ON NEUTROSOPHIC SETS AND TOPOLOGY

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Abstract

Recently, F.Smarandache generalized the Atanassov's intuitionistic fuzzy sets and other kinds of sets to neutrosophic sets. Also, this author defined the notion of neutrosophic topology on the non-standard interval. One can expect some relation between the intuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in this work that this is false.

Keywords: Logic, Set-Theory,Topology, Atanassov's IFSs.

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1 On neutrosophic Topology

1.1. Introduction.

The neutrosophic logic is a formal frame trying to measure the truth, indeterminacy, and falsehood.

Smarandache [36] remarks the differences between neutrosophic logic (NL) and intuitionistic fuzzy logic (IFL) and the corresponding neutrosophic sets and intuitionistic fuzzy sets. The main differences are:

a) Neutrosophic Logic can distinguish between absolute truth (i.e. that is an unalterable and permanent fact), and relative truth (where facts may vary depending on the circumstances), because

NL(absolute truth)=1^+ while NL(relative truth)=1. This has obvious application in philosophy. That's why the unitary standard interval $[0, 1]$ used in IFL has been extended to the unitary non-standard interval $]^-0, 1^+[$ in NL.

Similar distinctions for absolute or relative falsehood, and absolute or relative indeterminacy are allowed in NL.

b) In NL there is no restriction on T, I, F other than they are subsets of $]^{-}0, 1^{+}[$, thus:

$^{-}0 \leq \inf T + \inf I + \inf F \leq \sup T + \sup I + \sup F \leq 3^{+}$.

This non-restriction allows paraconsistent, dialetheist, and incomplete information to be characterized in NL (i.e. the sum of all three components if they are defined as points, or sum of superior limits of all three components if they are defined as subsets can be > 1, for paraconsistent information coming from different sources, or < 1 for incomplete information), while that information can not be described in IFL because in IFL the components T (truth), I (indeterminacy), F (falsehood) are restricted either to $t + i + f = 1$ if T, I, F are all reduced to the points t,i, f respectively, or to $\sup T + \sup I + \sup F = 1$ if T, I, F are subsets of $[0, 1]$.

c) In NL the components T, I, F can also be non-standard subsets included in the unitary non-standard interval $]^{-}0, 1^{+}[$, not only standard subsets, included in the unitary standard interval $[0, 1]$ as in IFL.

In various recent papers [35,38,39,40], F. Smarandache generalizes intuitionistic fuzzy sets (IFSs) and other kinds of sets to neutrosophic sets (NSs). In [39] some distinctions between NSs and IFSs are underlined.

The notion of intuitionistic fuzzy set defined by K.T. Atanassov [1] has been applied by Çoker [8] for study intuitionistic fuzzy topological spaces. This concept has been developed by many authors (Bayhan and Çoker[6], Çoker, [7,8], Çoker and Eş [9], Eş and Çoker[12], Gürçay, Çoker and Eş[13], Hanafy [14], Hur, Kim and Ryou [15], Lee and Lee [16]; Lupiáñez [17-21], Turanh and Çoker [41]).

A few years ago raised some controversy over whether the term "intuitionistic fuzzy set" was appropriate or not (see [11] and [4]). At present, it is customary to speak of "Atanassov' intuitionistic fuzzy set"

F. Smarandache also defined the notion of neutrosophic topology on the non-standard interval [35].

One can expect some relation between the inuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in this chapter that this is false. Indeed, the complement of an IFS A is not the complement of A in the neutrosophic operation, the union and the intersection of IFSs do not coincide with the corresponding operations for NSs, and finally an intuitionistic fuzzy topology is not necessarily a neutrosophic topology.

Clearly, for their various applications to many areas of knowledge, including philosophy, religion, sociology, .. (see [5,40,42]), the Atanassov' intuitionistic fuzzy sets and the neutrosophic sets are notions that use knowledge-based techniques to support human decision-making, learning and action.

