The Plithogenic set is a generalization of crisp, fuzzy, intuitionistic fuzzy, and Neutrosophic sets, it is a set whose elements are characterized by many attributes' values.
The authors have defined, described and developed Plithogenic graphs and given some applications of them to real world problems.

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PLITHOGENIC GRAPHS

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PREFACE

The plithogenic set is a generalization of crisp, fuzzy, intuitionistic fuzzy, and Neutrosophic sets, it is a set whose elements are characterized by many attributes' values. This book gives some possible applications of plithogenic sets defined by Florentin Smarandache (2018). The authors have defined a new class of special type of graphs which can be applied for plithogenic models. The main motivation for such new classes of special types of graphs comes from the fuzzy intuitionistic graphs (2017) and single valued Neutrosophic graphs (2018).

In case of fuzzy intuitionistic graphs the vertex sets are labelled as 1×2 row matrices, the values of the matrix is from the closed unit interval [0, 1], the first entry of the matrix being the fuzzy membership and the other entry being the fuzzy non-membership. The edges of the adjacent vertices are also fuzzy 1×2 matrices following certain rules.

In the case of Single Valued Neutrosophic graphs the vertex sets are labelled with 1×3 fuzzy row matrices, the values are the truth membership, the indeterminacy membership and the false membership, the values of this row matrices are also from the fuzzy unit interval [0, 1]. The edges of adjacent vertices are defined depending on the vertex row matrices which is again a fuzzy row matrix.

Authors in this book define plithogenic graphs for which vertex values are taken as a $1 \times n$ matrices and the entries can be real or complex or Neutrosophic or dual numbers or from the fuzzy interval; the relevant edges are not labelled with row matrices. However, the plithogenic graphs takes its edge and vertex values only from the $1 \times n$ row matrices, the entries can be real or complex or so on. That is for Plithogenic Crisp Graph, we have the row matrix as 0's and 1's; for Plithogenic Fuzzy Graph, when we will have the row matrix of numbers between 0 and 1. Similarly, for Plithogenic Intuitionistic Fuzzy Graph, we will have the row matrix of duplets of numbers between 0 and 1. In case of Plithogenic Neutrosophic Graph, we have the row matrix of triplets of numbers between 0 and 1 and in case of Plithogenic Real Number Graph, we will have the row matrix formed by real numbers. For Plithogenic Complex Number Graph, we will have the row matrix formed by complex numbers and in case of Plithogenic Neutrosophic Number Graph, we will have the row matrix formed by neutrosophic numbers. These graphs are modelled and their application to real world problems are provided. Certainly, in general these newly built plithogenic models can give a sensitive solution depending on the problem in hand.

We wish to acknowledge Dr. K Kandasamy for his sustained support and encouragement in the writing of this book.

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ILANTHENRAL K
FLORENTIN SMARANDACHE

Chapter One

INTRODUCTION

In this chapter we recall some basic definitions and properties essential to make this book a self-contained one. We have defined and developed several types of graphs whose vertex values are subsets of a set. We have also described multigraphs and subset vertex multigraphs. An interesting feature about these subset vertex graphs, and subset vertex multigraphs is they get only unique set of edges once the vertex subsets are given.

This has several advantages, the main among them is the Freeman index can be avoided when these subset vertex graphs are used in the study of social networks.

In this book we define Plithogenic graph G with vertex set $v_1, \ldots, v_n$ and its edges are weighted with a $1 \times m$ row matrix. The entries of the $1 \times m$ row matrix can be real or complex or neutrosophic or from the interval [0, 1]. The inspiration for such type of graphs comes from the intuitionistic fuzzy graphs whose edge weights as well as vertex sets are labeled as 1×2 row matrices. We use the definition from [3, 6].

However, we have defined 6 types of Plithogenic intuitionistic fuzzy graphs. On similar lines we can define 3 types of Plithogenic fuzzy graphs. We see the Plithogenic intuitionistic fuzzy graph can be generalized using any $1 \times m$ row matrices for vertex sets as well as edge weights. Given any set of vertices we can have only 6 types of edge weights. However, we can have only six such complete row weighted graphs, but there can be many in each type which is solely dependent on the problem in hand.

Chapter two deals with a Plithogenic graphs. Several interesting properties about them are derived.

In chapter three we introduce the notion of vertex row matrix labeled and edge row matrix labeled special graphs, which we choose to call as Plithogenic graphs. These types of graphs with 1×2 rows and 1×3 rows are defined and applied to problems and are known as Plithogenic intuitionistic fuzzy graphs and Plithogenic single valued neutrosophic graphs respectively. We study these Plithogenic graphs with $1 \times m$ row matrices, they are used in the plithogenic models.

The final chapter gives the applications of these Plithogenic graphs to model real world problems.

Chapter Two

PLITHOGENIC GRAPHS

In this chapter we for the first time define a new type of row matrix weighted graphs called Plithogenic graphs. We enumerate several of its properties and their related results. First, we describe them with a few examples.

Example 2.1. Let G be a graph given by the following figure:

G =

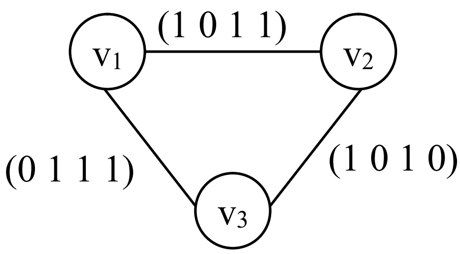


Figure 2.1

We call this Plithogenic graph, to be more precise as 1×4 Plithogenic triad. When the weights are only from the set {0, 1} we call it as Plithogenic crisp fuzzy graphs. So G is called as Plithogenic crisp triad.

Example 2.2: Let H be a graph given by the following figure.

H =

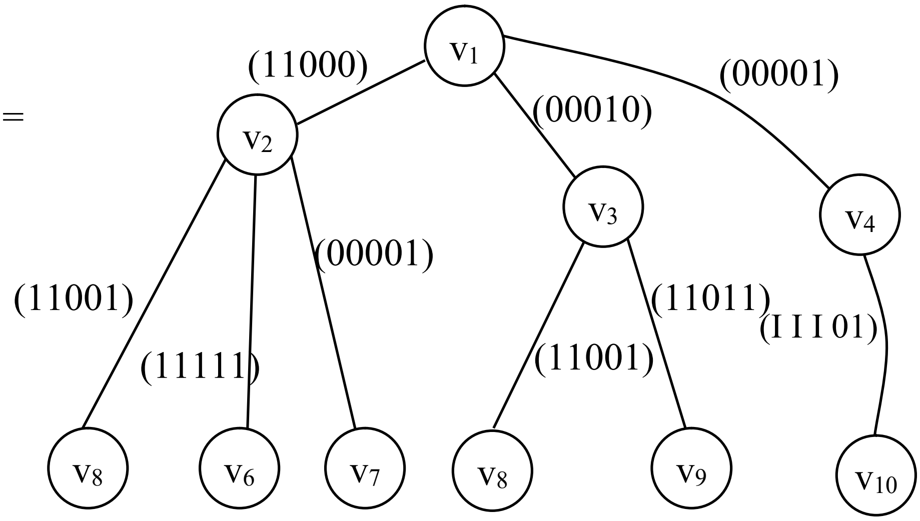


Figure 2.2

We call H as the Plithogenic crisp tree.

Example 2.3: Let K be the Plithogenic crisp star graph given by the following figure.

K =

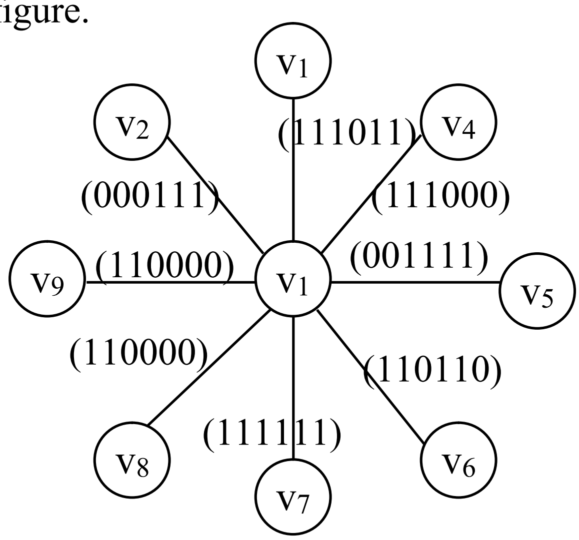


Figure 2.3

We see if the edge weight is the same the two concepts / nodes enjoy the same status with the ego centric node v_1. K is only a Plithogenic crisp graph. We now provide an example of a Plithogenic crisp complete graph by the following figure.

Example 2.4. Let G be a Plithogenic crisp complete graph given by the following figure.

G =

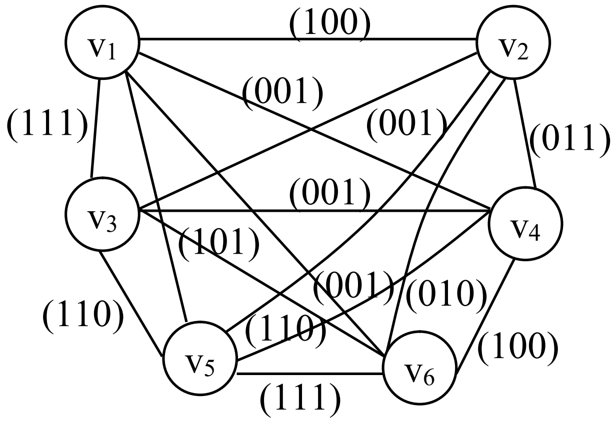


Figure 2.4

S is a Plithogenic crisp complete graph. We now illustrate a Plithogenic crisp line graph by the following example.

Example 2.5. Let G be a Plithogenic crisp line graph given by the following figure.

G =

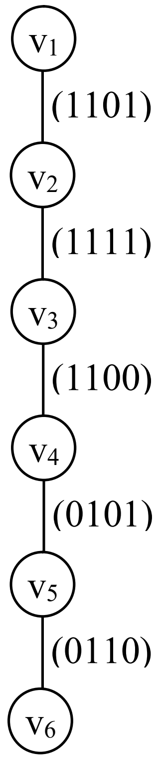


Figure 2.5

Now we proceed onto define this concept abstractly.

Definition 2.1. *Let $v_1, \ldots, v_m$ be m distinct vertices. Let G be a graph using these set of m vertices.*

Let the weights of the edges be taken from the set E = $\{(a_1, \ldots, a_n) / a_i \in \{0, 1\}; 1 \leq i \leq n\}$. We call G as the Plithogenic crisp graph as the edge weights are row matrices and the values of the row matrices are taken only from the set {0, 1}.

We have given several examples of them.

Now unlike in the case of usual graphs number of graphs that are Plithogenic crisp graphs depends on the order of the row matrix.

We will illustrate this situation by some examples.

Example 2.6: Let G be a Plithogenic crisp dyad given by the following figure,

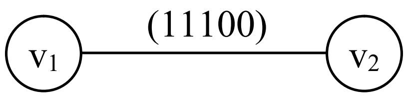


Figure 2.6

If crisp weights are from a 1×5 row matrix with values from {0, 1} we can have $2^5 - 1$ of them as (00000) weight between v_1 and v_2 means no edge exists between v_1 and v_2. They are given in the following.

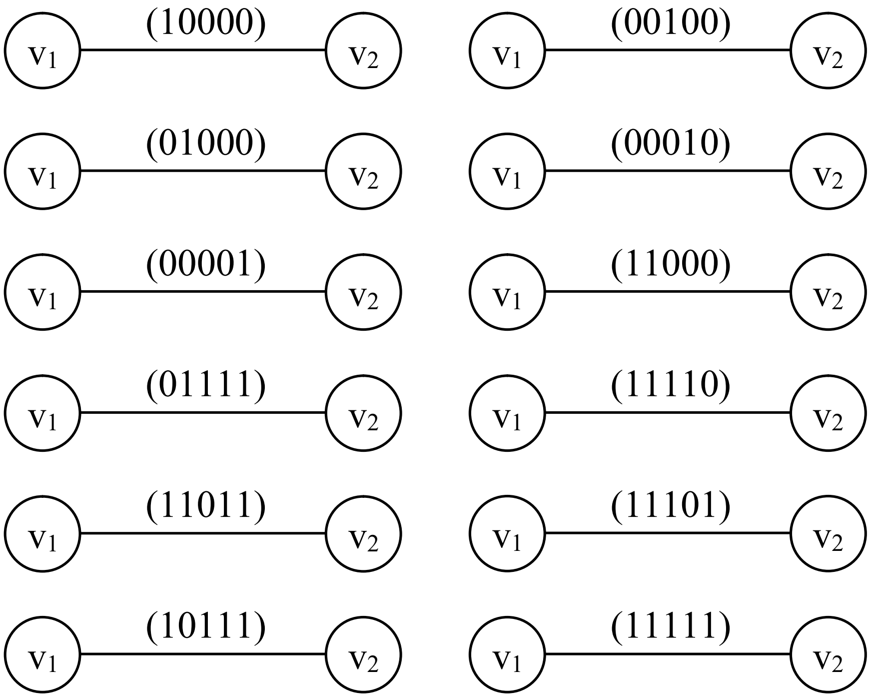
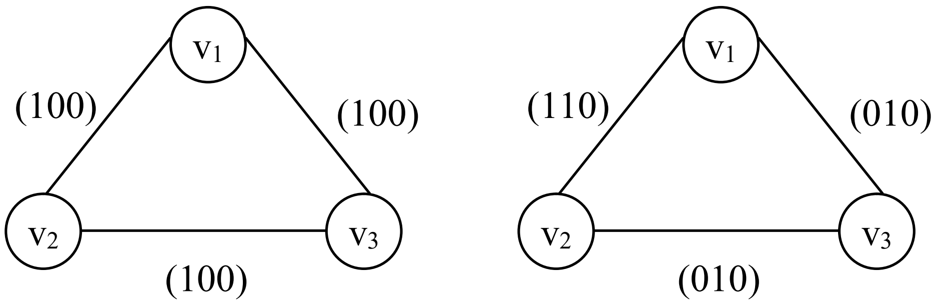


Figure 2.7

There will be $2^5 - 1$ number of crisp dyad of 1×5 row matrix. However in case of crisp dyads of order $1 \times n$ we can give, the number as $2^n - 1$.

But find the number in case of crisp triads happen to be a challenging one for any n; n a large number.

Consider a 1×3 Plithogenic crisp graphs which are complete triads. Some of them are given in the following.



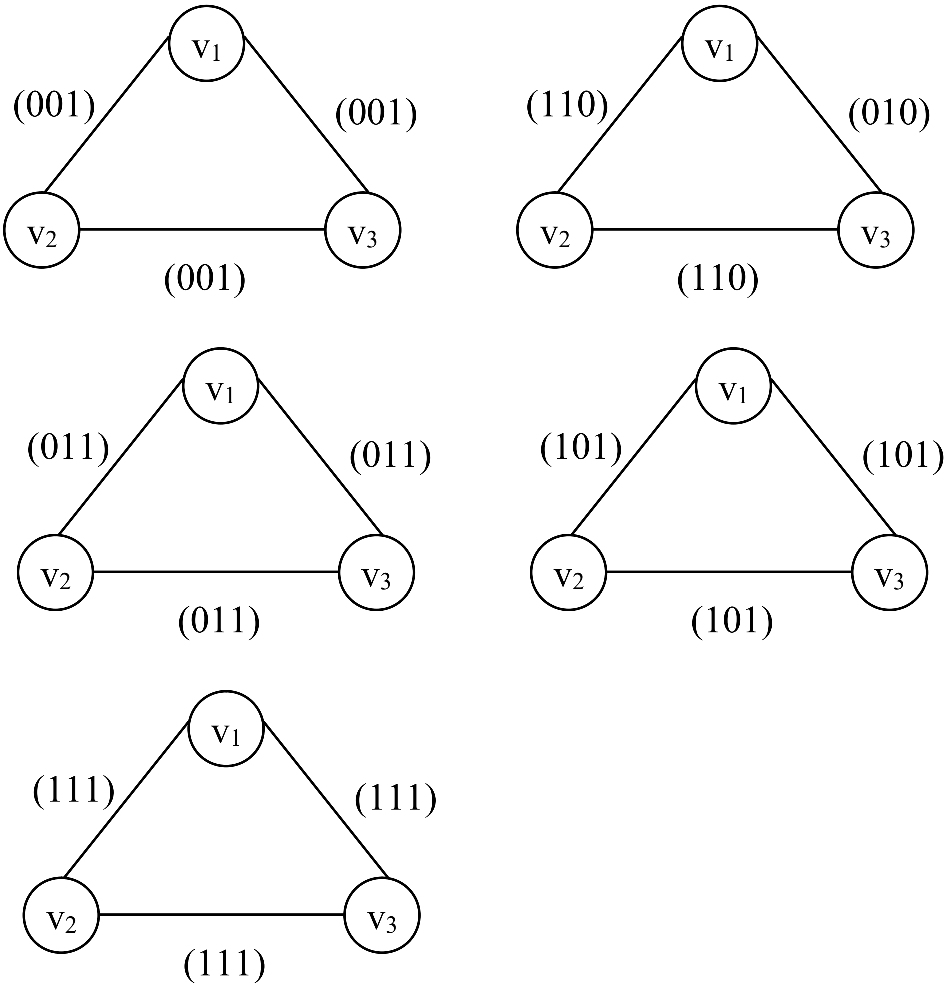
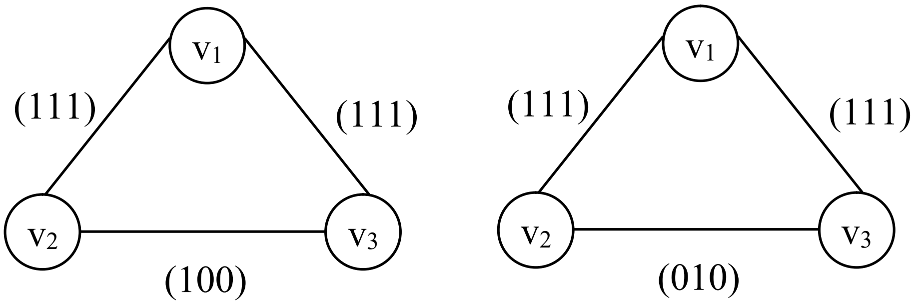


Figure 2.8

There are 7 such crisp triad with equal 1 × 3 row weights.

We now supply a few more of them.



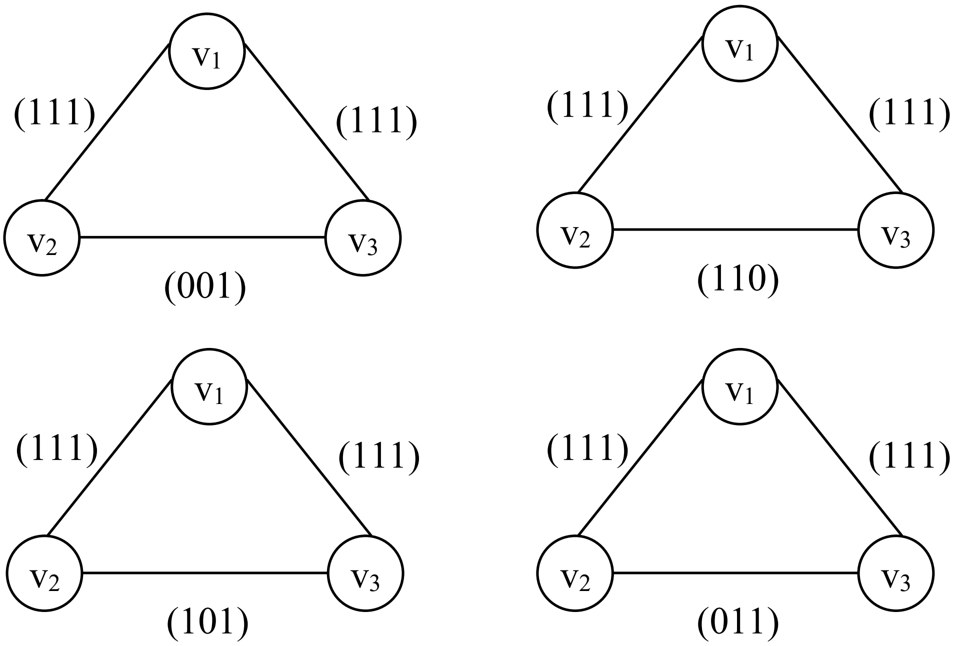
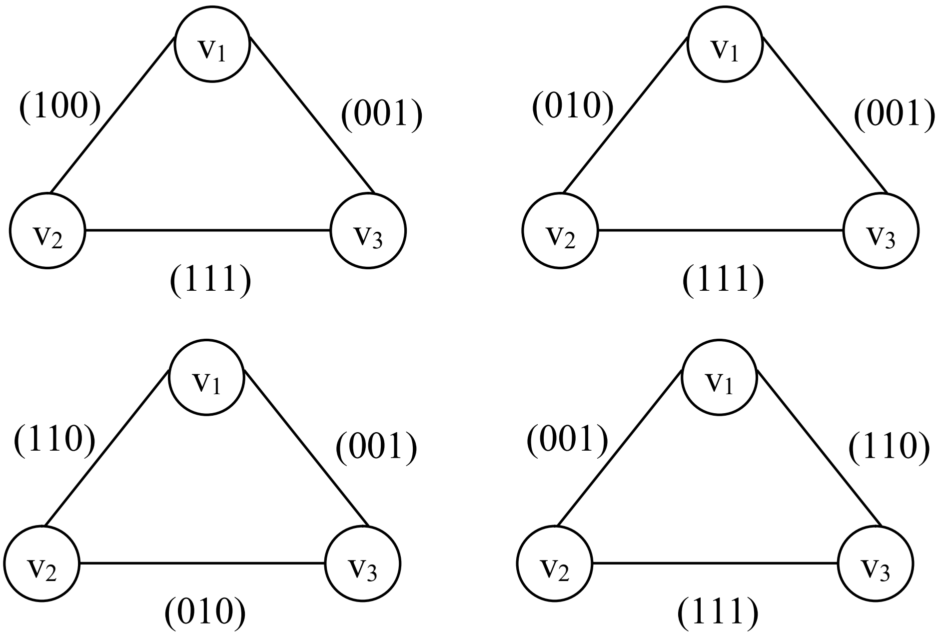


Figure 2.9

We can have 7 × 6 number of 1 × 3 Plithogenic crisp triads which has equal weights for two edges and a one edge with different weight. The other Plithogenic triads with different weights are enlisted in the following.



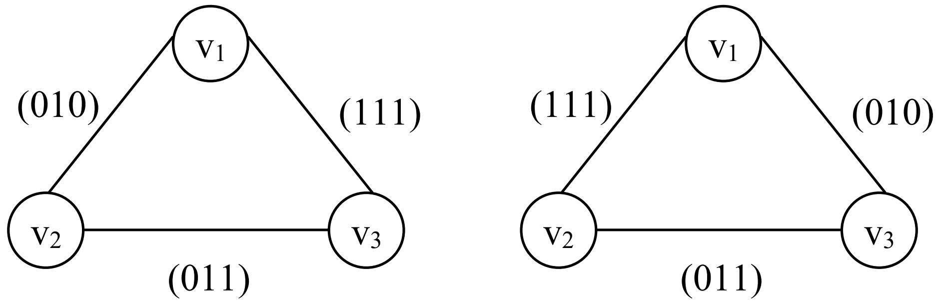
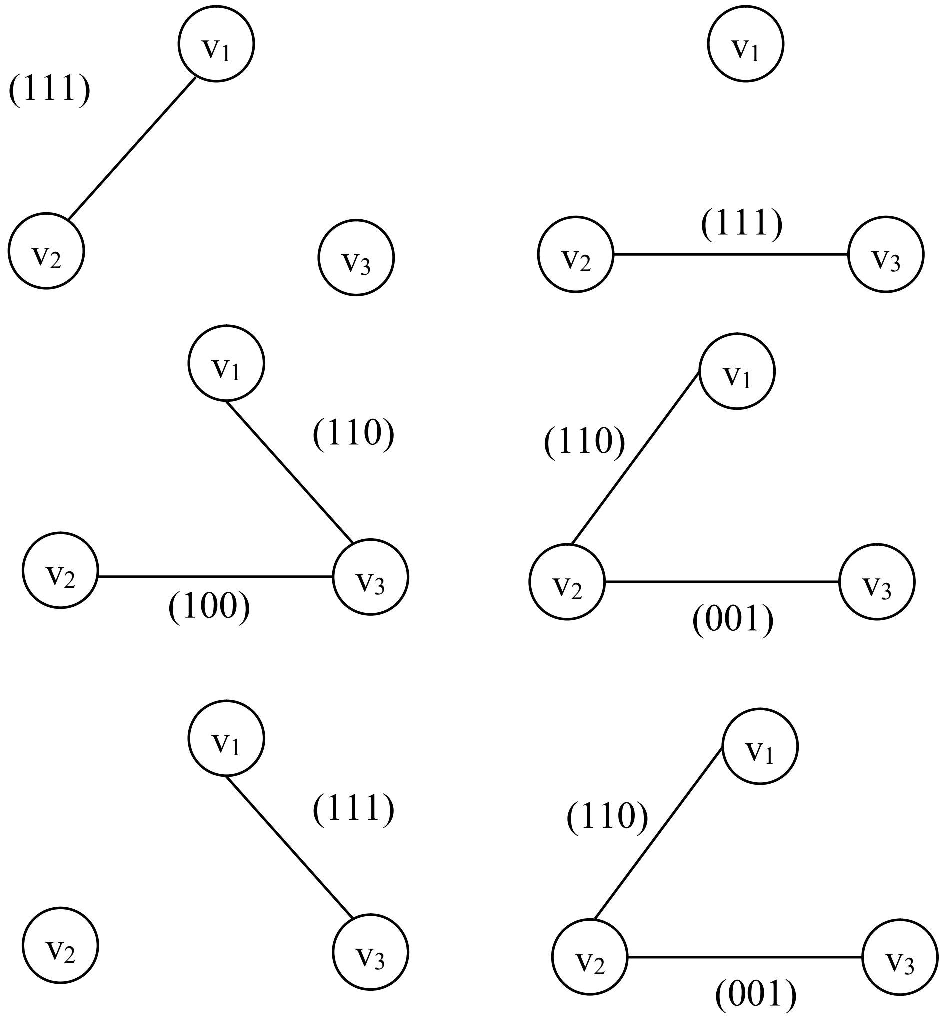


Figure 2.10

and so on. Finding the total number of 1 × 3 Plithogenic crisp triads happens to be a challenging one. Finding total number of 1 × 3 Plithogenic crisp graphs happens to be more challenging than this or we can have the following only a few are supplied.



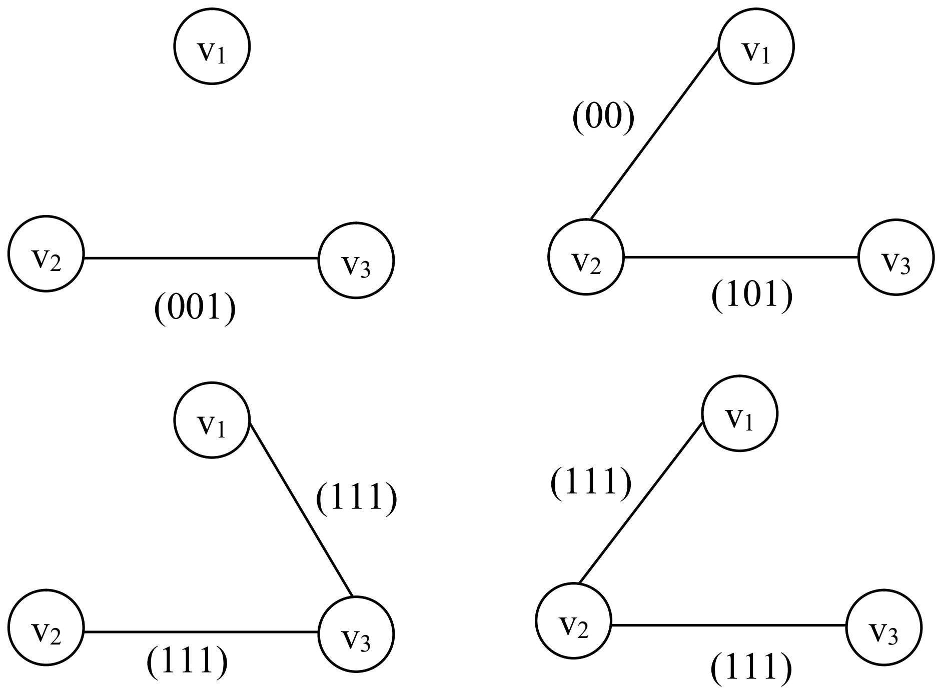


Figure 2.11

and so on.

So, working with 1 × 3 Plithogenic crisp triads itself is challenging, more so if we make rows from 1 × 3 to 1 × n for n a large number and the number of vertices from n to an arbitrarily large value say m.

We will provide a few more examples of them.

Example 2.7: Let G be a 1 × 4 Plithogenic crisp graph given by the following figure.

G =

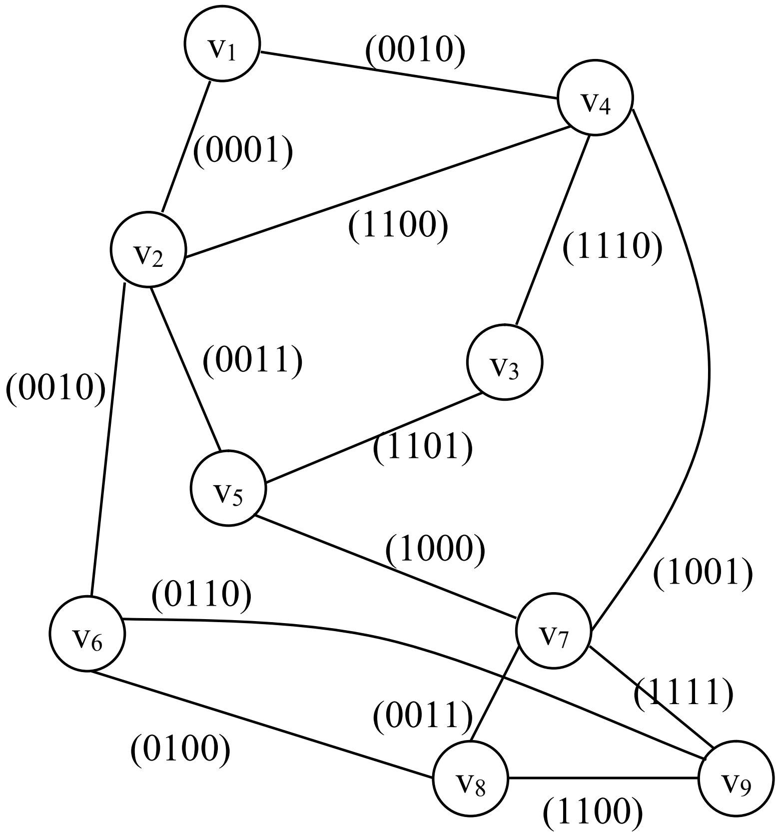


Figure 2.12

We can find the subgraphs of these Plithogenic crisp graph. It is pertinent to keep on record that subgraphs of Plithogenic crisp graphs will be Plithogenic crisp graphs only.

We provide a few examples of the Plithogenic crisp subgraph of the G in example 2.12.

Let H_1 be the Plithogenic crisp subgraph of G given by the following figure.

H_1 =

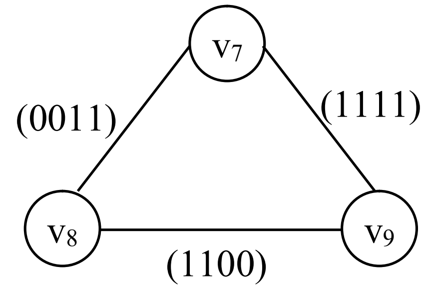


Figure 2.13

Clearly H_1 is a Plithogenic complete triad which is a Plithogenic crisp subgraph.

Let H_2 be a Plithogenic crisp subgraph of G given by the following figure.

H_2 =

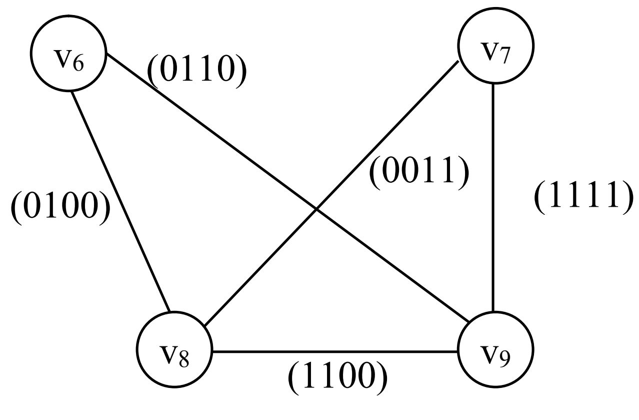


Figure 2.14

Clearly H_2 gives two adjacent Plithogenic crisp triads which is a Plithogenic crisp subgraphs of G. Let H_3 be the Plithogenic crisp subgraph of G given by the following figure.

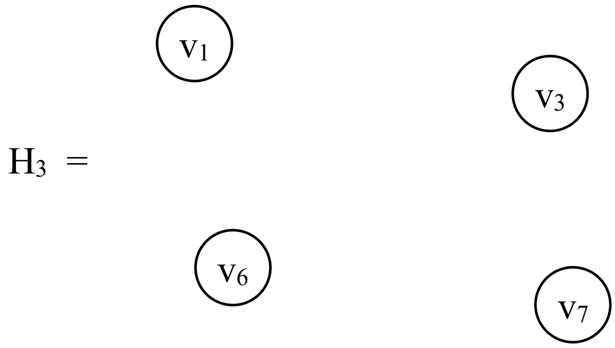


Figure 2.15

H_3 is an empty Plithogenic crisp subgraph of G.

Thus in some cases the Plithogenic crisp subgraph of G can also be an empty subgraph.

Definition 2.2. *Let G be a $1 \times m$ Plithogenic crisp graph. H is a subgraph of G if and only if H is also a $1 \times m$ Plithogenic crisp graph with vertex set being a subset of the vertex set of G the relevant edges remaining the same.*

Next, we proceed onto give examples of Plithogenic fuzzy graphs before we provide the abstract definition of it.

Example 2.8: Let G be a graph given by the following figure;

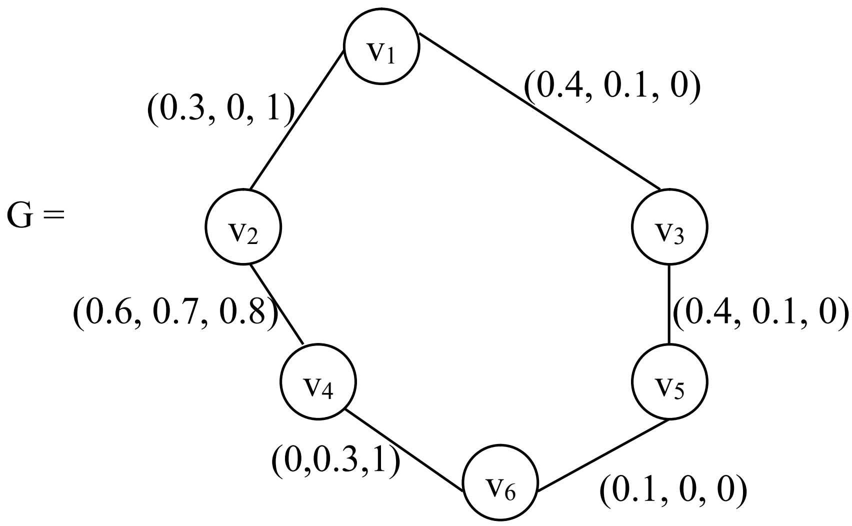


Figure 2.16

We see the weights of the edges are 1 × 3 row matrices with entries from the fuzzy interval [0, 1]. As the rows are fuzzy row vectors we call these graphs as Plithogenic fuzzy graphs.

Clearly G in the figure is a Plithogenic fuzzy circle graphs.

We provide another example of Plithogenic fuzzy graph.

Example 2.9: Let G_1 be a Plithogenic fuzzy graph given by the following figure.

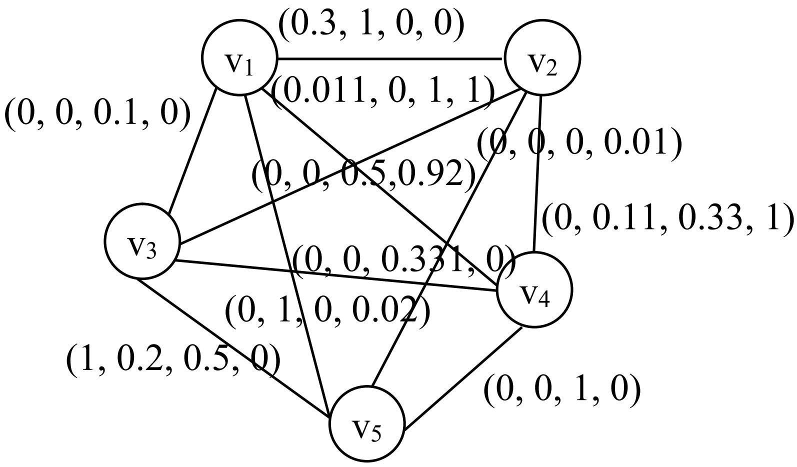
$G_1 =$ 

Figure 2.17

G_1 is a complete Plithogenic fuzzy graph of order 5.

We see all subgraphs of G_1 will be complete and also Plithogenic fuzzy graphs.

We now provide a Plithogenic fuzzy tree by the following example.

Example 2.10: Let G be a Plithogenic fuzzy tree given by the following figure.

G =

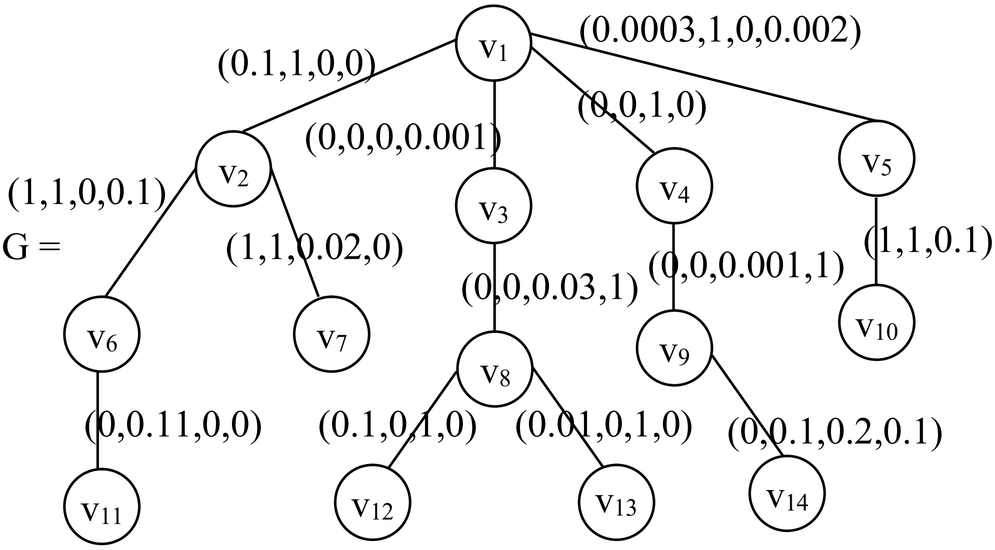


Figure 2.18

Clearly all the subgraphs of G will either be a Plithogenic fuzzy tree or a Plithogenic fuzzy line graph or empty and they will continue to be fuzzy row weighted.

H_1 =

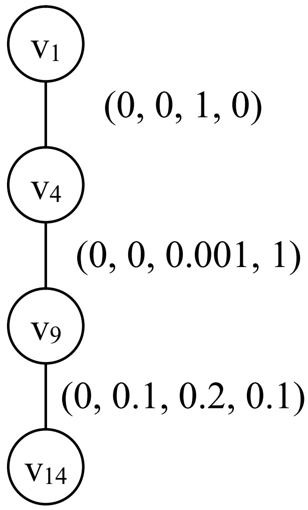


Figure 2.19

H_1 in the above figure is a Plithogenic fuzzy line subgraph of G.

Let H_2 be a Plithogenic fuzzy subgraph of G given by the following figure.

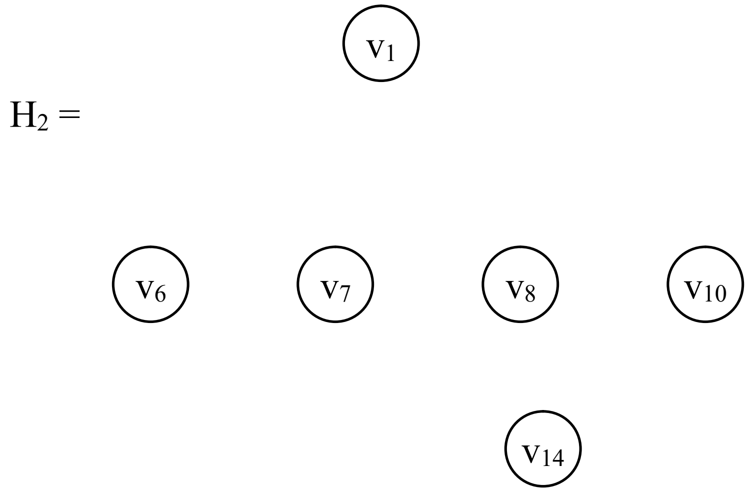


Figure 2.20

Clearly H_2 is a empty Plithogenic fuzzy subgraph of G.

Let H_3 be the Plithogenic fuzzy subgraph of G given by the following figure.

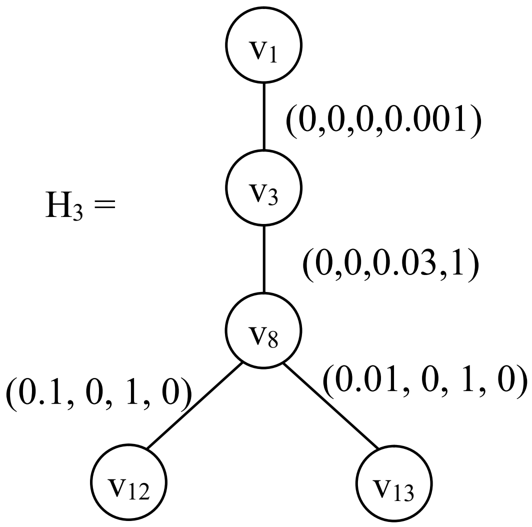


Figure 2.21

Let H_4 be the Plithogenic fuzzy subgraph of G given by the following figure.

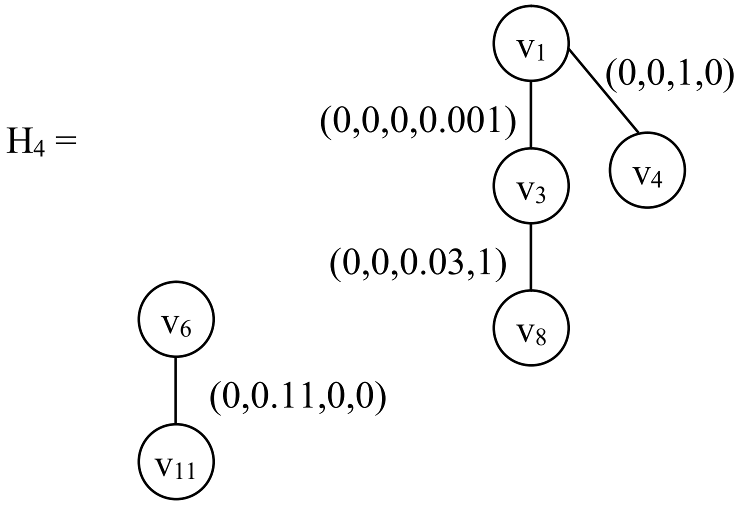


Figure 2.22

Clearly H_4 is a disconnected Plithogenic fuzzy subgraph of G and H_4 has two components one is a tree and other just a dyad. H_4 is in fact a Plithogenic fuzzy subgraph of G.

Let H_5 be the Plithogenic fuzzy graph of G given by the following figure.

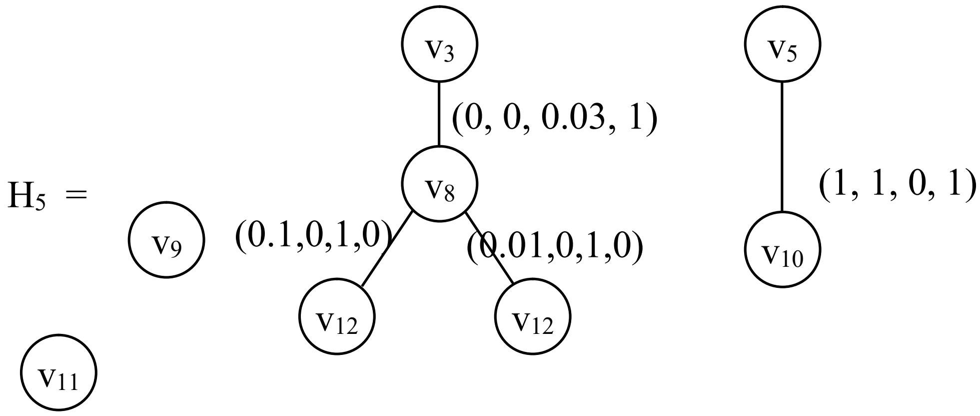


Figure 2.23

Clearly H_5 is a Plithogenic fuzzy subgraph of G which is disconnected and has four components one is a fuzzy row weighted.

One component is tree, one is Plithogenic fuzzy dyad other two are just vertices of G.

Now we make the abstract definition of the Plithogenic fuzzy graph in the following.

Definition 2.3. *Let G be a graph with n vertices say $v_1, v_2, \ldots, v_n$. If the relevant edges are weighted with fuzzy row matrices (fuzzy row matrices if the entries of the row matrix is from [0, 1]). We call G as a Plithogenic fuzzy graph.*

We have provided examples of them also we have given the subgraphs of them.

It is pertinent to keep on record that these Plithogenic fuzzy graphs are distinctly different from fuzzy graphs.

We find the number of Plithogenic fuzzy dyads for a fixed row say a 1×5 row.

We see the number of 1×3 Plithogenic fuzzy graphs are infinite in number for if G is a 1×3 Plithogenic fuzzy dyad given by the following figure.

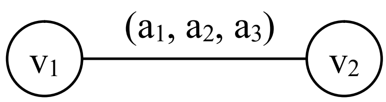
G = 

Figure 2.24

here v_1 and v_2 are the vertices of G and (a_1, a_2, a_3) is a fuzzy row matrix of order 1×3 where $a_i \in [0, 1]$; $1 \leq i \leq 3$.

We see number of 1×3 row matrices with varying values in [0, 1] are infinite in number hence the number of 1×3 Plithogenic fuzzy dyads are also infinite in number.

In fact the number of Plithogenic fuzzy graphs for any graph G with fixed number of vertices and a fixed number of edges is infinite as the number of fuzzy weights that can be given to any fuzzy row weighted graphs are infinite in number.

Just now we proved the result in case of dyads and as dyads are the fundamental units of any graph (that is network) any Plithogenic fuzzy graph for a given configuration is in fact infinite in number.

So, we see in case of Plithogenic fuzzy graphs even the dyads are infinite in number.

But the number of Plithogenic fuzzy subgraphs of a given Plithogenic fuzzy graph is finite in number so is the edge Plithogenic fuzzy subgraphs.

Next we proceed onto describe Plithogenic real graphs by some examples.

Example 2.11: Let G be a Plithogenic graph whose row weights are from the field of reals given by the following figure.

G =

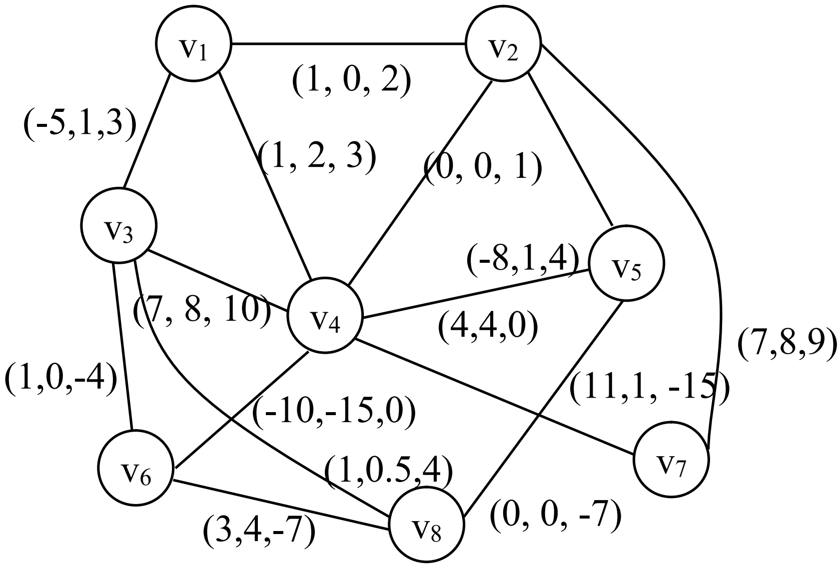


Figure 2.25

We see this Plithogenic real graph takes its row entries from the reals.

We make the abstract definition of the Plithogenic real graph in the following.

Definition 2.4. *Let G be a graph with n vertices. If the edges are weighted with 1 × m row matrices with entries from the real field R we call this graph G as real 1 × m Plithogenic real graph.*

The following observations are mandatory.

i) Every Plithogenic crisp graph is a Plithogenic real graph, however the converse is not true.

ii) Every Plithogenic fuzzy graph is a Plithogenic real graph, but the converse is not true

For both the cases the Plithogenic real graph given in figure is not crisp or not fuzzy, hence the above statements are valid.

It is important to record that the subgraphs of the Plithogenic real graph is again a Plithogenic real graph.

We provide an example or two to this effect.

Example 2.12: Let G be a Plithogenic real graph given by the following figure.

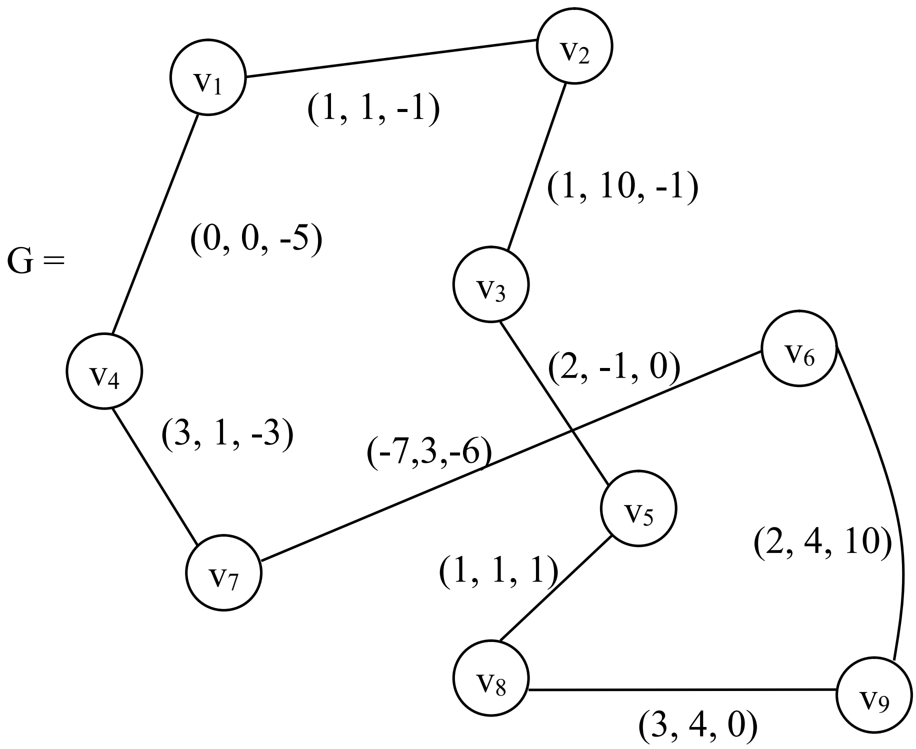


Figure 2.26

We give some of the subgraphs of G in the following.

Let H_1 be a subgraph of G given by the following figure.

H_1 =

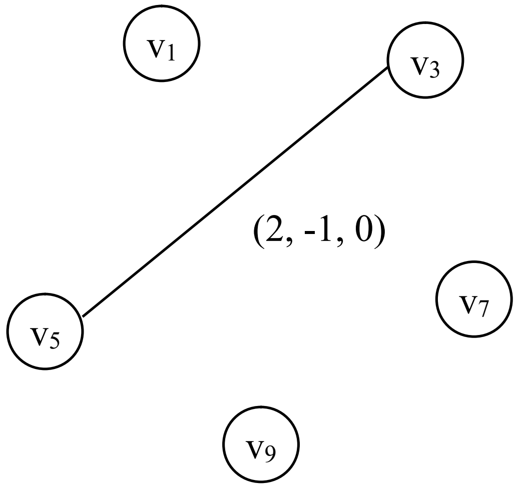


Figure 2.27

Clearly H_1 is a disconnected Plithogenic real subgraph with four components, one is just a dyad and other three are vertex sets.

Let H_2 be the subgraph of G given by the following figure.

H_2 =

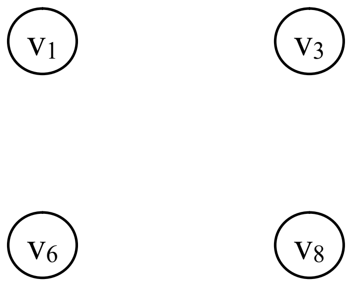


Figure 2.28

Clearly H_2 is a Plithogenic real empty subgraph of G.

Consider H_3 the subgraph of G given by the following figure.

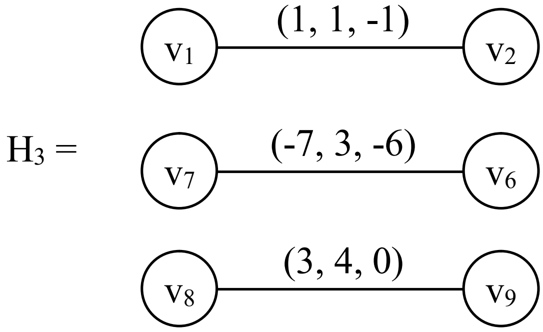


Figure 2.29

Clearly H_3 is a Plithogenic disconnected subgraph of G which has three components and all of them are dyads.

It is interesting to note that G has no clique only dyads of order two.

Now we proceed onto give examples of Plithogenic complex graphs.

Example 2.13: Let G be a graph given by the following figure.

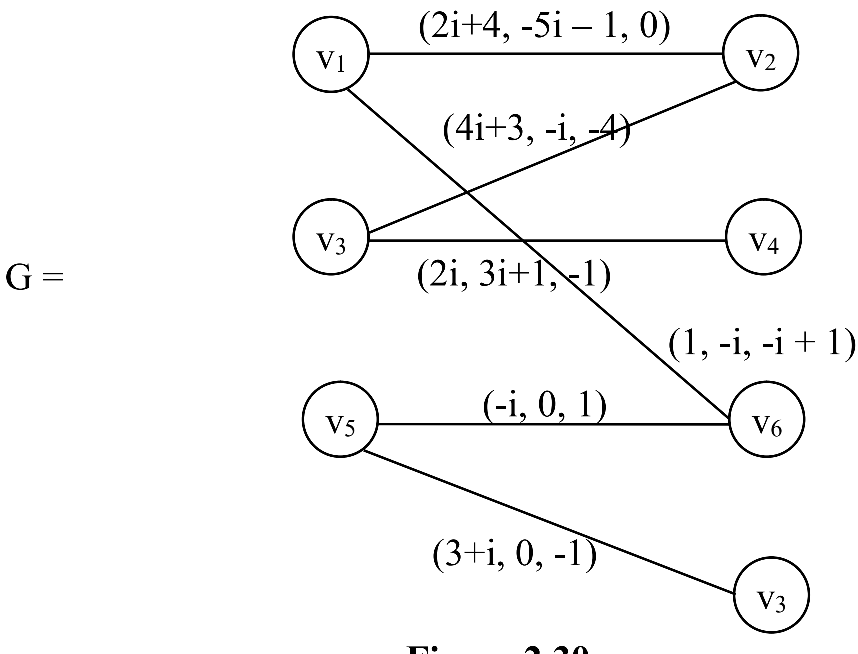


Figure 2.30

G is a Plithogenic complex graph. Clearly G is a Plithogenic complex bigraph.

Example 2.14: Let G be a Plithogenic complex star graph given by the following figure.

