Plithogenic Cubic Sets

S.P. Priyadharshini [1,*], F. Nirmala Irudayam [2] and F. Smarandache [3]

[1,2] Dept of Mathematics, Nirmala College for Women, India; priyadharshini125@gmail.com , nirmalairudayam@ymail.com
[3] Dept. Math and Sciences, University of New Mexico, Gallup, NM, USA; smarand@unm.edu

* Correspondence: priyadharshini125@gmail.com

Abstract

In this article, using the concepts of cubic set and plithogenic set, the ideas of plithogenic fuzzy cubic set, plithogenic intuitionistic fuzzy cubic set, Plithogenic neutrosophic cubic set are introduced and its corresponding internal and external cubic sets are discussed with examples. Primary properties of the Plithogenic neutrosophic cubic sets were also discussed.This concept is extremely suitable for addressing problems involving multiple attribute decision making as this plithogenic neutrosophic set are described by four or more value of attributes and the accuracy of the result is also so precise.

Keywords: Fuzzy set, Intuitionistic fuzzy set, Neutrosophic set, Cubic set, Plithogenic set, Plithogenic fuzzy cubic set, Plithogenic Intuitionistic fuzzy cubic set, Plithogenic neutrosophic cubic set.

1. Introduction

Zadeh implemented Fuzzy Sets[18].In [18] Zadeh gave the perception of a fuzzy set by an interval-valued fuzzy set , i.e. a fuzzy set with an intervalvalued belonging function. In conventional fuzzy concepts,numbers from the interval [0,1] are used to reflect, e.g., the degree of conviction of the expert in various statements.It is always tough for an expert to precisely enumerate their certainty; thus, rather than a particular value, it is more fitting to reflect this certainty degree by an interval or even by a fuzzy set. We obtain an interval valued fuzzy sets which were frequently used in real-life scenario.

Atanassov[2] introduced intuitionistic fuzzy sets(IFS), which is the generalisation of the fuzzy set. In IFS,each element is attached to both belonging and non-belonging grade with the constraint that the sum of these two grades is less than or equal to unity. If available knowledge is not sufficient to identify the inaccuracy of traditional fuzzy sets, IFS architecture can be viewed as an alternative / appropriate solution. Later on IFS were expanded to interval-valued IFS.

Smarandache [9, 10, 11] proposed Neutrosophic sets (NSs), a generalisation of FS and IFS, which is highly helpful for dealing with inadequate, uncertain, and varying data that exists in the real life. NSs are characterised by functions of truth (T), indeterminacy (I) and falsity (F) belonging functions. This concept is very essential in several areas of application since indeterminacy is clearly enumerated and the truth, indeterminacy, and falsity membership functions are independent.

DOI: 10.5281/zenodo.4030378

Received: May 11, 2020 Accepted: August 28, 2020

.Wang, Smarandache, Zhang and Sunderraman[17] proposed the definition of the interval valued neutrosophic set (INS) as an extension of the neutrosophic set.The INS could reflect indeterminate, inaccurate, inadequate and unreliable data that occurs in the reality.

Plithogeny is the origin, creation, production and evolution of new objects from the synthesis of conflicting or non-conflicting multiple old objects. Smarandache[12] introduced the plithogenic set as a generalisation of neutrosophy in 2017.

The elements of plithogenic sets are denoted by one or many number of attributes and each of it have several values.Each values of attribute has its respective (fuzzy, intuitionistic fuzzy or neutrosophic) appurtenance degree for the element x (say) to the plithogenic set P (say) with respect to certain constraints. For the first time, Smarandache[12] introduced the contradictory (inconsistency) degree between each value of attribute and the dominant value of attribute which results in getting the better accuracy for the plithogenic aggregation operators(fuzzy, intuitionistic fuzzy or neutrosophic).

Y.B. Jun et al [4] implemented a cubic set which is a combination of a fuzzy set with an interval valued fuzzy set. Internal and external cubic sets were also described and some properties were studied.