1.2. Basic definitions.

First, we present some basic definitions:

Definition 1 *Let X be a non-empty set. An intuitionistic fuzzy set (IFS for short) A, is an object having the form $A = \{< x, \mu_A, \gamma_A > /x \in X\}$ where the* *functions* $\mu_A : X \to I$ *and* $\gamma_A : X \to I$ *denote the degree of membership (namely* $\mu_A(x)$*) and the degree of nonmembership (namely* $\gamma_A(x)$*) of each element* $x \in X$ *to the set A, respectively, and* $0 \le \mu_A(x) + \gamma_A(x) \le 1$ *for each* $x \in X$*. [1].*

Definition 2 *Let* X *be a non-empty set, and the IFSs* $A = \{< x, \mu_A, \gamma_A > |x \in X\}$, $B = \{< x, \mu_B, \gamma_B > |x \in X\}$*. Let*

$\overline{A} = \{< x, \gamma_A, \mu_A > |x \in X\}$

$A \cap B = \{< x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B > |x \in X\}$

$A \cup B = \{< x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B > |x \in X\}$*.[3].*

Definition 3 *Let* X *be a non-empty set. Let* $0_\sim = \{< x, 0, 1 > |x \in X\}$ *and* $1_\sim = \{< x, 1, 0 > |x \in X\}$*.[8].*

Definition 4 *An intuitionistic fuzzy topology (IFT for short) on a non-empty set* X *is a family* τ *of IFSs in* X *satisfying:*

(a) $0_\sim, 1_\sim \in \tau$,

(b) $G_1 \cap G_2 \in \tau$ *for any* $G_1, G_2 \in \tau$,

(c) $\cup G_j \in \tau$ *for any family* $\{G_j | j \in J\} \subset \tau$.

In this case the pair (X, τ) *is called an intuitionistic fuzzy topological space (IFTS for short) and any IFS in* τ *is called an intuitionistic fuzzy open set (IFOS for short) in* X*. [8].*

Definition 5 *Let* T, I,F *be real standard or non-standard subsets of the non-standard unit interval* $]^{-}0, 1^{+}[$, *with*

$\sup T = t_{\sup}$, $\inf T = t_{\inf}$

$\sup I = i_{\sup}$, $\inf I = i_{\inf}$

$\sup F = f_{\sup}$, $\inf F = f_{\inf}$ *and* $n_{\sup} = t_{\sup} + i_{\sup} + f_{\sup}$ $n_{\inf} = t_{\inf} + i_{\inf} + f_{\inf}$,

T, I,F *are called neutrosophic components. Let* U *be an universe of discourse, and* M *a set included in* U*. An element* x *from* U *is noted with respect to the set* M *as* $x(T, I, F)$ *and belongs to* M *in the following way: it is* $t\%$ *true in the set,* $i\%$ *indeterminate (unknown if it is) in the set, and* $f\%$ *false, where* t *varies in* T, i *varies in* I, f *varies in* F*. The set* M *is called a neutrosophic set (NS). [40].*

Remark. *All IFS is a NS.*

Definition 6 *Let* S_1 *and* S_2 *be two (unidimensional) real standard or non-standard subsets, then we define:*

$S_1 \oplus S_2 = \{x | x = s_1 + s_2,$ *where* $s_1 \in S_1$ *and* $s_2 \in S_2\}$,

$S_1 \ominus S_2 = \{x | x = s_1 - s_2,$ *where* $s_1 \in S_1$ *and* $s_2 \in S_2\}$,

$S_1 \odot S_2 = \{x | x = s_1 \cdot s_2,$ *where* $s_1 \in S_1$ *and* $s_2 \in S_2\}$*. [36].*

Definition 7 *One defines, with respect to the sets A an B over the universe U:*
1. Complement: if $x(T_1, I_1, F_1) \in A$*, then*
$x(\{1^+\} \ominus T_1, \{1^+\} \ominus I_1, \{1^+\} \ominus F_1) \in C(A)$.
2. Intersection: if $x(T_1, I_1, F_1) \in A$*,* $x(T_2, I_2, F_2) \in B$*, then*
$x(T_1 \odot T_2, I_1 \odot I_2, F_1 \odot F_2) \in A \cap B$.
3.Union: if $x(T_1, I_1, F_1) \in A$*,* $x(T_2, I_2, F_2) \in B$*, then*
$x(T_1 \oplus T_2 \ominus T_1 \odot T_2, I_1 \oplus I_2 \ominus I_1 \odot I_2, F_1 \oplus F_2 \ominus F_1 \odot F_2) \in A \cup B$.
[40].

1.3. Results.

Proposition 1. Let A be an IFS in X, and $j(A)$ be the corresponding NS. We have that the complement of $j(A)$ is not necessarily $j(\overline{A})$.

