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A NOVEL APPROACH OF REFINED PLITHOGENIC NEUTROSOPHIC SETS IN MULTI CRITERIA DECISION MAKING

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ABSTRACT

In decision-making problems with sub-attributes Refined Plithogenic Neutrosophic Sets(RPNS) are efficiently used by considering their reality, indeterminacy and falsehood components. In this article a few concepts of RPNS along with union, intersection, complement and partial orders are introduced in a unique approach to cope up with uncertain and conflicting data. Some ideal properties of RPNS based on these perceptions have also been studied. We will implement the concept in order to evaluate the efficacy of the proposed methodology of RPNS in multi criteria decision making with the numerical examples.

Keywords: Plithogenic set, Neutrosophic Set, Refined Plithogenic Neutrosophic Set, Multi criteria decision making.

I. INTRODUCTION

Numerous concepts for dealing with ambiguity, inconsistency and incoherence have recently been implemented. Probabilistic theory, fuzzy set theory, intuitionist fuzzy set, rough set theory, etc. are constantly used as impactful techniques to handle with various types of complexities and inaccuracy in embedded method. All these hypotheses mentioned consequently have failed to deal with undefined and contradictory information in the worldview.

Florentin Smarandache introduced the Neutrosophic set (NS) theory in 1998 and it was developed from a new field of study, namely Neutrosophy. NS is capable of managing volatility, indeterminacy and contradictory information. NS approaches are appropriate for analysing complexity, indeterminacy and contradictory information problems in which technical expertise is required and human assessment is essential.

Yager [21] first proposed the multi-set theory as a representation of the framework of number theory, and after that the multi-set was established by Calude et al [3]. A variety of generalizations of the multi-set theory have been made by many scholars from different perspectives. Sebastian and Ramakrishnan [9, 10] introduced a new technique termed multi fuzzy sets, which is a multi-set generalization. Ever since, many researchers [4, 5, 6, 7] have addressed more multi-fuzzy set properties and they expanded the idea of fuzzy multi-sets to an intuitionistic fuzzy set called intuitionistic fuzzy multi-sets (IFMS). Since then, many exemplary methods have been suggested by the researchers in the IFMS analysis [2, 6, 12, 20]. A multi fuzzy set component can occur quite often with possibly the same or different membership functions, although an intuitionistic fuzzy multi-set technique requires membership and non-membership values to occur frequently. FMS and IFMS definitions fail to deal with indeterminacy.

In 2013, by refining each neutrosophical component t, i, f into $t_1, t_2, \ldots, t_s$ and $i_1, i_2, \ldots, i_r$ and $f_1, f_2, \ldots, f_m$. Smarandache [17] generalized traditional neutrosophical logic to n-value refined neutrosophical logic. The definition of refined (multi-set) neutrosophical sets(RNS) was recently used by Deli et al. [3] Ye [20] and some of their basic properties were studied by Said Broumi and Irfan Deli. RNS is the generalization of fuzzy multi-sets and intuitionistic fuzzy multi-sets.

Plithogeny which was introduced in 2017 by Florentine Smarandache is the origination, existence, development, growth and emergence of various entities from technologies and organic combinations of old objects that are conflicting and/or neutral and/or non-contradictory. A plithogenic set P is a set whose members are characterized by one or more attributes and there may be several values for each attribute.

The motive of this article is an attempt to study Plithogenic Neutrosophic Sets to Refined Plithogenic Neutrosophic Sets (RPNS). The manuscript is carried out in the following manner. In Section 2, we introduce some concepts and perceptions of PNS and RPNS theory which will support us in the subsequent study. We examine the operators of PNRS in Section 3. In section 4, we make an application for PNRS in decision-making. Finally, we conclude this article with the future work.