Y.B.Jun, Smarandache and Kim [4] introduced Neutrosophic cubic sets and the concept of internal and external for truth, falsity and indeterminacy values.Furthermore they provided so many properties of P (R) -union,P (R) –intersection for internal and external neutrosophic cubic sets.

In this article, using the principles of cubic sets and plithogenic set, we presented the generalisation of plithogenic cubic sets for fuzzy, Intuitionistic fuzzy and neutrosophic sets

2. Preliminaries

Definition 2.1 [15] Let Z be a universe of discourse and the fuzzy set $F=\{\langle z,\mu_f(z)\rangle \mid z\in Z\}$ is described by a belonging function μ_f as , $\mu_f : Z \to [0,1]$.

Definition 2.2 [10] Let Y be a non-empty set. Then, an interval valued fuzzy set B over Y is defined as $B=\{[B^-(y),B^+(y)]/y : y\in Y\}$ where $B^-(x)$ and $B^+(x)$ are stated as the inferior and superior degrees of belonging $y\in Y$ where $0\le B^-(y)+ B^+(y)\le 1$ correspondingly.

Definition 2.3 [2, 3] Let N be a non empty set.The set $A=\{\langle n,\lambda_A,\phi_A\rangle n\in N\}$ is called an intutionistic fuzzy set (in short, IFS) of N where the function $\lambda_A : N\to[0,1]$, $\phi_A : N\to[0,1]$ denotes the membership degree (say $\lambda_A(n)$) and non-membership degree (say $\phi_A(n)$) of each element $n\in N$ to the set A and satisfies the constraint that $0\le\lambda_A(n)+\phi_A(n)\le 1$.

Definition 2.4 [10] Let Y be a non-empty set. The set $\mathrm{A}=\{< y,M_A(y),N_A(y)>\mid y\in Y\}$ is called an interval valued intuitionistic fuzzy sets (IVIFS) A in Y where the functions $M_A(y):Y\to[0,1]$ and $N_A(y):Y\to[0,1]$ denotes the degree of belonging, non-belonging of A respectively. Also $M_A(y)=[M_{AL}(y),M_{AU}(y)]$ and $N_A(y)=[N_{AL}(y),N_{AU}(y)]$, $0\le M_{AU}(y)+M_{AU}(y)\le 1$ for each $y\in Y$

Definition 2.5 [6] Let N be a non empty set.The set $A=\{\langle n,\lambda_A,\phi_A,\gamma_A\rangle n\in N\}$ is called a neutrosophic set (in short, NS) of N where the function $\lambda_A : N\to[0,1]$, $\phi_A : N\to[0,1]$ and $\gamma_A : N\to[0,1]$ denotes the membership degree (say $\lambda_A(n)$), indeterminacy degree (say $\phi_A(n)$), and non- membership degree (say $\gamma_A(n)$)of each element $n\in N$ to the set A and satisfies the constraint that $0\le\lambda_A(n)+\phi_A(n)+\gamma_A(n)\le 3$.

DOI: 10.5281/zenodo.4030378

Definition 2.6 [1] Let R be a non-empty set. An interval valued neutrosophic set (INS) A in R is described by the functions of the truth-value (A_T), the indeterminacy (A_I) and the falsity-value (A_F) for each point $r \in R$, $A_T(r), A_I(r), A_F(r) \subseteq [0,1]$.

Definition 2.7 [4,5] Let E be a non-empty set. By a cubic set in E ,we construct a set which has the form $\Psi = \{< e, B(e), \mu(e) > | e \in E\}$ in which B is an interval valued fuzzy set (IVFS) in E and μ is a fuzzy set in E.

Definition 2.8 [4] Let E be a non-empty set. If $B^-(e) \le \mu(e) \le B^+(e)$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an internal cubic set (briefly IPS).

