic quadruple group.

NeutroSophication of the law and axioms of the neutrosophic quadruple

(NQ(G)1)
There exist some duplets $(x, y), (u, v), (p, q), \in NQ(G)$ such that $x * y \in G$ (inner-defined with degree of truth T) and [$u * v =$ indeterminate (with degree of indeterminacy I) or $p * q \notin NQ(G)$ (outer-defined/falsehood with degree of falsehood F)] [NeutroClosureLaw].
(NQ(G)2)
There exist some triplets $(x, y, z), (p, q, r), (u, v, w) \in NQ(G)$ such that $x * (y * z) = (x * y) * z$ (inner-defined with degree of truth T) and [$[p * (q * r)]$or $[(p * q) * r] =$ indeterminate (with degree of indeterminacy I) or $u * (v * w) \neq (u * v) * w$ (outer-defined/falsehood with degree of falsehood F)] [NeutroAxiom of associativity (NeutroAssociativity)].

NeutroSophication of the law and axioms of the neutrosophic quadruple

(NQ(G)3)

There exists an element $e \in NQ(G)$ such that $x * e = e * x = x$ (inner-defined with degree of truth T) and $[[x * e]\text{or}[e * x] =$ indeterminate (with degree of indeterminacy I) or $x * e \neq x \neq e * x$ (outer-defined/falsehood with degree of falsehood F)] for at least one $x \in NQ(G)$ [NeutroAxiom of existence of neutral element (NeutroNeutralElement)].

NeutroSophication of the law and axioms of the neutrosophic quadruple

(NQ(G)4)

There exists an element $u \in NQ(G)$ such that $x * u = u * x = e$ (inner-defined with degree of truth T) and $[[x * u]\text{or}[u * x)] =$ indeterminate (with degree of indeterminacy I) or $x * u \neq e \neq u * x$ (outer-defined/falsehood with degree of falsehood F)] for at least one $x \in G$ [NeutroAxiom of existence of inverse element (NeutroInverseElement)] where e is a NeutroNeutralElement in $NQ(G)$.

NeutroSophication of the law and axioms of the neutrosophic quadruple

(NQ(G)5)

There exist some duplets $(x, y), (u, v), (p, q) \in NQ(G)$ such that $x * y = y * x$ (inner-defined with degree of truth T) and $[[u*v]\text{or}[v*u] =$ indeterminate (with degree of indeterminacy I) or $p*q \neq q*p$ (outer-defined/falsehood with degree of falsehood F)] [NeutroAxiom of commutativity (NeutroCommutativity)].

NeutroQuadrupleGroup

A NeutroQuadrupleGroup $NQ(G)$ is an alternative to the neutrosophic quadruple group $Q(G)$ that has at least one NeutroLaw or at least one of $\{NQ(G)1, NQ(G)2, NQ(G)3, NQ(G)4\}$ with no AntiLaw or AntiAxiom.

NeutroCommutativeQuadrupleGroup

A NeutroCommutativeQuadrupleGroup $NQ(G)$ is an alternative to the commutative neutrosophic quadruple group $Q(G)$ that has at least one NeutroLaw or at least one of $\{NQ(G)1, NQ(G)2, NQ(G)3, NQ(G)4\}$ and $NQ(G)5$ with no AntiLaw or AntiAxiom.

Theorem

Let $\mathbb{U}$ be a nonempty finite or infinite universe of discourse and let S be a finite or infinite subset of $\mathbb{U}$. If n classical operations (laws and axioms) are defined on S where $n \geq 1$, then there will be $(2^n - 1)$ NeutroAlgebras and $(3^n - 2^n)$ AntiAlgebras (see [2]).

Theorem

Let $(NQ(G), *)$ *be a neutrosophic quadruple group. Then:*

(i) *there are 15 types of NeutroQuadrupleGroups,*

(ii) *there are 31 types of NeutroCommutativeQuadrupleGroups.*

Theorem

For positive integers $n = 2, 3, 4, \cdots,$

(i) $(NQ(\mathbb{Z}_n), -)$ *is a NeutroQuadrupleGroup.*

(ii) $(NQ(\mathbb{Z}_n), \times)$ *is a NeutroCommutativeQuadrupleGroup.*

Theorem

(i) $(NQ(\mathbb{Z}), -)$ *is a NeutroQuadrupleGroup.*

(ii) $(NQ(\mathbb{Z}), \times)$ *is a NeutroCommutativeQuadrupleGroup.*

(iii) $(NQ(\mathbb{Z}), \div)$ *is a NeutroCommutativeQuadrupleGroup.*

NeutroClosure of $\div$ over $NQ(\mathbb{Z})$

For the degree of truth, let $a = (0, 0T, I, 0F) \in NQ(\mathbb{Z})$. Then

$$a \div a = \frac{(0, 0T, I, 0F)}{(0, 0T, I, 0F)} = (1 - k_1 - k_2, 0T, k_1 I, k_2 F) \in NQ(\mathbb{Z}), k_1, k_2 \in \mathbb{Z}.$$