II. PRELIMINARY CONCEPTS

In this section, we refer to the noteworthy fundamental preliminaries and in particular, the work of Zadeh [23] Attanassov [1], Atiqe Ur Rahman et al [2], I.Deli et al [5] and Smarandache [15] who introduced the Plithogenic Set which investigates the complexities of several types of opposites and their neutrals and non-opposites.

Definition 2.1 [15] Plithogenic set (PS) is a generalisation of a crisp set, a fuzzy set (FS), an intuitionistic fuzzy set (IFS) and a neutrosophic set (NS), while these four categories are represented by a single attribute value (appurtenance): single value (belonging)-for a crisp set and a FS, two values (belonging, non-belonging)-for an IFS, or triple values (belonging, non-belonging and indeterminacy) for NS. In general, PS is a set whose members are determined by a set of elements with four or more values of attributes.

Definition 2.2 [8] Let U be a universe of discourse, and P a non-empty set of elements, P $\subseteq$ U, then

(P, Φ , R, d, C)is called a PNS where

The attribute is Φ = "appurtenance"

The set of attribute values R= {belonging, indeterminacy, non-belonging}, whose cardinal |R| = 3;

The Dominant Attribute Value = belonging;

The attribute value appurtenance grade function:

$d : P * R \to [0,1],\ d(y, belonging) \in [0,1], d(y, indeterminacy) \in [0,1], d(y, non\ belonging) \in [0,1]$

with $0 \le d(y, belonging) + d(y, indeterminacy) + d(y, nonbelonging) \le 3;$

and the attribute value contradiction grade function:

$C : R * R \to [0,1],$

$C(belonging, belonging) = C(indeterminacy, indeterminacy) = C(nonbelonging, nonbelonging = 0,$

$C(belonging, nonbelonging) = 0,$

$C(belonging, indeterminacy) = c(nonbelonging, indeterminacy) = \frac{1}{2},$

Which means that for the PNS aggregation operators (Intersection, Union, Complement etc.), if one applies the ℓ_{norm} on belonging function, then one has to apply the ℓ_{conorm} on non-belonging (and mutually), while on

indeterminacy one applies the average of ℓ_{norm} and ℓ_{conorm} (i.e)

$fuzzy\ \ell_{norm} = a \wedge_f b = ab \ \&\ fuzzy\ \ell_{conorm} = a \vee_f b = a + b - ab.$

III. REFINED PLITHOGENIC NEUTROSOPHIC SETS (RPNS) AND ITS OPERATORS

In this section we will consider some possible definitions for the fundamental concepts of the RPNS and its operators.

Definition 3.1 Let U be a universe of discourse, and P a non-empty set of elements, P $\subseteq$ U, then (P_{rpn}, ϕ, A, D, C) where 'rpn' stands for refined plithogenic neutrosophic is called RPNS if at least one of the values of attribute $a_i \in A$ is split into two or more sub values of attribute: $a_{i1}, a_{i2}, \ldots \in A$, with the sub value appurtenance degree function of attribute $D(z, a_{si}) \in P[0,1]^3,\ for\ s = 1,2,\ldots$

Definition 3.2 A single valued finitely refined PNS has the form

$(M_1, M_2, \ldots, M_b; H_1, H_2, \ldots H_x; N_1, N_2, \ldots N_l),$

$where\ b, x, l \ge 1\ are\ integers,\ with\ b + x + l \ge 4,$

$and\ all\ M_K, H_j, N_q \in [0,1],\ for\ k \in \{1,2,\ldots b\},\ j \in \{1,2,\ldots x\}\ and\ q \in \{1,2,\ldots l\}.$

The attribute ϕ = "appurtenance".

The set of attribute values $A = \{t_1, t_2, \ldots t_b; e_1, e_2, \ldots e_x; y_1, y_2, \ldots y_l\}$

Where "t" means sub membership, "e" means sub indeterminacy, "y" means sub non-membership.