Definition 2.9 [4] Let E be a non-empty set. If $\mu(e) \notin (B^-(e), B^+(e))$ for all $e \in E$ then the cubic set $\Psi =< B, \mu >$ in E is called an external cubic set (briefly ECS)

Definition 2.10 [10] Plithogenic Fuzzy set for an interval valued (IPFS) is defined as $\forall c \in C, d : C \times W \to C([0,1])$, $\forall w \in W$ and $d(c, w)$ is an open, semi-open, closed interval included in [0, 1] and C([0, 1]) is the power set of the unit interval [0,1] (i.e) all subsets of [0,1].

Definition 2.11 [10] A set which has the form $\forall c \in C, d : C \times W \to C([0,1]^2)$ and $\forall w \in W$ is called interval valued plithogenic intuitionistic fuzzy set (IPIFS) where $d(c, w)$ is an open, semi-open, closed for the interval included in $[0,1]$.

Definition 2.12 [10] A set which has the form $\forall c \in C, d : C \times W \to C([0,1]^3)$ and $\forall w \in W$ is called interval valued plithogenic neutrosophic set (IPNS) where $d(c, w)$ is an open, semi-open, closed for the interval included in $[0,1]$.

3. Plithogenic Cubic sets

3.1 Plithogenic fuzzy Cubic sets

Definition 3.1.1 For a non empty set Y, the Plithogenic fuzzy cubic set (PFCS) is defined as $\Lambda = \{< y, B(y), \lambda(y) > | y \in Y\}$ in which B is an interval valued plithogenic fuzzy set in Y and λ is a fuzzy set in Y.

Example 3.1.2 The following values of attribute represents the criteria "Musical instruments" : Piano (the dominant one), guitar, saxophone, violin.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Piano	Guitar	Saxophone	Violin
Appurtenance Degree $B(y)$	[0.5, 0.6]	[0.2, 0.8]	[0.4, 0.7]	[0.1, 0.3]
$\lambda(y)$	0.3	0.4	0.6	0.2

Definition 3.1.3 For a non-empty set Y, the PFCS $\Lambda =< B, \lambda >$ in Y is called an internal plithogenic fuzzy cubic set (briefly IPFCS) if $B_{d_i}{}^-(y) \le \lambda_i(y) \le B_{d_i}{}^+(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.1.4 The following values of attribute represents the criteria "Color" : Yellow(the dominant one), green, orange and red.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Yellow	Green	Orange	Red

Appurtenance Degree $B(y)$	[0.2, 0.3]	[0.6, 0.8]	[0.3, 0.6]	[0.4, 0.9]
$\lambda(y)$	0.2	0.7	0.5	0.8

Definition 3.1.5 For a non-empty set Y, the PFCS $\Lambda = < B, \lambda >$ in Y is called an external plithogenic fuzzy cubic set (briefly EPFCS) if $\lambda_i(y) \notin (B_{d_i}^{-}(y), B_{d_i}^{+}(y))$ for all $y \in Y$ and and d_i represents the contradictory degree and their respective value of attributes.

Example 3.1.6 The following values of attribute represents the criteria "Subjects" : Mathematics(the dominant one), Physics, Chemistry and Biology

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Mathematics	Physics	Chemistry	Biology
Appurtenance Degree $B(y)$	[0.4, 0.6]	[0.5, 0.8]	[0.2, 0.3]	[0.5, 0.9]
$\lambda(y)$	0.7	0.1	0.6	0.3

Remark 3.1.7 If any one of the attribute value $\lambda_i(y) = y$ for all $y \in Y$ and *i* represents the values of attribute then Λ is neither an IPFCS nor an EPFCS.

3.2 Plithogenic Intuitionistic fuzzy cubic set

Definition 3.2.1 For a non empty set Y, the Plithogenic Intuitionistic fuzzy cubic set (PIFCS) is defined as $\Lambda = \{< y, B(y), \lambda(y) > \mid y \in Y\}$ in which *B* is an interval valued Plithogenic Intuitionistic fuzzy set in Y and λ is a intuitionistic fuzzy set in Y.