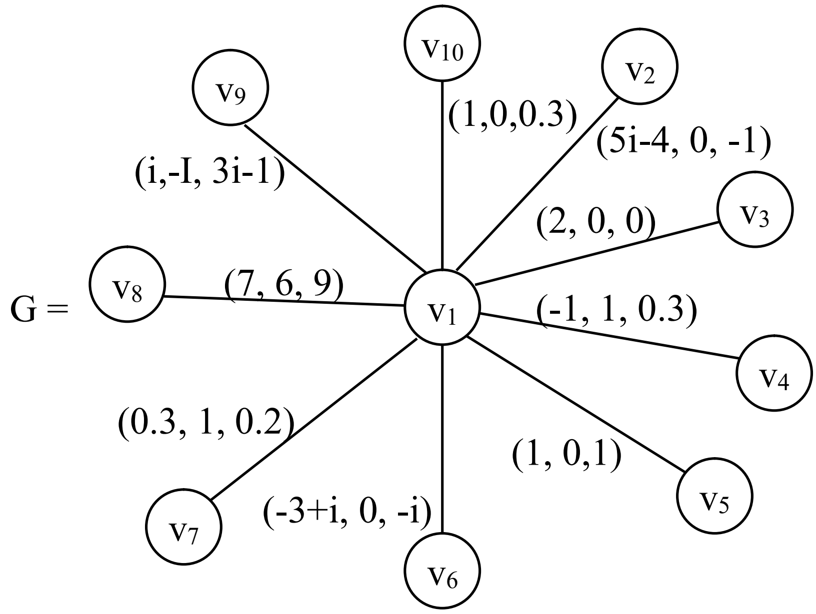


Figure 2.31

G is a egocentric Plithogenic complex star graph.

We see some of the persons are related with imaginary row weight. So, when studying egocentric graph if we use complex (imaginary) values one can understand some characters who are related are admiration or attractive is more imaginary than realistic.

Next, we provide one more example of some complex row weighted graph in the following.

Example 2.15: Let G be a Plithogenic complex graph given by the following figure. We also give a few of its subgraphs.

G =

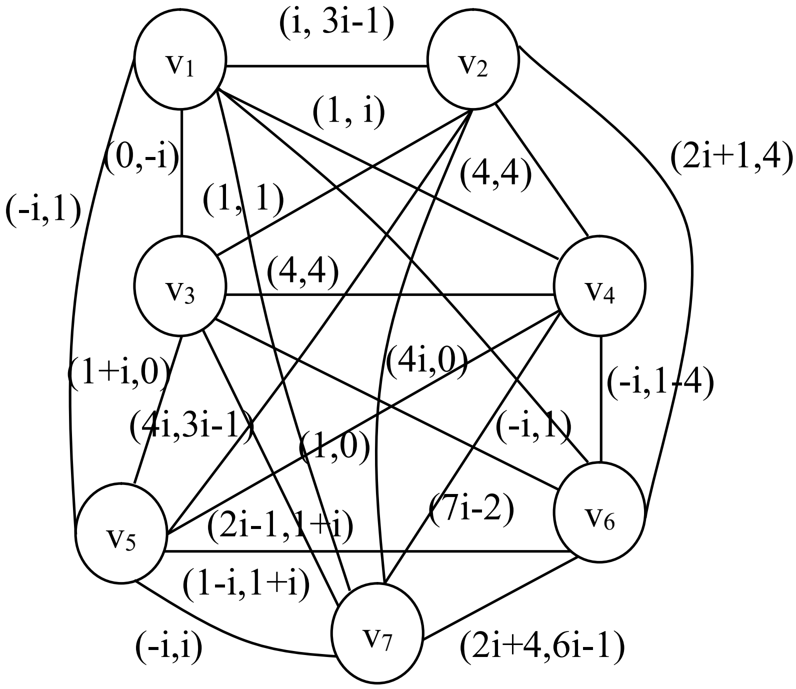


Figure 2.32

We see all subgraphs of G are complete as basically G is a Plithogenic complete graph of order 7.

We see there are several Plithogenic complete subgraphs in fact there are $7C_2 + 7C_3 + \ldots + 7C_6$ such Plithogenic complete subgraphs in G.

Like usual graphs we can find the number of edge Plithogenic subgraphs of G.

We now give the abstract definition of Plithogenic complex graph in the following.

Definition 2.5. *Let G be a graph with n-vertices if the relevant edges are weighted by $1 \times m$ row complex valued matrices then we define G as the Plithogenic complex graph.*

We have provided examples of them.

Now we proceed onto describe Plithogenic Neutrosophic graphs by some examples.

Example 2.16: Let G be a Plithogenic Neutrosophic graph given by the following figure.

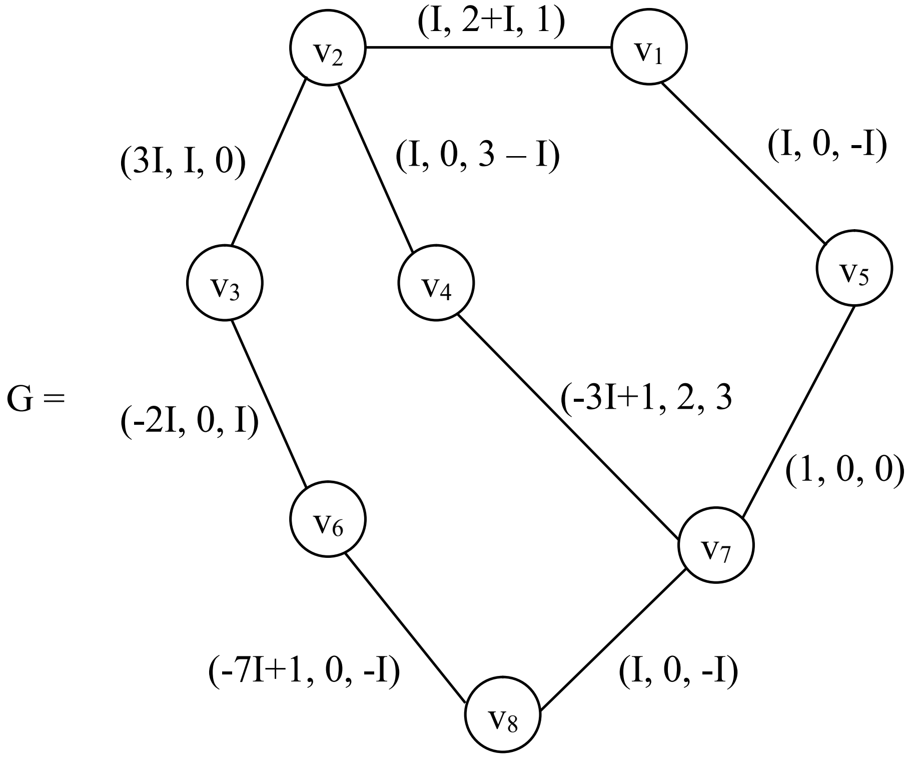


Figure 2.33

We find some of the subgraphs of G.

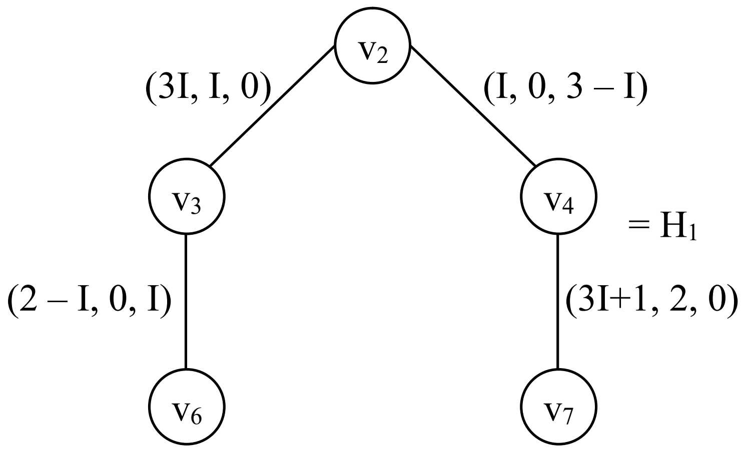


Figure 2.33a

H_1 is a Plithogenic neutrosophic subgraph of G which is a tree.

We see this G has no clique. In fact G has no nontrivial Plithogenic neutrosophic \subgraph which is complete. Even G has no triads, that is complete graph of order three. H_1 is a tree.

We give yet another example of a Plithogenic neutrosophic graph.

Example 2.17: Let G be a Plithogenic neutrosophic graph which is given by the following figure.

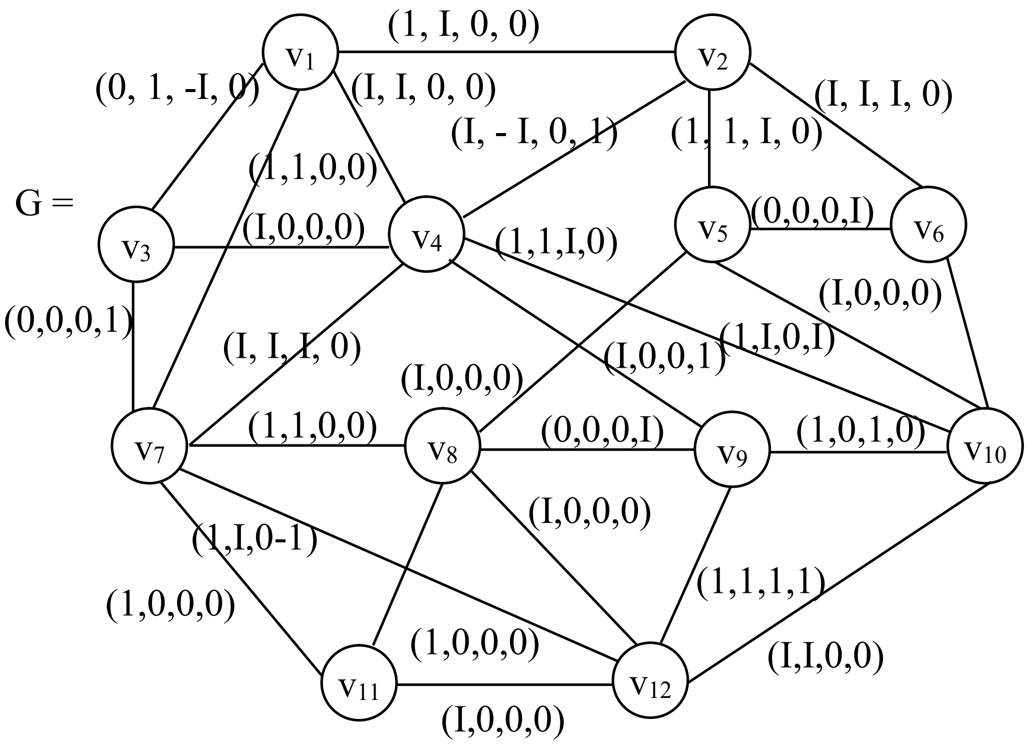


Figure 2.34

We can find subgraphs of this Plithogenic neutrosophic graph G. This task is left for the reader.

Now we can likewise define the notion of Plithogenic dual numbers graphs, Plithogenic complex neutrosophic numbers graphs, Plithogenic special quasi dual number graphs and so on.

We see all these Plithogenic graphs are infinite in number.

These situations will be represented by some examples.

Example 2.18: Let G be a Plithogenic dual number graph with row weights from $\langle R \cup g \rangle = \{a + bg \mid a, b \in R \text{ and } g^2 = 0\}$ given by the following figure.

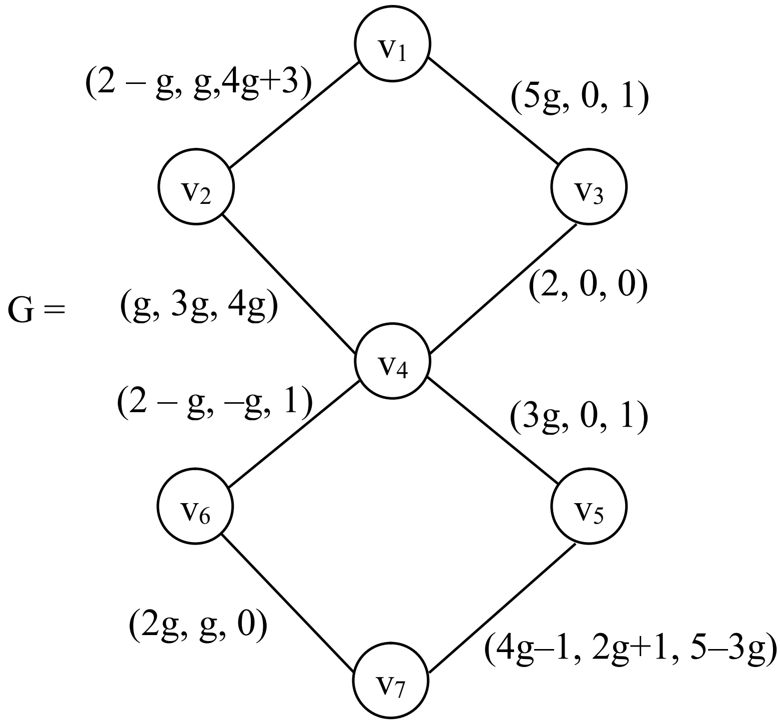


Figure 2.35

Clearly G is a dual number row weighted graph of order 7.

We can have subgraphs of G which are also dual number row weighted subgraphs of G.

Next we give one example of the Plithogenic special quasi dual number graph.

Example 2.19: Let G be a Plithogenic graph with row weights from the set $\langle R \cup h\rangle = \{a + bh / a, b \in R, h^2 = -h\}$. G will be known as the 1×4 Plithogenic special quasi dual number graph with m vertices.

Here we take n = 5 and m = 7, so G is a 1×5 Plithogenic special quasi dual number graph given by the following figure.

G =

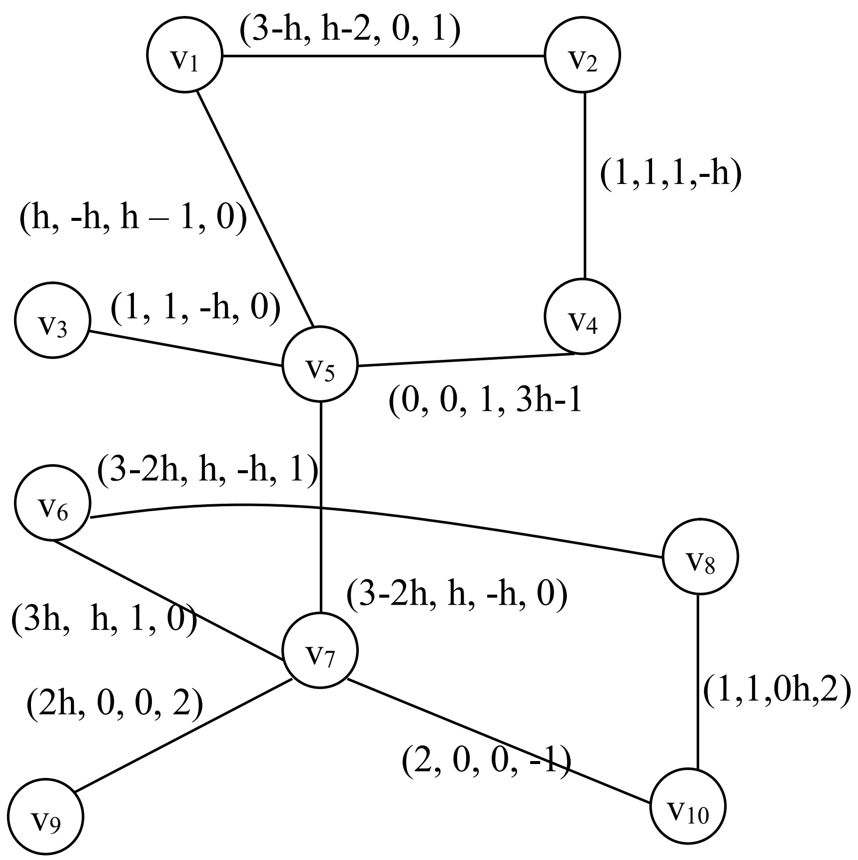


Figure 2.36

G is a Plithogenic 1×4 special quasi dual graph with 10 vertices.

This has several subgraphs all of which are 1×4 Plithogenic special quasi dual graphs.

It is important to revoke all these Plithogenic graphs are infinite in number for a given set of vertex sets.

We can have finite such collection if we use the finite complex modulo integers, mod integers, dual mod integers neutrosophic mod integers and so on. We will illustrate this situation by some examples.

Example 2.20: Let G be a Plithogenic graph given by the following figure.

G =

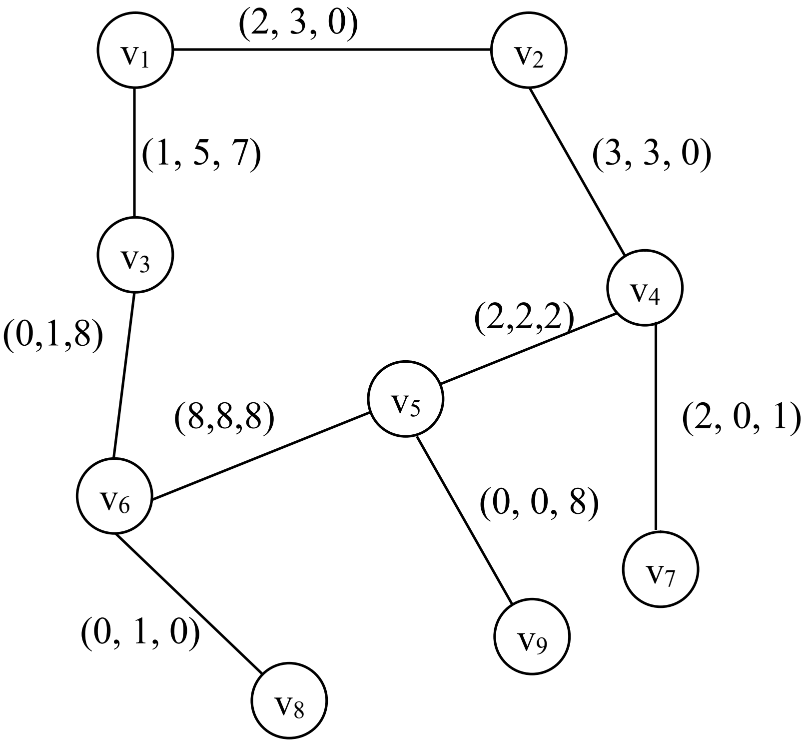


Figure 2.37

The row entries are from Z_9.

The number of 1×3 row elements is finite.

If the row weights are taken from Z_9 we see for a given set of n vertices the number of graphs with a fixed number of row order say $1 \times m$ we have only a finite collection of Plithogenic special modulo integer graphs.

We will give a few more examples.

Example 2.21: Let G be the Plithogenic modulo integer special graph given by the following figure.

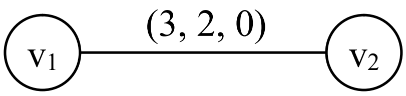


Figure 2.38

where $(3, 2, 0) \in Z_4 \times Z_4 \times Z_4$

We can have 63 such graphs for the edges weights and all the 63 of them are different. To be more precise we can say that there are 63 dyads with row weights from $Z_4 \times Z_4 \times Z_4$.

Thus we can say if the row weights are taken from $Z_n \times Z_n \times Z_n$ we can have $(n^3 - 1)$ number of distinct dyads.

Consider G_1 to be a triad given by the following figure.

$G_1 =$

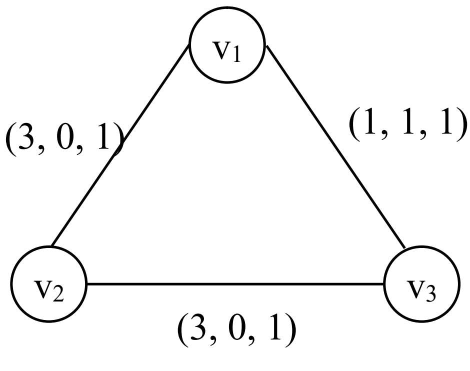


Figure 2.39

where G_1 is a Plithogenic triad and the row edge weights (1, 1, 1) and (3, 0, 1) $\in Z_4 \times Z_4 \times Z_4 = S$.

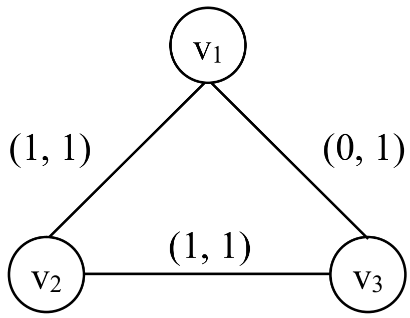
There are 63 × 63 × 63 number of triads with row weights taken from S.

Suppose we replace Z_4 by Z_2 we will have 7 × 7 × 7 number of distinct triads with row weights from $Z_2 \times Z_2 \times Z_2$.

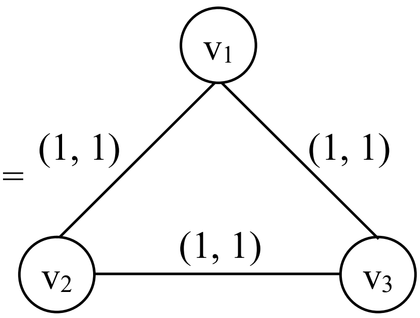
We provide yet another example.

Example 2.22: Suppose G_i's are Plithogenic triads with row weights from $Z_2 \times Z_2$; how many such triads exist

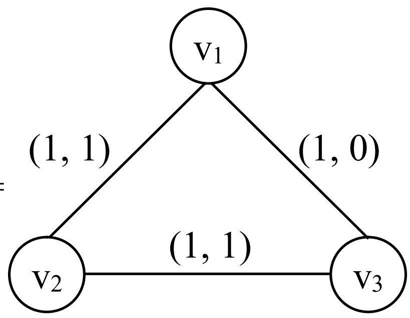
G_1 =



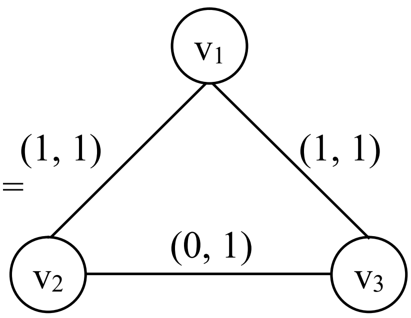
G_2 =



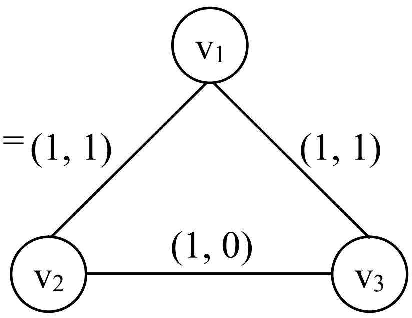
G_3 =



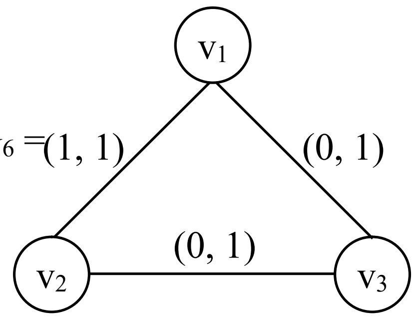
G_4 =

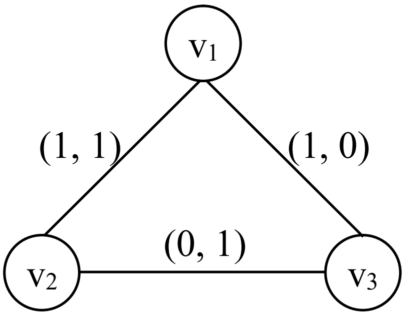


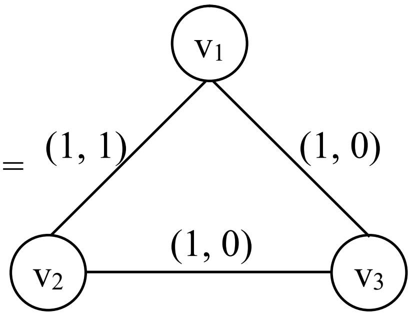
G_5 =

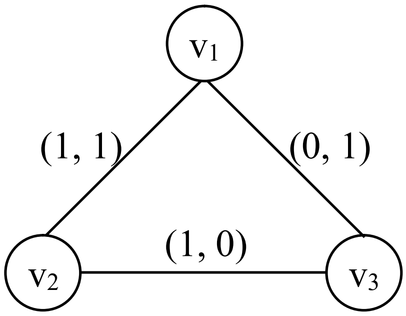


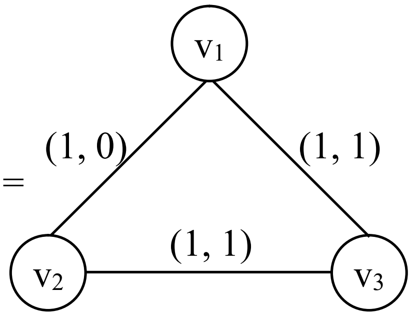
G_6 =

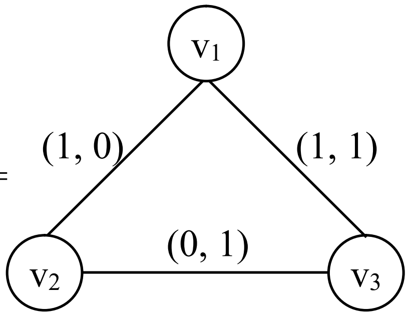


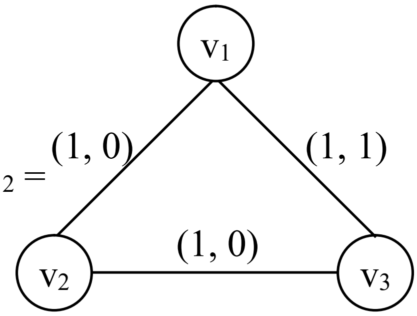
$G_7 =$


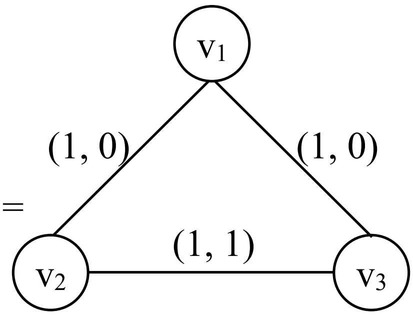
$G_8 =$


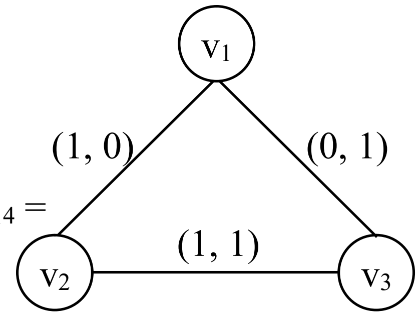
$G_9 =$


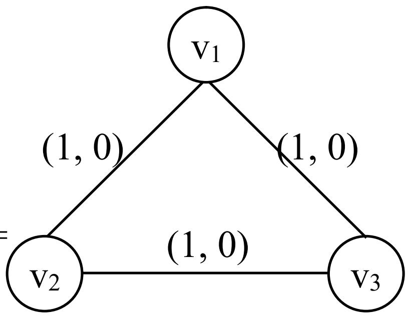
$G_{10} =$


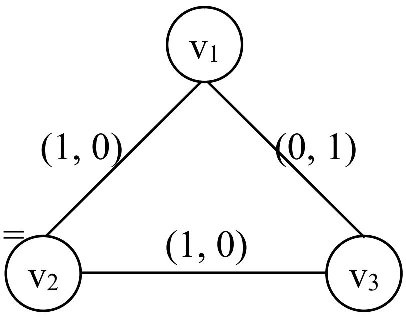
$G_{11} =$


$G_{12} =$


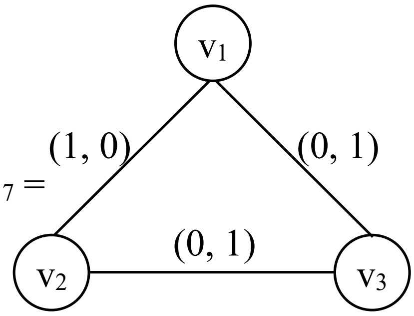
$G_{13} =$


$G_{14} =$


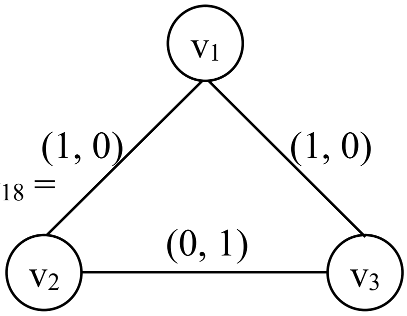
$G_{15} =$


$G_{16} =$


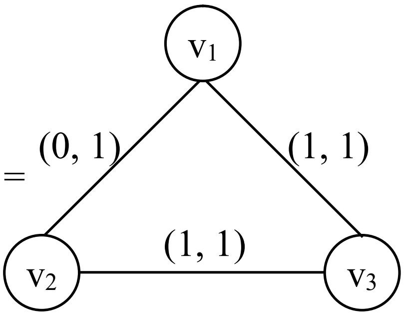
G_{17} =



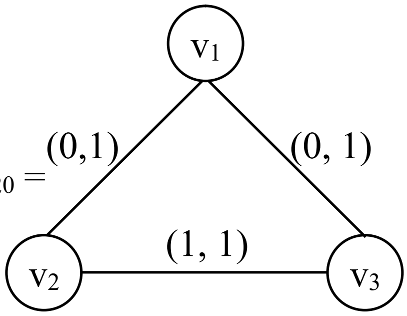
G_{18} =



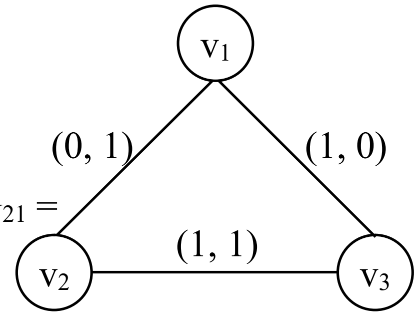
G_{19} =



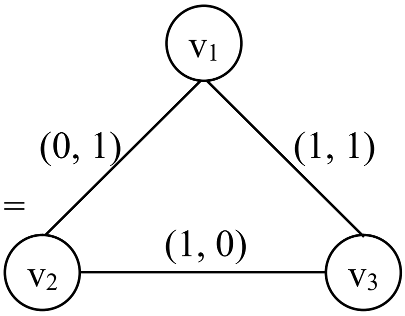
G_{20} =



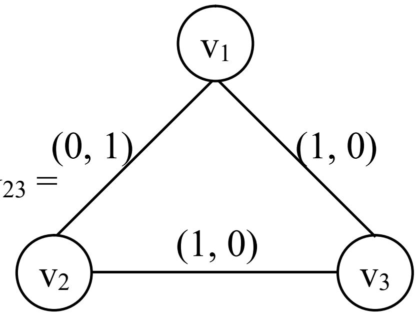
G_{21} =



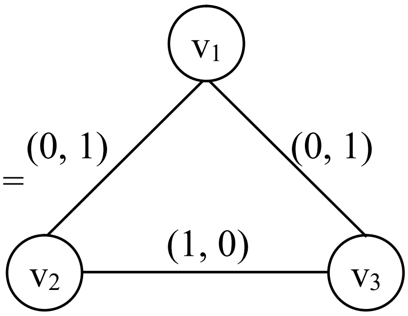
G_{22} =



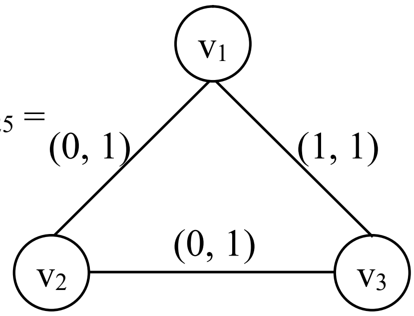
G_{23} =



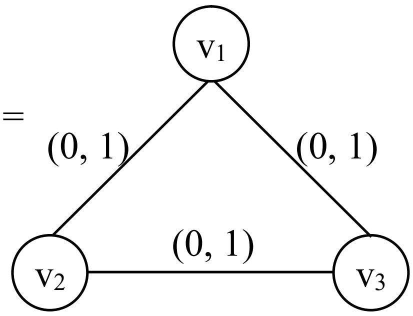
G_{24} =



G_{25} =



G_{26} =



G_{27} =

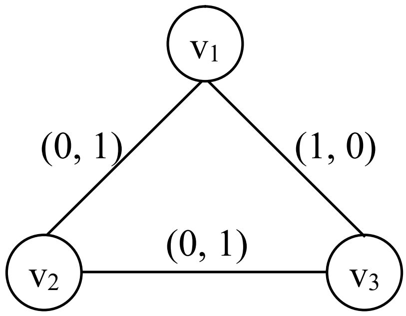


Figure 2.40

It is left as an exercise for the reader to find using the $Z_2 \times Z_2$ as edge weights. Study of finding the number of distinct Plithogenic triads for any n-row vector with entries from $Z_m \times Z_m \times \ldots \times Z_m$ is an interesting as an exercise and left for the reader.

Thus row weighted modulo integer special matrices yield a very different large number of Plithogenic graphs but a finite order as we have taken the row vectors from Z_m's $2 \leq m < \infty$ the ring / field of modulo integers.

We can derive many interesting results in this direction.

We will provide some more examples of them.

Example 2.23: Let G be a Plithogenic modulo integer special graph where the 1×3 row values are from $Z_5 \times Z_5 \times Z_5$ given by the following figure.

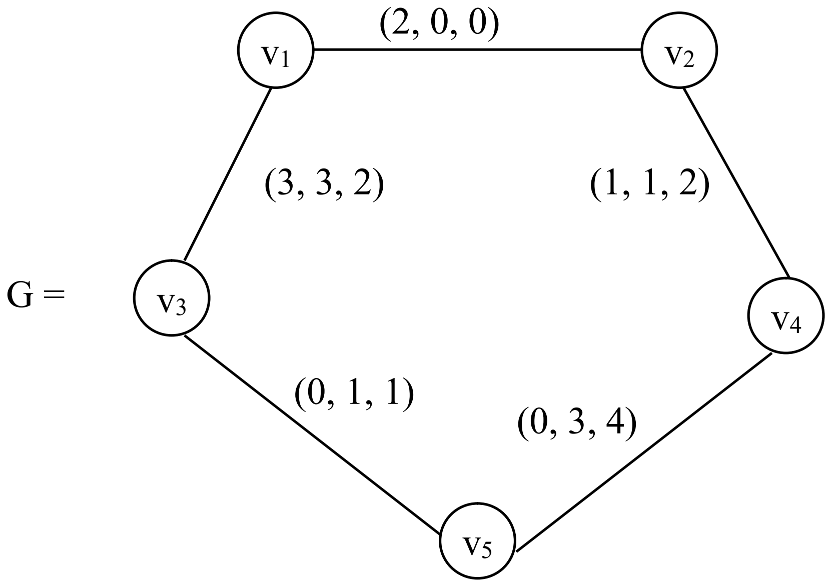


Figure 2.41

For the given 5 vertices there can be several 1×3 Plithogenic modulo integer special ring graphs. Finding the total number of them even in case of Z_5 happens to be a challenging problem.

Example 2.24: Let G be a 1×2 Plithogenic modulo integer special complete graph of order six with row vectors from $Z_9 \times Z_9$ given by the following figure.

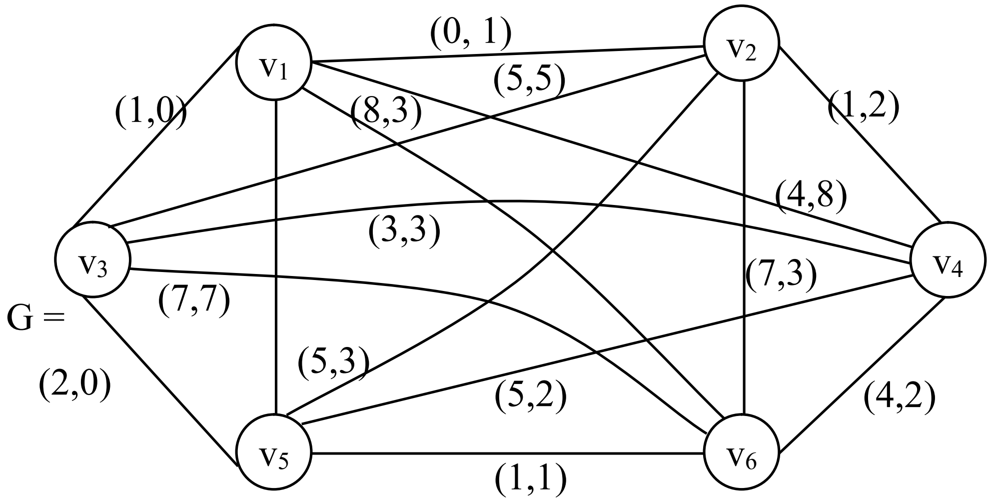


Figure 2.42

Finding the total number of 1×2 Plithogenic modulo integer special complete graphs of order six with edge weights from $Z_9 \times Z_9$ happens to be a challenging one.

Example 2.25: Let G be a Plithogenic modulo integer special star graph of order 9 given by the following figure.

G =

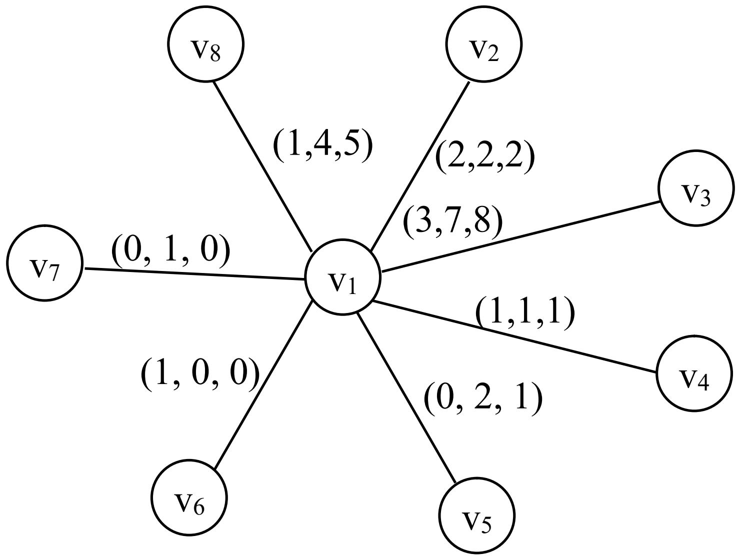


Figure 2.43

The row vectors are from $S = Z_{18} \times Z_{18} \times Z_{18}$.

We see finding the total number of 1×3 Plithogenic special star graphs with row weights from S happens to be a challenging problem.

In view of all these we propose the following problems.

Problem 2.1. Let G be a $1 \times n$ Plithogenic modulo integer special complete graph with s vertices. The row vectors are from $(Z_m)^n$. $2 \leq n, s, m < \infty$.

Find the number of such $1 \times n$ Plithogenic special complete graphs (for $s \neq 2$ or 3).

Problem 2.2. Let H be a Plithogenic modulo integer special ($1 \times n$ row vectors) star graph with s vertices. The row weights are from

$$\underbrace{Z_m \times Z_m \times \ldots \times Z_m}_{n-\text{times}}.$$

Find the total number of such special star graphs.

Prove the number of such Plithogenic star graphs are less than the number of Plithogenic complete graphs with same number of vertices for both problems.

Problem 2.3. Let K be a $1 \times s$ Plithogenic modulo integer special ring graph with n vertices.

The $1 \times s$ row vectors are taken from $(Z_m)^s$, $2 \leq n, m, s < \infty$.

a) Find the number of such ring graphs for the fixed number of vertices.

b) Compare for the fixed number of vertices as n and $1 \times s$ row vectors from $(Z_m)^s$ with complete graphs and star graphs and prove complete graphs are more in number than star graphs and ring graphs and star graphs happens to be of least number.

Now on similar lines we can define and develop Plithogenic modulo integer finite complex graphs using $C(Z_m)$, m a finite positive number.

We will illustrate this situation by some examples.

Example 2.26: Let G be a finite 1 × 3 Plithogenic complex special graph given by the following figure. The row vectors are from $C(Z_9) \times C(Z_9) \times C(Z_9)$.

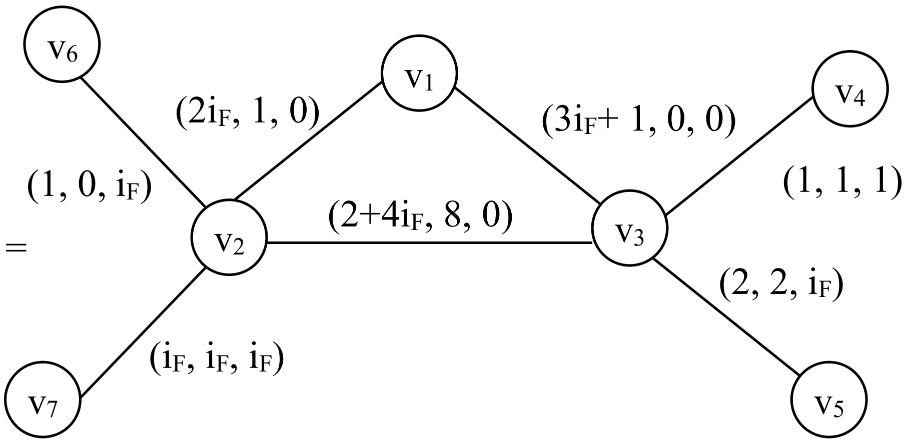


Figure 2.44

We see G is a 1 × 3 Plithogenic finite complex modulo integer graph with 7 vertices.

It is interesting to note that G can have subgraphs which are real.

For instance, this G has only

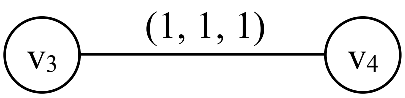


Figure 2.45

one dyad which is real modulo integer valued one.

Example 2.27: Let G be a finite 1 × 4 Plithogenic complex special graph with 5 vertices with row entries from $C(Z_6) \times C(Z_6) \times C(Z_6) \times C(Z_6)$ given by the following figure.

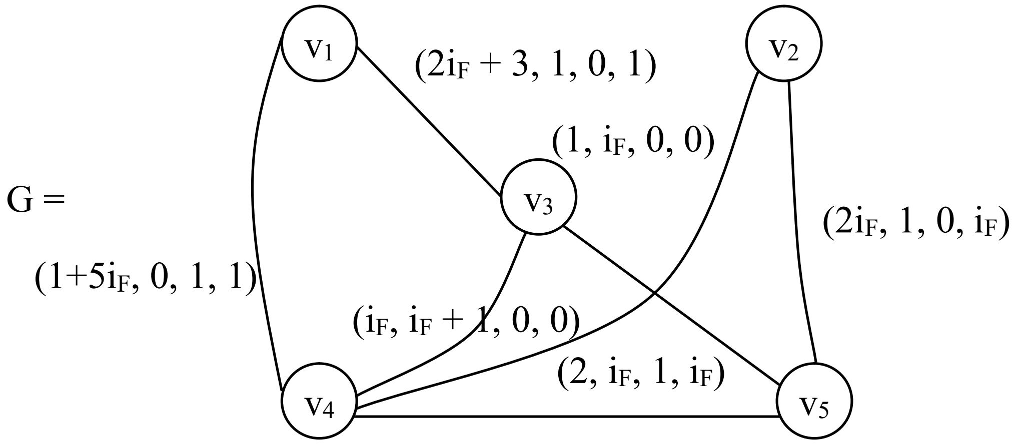


Figure 2.46

We see all subgraphs of G are finite Plithogenic complex special graphs. Next we proceed onto describe Plithogenic neutrosophic modulo integer special graphs by examples.

Example 2.28: Let G be a Plithogenic neutrosophic modulo integer graph given by the following figure. The row elements are taken from $\langle Z_7 \cup I\rangle \times \langle Z_7 \cup I\rangle \times \langle Z_7 \cup I\rangle \times \langle Z_7 \cup I\rangle = S$ where $\langle Z_7 \cup I\rangle = \{a + bI/a, b \in Z_7, I^2 = I\}$.

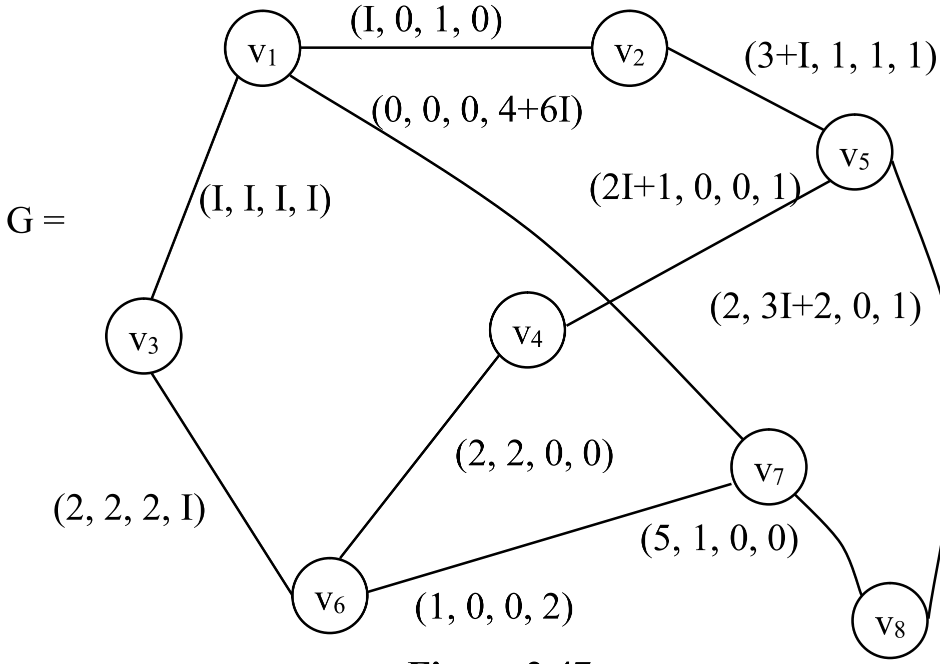


Figure 2.47

G has several Plithogenic neutrosophic modulo integer row valued subgraphs.

We give a few subgraphs of G. Let H_1 be a subgraph given by the following figure.

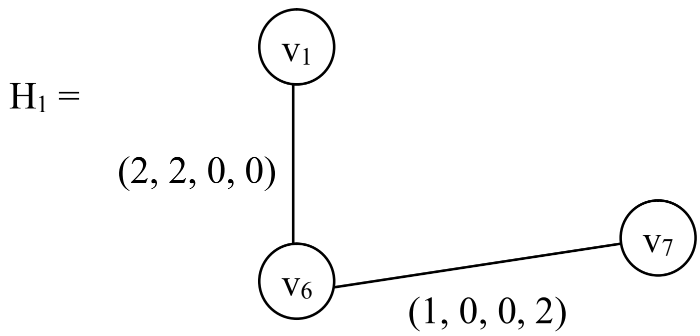


Figure 2.48

Clearly H_1 is Plithogenic real subgraph of G.

Let H_2 be a subgraph given by the following figure.

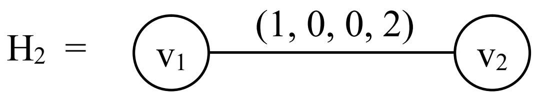


Figure 2.49

Let H_3 be the subgraph of G given by the following figure.

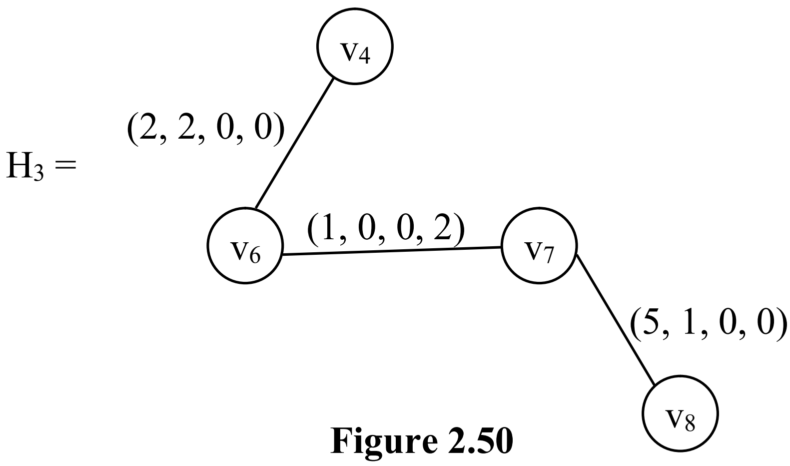


Figure 2.50

We see H_3 is also a real subgraph of G but H_3 is also real.

We will be defining the notion of real hyper subgraphs of these types of Plithogenic special modulo integer graphs.

Let H_4 be a Plithogenic special subgraph of G given b y the following figure.

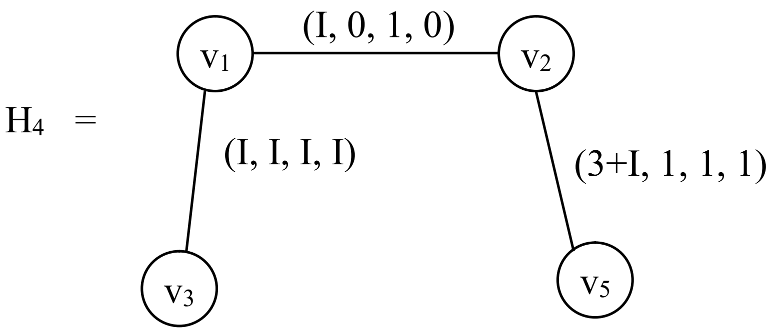


Figure 2.51

We see H_4 is a Plithogenic neutrosophic special subgraph of G which has no real subgraph.

This type of subgraph will also be distinctly defined in the following sections.

Consider H_5 is subgraph of G given by the following figure.

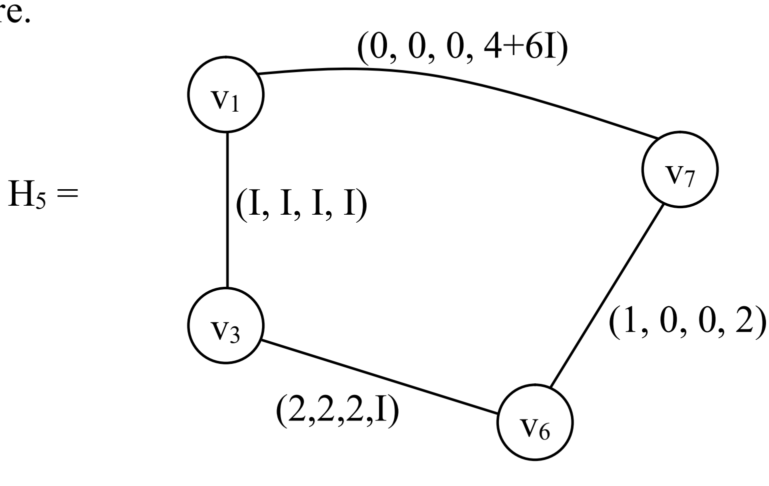


Figure 2.52

Clearly H_5 is a Plithogenic neutrosophic modulo integer special subgraph of G different from H_1, H_2, H_3 and H_4. Further H_5 is ring subgraph.

We see using these properties we define special substructures of these subgraphs.

We can define on similar lines the notion of Plithogenic complex neutrosophic modulo integer graphs. $(C(\langle Z_n \cup I\rangle) = \{a + bI + Ci_F + dIi_F / a, b, c, d \in Z_n, I^2 = I, i_F^2 = (n-1), (Ii_F)^2 = (n-1) I; 2 \leq n < \infty\}$ where the row values are taken from $C(\langle Z_n \cup I\rangle)$.

We will first illustrate this situation by some examples.

Example 2.29: Let G be a 1×3 Plithogenic complex neutrosophic modulo integer graph given by the following figure.

G =

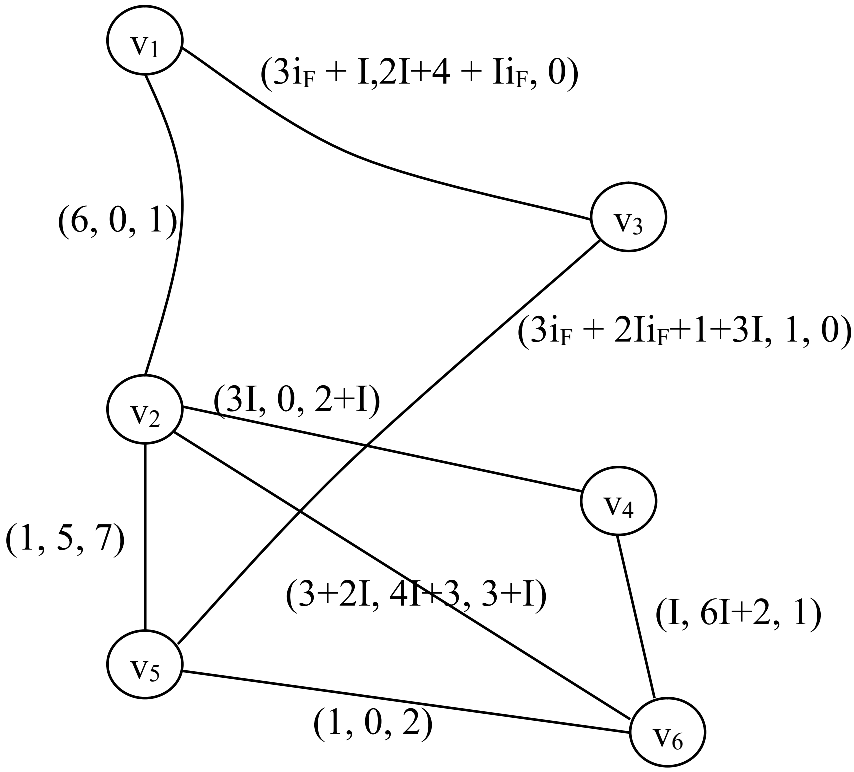


Figure 2.53

The row vectors are from $C(\langle Z_9 \cup I\rangle) \times C(\langle Z_9 \cup I\rangle) \times C(\langle Z_9 \cup I\rangle)$. Consider the following subgraphs of G.

K_1 =

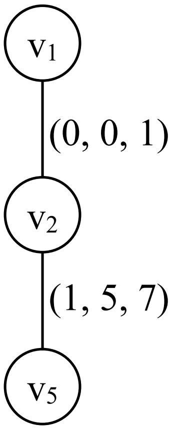


Figure 2.54

Clearly K_1 is just a Plithogenic modulo integer subgraph of G as none of the edges of K_1 has neutrosophic and (or) complex valued row valued matrix.

Let K_2 be the subgraph of G given by the following figure.

K_2 =

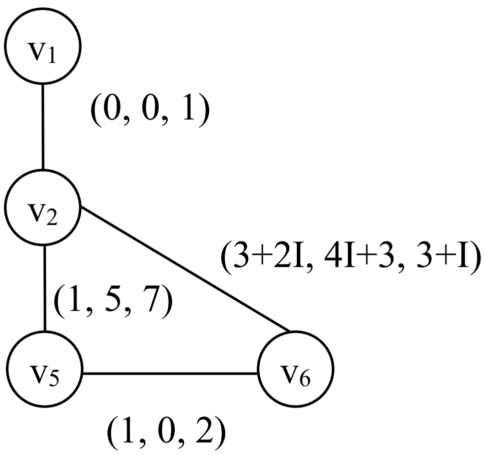


Figure 2.55

When we try to take all vertices of G which has real row valued edges then K_2 has also an edge in this case which is neutrosophic. So K_2 is not real modulo integer edges however K_1 is a subgraph which is a real modulo integer edge but it does not take into account all edges but if all edges are to be taken into account it fails to be real.

However we have subgraphs of G which has all edges to be neutrosophic. Consider K_3 the subgraph of G given by the following figure.

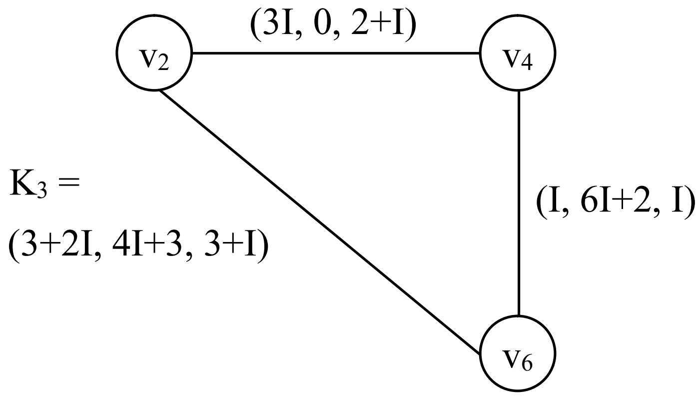


Figure 2.56

K_3 is a Plithogenic neutrosophic modulo integer special subgraph of G.

All the 3 edges are neutrosophic.

We list out some of the special features associated with subgraphs of Plithogenic neutrosophic modulo integer special graphs.