Proof. If $A =< x, \mu_A, \gamma_A >$ is $x(\mu_A(x), 1 - \mu_A(x) - \nu_A(x), \nu_A(x)) \in j(A)$.
Then ,
for $0_\sim =< x, 0, 1 >$ is $x(0, 0, 1) \in j(0_\sim)$
for $1_\sim =< x, 1, 0 >$ is $x(1, 0, 0) \in j(1_\sim)$
and for $\overline{A} =< x, \gamma_A, \mu_A >$ is $x(\gamma_A(x), 1 - \mu_A(x) - \nu_A(x), \mu_A(x)) \in j(\overline{A})$.
Thus, $1_\sim = \overline{0_\sim}$ and $j(\ 1_\sim) \neq C(j(0_\sim))$ because $x(1, 0, 0,) \in j(\ 1_\sim)$ but $x(\{1^+\}, \{1^+\}, \{0^+\}) \in C(j(0_\sim))$.

Proposition 2. Let A and B be two IFSs in X,and $j(A)$ and $j(B)$ be the corresponding NSs. We have that $j(A)\cup\ j(B)$ is not necessarily $j(A \cup B)$, and $j(A)\cap\ j(B)$ is not necessarily $j(A \cap B)$.

Proof. Let $A =< x, 1/2, 1/3 >$ and $B =< x, 1/2, 1/2 >$ (i.e. μ_A, ν_A, μ_B, ν_B are constant maps).

Then, $A \cup B =< x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B >=< x, 1/2, 1/3 >$ and $x(1/2, 1/6, 1/3) \in j(A \cup B)$. On the other hand, $x(1/2, 1/6, 1/3) \in j(A), x(1/2, 0, 1/2) \in j(B), x(1, 1/6, 5/6) \in j(A) \oplus\ j(B), x(1/4, 0, 1/6) \in j(A) \odot\ j(B)$ and $x(3/4, 1/6, 2/3) \in j(A) \cup\ j(B)$ .Thus $j(A \cup B) \neq j(A) \cup\ j(B)$.

Analogously, $A \cap B =< x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B >=< x, 1/2, 1/2 >$ and $x(1/2, 0, 1/2) \in j(A \cap B)$, but $x(1/4, 0, 1/6) \in j(A) \cap j(B)$.Thus, $j(A \cap B) \neq j(A) \cap j(B)$.

Definition 8 *Let's construct a neutrosophic topology on* $NT =]^-0, 1^+[$*, considering the associated family of standard or non-standard subsets included in* NT*, and the empty set which is closed under set union and finite intersection neutrosophic. The interval* NT *endowed with this topology forms a neutrosophic topological space. [35].*

Proposition 3. Let (X, τ) be an intuitionistic fuzzy topological space. Then, the family $\{j(U)|U \in \tau\}$ is not necessarily a neutrosophic topology.

Proof. Let $\tau = \{1_\sim, 0_\sim, A\}$ where $A =< x, 1/2, 1/2 >$ then $x(1, 0, 0) \in j(1_\sim)$, $x \in (0, 0, 1) \in j(0_\sim)$ and $x(1/2, 0, 1/2) \in j(A)$. Thus $\{j(1_\sim), j(0_\sim), j(A)\}$ is not a neutrosophic topology, because this family is not closed by finite intersections, indeed, $x(1/2, 0, 0) \in j(1_\sim) \cap j(A)$, and this neutrosophic set is not in the family.

2 Other neutrophic topologies

2.1. Introduction.

F. Smarandache also defined various notions of neutrosophic topologies on the non-standard interval [35,40].

One can expect some relation between the intuitionistic fuzzy topology on an IFS and the neutrosophic topology. We show in this chapter that this is false. Indeed, the union and the intersection of IFSs do not coincide with the corresponding operations for NSs, and an intuitionistic fuzzy topology is not necessarilly a neutrosophic topology on the non-standard interval, in the various senses defined by Smarandache.