Example 3.2.2 The following values of attribute represents the criteria "Mobile phone brands" : Apple (the dominant one), Samsung, Nokia, Lenova.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Apple	Samsung	Nokia	Lenova
Appurtenance Degree $B(y)$	([0.2, 0.5], [0.3, 0.6])	([0.5, 0.8], [0.1, 0.7])	([0.4, 0.7], [0.2, 0.6])	([0.1, 0.3], [0.4,0.7])
$\lambda(y)$	([0.1,0.7])	([0.2, 0.4])	([0.1, 0.8])	([0.2, 0.5])

Definition 3.2.3 For a non-empty set Y, the PIFCS $\Lambda = < B, \lambda >$ in Y is called an internal plithogenic intuitionistic fuzzy cubic set (briefly IPIFCS) if $B_{d_i}^{-}(y) \le \lambda_i(y) \le B_{d_i}^{+}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.2. The following values of attribute represents the criteria " Proficiency " : Excellent (the dominant one), good, average, poor

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Excellent	Good	Average	Poor
Appurtenance Degree $B(y)$	([0.3, 0.5], [0.1, 0.6])	([0.4, 0.6], [0.5, 0.8])	([0.1, 0.8], [0.7, 0. 8])	([0.2, 0.3], [0.1,0.6])
$\lambda(y)$	([0.4,0.5])	([0.6, 0.7])	([0.2, 0.5])	([0.3, 0.5])

DOI: 10.5281/zenodo.4030378

Definition 3.2.5 For a non-empty set Y, the PIFCS $\Lambda = < B, \lambda >$ in Y is called an external plithogenic intuitionistic fuzzy cubic set (briefly EPIFCS) if $\lambda_i(y) \notin (B_{d_i}^{-}(y), B_{d_i}^{+}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes.

Example 3.2.6 The following values of attribute represents the criteria " Mode of Transport" : Bus (the dominant one), Car, Lorry, Train

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Bus	Car	Lorry	Train
Appurtenance Degree $B(y)$	([0.4, 0.6], [0.3, 0.5])	([0.5, 0.7], [0.2, 0.4])	([0.2, 0.6], [0.3, 0.7])	([0.3, 0.6], [0.1,0.4])
$\lambda(y)$	([0.1,0.6])	([0.4, 0.5])	([0.7, 0.2])	([0.7, 0.6])

3.3 Plithogenic Neutrosophic cubic set

Definition 3.3.1 Let Ω be an universal set and Y be a non empty set. The structure $\Lambda = \{< y, B(y), \lambda(y) > | \, y \in Y\}$ is said to be Plithogenic Neutrosophic cubic set (PNCS) in Y, where $B = \{(B_{d_i}^{T}(y), B_{d_i}^{I}(y), B_{d_i}^{F}(y))\}$ is an interval valued Plithogenic Neutrosophic set in Y and $\lambda = \{(\lambda_i^{T}(y), \lambda_i^{I}(y), \lambda_i^{F}(y)\}$ is a neutrosophic set in Y.

The pair $\Lambda = < B, \lambda >$ is called plithogenic neutrosophic cubic set over Ω where Λ is a mapping given by $\Lambda : B \to NC(\Omega)$ .The set of all plithogenic neutrosophic cubic sets over Ω will be denoted by P_N^{Ω}

Example 3.3.2 consider the attribute "Size" which has the following values: Small (the dominant one), medium, big, very big.

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Small	Medium	Big	Very big
Appurtenance Degree $B(y)$	([0.3, 0.5], [0.1, 0.8], [0.1, 0.9])	([0.1, 0.3], [0.4, 0.8], [0.4, 0.9])	([0.1, 0.5], [0.2, 0. 6], [0.6, 0.9])	([0.2, 0.6], [0.2,0.5], [0.7, 0.9])
$\lambda(y)$	([0.4,0.5, 0.8])	([0.3, 0.7, 0.4])	([0.2,0.5,0.8])	([0.5,0.2,0.8])

Definition 3.3.3 For a non-empty set Y, the plithogenic neutrosophic cubic set $\Lambda = < B, \lambda >$ in Y is called

(i) truth internal if $B_{d_i}^{-T}(y) \le \lambda_i^{T}(y) \le B_{d_i}^{+T}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.1)
(ii) indeterminacy internal if $B_{d_i}^{-I}(y) \le \lambda_i^{I}(y) \le B_{d_i}^{+I}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.2)
(iii) falsity internal if $B_{d_i}^{-F}(y) \le \lambda_i^{F}(y) \le B_{d_i}^{+F}(y)$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.3).