1. We can have subgraphs whose edges are only mod real valued row matrices.

2. We can have subgraphs all of whose edges are mod neutrosophic row valued matrices.

3. We can have subgraphs which has both real and mod neutrosophic row valued matrices.

Now how to specify the existence of largest ones in case of (1) and (2).

In case we get a Plithogenic subgraph which includes all real modulo row valued edges and no neutrosophic valued edge then we call such Plithogenic subgraphs as Plithogenic hyper pseudo neutrosophic real subgraphs.

Given a Plithogenic neutrosophic modulo integer graph in general we cannot always say a hyper pseudo neutrosophic real subgraphs exist or not. The reality is it may exist or it may not exist.

In the example we are discussing about such Plithogenic hyper pseudo neutrosophic real subgraph does not exist.

Similarly, we call the subgraph of a neutrosophic graph to be a Plithogenic hyper neutrosophic subgraph if all the row matrix edges take neutrosophic values and all those neutrosophic edges of G are also present in the subgraph. Such situations may occur or may not occur. It is pertinent to keep on record that hyper subgraphs if they exist are unique unlike hyper planes of a plane in vector spaces.

Further it is important to record that these notions of hyper subgraphs used in this book is very different from the classical hyper subgraphs that are defined.

We see in the above-mentioned example we do not have either Plithogenic hyper neutrosophic subgraphs or Plithogenic hyper pseudo neutrosophic real subgraphs. So these are Plithogenic neutrosophic graph which has no hyper subgraphs of both the types mentioned.

Finally, we wish to add on that if the Plithogenic neutrosophic graph has no real edges then it cannot contain Plithogenic hyper pseudo neutrosophic subgraphs. In this case we also do not talk about Plithogenic hyper neutrosophic subgraphs.

Example 2.30: Let G be a Plithogenic neutrosophic graph given by the following figure.

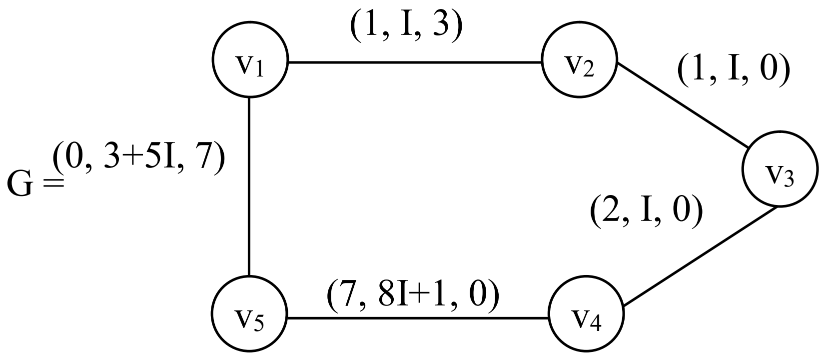


Figure 2.57

G has no hyper subgraph of both types. We call such Plithogenic neutrosophic graphs as simple.

Definition 2.6. *G be a Plithogenic neutrosophic graph. We say G is simple if G has no Plithogenic neutrosophic hyper subgraph and no Plithogenic pseudo neutrosophic real subgraph.*

We have already given example of them. We will give some more examples before we proceed on to get some special properties about these simple neutrosophic graphs.

Example 2.31: Let G be a Plithogenic neutrosophic graph given by the following figure.

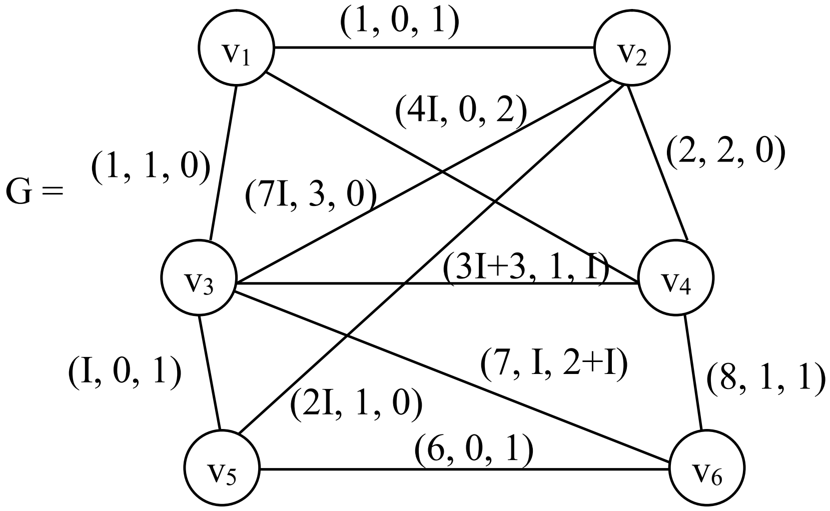


Figure 2.58

We can get only dyads to be either Plithogenic neutrosophic subgraphs or Plithogenic real pseudo neutrosophic graphs.

We provide some of them.

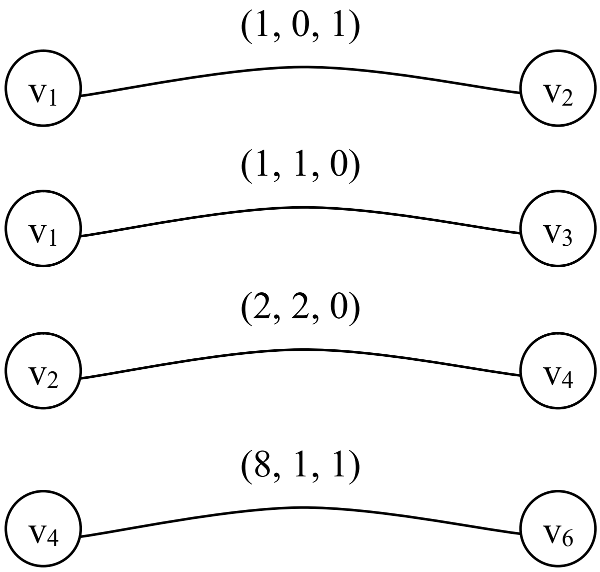


Figure 2.59

Likewise, we can have some forbidden triads which are Plithogenic neutrosophic subgraphs given by the following figures.

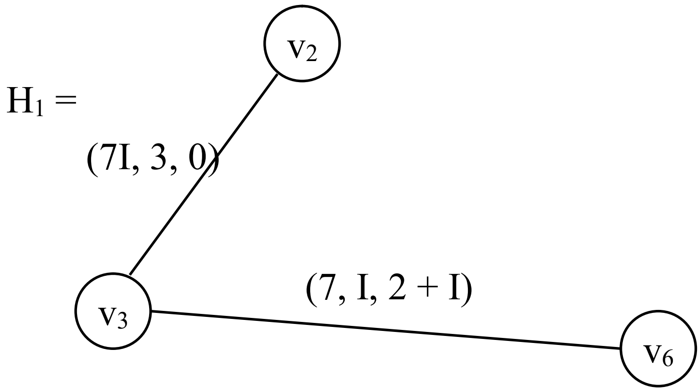


Figure 2.60

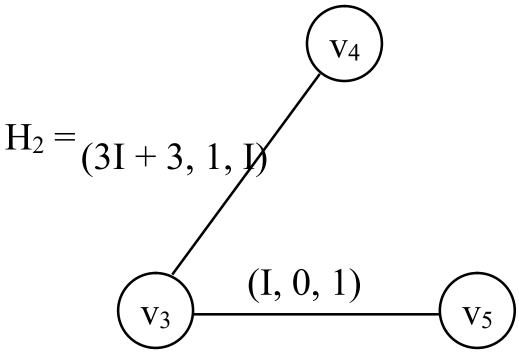


Figure 2.61

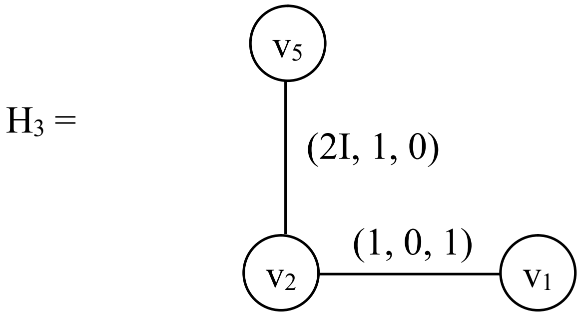


Figure 2.62

and so on.

We see this G is simple as it has no Plithogenic hyper neutrosophic subgraph or Plithogenic hyper pseudo neutrosophic real subgraph.

We provide yet another example.

Example 2.32: Let G be a Plithogenic neutrosophic graph given by the following figure.

G =

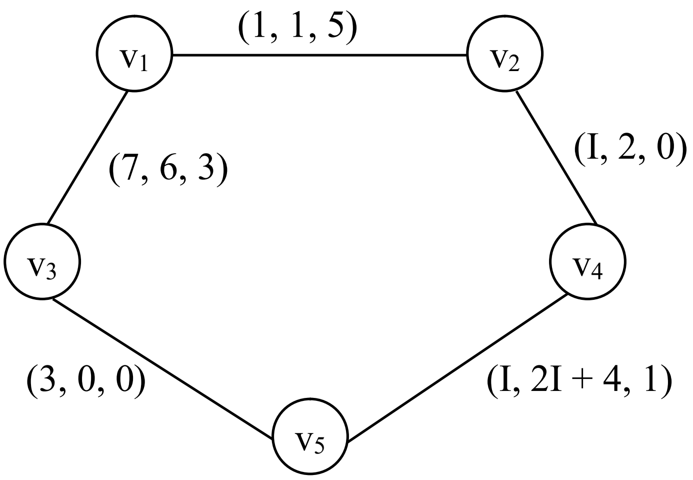


Figure 2.63

We see G is not simple for the Plithogenic hyper neutrosophic subgraph H_1 of G is as follows.

H_1 =

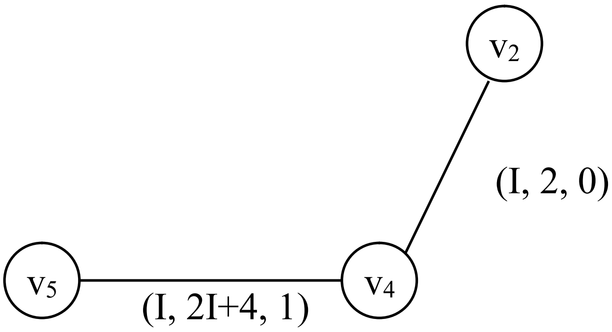


Figure 2.64

H_2 be the Plithogenic pseudo hyper real neutrosophic subgraph H_2 of G given by the following figure.

$H_2 =$

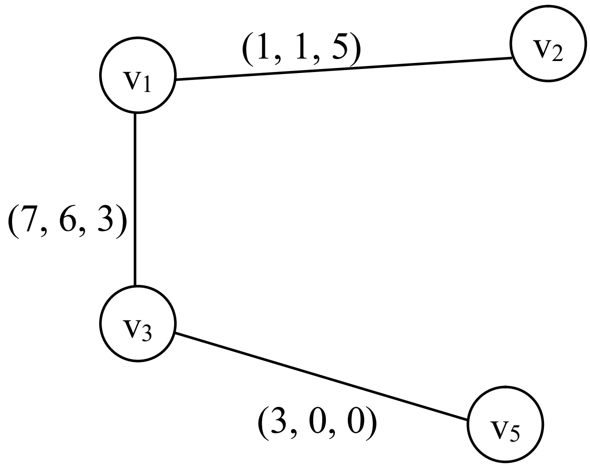


Figure 2.65

Thus, we can have both types of Plithogenic neutrosophic graphs which are simple or otherwise.

Of course, G has subgraphs which are not hyper.

We supply a few subgraphs in the following.

K_1 is a subgraph of G given by the following figure.

$K_1 =$

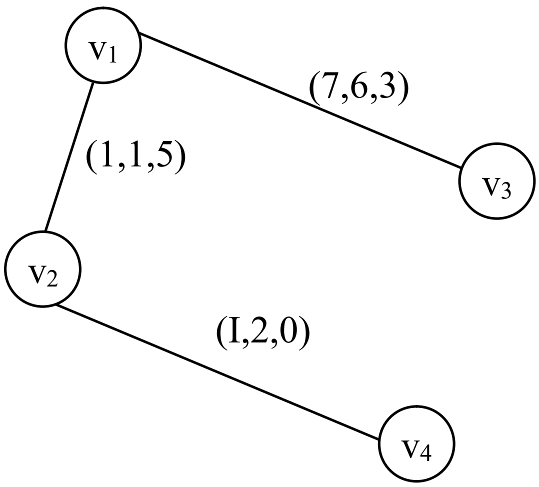


Figure 2.66

In fact this Plithogenic neutrosophic graph has no cliques. It has only the basic units of graphs viz dyads.

We now give some more examples of them.

Example 2.33: Let G be Plithogenic neutrosophic graph given by the following figure.

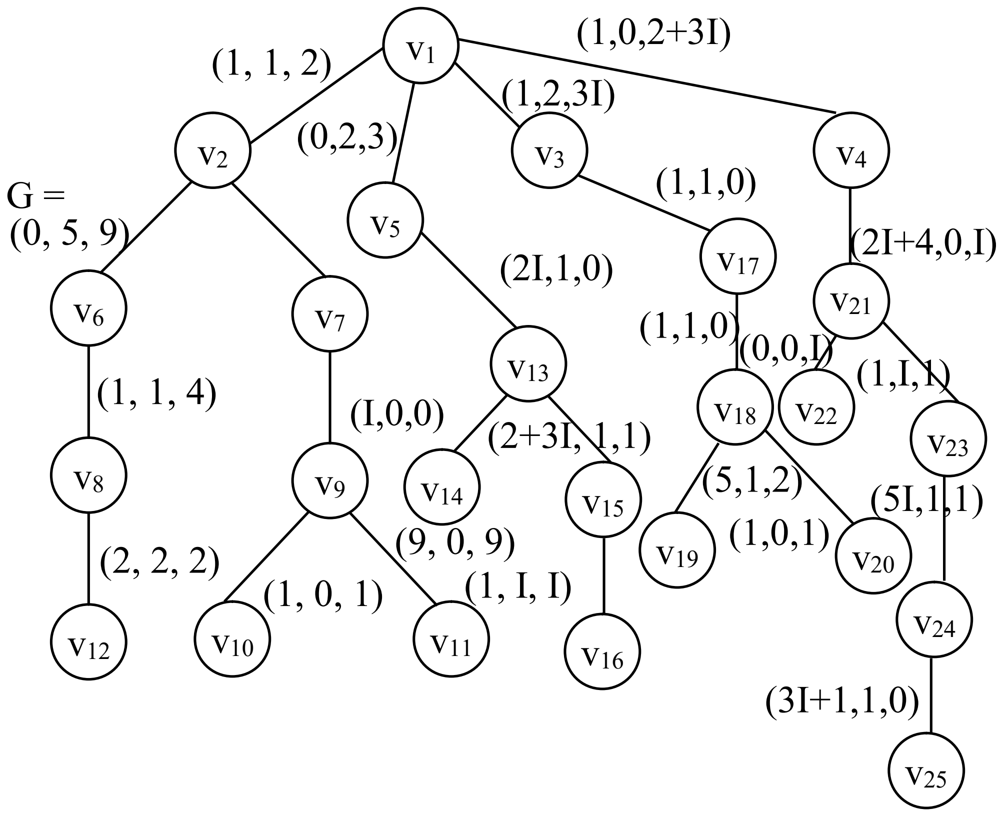


Figure 2.67

G is a simple Plithogenic neutrosophic graph. It has subgraphs some of which we describe in the following.

Let H_1 be a Plithogenic neutrosophic subgraph of G given by the following figure.

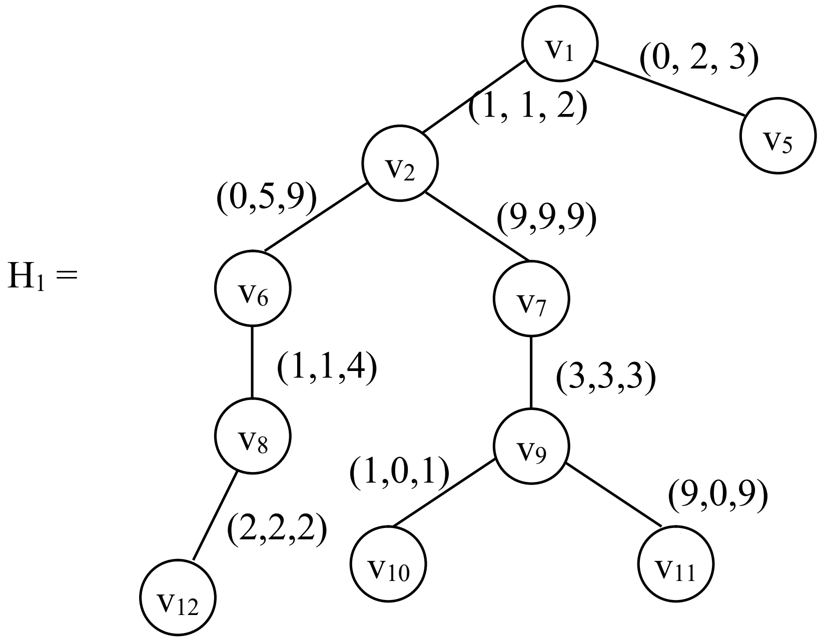


Figure 2.68

Clearly H_1 is a tree which has all its edges to be real. H_1 is also a tree. Consider H_2 a subgraph given by the following figure.

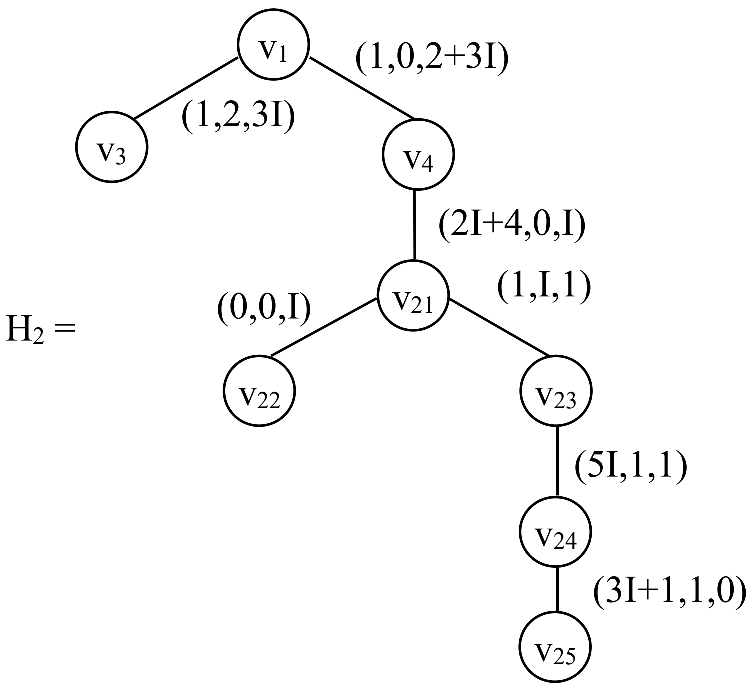


Figure 2.69

H_2 is a subgraph in which the edges are neutrosophic row vectors.

Let H_3 be the subgraph given by the following figure.

H_3 =

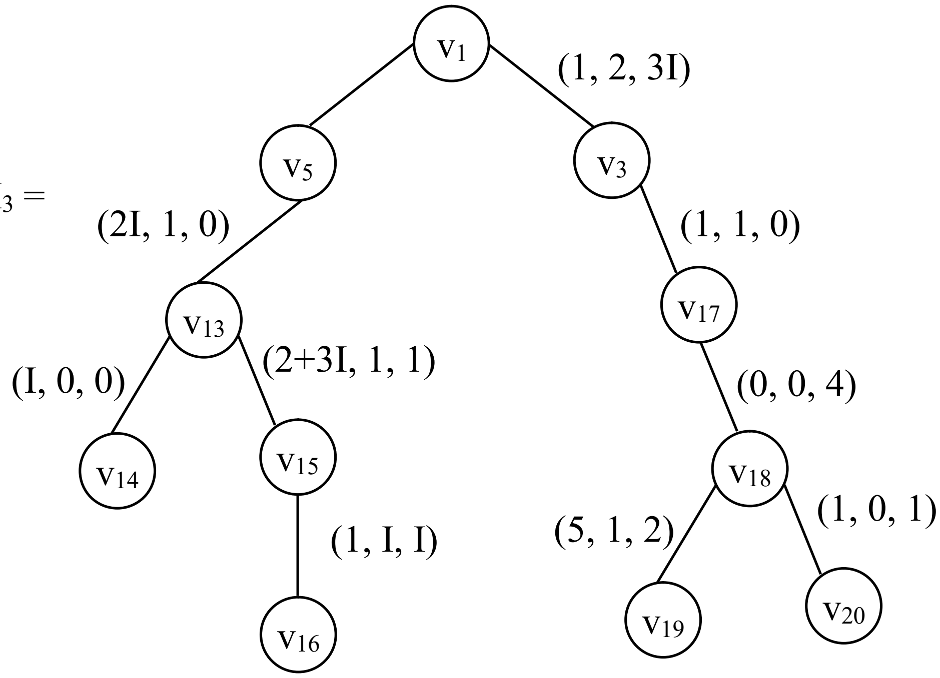


Figure 2.70

H_3 is a Plithogenic neutrosophic subgraph of G has edges to be both real row matrix vectors as well as neutrosophic row matrix vectors.

So we have simple Plithogenic neutrosophic trees.

Next, we describe a complete Plithogenic neutrosophic graph G given by the following figure.

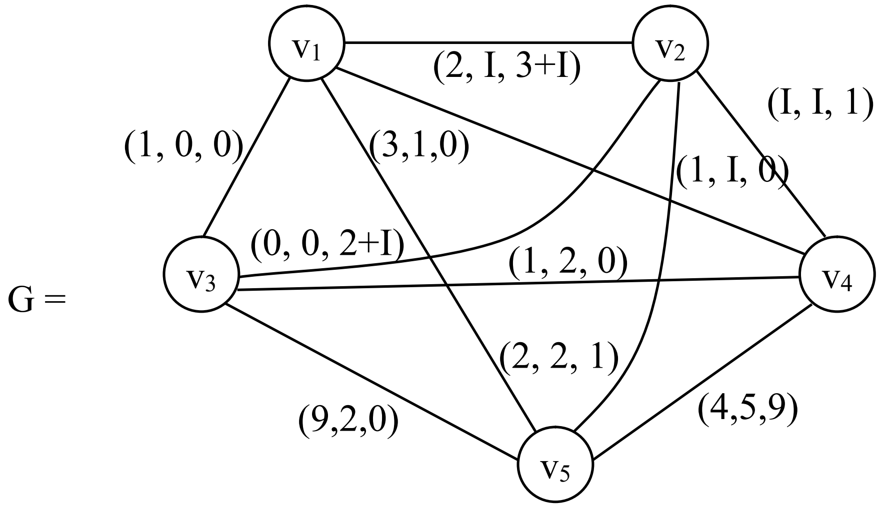


Figure 2.71

This is not simple for it has a Plithogenic pseudo hyper neutrosophic real subgraph P_1, given by the following figure.

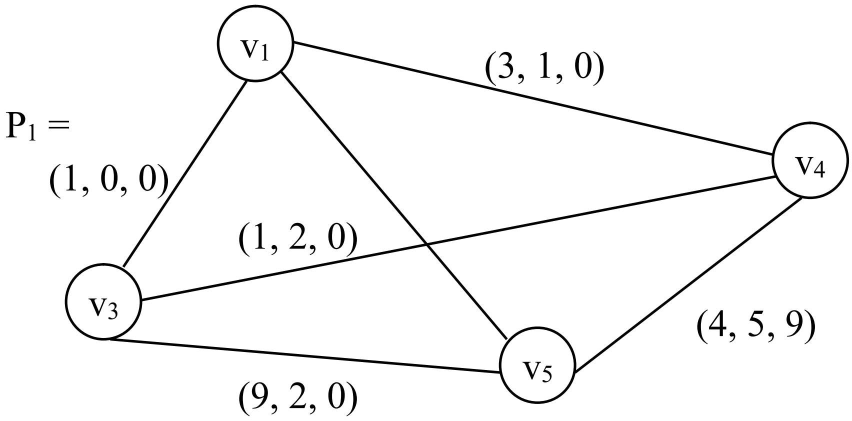


Figure 2.72

However, this G has no Plithogenic hyper special neutrosophic subgraph. It has only Plithogenic neutrosophic dyads.

We call such type of Plithogenic neutrosophic graphs as semi simple Plithogenic neutrosophic pseudo graphs.

Consider the following example.

Example 2.34: Let G be a complete Plithogenic neutrosophic graph given by the following figure.

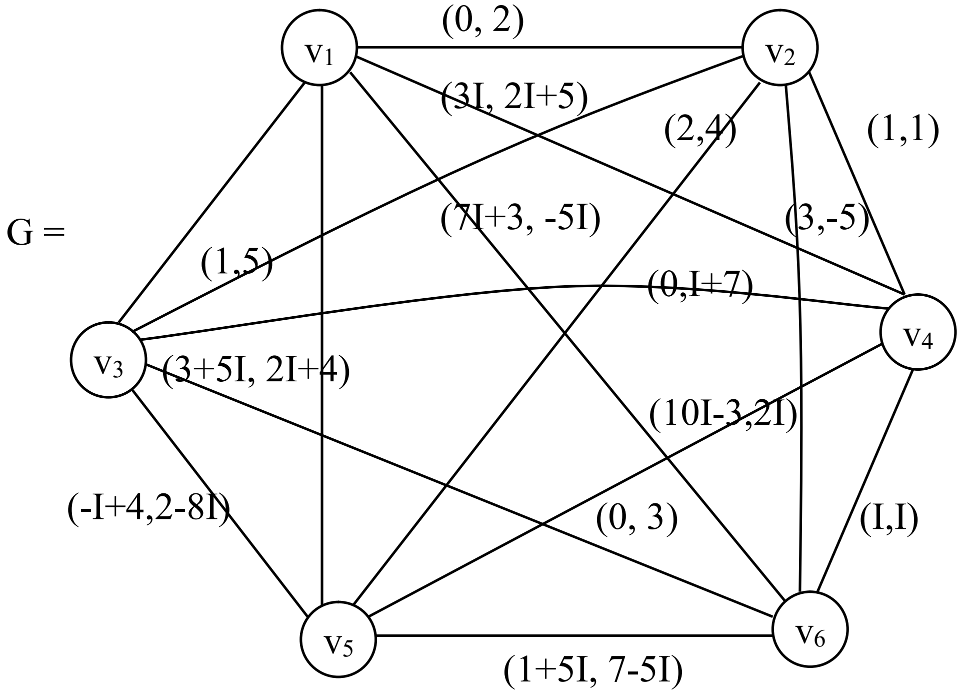


Figure 2.73

Consider the subgraph B given by the following figure.

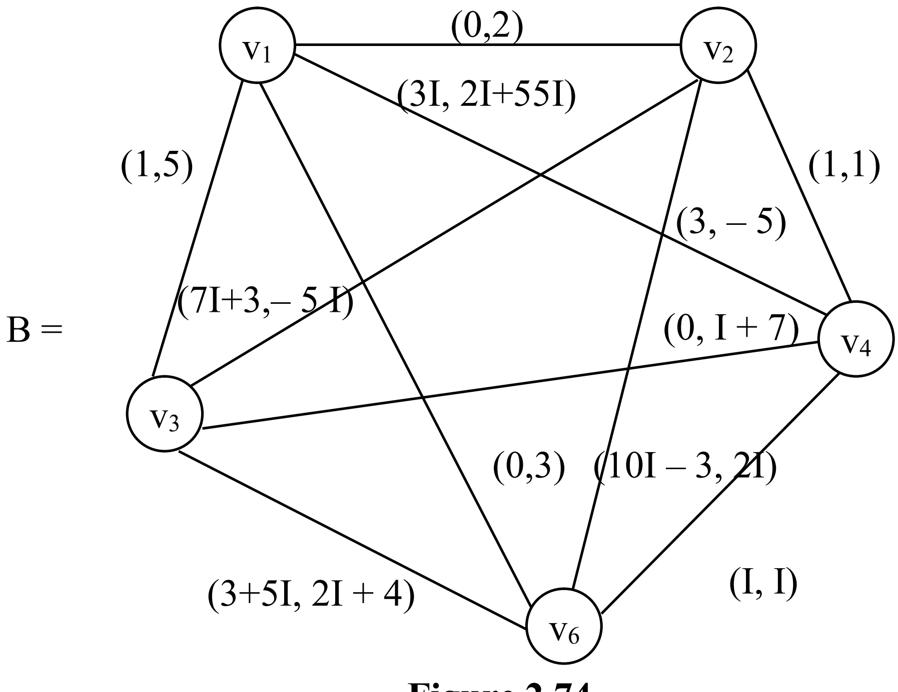


Figure 2.74

B is a complete Plithogenic neutrosophic subgraph of G in which all the edges are neutrosophic. In fact, B is the largest such subgraph which is a Plithogenic hyper neutrosophic subgraph of G which is also unique. However, G has no Plithogenic hyper neutrosophic real pseudo subgraph.

G has only 5 real dyads. We define such graphs which has only Plithogenic hyper neutrosophic subgraphs as semi simple Plithogenic neutrosophic special graphs.

Only when a Plithogenic neutrosophic graph does not contain both real and neutrosophic hyper subgraphs we call them simple otherwise only semi simple.

We give yet another example.

Example 2.35: Let G be a Plithogenic neutrosophic graph given by the following figure.

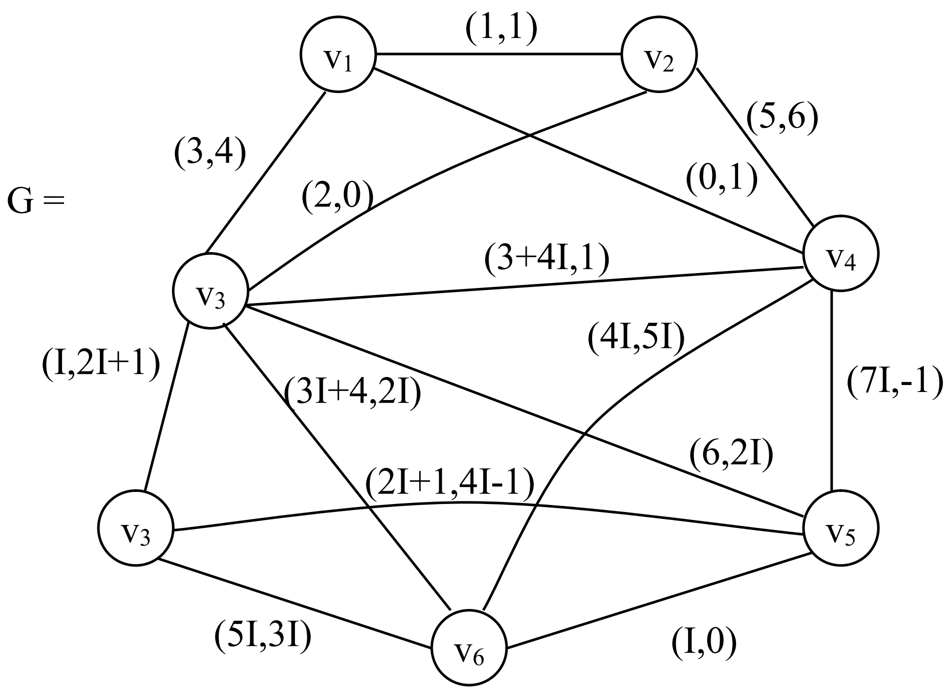


Figure 2.75

We see G is only semi-simple as it has a Plithogenic neutrosophic hyper subgraph S given by the following figure.

S =

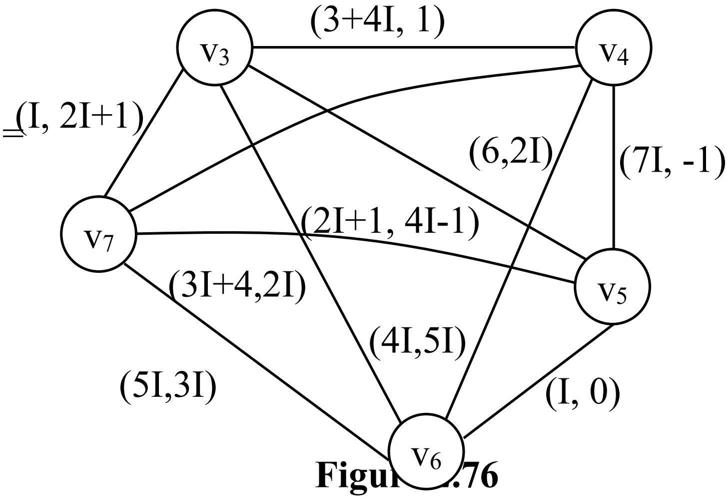


Figure 2.76

S is a Plithogenic hyper neutrosophic subgraph of G which has all 15 edges to be neutrosophic row matrix vectors.

In fact all the neutrosophic edges of S is the same as that of G. However we have no Plithogenic neutrosophic hyper real subgraph of G as there is no subgraph which can have all real row matrix (vector) edges. Hence only we claim G is semi simple.

Here we give a few of the Plithogenic neutrosophic special real subgraphs of G in the following figures.

H_1 =

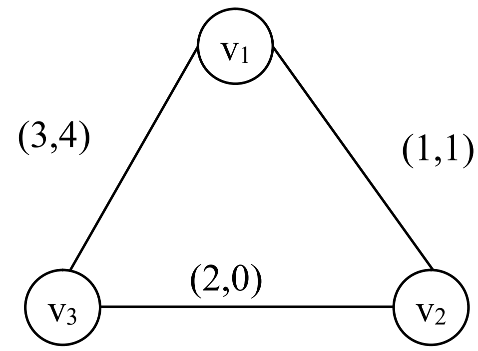


Figure 2.77

$H_2 =$

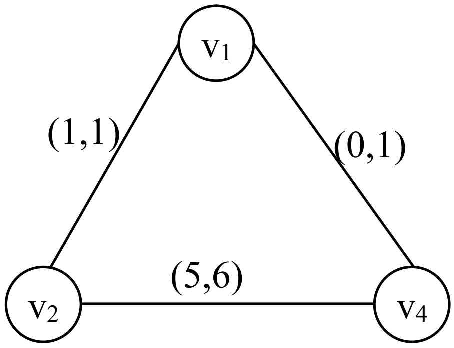


Figure 2.78

However, using all the four vertices v_1, v_2, v_3 and v_4 we get K to be the related subgraph given by the following figure.

K =

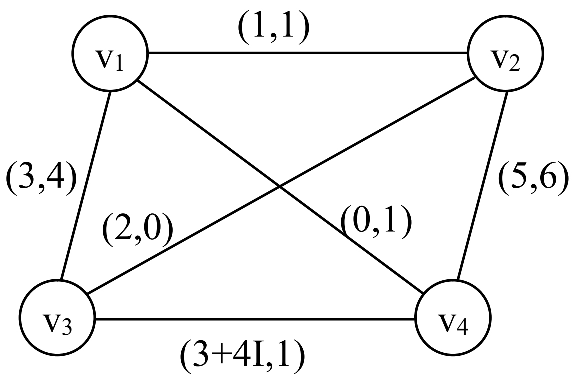


Figure 2.79

Clearly K is a Plithogenic neutrosophic subgraph. It is evident we cannot get a subgraph of G with all the real edges so only G is semi simple as there is no Plithogenic pseudo neutrosophic real subgraph of G.

We give yet another example a Plithogenic neutrosophic graph which is not simple or semi simple or pseudo semi simple by the following example.

Example 2.36: Let G be a Plithogenic neutrosophic graph given by the following figure.

G =

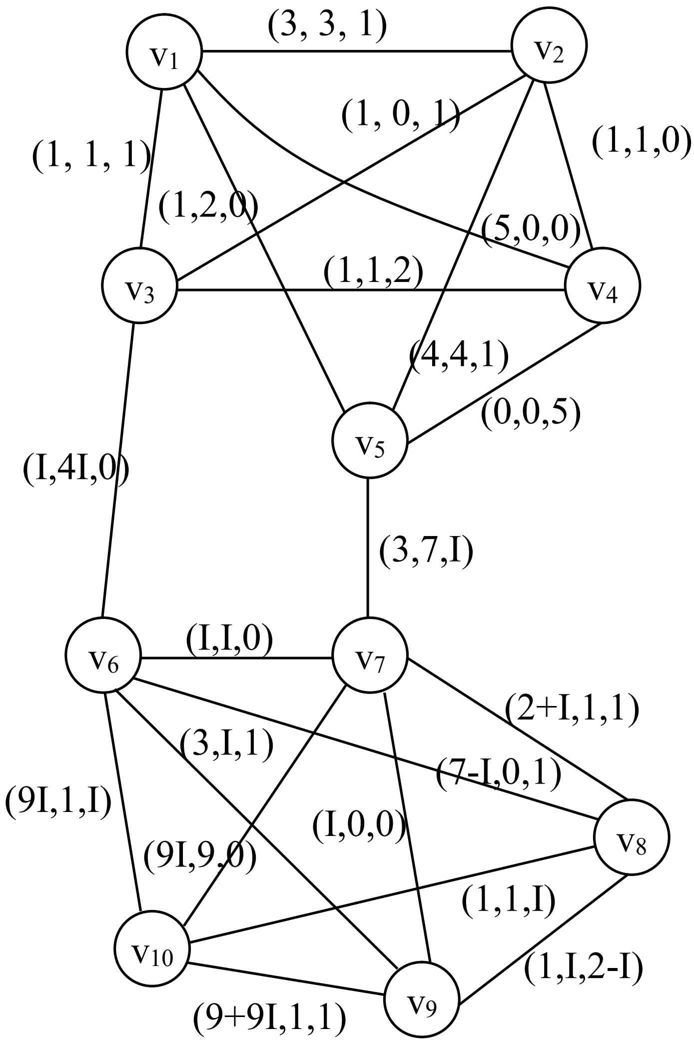


Figure 2.80

G is not simple or semi-simple or pseudo-semi-simple.

Let K_1 be the Plithogenic neutrosophic subgraph of G given by the following figure.

K_1 =

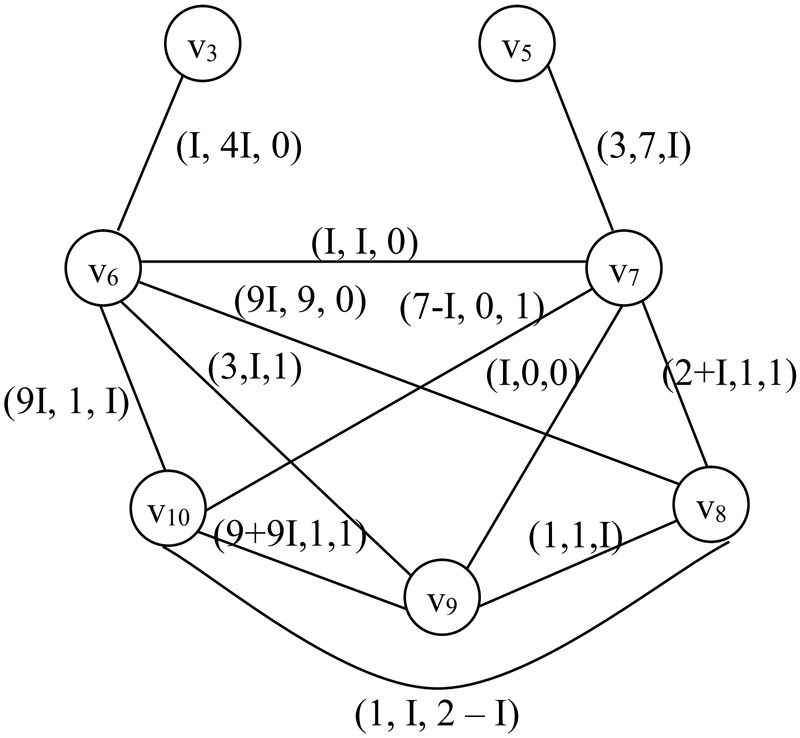


Figure 2.81

K_1 is a Plithogenic hyper neutrosophic subgraph of G. K_1 is the unique hyper subgraph of G. Further all neutrosophic row matrix valued edges of G are in K_1.

Consider K_2 the Plithogenic neutrosophic subgraph of G given by the following figure.

K_2 =

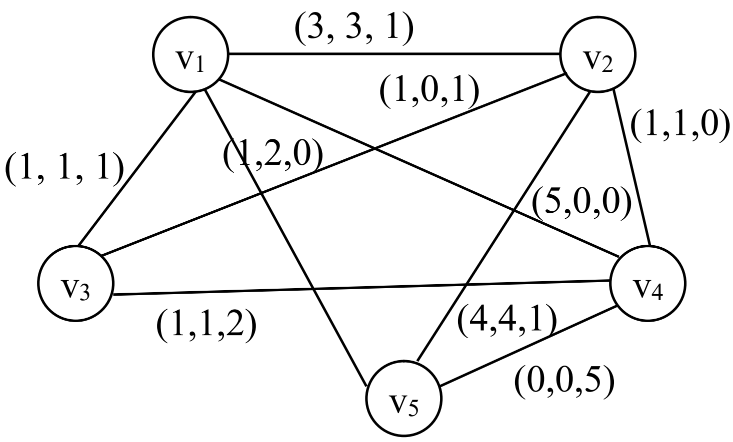


Figure 2.82

Clearly K_2 is the Plithogenic pseudo hyper real neutrosophic subgraph of G. All the real row matrix valued edges of G are present in K_2 and no other edges which are neutrosophic row matrix valued. Thus, G is not simple, semi simple or pseudo semi simple.

It is in fact a difficult problem to characterize simple or semi simple or Plithogenic pseudo semi simple neutrosophic graphs.

We next proceed onto describe few other types of Plithogenic graphs by some examples.

Example 2.37: Let G be a Plithogenic dual number graph given by the following figure. The row vectors are dual numbers from $\langle Z \cup g\rangle = \{a + bg / a, b \in Z, g^2 = 0\}$.

G =

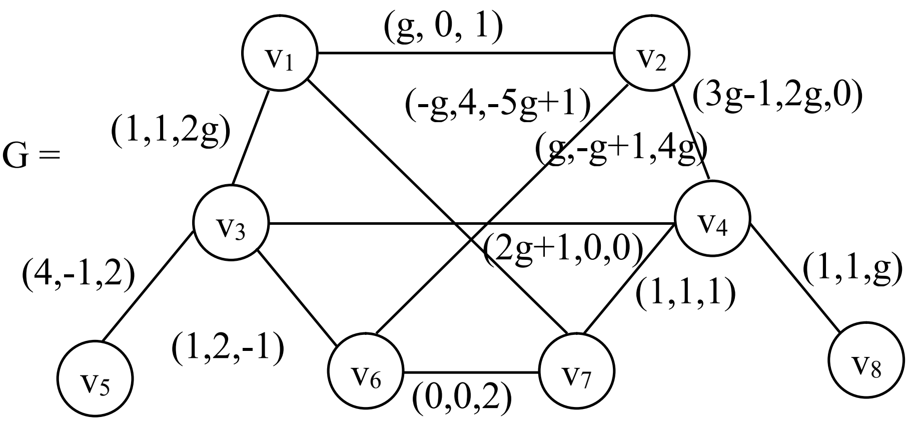


Figure 2.83

We define this G as Plithogenic dual number graph. Consider H_1 the subgraph of G given by the following figure.

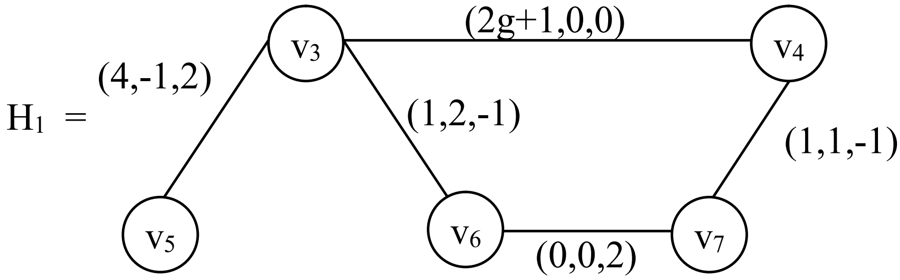


Figure 2.84

Clearly H_1 is a Plithogenic dual number subgraph of G. Only one edge is a dual number row vector. All other edges are real row vectors. In fact all real row valued vector edges of G are also present in H_1.

However, H_1 is not a Plithogenic pseudo real hyper dual number subgraph of G. Also G has no Plithogenic pseudo real hyper dual number subgraph.

Consider the subgraph H_2 of G given by the following figure.

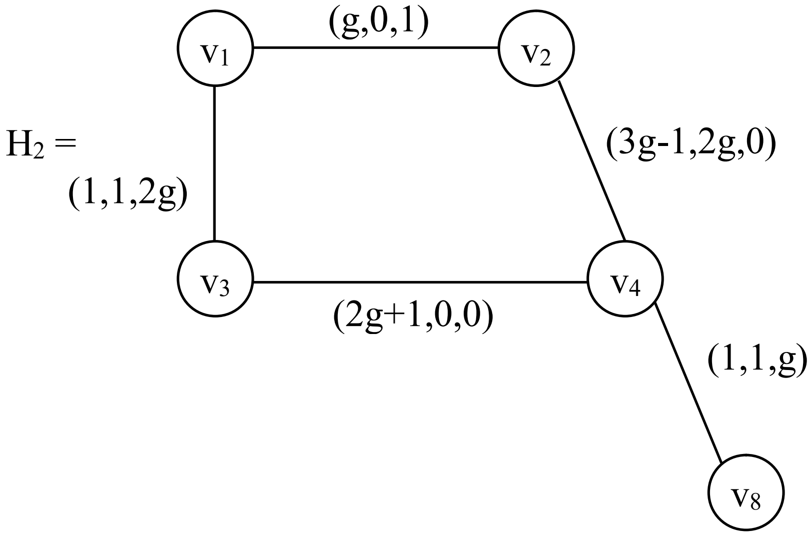


Figure 2.85

H_2 is a Plithogenic dual number subgraph of G. All the edges of H_2 are dual number row vectors but all dual number row vectors of G are not present in H_2. Hence H_2 is not a Plithogenic hyper dual number subgraph of G.

Thus G is simple for it has no Plithogenic hyper subgraphs of both the types.

The notion of simple graph in case of Plithogenic dual number graphs are defined in a similar way as in case of Plithogenic neutrosophic graph.

We provide a few more examples before we proceed to describe Plithogenic complex graphs.

Example 2.38: Let G be a Plithogenic dual number graph given by the following figure.

G =

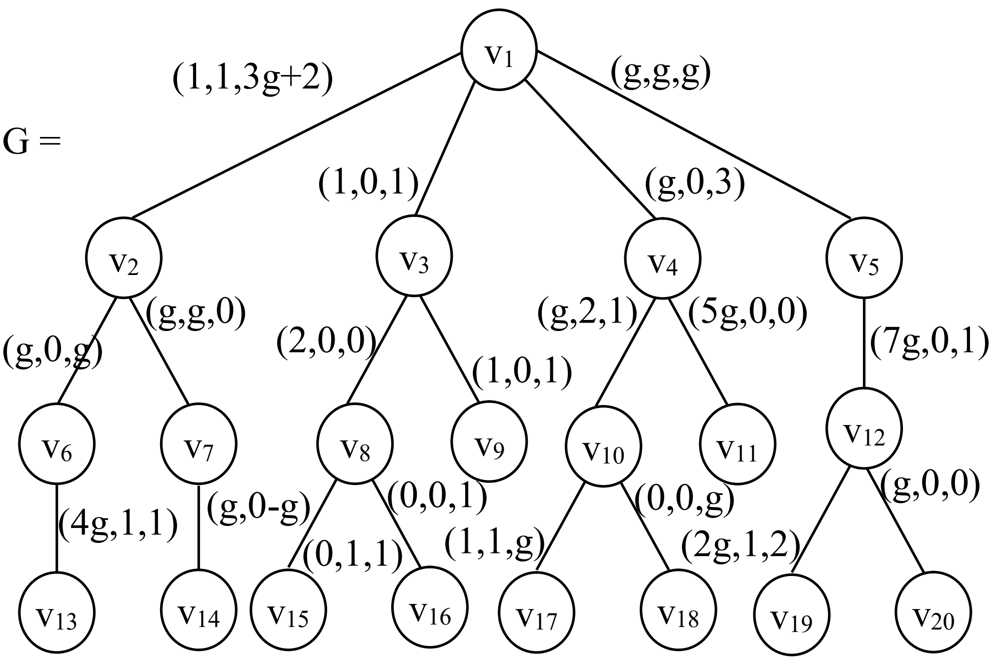


Figure 2.86

G is a Plithogenic dual number tree.

Consider the subgraph H_1 of is G given in the following

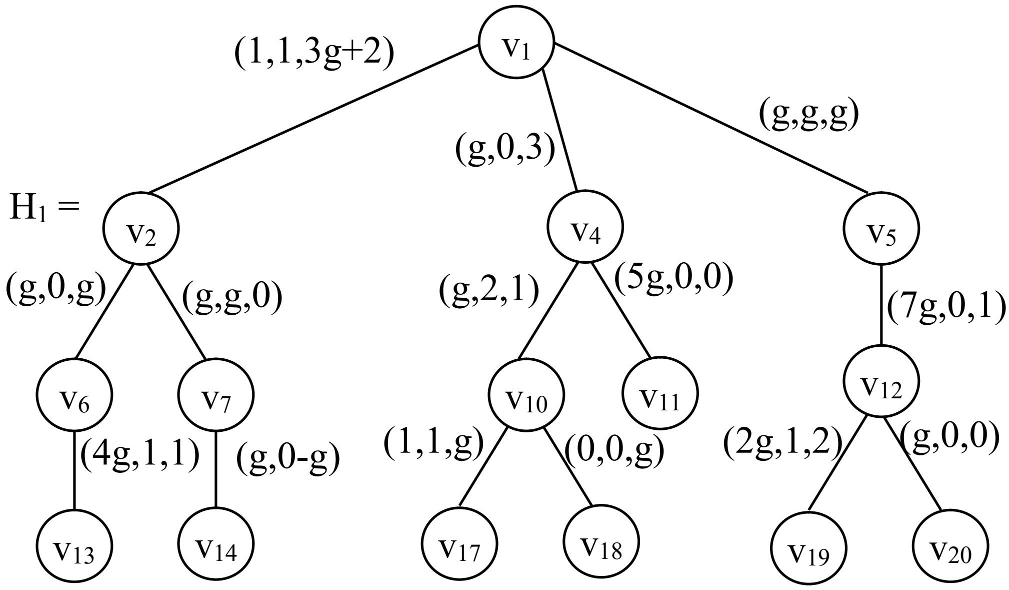


Figure 2.87

We see H_2 is a Plithogenic dual number hyper subgraph G. All dual number row vectors which are edges of G are also present as edges in H_1. In fact all edges of H_1 are dual number row vectors. H_1 is unique and it is the Plithogenic dual number subgraph of G.

Now consider H_2 is a Plithogenic dual number subgraph of G given by the following figure.

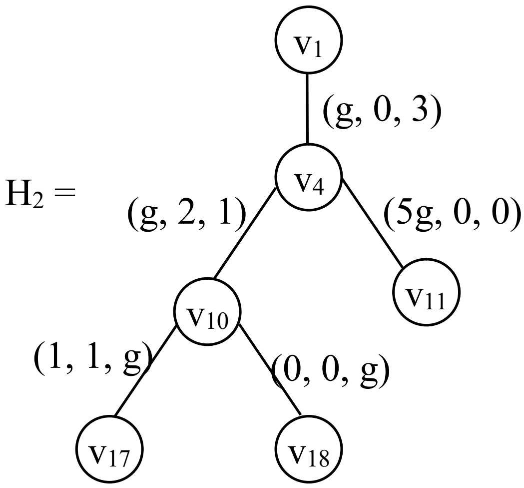


Figure 2.88

H_2 is also a Plithogenic dual number subgraph of G all of its edge row vectors are dual numbers, how ever H_2 is not hyper as H_2 does not contain all the dual number row vectors of G.

Let H_3 be the Plithogenic dual number subgraph of G given by the following figure.

H_3 =

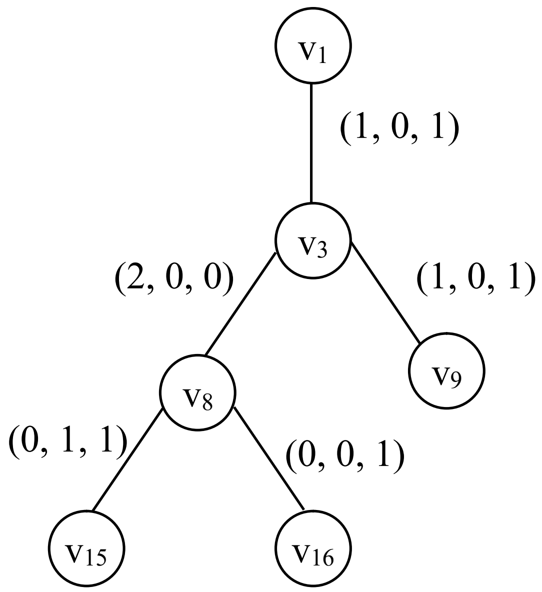


Figure 2.89

We see H_3 has all its edges which are row vectors are real and in fact H_3 has all the real row vectors of G.

Thus H_3 is a Plithogenic pseudo dual number real hyper subgraph of G.

We see all the 3 subgraphs are trees and in fact G is not simple or semi simple or pseudo semi simple.

Now we describe a Plithogenic complex graphs whose row weights are from C = {a + bi / a, b ∈ R and $i^2 = -1$} by an example.

Example 2.39: Let G be a Plithogenic complex graph given by the following figure.

G =

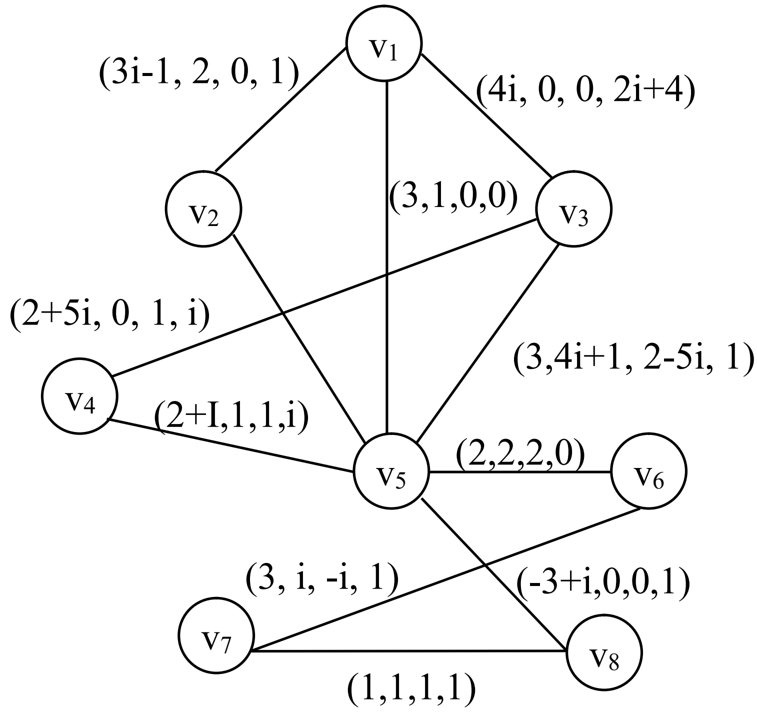


Figure 2.90

G has no Plithogenic pseudo hyper complex subgraph. So G is pseudo semi-simple. G has no Plithogenic hyper complex subgraph. Hence G is simple.

Thus, a Plithogenic complex graph G can be simple or otherwise.

We can also define Plithogenic neutrosophic complex graphs whose edges have row weights from the set $\langle C \cup I \rangle = \{a + bi + cI + diI / a, b, c, d \in R$ (reals), $i^2 = -1$, $I^2 = I$ and $(iI)^2 = -I\}$.

In this case we can define 4 types of Plithogenic hyper subgraphs.

i) All the edges of G which are real should be present as the only edges of the subgraph H_1 of G. We call this H_1 a Plithogenic pseudo complex neutrosophic real subgraph of G.

ii) All the edges of G which are complex and neutrosophic valued must be the only edges of a subgraph H_2 of G we call H_2 the Plithogenic complex neutrosophic hyper subgraph of G.

iii) The subgraph H_3 of G which has all complex row valued edges of G to be edges of H_3 then H_3 is defined as Plithogenic complex neutrosophic pseudo complex hyper subgraph of G.

iv) Similarly we define Plithogenic complex neutrosophic pseudo neutrosophic hyper subgraph of G.

A Plithogenic complex neutrosophic graph of G which has none of the four types of hyper subgraphs is defined as the simple Plithogenic complex neutrosophic graph.