2.2. Basic definitions.

First, we present some basic definitions:

Definition 9 *Let $J \in \{T, I, F\}$ be a component. Most known N-norms are:*
The ***algebraic product N-norm****:* $N_{n-a\lg ebraic}J(x,y) = x \cdot y$
The ***bounded N-norm****:* $N_{n-bounded}J(x,y) = \max\{0, x+y-1\}$
The ***default (min) N-norm****:* $N_{n-\min}J(x,y) = \min\{x,y\}$
N_n *represent the intersection operator in neutrosophic set theory. Indeed* $x \wedge y = (T_\wedge, I_\wedge, F_\wedge)$*.*
[40]

Definition 10 *Let $J \in \{T, I, F\}$ be a component. Most known N-conorms are:*
The ***algebraic product N-conorm****:* $N_{c-a\lg ebraic}J(x,y) = x + y - x \cdot y$
The ***bounded N-conorm****:* $N_{c-bounded}J(x,y) = \min\{1, x+y\}$
The ***default (max) N-conorm****:* $N_{c-\max}J(x,y) = \max\{x,y\}$
N_c *represent the union operator in neutrosophic set theory.Indeed* $x \vee y = (T_\vee, I_V, F_\vee)$
[40]

2.3. Results.

Proposition 1. Let A and B be two IFSs in X, and $j(A)$ and $j(B)$ be the corresponding NSs. We have that $j(A) \cup j(B)$ is not necessarily $j(A \cup B)$, and $j(A) \cap j(B)$ is not necessarily $j(A \cap B)$, for any of three definitions of intersection of NSs.

Proof. Let $A =< x, 1/2, 1/3 >$ and $B =< x, 1/2, 1/2 >$ (i.e. μ_A, ν_A, μ_B, ν_B are constant maps).

Then, $A \cup B =< x, \mu_A \vee \mu_B, \gamma_A \wedge \gamma_B >=< x, 1/2, 1/3 >$ and $x(1/2, 1/6, 1/3) \in j(A \cup B)$. On the other hand, $x(1/2, 1/6, 1/3) \in j(A), x(1/2, 0, 1/2) \in j(B)$.

Then, we have that:

1) for the union operator defined by the algebraic product N-conorm $x(3/4, 1/6, 2/3) \in j(A) \cup j(B)$.

2) for the union operator defined by the bounded N-conorm $x(1, 1/6, 5/6) \in j(A) \cup j(B)$.

3) for the union operator defined by the default (max) N-conorm $x(1/2, 1/6, 1/2) \in j(A) \cup j(B)$.

Thus $j(A \cup B) \neq j(A) \cup j(B)$,with the three definitions.

Analogously, $A \cap B = < x, \mu_A \wedge \mu_B, \gamma_A \vee \gamma_B > = < x, 1/2, 1/2 >$ and $x(1/2, 0, 1/2) \in j(A \cap B)$.

And, we have that:

1) for the intersection operator defined by the algebraic product N-norm $x(1/4, 0, 1/6) \in j(A) \cap j(B)$.

2) for the intersection operator defined by the bounded N-norm $x(0, 0, 0) \in j(A) \cap j(B)$.

3) for the intersection operator defined by the default (min) N-norm $x(1/2, 0, 1/3) \in j(A) \cap j(B)$.

Thus $j(A \cap B) \neq j(A) \cap j(B)$,with the three definitions.

Definition 11 *Let's construct a neutrosophic topology on $NT =]^{-}0, 1^{+}[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space. There exist various notions of neutrosophic topologies on NT , defined by using various N-norm/N-conorm operators. [35, 40].*

Proposition 2. Let (X, τ) be an intuitionistic fuzzy topological space. Then, the family $\{j(U) | U \in \tau\}$ is not necessarily a neutrosophic topology on NT (in the three defined senses).

Proof. Let $\tau = \{1_\sim, 0_\sim, A\}$ where $A = < x, 1/2, 1/2 >$ then $x(1, 0, 0) \in j(1_\sim)$, $x \in (0, 0, 1) \in j(0_\sim)$ and $x(1/2, 0, 1/2) \in j(A)$. Thus $\tau^* = \{j(1_\sim), j(0_\sim), j(A)\}$ is not a neutrosophic topology, because this family is not closed by finite intersections, for any neutrosophic topology on NT. Indeed,

1) For the intersection defined by the algebraic product N-norm, we have that $x(1/2, 0, 0) \in j(1_\sim) \cap j(A)$, and this neutrosophic set is not in the family τ^*.

2) For the intersection defined by the bounded N-norm, we have also that $x(1/2, 0, 0) \in j(1_\sim) \cap j(A)$, and this neutrosophic set is not in the family τ^*.

3) For the intersection defined by the default (min) N-norm, we have also that $x(1/2, 0, 0) \in j(1_\sim) \cap j(A)$, and this neutrosophic set is not in the family τ^*.