If a plithogenic neutrosophic cubic set in Y satisfies the above equations we conclude that Λ is an internal plithogenic neutrosophic cubic set (IPNCS) in Y.

Example 3.3.4 The following values of attribute represents the criteria "Sports": Volley ball(the dominant one), Basket ball, Cricket, Bat-minton

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Volley ball	Basket ball	Cricket	Bat-minton

Appurtenance Degree $B(y)$	([0.2, 0.5], [0.6, 0.8], [0.1, 0.5])	([0.1, 0.3], [0.4, 0.5], [0.6, 0.9])	([0.5, 0.8], [0.1, 0. 4], [0.6, 0.9])	([0.3, 0.6], [0.1,0.5], [0.7, 0.9])
$\lambda(y)$	([0.3,0.6, 0.4])	([0.2, 0.5, 0.8])	([0.7,0.2, 0.8])	([0.4, 0.3, 0.9])

Definition 3.3.5 For a non-empty set Y, the plithogenic neutrosophic cubic set $\Lambda = <B, \lambda>$ in Y is called

(i) truth external if $\lambda_i^T(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.4)

(ii) indeterminacy external if $\lambda_i^I(y) \notin (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.5)

(ii) falsity external if $\lambda_i^F(y) \notin (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$ for all $y \in Y$ and d_i represents the contradictory degree and their respective value of attributes. (3.6)

If a plithogenic neutrosophic cubic set in Y satisfies the above equations, we conclude that Λ is an external plithogenic neutrosophic cubic set (EPNCS) in Y.

Example 3.3.6 The following values of attribute represents the criteria "Seasons" : Spring (the dominant one), Winter, Summer, Autumn

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Spring	Winter	Summer	Autumn
Appurtenance Degree $B(y)$	([0.3, 0.6], [0.7, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.3], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.1,0.2, 0.9])	([0.4, 0.5, 0.9])	([0.7, 0.9, 0.6])	([0.1, 0.8, 0.3])

Theorem 3.3.7 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y which is not external. Then there exists $y \in Y$ such that $\lambda_i^T(y) \in (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$, $\lambda_i^I(y) \in (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$ or $\lambda_i^F(y) \in (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$ where d_i represents the contradictory degree and their respective value of attributes..

Proof. Direct proof

Theorem 3.3.8 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both T-internal and T-external, then $(\forall y \in Y)\left(\lambda_i^T(y) \in \{B_{d_i}^{-T}(y) \mid y \in Y\} \cup \{B_{d_i}^{+T}(y) \mid y \in Y\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Proof. The conditions (3.1) and (3.4) implies that $B_{d_i}^{-T}(y) \le \lambda_i(y) \le B_{d_i}^{+T}(y)$ and $\lambda_i^T(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and i = 1, 2, 3,…n.

Then it indicates that $\lambda_i^T(y) = B_{d_i}^{-T}(y)$ or $\lambda_i^T(y) = B_{d_i}^{+T}(y)$, and hence

$\lambda_i^T(y) \in \{B_{d_i}^{-T}(y) \mid y \in Y\} \cup \{B_{d_i}^{+T}(y) \mid y \in Y\}$ where d_i represents the contradictory degree and their respective value of attributes.

Hence the proof.

DOI: 10.5281/zenodo.4030378

Correspondingly the subsequent theorems hold for the indeterminate and falsity values.