The dominant attribute value (DAV)= $t_1, t_2, \ldots t_b$;

The value of attribute appurtenance degree function: $D: P \times A \rightarrow [0,1]$,

$D(z,t_x) \in [0,1], D(z,e_j) \in [0,1], D(z,y_q) \in [0,1],$ for all x, j, q with

$$0 \le \sum_{k=1}^{b} D_z(t_x) + \sum_{j=1}^{x} D_z(e_j) + \sum_{q=1}^{l} D_z(y_q) \le b + x + l;$$

And the value of attribute dissimilarity function:

$C(t_{k_1}, t_{k_1}) = C(e_{j_1}, e_{j_2}) = C(y_{q_1}, y_{q_2}) = 0,$ forall

$k_1, k_2 \in \{1, 2, \ldots b\},\ j_1, j_2 \in \{1, 2, \ldots x\}$ and $q_1, q_2 \in \{1, 2, \ldots l\}$;

$C(t_k, y_q) = 1,\ C(t_k, e_j) = 1,\ C(y_q, e_j) = 0.5,$ for all x, j, q.

3.2.1 RPNS intersection

$$(r_x, 1 \le x \le u; o_j 1 \le j \le x; v_q, 1 \le q \le l) \wedge_{RPNS} (g_x, 1 \le x \le u; w_j 1 \le j \le x; h_q, 1 \le q \le l)$$
$$= \left(r_x \wedge_f g_x, 1 \le x \le u; 0.5 * \left([o_j \wedge_f w_j] + [o_j \vee_f w_j]\right), v_q \vee_f h_q, 1 \le q \le l\right)$$

3.2.2 RPNS union

$$(r_x, 1 \le x \le u; o_j 1 \le j \le x; v_q, 1 \le q \le l) \vee_{RPNS} (g_x, 1 \le x \le u; w_j 1 \le j \le x; h_q, 1 \le q \le l)$$
$$= \left(r_x \vee_f g_x, 1 \le x \le u; 0.5 * \left([o_j \vee_f w_j] + [o_j \wedge_f w_j]\right), v_q \wedge_f h_q, 1 \le q \le l\right)$$

3.2.3 RPNS Complement

$$\neg RPNS\left(M_x = r_x, 1 \le x \le b; H_j = o_j, 1 \le j \le x;\ N_q = v_q, 1 \le q \le l\right)$$
$$= (M_x = v_q, 1 \le q \le l; H_j = o_j, 1 \le j \le x; v_q, 1 \le q \le l;\ N_q = r_x, 1 \le x \le u),$$

Where all M_x =sub truths, all H_j =sub indeterminacies, and all N_q =sub falsehoods.

3.2.4 RPNS Inclusions (Partial orders)

$$(r_x, 1 \le x \le u; o_j 1 \le j \le x; v_q, 1 \le q \le l) \le_{RPNS} (g_x, 1 \le x \le u; w_j 1 \le j \le x; h_q, 1 \le q \le l)$$

if and only if

$$all\ r_x \le (1 - C_a) g_x, all\ o_j \ge (1 - C_a) w_j\ and\ all\ v_q \ge (1 - C_a) h_q, where\ C_a \in [0, 0.5)$$

is the dissimilarity degree between the values of attribute a. If C_a does not occur, we will take it as zero by default.

IV. NUMERICAL EXAMPLE

In this section, we refine the existing attributes into sub-attributes in order to achieve a better accurateness which really helps to make decisions when attempting to deal with multi-attribute scenarios.