Otherwise if it contains only one or two or three types it is said to be pseudo simple or semi simple.

We will first provide examples to this effect.

Example 2.40: Let G be a Plithogenic complex neutrosophic graph given by the following figure.

G =

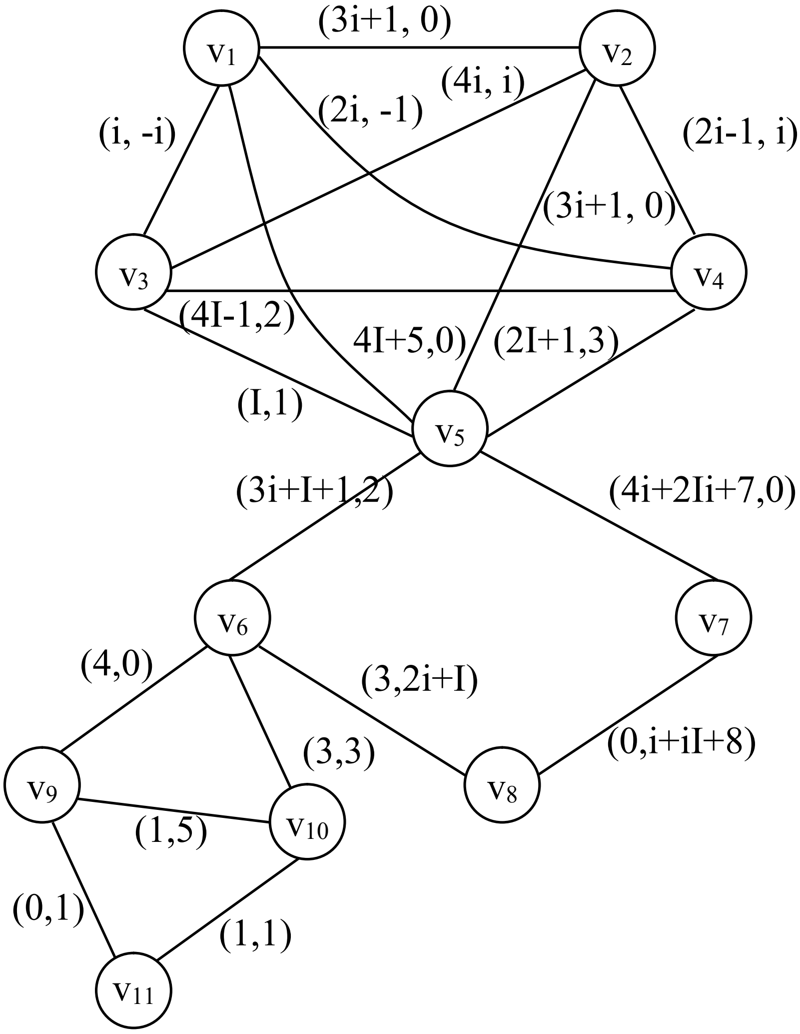


Figure 2.91

We see G has all the four types of hyper subgraphs.

Let H_1 be the Plithogenic complex neutrosophic subgraph of G given by the following figure.

$H_1 =$

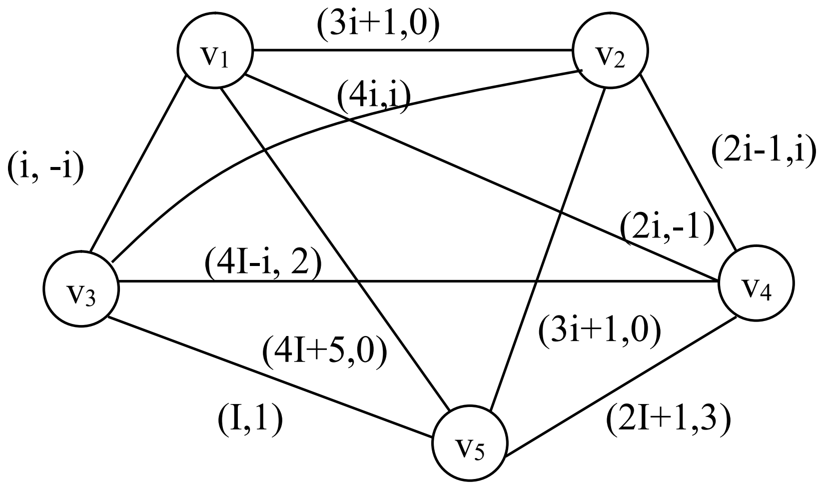


Figure 2.92

H_1 is only a Plithogenic neutrosophic complex subgraph of G.

Let H_2 be a subgraph of G given by the figure.

$H_2 =$

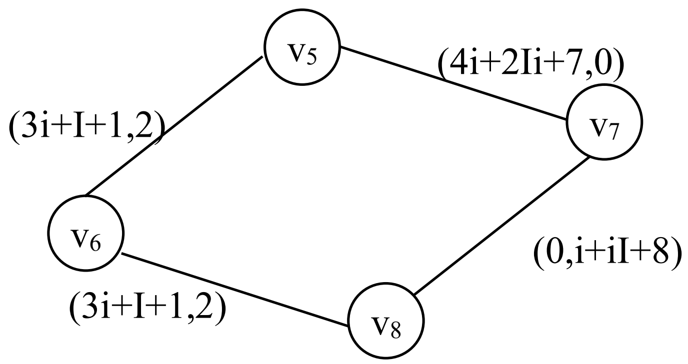


Figure 2.93

H_2 is a Plithogenic complex neutrosophic hyper subgraph of G.

Let H_3 be the Plithogenic pseudo hyper complex neutrosophic real subgraph of G given by the following figure.

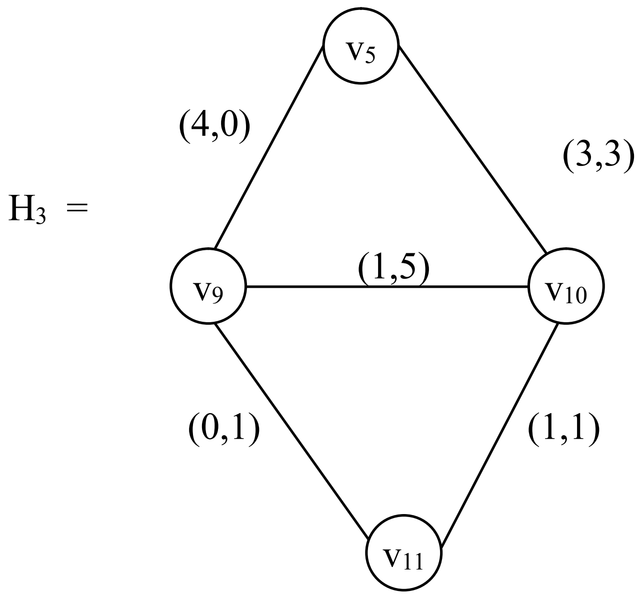


Figure 2.94

Clearly H_3 contains all the real row vectors of G, hence H_3 is a Plithogenic pseudo hyper complex neutrosophic real subgraph of G and it is unique.

H_2 also contains all complex and neutrosophic valued row vectors of G hence H_2 is the unique Plithogenic hyper neutrosophic complex subgraph of G. G is only a pseudo semi simple graph.

Next we give another example of a Plithogenic complex neutrosophic graph G.

Example 2.41: Let G be a Plithogenic neutrosophic complex graph given by the following figure.

G =

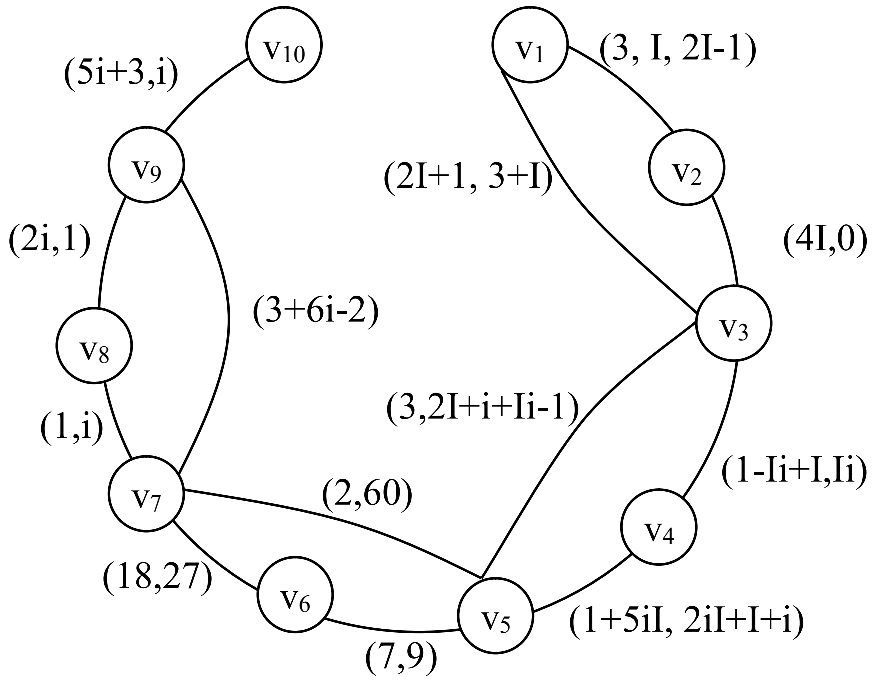


Figure 2.95

Clearly G is a Plithogenic complex neutrosophic graph with row vector from $\langle C \cup I\rangle \times \langle C \cup I\rangle = \{(a_1 + a_2I + a_3i + a_4Ii, b_1 + b_2I + b_3i + b_4iI) / a_i, b_i \in R\ 1 \leq i \leq 4\}$ given by the above figure.

We just give a few subgraphs of G.

Let H_1 be a subgraph of G given by the following figure.

H_1 =

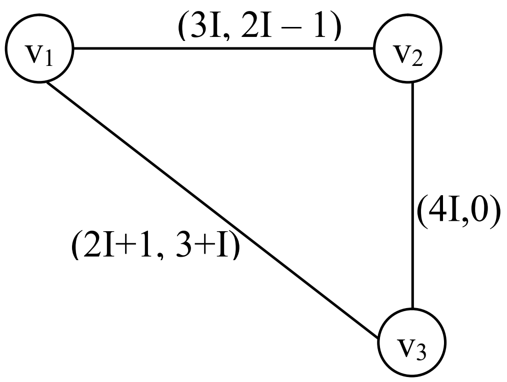


Figure 2.96

H_1 contains all the neutrosophic row vector values edges of G and no other edge. Hence H_1 is a Plithogenic pseudo hyper neutrosophic complex subgraph of G.

Let H_2 be a subgraph of G given by the following figure.

H_2 =

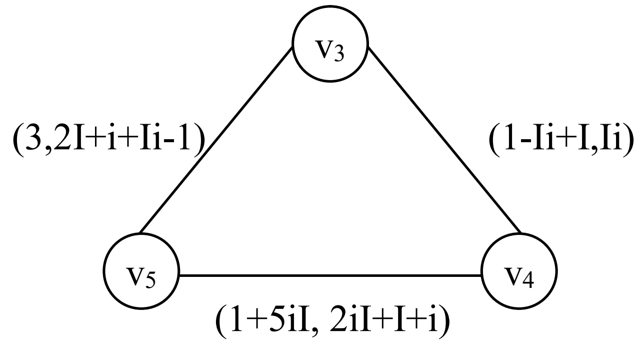


Figure 2.97

The subgraph H_2 of G contains all the complex neutrosophic row vector valued edges of G and nothing more. Hence H_2 is a Plithogenic hyper complex neutrosophic subgraph of G and it is unique.

Consider H_3 the subgraph of G given by the following figure.

H_3 =

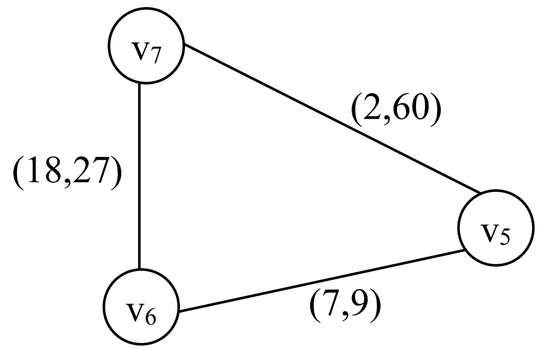


Figure 2.98

H_3 is a Plithogenic pseudo hyper complex neutrosophic real subgraph of G. In fact all real edges present in G and present in H_3 and nothing more.

We see all the three hyper subgraphs H_1, H_2 and H_3 are triads.

Now consider the subgraph H_4 of G given by the following figure.

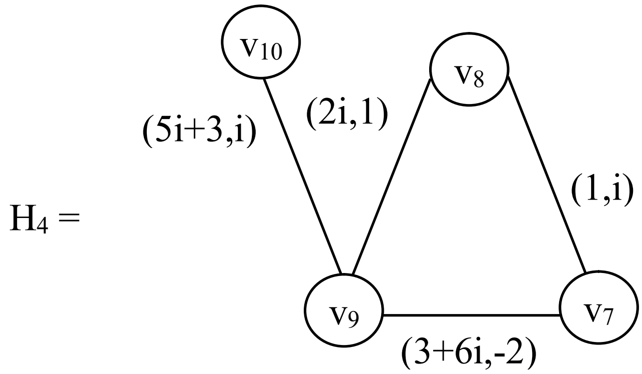


Figure 2.99

All the complex valued row vector edges of G are present in H_4 and nothing more; hence H_4 is a Plithogenic pseudo hyper complex neutrosophic subgraph of G.

Thus G is not a simple or semi simple Plithogenic complex neutrosophic graph. It has all four Plithogenic hyper and Plithogenic pseudo hyper subgraphs of G. Consider the subgraph K_1 of G given by the following figure.

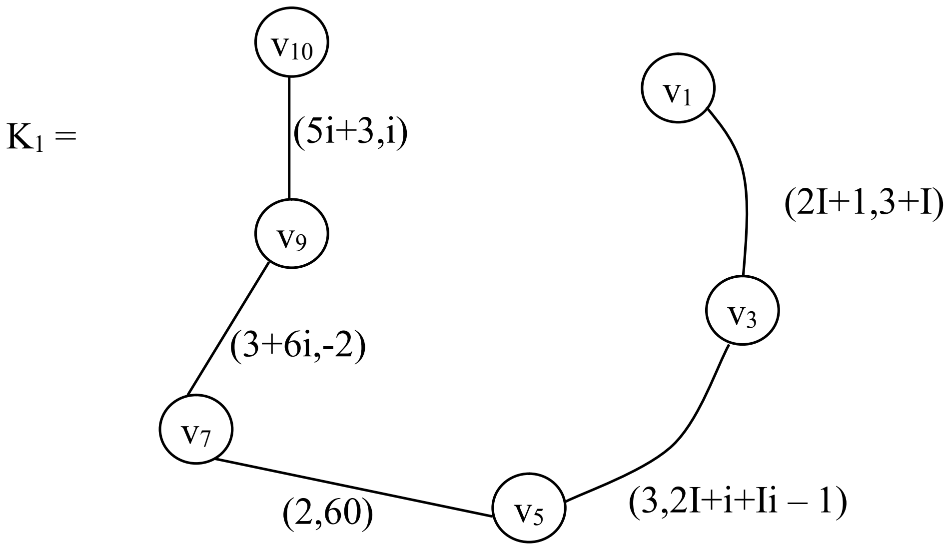


Figure 2.100

Clearly K_1 is not a hyper subgraph or a pseudo hyper subgraph just a Plithogenic neutrosophic complex subgraph of G. Thus we can have Plithogenic graphs which are not simple or pseudo simple. Characterization of simple or pseudo simple graphs in case of appropriate Plithogenic graphs happens to be a difficult problem.

Also the condition for Plithogenic subgraphs of G to be structure preserving also happens to be a difficult problem.

However, we can as in case of usual graphs define and develop cliques. This task is left as an exercise to the reader.

We provide a few examples of them.

Example 2.42: Let G be a Plithogenic graph given by the following figure.

G =

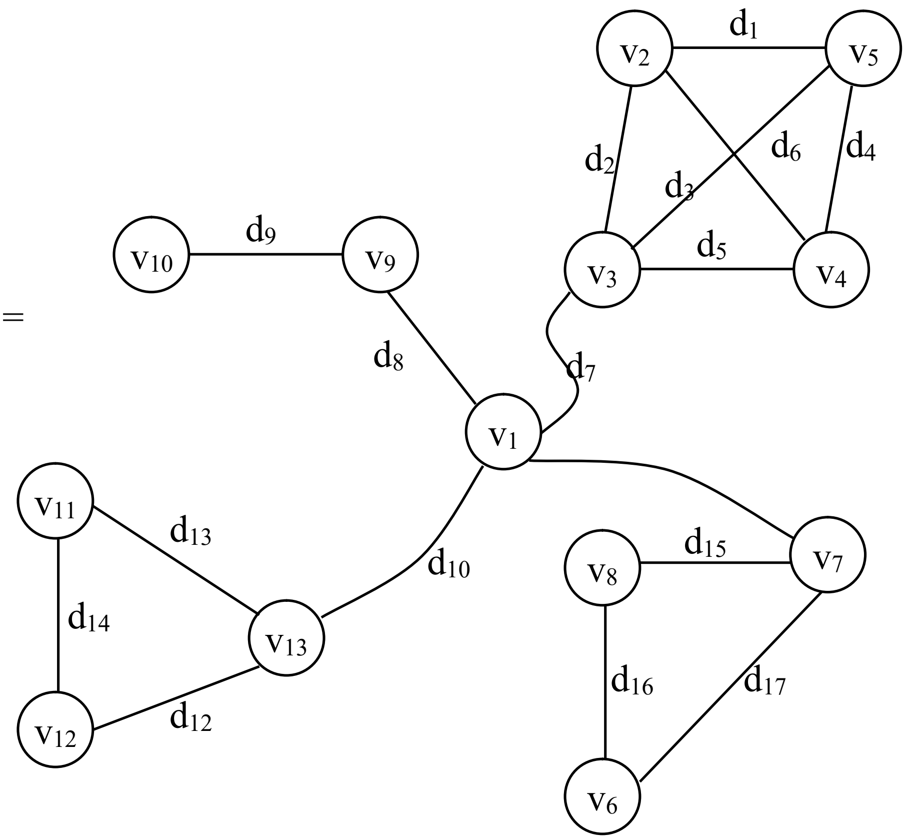


Figure 2.101

With row weights from P = {(x, y, z) / x, y, z ∈ $\langle C \cup g\rangle$ = $a_1 + a_2g + a_3i + a_4ig$ where $a_i \in R$, $i^2 = -1$, $g^2 = 0$, $(ig)^2 = 0$; $1 \leq i \leq 4$} where $d_i \in P$; $1 \leq i \leq 14$. This has a clique B of order four given by

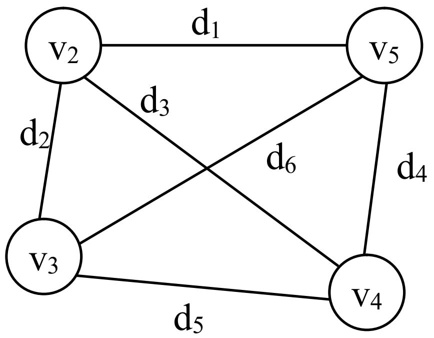
B = 

Figure 2.102

It has two different triads apart from this clique. So as in case of usual graphs we can in case of Plithogenic graphs also have the concept of clique the largest complete subgraph of G. In this case G has only one clique.

Further as in case of usual graphs in case of Plithogenic ring graph, Plithogenic tree (graph) and Plithogenic wheel graph we do not have the concept of clique. Clique may not be present in other types of special row vector valued graphs which is difficult to characterize.

We prove an example before we proceed to develop the concept of edge Plithogenic subgraph of a Plithogenic graph G.

Example 2.43: Let G be a Plithogenic graph given by the following figure.

G =

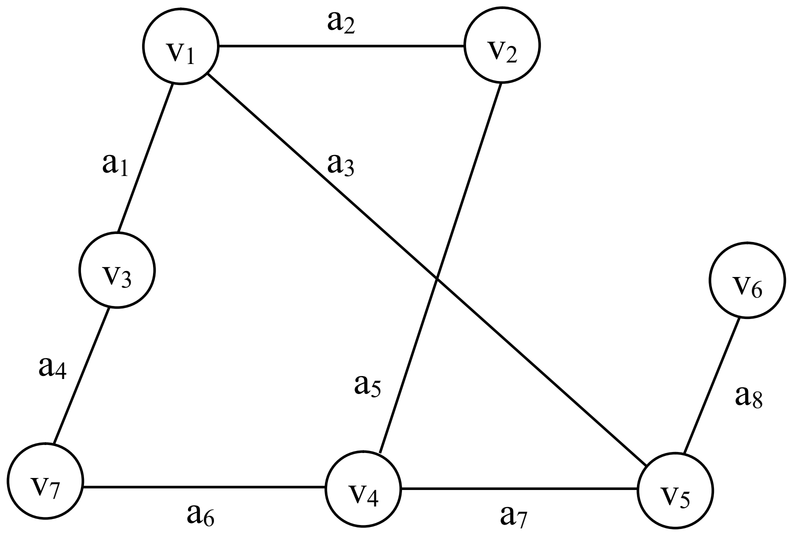


Figure 2.103

G has no clique. This Plithogenic subgraph does not contain even a triad. Here $a_j \in \{\langle C \cup I\rangle \times \langle C \cup I\rangle \times \langle C \cup I\rangle \times \langle C \cup I\rangle\} = \{(x, y, z, \omega) / x, y, z, \omega \in \langle C \cup I\rangle = \{a + bI + ci + dIi;\ a, b, c, d \in R\}\ 1 \leq j \leq 8$. Hence we see as in case of usual graphs Plithogenic graphs may not always contain a clique.

What is interesting is when we remove the edges and edges are not only real complex or neutrosophic or dual numbers we find edge subgraphs behave differently.

In this chapter we further study the edge Plithogenic subgraphs of G by some examples then discuss the specialty about them in case of Plithogenic neutrosophic graphs, Plithogenic complex graphs, Plithogenic neutrosophic complex graphs, Plithogenic dual number graphs and so on.

Example 2.44: Let G be a Plithogenic graph given by the following figure.

G =

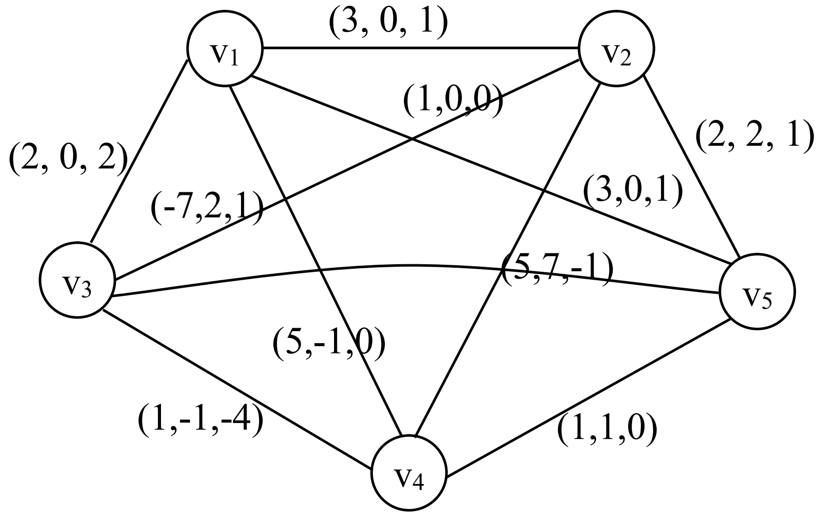


Figure 2.104

Suppose the edge (3, 0, 1) is removed from G then let $H_1 = G \setminus \{(3, 0, 1)\}$ we get a Plithogenic subgraph and no special property is enjoyed by these Plithogenic graphs. So the study in this direction is similar to that of usual graphs.

The only warning is instead of writing $H_1 = G \setminus \{(3, 0, 1)\}$ we should write as only $G \setminus \{v_1v_2\}$ or $G \setminus \{v_1 v_5\}$ to avoid this problem it is appropriate we mention only the edges and not the row vector.

As far as Plithogenic graphs with entries from real row vectors are concerned there is no difference between usual graphs and these graphs. Only in case when the row vectors are not real or modulo integers we see it has a strong way in predicting the hyper and pseudo hyper appropriate subgraphs.

Hence we will proceed onto describe a few properties and more onto Plithogenic graphs whose vector row vector values are not real.

Example 2.45: Let G be a Plithogenic ring graph given by the following figure.

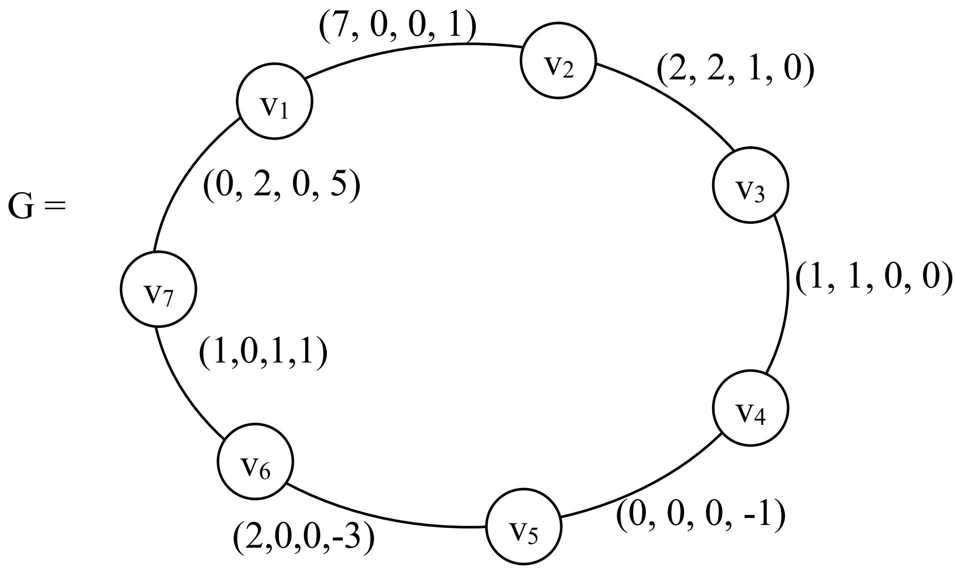


Figure 2.105

Any subgraph got as $G \setminus \{v_iv_j\}$; $i \neq j$ is not a ring graph.

Example 2.46: Let G be a Plithogenic star graph given by the following figure.

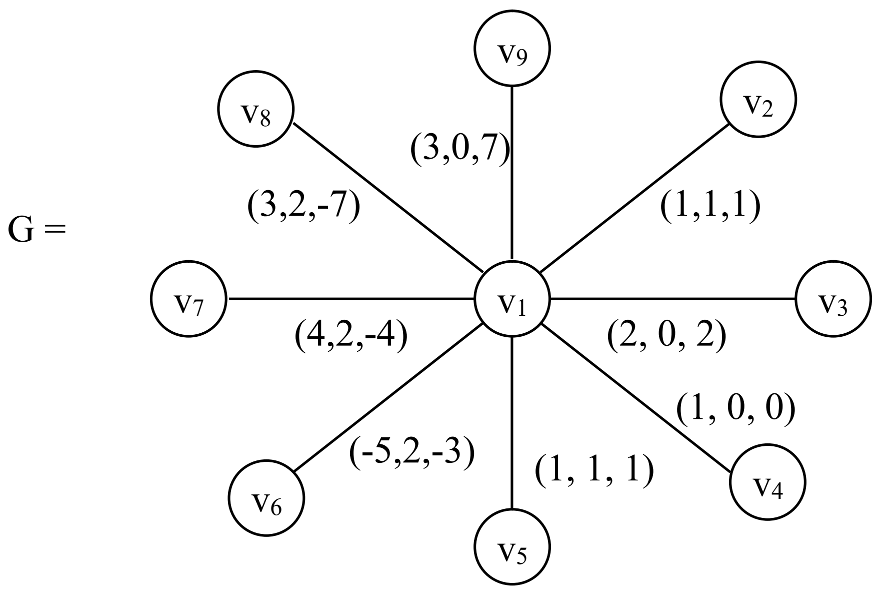


Figure 2.106

Removal of any edge results in a subgraph which is again a star subgraph of G. This is the marked difference between the circle graph and a star graph.

The structure is preserved in case of subgraphs of star graphs whereas structure is not preserved by subgraphs in case of ring graphs.

Consider the following example of a wheel.

Example 2.47: Let G be a Plithogenic wheel graph given by the following figure.

G =

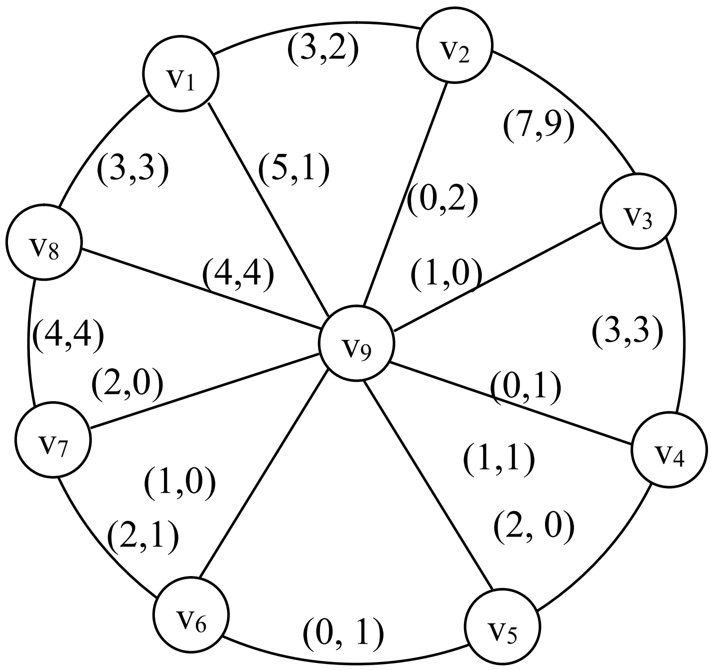


Figure 2.107

Removal of any edge destroys the structure of the wheel. Thus edge subgraphs of wheel is not a wheel.

Finally we give the following theorem.

Theorem 2.1. *Let G be a Plithogenic star graph every subgraph of G is again a Plithogenic star graph. Thus, structure is preserved.*

Proof is direct and hence left as an exercise to the reader.

Theorem 2.2. *Let G be Plithogenic ring graph. None of the subgraphs of G is a ring graph. So, structure is not preserved.*

Proof is direct and is left as an exercise to the reader.

Theorem 2.3. *Let G be a Plithogenic wheel graph. None of the subgraphs of G is a wheel.*

Proof is left as an exercise to the reader.

Next when we study the case of tree or line of Plithogenic graphs then subgraphs can be trees or lines respectively in some cases.

The study in this direction is also a routine as in case of usual graph so left as an exercise to the reader.

Now in case of complete Plithogenic graphs we may or may not have the subgraph to be complete always it may be complete depending on the edges which are removed.

We may in case of certain Plithogenic graphs which are connected may become disconnected by removal of some edges and so on. All this study is like usual graphs in case the row vector valued vectors which forms the edge weights are taken to be real.

Now if they are neutrosophic we discuss the subgraph properties by some examples. For our motivation is to relate it to simple Plithogenic graphs or Plithogenic hyper subgraphs.

Example 2.48: Let G be a Plithogenic neutrosophic graph given by the following figure.

G =

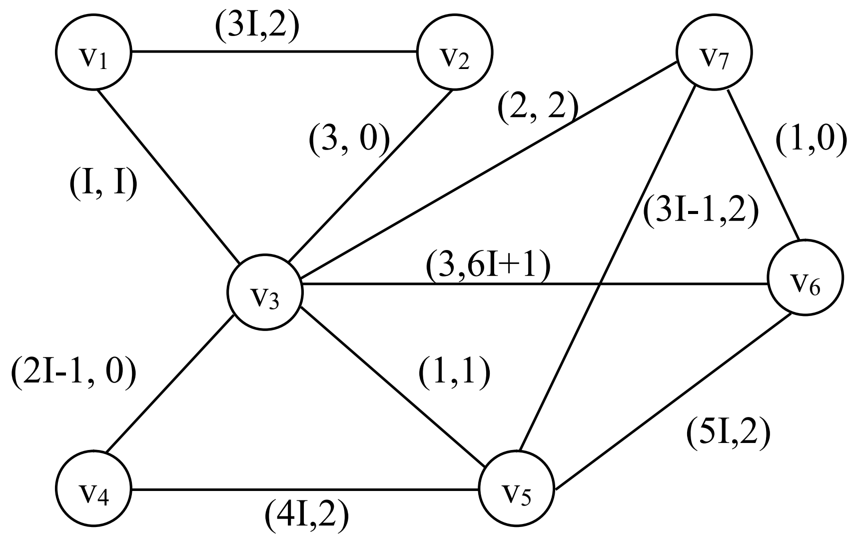


Figure 2.108

Now this has no Plithogenic hyper neutrosophic subgraph or a Plithogenic pseudo hyper neutrosophic subgraph. Thus, it is simple.

However, if we try to find the Plithogenic hyper neutrosophic edge subgraphs is it simple? Can G have both Plithogenic hyper neutrosophic edge subgraph as well as Plithogenic pseudo hyper neutrosophic edge subgraph, we try to describe this by the following figures.

Consider H = G \ $\{v_1v_2, v_1v_3, v_3v_6, v_5v_7, v_5v_6, v_3v_4, v_4v_5, v_7v_5\}$.

H is described in the following.

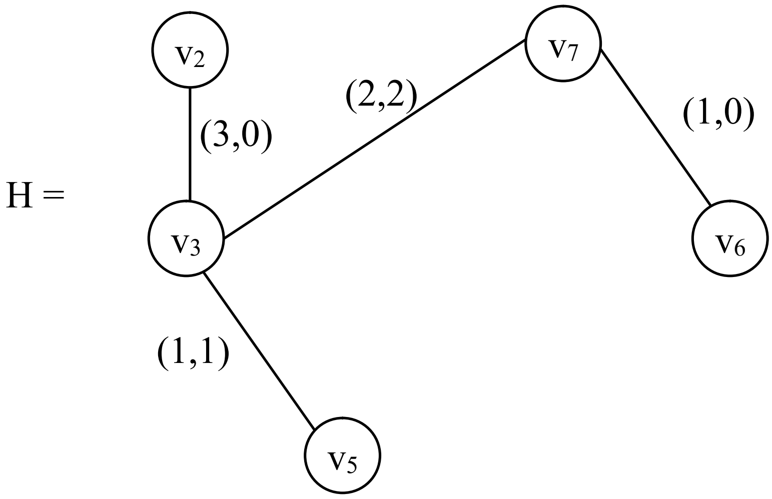


Figure 2.109

H is the Plithogenic pseudo hyper subgraph of G which contains all real edges of G; in fact has only all edges to be real and no other edges.

Consider K = G \ $\{v_2v_3, v_3v_7, v_3v_5, v_7v_6\}$, in which is the edge subgraph of G got by removing all the real edges from G and is given by the following figure.

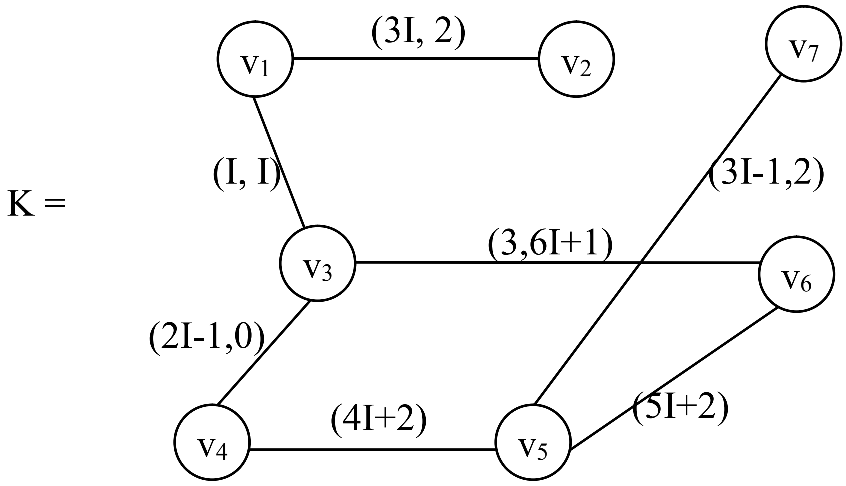


Figure 2.110

Clearly K is a Plithogenic hyper neutrosophic edge subgraph of G. Thus G is not simple or pseudo simple as edge hyper subgraphs but G is simple as realized as usual subgraph.

Thus it is advantageous to use edge subgraphs to get hyper subgraphs. For we see K contains all the neutrosophic row vector valued vector edges of G and nothing more.

Thus K is the unique Plithogenic hyper edge subgraph of G. So G is not edge simple. In fact we can say none of the Plithogenic graphs are edge simple or edge pseudo simple.

We proceed to describe more examples of the same.

***Example 2.49*:** Let G be a Plithogenic dual number graph given by the following figure.

G =

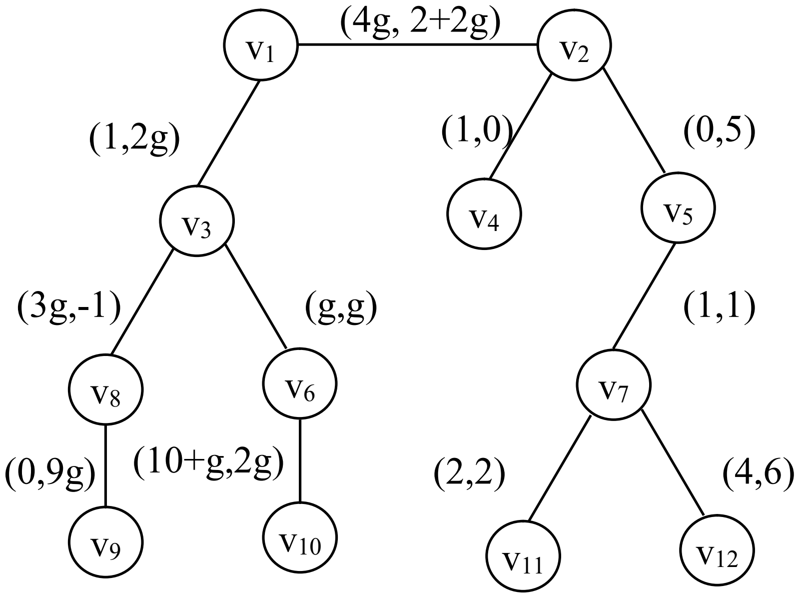


Figure 2.111

If the edge v_1v_2 is removed from G we get the dual number special row vector valued subgraphs K with two components one Plithogenic pseudo hyper dual number subgraph of G and other is just a Plithogenic dual number subgraph of G given by the figure K in the following.

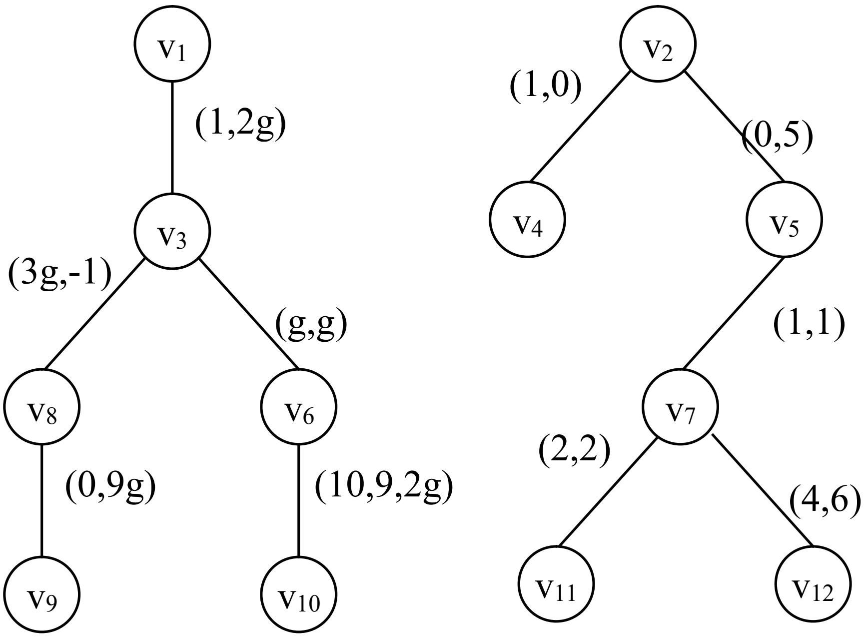


Figure 2.112

However if we find the edge subgraph H = G \ {v_2v_4, v_2v_5, v_5v_7, v_7v_{11} and v_7v_{12}}; we get a Plithogenic hyper dual number subgraph H given by the following figure.

H =

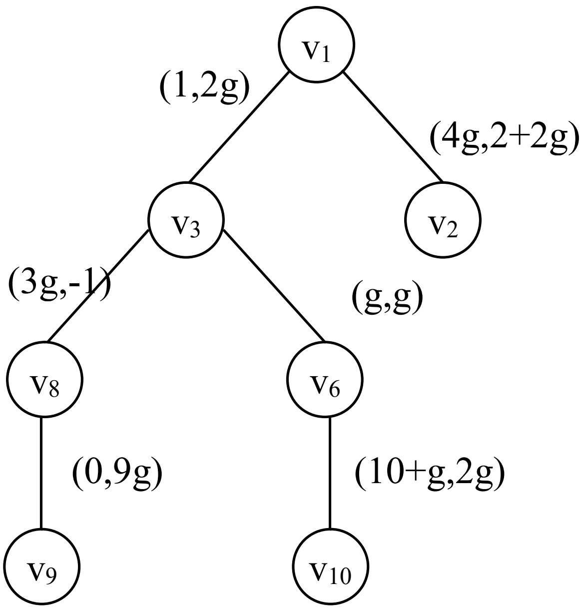


Figure 2.113

Consider the subgraph K = G \ {v_1v_2, v_1v_3, v_3v_8, v_3v_6, v_8v_9, v_6v_{10}} got by removing the edges mentioned above.

The Plithogenic pseudo hyper subgraph of G is given by the following figure.

K =

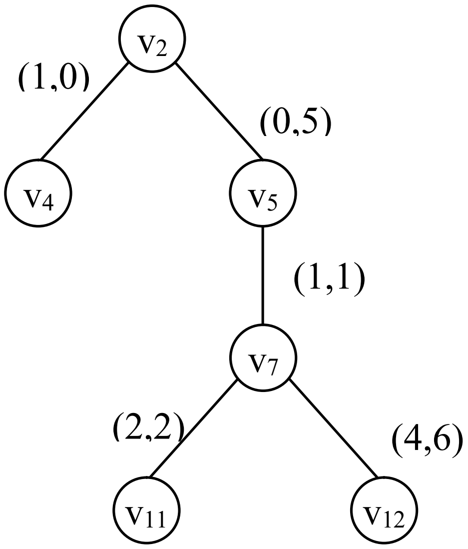


Figure 2.114

Thus G is not simple. It has also other subgraphs got by removing some edges. Edge removing always entail in hyper subgraphs when done appropriately.

However usual vertex subgraphs cannot satisfy this.

In view of this we have the following theorem.

Theorem 2.4. *Let G be a Plithogenic neutrosophic graph. There exist an Plithogenic edge hyper subgraph and Plithogenic pseudo edge hyper real subgraph.*

Proof. Let G be the given Plithogenic neutrosophic graph. Obtain H a edge subgraph of G by removing all real edges from G that is H = G \ {real edges of G}. Then H is the Plithogenic unique pseudo neutrosophic hyper subgraphs of G.

Let K = G \ {all neutrosophic edges of G}; that is K is got by removing all neutrosophic edges of G. Then K is the Plithogenic pseudo hyper real subgraph of G.

Hence the claim.

So we have the following corollary.

Corollary 2.1: All Plithogenic neutrosophic graph are not edge simple or edge pseudo simple.

Proof. Obvious from the above theorem.

Thus when we discuss about Plithogenic neutrosophic hyper graphs we have simple and edge simple together with pseudo simple and pseudo edge simple. An interesting study is to find those Plithogenic neutrosophic graphs which are both not simple and edge simple. Can we characterize such Plithogenic neutrosophic graphs?

Here it is pertinent to keep on record that the notion of Plithogenic neutrosophic graphs can be replaced by Plithogenic neutrosophic complex graphs or Plithogenic dual complex graphs and so on and so forth. Study and research in this direction is both interesting and innovative.

Next we proceed onto study edge-vertex subgraphs of Plithogenic subgraphs. Before we make such a study we first describe edge vertex subgraph of usual graphs and define and describe them in case of classical graphs.

Example 2.50: Let G be a graph given by the following figure.

G =

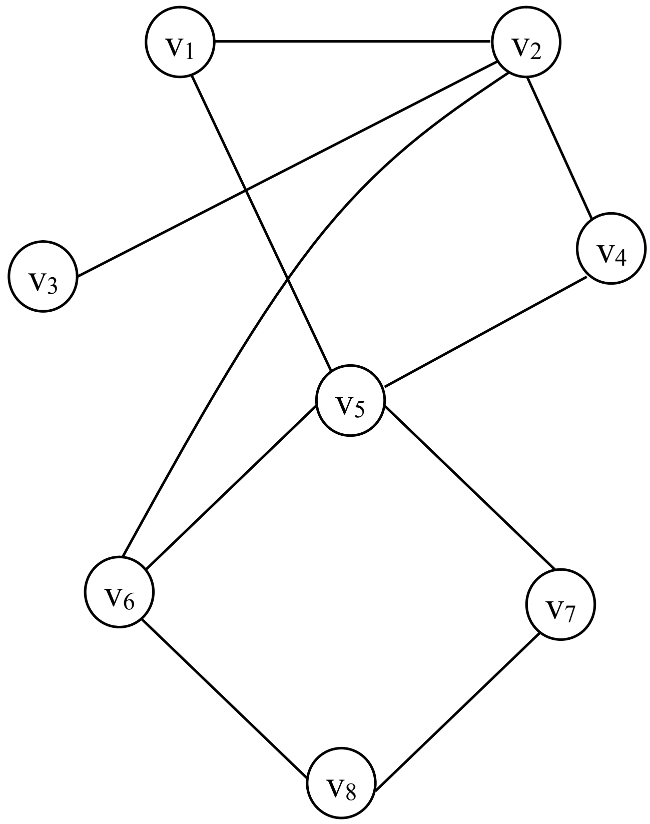


Figure 2.115

Let H be an edge-vertex subgraph got by removing the edges v_6v_8 and v_7v_8 and the vertices v_1 and v_2.

H =

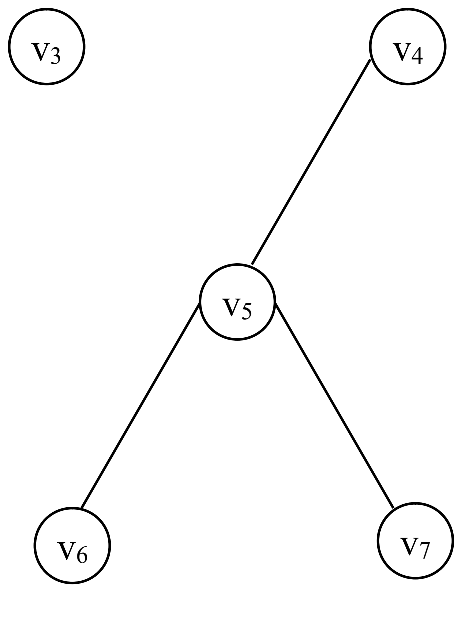


Figure 2.116

We see the vertex subgraph H_1 of G got by removing the vertices is

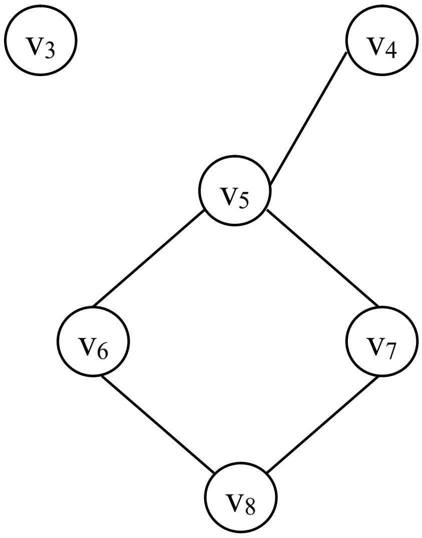
H_1 = 

Figure 2.117

The vertex subgraph H_1 of G.

Let H_2 be the edge subgraph got by removing the edges v_7v_8 and v_6v_8.

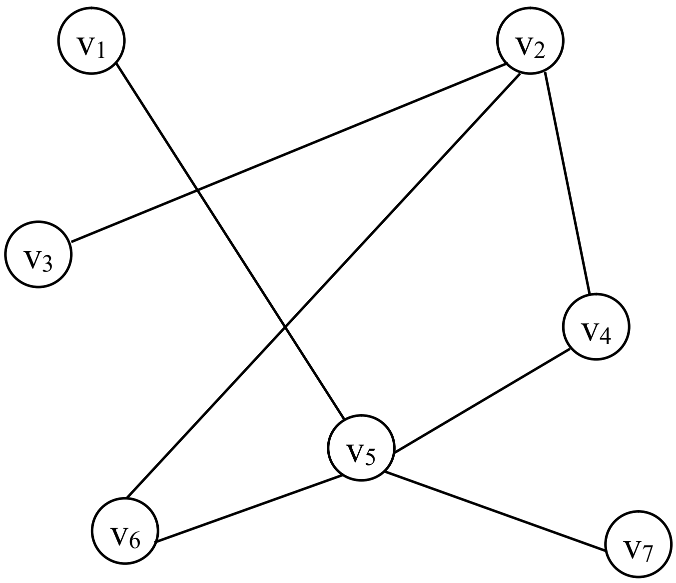
H_2 = 

Figure 2.118

We see H_2 is connected but H_1 is disconnected. However, H is also a disconnected subgraph.

We provide a few more examples of them.

Example 2.51: Let G be a Plithogenic star graph given by the following figure.

G =

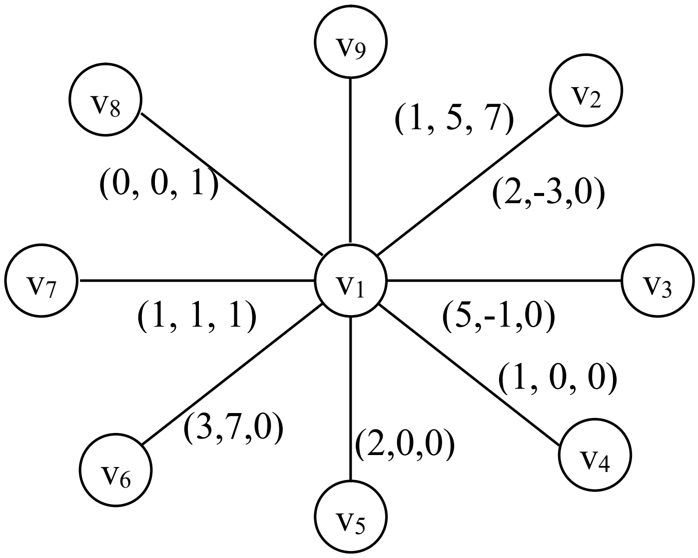


Figure 2.119

Suppose H_1 is an edge subgraph of G got by removing edges v_1v_9 and v_1v_2. $H_1 = G \setminus \{v_1v_9, v_1v_2\}$ is given by the figure.

H_1 =

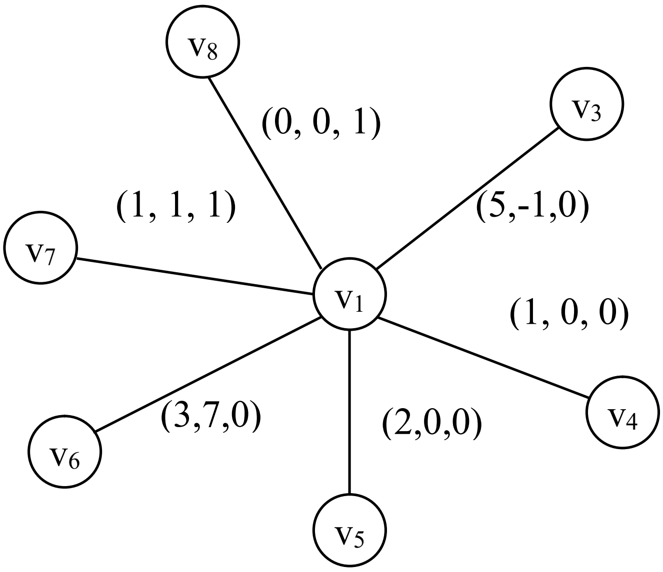


Figure 2.120

Let H_2 be the vertex subgraph by taking away the vertex sets v_2 and v_9. H_2 the vertex subgraph of G is given by the following figure.

H_2 =

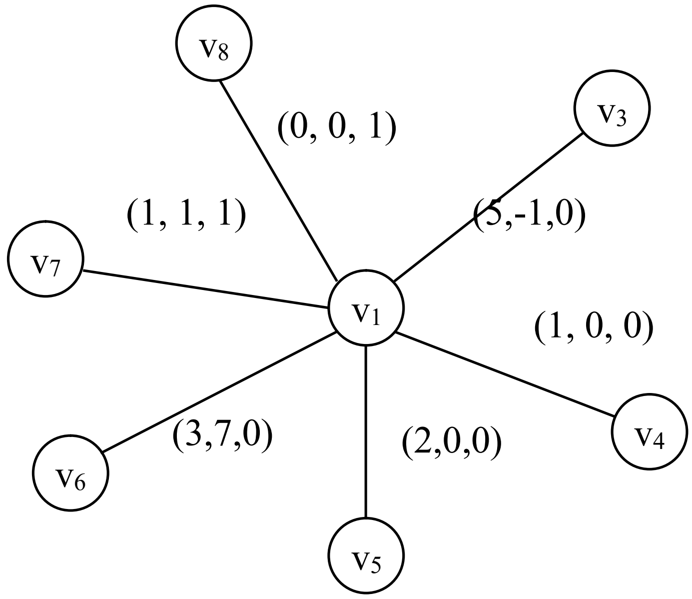


Figure 2.121

Let H_3 be the edge-vertex subgraph of G got by removing the edges v_1v_2 and v_1v_9 and the vertices v_2 and v_9.

H_3 is given by the following figure.

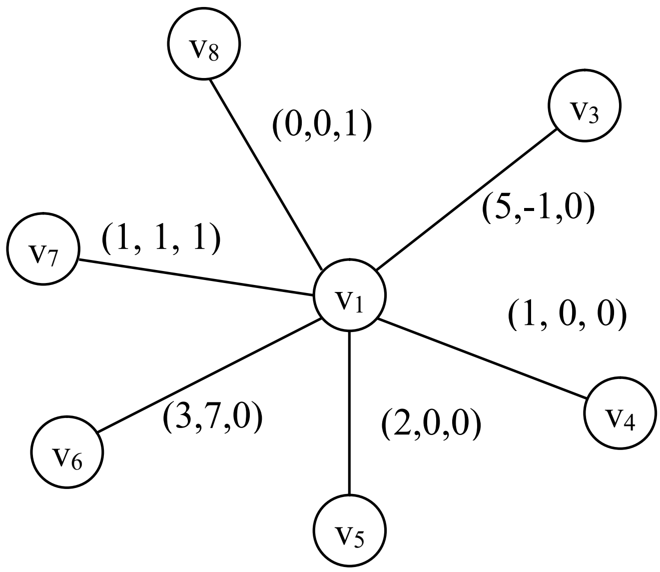


Figure 2.122

We observe that all the three subgraphs H_1, H_2 and H_3 are identical. Thus we see in this case the edge subgraph, vertex subgraph and the edge-vertex subgraph are identical.

Let K_1 be the edge vertex subgraph of G given by the following figure with vertex set v_1, v_3, v_5, v_7, v_9 and edges v_1v_2, v_1v_4, v_1v_6 are removed from G.

K_1 =

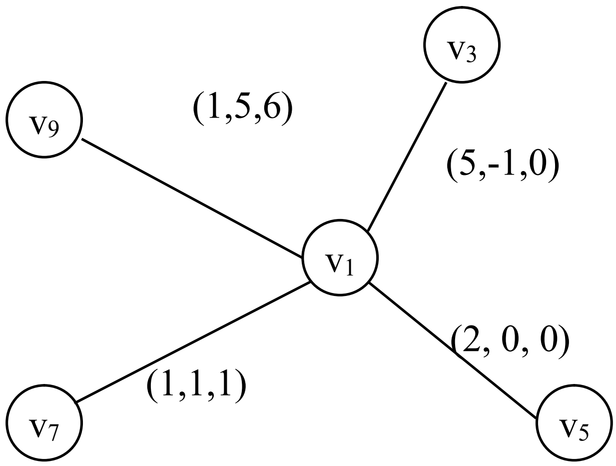


Figure 2.123

Now the vertex subgraph K_2 of G with vertex sets v_1, v_3, v_5, v_7, and v_9 is given by the following figure.