NeutroAssociativity of $\div$ over $NQ(\mathbb{Z})$

For the degree of indeterminacy, let $a = (4, 5T, -2I, -7F)$, $b = (0, -6T, I, 3F) \in NQ(\mathbb{Z})$. Then

$$a \div b = \frac{(4, 5T, -2I, -7F)}{(0, -6T, I, 3F)} = \left(\frac{4}{0}, ?T, ?I, ?F\right) \notin NQ(\mathbb{Z}).$$

NeutroAssociativity of $\div$ over $NQ(\mathbb{Z})$

For the degree of falsehood, let $a = (0, 0T, 0I, F)$, $b = (0, 0T, 0I, 2F) \in NQ(\mathbb{Z})$. Then

$$a \div b = \frac{(0, 0T, 0I, F)}{(0, 0T, 0I, 2F)} = \left(\frac{1}{2} - k, 0T, 0I, kF\right) \notin NQ(\mathbb{Z}), k \in \mathbb{Z}.$$

NeutroAssociativity of $\div$ over $NQ(\mathbb{Z})$

For the degree of truth, let $a = (6, 6T, 6I, 6F)$, $b = (2, 2T, 2I, 2F)$, $c = (-1, 0T, 0I, 0F) \in NQ(\mathbb{Z})$. Then

$$\begin{aligned} a \div (b \div c) &= (6, 6T, 6I, 6F) \div ((2, 2T, 2I, 2F) \div (-1, 0T, 0I, 0F)) \\ &= (6, 6T, 6I, 6F) \div (-2, 0T, 0I, 0F) \\ &= (-3, 0T, 0I, 0F). \\ (a \div b) \div c &= ((6, 6T, 6I, 6F) \div (2, 2T, 2I, 2F)) \div (-1, 0T, 0I, 0F) \\ &= (3, 0T, 0I, 0F) \div (-1, 0T, 0I, 0F) \\ &= (-3, 0T, 0I, 0F). \end{aligned}$$

NeutroAssociativity of $\div$ over $NQ(\mathbb{Z})$

For the degree of indeterminacy, let $a = (4, -T, 2I, -7F)$, $b = (0, T, 0I, -8F)$, $c = (0, 0T, 9I, -F) \in NQ(\mathbb{Z})$. Then

$$\begin{aligned}
a \div (b \div c) &= (4, -T, 2I, -7F) \div ((0, T, 0I, -8F) \div (0, 0T, 9I, -F)) \\
&= (4, -T, 2I, -7F) \div \left(8 - k, \frac{1}{8}T, -9I, kF\right), k \in \mathbb{Z} \\
&= (?, ?T, ?I, ?F). \\
(a \div b) \div c &= ((4, -T, 2I, -7F) \div (0, T, 0I, -8F)) \div (0, 0T, 9I, -F) \\
&= \left(\frac{4}{0}, ?T, ?I, ?F\right) \div (0, 0T, 9I, -F) \\
&= (?, ?T, ?I, ?F).
\end{aligned}$$

NeutroAssociativity of $\div$ over $NQ(\mathbb{Z})$

For the degree of falsehood, let $a = (0, 5T, 0I, 0F)$, $b = (0, T, 0I, 0F)$, $c = (5, 0T, 0I, 0F) \in NQ(\mathbb{Z})$. Then

$$
\begin{aligned}
a \div (b \div c) &= (0, 5T, 0I, 0F) \div ((0, T, 0I, 0F) \div (5, 0T, 0I, 0F)) \\
&= (0, 5T, 0I, 0F) \div \left(0, \frac{1}{5}T, 0I, 0F\right) \\
&= (25 - k_1 - k_2 - k_3, k_1T, k_2I, k_3F) \in NQ(\mathbb{Z}), \\
(a \div b) \div c &= ((0, 5T, 0I, 0F) \div (0, T, 0I, 0F)) \div (5, 0T, 0I, 0F) \\
&= (5 - k_1 - k_2 - k_3, k_1T, k_2I, k_3F) \div (5, 0T, 0I, 0F), \\
&= \left(\frac{1}{5}(5 - k_1 - k_2 - k_3), \frac{1}{5}k_1T, \frac{1}{5}k_2I, \frac{1}{5}k_3F\right) \notin NQ(\mathbb{Z})
\end{aligned}
$$

$k_1, k_2, k_3 \in \mathbb{Z}$.