(i) Consider a Uni attribute "Evaluation" (of 5 attribute values) PNS for selecting the Candidate in an interview is represented by

$$z\begin{pmatrix} disimilarity\ \deg ree & 0 & 0.25 & 0.5 & 0.75 & 1 \\ Attribute\ value & very\ good, & good & Fair & bad & very\ bad \\ Degree\ of\ Appurtenance & [0.2, 0.3, 0.1] & [0.4, 0.6, 0.3] & [0.2, 0.8, 0.5] & [0.9, 0.1, 0.6] & [0.2, 0.5, 0.7] \end{pmatrix}$$

The PNS negation and RPN Attribute Value Set for the above data is given below.

$$\neg_{PN} z\begin{pmatrix} disimilarity\ \deg ree & 0 & 0.25 & 0.5 & 0.75 & 1 \\ Attribute\ value & very\ good, & good & Fair & bad & very\ bad \\ Degree\ of\ Appurtenance & [0.2, 0.3, 0.1] & [0.4, 0.6, 0.3] & [0.2, 0.8, 0.5] & [0.9, 0.1, 0.6] & [0.2, 0.5, 0.7] \end{pmatrix}$$

$$= \neg_{PN} z\begin{pmatrix} disimilarity\ \deg ree & 0 & 0.25 & 0.5 & 0.75 & 1 \\ Attribute\ value & very\ good, & above\ bad & Fair & below\ good & very\ bad \\ Degree\ of\ Appurtenance & [0.2, 0.3, 0.1] & [0.4, 0.6, 0.3] & [0.2, 0.8, 0.5] & [0.9, 0.1, 0.6] & [0.2, 0.5, 0.7] \end{pmatrix}$$

Therefore, the original attribute set

$A = \{very\ good,\ good,\ fair, bad, very\ bad\}$

And has been partially refined into

$\text{Re}\ fined\ A = \{very\ good,\ above bad,\ fair, below good, very\ bad\}$

Where $above\ bad, below good \in [good,\ bad]$

(ii) Consider a uni-attribute (of 4 attribute values) PNS for the attribute "Proficiency" of selecting the students in a class that has the following attribute values: Excellent (the dominant one), good, average, poor.

$$\begin{pmatrix} Degrees\ of\ contradiction & 0 & 0.5 & 0.75 & 1 \\ Attribute\ Values & poor & average & good & Excellent \\ Degrees\ of\ Appurtenance & (0.2,0.3,0.4) & (0.4,0.8,0.6) & (0.9,0.3,0.4) & (0.8,0.9,0.6) \end{pmatrix}$$

Now let us introduce the component 'best' as a refinement of the preceding table which is tabulated below

$$\begin{pmatrix} Degree\ of\ contradiction & 0 & 0.1 & 0.3 & 0.5 & 0.8 & 0.9 & 1 \\ Attribute values & poor & \begin{matrix} above\ poor \\ (anti-best) \end{matrix} & \begin{matrix} less average \\ (anti-good) \end{matrix} & Average & good & best & Excellent \\ Degree\ of\ Appurtenance & (0.2,0.3,0.4) & (0.4,0.5,0.6) & (0.8,0.2,0.4) & (0.4,0.8,0.6) & (0.9,0.3,0.4) & (0.5,0.6,0.7) & (0.8,0.9,0.6) \end{pmatrix}$$

The complement of the attribute value "Excellent" which is 80% dissimilarity with "good", will be an attribute value which is 1-0.8 =0.2=20% dissimilarity with "Excellent", so it will be equal to $\frac{1}{2}[Excellent + Average]$. Let us call it "less average", whose degree of appurtenance is $1-(0.9,0.3,0.4) = (0.8,0.2,0.3)$.

If the attribute "Proficiency" has other values "Excellent" being the dominant.

The negation of best is 1-0.9 = 0.1 = 10% dissimilarity degree with the dominant attribute value "Excellent", so it is in between Excellent and Average, we may say it is included into the attribute value interval $[Excellent, average]$ much closer to Excellent than to Average. Let us call "above poor", whose degree of appurtenance is $1-(0.8,0.9,0.4) = (0.7,0.8,0.3)$.

V. CONCLUSION AND FUTURE WORK

In this article, we have defined the RPNS by introducing its core operators. We have present an application of RPNS in multi criteria decision making. In future work, we will extend this concept to study the correlation measures of RPNS and also its elementary properties.

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