K_2 =

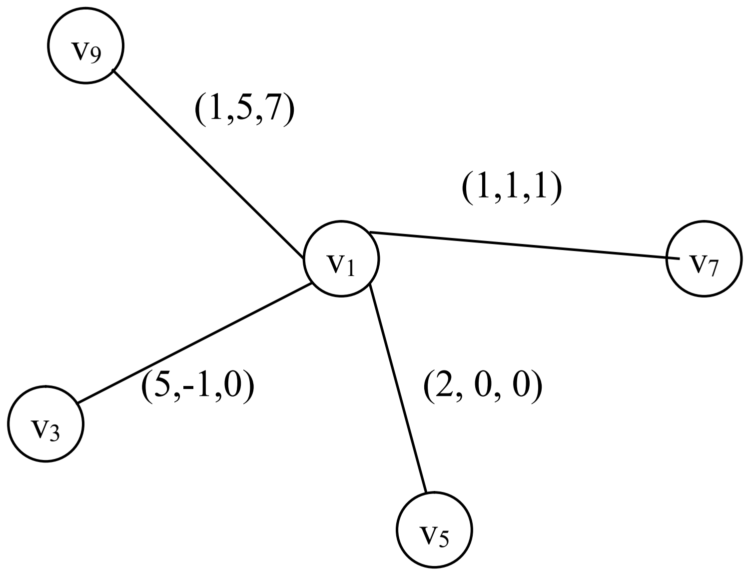


Figure 2.124

Let K_3 be the edge subgraph of G where $K_3 = G \setminus \{v_1v_2, v_1v_4, v_1v_6, v_1v_8\}$ given by the following figure.

K_3 =

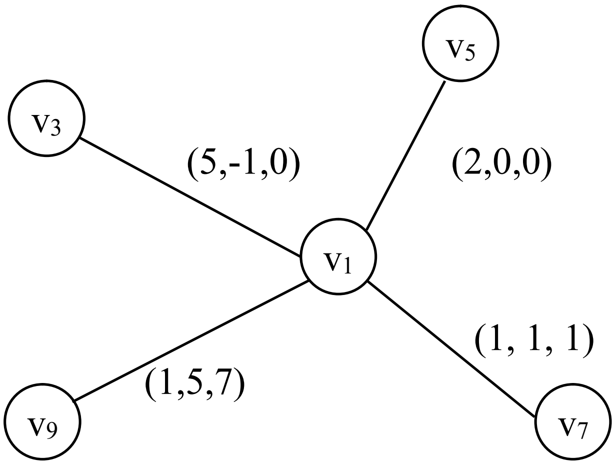


Figure 2.125

We see in this case also all the three subgraphs given in figures K_1, K_2 and K_3 are identical.

From these examples it is clear we can have edge-vertex subgraphs which are identical with edge subgraph and vertex subgraph. That is all the 3 subgraphs are identical.

So, it is left for the reader to find conditions for all the three subgraphs to different.

Example 2.52: Let G be a Plithogenic edge graph given by the following figure.

G =

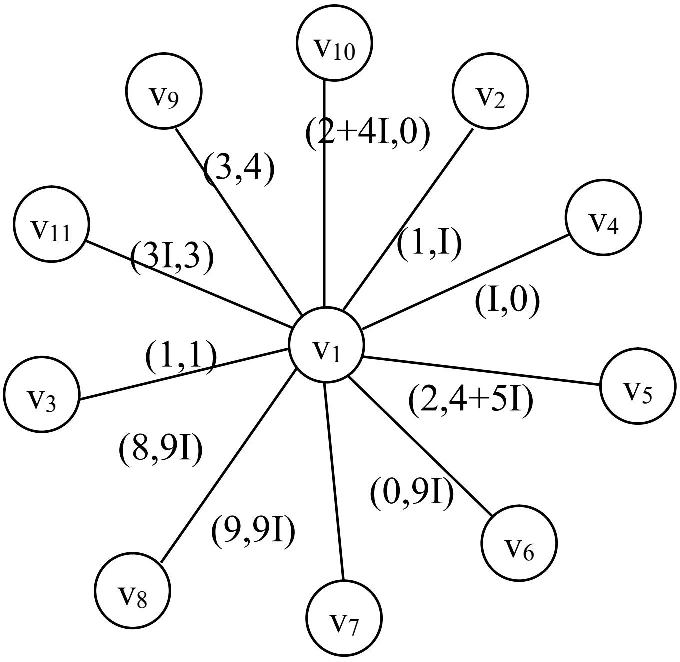


Figure 2.126

Let H_1 be a edge subgraph of G given by the following figure where $H_1 = G - \{v_1v_5, v_1v_9, v_1v_3\}$

H_1 =

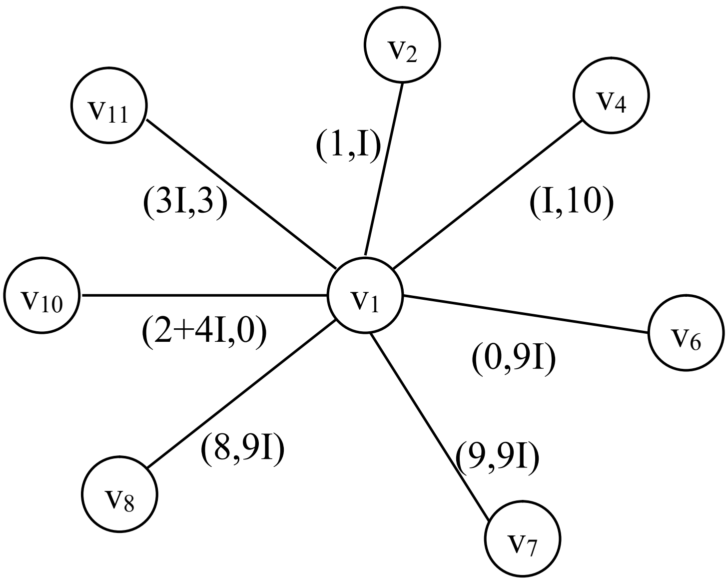


Figure 2.127

Let H_2 be the vertex subgraph of G using the vertices v_1, v_2, v_4, v_6, v_8, v_{10} and v_{11} which is given by the following figure.

$H_2 =$

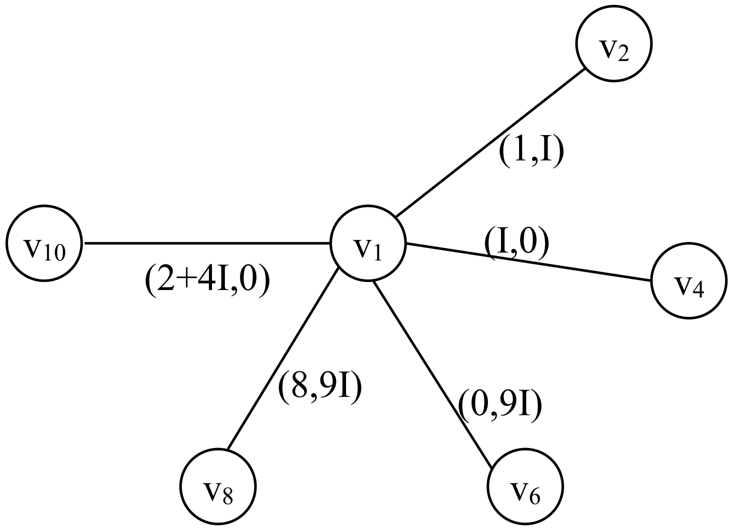


Figure 2.128

Clearly H_1 and H_2 are distinct. Now we find the edge vertex subgraph H_3 got by removing the edges v_1v_5, v_1v_9 and v_1v_3 and the

$H_3 =$

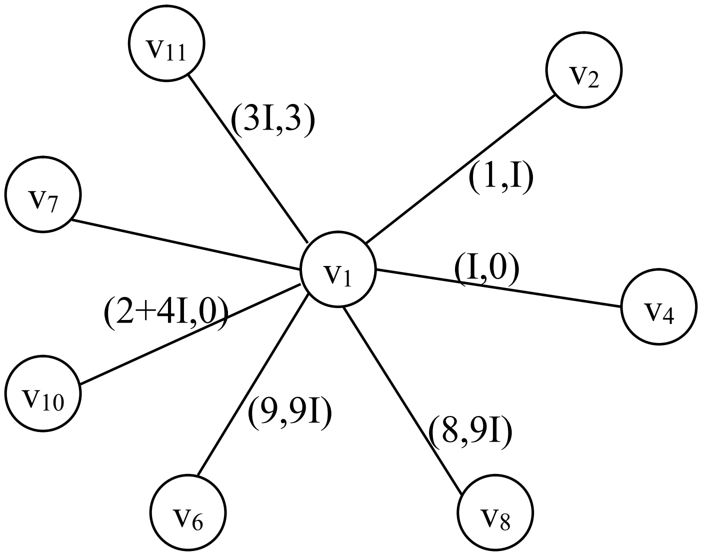


Figure 2.129

We see H_3 and H_1 are identical, however H_2 is different from H_1 and H_3.

But it is interesting to observe that H_2 is a subgraph of H_1 (as well as H_3 as $H_1 = H_3$).

Such situations are interesting, and one needs to study them.

Consequent of the above example we propose the following problem.

Problem 2.4. Let G be a special row weighted graph. Suppose H_1, H_2 and H_3 are the vertex subgraph, edge subgraph and edge-vertex subgraph respectively of G.

i) Find conditions under which all the three subgraphs H_1, H_2 and H_3 are distinct.

ii) Find conditions under which $H_1 = H_2 = H_3$ that is all the three subgraphs are identical.

iii) Find conditions under which one pair is identical and other is a subgraph of them.

The study in this direction is interesting and innovative. Author suggest that this can be done in case of special graph like star graph, ring graph, complete graph, tree etc.

Example 2.53: Let G be a special row weighted graph given by the following figure.

G =

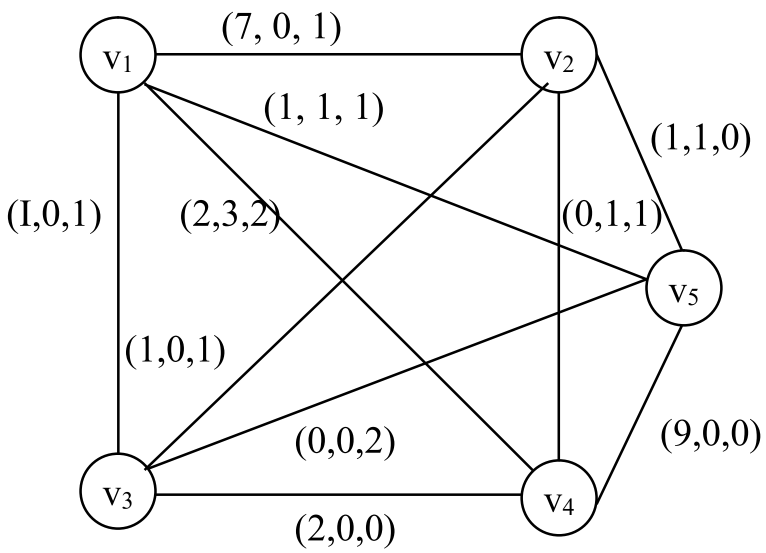


Figure 2.130

Let H_1 be the vertex subgraph of G using the vertex sets v_1,v_2,v_3 and v_5 given by the following figure.

H_1 =

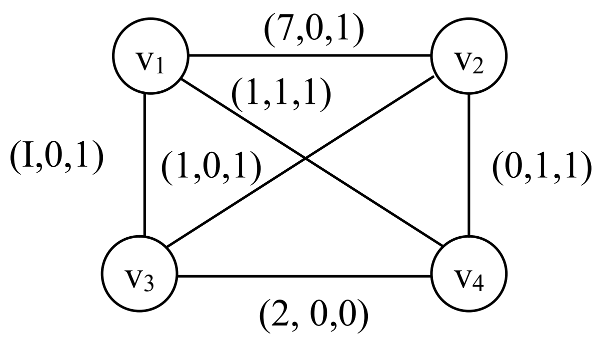


Figure 2.131

Let H_2 be the edge subgraph of G given by G \ $\{v_1v_2, v_3v_4, v_5v_1, v_3v_5\}$ given by the following figure.

$H_2 =$

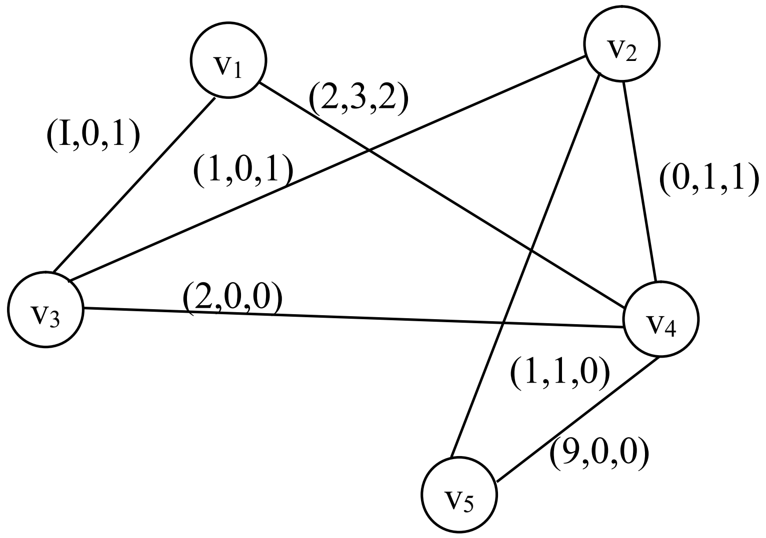


Figure 2.132

We see H_1 and H_2 are entirely different. Let H_3 be the edge vertex subgraph of G given by the following figure. H_3 has vertex set v_1, v_2, v_3 and v_5 and edges v_1v_2, v_3v_4, v_5v_1 and v_3v_5 are removed.

$H_3 =$

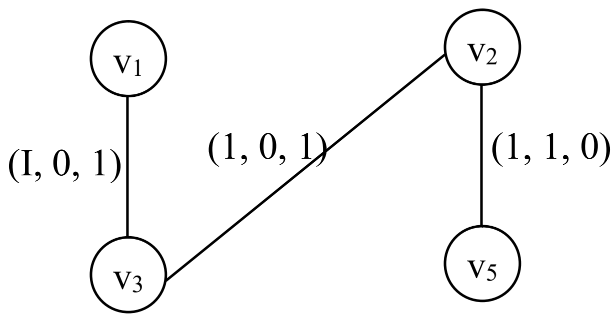


Figure 2.133

Clearly H_3 is distinct from H_1 and H_2; all the three subgraphs H_1, H_2 and H_3 of G are distinct. So we can have situations under which all the 3 subgraphs, viz; edge - special row matrix weighted subgraph, vertex special row vector weighted subgraph and edge vertex special row vector weighted subgraph are distinct, situations in which two of them identical and situations in which all the 3 subgraphs are identical.

Study in this direction is interesting and it is difficult to characterize the 3 situations mentioned above.

We can define path walk etc and give the value of them as sum of the row vectors. We just illustrate this situation by an example or two.

Example 2.54: Let G be special row weighted graph given by the following figure.

G =

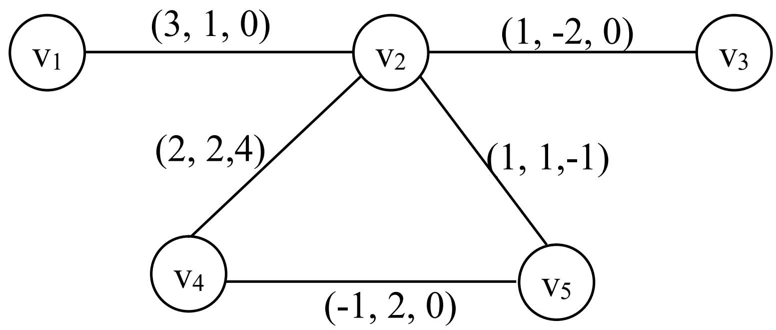


Figure 2.134

We see for this G, $v_1\ v_2\ v_5\ v_2\ v_3$ is a walk.

Suppose we want to find the total weight or total cost or the sum of weight of row vectors associated with G we get it as

$$(3, 1, 0) + (1, 1, -1) + (1, 1, -1) + (1, -2, 0)$$

$$= (6, +1, -2) \qquad \text{(w)}$$

For the same G the path is $v_1v_2v_5v_4$.

Now the same row weight of the path $v_1v_2v_5v_4$ is ;

$(3, 1, 0) + (1, 1, –1) + (–1 , 2, 0)$

$$= (3,4, –1) \qquad (p)$$

We see w and p are distinct row weights.

Next we proceed onto find the trail of this G. $v_1v_2v_5v_4v_2v_3$ is a trail and this is not a path.

The row weight vector associated with this trail is

$(3, 1, 0) + (1, 1, –1) + (–1, 2, 0) + (2, 2, 4) + (1, –2, 0)$

$$= (6, 4, 3) \qquad (t)$$

Clearly the values s, t, w and p are different.

The cycle of G is given by $v_2v_4v_5v_2$ and the row weight vector associated with G is given by

$(2, 2, 4) + (–1, 2, 0) + (1, 1, 1)$

$$= (2, 5, 3) \qquad (c)$$

Clearly the row weights of c, t, p and w are distinct.

Finding the cost or time or distance weights in a Plithogenic graphs give vector as answer for the value of path or trail or so on.

Clearly the G we have used is not a directed graph certainly in case of directed Plithogenic graphs they will vary in a very different way.

Study in this direction is interesting. Just for the sake of completeness we just recall the definition of path, walk etc in case of directed graph G.

A directed walk in a directed graph G is an alternating sequence of points and arcs, v_0, x_1, v_1, …, x_n, v_n. The length of this directed walk is n, the number of occurrences of arcs in it.

A closed walk has the same first and last points and the spanning walk contains all points.

A path is a walk in which all points are distinct and a cycle is a nontrivial closed walk with all points distinct (except the first and last).

A semi walk is again an alternating sequence v_0,x_1,v_1, … x_n, v_n of points.

Study in this direction is interesting as to find uniform cost or cost we may use these notions.

Further the study of directed Plithogenic graphs for these properties may give innovative results.

Finally, one can call these graphs when used in networks as Plithogenic networks.

The main aim of defining and developing the concept of Plithogenic graphs are to use this in single valued neutrosophic graphs, neutrosophic quadruples, neutrosophic refined sets and finally in the study of graphs for plithogenic sets recently developed by Smarandache [44].

We develop in the next chapter the line graph of Plithogenic graphs and analyse them.

Single valued neutrosophic graphs have been studied by [5, 6]. They are nothing but these Plithogenic graphs where edge weights are (a_1, a_2, a_3) with $a_i \in [0, 1]$; $1 \leq i \leq 3$.

On similar lines we can define Plithogenic quadruple neutrosophic graphs whose edge weights $(a, b, c, d) \in R^4$ or $[0, 1]^4$.

Study in this direction is still open interested researcher can pursue it.

We now suggest some problems to the reader.

Problems

1. Give an example of a integer 1×5 Plithogenic graph with 4 vertices.
2. How many integer 1×3 Plithogenic graphs with 3 vertices can be drawn? (Justify your claim).
3. Show the number of real 1×3 Plithogenic graphs with 3 vertices is greater than the number of integer Plithogenic graphs with 3 vertices (Prove your claim).
4. Give an example of a real 1×7 Plithogenic star graph with 18 vertices.
5. Give an example of a 1×5 Plithogenic binary tree with 5 layers.

6. Give an example of a circle 1 × 3 Plithogenic graph with 9 vertices.

7. Give an example of a 1 × 6 Plithogenic wheel G with 9 vertices and find all subgraphs of G.

8. Let G be a 1 × 2 Plithogenic graph given by the following figure.

G =

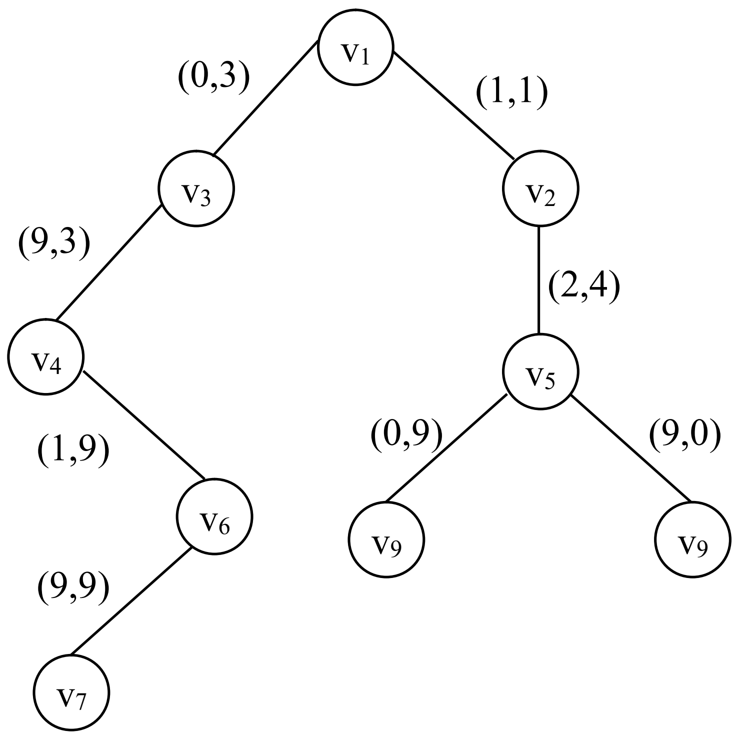


Figure 2.135

i) Find all Plithogenic subtrees of G.

ii) How many of the Plithogenic subgraphs are disconnected?

iii) How many Plithogenic edge subgraphs of G are disconnected?

iv) Find a walk from v_9 to v_8.

v) Find a path from v_6 to v_6.

vi) Find a trail from v_1 to v_8.

vii) Can G have a clique?

9. Let G be a complete 1 × 3 Plithogenic graph of order 7.

i) Can we say all Plithogenic subgraphs of G are also complete? (Justify your claim).

ii) Find all edge Plithogenic subgraphs of G.

iii) How many in (ii) are complete?

iv) Find all disconnected Plithogenic subgraphs of G.

10. Let G be a Plithogenic graph given by the following figure.

G =

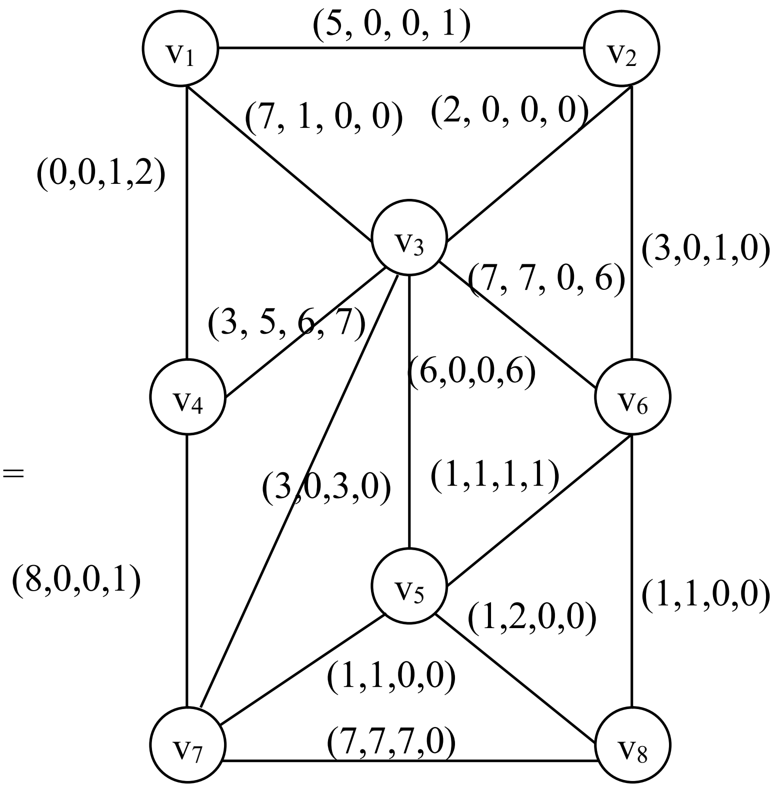


Figure 2.136

i) Answer all the questions (i) to (vii) of problem 8.

ii) Find the largest clique of G.

iii) How many Plithogenic triads are in G?

iv) Find all adjacent triads in G.

11. Let G be a Plithogenic neutrosophic graph given by the following figure.

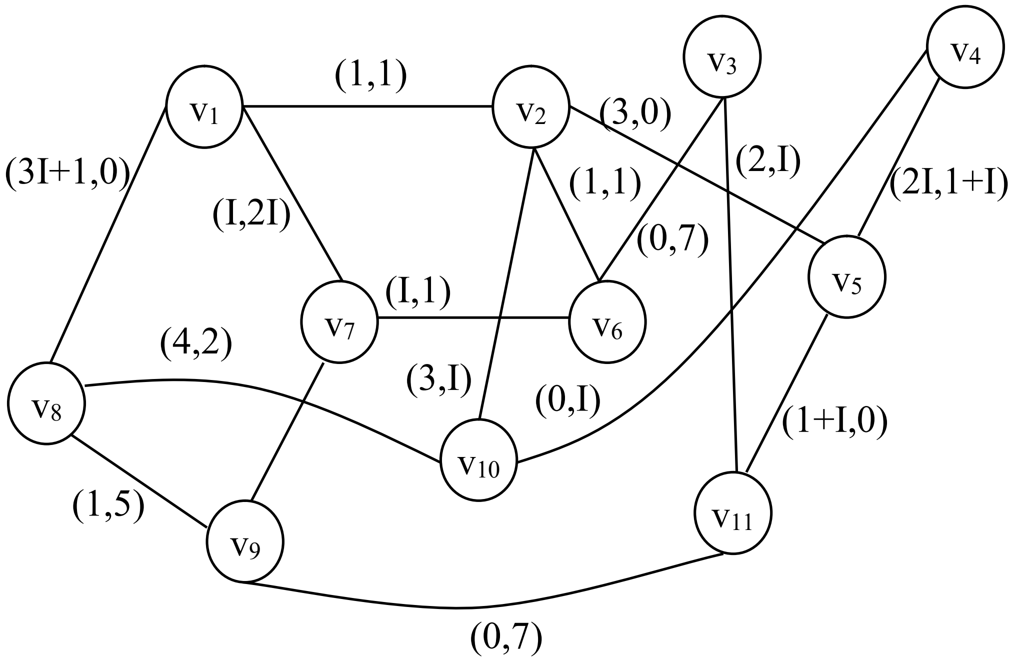


Figure 2.137

i) Study all questions (i) to (vii) of problem 8.

ii) Find all Plithogenic neutrosophic subgraphs of G which have only neutrosophic row vectors.

iii) Find all Plithogenic subgraphs of G which has only real row vector weights.

iv) Find the clique of G?

v) What is the order of the clique?

13. Give some real-world problem applications of these Plithogenic graphs.

14. Find for the G in problem (11) the adjacency matrix.

15. What will be the form of the distance matrix in case of Plithogenic graphs?

16. For the graph G given in problem (11) can one get the largest Plithogenic neutrosophic subgraphs?

    (Is it possible as subset Plithogenic subgraph or only as Plithogenic edge subgraph?)

17. For G in problem (11) find a walk, path and trail from the vertex v_1 to v_2.

18. Find for a Plithogenic neutrosophic tree with 5 layers and 20 nodes which has no Plithogenic subset neutrosophic subgraphs.

19. Can these Plithogenic graphs find applications in social network problems?

20. How are Plithogenic graphs different from single valued neutrosophic graphs?

Chapter Three

PLITHOGENIC VERTEX GRAPHS

In this chapter we define the new notion of Plithogenic vertex graphs where the vertices are labelled with row matrix. Study of these graphs are very essential for the case of single valued neutrosophic graphs and Plithogenic vertex graphs.

Example 3.1. Let G be Plithogenic vertex graph given by the following figure. The row matrix for vertex and edge labels are of the same order.

G =

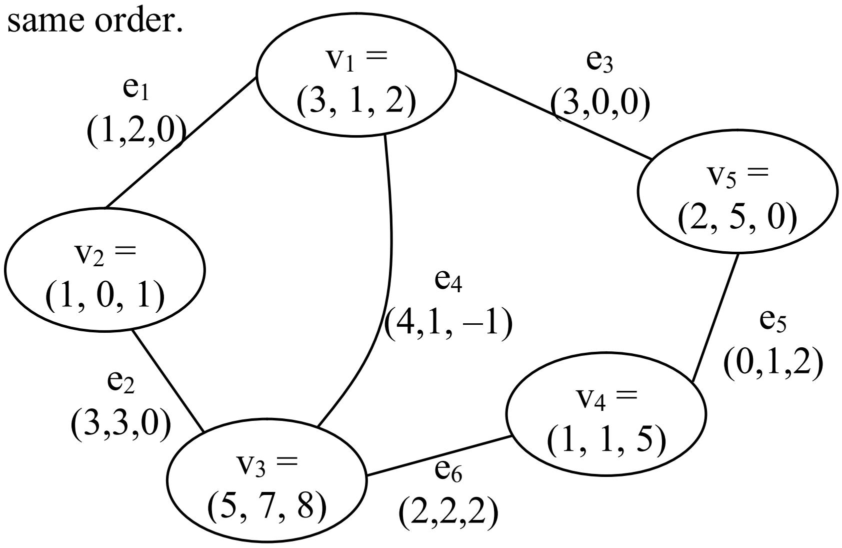


Figure 3.1

Example 3.2. Let G be a Plithogenic vertex graph given by the following figure.

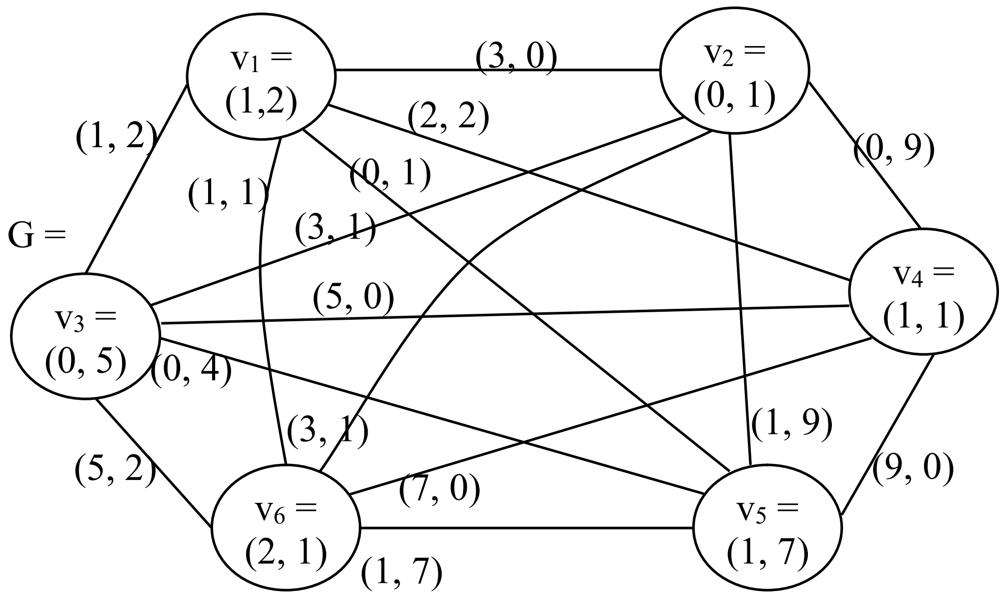


Figure 3.2

We see G is a Plithogenic vertex graph with labels of same order, we call it as same Plithogenic vertex graph.

Example 3.3. Let G be a special graph,

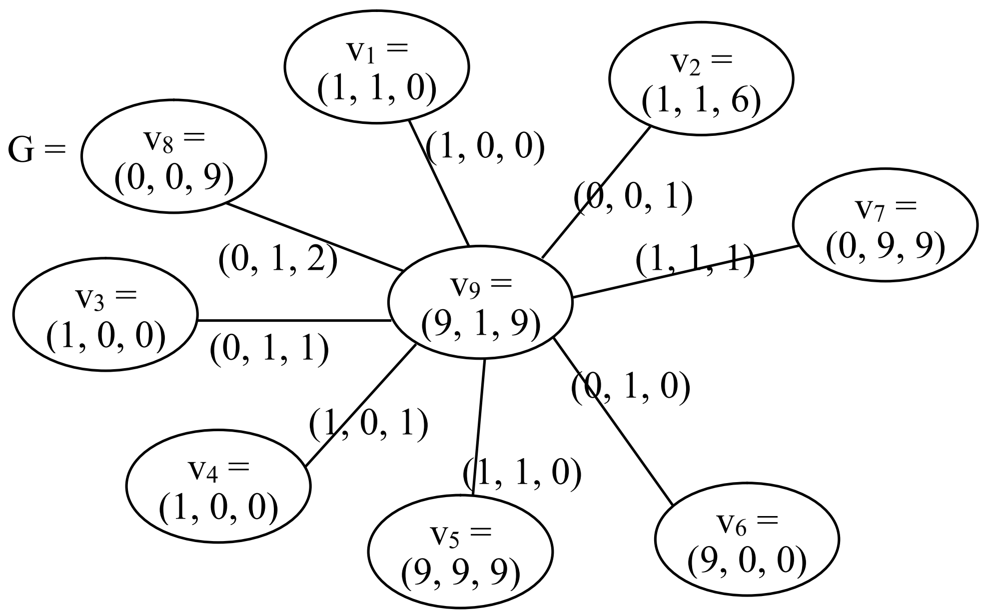


Figure 3.3

Clearly G is a same Plithogenic vertex star graph.

Example 3.4. Let G be a same Plithogenic vertex ring or circle graph given by the following figure.

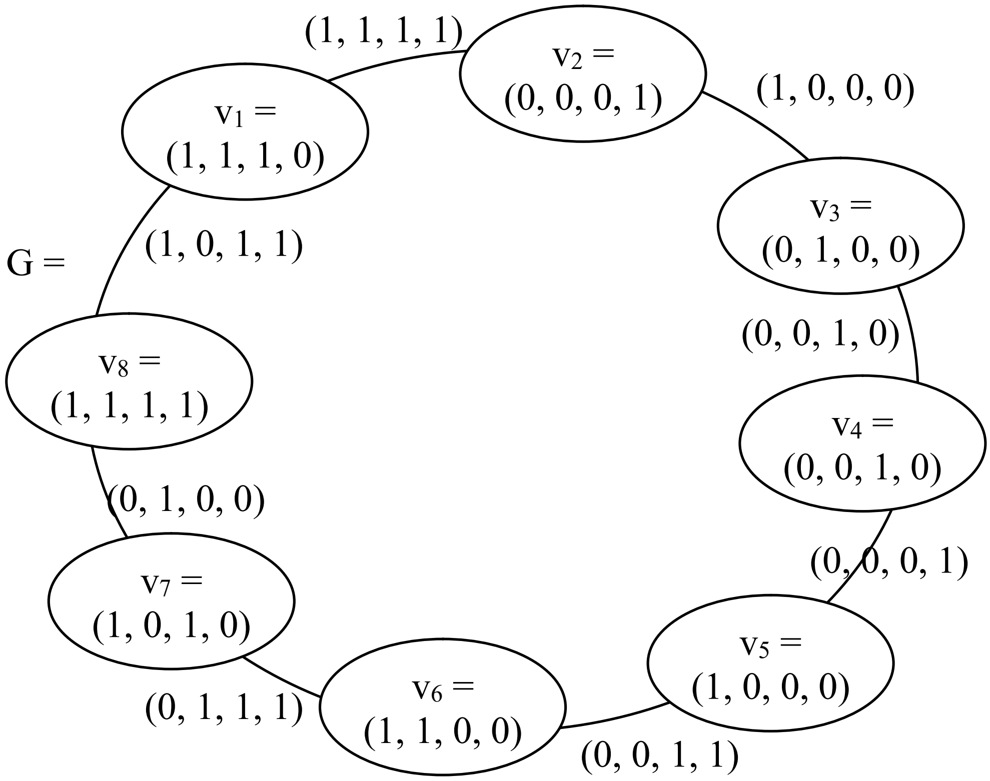


Figure 3.4

Similarly, we can have the same Plithogenic vertex line graph which is given by the following figure.

Example 3.5. Let L be the same Plithogenic vertex graph given by the following figure.

L =

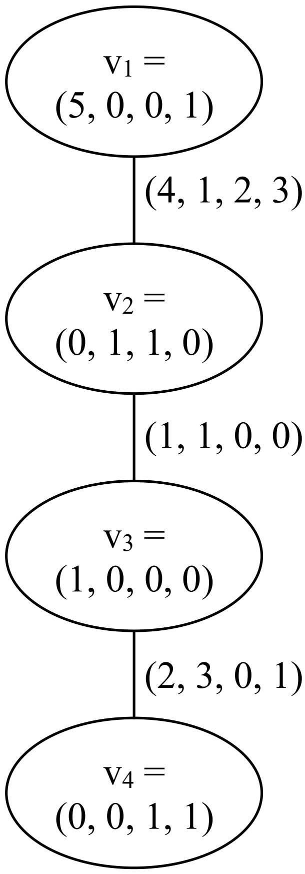


Figure 3.5

Now we make the formal definition of the same.

Definition 3.1. *Let G be the graph where vertex sets $v_1, v_2, \ldots, v_n$ are labeled values form the collection of $1 \times m$ row vectors where $M = \{(a_1, \ldots, a_m) \mid a_i \in R$ or C or Z or $\langle R \cup I\rangle$ or Z_n or $C(Z_n)$ or $\langle Z_n \cup I\rangle\}$. The edges are also labeled from the elements of M. We call such graph G as Plithogenic vertex graph.*

Researchers / readers may wonder about the use of such graphs. Basically they are graphs related with intuitionistic fuzzy graphs and single valued neutrosophic graphs.

We will be elaborately discussing in the final chapter of this book the applications of these Plithogenic vertex graphs in several models.

First we define various types of Plithogenic vertex graphs. In Definition 3.1 we have defined the new notion of Plithogenic vertex graphs. We have also given examples of them. The edges are given values with no fixed way it is left at the choice of the researchers.

Sometimes they may represent a social network. We have still not given the directed same Plithogenic vertex graph.

We now proceed onto supply some examples of special subgraphs of these graphs.

Example 3.6. Let G be a same Plithogenic vertex graph given by the following figure.

G =

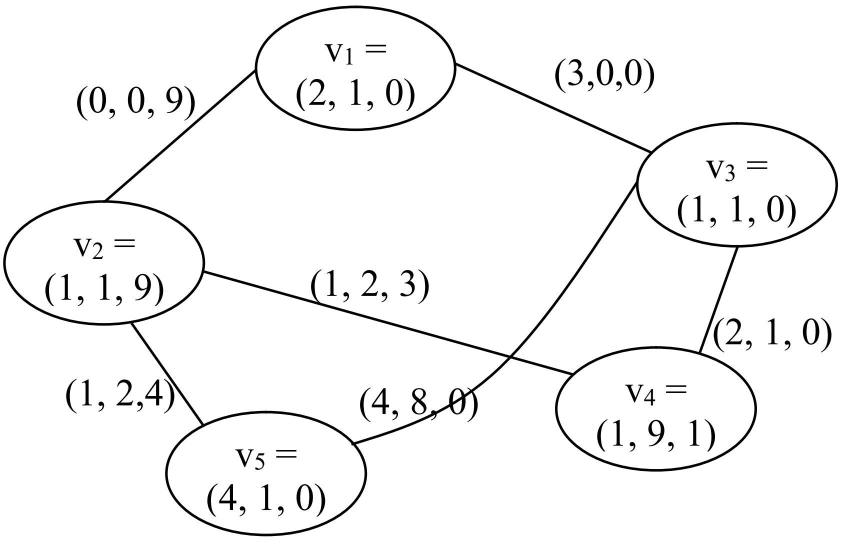


Figure 3.6

The subgraphs of G are

H_1 =

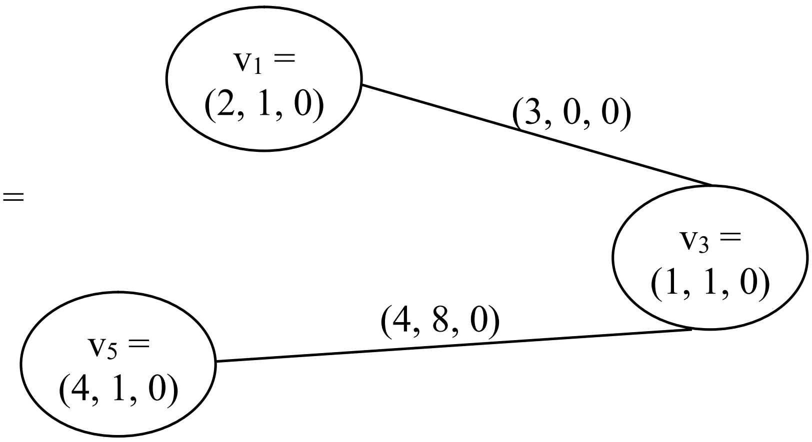


Figure 3.7

The subgraph H_1 is again a same Plithogenic vertex graph.

H_2 be a subgraph of G given by the following figure.

H_2 =

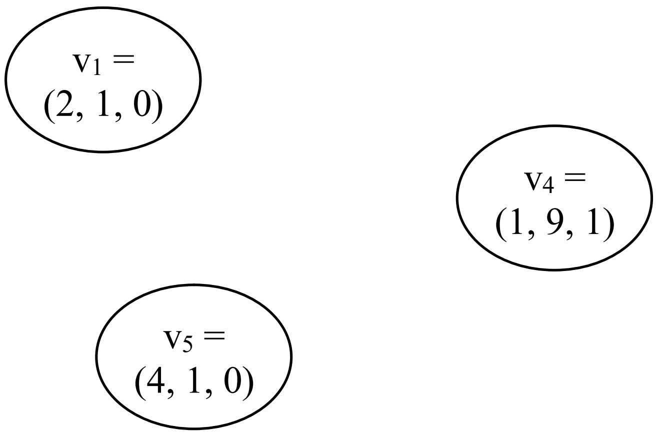


Figure 3.8

Clearly H_2 is an empty same Plithogenic vertex subgraph of G.

Let H_3 be the special subgraph of G given by the following figure.

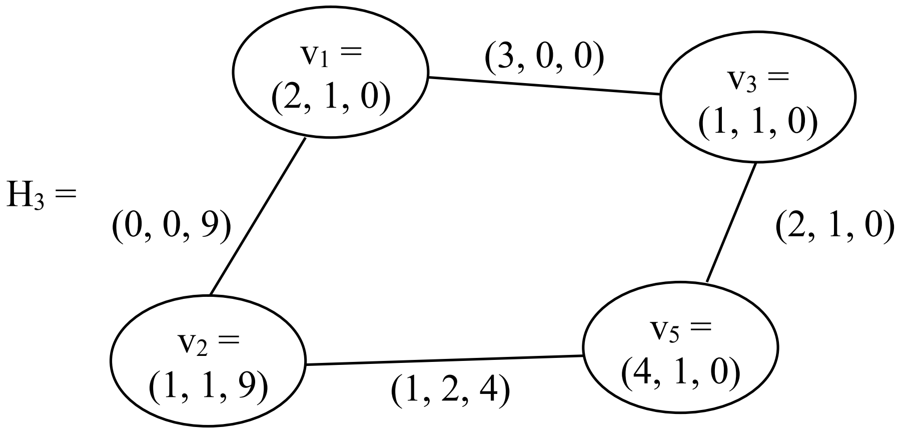


Figure 3.9

H_3 is a same Plithogenic vertex subgraph of G. G two such rings but G has no cliques.

The only maximal subgraphs of G just dyads and no proper complete subgraphs.

Example 3.7. Let G be a same Plithogenic vertex graph given by the following figure.

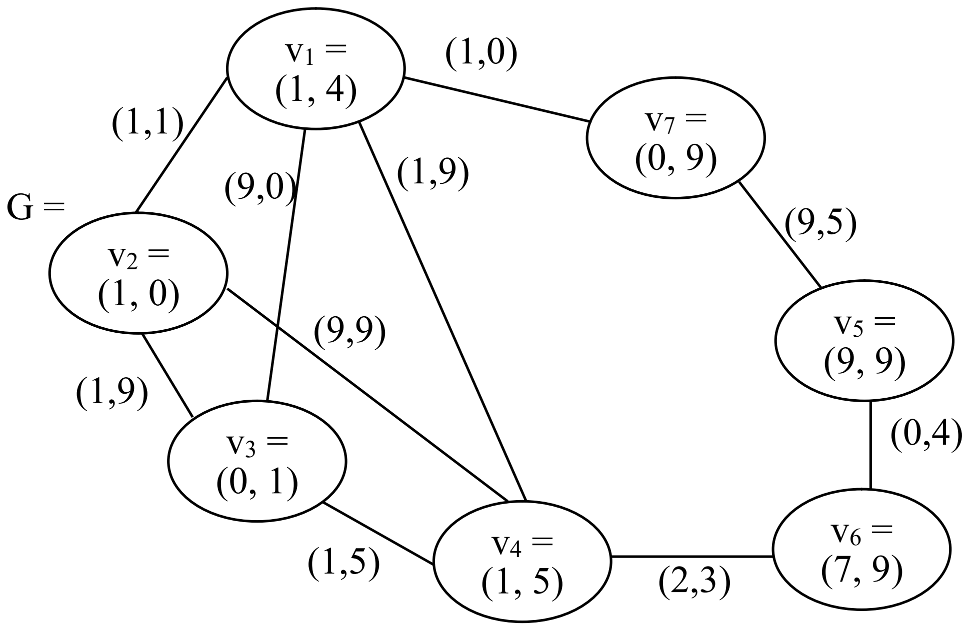


Figure 3.10

G is a same Plithogenic vertex graph.

This has a clique of order four.

We define a clique as a maximal complete subgraph of G. It need not necessarily be a maximal subgraph.

For the maximal subgraphs of G are given by the vertex sets $\{v_1, v_2, v_3, v_4, v_5, v_6\}$, $\{v_1, v_2, v_3, v_4, v_5, v_7\}$, $\{v_1, v_2, v_3, v_4, v_6, v_7\}$, $\{v_1, v_2, v_3, v_5, v_6, v_7\}$, $\{v_1, v_5, v_3, v_4, v_6, v_7\}$, $\{v_1, v_2, v_4, v_5, v_6, v_7\}$ and $\{v_2, v_3, v_4, v_5, v_6, v_7\}$.

However none of them can contribute for a complete subgraph of G and G has only one subgraph of order four which is complete given by

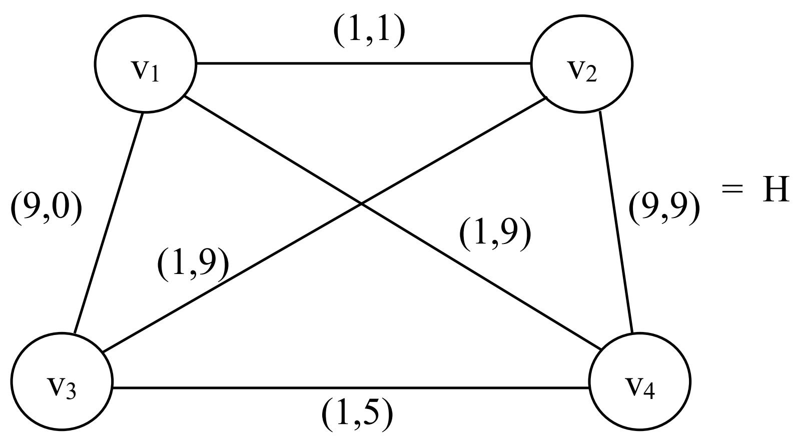


Figure 3.11

H is a clique of order four however it is not a maximal subgraph of G.

We see some graphs can have more than one clique of same order.

We give an example of the same.

Example 3.8. Let G be a graph given by the following figure.

G =

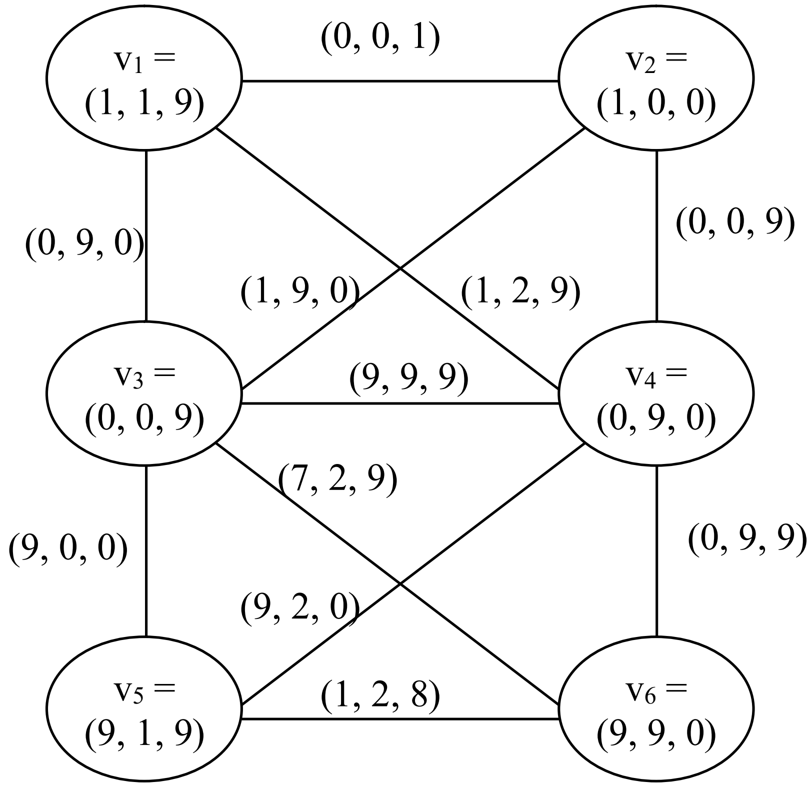


Figure 3.12

Clearly G is a Plithogenic vertex graph.

This two subgraphs H_1 and H_2 of G of order four which are complete. They are as follows.

H_1 =

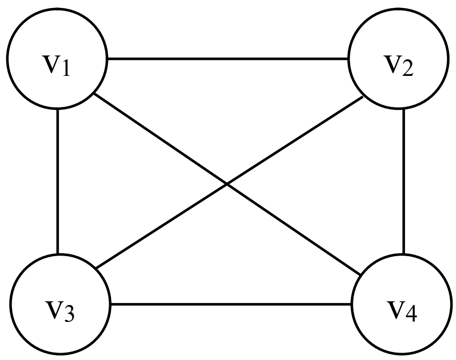


Figure 3.13

$H_2 =$

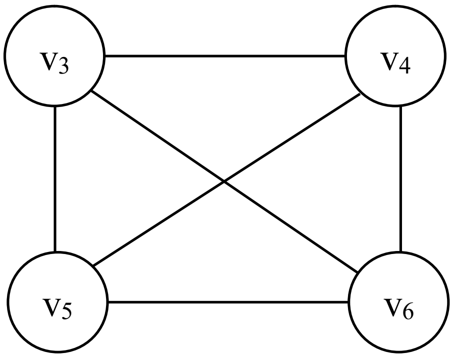


Figure 3.14

Let H_3 be the subgraph of G with 5 vertices given by the following figure.

$H_3 =$

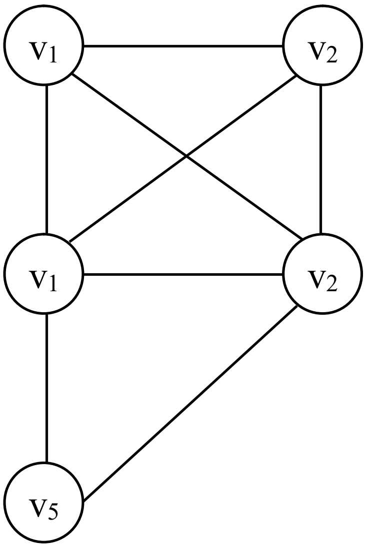


Figure 3.15

We see this has 5 vertices however it is not a clique but has a clique of order four.

Let H_4 be the subgraph of G given by the following figure.

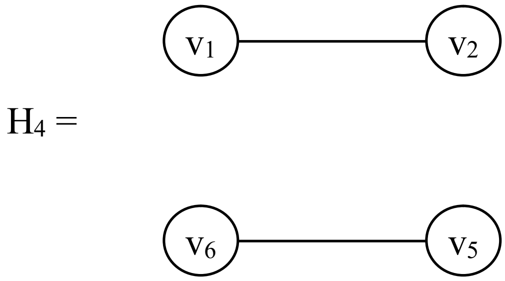


Figure 3.16

H_4 is a disconnected subgraph of G.

Now we proceed onto describe the edge removed Plithogenic vertex graphs of G by some examples.

Example 3.9. Let G be a same Plithogenic vertex graph given by the following figure.

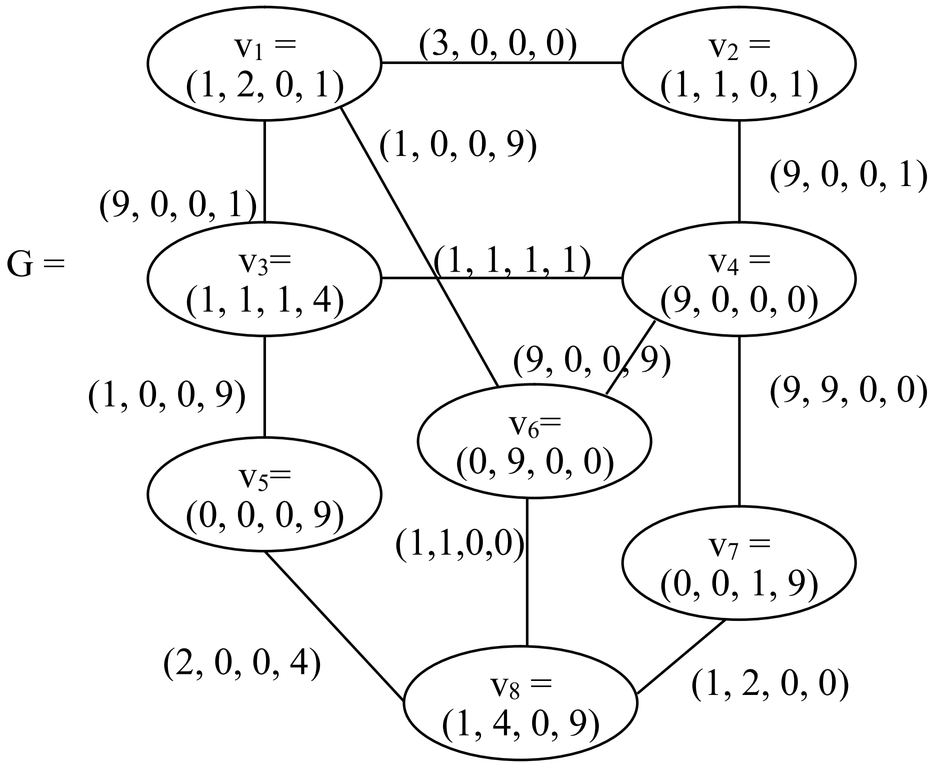


Figure 3.17

Suppose the edge v_1v_2 with associated row matrix (3, 0, 0, 0) and the edge v_2v_4 with row matrix (9, 0, 0, 1) is removed. The resultant edge same Plithogenic vertex subgraph H_1 is as follows.