Theorem 3.3.9 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both I-internal and I-external, then $(\forall y \in Y)\left(\lambda_i^{I}(y) \in \left\{B_{d_i}^{-I}(y) \mid y \in Y\right\} \cup \left\{B_{d_i}^{+I}(y) \mid y \in Y\right\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Theorem 3.3.10 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *F*-internal and *F*-external, then $(\forall y \in Y)\left(\lambda_i^{F}(y) \in \left\{B_{d_i}^{-F}(y) \mid y \in Y\right\} \cup \left\{B_{d_i}^{+F}(y) \mid y \in Y\right\}\right)$ where d_i represents the contradictory degree and their respective value of attributes.

Definition 3.3.11 Let Y be a non empty set, The complement of $\Lambda = <B, \lambda>$ is said to be the PNCS $\Lambda^c =< B^c, \lambda^c >$ where $B^c = \left\{\left(B^c{}_{d_i}{}^{T}(y), B^c{}_{d_i}{}^{I}(y), B^c{}_{d_i}{}^{F}(y)\right)\right\}$ is an interval valued PNCS in Y and $\lambda = \{(\lambda^c{}_i{}^{T}(y), \lambda^c{}_i{}^{I}(y), \lambda^c{}_i{}^{F}(y)\}$ is a neutrosophic set in Y.

Theorem 3.3.12 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *T*-internal and *T*-external, then the complement $\Lambda^c =< B^c, \lambda^c >$ of $\Lambda =< B, \lambda >$ is an *T*-Internal and *T*-external PNCS in Y.

Proof. Let Y be a non empty set .If $\Lambda =< A, \lambda >$ is an *T*-internal and *T*-external PNCS in Y, then $B_{d_i}^{-T}(y) \le \lambda_i(y) \le B_{d_i}^{+T}(y)$ and $\lambda_i^{T}(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ and i = 1, 2, 3,…,n. It follows that $1 - B_{d_i}^{-T}(y) \le 1 - \lambda_i(y) \le 1 - B_{d_i}^{+T}(y)$ and. $1 - \lambda_i^{T}(y) \notin (1 - B_{d_i}^{-T}(y), 1 - B_{d_i}^{+T}(y))$. Therefore $\Lambda^c =< B^c, \lambda^c >$ is an *T*-Internal and *T*-external PNCS in Y.

Correspondingly the subsequent theorems hold for the indeterminate and falsity values.

Theorem 3.3.13 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *I*-internal and *I*-external, then the complement $\Lambda^c =< B^c, \lambda^c >$ of $\Lambda = <B, \lambda>$ is an *I*-internal and *I*-external PNCS in Y.

Theorem 3.3.14 Let Y be a non empty set and $\Lambda = <B, \lambda>$ be a PNCS in Y. If Λ is both *F*-internal and *F*-external, then the complement $\Lambda^c =< B^c, \lambda^c >$ of $\Lambda = <B, \lambda>$ is an *F*-internal and *F*-external PNCS in Y.

Definition 3.3.15 Let $\Lambda =< B, \lambda > \in P^{\Omega}{}_N$.

If $B_{d_i}^{-T}(y) \le \lambda_i^{T}(y) \le B_{d_i}^{+T}(y)$, $B_{d_i}^{-I}(y) \le \lambda_i^{I}(y) \le B_{d_i}^{+I}(y)$, $\lambda_i^{F}(y) \notin (B_{d_i}^{-F}(y), B_{d_i}^{+F}(y))$

or

$B_{d_i}^{-T}(y) \le \lambda_i^{T}(y) \le B_{d_i}^{+T}(y)$, $B_{d_i}^{-F}(y) \le \lambda_i^{F}(y) \le B_{d_i}^{+F}(y)$, $\lambda_i^{I}(y) \notin (B_{d_i}^{-I}(y), B_{d_i}^{+I}(y))$

or

$B_{d_i}^{-F}(y) \le \lambda_i^{F}(y) \le B_{d_i}^{+F}(y)$, $B_{d_i}^{-I}(y) \le \lambda_i^{I}(y) \le B_{d_i}^{+I}(y)$, $\lambda_i^{T}(y) \notin (B_{d_i}^{-T}(y), B_{d_i}^{+T}(y))$ for all $y \in Y$ .Then Λ is called $\frac{2}{3}$ IPNCS or $\frac{1}{3}$ EPNCS.

