From Zeldovich Approximation to Burgers' equation: A Plausible Route to Cellular Automata Adhesion Universe

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ABSTRACT

Some years ago, Hidding *et al*. suggest that the emergence of intricate and pervasive weblike structure of the Universe on Megaparsec scales can be approximated by a well-known equation from fluid mechanics, the Burgers' equation. The solution to this equation can be obtained from a geometrical formalism. The resulting Adhesion formalism provides deep insight into the dynamics and topology of the Cosmic Web. It uncovers a direct connection between the conditions in the very early Universe and the complex spatial patterns that emerged out of these under the influence of gravity. In the present paper, we describe a cellular automaton model of the Burgers' equation, which can be investigated via a fast computer simulation. In the end, this suggests a Cellular Automata Adhesion Model of the Universe.

Keywords: discrete physics, Cosmology, Large Scale Structure of the Universe, Numerical Methods, Cellular Automata, Burgers equation, Zeldovich approximation

1. Introduction

The Cosmic Web is the fundamental spatial organization of matter on scales of a few up to a hundred Megaparsec. Galaxies and intergalactic gas matter exist in a wispy weblike arrangement of dense compact clusters, elongated filaments, and sheetlike walls, amidst large near-empty void regions. The filaments are the transport channels along which matter and galaxies flow into massive high-density cluster located at the nodes of the web. The weblike network is shaped by the tidal force field accompanying the inhomogeneous matter distribution.[1]

Structure in the Universe has risen out of tiny primordial (Gaussian) density and velocity perturbations by means of gravitational instability. The large-scale anisotropic force field induces anisotropic gravitational collapse, resulting in the emergence of elongated or flattened matter configurations. The simplest model that describes the emergence of structure and complex patterns in the Universe is the Zeldovich Approximation (ZA).[1]

It is our hope that the new approach of CA Adhesion model of the Universe can be verified either with lab experiments, computer simulation, or by large-scale astronomy observation data.

2. From Zeldovich Approximation to Burgers' equation to Cellular Automaton model

In this section, we will outline a route from ZA to Burgers' equation and then to CA model.

The simplest model that describes the emergence of structure and complex patterns in the Universe is the Zeldovich Approximation (ZA). In essence, it describes a ballistic flow, driven by a constant (gravitational) potential. The resulting Eulerian position x(t) at some cosmic epoch t is specified by the expression:[1]

$$x(t) = q + D(t)u_o(q), \qquad (1)$$

where q is the initial "Lagrangian" position of a particle, D(t) the time-dependent structure growth factor and

$$u_0 = -\nabla_q \Phi_0 \qquad (2)$$

its velocity. The nature of this approximation may be appreciated by the corresponding source-free equation of motion,

$$\frac{\partial u}{\partial D} + (u \cdot \nabla_x)u = 0. \tag{3}$$

The use of ZA is ubiquitous in cosmology. One major application is its key role in setting up initial conditions in cosmological N-body simulations. Of importance here is its nonlinear extension in terms of *Adhesion Model*.[1]

The ZA breaks down as soon as self-gravity of the forming structures becomes important. To 'simulate' the effects of self-gravity, Gurbatov *et al*. included an artificial viscosity. This results in the Burgers' equation as follows:[1]

$$\frac{\partial u}{\partial D} + (u \cdot \nabla_x)u = \nu . \nabla_x^2 u, \tag{4}$$

a well known PDE from fluid mechanics. This equation has an exact analytical solution, which in the limit of $\nu \to 0$, the solution is: [1]

$$\phi(x, D) = \max_q \left[\Phi_0(q) - \frac{(x-q)^2}{2D} \right]. \tag{5}$$

This leads to a geometric interpretation of the Adhesion Model. The solution follows from the evaluation of the convex hull of the velocity potential modified by a quadratic term. We found that the solution can also be found by computing the weighted Voronoi diagram of a mesh weighted with the velocity potential. For more detailed discussion on Adhesion Model of the Universe, see for example [4].

Now, let us consider another routes to solve Burgers equation: (a) by numerical computation with *Mathematica*, see [3]; and (b) by virtue of CA approach. Let us skip route (a), and discuss less known approach of cellular automata.

We start with the Burgers' equation with Gaussian white noise which can be rewritten as follows:[2]

$$\frac{\partial u}{\partial t} + \xi = 2u\frac{\partial u}{\partial x} + \frac{\partial^2 u}{\partial x^2} + \eta. \tag{6}$$

By introducing new variables and after straightforward calculations, we have the automata rule:[2]

$$\begin{aligned} \phi_i^{t+1} &= \phi_{i-1}^t + \max[0, \phi_i^t - A, \phi_i^t + \phi_{i+1}^t - B, \Psi_i^t - \phi_{i-1}^t] \\ &- \max[0, \phi_{i-1}^t - A, \phi_{i-1}^t + \phi_i^t - B, \Phi_i^t + \phi_{i-1}^t] \end{aligned} \tag{7}$$

In other words, in this section we give an outline of a plausible route from ZA to Burgers' equation then to CA model, which suggests that it appears possible –at least in theory- to consider a nonlinear cosmology based on CA Adhesion model.

3. Concluding Remarks

The use of ZA is ubiquitous in cosmology. One major application is its key role in setting up initial conditions in cosmological N-body simulations. Of importance here is its nonlinear extension in terms of *Adhesion Model*. In this paper, we give an outline of a plausible route from ZA to Burgers' equation then to CA model, which suggests that it appears theoretically possible to consider a nonlinear cosmology based on CA Adhesion model.

This paper is part of our theoretical investigation of plausible nonlinear cosmology models beyond Navier-Stokes-inspired approaches.

It is our hope that the proposed approach can be verified with more extensive computer simulation and (astronomy) observation data.

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