3 Interval neutrosophic sets and Topology

3.1. Introduction.

Also, Wang, Smarandache, Zhang, and Sunderraman [42] introduced the notion of interval neutrosophic set, which is an instance of neutrosophic set and studied various properties. We study in this chapter relations between interval neutrosophic sets and topology.

3.2. Basic definitions.

First, we present some basic definitions. For definitions on non-standard Analysis, see [33] :

Definition 12 *Let X be a space of points (objects) with generic elements in X denoted by x. An interval neutrosophic set (INS) A in X is characterized by thuth-membership function T_A, indeteminacy-membership function I_A and falsity-membership function F_A. For each point x in X, we have that $T_A(x)$, $I_A(x)$, $F_A(x) \in [0,1]$. [42].*

Remark. *All INS is clearly a NS.*

When X is continuous, an INS A can be written as

$$A = \int_X \langle T(x), I(x), F(x) \rangle /x, \quad x \in X$$

When X is discrete, an INS A can be written as

$$A = \sum_{i=1}^{n} \langle T(x_i), I(x_i) F(x_i) \rangle /x_i \ , \quad x_i \in X$$

Definition 13 *a) An interval neutrosophic set A is empty if $\inf T_A(x) = \sup T_A(x) = 0$, $\inf I_A(x) = \sup I_A(x) = 1$, $\inf F_A(x) = \sup F_A(x) = 0$ for all x in X.*

b) Let $\underline{0} =< 0,1,1 >$ and $\underline{1} =< 1,0,0 >$.[42].

Definition 14 *(Complement) Let C_N denote a neutrosophic complement of A.*

Then C_N is a function $C_N : N \to N$ and C_N must satisfy at least the following three axiomatic requirements:

1. $C_N(\underline{0}) = \underline{1}$ and $C_N(\underline{1}) = \underline{0}$ (boundary conditions).

2. Let A and B be two interval neutrosophic sets defined on X, if $A(x) \leq B(x)$, then $C_N(A(x)) \geq C_N(B(x))$, for all x in X. (monotonicity).

3. Let A be an interval neutrosophic set defined on X, then $C_N(C_N(A(x))) = A(x)$, for all x in X. (involutivity).[42].

Remark. *There are many functions which satisfy the requirement to be the complement operator of interval*

neutrosophic sets. Here we give one example.

Definition 15 *(Complement C_{N_1}) The complement of an interval neutrosophic set A is denoted by $\bar{A}$ and is defined by*

$T_{\bar{A}}(x) = F_A(x)$;

$\inf I_{\bar{A}}(x) = 1 - \sup I_A(x)$;

$\sup I_{\bar{A}}(x) = 1 - \inf I_A(x)$;

$F_{\bar{A}}(x) = T_A(x)$; *for all x in X.*

Definition 16 *(N-norm) Let I_N denote a neutrosophic intersection of two interval neutrosophic sets A and B. Then I_N is a function $I_N : N \times N \rightarrow N$ and I_N must satisfy at least the following four axiomatic requirements:*

1. $I_N\ (A(x), \underline{1}) = A(x)$, for all x in X. (boundary condition).

2. $B(x) \leq C(x)$ implies $I_N\ (A(x), B(x)) \leq I_N(A(x), C(x))$, for all x in X. (monotonicity).

3. $I_N\ (A(x), B(x)) = I_N(B(x), A(x))$, for all x in X. (commutativity).

4. $I_N\ (A(x), I_N(B(x), C(x))) = I_N(I_N(A(x), B(x)), C(x))$, for all x in X. (associativity).[42].

Remark. *Here we give one example of intersection of two interval neutrosophic sets which satis es above N-norm axiomatic requirements. Other diferent definitions can be given for different applications*

Definition 17 *(Intersection I_{N_1}) The intersection of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cap B$, whose truth-membership, indeterminacy-membership, and false-membership are related to those of A and B by*

$\inf\ T_C(x) = \min(\inf T_A(x); \inf T_B(x))$,
$\sup T_C(x) = \min(\sup T_A(x); \sup T_B(x))$,
$\inf I_C(x) = \max(\inf I_A(x); \inf I_B(x))$,
$\sup I_C(x) = \max(\sup I_A(x); \sup I_B(x))$,
$\inf F_C(x) = \max(\inf F_A(x); \inf F_B(x))$,
$\sup F_C(x) = \max(\sup F_A(x); \sup F_B(x))$; *for all x in X.*

Definition 18 *(N-conorm) Let U_N denote a neutrosophic union of two interval neutrosophic sets A and B. Then U_N is a function $U_N : N \times N \rightarrow N$*

and U_N must satisfy at least the following four axiomatic requirements:

1. $U_N(A(x), \underline{0}) = A(x)$, for all x in X. (boundary condition).