H_1 =

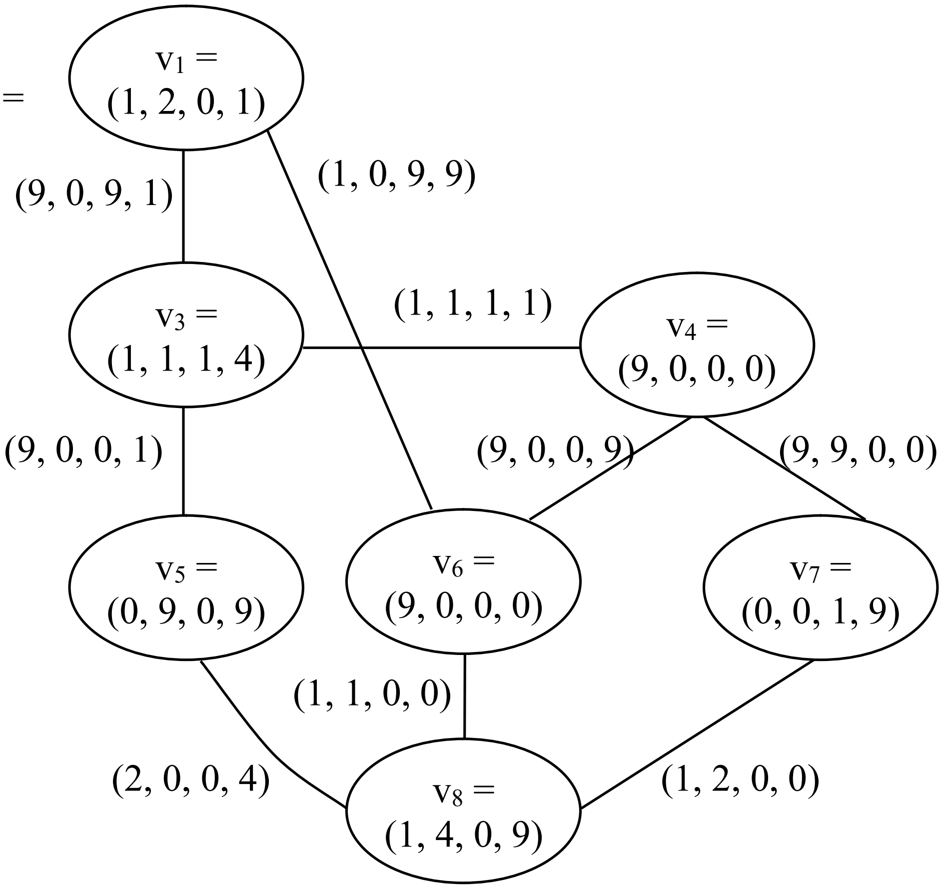


Figure 3.18

H_1 looses a vertex v_2.

So this is the same as vertex same Plithogenic vertex subgraph for which the vertex v_2 is removed.

Let H_2 be a Plithogenic vertex subgraph got by removing the edges (2, 0, 0, 4) (i.e. v_5v_8) and (9, 0, 0, 9) (i.e. v_6v_4).

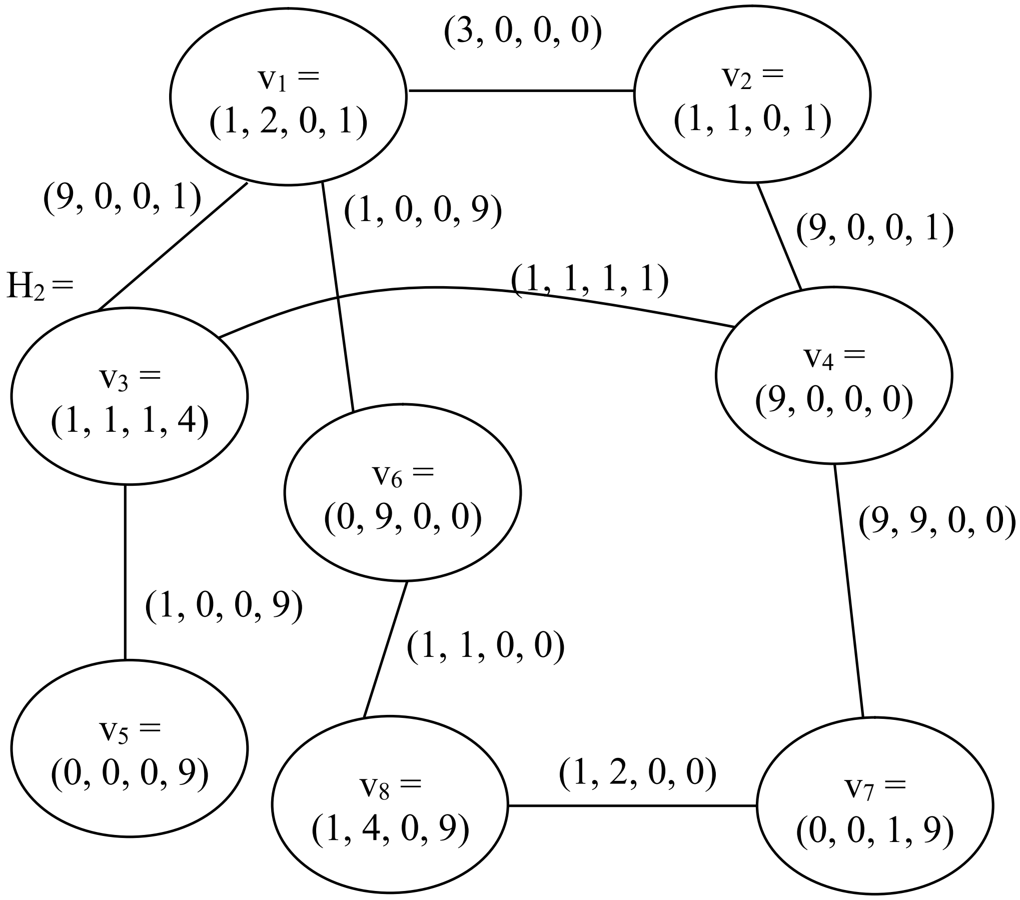


Figure 3.19

Clearly H_2 cannot be got as Plithogenic vertex subgraph of G. Thus, H_2 the edges subgraph is different from the vertex subgraph.

These edge-subgraphs play at times a vital role when the network can afford to lose some edges but not vertices.

Study in this direction is a matter of routine so left as an exercise to the reader. In a way we can say they are same as working with usual graphs.

So, we do not discuss the other properties. We can see the only difference being in the study of length of the walk or trail and so on where if we associated a final value to it.

That value will be again a row matrix.

Next, we find the adjacency matrix of a same Plithogenic vertex graph G.

We will first illustrate this situation by some examples.

Example 3.10. Let G be a same Plithogenic vertex graph given by the following figure.

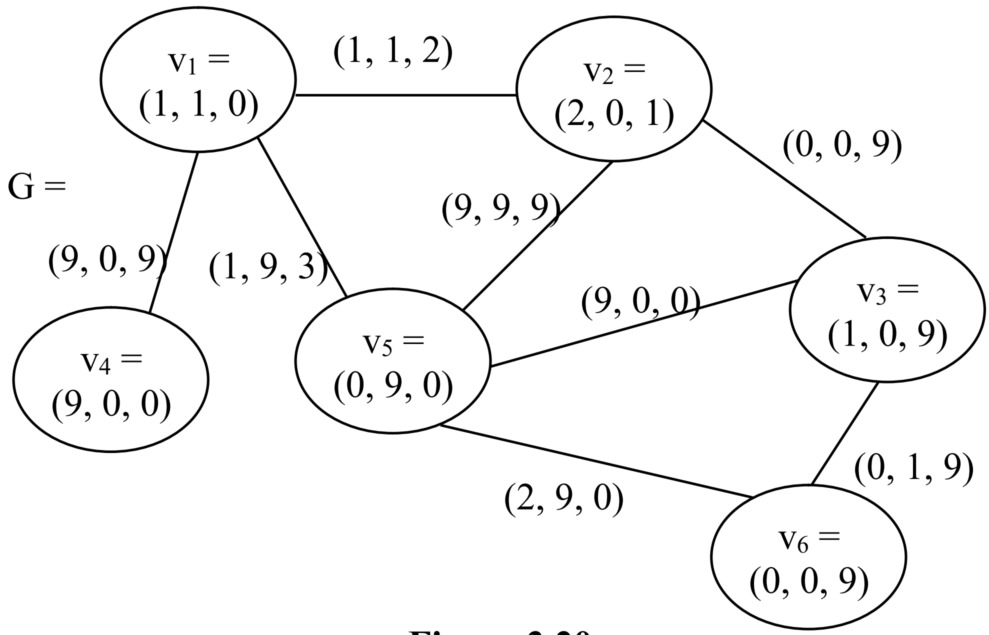


Figure 3.20

Let M be the adjacency matrix associated with G.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \end{array} \begin{array}{c} \begin{array}{cccccc} \quad v_1 \quad & \quad v_2 \quad & \quad v_3 \quad & \quad v_4 \quad & \quad v_5 \quad & \quad v_6 \quad \end{array} \\ \begin{pmatrix} (0,0,0) & (1,1,2) & (0,0,0) & (9,0,9) & (1,9,3) & (0,0,0) \\ (1,1,2) & (0,0,0) & (0,0,9) & (0,0,0) & (9,9,9) & (0,0,0) \\ (0,0,0) & (0,0,9) & (0,0,0) & (0,0,0) & (9,0,0) & (0,1,9) \\ (9,0,9) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) & (0,0,0) \\ (1,9,3) & (9,9,9) & (9,0,0) & (0,0,0) & (0,0,0) & (2,9,0) \\ (0,0,0) & (0,0,0) & (0,0,9) & (0,0,0) & (2,9,0) & (0,0,0) \end{pmatrix} \end{array}$$

We make the following observations:

1. The entries of the adjacency matrix M has its entries from the collection of 1×3 row matrices.

2. The diagonal entries are zero 1×3 matrices.

3. The matrix is symmetric.

So it is a matter of routine to find the adjacency matrix of any same Plithogenic vertex graph G.

However, if the vertices alone are row matrix labeled then we get the adjacency matrix as the usual matrix. We will illustrate this situation also by an example or two.

Example 3.11. Let G be a same 1×5 Plithogenic vertex graph given by the following figure.

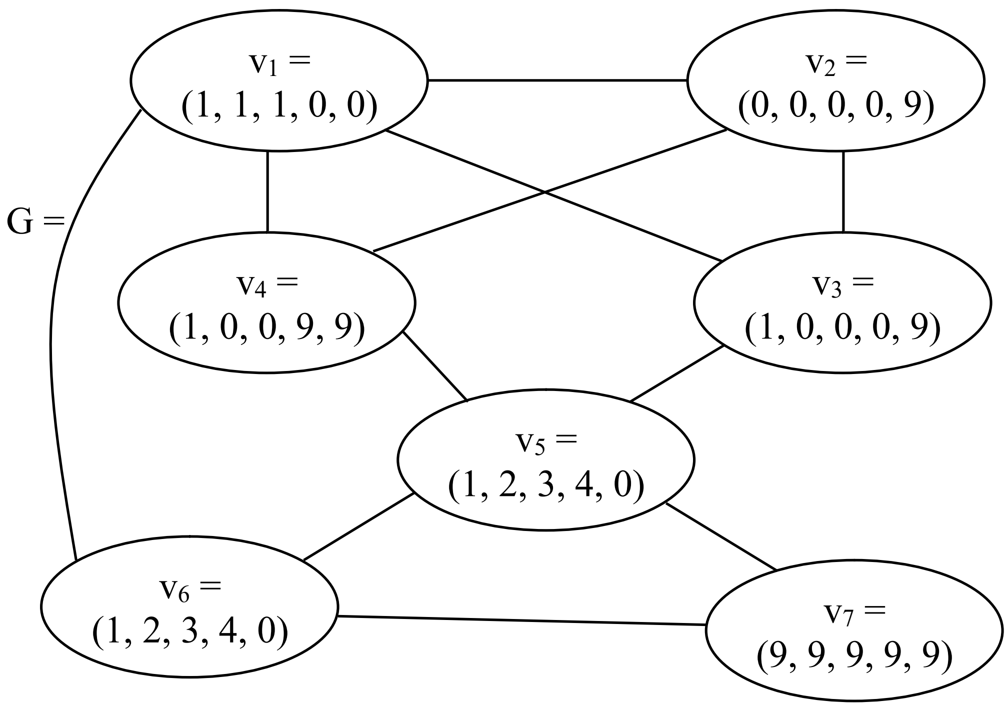


Figure 3.21

The adjacency matrix M associated with G is as follows.

$$M = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \end{array} \begin{array}{c} \begin{array}{ccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 \end{array} \\ \begin{pmatrix} 0 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 1 & 0 \end{pmatrix} \end{array}.$$

M is like any adjacency matrix. Clearly M is also symmetric. In case of Plithogenic vertex graph we do not get a different adjacency matrix. It happens to be identical like that of the usual graphs.

So, study of only vertex labeled with row matrices may find applications when we study them as multigraphs in some modeling.

However, we give applications of them in the final chapter of this book. We see all the graphs which are vertex labeled by row matrices (Plithogenic vertex) behave in a similar way as that of the usual simple vertex labeled graphs. But if we construct a special type of multigraphs with vertex labeled as row matrices the results and the outcome would be entirely different.

We will just illustrate by an example this situation.

Example 3.12. Let G be a row matrix vertex labeled graph given by the following figure.

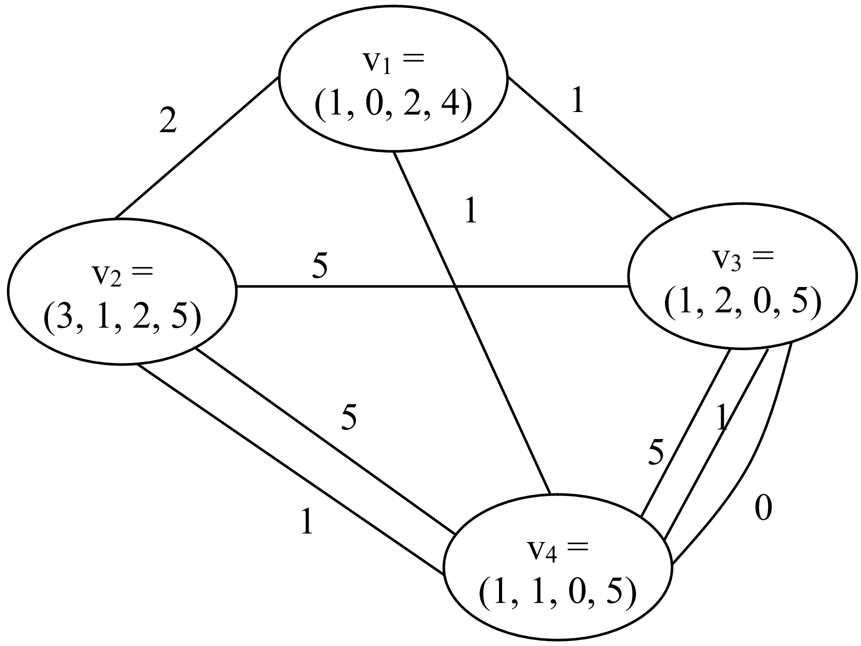


Figure 3.22

The mapping or edges are fixed depending on the elements of the row matrix to be identical we can have edges marked as zero also. So the above example is one such the resultant is a multigraph.

Clearly the maximum number of edges between any two vertices is only three. They are akin to subset vertex multigraphs but they are some what different for we can say the number of edges is P(S) where P(S) is the powerset of S. Now these type are a special type of multigraphs where the vertex sets are labeled as row matrices. In this situation it is not possible to get a complete uniform multigraph.

We can have only pseudo complete non uniform multigraph.

We will illustrate this situation by an example.

Example 3.13. Let G be a Plithogenic multigraph given by the following figure.

G =

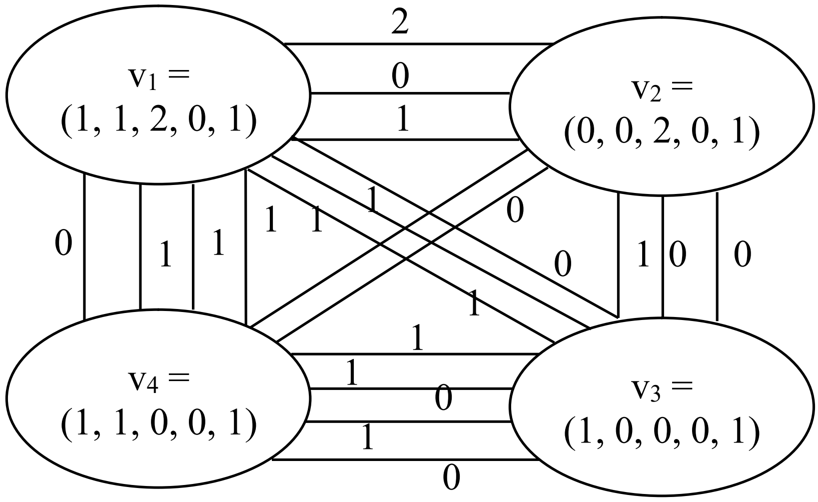


Figure 3.23

We see G is a pseudo nonuniform complete special multigraph of order four. All these multigraphs are different from subset vertex multigraphs or for the fixing (or for existing of an) edge is possible only if the entries of elements are the same in the row vector at the i^{th} component $1 \leq i \leq n$; ($n = 4$ here) the $1 \times n$ row matrix otherwise no edge exist. If some three components say i, j, k in two vertex sets (row matrices) are the same we will have 3 edges connecting them.

In view of all these we put forth the following theorem.

Theorem 3.1. Let G be a $1 \times n$ Plithogenic multigraph of order M.

1) G can never be a complete uniform multigraph of order m.
2) G has no clique of any order which has n edges.

3) G has no dyad which has n edges.

Proof: Given G is a $1 \times n$ Plithogenic multigraph of order M.

Proof of (1) to prove G has no uniform complete graph with n edges.

We know for any two vertices v_i and v_j we can have a maximum of only $(n – 1)$ edges for n edges are common if and only if $v_i = v_j$ that is both of the vertices have the same label in $1 \times n$ row matrices. We see there is a lot of difference between subset vertex multigraphs and Plithogenic multigraphs.

Here in the row matrix the element 0 or zero entry component if two row vectors have the edge is marked 0. However if we have to be more specific we to have to say it is a label and an edge is connected with label zero. We see the graph given in this example cannot be transformed into a subset vertex multigraph.

For the set $v_1 = \{0, 2, 1, 1, 1\}$, $v_2 = \{0, 0, 2, 0, 1\}$, $v_3 = \{0, 0, 0, 1, 1\}$ and $v_4 = \{1, 1, 1, 0, 0\}$ so none of these sets are subsets they are multisets as 1repeats in v_1 3 times and so on.

Thus we have to define only the new notion of multiset vertex multigraphs to transform the row matrix vertex labeled into subset vertex graphs.

So the concept of Plithogenic graphs are distinctly different from other vertex labeled graphs like subset vertex graphs, subset vertex multigraphs and so on.

In this case of Plithogenic multigraphs the edges are fixed once the vertex sets are fixed.

We will provide one more example of the same.

Example 3.14. Let G be a Plithogenic star multistar graph given by the following figure.

G =

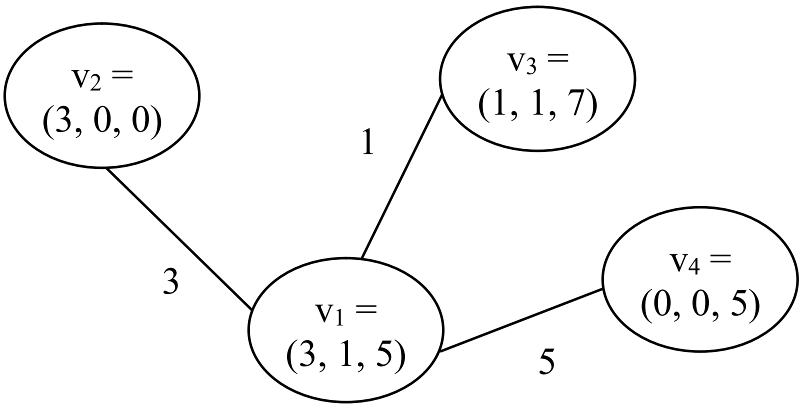


Figure 3.24

Clearly G is not a multistar graph.

Example 3.15. Let G be a Plithogenic multistar graph given by the following figure.

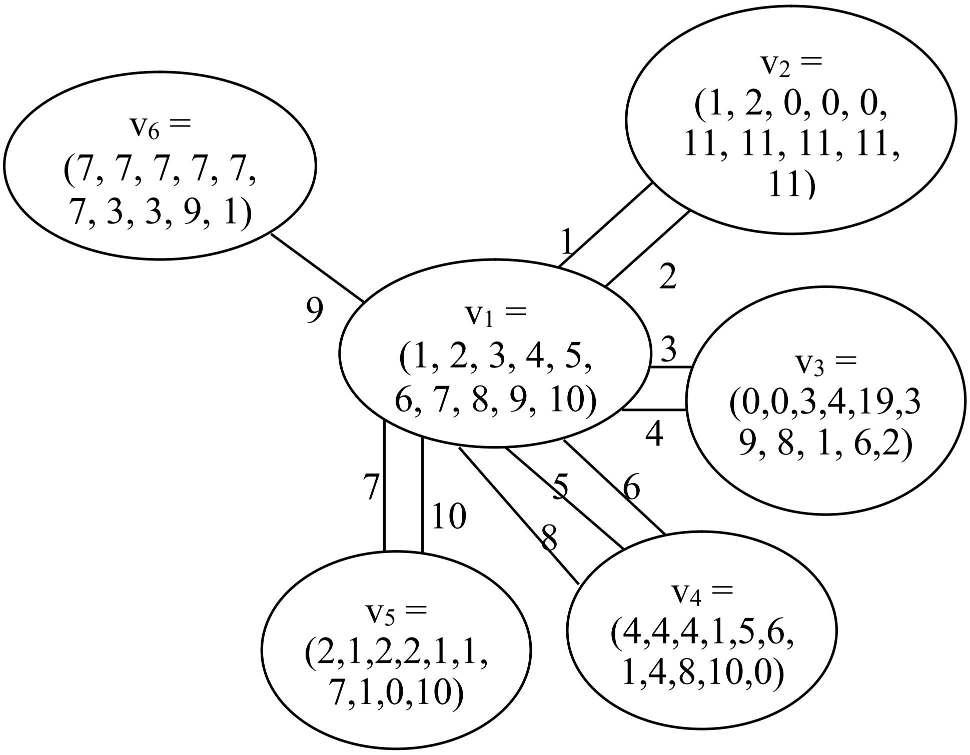


Figure 3.25

We can have several vertices associated with star graph only if the row matrix which labels the vertices is large.

We can also have the concept of Plithogenic multiring graph.

Example 3.16. Let G be a Plithogenic multiring graph given by the following figure.

G =

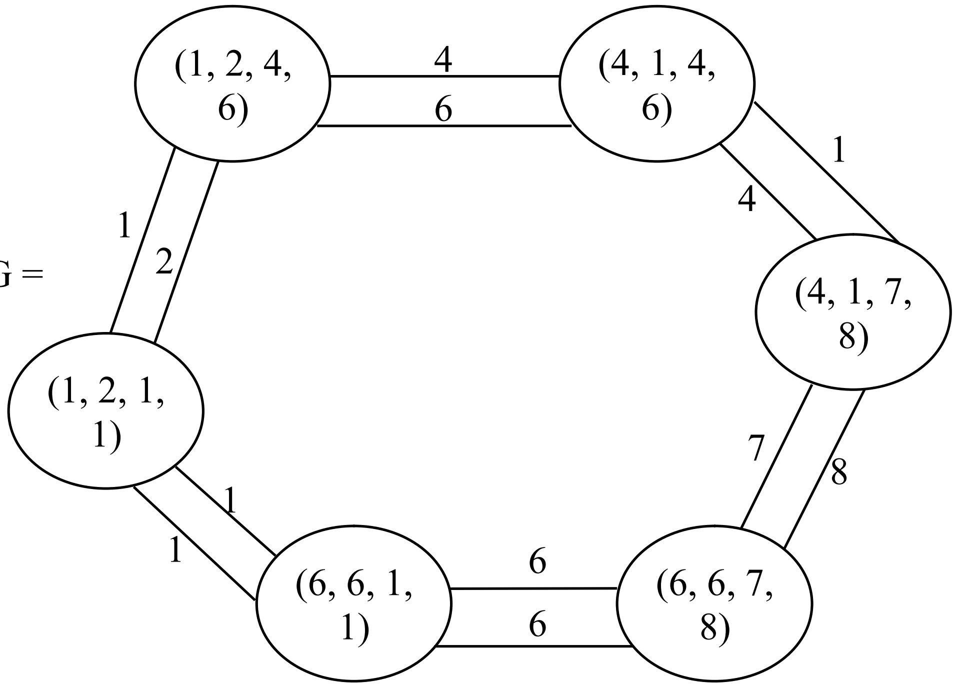


Figure 3.26

We can also have Plithogenic multitrees.

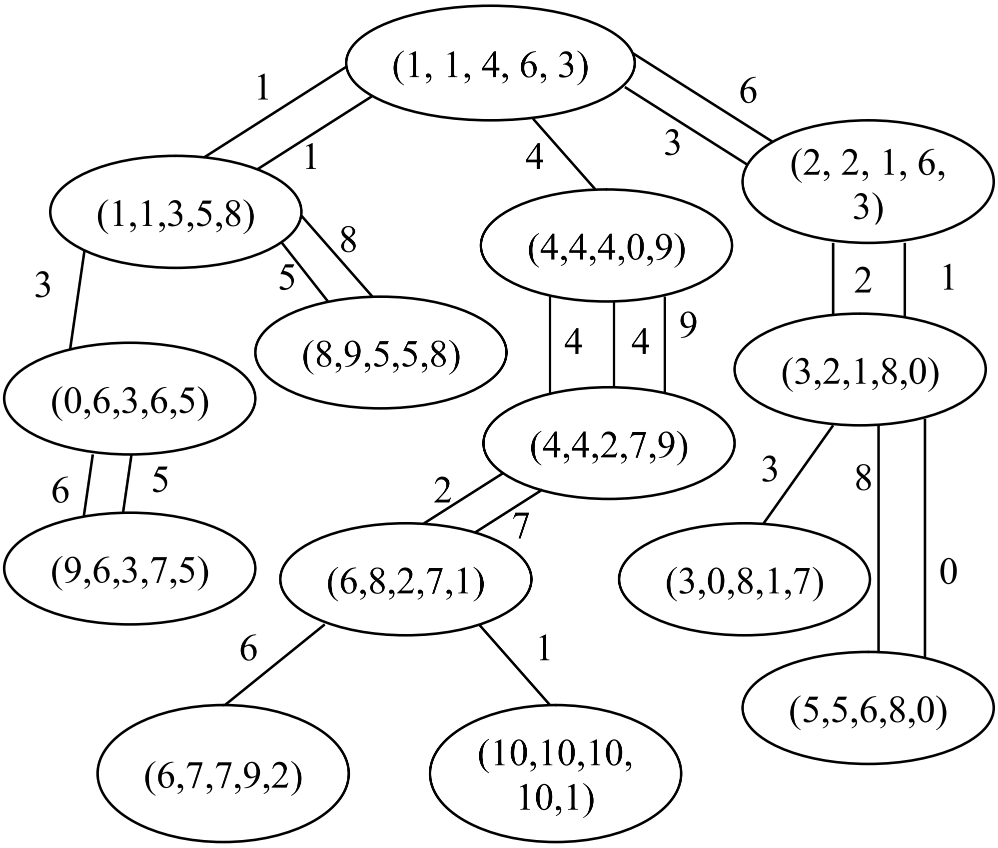


Figure 3.27

We can also have Plithogenic line multigraph given by the following example.

Example 3.17. Let G be the Plithogenic line multigraph given by the following figure.

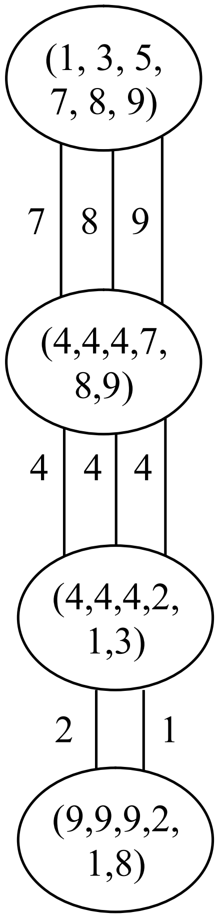
G = 

Figure 3.28

We cannot transform this into a subset vertex graph as the set associated with (4, 4, 4, 7, 8, 9) is {4, 4, 4, 7, 8, 9} which is a multiset. Similarly, the set associated with the row matrix (9, 9, 9, 2, 1, 8) is {9, 9, 9, 2, 1, 8} again a multiset.

Next, we proceed onto describe a few Plithogenic multisubgraphs of a Plithogenic multigraph G by some examples.

Example 3.18. Let G be a Plithogenic multigraph given by the following figure.

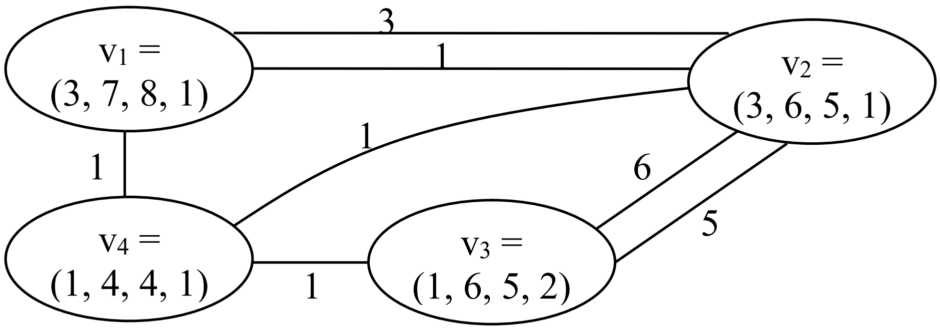


Figure 3.29

The way edge removed Plithogenic graph are formed is described in the following.

They are not formed as in case of edge subset vertex multigraphs where the edges are removed and correspondingly the elements are removed. Here if the edges are removed then the corresponding value of the component in the row matrix of that particular vertex is put as '–' which implies the value is undefined that is it does not exist or unknown.

We just define this notion.

Definition 3.2. *Let $x = (a_1, a_2, .., a_n)$ be a $1 \times n$ matrix $a_i \in R$ or C or Z or Z_n $(1 \leq i \leq n)$. If some a_i are not found just a blank is present indicated by a '–' then we call the $1 \times n$ row matrix as a row matrix with undefined components.*

We will first illustrate this situation by some examples.

Example 3.19. Let G be a Plithogenic multigraph given by the following figure.

G =

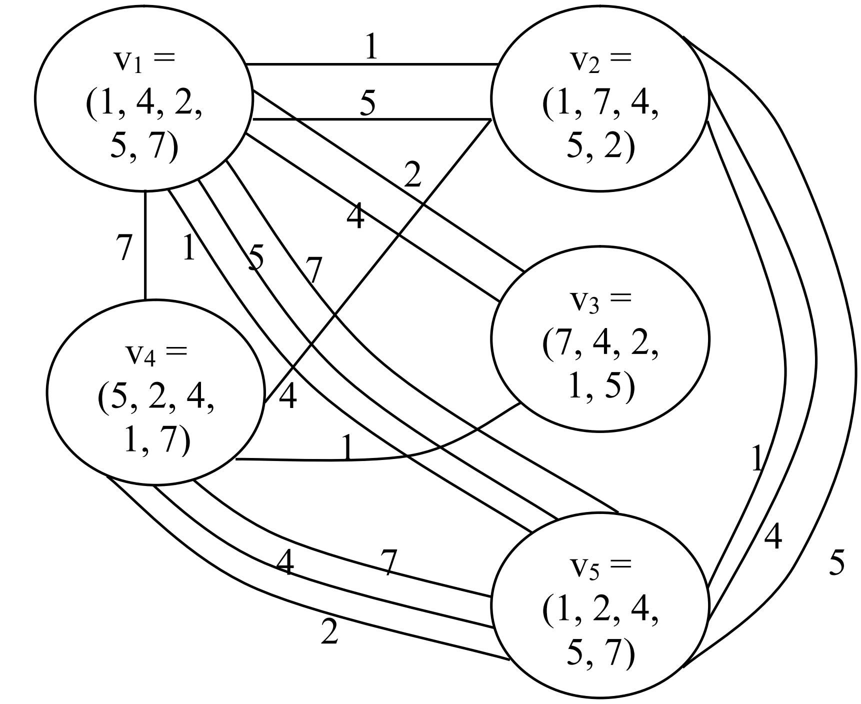


Figure 3.30

Suppose the edge v_1v_5 with weight 1 is removed from G. Let H be the (edge v_1v_5 with weight 1 removed) multisubgraph of G given by the following figure.

H =

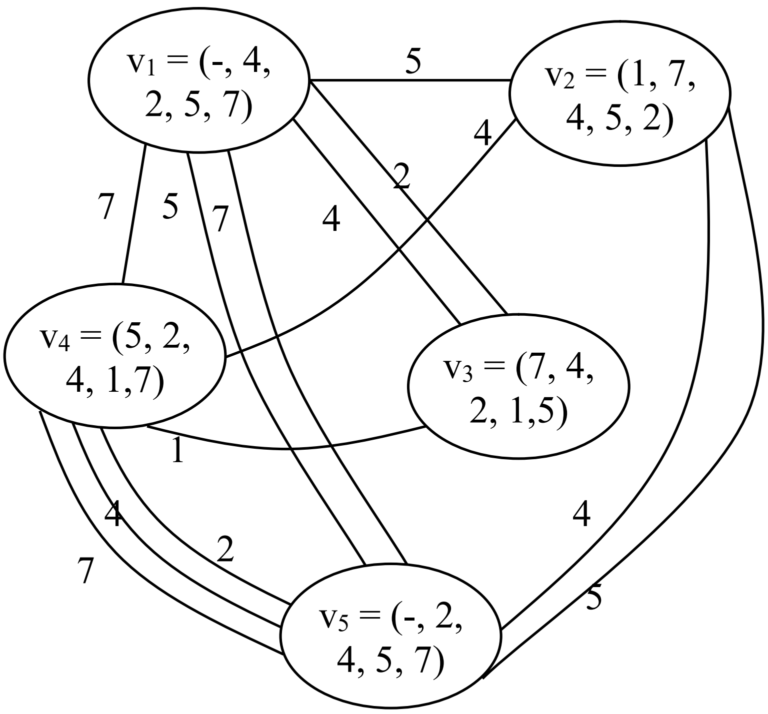


Figure 3.31

H is a multisubgraph which has two vertex sets which are labeled with row matrix having undefined components. We call the vertex sets v_1 and v_5 as 1×5 row matrices with undefined components.

Thus in view of this we proceed onto define a new notion.

Let us consider a multisubgraph K in which we remove 3 edges from G given by the following figure.

G \ {v_1v_4 with weight 7, v_2v_5 with weight 5 and v_4v_5 with weight 7} = K.

The multigraph K is as follows.

K =

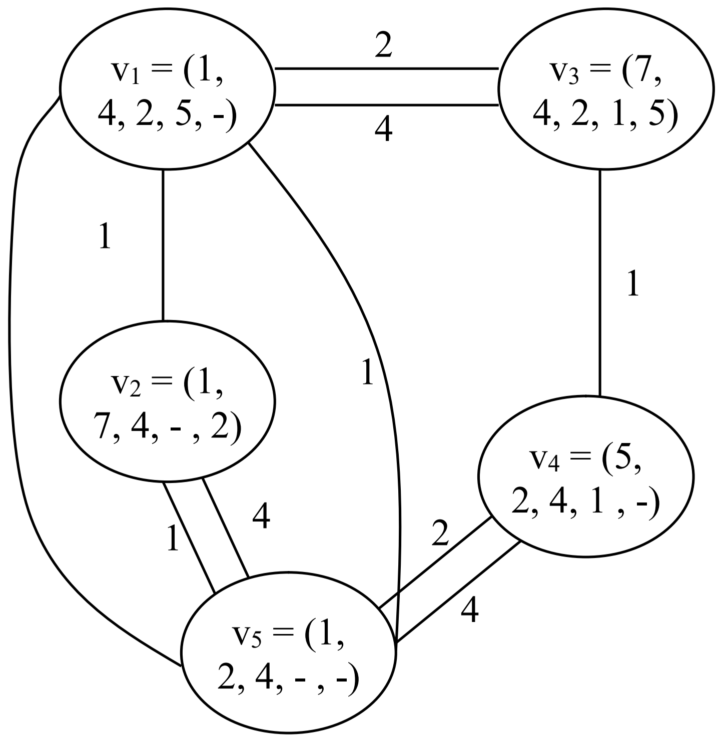


Figure 3.32

K is an edge Plithogenic subgraph of G. This has lost at least 3 edges.

Study in this direction is interesting and important in case of social multinetworks and in the study of node connectivity and edge connectivity in case of row matrix vertex labeled multigraphs when any of the attribute in that community or society becomes undefined.

It is pertinent to keep in mind that in these multigraphs the edges get labeled automatically getting its value as the same component in these two vertices.

To make this information or property clear we give this concept in case of a multi dyad whose vertex sets are row matrices by the following example.

Example 3.20. Let G be a Plithogenic multigraph given by the following figure.

G =

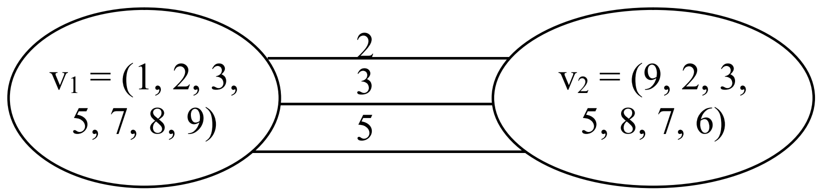


Figure 3.33

Clearly this is a multi-dyad where the 2nd component of v_1 and v_2 are the same so as the edge v_1v_2 marked by 2. The 3rd component of v_1 and v_2 are the same edge marked by 3 and the fourth components of v_1 and v_2 and is marked by 5.

Thus G is a multidyad with 3 edges.

If we find the edge Plithogenic multisubgraph H_1 of the dyad then we see if edge 2 is removed we get

H_1 =

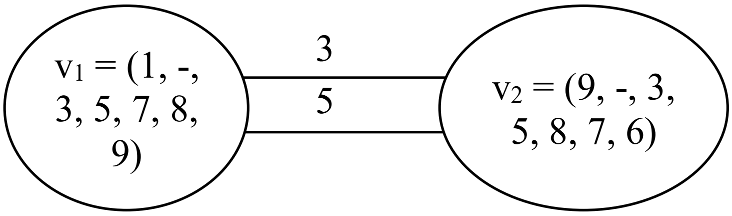


Figure 3.34

If edge labeled 3 is removed, we get the multisubgraph H_2 is follows.

H_2 =

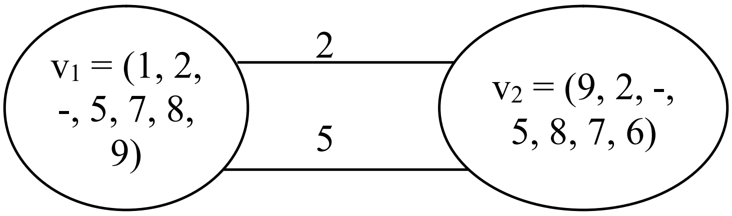


Figure 3.35

H_3 be the multidyad in which edges with labeled 5 and 2 are removed; H_3 is as follows.

H_3 =

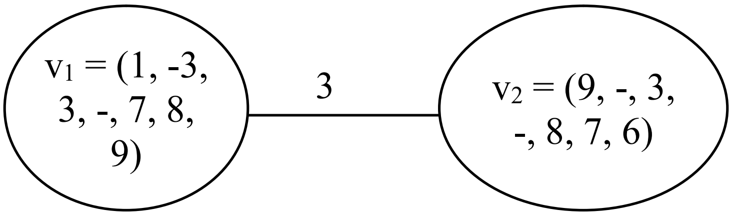


Figure 3.36

If all the 3 edges labeled 2, 3, and 5 are removed we see this G becomes a null multidyad given by the H_4 given in the following.

H_4 =

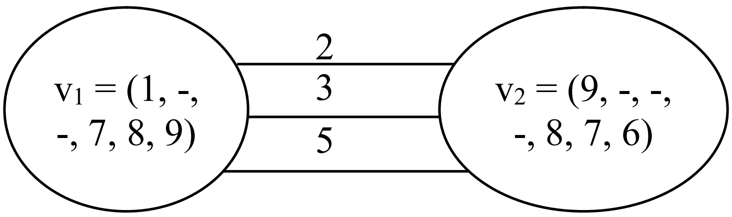


Figure 3.37

H_4 is a null dyad so 3 edges must be removed or 3 relevant components of the row matrices of the vertex sets v_1 and v_2 must removed to become a disconnected.

So in case of Plithogenic multigraph the study is interesting for it shows how many vertex sets becomes undefined row matrices and gives both the number of components of the pair of row matrices that must be removed so that none of the edges between the pair of vertices exist.

The Plithogenic multigraphs have the row matrices to be well defined. If the row matrices are not well defined then we call them deficient Plithogenic multigraphs. So when we define edge Plithogenic multisubgraphs of the graph G they are always deficient.

We provide some examples of them. A word of caution if all the entries of a row matrix happens to be undefined then the deficient multisubgraph will ignore that vertex.

Example 3.21. Let G be a 1×5 Plithogenic multigraph with six vertices given by the following figure.

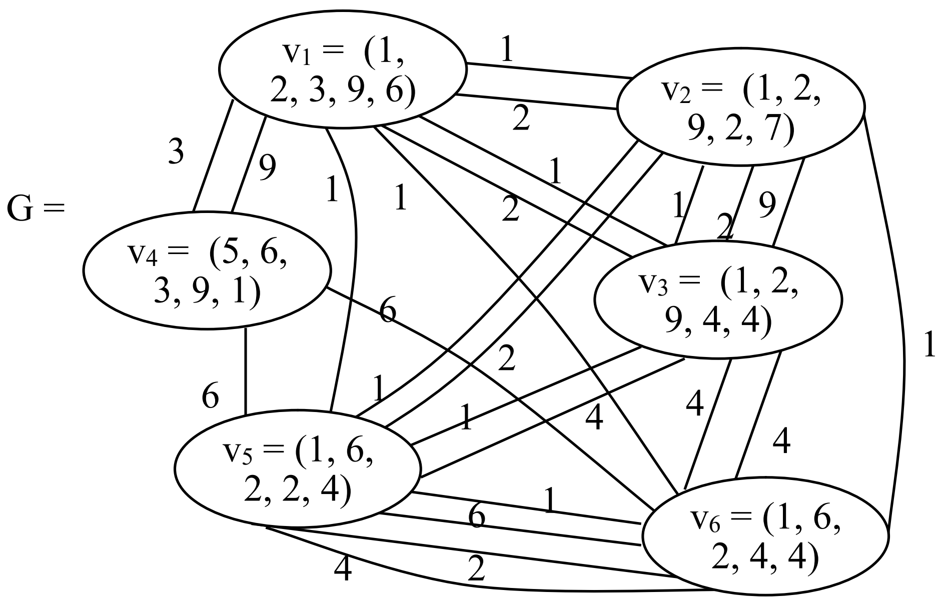


Figure 3.38

Suppose we are interested in an edge Plithogenic multisubgraph H_1 of G with removal of all edges with label 9 being removed. H_1 is as follows.

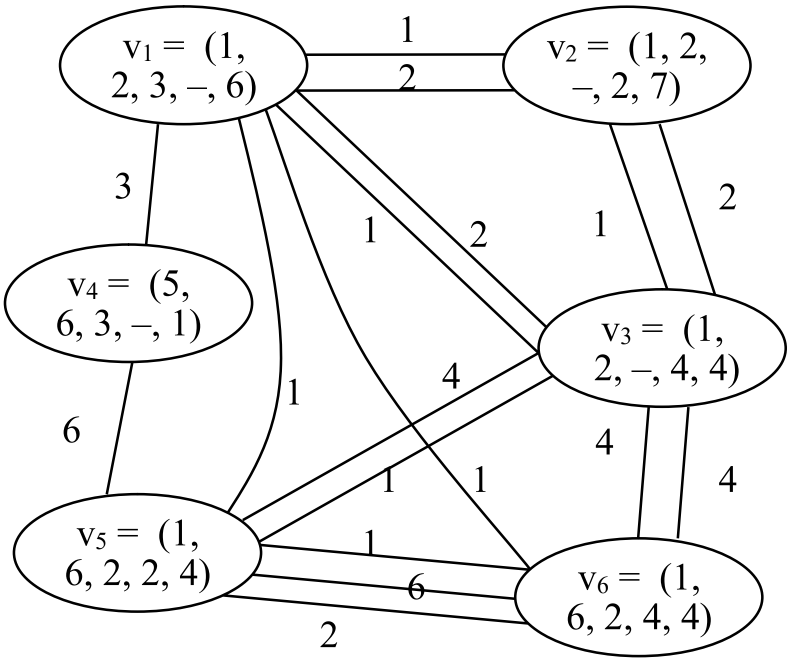


Figure 3.39

In this course of study, we see 4 vertices v_1, v_2, v_4 and v_3 become undefined row matrices.

We call H_1 only as a deficient Plithogenic multisubgraph of G.

We now get a deficient edge Plithogenic multisubgraph H_2 of G be removing all edges labeled 2 and 1. H_2 is given by the following figure.

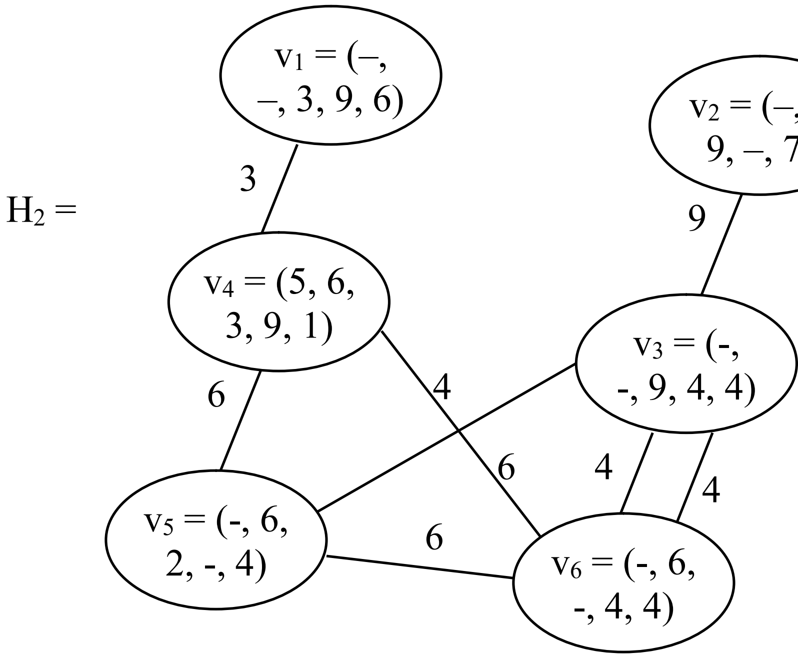


Figure 3.40

We see still the edge multisubgraph of G is connected.

Let H_3 be the deficient edge Plithogenic multisubgraph of G got by removing the edges labeled 1, 2, 4 and 6. H_3 is given in the following.

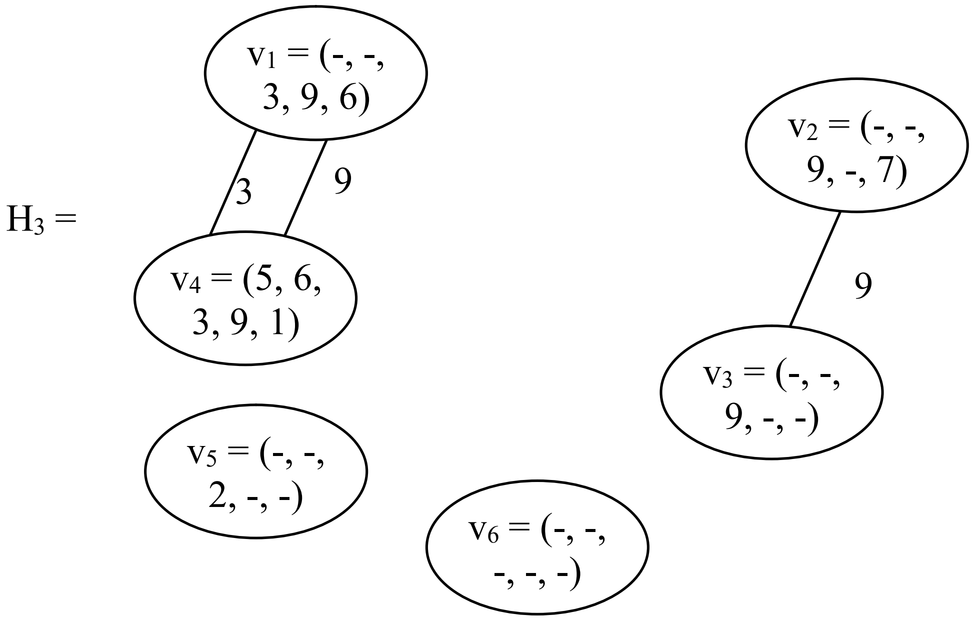


Figure 3.41

This becomes a multisubgraph only with 5 vertices which is disconnected as the vertex v_6 becomes fully undefined, so the vertex does not exist.

We have seen earlier Plithogenic graphs with row matrix labeled edges and Plithogenic multigraphs in which the multi edges are labeled automatically.

In the following we define the new graph called permutation Plithogenic multigraphs on the set (1, 2, …, n). Here these are not classical permutation graphs. Also the edges get automatically labeled. Further they are also different from groups represented as graphs [72-78].

We will treat each permutation as a row matrix for in case of permutation of (1, 2, 3) all possible row matrices are (1, 2, 3), (1, 3, 2), (3, 2, 1), (2, 1, 3), (2, 3, 1) and (3, 1, 2). So by default of notation we call them as permutation Plithogenic multigraphs.

In fact these will be a subcollection of Plithogenic multigraphs.

We will illustrate this situation by some examples.

Example 3.22. Let G be a Plithogenic multigraph with row matrix vertex sets (1, 2) and (2, 1). The permutation group on S_2 = {(1, 2), (2, 1)}.

Now the Plithogenic multigraph is the empty graph given by two vertex sets.

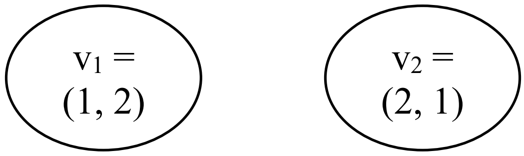


Figure 3.42

Example 3.23. Let G be the Plithogenic multigraph with row matrices {(1, 2, 3), (1, 3, 2), (3, 2, 1), (2, 1, 3), (2, 3, 1), (3, 1, 2)} which is the group elements of S_3, given by the following figure.

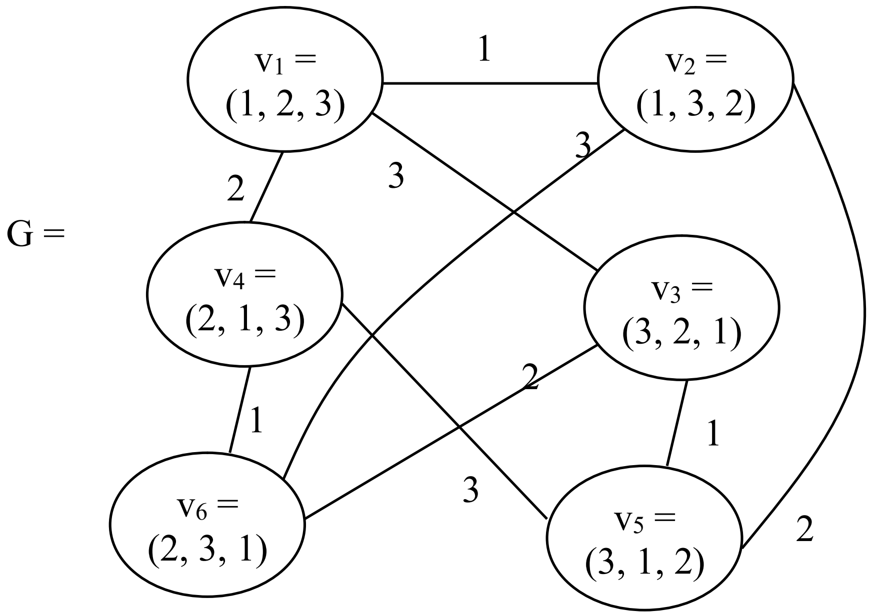


Figure 3.43

We see this is a special type of 3-regular graph on six vertices.

The subgraphs of G taking subgroups of S_3 is given in the following.

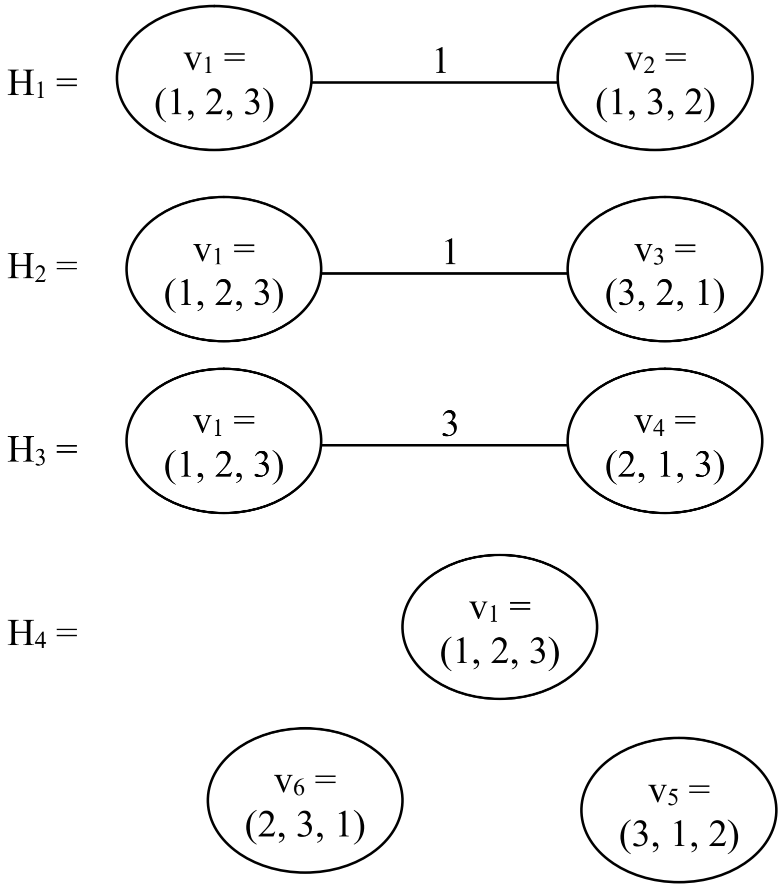


Figure 3.44

H_4 is a empty subgraph though it is the largest subgraph of S_3. Clearly S_3 taken as 1×3 matrices do not contribute to any Plithogenic multigraph.

We next analysis S_4 that is 24, 1×4 row matrices as vertex sets.

Let G be the 1×4 permutation Plithogenic multigraph associated with S_4.

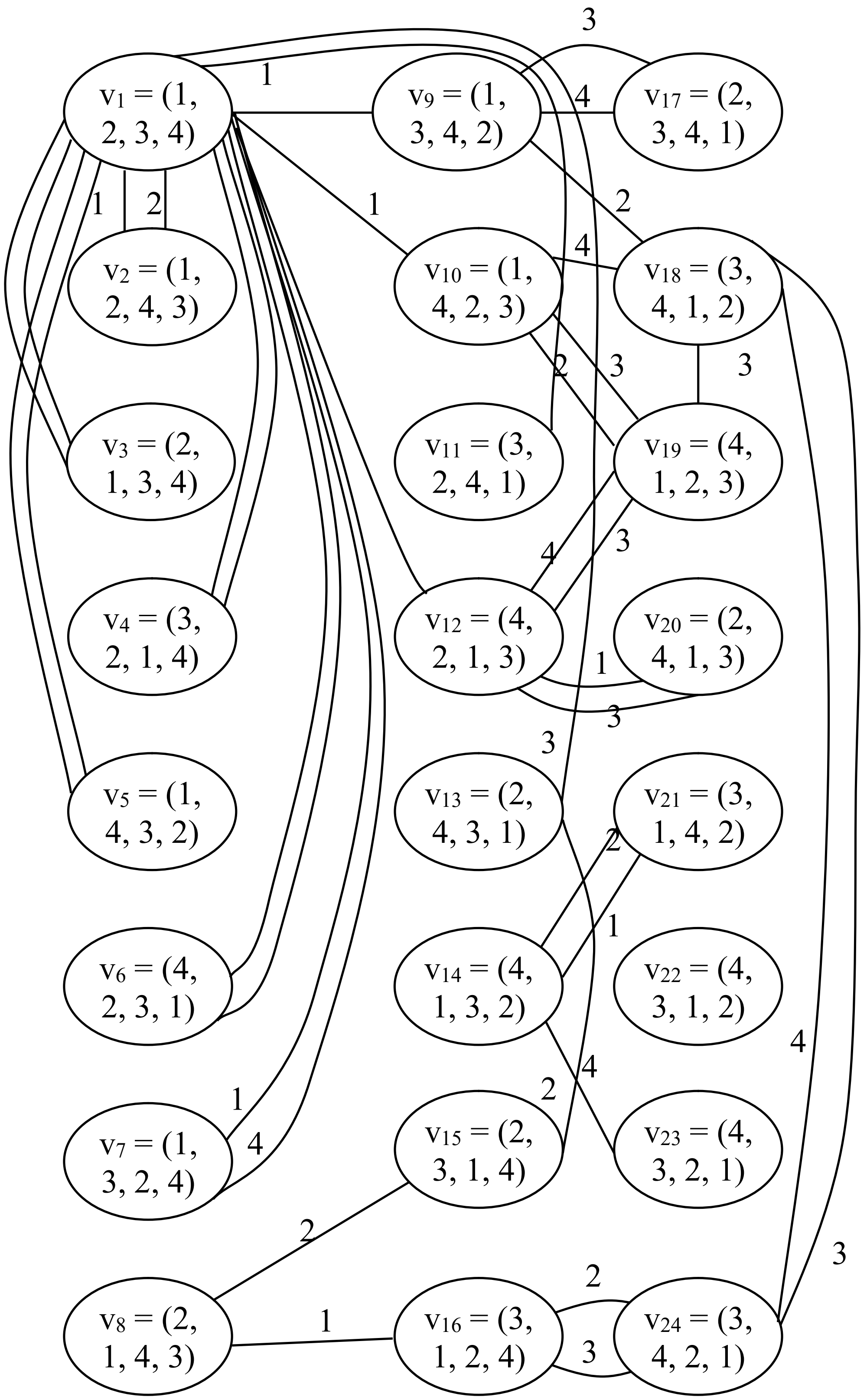

v1 = (1, 2, 3, 4)
v2 = (1, 2, 4, 3)
v3 = (2, 1, 3, 4)
v4 = (3, 2, 1, 4)
v5 = (1, 4, 3, 2)
v6 = (4, 2, 3, 1)
v7 = (1, 3, 2, 4)
v8 = (2, 1, 4, 3)
v9 = (1, 3, 4, 2)
v10 = (1, 4, 2, 3)
v11 = (3, 2, 4, 1)
v12 = (4, 2, 1, 3)
v13 = (2, 4, 3, 1)
v14 = (4, 1, 3, 2)
v15 = (2, 3, 1, 4)
v16 = (3, 1, 2, 4)
v17 = (2, 3, 4, 1)
v18 = (3, 4, 1, 2)
v19 = (4, 1, 2, 3)
v20 = (2, 4, 1, 3)
v21 = (3, 1, 4, 2)
v22 = (4, 3, 1, 2)
v23 = (4, 3, 2, 1)
v24 = (3, 4, 2, 1)

Figure 3.45

We leave the task of completing the missing edges. The following observations are mandatory;

1) In case of S_3 the permutation Plithogenic graph has only one edge so it is a simple graph not a multigraph.
2) However S_4 contributes to a multigraph but the maximum number of multi edges of any relevant pair of vertices is 2 and in case only one and in some cases no edge exists.
3) We see none of the cyclic groups of order 3 in S_3 or order 4 in S_4 are connected.

