Reply to "Notes on Pioneer Anomaly Explanation by Satellite-Shift Formula of Quaternion Relativity"

Vic Christianto* and Florentin Smarandache†

*Sciprint.org — a Free Scientific Electronic Preprint Server, http://www.sciprint.org
E-mail: admin@sciprint.org
†Department of Mathematics, University of New Mexico, Gallup, NM 87301, USA
E-mail: smarand@unm.edu

In the present article we would like to make a few comments on a recent paper by A. Yefremov in this journal [1]. It is interesting to note here that he concludes his analysis by pointing out that using full machinery of Quaternion Relativity it is possible to explain Pioneer XI anomaly with excellent agreement compared with observed data, and explain around 45% of Pioneer X anomalous acceleration. We argue that perhaps it will be necessary to consider extension of Lorentz transformation to Finsler-Berwald metric, as discussed by a number of authors in the past few years. In this regard, it would be interesting to see if the use of extended Lorentz transformation could also elucidate the long-lasting problem known as Ehrenfest paradox. Further observation is of course recommended in order to refute or verify this proposition.

1 Introduction

We are delighted to read A. Yefremov's comments on our preceding paper [3], based on his own analysis of Pioneer anomalous "apparent acceleration" [1]. His analysis made use of a method called Quaternion Relativity, which essentially is based on $\mathrm{SO}(1,2)$ form invariant quaternion square root from space-time interval rather than the interval itself [1, 2]. Nonetheless it is interesting to note here that he concludes his analysis by pointing out that using full machinery of Quaternion Relativity it is possible to explain Pioneer XI anomaly with excellent agreement compared with observed data, and explain around 45% of Pioneer X anomalous acceleration [1].

In this regard, we would like to emphasize that our preceding paper [3] was based on initial "conjecture" that in order to explain Pioneer anomaly, it would be necessary to generalize pseudo-Riemann metric of General Relativity theory into broader context, which may include Yefremov's Quaternion Relativity for instance. It is interesting to note here, however, that Yefremov's analytical method keeps use standard Lorentz transformation in the form Doppler shift effect (Eq. 6):

$$f = \frac{f'}{\sqrt{1-\left(\frac{v_D}{c}\right)^2}}\left(1-\frac{v_D}{c}\cos\beta\right). \qquad (1)$$

While his method using relativistic Doppler shift a la Special Relativity is all right for such a preliminary analysis, in our opinion this method has a drawback that it uses "standard definition of Lorentz transformation" based on 2-dimensional problem of *rod-on-rail* as explained in numerous expositions of relativity theory [5]. While this method of rod-on-rail seems sufficient to elucidate why "simultaneity" is ambiguous term in physical sense, it does not take into consideration 3-angle problem in more general problem. This is why we pointed out in our preceding paper that apparently General Relativity inherits the same drawback from Special Relativity [3].

Another problem of special relativistic definition of Lorentz transformation is known as "reciprocity postulate", because in Special Relativity it is assumed that: $x \leftrightarrow x'$, $t \leftrightarrow t'$, $v \leftrightarrow -v'$ [6]. This is why Doppler shift can be derived without assuming reciprocity postulate (which may be regarded as the "third postulate" of Special Relativity) and without special relativistic argument, see [7]. Nonetheless, in our opinion, Yefremov's Quaternion Relativity is free from this "reciprocity" drawback because in his method there is difference between moving-observer and static-observer [2].

An example of implications of this drawback of 1-angle problem of Lorentz transformation is known as Ehrenfest paradox, which can be summarized as follows: "According to Special Relativity, a moving rod will exhibit apparent length-reduction. This is usually understood to be an observational effect, but if it is instead considered to be a real effect, then there is a paradox. According to Ehrenfest, the perimeter of a rotating disk is like a sequence of rods. So does the rotating disk shatter at the rim?" Similarly, after some thought Klauber concludes that "*The second relativity postulate does not appear to hold for rotating systems*" [8].

While it is not yet clear whether Quaternion-Relativity is free from this Ehrenfest paradox, we would like to point out that an alternative metric which is known to be nearest to Riemann metric is available in literature, and known as Finsler-Berwald metric. This metric has been discussed adequately by Pavlov, Asanov, Vacaru and others [9–12].

2 Extended Lorentz-transformation in Finsler-Berwald metric

It is known that Finsler-Berwald metric is subset of Finslerian metrics which is nearest to Riemannian metric [12], therefore it is possible to construct pseudo-Riemann metric based on Berwald-Moor geometry, as already shown by Pavlov [4]. The neat link between Berwald-Moor metric and Quaternion Relativity of Yefremov may also be expected because Berwald-Moor metric is also based on analytical functions of the H4 variable [4].