2. $B(x) \leq C(x)$ implies $U_N(A(x), B(x)) \leq U_N(A(x), C(x))$, for all x in X. (monotonicity).

3. $U_N(A(x), B(x)) = U_N(B(x), A(x))$, for all x in X. (commutativity).

4. $U_N(A(x), U_N(B(x), C(x))) = U_N(U_N(A(x), B(x)), C(x))$, for all x in X. (associativity). [42].

Remark. *Here we give one example of union of two interval neutrosophic sets which satis es above N-conorm axiomatic requirements. Other different definitions can be given for different applications.*

Definition 19 *(Union U_{N_1}) The union of two interval neutrosophic sets A and B is an interval neutrosophic set C, written as $C = A \cup B$, whose truth-membership, indeterminacy-membership, and false-membership are related to those of A and B by*

$\inf T_C(x) = \max(\inf T_A(x); \inf T_B(x))$,
$\sup T_C(x) = \max(\sup T_A(x); \sup T_B(x))$,
$\inf I_C(x) = \min(\inf I_A(x); \inf I_B(x))$,
$\sup I_C(x) = \min(\sup I_A(x); \sup I_B(x))$,
$\inf F_C(x) = \min(\inf F_A(x); \inf F_B(x))$,

$\sup F_C(x) = \min(\sup F_A(x); \sup F_B(x))$, *for all x in X.*

3.3. Results.

Proposition 1. Let A be an IFS in X, and $j(A)$ be the corresponding INS. We have that the complement of $j(A)$ is not necessarily $j(\overline{A})$.

Proof. If $A =< x, \mu_A, \gamma_A >$ is $j(A) =< \mu_A, 0, \gamma_A >$.

Then ,

for $0_\sim =< x, 0, 1 >$ is $j(0_\sim) = j(< x, 0, 1 >) =< 0, 0, 1 >\neq \underline{0} =< 0, 1, 1 >$

for $1_\sim =< x, 1, 0 >$ is $j(1_\sim) = j(< x, 1, 0 >) =< 1, 0, 0 >= \underline{1}$

Thus, $1_\sim = \overline{0_\sim}$ and $j(1_\sim) = \underline{1} \neq C_N(j(0_\sim))$ because $C_N(\underline{1}) = \underline{0} \neq j(0_\sim)$.

Definition 20 *Let's construct a neutrosophic topology on $NT =]^-0, 1^+[$, considering the associated family of standard or non-standard subsets included in NT, and the empty set which is closed under set union and finite intersection neutrosophic. The interval NT endowed with this topology forms a neutrosophic topological space. [35].*

Proposition 2. Let (X, τ) be an intuitionistic fuzzy topological space. Then, the family of INSs $\{j(U)|U \in \tau\}$ is not necessarily a neutrosophic topology.

Proof. Let $\tau = \{1_\sim, 0_\sim, A\}$ where $A =< x, 1/2, 1/2 >$ then $j(1_\sim) = \underline{1}$, $j(0_\sim) =< 0, 0, 1 >\neq \varnothing$ and $j(A) =< 1/2, 0, 1/2 >$. Thus $\{j(1_\sim), j(0_\sim), j(A)\}$ is not a neutrosophic topology, because the empty INS is not in this family.

4 Neutrosophic paraconsistent Topology

The history of paraconsistent logic is not very long. It was designed by S. Jaskowski in 1948. Without knowing the work of this author, N.C. A. da Costa, from 1958, using different methods and ideas, began to make statements about this type of logic. After other logicians have developed independently, new systems of paraconsistent logic, as Routley, Meyer, Priest, Asenjo, Sette, Anderson and Benalp, Wolf (with da Costa himself), ... At present there is a thriving movement dedicated to the study of paraconsistent logic in several countries. In the philosophical aspect has meant, in some cases, a real opening of horizons, for example, in the treatment of the paradoxes, in efforts to treat rigorously dialectical thinking, in fact possible to develop a set theory inconsistent. .. Because of this, there is growing interest in understanding the nature and scope.