They are empty subgraphs.

We will describe the multigraph associated with A_4 in the following example.

Example 3.24. The permutation Plithogenic multigraph of A_4 is given by the following figure.

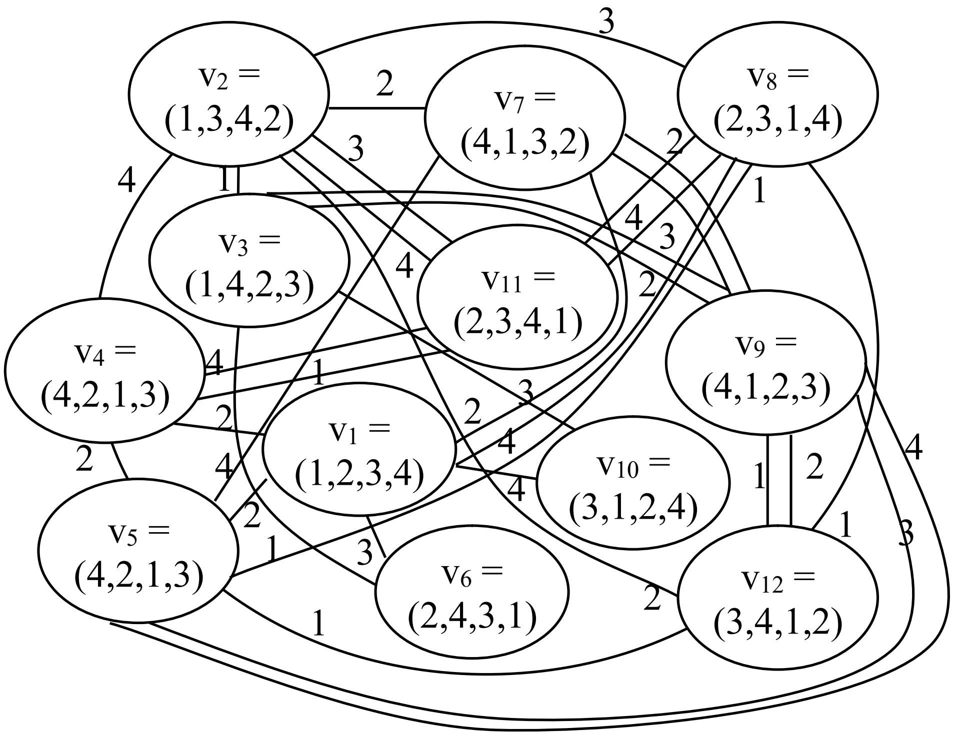


Figure 3.46

The main observation is that these permutation Plithogenic multigraphs are connected. However, all multisubgraphs of these graphs may be connected or may not be connected.

Consider the permutation Plithogenic multisubgraph H_1 of G associated with S_4 be given by the following figure.

H_1 =

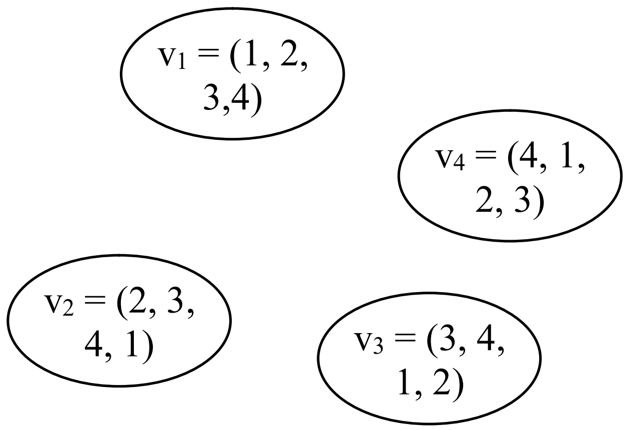


Figure 3.47

Clearly H_1 is a empty multigraph.

Let H_2 be a permutation Plithogenic subgraph of G given by the following figure.

H_2 =

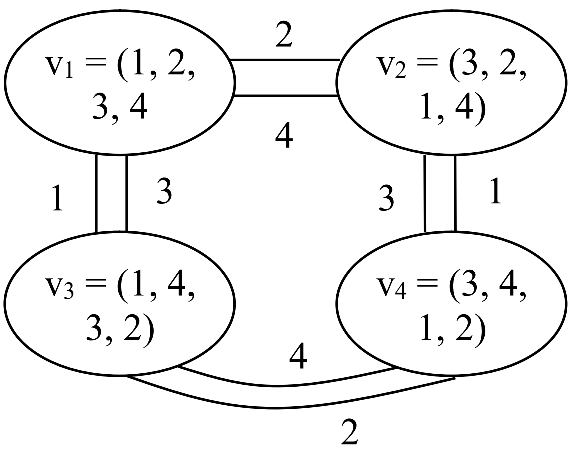


Figure 3.48

H_2 is a multisubgraph which is uniform as all edges are of two. H_2 is not a complete subgraph of G. It is a ring. The group associated with H_2 is not a cyclic group but a Klein group of order four.

In view of all these we have the following theorem.

Theorem 3.2. *Let G be a permutation Plithogenic multigraph with entries from S_n, (S_n the symmetric group on permutation of (1, 2, …, n); $2 \leq n < \infty$).*

1) *The maximum number of edges between any possible adjacent vertices is only (n – 2).*
2) *Minimum possible edges that can exist between adjacent vertices is 1.*
3) *There can be no edge or the vertex sets may not be adjacent.*

Proof is direct using the fact the permutation of (1, 2, …, n) can produce a pair in which maximum only (n – 2) entries of (1, 2, …, n) can be fixed and two entries are permuted.

Similarly the other extreme situation between adjacent vertices is one entry is fixed and all the other (n – 1) entries of (1, 2, …, n) are permuted.

There can be vertices like (2, 3, …, n – 1, n, 1) and (3, 4, 5, .., n – 1, n, 1, 2) which are never adjacent vertices.

Hence the claim.

Next we proceed onto show.

Theorem 3.3:*Let S_n be the symmetric group of degree n; $3 \leq n < \infty$.*

Let G be the permutation Plithogenic multigraph associated with S_n.

1) *The cyclic subgroup of S_n of order n results in an empty multigraph.*
2) *Every cyclic subgroup of order $p < n$ results in a multisubgraph which has only $(n – p)$ multiedges for every p; $2 \leq p < n$.*

Proof of (1) Now any cyclic subgroup P of order n of S_n will be of the form P = {(1, 2, …, n), (2, 3, …, n, 1), (3, 4, 5, …, n, 1, 2), …, (n – 1, n, 1, 2, .., n – 2), …, (n, 1, 2, 3, …, n – 1)} $\subseteq S_n$.

Clearly we cannot find any pair of elements \ vertices in P which can be adjacent. Hence P yields a empty multisubgraph of the multigraph G associated with S_n. Hence (1) of the theorem is true.

Proof of (2): Suppose there is a cyclic subgroup of S_n order p, $2 \leq p < n$ then we see in e = (1, 2, …, n), n – p of the elements in e are fixed and rest of the p elements in e are permuted; to form a cyclic group. The Plithogenic multisubgraph is such that it is a uniform complete multisubgraph with (n – p) multiedges.

We will illustrate both the situations by some examples.

Example 3.25. Let G be the permutation Plithogenic multigraph associated with S_9. Let H be the Plithogenic multisubgraph of G associated with the cyclic subgroup P of order 6. S_9 is given by the following elements.

p = {(1, 2, 3, 4, 5, 6, 7, 8, 9) = e, p_1 = (2, 3, 4, 5,6 1, 7, 8, 9)

p_2 = (3, 4, 5, 6, 1, 2, 7, 8, 9), p_3 = (4, 5, 6, 1, 2, 3, 7, 8, 9),

p_4 = (5, 6, 1, 2, 3, 4, 7, 8, 9), p_5 = (6, 1, 2, 3, 4, 5, 7, 8, 9)} $\subseteq S_9$.

The multisubgraph H associated with P is as follows.

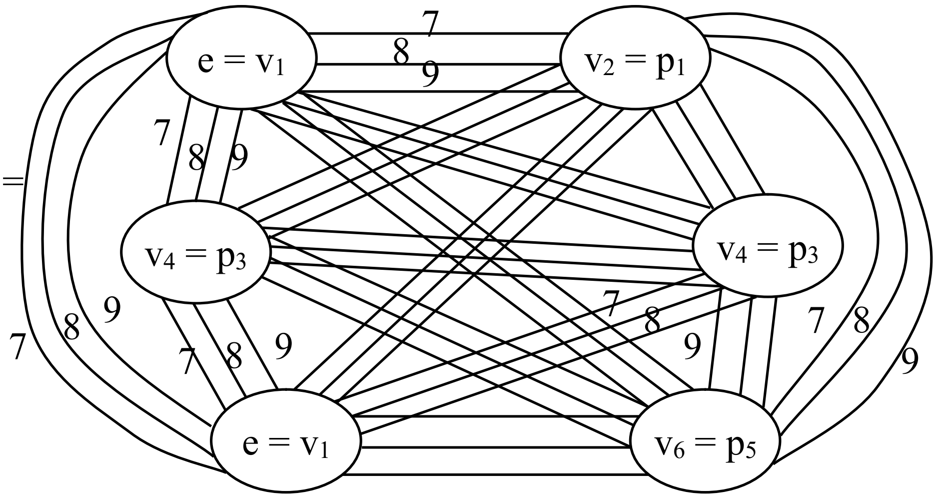
H = 

Figure 3.49

Clearly H is a Plithogenic multisubgraph which is a strong uniform complete multisubgraph all the multiedges are labeled with the same label as 7, 8 and 9.

That is why we use the term strong uniform multisubgraph.

Next we give a multisubgraph, L associated with the subgraph K of G where K is a cyclic subgroup of G of order 3 where K = {e = (1, 2, …, 9), (2, 3, 4, 5, …, 9) = k_1, k_2 = (3, 1, 2, 4, 5, …,9)}.

The multisubgraph L of G is as follows.

L =

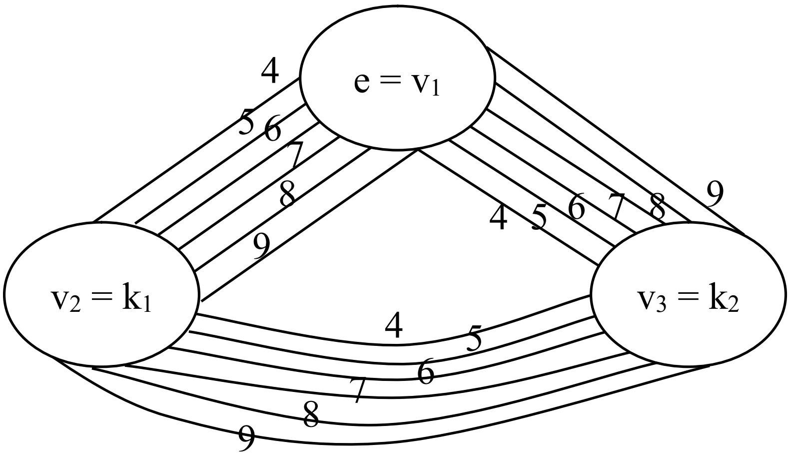


Figure 3.50

We see L is a strong uniform multisubgraph with 6 multiedges.

We see larger the order of the cyclic subgroup of S_n we have lesser number of multiedges. The multisubgraph reduces to a simple subgraph if in S_n we take the cyclic subgroup of order (n – 1).

We will illustrate this situation by an example.

Example 3.26. Let S_{12} be the symmetric group of degree 12. Let H be the cyclic group of order 11.

H = {(1, 2, …, 12) = h, (1, 3, 4, …, 12, 2) = h_1

h_2 = (1, 4, 5, 6, …, 12, 2, 3), h_3 = (1, 5, 6, 7, .., 12, 2, 3, 4), h_4 = (1, 6, 7, ..., 12, 2, 3, 4, 5), h_5 = (1, 7, 8, …, 12, 2, 3, 4, 5, 6), h_6 = (1, 8, 9, …, 12, 2, 3, 4, …., 7), h_7 = (1, 9, 10, 1, 12, 2, …, 8), h_8 = (1, 10, 11, 12, 2, …, 9), h_9 = (1, 11, 12, 2, …, 10), h_{10} = (1, 12, 2, …, 11)}

Let W be the Plithogenic multisubgraph associated with H given by the following figure.

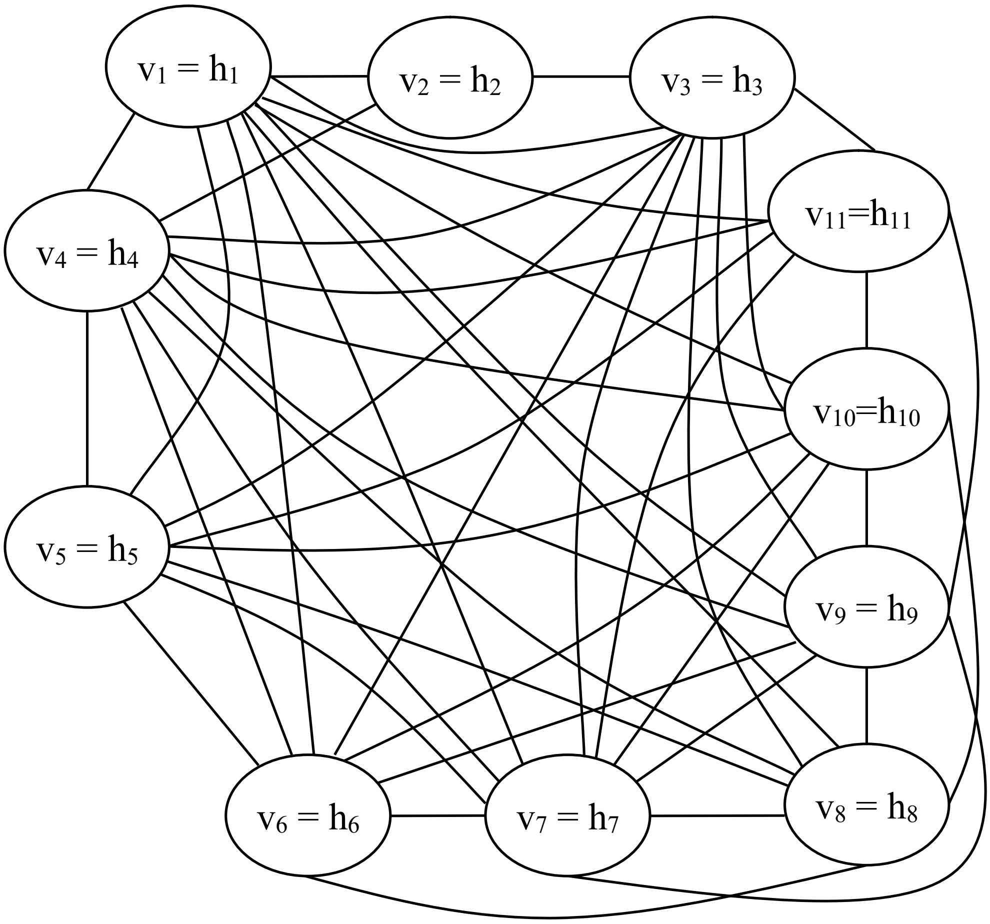


Figure 3.51

W is a simple complete graph K_{11}. Clearly W is not a multisubgraph of the symmetric group degree 12 graph G.

In view of all these we have the following theorem.

Theorem 3.4. *Let G be the permutation Plithogenic multigraph associated with S_n.*

i) *There are n-number of K_{n-1} simple complete subgraphs associated with the cyclic groups $\{(1, 2, .., n), (1, 3, 4, \ldots, n, 2), \ldots (1, n, n-1, \ldots, 2)\}$,* *{(1, 2, …, n), (3, 2, 4, …, n), … (n2, n – 1, …, 1)}, … so on {(1, 2, …, n), (2, 3, …, 1, n), (3, 4, …, 1, 2, n), …, (n – 1, 1, 2, …, n – 1, n)} which are simple Plithogenic subgraphs.*

ii) *There are nC_2 number of uniform complete multisubgraphs K_{n-2} of G associated with cyclic subgroups of S_n of order n – 2.*

iii) *There are nC_r. $2 \leq r < n - 2$ number of uniform complete multisubgraphs of G associated with cyclic subgroups of S_n of order n – r.*

Proof is direct and hence left as an exercise to the reader.

To this effect we provide an example.

Example 3.27. Let S_9 be the symmetric group of degree 9.

Let G be the Plithogenic multigraph associated with S_9 where row matrices are the permutation of (1, 2, …, 9) which form the vertex labels of G.

Consider H_1 = {r_1 = (1, 2, …, 9), r_2 = (1, 3, 4,…, 9, 2), …, r_8 = (1, 9, 2, …, 8)} be the cyclic subgroup of S_9 of order 8.

The associated subgraph P_1 of G with H_1 is as follows.

$P_1 =$

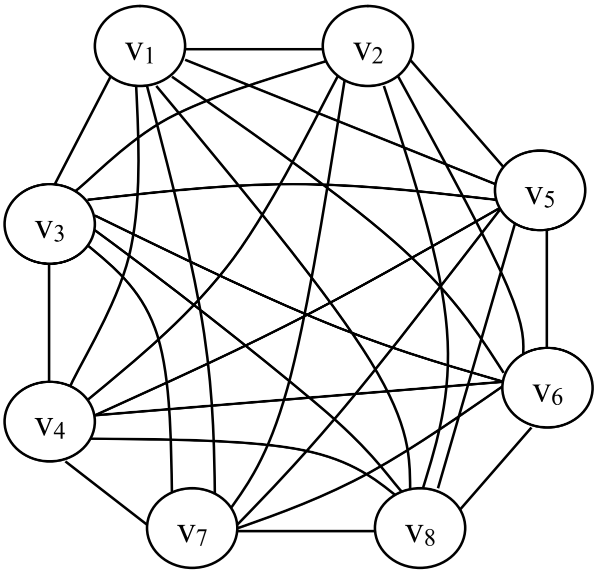


Figure 3.52

Let $H_2 = \{(1, 2, \ldots, 9) = u_1, u_2 = (3, 2, 4, \ldots, 9, 1), \ldots, u_8 = (9, 2, 1, \ldots, 8)\}$ be the cyclic subgroup of S_9 got by fixing 2 and permuting all the elements cyclically. Let P_2 be the subgraph of G associated with H_2 given by the following figure.

$P_2 =$

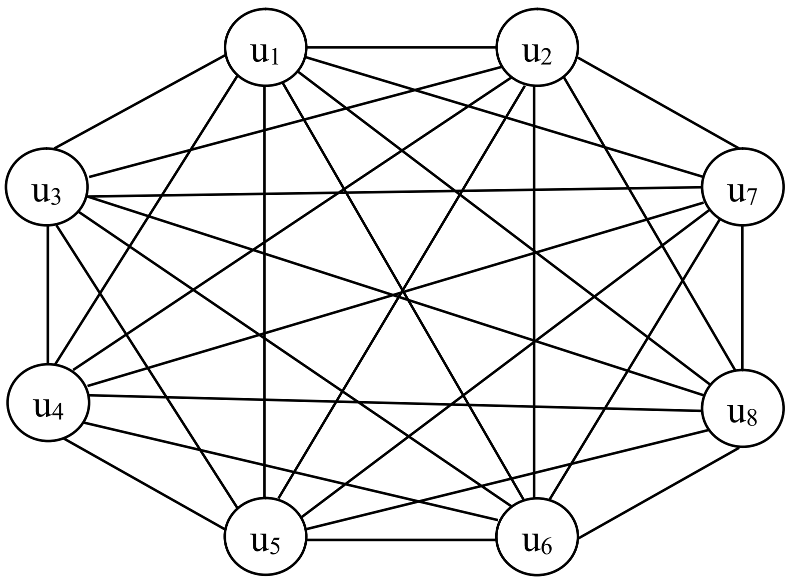


Figure 3.53

P_2 is a simple complete subgraph of G isomorphic with K_8.

Let H_9 be the cyclic subgroup of S_9 given in the following figure got by fixing 9 and permuting the rest of the numbers $H_9 = \{(1, 2, \ldots, 9) = a_1, (2, 3, 4, \ldots, 8, 1, 9) = a_2, a_3 = (3, 4, 5, \ldots, 8, 1, 3), a_4 = (4, 5, \ldots, 8, 1, 2, 3, 9)\ldots, a_8 = (8, 1, 2, \ldots, 7, 9)\} \subseteq S_9$.

Now the simple subgraph P_9 of G associated with the cyclic subgroup H_9 is as follows.

$P_9 =$

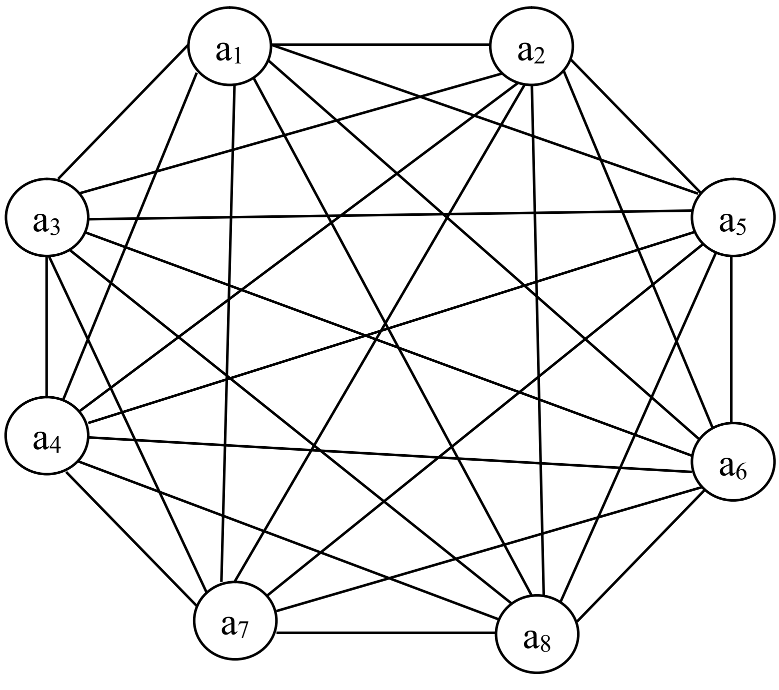


Figure 3.54

P_9 is a complete subgraph of G isomorphic with K_8.

Let B_1 be the cyclic subgroup of S_9 of order 7.

$B_1 = \{(1, 2, \ldots, 9) = b_1, b_2 = (1, 2, 4, 5, 6, 7, 8, 9, 3), \ldots, b_7 = (1, 2, 9, 3, 4, 5, 6, 7, 8)\}$. Let L_1 be the multisubgraph of G associated with B_1 given by the following figure.

L_1 =

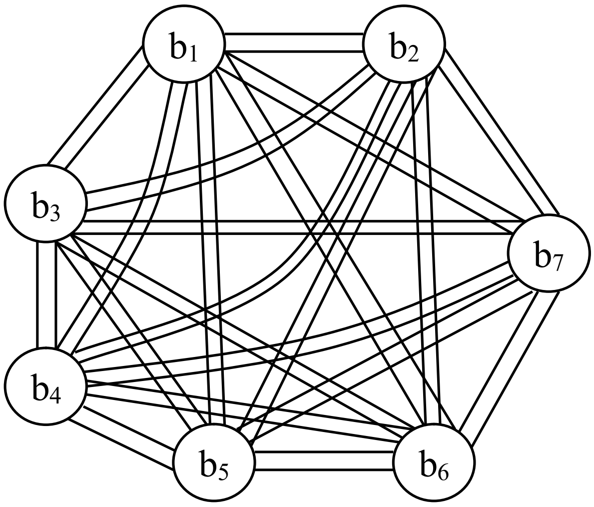


Figure 3.55

Clearly L_1 is isomorphic with K_7.

Let $B_2 = \{(1, 2, 3, \ldots, 9) = t_1, t_2 = (1, 4, 3, \ldots, 9, 2), t_3 = (1, 5, 3, \ldots, 9, 2, 4), \ldots, t_7 = (1, 9, 3, 2, \ldots, 8)\} \subseteq S_9$ be the cyclic subgroup of S_9 of order 7. Let L_2 be the multisubgraph of G associated with B_2 given by the following figure.

L_2 =

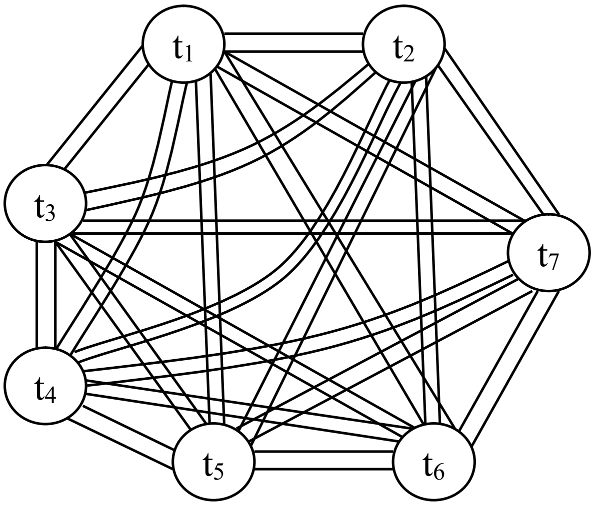


Figure 3.56

L_2 is isomorphic with the uniform complete multisubgraph of order 7.

Let $B_3 = \{(1, 2, \ldots, 9) = d_1, d_2 = (1, 3, 4, \ldots, 8, 2, 9), d_3 = (1, 4, 5, 6, 7, 8, 2, 3, 9), \ldots, d_7 = (1, 8, 2, 3, 4, 5, 6, 7, 9)\} \subseteq S_9$ be the cyclic subgroup of S_9 of order 7. Let L_{36} be the multisubgraph of G associated with B_{36} given by the following figure.

$L_{36} =$

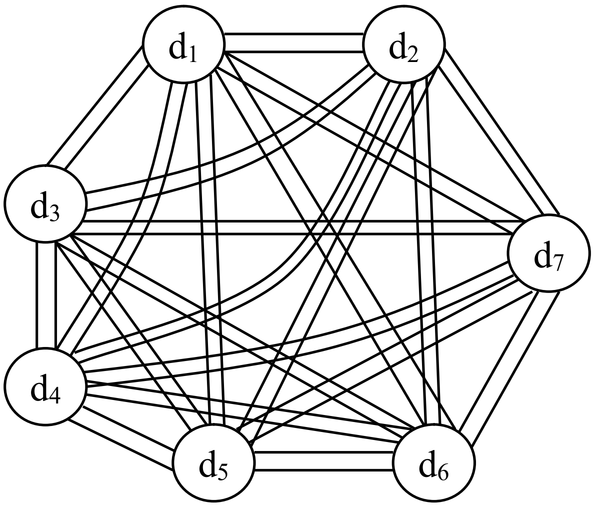


Figure 3.57

We can call these uniform complete multisubgraphs as 2-uniform complete multisubgraph.

Now we give some more complete multisubgraphs of G using which we make some important observation.

Let $D_1 = \{(1, 2, \ldots, 9) = p_1\ (1, 2, 3, 5, 6, 7, 8, 9, 4) = p_2, p_3 = (1, 2, 3, 6, 7, 8, 9, 4, 5), \ldots, p_6 = (1, 2, 3, 9, 4, 5, 6, 7, 8)\} \subseteq S_9$ be the cyclic subgroup of S_9 of order 6 got by fixing the elements 1, 2 and 3 and permuting the rest of the six elements cyclically.

Let M_1 be the multisubgraph of given by the following figure.

M_1 =

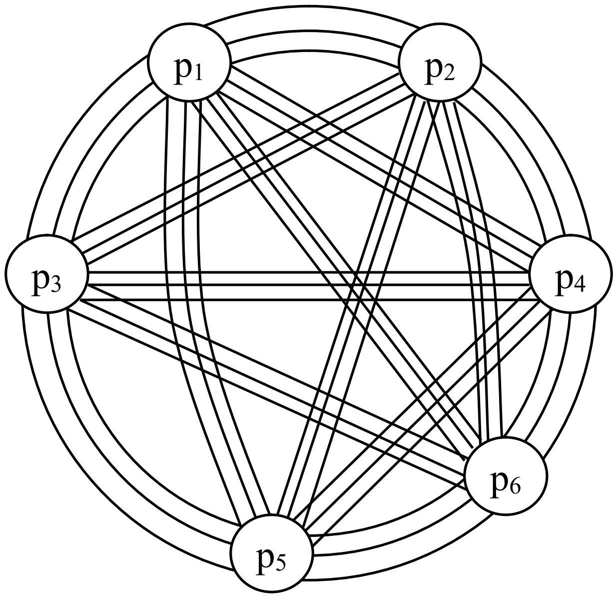


Figure 3.58

M_1 is a 3-uniform complete multisubgraph of G.

Let $T_1 = \{(1, 2, 3, 4, 6, 7, 8, 9, 5) = t_1, t_2 = (1,2,3,4, \ldots, 9)$, $t_3 = (1, 2, 3, 4, 7, 8, 9, 5, 6)$, $t_4 = (1, 2, 3, 4, 8, 9, 5, 6, 7)$, $t_5 = (1, 2, 3, 4, 9, 5, 6, 7, 8)\} \subseteq S_9$ be the cyclic subgroup of S_9 order five got by fixing 4 elements and permuting cyclically the rest. Let A_1 be the multisubgraph of G associated with T_1 given by the following figure.

A_1 =

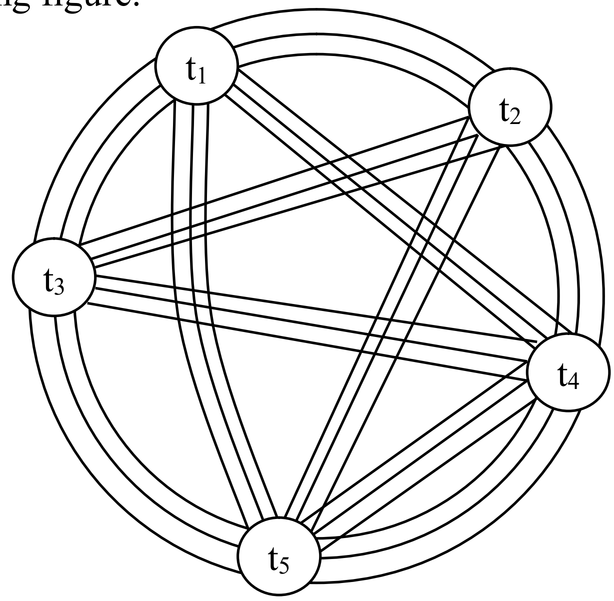


Figure 3.59

A_1 is a 4-uniform complete multisubgraph of G which is of order 5.

Consider the cyclic subgroup W_1 of order four in S_9 got by fixing five elements and permuting the rest cyclically.

$w_1 = \{(1, 2, \ldots, 9) = w_1$, $w_2 = (1, 2, 3, 4, 5, 7, 8, 9, 6)$, $w_3 = (1, 2, 3, 4, 5, 8, 9, 6, 5)$, $w_4 = (1, 2, 3, 4, 5, 9, 6, 5, 8)\} \subseteq S_9$ is a cyclic subgroup of S_9 of order 4. Let V_1 be the multisubgraph of G associated with W_1 given by the following figure.

$V_1 =$

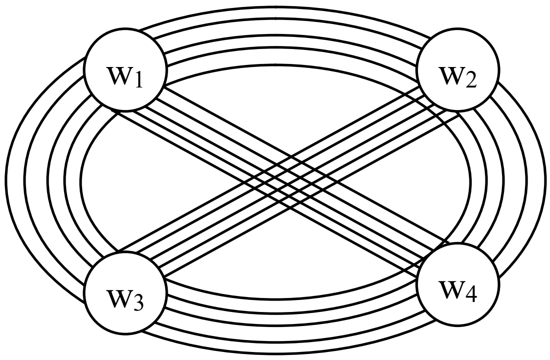


Figure 3.60

V_1 is a_5 uniform complete multisubgraph of G of order 4.

Let $X_1 = \{(1, 2, 3, 4, \ldots, 9) = x_1$, $x_2 = (1, 2, 3, 4, 5, 6, 8, 9, 7)$, $x_3 = (1, 2, 3, 4, 5, 6, 9, 7, 8)\} \subseteq S_9$ be the cyclic subgroup of G of order 3 given by the following figure.

Let Y_1 be the multisubgraph of G associated with the subgroup X_1.

$Y_1 =$

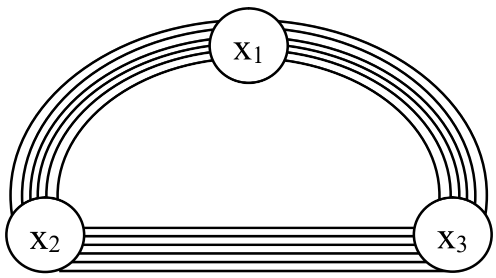


Figure 3.61

All cyclic subgroups of order 2 of S_9 got by fixing 7 elements and permuting only two elements results in a symmetric multidyad.

For instance let

$N_1 = \{(1, 2, 3, 4, 5, 6, 7, 8, 9) = s_1$

$s_2 = (1, 2, 3, 4, 5, 6, 7, 9, 8)\} \subseteq S_9$ is a cyclic subgroup of order 2.

The multisubgraph N_1 of G associated with S_1 is as follows.

$N_1 =$

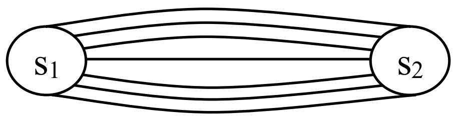


Figure 3.62

Thus, we see these specially formed permutation of a row matrices as vertex labels can yield many t-uniform complete multigraphs, $1 \leq t < n$ where order of the permuted row matrix is $1 \times n$.

We see as the value of t increases the number of vertices decreases.

We can visualize this for S_9 from the following table.

As the components of the vertices which are fixed increases the number of vertices decreases but number of multiedges increases. Clearly seen from the table.

We by default of notation. Cyclic subgroup table for the special row matrix $(a_1, \ldots, a_9) = (1, \ldots, 9)$ permuted as the group S_9 which are labeled as vertices.

S. No.	Cyclic subgroups of various order	No. of Cyclic subgroups	No.of vertices	Number of multiedges between nodes
1.	8	9	8	1
2.	7	$9C_2$	7	2
3.	6	$9C_3$	6	3
4.	5	$9C_4$	5	4
5.	4	$9C_3$	4	5
6.	3	$9C_6$	3	6
7.	2	$9C_7$	2	7

A similar table in case of S_n when the vertex are labeled by the elements of S_n which by default of notation are taken as (1, 2, …, n) as row matrix.

Cyclic subgroup table for the special row matrix (1, 2, …,n) of S_n which are labeled as vertices.

S. No.	Cyclic subgroups of various order in S_n	No. of Cyclic subgroups in S_n	No.of vertices	Number of multiedges between nodes
1.	(n – 1)	n	(n – 1)	Nil (simple complete sub graph (K_{n-1})
2.	n – 2	nC_2	(n – 2)	2
3.	n – 3	nC_3	(n – 3)	3
4.	n – 4	nC_4	(n – 4)	4
. . .				
r	n – r	nC_r	(n – r)	R
. . .				
(n – 2)	2	$nC_{(n-2)}$	2	(n – 2) multidyads

Thus, if one wants to study a social information multinetwork with many complete multisubgraphs and n-cliques can use these graphs. A study of these complete multisubgraphs can also throw some light on the communities in these social information multinetworks.

Next we proceed onto work with row matrices from the symmetric semigroup S(n); where S(n) = {all mappings of (1, 2, 3, …, n) to itself}.

We first illustrate this situation by some examples.

Example 3.28. Let S(2) = {(1,2), (1, 1), (2, 2), (2, 1)} be the symmetric semigroup of order two.

The special row matrix vertex labeled multigraph associated with S(2) is as follows.

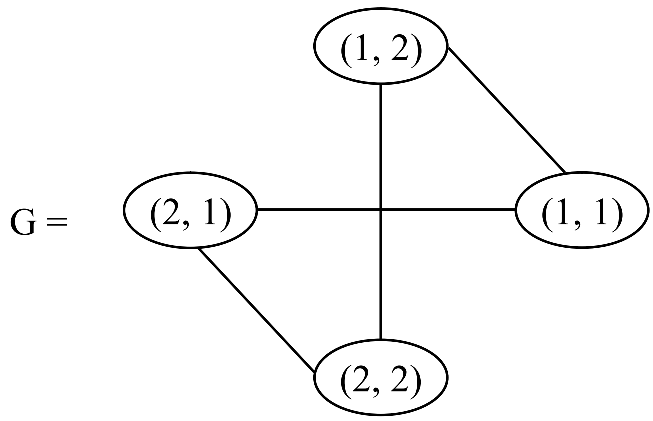


Figure 3.63

Example 3.29. Let S(3) be the symmetric semigroup of order 3.

Let G be the special symmetric semigroup vertices as row matrix vertex labeled multigraph associated with S(3).

The multigraph of G is as follows.

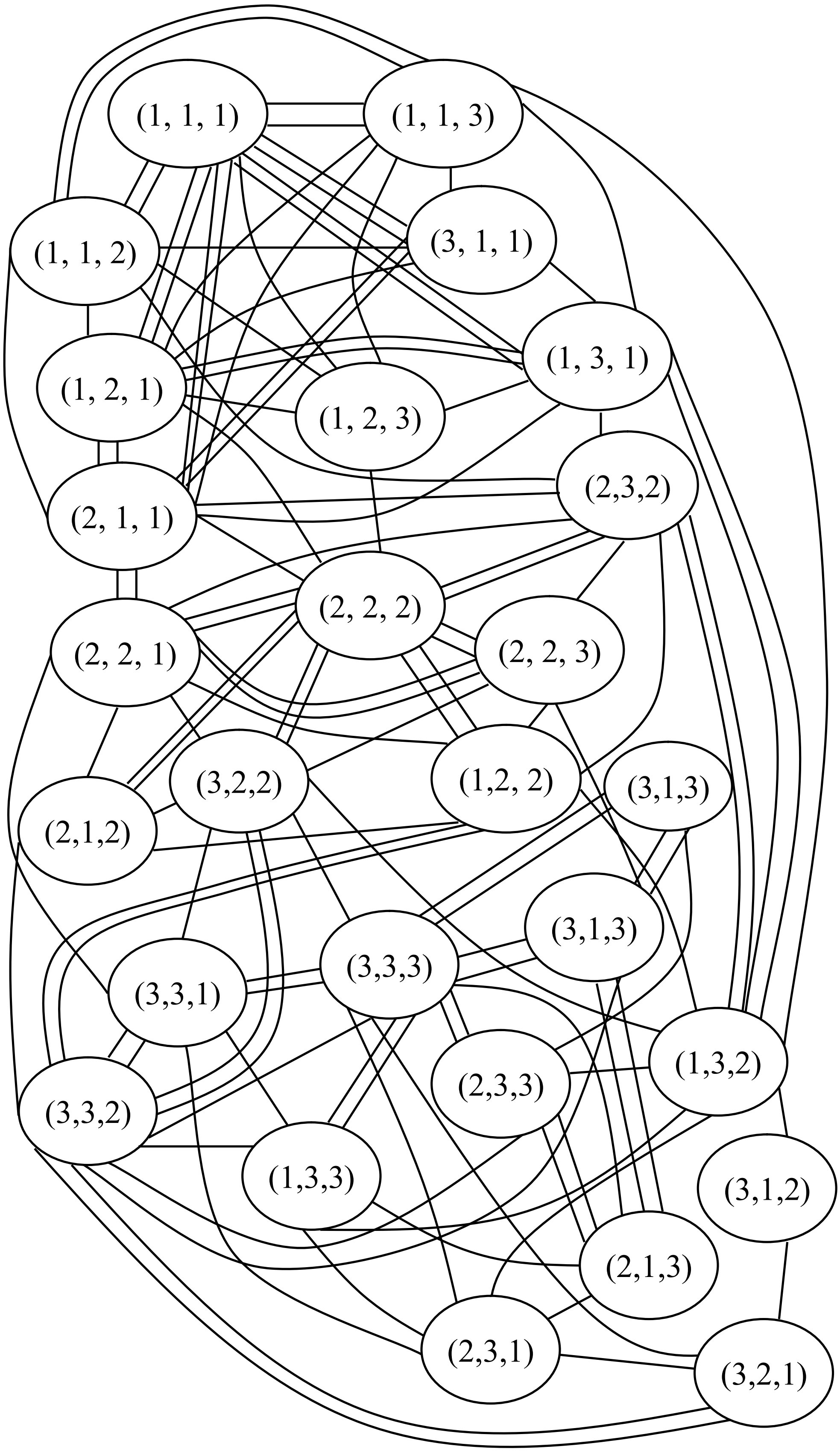

(1, 1, 1)
(1, 1, 3)
(1, 1, 2)
(3, 1, 1)
(1, 2, 1)
(1, 3, 1)
(1, 2, 3)
(2,3,2)
(2, 1, 1)
(2, 2, 2)
(2, 2, 1)
(2, 2, 3)
(3,2,2)
(1,2, 2)
(3,1,3)
(2,1,2)
(3,1,3)
(3,3,3)
(3,3,1)
(1,3,2)
(2,3,3)
(3,3,2)
(1,3,3)
(3,1,2)
(2,1,3)
(2,3,1)
(3,2,1)

Figure 3.64

We have not given all the edges and multiedges of the graph G. It is not complete for several edges we have not connected. We make the following observations. The multigraph associates with the symmetric semigroup S(3) contains the multigraph H associated with S_3 ($S_3 \subseteq S(3)$) viz given by the following figure.

H =

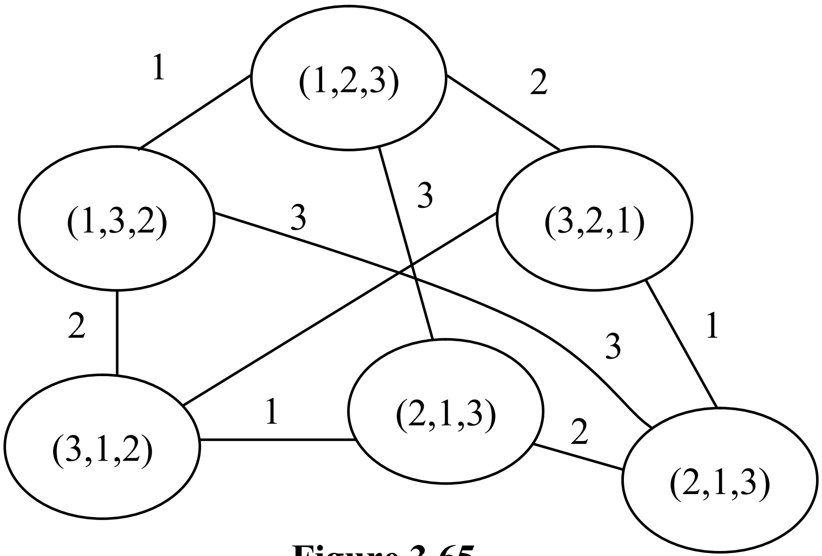


Figure 3.65

We see the multigraph associated with S(3) has multisubgraphs which are complete given by the following figure;

B_1 =

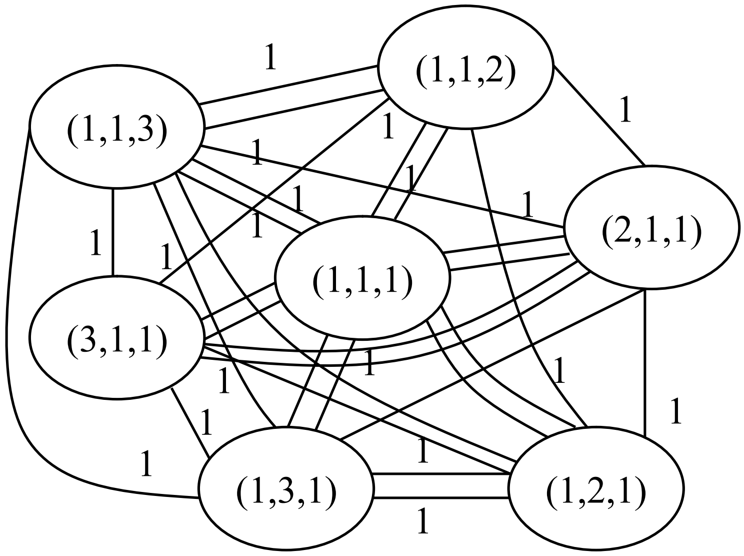


Figure 3.66

Clearly B_1 is a non uniform complete multisubgraph of G of order 7.

Let B_2 be another non uniform multisubgraph of G given by the following figure.

$B_2 =$

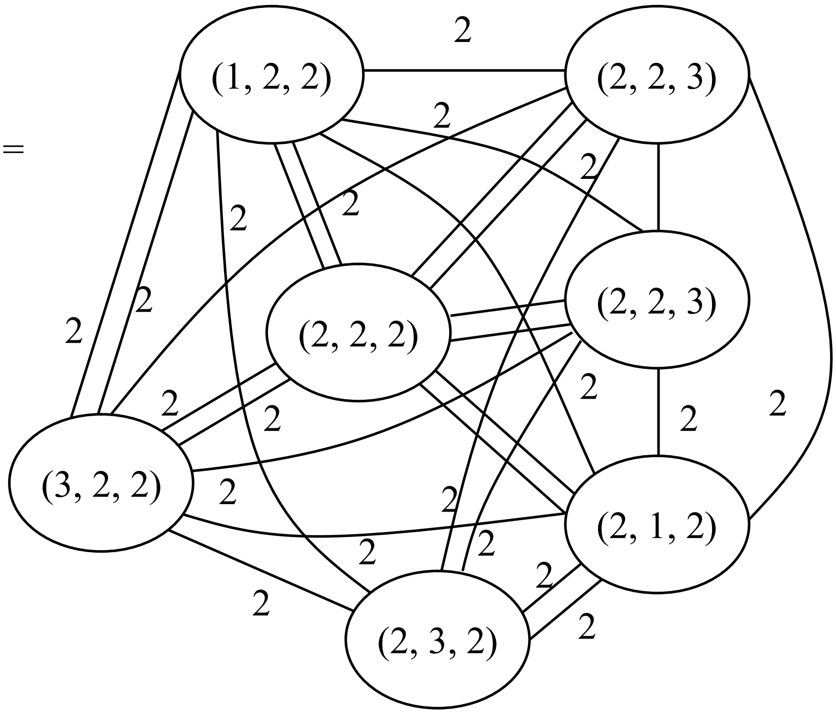


Figure 3.67

Clearly B_1 and B_2 are multigraphs but the edges and vertices are labeled differently.

Let B_2 be the nonuniform complete multisubgraph of G given by the following figure.

B3 =

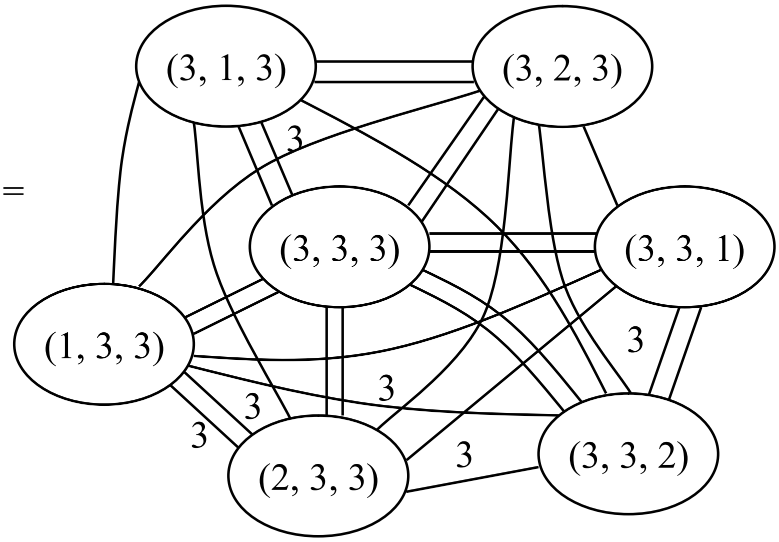


Figure 3.68

B_3 is also non uniform complete multisubgraph of order 7.

All the three multisubgraphs B_1, B_2 and B_3 are structurally identical except for the edge and vertex labels.

Z1 =

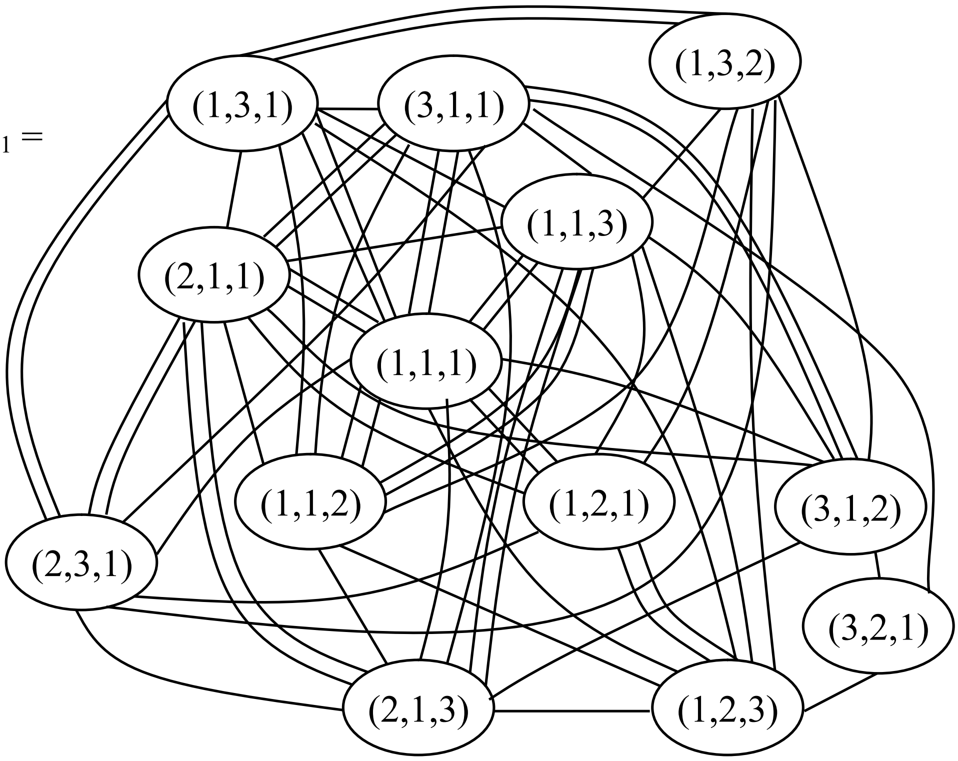


Figure 3.69

Now we have given the multisubgraph Z_1 with vertex sets of B_1 and S_3 in the Figure 3.69.

We see all the vertices associated with S_3 are densely related with B_1 as seen from the multisubgraph Z_1 of G.

Similar multisubgraphs Z_2 and Z_3 can be got using B_2 and B_3 with the vertex set {(1, 2, 3), (1, 3, 2), (3, 2, 1), (2, 1, 3), (2, 3, 1), (3, 1, 2)} respectively.

However, these multigraphs using S(n), the symmetric semigroups as vertex labels happens to be have many multiedges and has several complete multisubgraphs which may not in general be uniform.

We illustrate these situations in case of S(4) and S(5).

Example 3.30. Let G be the Plithogenic multigraph associated with the symmetric semigroup S(4).

We give a few of the multisubgraphs of G in the following.

Let H_1 be the multisubgraph with vertex sets from S(4) given by the following figure.

H_1=

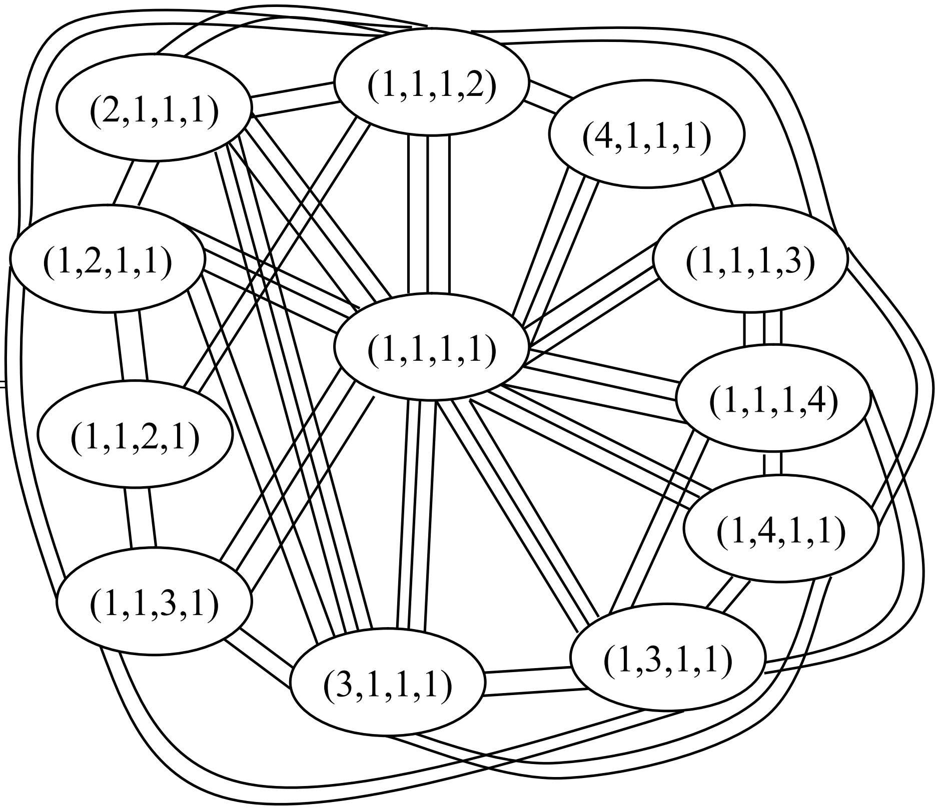


Figure 3.70

We have not completed this multigraph H_1. In fact it is a non-uniform complete multigraph with a maximum of 3 multiedges and a minimum of 2 multiedges. Order of H_1 is 13 and however the pattern we get is beautiful.

One can imagine the same situation in case of S(5) and generalize for S(n). These multisubgraphs are structurally very powerful and can be used as multinets when the connections of same type is needed.

Even in case of computer networks where several computers share the same type of information.

In case of S(4) we see if we are trying to use two elements 1 and 2 and form the multisubgraph B of G, we get the following structure.