More interestingly, there was an attempt in recent years to extend 2d-Lorentz transformation in more general framework on H4 of Finsler-Berwald metric, which in limiting cases will yield standard Lorentz transformation [9, 10]. In this letter we will use extension of Lorentz transformation derived by Pavlov [9]. For the case when all components but one of the velocity of the new frame in the old frame coordinates along the three special directions are equal to zero, then the transition to the frame moving with velocity V_1 in the old coordinates can be expressed by the new frame as [9, p.13]:

$$\begin{pmatrix} x_0 \\ x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{bmatrix} [F] & [0] \\ [0] & [F] \end{bmatrix} = \begin{pmatrix} x'_0 \\ x'_1 \\ x'_2 \\ x'_3 \end{pmatrix} \tag{2}$$

where the transformation matrix for Finsler-Berwald metric is written as follows [9, p.13]:

$$[F] = \begin{pmatrix} \frac{1}{\sqrt{1-V_1^2}} & \frac{V_1}{\sqrt{1-V_1^2}} \\ \frac{V_1}{\sqrt{1-V_1^2}} & \frac{1}{\sqrt{1-V_1^2}} \end{pmatrix} \tag{3}$$

and

$$[0] = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}. \tag{4}$$

Or

$$x_0 = \frac{x'_0 + V x'_1}{\sqrt{1-V_1^2}} \, x_1 = \frac{V x'_0 + x'_1}{\sqrt{1-V_1^2}}\,, \tag{5}$$

and

$$x_2 = \frac{x'_2 + V x'_3}{\sqrt{1-V_1^2}} \, x_3 = \frac{V x'_2 + x'_3}{\sqrt{1-V_1^2}}. \tag{6}$$

It shall be clear that equation (5) $(x'_0, x'_1) \leftrightarrow (x_0, x_1)$ coincides with the corresponding transformation of Special Relativity, while the transformation in equation (6) differs from the corresponding transformation of Special Relativity where $x_2 = x'_2$, $x_3 = x'_3$ [9].

While we are not yet sure whether the above extension of Lorentz transformation could explain Pioneer anomaly better than recent analysis by A. Yefremov [1], at least it can be expected to see whether Finsler-Berwald metric could shed some light on the problem of Ehrenfest paradox. This proposition, however, deserves further theoretical considerations.

In order to provide an illustration on how the transformation keeps the Finslerian metric invariant, we can use Maple algorithm presented by Asanov [10, p.29]:

```
> c1:=cos(tau);c2:=cos(psi);c3:=cos(phi);
> u1:=sin(tau);u2:=sin(psi);u3:=sin(phi);
> l1:=c2*c3−c1*u2*u3;l2:=−c2*u3−c1*u2*c3;l3:=u1*u2;
> m1:=u2*c3+c1*c2*u3;m2:=−u2*u3+c1*c2*c3;m3:=−u1*c2;
> n1:=u1*u3; u1*c3; c1;
> F1:=(e1)ˆ((l1+m1+n1+l2+m2+n2+l3+m3+n3+1)/4)*
(e2)ˆ((−l1−m1−n1+l2+m2+n2−l3−m3−n3+1)/4)*
(e3)ˆ((l1+m1+n1−l2−m2−n2−l3−m3−n3+1)/4)*
(e4)ˆ((−l1−m1−n1−l2−m2−n2+l3+m3+n3+1)/4):
> F2:=(e1)ˆ((−l1+m1−n1−l2+m2−n2−l3+m3−n3+1)/4)*
(e2)ˆ((l1−m1+n1−l2+m2−n2+l3−m3+n3+1)/4)*
(e3)ˆ((−l1+m1−n1+l2−m2+n2+l3−m3+n3+1)/4)*
(e4)ˆ((l1−m1+n1+l2−m2+n2−l3+m3−n3+1)/4):
> F3:=(e1)ˆ((l1−m1−n1+l2−m2−n2+l3−m3−n3+1)/4)*
(e2)ˆ((−l1+m1+n1+l2−m2−n2−l3+m3+n3+1)/4)*
(e3)ˆ((l1−m1−n1−l2+m2+n2−l3+m3+n3+1)/4)*
(e4)ˆ((−l1+m1+n1−l2+m2+n2+l3−m3−n3+1)/4):
> F4:=(e1)ˆ((−l1−m1+n1−l2−m2+n2−l3−m3+n3+1)/4)*
(e2)ˆ((l1+m1−n1−l2−m2+n2+l3+m3−n3+1)/4)*
(e3)ˆ((−l1−m1+n1+l2+m2−n2+l3+m3−n3+1)/4)*
(e4)ˆ((l1+m1−n1+l2+m2−n2−l3−m3+n3+1)/4):
> a:=array(1..4,1..4):
for i from 1 to 4
do
for j from 1 to 4
do
a[i,j]:=diff(F||i,e||j);
end do:
end do:
> b:=array(1..4,1..4):
for i from 1 to 4
do
for j from 1 to 4
do
b[i,j]:=simplify(add(1/F||k*diff(a[k,i],e||j),k=1..4),symbolic);
end do:
end do:
> print(b);
```

The result is as follows:

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

This result showing that all the entries of the matrix are zeroes support the argument that the metricity condition is true [10].

3 Concluding remarks

In the present paper we noted that it is possible to generalise standard Lorentz transformation into H4 framework of Finsler-Berwald metric. It could be expected that this extended Lorentz transformation could shed some light not only to Pioneer anomaly, but perhaps also to the long-lasting problem of Ehrenfest paradox which is also problematic in General Relativity theory, or by quoting Einstein himself:

> "... Thus all our previous conclusions based on general relativity would appear to be called in question. In reality we must make a subtle detour in order to be able to apply the postulate of general relativity exactly" [5].

This reply is not intended to say that Yefremov's preliminary analysis is not in the right direction, instead we only highlight a possible way to improve his results (via extending Lorentz transformation). Furthermore, it also does not mean to say that Finsler-Berwald metric could predict better than Quaternion Relativity. Nonetheless, further observation is of course recommended in order to refute or verify this proposition.

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