Jaskowski deductive logic led her to refer to several problems that caused the need for paraconsistent logic:

1) The problem of organizing deductive theories that contain contradictions, as in the dialectic: "The principle that no two contradictory statements are both true and false is the safest of all."

2) To study theories that there are contradictions engendered by vagueness: "The contemporary formal approach to logic increases the accuracy of research in many fields, but it would be inappropriate to formulate the principle of contradiction of Aristotle thus: *"Two contradictory propositions are not true"*. We need to add: *"in the same language"*or *"if the words that are part of those have the same meaning"*. This restriction is not always found in daily use, and also science, we often use terms that are more or less vague.

3) To study directly some postulates or empirical theories whose basic meanings are contradictory. This applies, for example, the physics at the present stage.

Objectives and method of construction of paraconsistent logics can be mentioned, besides those mentioned by Jaskowski:

1) To study directly the logical and semantic paradoxes, for example, if we directly study the paradoxes of set theory (without trying to avoid them, as it normally is), we need to construct theories of sets of such paradoxes arising, but without being formal antinomies. In this case we need a paraconsistent logic.

2) Better understand the concept of negation.

3) Have logic systems on which to base the paraconsistent theories. For example, set up logical systems for different versions and possibly stronger than standard theories of sets, of dialectics, and of certain physical theories that , perhaps, are inconsistent (some versions of quantum mechanics).

Various authors [31] worked on "paraconsistent Logics", that is, logics where some contradiction is admissible. We remark the theories exposed by Da Costa [10], Routley and other [34], and Peña [29,30].

Smarandache defined also the neutrosophic paraconsistent sets [Sm5] and he proposed a natural definition of neutrosophic paraconsistent topology.

A problem that we consider is the possible relation between this concept of neutrosophic paraconsistent topology and the previous notions of general neutrosophic topology and intuitionistic fuzzy topology. We show in this chapter that neutrosophic paraconsistent topology is not an extension of intuitionistic fuzzy topology.

First, we present some basic definitions:

Definition 21 *Let M be a non-empty set. A* ***general neutrosophic topology*** *on M is a family Ψ of neutrosophic sets in M satisfying the following axioms:*

(a) $0_{\sim} = x(0,0,1)$,$1_{\sim} = x(1,0,0) \in \Psi$

(b) If $A, B \in \Psi$, then $A \cap B \in \Psi$

(c) If a family $\{A_j | j \in J\} \subset \Psi$,then $\cup A_j \in \Psi$.

[40]

Definition 22 *A neutrosophic set $x(T, I, F)$ is called* ***paraconsistent*** *if $inf(T)+inf(I) + inf(F) > 1$.[39]*

Definition 23 *For neutrosophic paraconsistent sets $0\_ = x(0,1,1)$ and $1\_ = x(1,1,0)$.(Smarandache).*

Remark. *If we use the unary neutrosophic negation operator for neutrosophic sets [40], $n_N(x(T,I,F)) = x(F,I,T)$ by interchanging the thuth T and falsehood F components, we have that $n_N(0\_) = 1\_$.*

Definition 24 *Let X be a non-empty set. A family Φ of neutrosophic paraconsistent sets in X will called a **neutrosophic paraconsistent topology** if:*
(a) $0\_$ and $1\_ \in \Phi$
(b) If $A, B \in \Phi$, then $A \cap B \in \Phi$
(c) Any union of a subfamily of paraconsistent sets of Φ is also in Φ.
(Smarandache).
Results.

Proposition 1. The neutrosophic paraconsistent topology is not an extension of intuitionistic fuzzy topology.

Proof. We have that $0_{\sim} =< x, 0, 1 >$ and $1_{\sim} =< x, 1, 0 >$ are members of all intuitionistic fuzzy topology, but
$x(0,0,1) \in j(0_{\sim}) \neq 0\_$, and, $x(1,0,0) \in j(1_{\sim}) \neq 1\_$.

Proposition 2. A neutrosophic paraconsistent topology is not a general neutrosophic topology.

Proof. Let the family $\{1\_, 0\_\}$. Clearly it is a neutrosophic paraconsistent topology, but $0_{\sim}$,$1_{\sim}$ are not in this family.

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