Example 3.3.16 The following values of attribute represents the criteria “Types of humans” : Fun loving (the dominant one), Sensitive, Determined, Serious

Contradictory Degree	0	0.50	0.75	1

DOI: 10.5281/zenodo.4030378

Value of attributes	Fun loving	Sensitive	Determined	Serious
Appurtenance Degree $B(y)$	([0.1, 0.6], [0.3, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.3], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.1,0.4, 0.9])	([0.2, 0.5, 0.7])	([0.7, 0.5, 0.8])	([0.3,0.4, 0.3])

Definition 3.3.16 Let $\Lambda = < B, \lambda > \in P^{\Omega}{}_N$.

If $B_{d_i}{}^{-T}(y) \le \lambda_i{}^T(y) \le B_{d_i}{}^{+T}(y)$, $\lambda_i{}^I(y) \notin (B_{d_i}{}^{-I}(y), B_{d_i}{}^{+I}(y))$, $\lambda_i{}^F(y) \notin (B_{d_i}{}^{-F}(y), B_{d_i}{}^{+F}(y))$

or

$B_{d_i}{}^{-F}(y) \le \lambda_i{}^F(y) \le B_{d_i}{}^{+F}(y)$, $\lambda_i{}^T(y) \notin (B_{d_i}{}^{-T}(y), B_{d_i}{}^{+T}(y))$, $\lambda_i{}^I(y) \notin (B_{d_i}{}^{-I}(y), B_{d_i}{}^{+I}(y))$

or

$B_{d_i}{}^{-I}(y) \le \lambda_i{}^I(y) \le B_{d_i}{}^{+I}(y)$, $\lambda_i{}^F(y) \notin (B_{d_i}{}^{-F}(y), B_{d_i}{}^{+F}(y))$, $\lambda_i{}^T(y) \notin (B_{d_i}{}^{-T}(y), B_{d_i}{}^{+T}(y))$ for all $y \in Y$.

Then Λ is called $\frac{1}{3}$ IPNCS or $\frac{2}{3}$ EPNCS.

Example 3.3.17 The following values of attribute represents the criteria " Social Networks " : Whatsapp, Facebook (the dominant one), Instagram, Linkedinn

Contradictory Degree	0	0.50	0.75	1
Value of attributes	Whatsapp	Facebook	Instagram	Linkedin
Appurtenance Degree $B(y)$	([0.2, 0.6], [0.7, 0.5], [0.4, 0.8])	([0.1, 0.3], [0.2, 0.4], [0.6, 0.8])	([0.1, 0.4] , [0.2, 0. 6], [0.8, 0.9])	([0.3, 0.5], [0.2,0.7], [0.6, 0.8])
$\lambda(y)$	([0.3,0.2, 0.9])	([0.4, 0.3, 0.9])	([0.7, 0.2, 0.6])	([0.2, 0.5, 0.4])

Theorem 3.3.14 Let $\Lambda = < B, \lambda > \in P^{\Omega}{}_N$. Then

(i) All IPNCS is a generalisation of ICS.
(ii) All EPNCS is a generalisation of ECS.
(iii) All PNCS is a generalisation of CS.

Proof. Direct proof.

4. Conclusions and future work

In this article,We have introduced the plithogenic fuzzy cubic set, plithogenic intuitionistic fuzzy cubic set, plithogenic neutrosophic cubic sets and their corresponding internal and external cubic sets are defined with examples. Furthermore some of the properties of plithogenic neutrosophic cubic sets are investigated. In the consecutive research, we will study the P-Union, P-Intersection, R-Union, R-Intersection of plithogenic neutrosophic cubic sets.

