# The Smarandache P and S persistence of a prime

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In [1], Sloane has defined the multiplicative persistence of a number in the following manner.
Let's N be any n-digits number with $N = x_1 x_2 x_3 .....x_n$ in base 10. Multiplying together the digits of that number ($x_1 \cdot x_2 \cdot ...... \cdot x_n$), another number $N'$ results. If this process is iterated, eventually a single digit number will be produced. The number of steps to reach a single digit number is referred to as the persistence of the original number N. Here an example:

679 → 378 → 168 → 48 → 32 →6

In this case, the persistence of 679 is 5.
Of course, that concept can be extended to any base b. In [1], Sloane conjectured that, in base 10, there is a number c such that no number has persistence greater than c. According to a computer search no number smaller then $10^{50}$ with persistence greater than 11 has been found.
In [2], Hinden defined in a similar way the additive persistence of a number where, instead of multiplication, the addition of the digits of a number is considered. For example, the additive persistence of 679 is equal to 2.

679 → 22 → 4

Following the same spirit, in this article we introduce two new concepts: the Smarandache P-persistence and the Smarandache S-persistence of a prime number.
Let's X be any n-digits prime number and let's suppose that $X = x_1 x_2 x_3 .....x_n$ in base 10.
If we multiply together the digits of that prime ($x_1 \cdot x_2 \cdot ...... \cdot x_n$) and add them to the original prime ($X + x_1 \cdot x_2 \cdot ..... \cdot x_n$) a new number results, which may be a prime. If it is a prime then the process will be iterated otherwise not. The number of steps required to X to collapse in a composite number is called the Smarandache P-persistence of prime X.
As an example, let's calculate the Smarandache P-persistence of the primes 43 and 23:

43 → 55
23 → 29 → 47 → 75

which is 1 and 3, respectively. Of course, the Smarandache P-Persistence minus 1 is equal to the number of primes that we can generate starting with the original prime X.
Before proceeding, we must highlight that there will be a class of primes with an infinite Smarandache P-persistence; that is, primes that will never collapse in a composite number. Let's give an example:

61 → 67 → 109 → 109 → 109…….

In this case, being the product of the digits of the prime 109 always zero, the prime 61 will never reach a composite number. In this article, we shall not consider that class of primes since it is not interesting.
The following table gives the smallest multidigit primes with Smarandache P-persistence less than or equal to 8:

| **Smarandache P-persistence** | **Prime** |
|---|---|
| 1 | 11 |
| 2 | 29 |
| 3 | 23 |
| 4 | 347 |
| 5 | 293 |
| 6 | 239 |
| 7 | 57487 |
| 8 | 486193 |

By looking in a greater detail at the above table, we can see that, for example, the second term of the sequence (29) is implicitly inside the chain generated by the prime 23. In fact:

29 → 47 → 75
23 → 29 → 47 → 75

We can slightly modify the above table in order to avoid any prime that implicitly is inside other terms of the sequence.

| **Smarandache P-persistence** | **Prime** |
|---|---|
| 1 | 11 |
| 2 | 163 |
| 3 | 23 |
| 4 | 563 |
| 5 | 1451 |
| 6 | 239 |
| 7 | 57487 |
| 8 | 486193 |

Now, for example, the prime 163 will generate a chain that isn't already inside any other chain generated by the primes listed in the above table.
What about primes with Smarandache P-persistence greater than 8? Is the above sequence infinite?
We will try to give an answer to the above question by using a statistical approach.
Let's indicate with L the Smarandache P-persistence of a prime. Thanks to an u-basic code the occurrrencies of L for different values of N have been calculated. Here an example for $N = 10^7$ and $N = 10^8$:

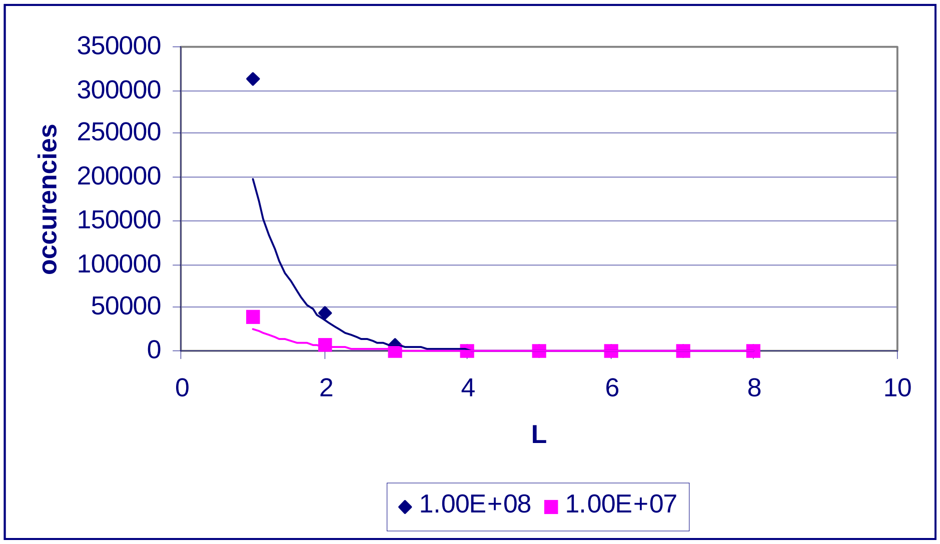


**Fig 1.** Plot of the occurrencies for each P-persistence at two different values of N.

The interpolating function for that family of curves is given by:

$$a(N) \cdot e^{-b(N) \cdot L}$$

where a(n) and b(n) are two function of N.

To determine the behaviour of those two functions, the values obtained interpolating the histogram of occurencies for different N have been used:

| N | a | b |
|---|---|---|
| 1.00E+04 | 2238.8 | 1.3131 |
| 1.00E+05 | 17408 | 1.4329 |
| 1.00E+06 | 121216 | 1.5339 |
| 1.00E+07 | 1.00E+06 | 1.6991 |
| 1.00E+08 | 1.00E+07 | 1.968 |

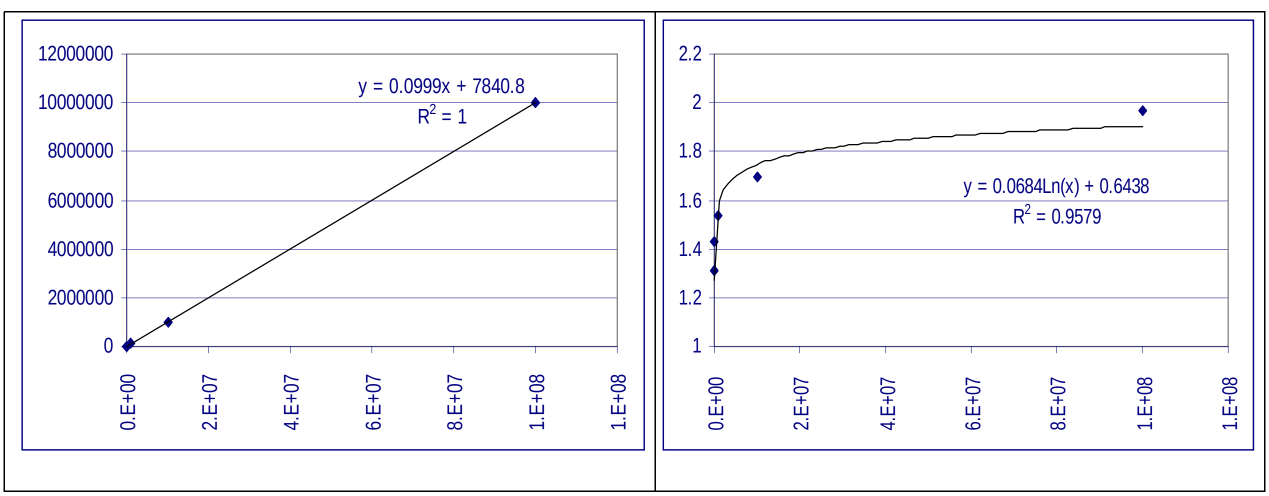


**Fig. 2** Plot of the two functions a(N) and b(N) versus N

According to those data we can see that :

$$a(N) \approx k \cdot N \qquad b(N) \approx h \cdot \ln(N) + c$$

where k ,h and c are constants (see fig. 2).
So the probability that $L \geq M$ (where M is any integer) for a fixed N is given by:

$$P(L \geq M) \approx \frac{\int_M^\infty kN \cdot e^{-(h \ln N + c) \cdot L}\, dL}{\int_0^\infty kN \cdot e^{-(h \ln N + c) \cdot L}\, dL} = e^{-(h \cdot \ln N + c) \cdot M}$$

and the counting function of the primes with Smarandache P-persistence L=M below N is given by $N \cdot P(L = M)$. In fig 3, the plot of counting function versus N for 4 different L values is reported. As we can see, for L< 15 and $L \geq 15$ there is a breaking in the behaviour of the occurrencies. For $L \geq 15$, the number of primes is very very small (less than 1) regardless the value of N and it becomes even smaller as N increases. The experimental data seem to support that L cannot take any value and that most likely the maximum value should be L=14 or close to it. So the following conjecture can be posed:

**Conjecture 1.** T*here is an integer M such that no prime has a Smarandache P-persistence greater than M. In other words the maximum value of Smarandache P-persistence is finite*

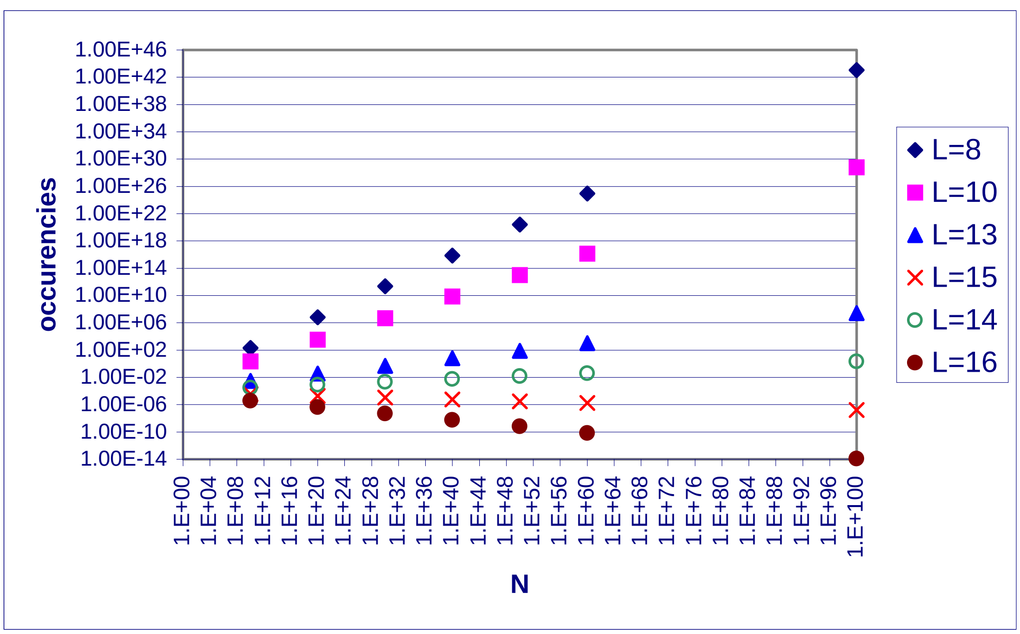


**Fig. 3** Counting function for the P-persistence for difference values of N

Following a similar argumentation the Smarandache S-persistence of a prime can be defined. In particular it is the number of steps before a prime number collapse to a composite number considering the sum of the digits instead of the product as done above. For example let's calculate the Smarandache S-persistence of the prime 277:

277 → 293 → 307 → 317 → 328

In this case we have a Smarandache S-persistence equal to 4. The sequence of the smallest multi-digit prime with Smarandache S-persistence equal to 1,2,3,4….. has been found by Rivera [3]. Anyway no prime has been found with the Smarandache S-persistence greater than 8 up to N=18038439735. Moreover by following the same statistical approach used above for the Smarandache P-persistence the author has found a result similar to that obtained for the Smarandache P-persistence (see [3] for details).
Since the statistical approach applied to the Smarandache P and S persistence gives the same result (counting function always smaller than 1 for $L \geq 15$) we can be confident enough to pose the following conjecture:

**Conjecture 2.** *The maximum value of the Smarandache P and S persistence is the same.*

## References

[1] N. Sloane, "The persistence of a number", J. Recreational Mathematics, Vol 6, No 2, Spring 1973
[2] Hinden, H. J. "The Additive Persistence of a Number." *J. Recr. Math.* **7**, 134-135, 1974.
[3] C. Rivera, Puzzle 163: P+SOD(P), http://www.primepuzzles.net/puzzles/puzz_163.htm