Existence of NeutroUnitaryElement and NeutroInverseElement in $NQ(\mathbb{Z})$

Let $a = (0, T, 0I, 0F)$, $b = (0, 0T, I, 0F)$, $c = (0, 0T, 0I, F) \in NQ(\mathbb{Z})$. Then

$$a \div a = \frac{(0, T, 0I, 0F)}{(0, T, 0I, 0F)} = (1 - k_1 - k_2 - k_3, k_1 T, k_2 I, k_3 F) \quad (10)$$

$$b \div b = \frac{(0, 0T, I, 0F)}{(0, 0T, I, 0F)} = (1 - k_1 - k_2, 0T, k_1 I, k_2 F) \quad (11)$$

$$c \div c = \frac{(0, 0T, 0I, F)}{(0, 0T, 0I, F)} = (1 - k, 0T, 0I, kF) \quad (12)$$

$$a \div b = \frac{(0, T, 0I, 0F)}{(0, 0T, I, 0F)} = (-(k_1 + k_2), T, k_1 I, k_2 F) \quad (13)$$

$$b \div a = \frac{(0, 0T, I, 0F)}{(0, T, 0I, 0F)} = (-(k_1 + k_2 + k_3), k_1 T, k_2 I, k_3 F) \quad (14)$$

$k, k_1, k_2, k_3 \in \mathbb{Z}$.

Existence of NeutroUnitaryElement and NeutroInverseElement in $NQ(\mathbb{Z})$

For the degree of truth, putting $k_1 = 1, k_2 = k_3 = 0$ in Equation (10), $k_1 = 1$, $k_2 = 0$ in Equation (11) and $k = 1$ in Equation (12) we will obtain $a \div a = a$, $b \div b = b$ and $c \div c = c$. These show that a, b, c are respectively NeutroUnitaryElements and NeutroInverseElements in $NQ(\mathbb{Z})$.

Existence of NeutroUnitaryElement and NeutroInverseElement in $NQ(\mathbb{Z})$

For the degree of falsehood, putting $k_1 \neq 1, k_2 \neq k_3 \neq 0$ in Equation (10), $k_1 \neq 1, k_2 \neq 0$ in Equation (11) and $k \neq 1$ in Equation (12) we will obtain $a \div a \neq a, b \div b \neq b$ and $c \div c \neq c$. These show that a, b, c are respectively not NeutroUnitaryElements and NeutroInverseElements in $NQ(\mathbb{Z})$.

NeutroCommtativity of $\div$ over $NQ(\mathbb{Z})$

For the degree of truth, putting $k_1 = 1, k_2 = k_3 = 0$ in Equation (10), $k_1 = 1$, $k_2 = 0$ in Equation (11) and $k = 1$ in Equation (12) we will obtain $a \div a = a, b \div b = b$ and $c \div c = c$. These show the commutativity of $\div$ wrt a, b and c $NQ(\mathbb{Z})$.

For the degree of falsehood, putting $k_1 = k_2 = k_3 = 1$ in Equation (13) and Equation (14), we will obtain $a \div b = (-2, T, I, F)$ and $b \div a = (-3, T, I, F) \neq a \div b$. Hence, $\div$ is NeutroCommutative in $NQ(\mathbb{Z})$.

NeutroQuadrupleSubgroup

Let $(NQ(G), *)$ be a neutrosophic quadruple group. A nonempty subset $NQ(H)$ of $NQ(G)$ is called a NeutroQuadrupleSubgroup of $NQ(G)$ if $(NQ(H), *)$ is a neutrosophic quadruple group of the same type as $(NQ(G), *)$.

Example

(i) For $n = 2, 3, 4, \cdots$ $(NQ(n\mathbb{Z}), -)$ is a NeutroQuadrupleSubgroup of $(NQ(\mathbb{Z}), -)$.

(ii) For $n = 2, 3, 4, \cdots$ $(NQ(n\mathbb{Z}), \times)$ is a NeutroQuadrupleSubgroup of $(NQ(\mathbb{Z}), \times)$.

Example

(i) Let $NQ(H) = \{(a, bT, cI, dF) : a, b, c, d \in \{1, 2, 3\}\}$ be a subset of the NeutroQuadrupleGroup $(NQ(\mathbb{Z}_4), -)$. Then $(NQ(H), -)$ is a NeutroQuadrupleSubgroup of $(NQ(\mathbb{Z}_4), -)$.

(ii) Let $NQ(K) = \{(w, xT, yI, zF) : a, b, c, d \in \{1, 3, 5\}\}$ be a subset of the NeutroQuadrupleGroup $(NQ(\mathbb{Z}_6), \times)$. Then $(NQ(H), \times)$ is a NeutroQuadrupleSubgroup of $(NQ(\mathbb{Z}_6), \times)$.

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