Funding: "This research received no external funding"

Conflicts of Interest: "The authors declare no conflict of interest."

DOI: 10.5281/zenodo.4030378

References

[1] R.Anita Cruz and F.Nirmala Irudayam, "Neutrosophic Soft Cubic set", International Journal of Mathematics trends and Technology 46 (2), pp.88-94, 2017.

[2] K.T. Atanassov, "Intuitionistic fuzzy sets", Fuzzy sets and Systems 20, pp.87- 96,1986.

[3] Y.B.Jun, C.S.Kim and K.O.Yang, "Cubic sets", Annals of Fuzzy Mathematics and Informatics 4(3),pp.83-98, 2012.

[4] Y.B. Jun, F.Smarandache and C.S. Kim, "Neutrosophic Cubic set", New Mathematics and Natural Computation, 13 (1), pp.41-54, 2017.

[5] Majid Khan , Ismat Beg and Muhammad Gulistan, "Exponential Laws and Aggregation Operators on Neutrosophic Cubic Sets", International Journal of Neutrosophic Science (IJNS), Vol. 4, No. 1,pp. 47-71, 2020

[6] Nivetha Martin and Florentin Smarandache, "Plithogenic Cognitive Maps in Decision Making", International Journal of Neutrosophic Science (IJNS) ,Vol. 9, No. 1,pp. 09-21, 2020

[7] Prem Kumar Singh, "Plithogenic set for multi-variable data analysis", International Journal of Neutrosophic Science (IJNS), Vol. 1, No. 2, pp. 81-89, 2020

[8] F.Smarandache, "A Unifying Field in Logics. Neutrosophy: Neutrosophic Probability, Set and Logic", Rehoboth: American Research Press, 1999.

[9] F.Smarandache, "Neutrosophic set, a generalisation of the intuitionistic fuzzy sets", Inteer J. PuBre Appl. Math. 24, pp.287-297,2005.

[10]F.Smarandache, "Neutrosophy and Neutrosophic Logic, First International Conference on Neutrosophy,NeutrosophyLogic, Set, Probability and Statistics" University of New Mexico,Gallup,NM 87301,USA, 2002.

[11] F.Smarandache "Neutrosophy, A new Branch of Philosophy",= Multiple Valued Logic /An International Journal, USA, ISSN 1023-6627, Vol;. 8, No.3, pp.297-384, 2002.

[12] F.Smarandache, "Plithogeny, PlithogenicSet, Logic,Probability, and Statistics", 141 pages, Pons Editions,Brussels,Belgium,2017, arXiv.org (Cornell University), Computer Science-Artificial Intelligence, 03Bxx:abstract:https://arxiv.org/abs/1808.03948,book:https://arxiv.org/ftp/arxiv/papers/1808/1808.03948.pdf,HarvardSAO/NASAADS: http://adsabs.harvard.edu/cgi-bin/bib_query?arXiv:1808.03948.

[13] Turksen, "Interval valued fuzzy sets based on normal forms", Fuzzy sets and Systems, 20, pp.191-210,1968.

[14] B. Turksen," Interval-valued fuzzy sets and compensatory" Fuzzy Sets and Systems, 51, pp.295–307,1992.

[15] I. B. Turksen, "Interval-valued strict preference with Zadeh triples", Fuzzy Sets and Systems 78, pp.183–195,1996.

[16] Victor Christianto, Robert N. Boyd and Florentin Smarandache "Three Possible Applications of Neutrosophic Logic in Fundamental and Applied Sciences", International Journal of Neutrosophic Science (IJNS)Vol. 1, No. 2, pp. 90-95, 2020

[17] Wang, H.; Smarandache, F.; Zhang, Y. Q.; Sunderraman, R. "Interval Neutrosophic Sets and logic: Theory and Applications in Computing"; Hexis, Phoenix, Ariz, USA, 2005.

[18] L.A Zadeh, "Fuzzy sets", Inform and Control 8, pp.338-353,1965.

DOI: 10.5281/zenodo.4030378