B =

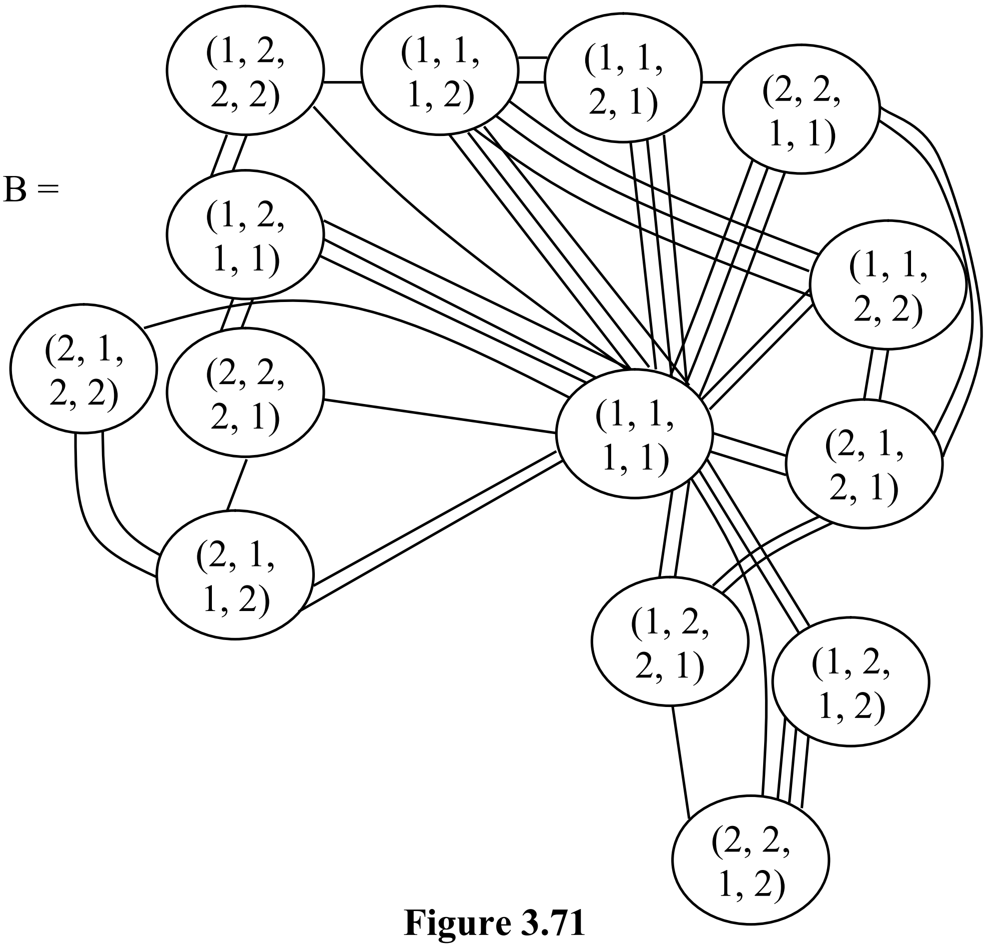


Figure 3.71

B is not even a nonuniform complete multisubgraph of G.

However we have said earlier the multigraph associated with S_4 will be a multisubgraph of G where G is the multigraph associated with the symmetric semigroup S(4) of degree 4.

This multigraph G has several complete uniform multisubgraphs contributed by cyclic subgroups of S_4 and some non uniform complete multisubgraphs contributed by elements of the form P = {(1, 1, 1, 1), (1, 1, 1, 2), (1, 1, 1, 3), (1, 1, 1, 4), (2, 1, 1, 1), …., (1, 4, 1, 1), (4, 1, 1, 1)} $\subseteq$ S(4), where |P| = 13.

We see in case of S(3) we have P = {(1, 1, 1), (1, 1, 2), (1, 2, 1), (2, 1, 1), (3, 1, 1), (1, 3, 1), (1, 1, 3)} and |P| = 7.

In case of S(5) we have,

P = {(1, 1, 1, 1, 1), (1, 1, 1, 1, 2), (1, 1,1 , 2, 1), (1, 1, 2, 1, 1), (1, 2, 1, 1, 1), (2, 1, 1, 1, 1), (1, 1, 1, 1, 3), (1, 1, 1, 3, 1), (1, 1, 3, 1, 1), (1, 3, 1, 1, 1), (3, 1, 1, 1, 1), (1, 1, 1, 1, 4), (1, 1, 1, 4, 1), (1, 1, 4, 1, 1), (1, 4, 1, 1, 1), (4, 1, 1, 1, 1), (1, 1, 1, 1, 5), (1, 1, 1, 5, 1), (1, 1, 5, 1, 1), (1, 5, 1, 1, 1), (5, 1, 1, 1, 1)} $\subseteq$ S(5), |P| = 21. We get a non uniform complete multisubgraph of order 21 = 5 × 4 + 1 with maximum of 4 multiedges and a minimum of 3 multiedges.

In view of all these we have the following result.

Theorem 3.5. *Let S(n) be the symmetric semigroup of degree n. G be the Plithogenic multigraph associated with S(n).*

i) G has non uniform complete multisubgraphs of G of order n (n – 1) + 1 and the maximum number of edges is (n – 1) and minimum number of edges is (n – 2).

ii) There are n such multisubgraphs of G of order n(n – 1) + 1.

Proof follows from the fact that if P = {(1, 1, …, 1), (1, 1, …, 2), …, (2, 1, 1, …, 1), (1, 1, …, 3), …, (3, 1, 1, …, 1), …, (1, 1, 1…, 1, n), …, (n, 1, …, 1)} $\subseteq$ S(n) is a subset of S(n) and |P| = n(n – 1) + 1.

Now it is easily verified we can have maximum (n – 1) edges and a minimum of (n – 3) edges, hence the claim.

Proof of (ii). Instead of (1, …, 1), we can with any (r, r, .., r); $1 \leq r \leq n$; so there are n such non uniform multisubgraphs of order n (n – 1) + 1.

The number of uniform complete multisubgraphs are got from the cyclic subgroups of $S_n \subseteq S(n)$.

Now we in case of S(5) make changes in two variables and the rest of the 3 remains the same.

Example 3.31. Let S(5) be the symmetric semigroup of degree 5. Let G be the associated Plithogenic multigraph.

Consider M = {(11111), (22111), (21112), (11122), (12121), (21211), (12211), (11221), (21121), (12112), (11212), (31111), (13311), (11331), (11133), (31311), (31131), (31113), (13131), (13113), (11313), (11144), (11441), (14411), (44111), (41114), (14114), (14141), (11414), (41141), (41411), (11155), (11551), (15511), (55111), (51115), (51511), (15151), (11515), (51151), (15115)} |M| = 41 = (4 × 5C2 + 1).

We see the multisubgraph associated with M is a non uniform complete multisubgraph with maximum of 3 multi edges and a minimum of 1 edge.

Now with the same multisubgraph we work using S(6).

Let N {(1, 1, 1, 1, 1, 1), (1, 1, 1, 1, 2, 2), (1, 1, 1, 2, 2, 1), (1, 1, 2, 2, 1, 1), (1, 2, 2, 1, 1, 1), (2, 2, 1, 1, 1, 1), (2, 1, 2, 1, 1, 1), (2, 1, 1, 2, 1, 1), (2, 1, 1, 1, 2, 1), (1, 2, 1, 2, 1, 1), (1, 2, 1, 1, 2, 1), (1, 2, 1, 1, 1, 2), (1, 1, 2, 1, 1, 2), (1, 1, 2, 1, 2, 1), (1, 1, 1, 2, 1, 2), (2, 1, 1, 1, 1, 2), (1, 1, 1, 1, 3, 3), …, (3, 1, 1, 1, 1, 3), (1, 1, 1, 1, 4, 4), …, (4, 1, 1, 1, 1, 4), (5, 1, 1, 1, 1, 5), …, (1, 1, 1, 1, 5, 5), (6, 1, 1, 1, 1, 6), .., (1, 1, 1, 1, 6, 6)} |N| = 76 = 5 × $6C_2$ + 1 number of vertices.

We see the associated multisubgraph with N is a non uniform complete multisubgraph with a maximum multiedge of four and a minimum multiedges are two.

In view of all these we can obtain some conditions for the non-uniform multisubgraphs to be complete that are of bigger order in G.

Theorem 3.6. *Let S(n) be the symmetric semigroup of degree n. Let G be the permutation Plithogenic multigraph associated with S(n).*

There is a non-uniform complete multisubgraph of order $(n - 1) \times nC_2 + 1$.

Proof: Let G be the given Plithogenic multigraph associated with S(n). To prove there exist one such multisubgraph. We take P = {(1, 1, …, 1), (1, 1, 1, 1, …, 1, 2,2), (1, 1, 1, 1, 1, …, 1, 2, 1, 2), …, (n – 1, n – 1, 1, 1, 1, …, 1)} $\subseteq$ S(n). Clearly |P| = $(n - 1)\, nC_2 + 1$ and it has a maximum of (n – 2) multiedges and minimum of (n – 4) multiedges.

In fact there are n number of such non uniform complete multisubgraphs in G.

The following observations are mandatory.

In case of S(6) if we take elements of the form {(1, 1, 1, 2, 2, 2), (2, 1, 2, 1, 2, 1), (1, 1, 1, 1, 1, 1), (2, 2, 2, 1, 1, 1) and so on} = B. The multisubgraph associated with B will not be non uniform complete multisubgraph as there is no edges between (1, 1, 1, 2, 2, 2) and (2, 2, 2, 1, 1, 1).

Next we consider S(7) and its permutation Plithogenic multigraph in the following.

Example 3.32. Let S(7) be the symmetric semigroup of degree 7. Let H = {(1111111), (1111222), (1112221), (1122211), (1222111), (2221111), (1112212), (1112122), (1121122), (1121212), (1122121), (1122112), (1221112), (1221121), (1221211), (2212111), (2211112), (2211121), (2211211), (211,1122), (2111212), (2112211), …, (1212121), …, (6661111), …, (6161616), (7771111), ..., (1717171)}.

We see the associated multisubgraph of G is a non uniform complete multisubgraph of order 211 = 6 × 7C_3 + 1. There are 7 such non uniform complete multisubgraphs in G.

In view of this we have the following theorem.

Theorem 3.7. *Let G be the permutation Plithogenic multigraph associated with S(n); n ≥ 7.*

There exists n multisubgraphs of G which are non uniform complete multisubgraphs of order (n – 1) nC_3 + 1 with maximum of (n – 3) multiedges and minimum of (n – 6) edges.

Proof is as in case of other theorems so left as an exercise to the reader.

In view of all these we put forth the following results.

Theorem 3.8. *Let G be the permutation Plithogenic multigraph associated with the symmetric semigroup S(n).*

i) The maximum number of multiedges multisubgraphs of G which are non uniform are given by those (n – 1) (nC_r) + 1 vertices where $r \begin{cases} \leq \frac{n}{2} - 1 & \text{if } n \text{ is even} \\ \leq \frac{n-1}{2} & \text{if } n \text{ is odd} \end{cases}$.

Proof is left as an exercise to the reader.

However, if one wishes to work with non-uniform complete multinetworks the researcher can label the vertices as per the theorem and get these complete non uniform multigraphs of desired order depending upon the need.

The condition on r given in the theorem is mandatory for the non-uniform complete multigraphs to exist. For otherwise these multigraphs will not be non-uniform complete multisubgraphs of G.

We further see this Plithogenic multigraphs G associated with the symmetric semigroup S(n) has complete multisubgraphs of G generated by cyclic subgroups of $S_n \subseteq S(n)$ of order (n – 1), (n – 2) and so on.

Thus if any researcher needs multinetworks which are complete or non uniform complete one can seek the Plithogenic multisubgraphs of the Plithogenic multigraphs associated with S(n) the symmetric semigroup of S(n).

There are several other properties associated with these symmetric semigroup graphs [24] we do not deal with them in this book.

We aspire to work through the applications of these to multinetworks of plithogenic models which is a generalization of SVNS model.

Study in this direction is innovative and interesting. This analysis is carried out in the last chapter of this book.

We just give examples Plithogenic fuzzy graphs.

Example 3.33. Let G be a Plithogenic fuzzy graph given by the following figure.

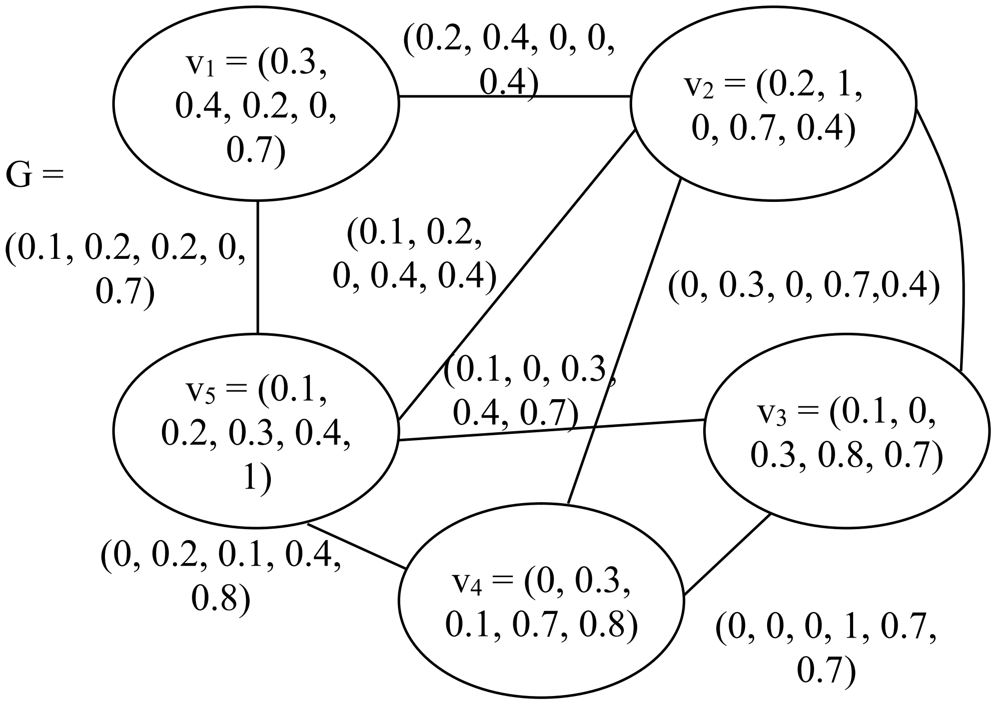


Figure 3.72

The edges are given row matrix label for v_i v_j we denote the label by min (v_i, v_j); for instance min (v_1, v_2) = (0.2, 0.4, 0, 0, 0.4).

We call this graph G as a min Plithogenic graph.

The same graph H which is max but has the same structure as that of G is given below.

H =

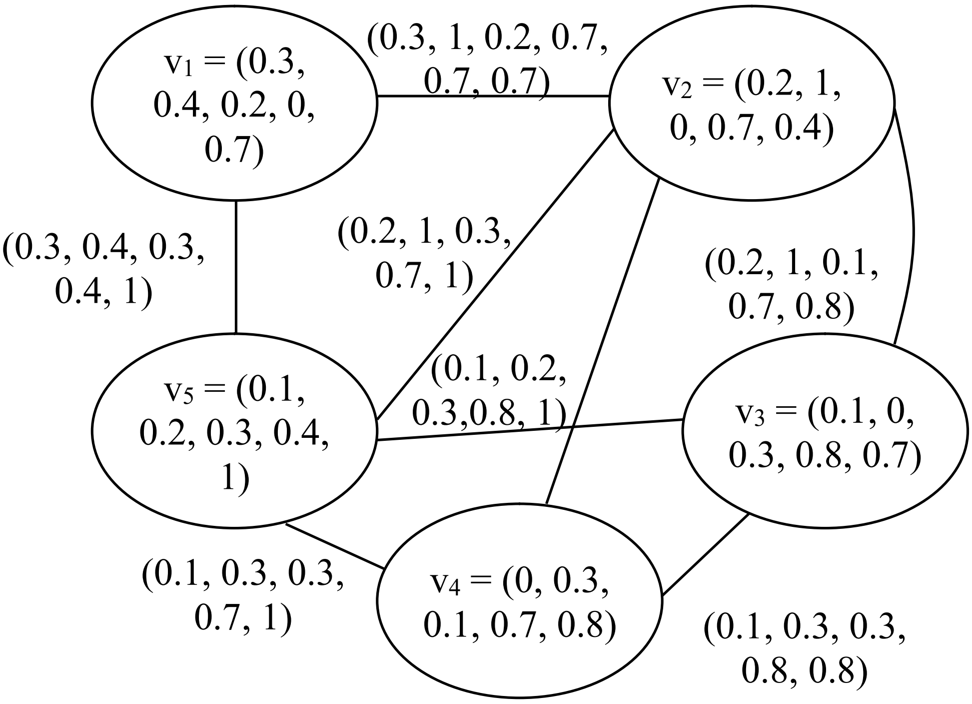


Figure 3.73

Both the Plithogenic fuzzy graphs are structurally the same with the difference that the edge labels are different.

The number of edges in these graphs are in the hands of the researcher more so is the operation which they use to define the edge label max or min.

This sort of single valued neutrosophic graphs have been defined and applied in medical science [87]. The only difference is that in case of SVNS they use only 1×3 row matrices but when we try to apply for plithogenic models the row matrices can be $1 \times n$ where n need not in general be three.

We also by default of notation call these as row matrix vertex and edges labeled graph for these are the membership values taken from the interval [0, 1].

However, it is pertinent to keep on record that these values of 1 × n row matrices can be real or complex or fuzzy or neutrosophic. Study in this direction will be dealt in the following chapter.

Finally, we give one example of a min Plithogenic neutrosophic graph.

Example 3.34. Let G be the neutrosophic graph given by the following figure.

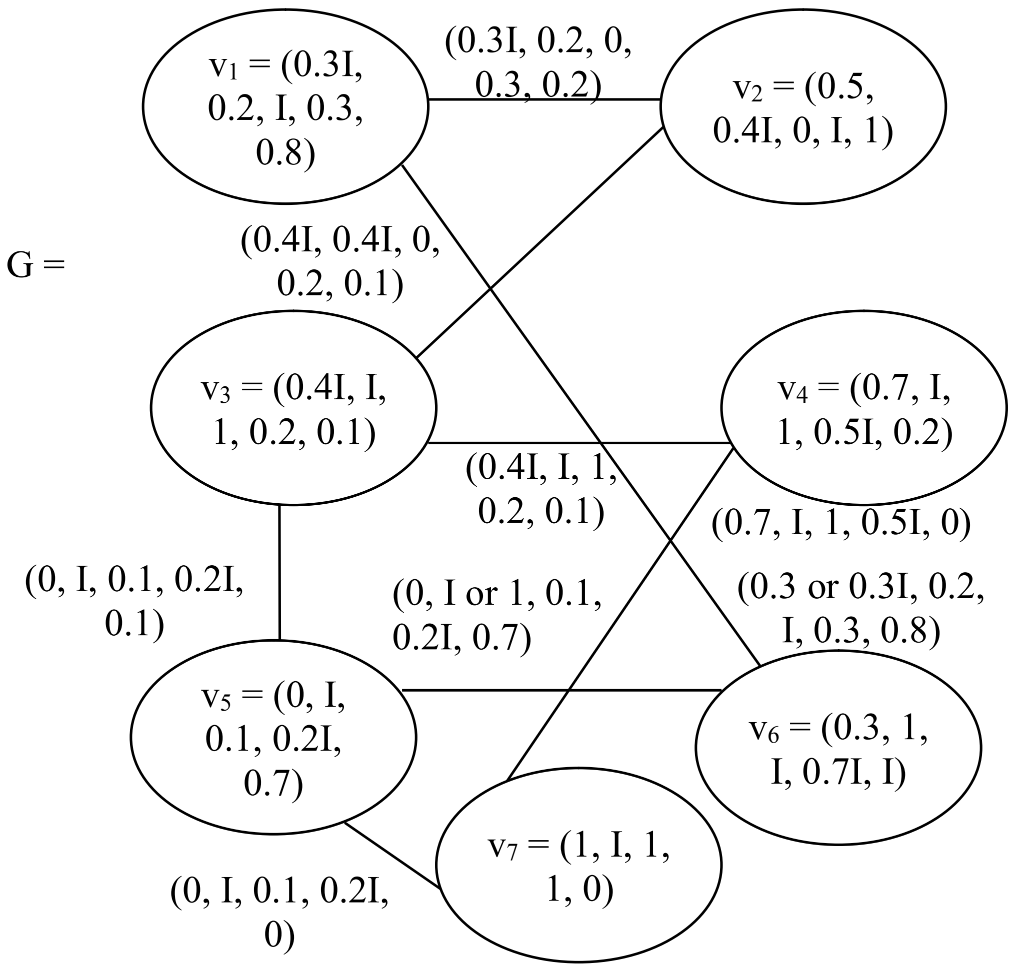


Figure 3.74

The following observations are important.

i) min (aI, a) is a or aI according to the wishes of the expert whether he/she want to emphasize on neutrosophic value or real value ($a \in R$).

ii) $\min (aI, bI), = \begin{cases} aI & \text{if } a < b \\ bI & \text{if } b < a. \end{cases}$

iii) $\min (aI, b) = \begin{cases} aI & \text{if } a < b \\ b & \text{if } b < a \end{cases}$

iv) $\min (a, bI) = \begin{cases} a & \text{if } a < b \\ bI & \text{if } b < a. \end{cases}$

Next we proceed onto describe a few Plithogenic bipartite multigraphs in the following by some examples.

Example 3.35. Let M be a Plithogenic multibigraph using the vertex labels from S(4).

M =

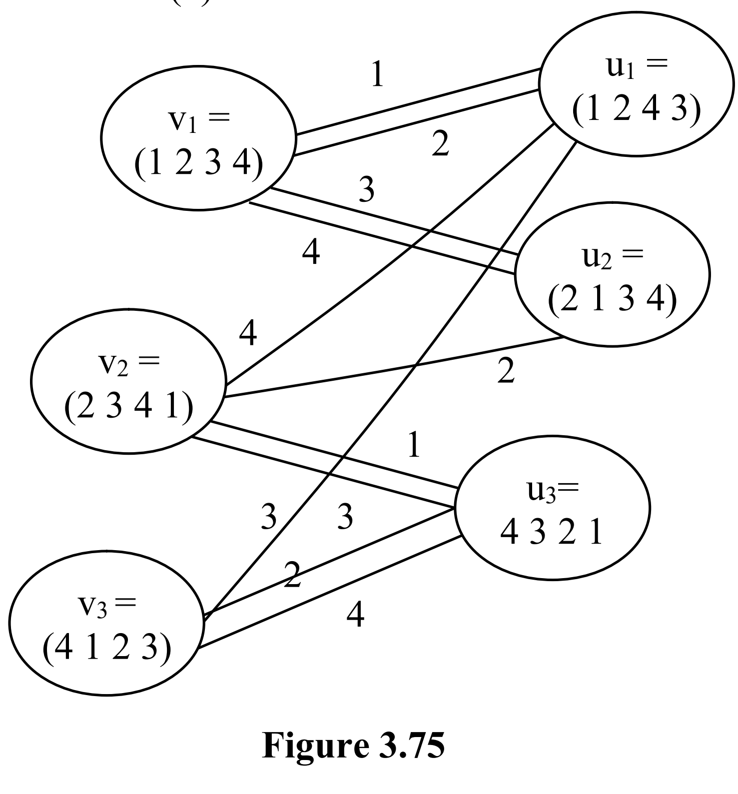


Figure 3.75

Clearly B is a Plithogenic bipartite multigraph. It is important to note that the multiedges are labeled automatically so it is mandatory to maintain none of the domain space of the multigraph has edges that automatically fixes. Similarly, none of the range space of the multigraph has common edges.

That is we see in this case the vertices v_1, v_2 and v_3 have no edges in common. Similarly, the vertex set $\{u_1, u_2, u_3\}$ have no edges in common.

We give yet another example of bipartite multigraph associated with S_6 in the following.

Example 3.36. Let N be Plithogenic multibipartite graph given by the following figure.

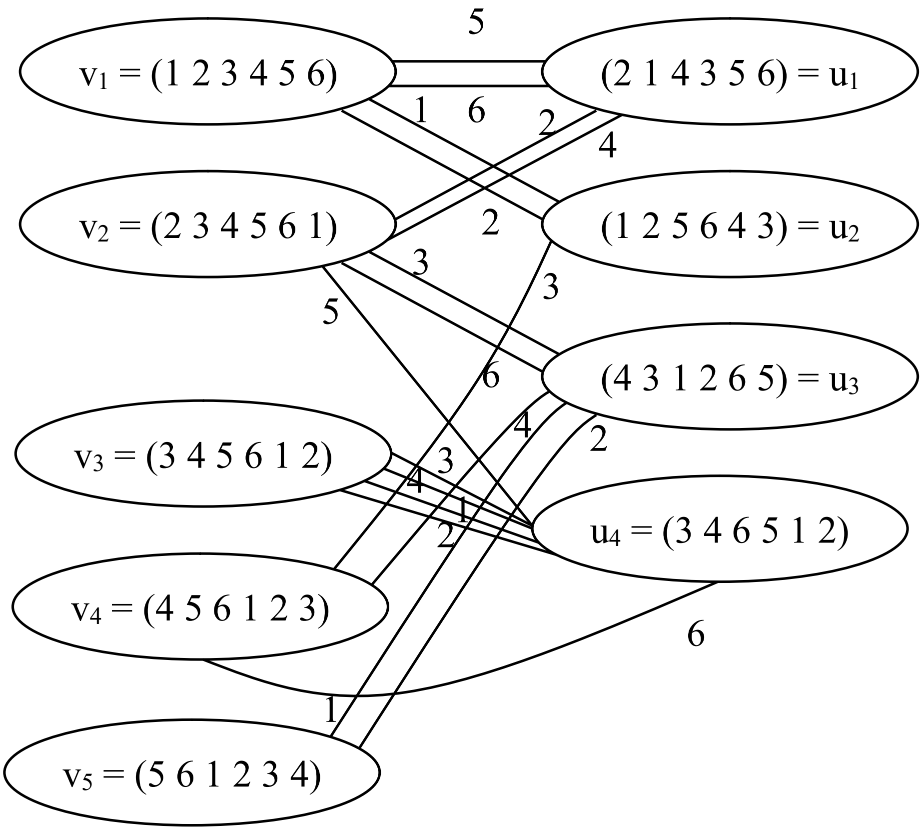


Figure 3.76

This is a multibipartite graph. Clearly, we see the vertex set $\{v_1, v_2, v_3, v_4, v_5\}$ are not adjacent taken in pairs.

Similarly the vertex set $\{u_1, u_2, u_3, u_4\}$ are such that none of them are adjacent taken in pairs. Hence N is a bipartite multigraph.

It is interesting to note every row matrix vertex labeled multigraph associated with the symmetric group S_n has a multisubgraph which is a bipartite multisubgraph.

Theorem 3.8. *Let G be a Plithogenic multigraph associated with the symmetric group S_n.*

G has a multisubgraph which is a multibipartite subgraph.

Proof is direct can be given by the examples.

However we proceed onto suggest a open conjecture.

Conjecture 3.1. Let G be the Plithogenic multigraph associated with the symmetric group S_n.

i) What is the largest order of the Plithogenic multibipartite subgraph of G?

ii) What is the biggest size of the Plithogenic multibipartite subgraph of G? $(2 \leq n < \infty)$.

Next we study by examples the bipartite multisubgraphs of G, where G is the multigraph associated with the symmetric semigroup S(n). $2 \leq n < \infty$.

Example 3.37. Let S(4) be the symmetric semigroup of degree 4. Let G be Plithogenic multigraph of S(4). Let H be a multipartite subgraph of G given by the following figure.

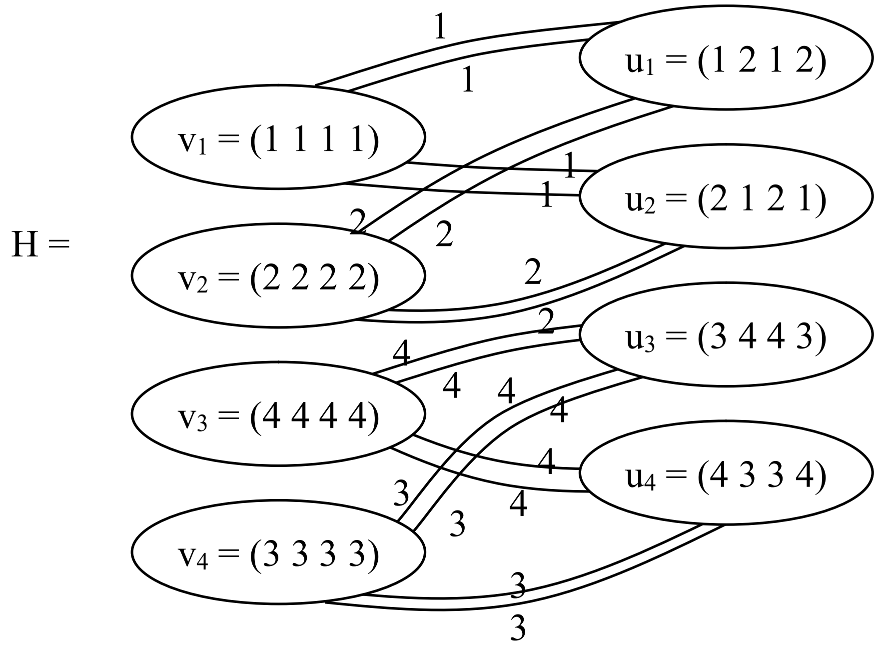


Figure 3.77

H is a bipartite multisubgraph of G.

We can have several such multisubgraphs which are bipartite multisubgraphs.

The main problem which we encounter here is finding the biggest or largest order of such multibipartite subgraph of G and the largest size of such multibipartite subgraph of G for any S(n); $2 \leq n < \infty$.

To this effect we propose the following open conjectures.

Conjecture 3.2. Let S(n) be the symmetric semigroup of degree n.

Let G be the permutation Plithogenic multigraph associated with the symmetric semigroup S(n); $2 \leq n < \infty$.

i) What is the order of the largest multibipartite subgraph of G?

ii) What is the size of the largest multibipartite subgraph of G?

iii) Are these largest multibipartite subgraphs unique?

Next we proceed onto describe the notion of min or max Plithogenic graphs by examples.

Example 3.38. Let G be a min edge fuzzy bipartite graph given by the following figure.

G =

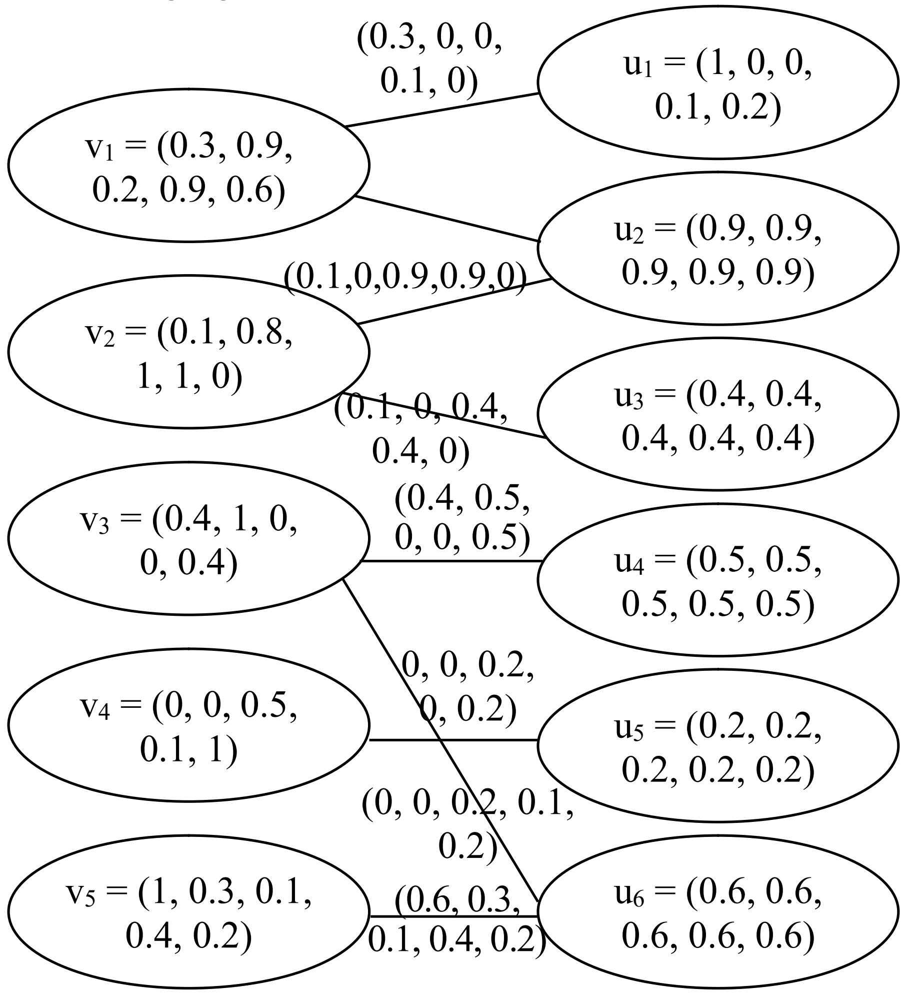


Figure 3.78

The concerned has taken $v_1, v_2, \ldots, v_5$ as the nodes of the domain space and $(u_1, u_2, \ldots, u_6)$ as the nodes of the range space so we do not have any edges defined between the vertices $v_1, v_2, \ldots, v_5$ similarly there are no edges between $u_1, u_2, \ldots, u_6$.

However there are edges defined as minimum of (v_i, u_j) for appropriate i and j; $1 \leq i \leq 5$ and $1 \leq j \leq 6$. We see if we defines edges between every pair (v_i, v_j) then the resulting bipartite graph will be complete otherwise it will be not be a complete bipartite graph.

We give an example of a minimum edge complete bipartite graph.

Example 3.39. Let G be a min edge complete bipartite graph given by the following figure.

G =

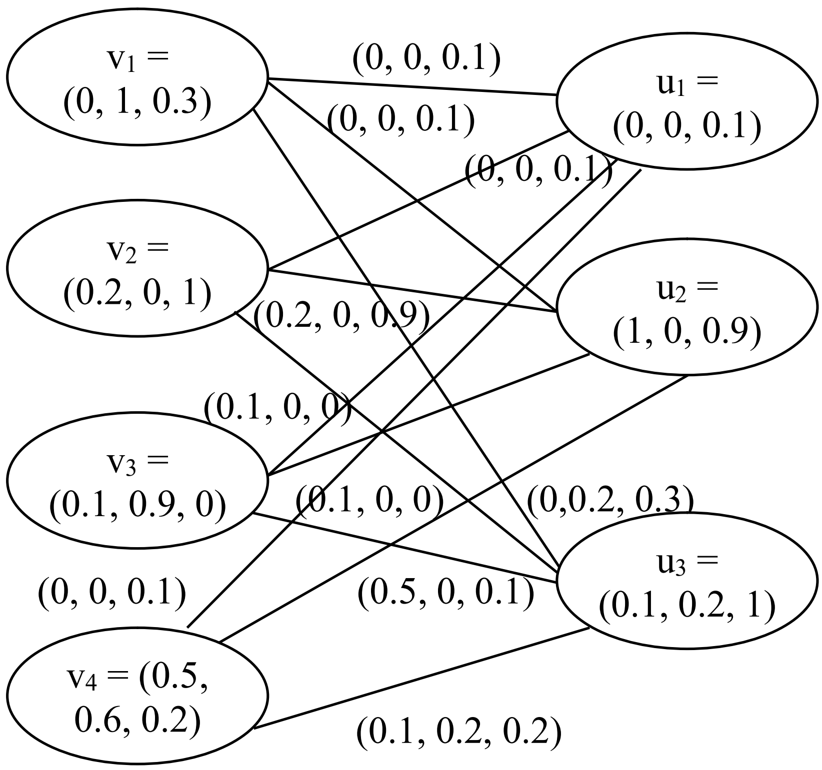


Figure 3.79

G is a complete min edge bipartite graph.

We give one illustration of each max edge complete bipartite Plithogenic graph and one not a complete max edge bipartite Plithogenic graph by examples.

Example 3.40. Let G be a complete max edge Plithogenic graph given by the following figure.

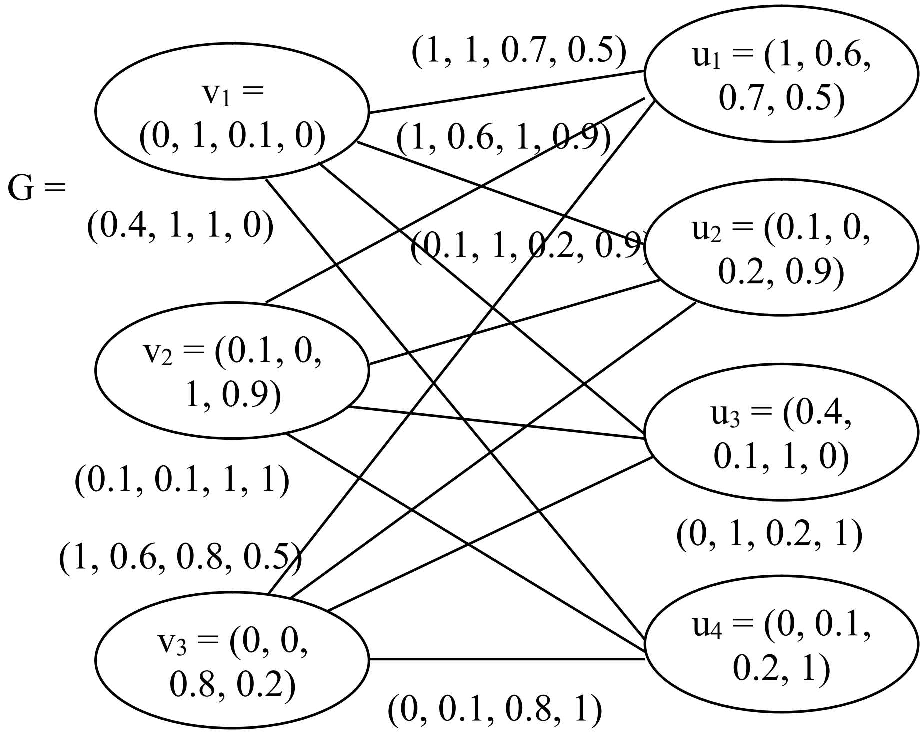


Figure 3.80

Clearly G is a max edge complete bipartite graph. We have not labeled the graph, the reader is expected to label edges using max operator.

Now an example of not a complete bipartite graph is provided.

Example 3.41. Let G be a not complete max edge bipartite graph given by the following figure.

G =

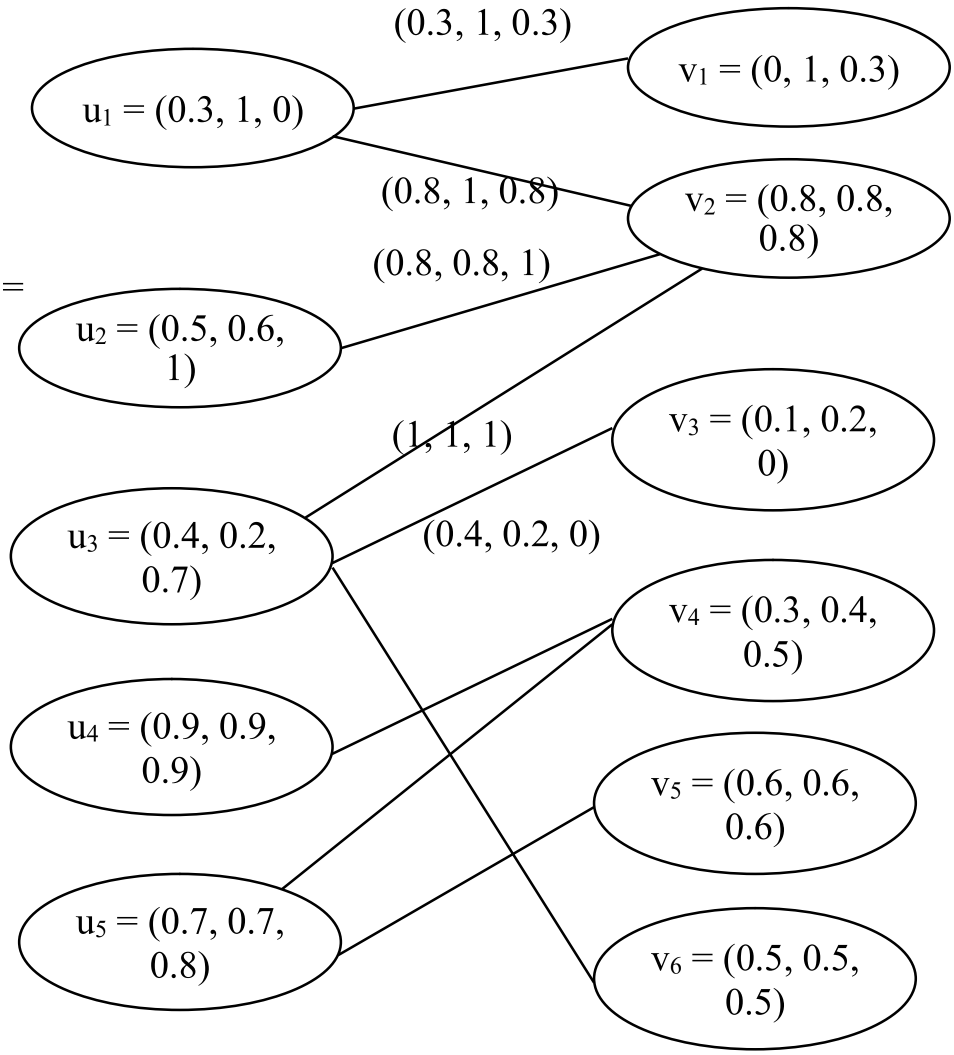


Figure 3.81

Clearly G is a max non complete bipartite graph with row matrix labeled as edges and vertices.

We now provide one example of each max and min tripartite graph of this type.

Example 3.42. Let G be the min tripartite graph given by the following figure.

G =

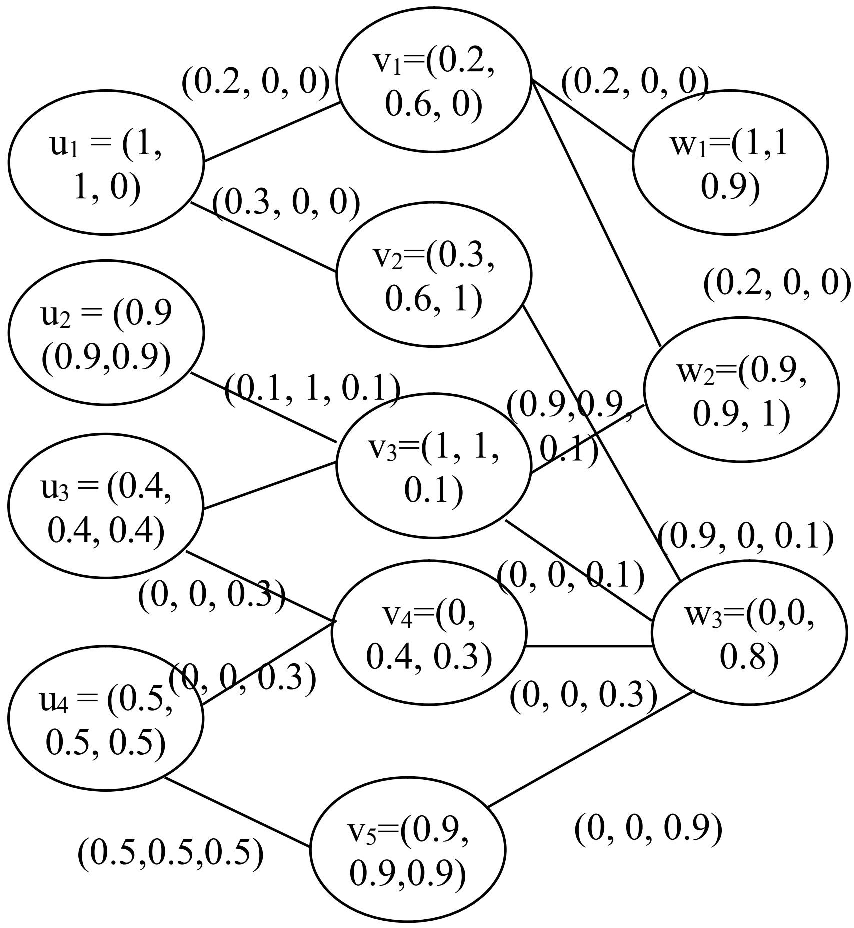


Figure 3.82

G is a min Plithogenic tripartite graph. It has applications in the field of medicine, social networks, psychology and so on.

Next we provide an example of a max row matrix vertex and edge labeled tripartite graph.

Example 3.43. Let G be a max Plithogenic tripartite graph given by the following figure.

It is pertinent to mention that the edge labels are automatically fixed once the vertex labels are provided.

G =

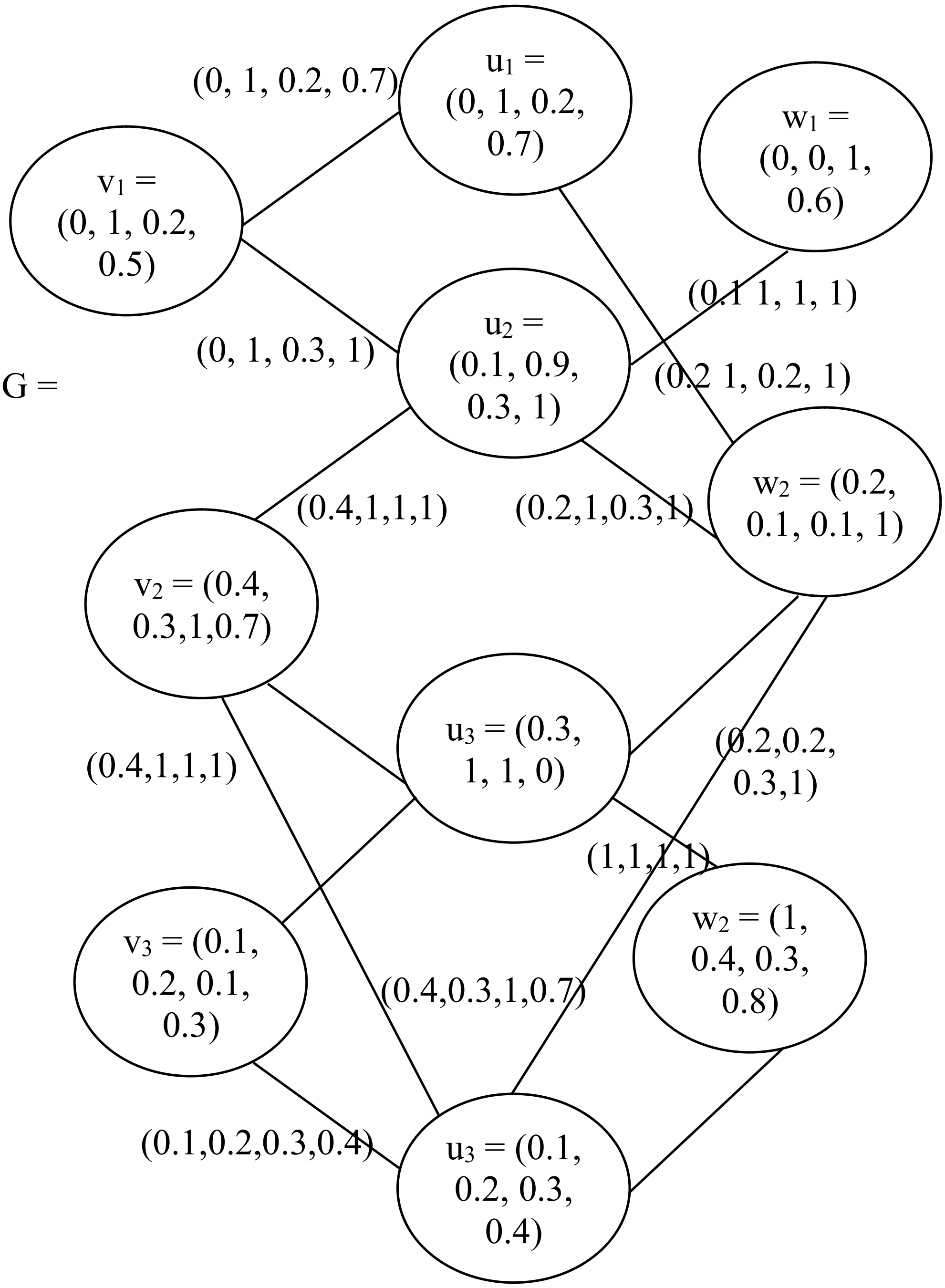


Figure 3.83

Clearly G is a max edge bipartite Plithogenic graph.

Next we proceed onto give as exercise to the reader finding complete n partite graphs for $n \geq 4$.

Also find those m-partite multisubgraphs of G associated with S_n and S(n). Characterize them in these special cases.

Next we proceed onto give some problems some of which are at research level and some simple problems in the following.

Problems

1. Give an example of a same 1×5 Plithogenic vertex graph with 6 vertices.

    a) A complete graph

    b) A star graph

    c) A ring / circle graph

    d) A line graph.

2. How are these same Plithogenic vertex different from other graphs?

3. Let G be a Plithogenic multigraph associated with the symmetric group S_7.

i) Find all complete Plithogenic multisubgraphs of G. (Find the associated subset in S_7).

ii) Does G contain Plithogenic non uniform multisubgraphs? (Justify your claim).

iii) What is the largest mutlibipartite subgraph in G?

iv) Can G have complete multi bipartite subgraphs? (Justify your claim).

v) How many empty multisubgraphs of G exist? (Find its vertex subset in S_7).

4. Let S(6) be the symmetric semigroup of degree 6. Let G be the Plithogenic multigraph.

i) Prove the edge labels are fixed once vertex labels are given.

ii) Find the highest number of multiedges that can exist between any two relevant vertex sets.

iii) Can G have uniform complete special multisubgraphs? (What is the vertex sets of such multisubgraphs?)

iv) Does the vertex sets mentioned in problem (iii) enjoy any special properties as subsets?

v) Find all non-uniform complete special multisubgraphs of G. (Find the vertex sets associated with them, do they enjoy any special features associated with S(6)) the largest uniform complete multisubgraph of G.

vi) Find the largest non uniform complete multisubgraph of G.

vii) Find all uniform complete bipartite multisubgraphs of G.

viii) Find all nonuniform complete bipartite multisubgraphs of G.

ix) What are the probable applications of these in social information multinetworks?

5. Give any innovative application of Plithogenic multigraph.

6. Let G be a Plithogenic multigraph given by the following figure.

G =

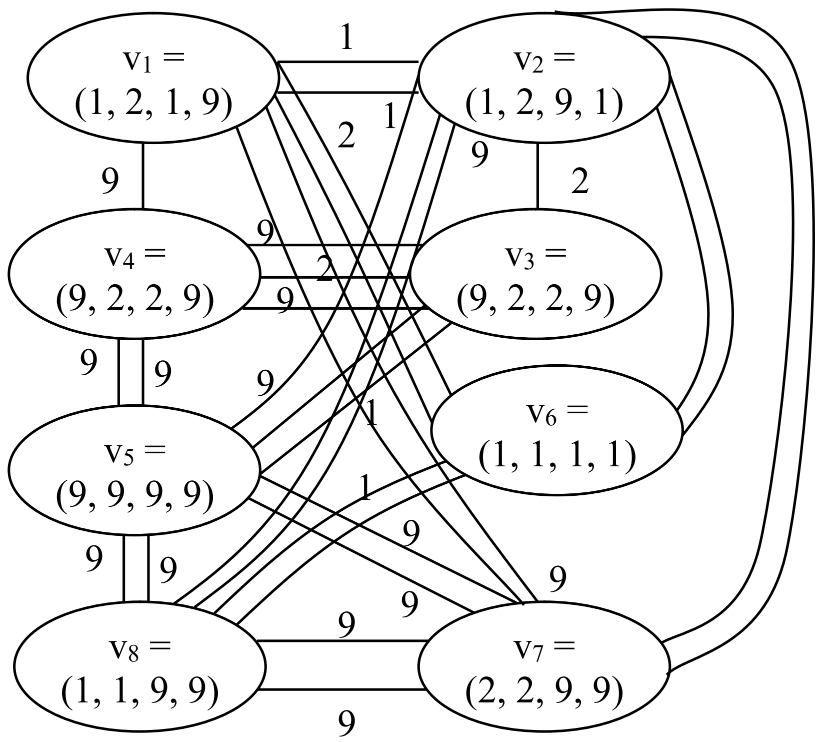


Figure 3.84

i) Find all multisubgraphs of G.

ii) What is the size of the multiclique?

iii) Does G contain K-clique? (what is the value of that K if one such exist)

iv) Can G contain a bipartite multigraph?

v) What is the size of the largest uniform complete multisubgraph of G?

vi) Does G contain nonuniform complete multisubgraphs?

vii) What are the special features associated with this multigraph?

7. Let S(9) be the symmetric semigroup of degree 9. G be the Plithogenic multigraph associated with S(9).

i) Find the maximum number of multiedges between any two nodes.

ii) Find the multisubgraph H of G associated with S_9.

iii) What is the largest order of a nonuniform complete multisubgraph of G?

iv) Does G contain a uniform complete multisubgraph?

v) What is the structure enjoyed by the multisubgraph in (iv)?

vi) What is the order of the largest complete simple subgraph of G?

vii) Find all bipartite multisubgraphs of G.

viii) What is the highest order of the multibipartite subgraph of G?

ix) What can be the biggest size of the non-uniform complete multisubgraph of G?

x) Find the largest size of a non-uniform complete bipartite multisubgraph of G.

8. Find some applications of these Plithogenic graphs in plithogenic models.

9. For the vertex sets V = {(0, 0.7, 1, 0.3, 0.5, 0), (0, 0, 0, 1, 1, 1), (0.5, 0.6, 0.7, 0, 1, 0), (0.7, 0.9, 0. 1, 0.9, 0.9), (1, 1, 0.1, 0.3, 0.4, 0.6)} draw a min edge complete Plithogenic graph.

i) How many complete graphs can be drawn using subsets of the V?

ii) Find the total number of min edge Plithogenic graphs that can be drawn using all the 6 vertices of the vertex set V.

iii) Is the number in (ii) same in case of max edge Plithogenic graphs also? Justify your claim.

iv) How many bipartite complete row matrix with min operator to label edges can be drawn using the 6 vertices?

v) Obtain any other special features associated with min (or max) edge Plithogenic graphs.

10. Draw G the Plithogenic multigraph using the vertex set V = {(1, 1, 6, 6, 5, 2, 4), (6, 1, 6, 1, 5, 4, 2), (3, 3, 4, 5, 6, 2, 1), (6, 6, 6, 6, 5, 5, 5), (4, 4, 4, 6, 6, 6, 1), (3, 3, 6, 6, 2, 2, 4), (6, 6, 5, 5, 4, 4), (1, 1, 6, 6, 4, 5, 5), (1, 1, 1, 6, 6, 6, 2), (1, 2, 3, 6, 4, 6)}.

i) Find the maximum number of multiedges in G.

ii) Does G contain a uniform complete multisubgraph?

iii) Can G have non uniform complete multisubgraphs?

iv) Define for these multigraphs the notion of

k – clique
k – clan
k – plex
k – cone and
k – club

v) What can be the maximum k and minimum k so problem (iv) exists?

vi) Can G contain complete uniform multibipartite subgraphs?

vii) Can G have bipartite multisubgraphs?

viii) What is the biggest size of the bipartite multisubgraph of G?

ix) What is the largest order of the complete bipartite multisubgraph of G?

x) Can G have a multitripartite subgraph?

Chapter Four

APPLICATIONS OF PLITHOGENIC GRAPHS

In this chapter we first give some applications of these newly built Plithogenic graphs to models using fuzzy graphs, single valued neutrosophic graphs. We just for the sake of completeness describe the above-mentioned concepts together with some applications.

In this section we proceed onto describe how Plithogenic graphs are applied to fuzzy graphs. To this end we define the notion of fuzzy graphs.

Definition 4.1. *Let s be a fuzzy subset of a nonempty set W and μ is a fuzzy relation which is symmetric on s that is*

$$s : W \to [0, 1] \text{ and } \mu: W \times W \to [0, 1]$$

such that $\mu(w, t) \leq s(w) \wedge s(t)$ for all $w, t \in W$ and wt denotes the edge between w and t and $s(t) \wedge s(w)$ denotes the minimum of s(t) and s(w). s is called the fuzzy vertex set of W and μ the fuzzy edge set of E.

We will illustrate this situation by an example.

Example 4.1. Let v_1, v_2, v_3, v_4, v_5 and v_6 be the four vertices of the fuzzy graphs G = G(s, μ) where s, W, μ are as given in definition.

G =

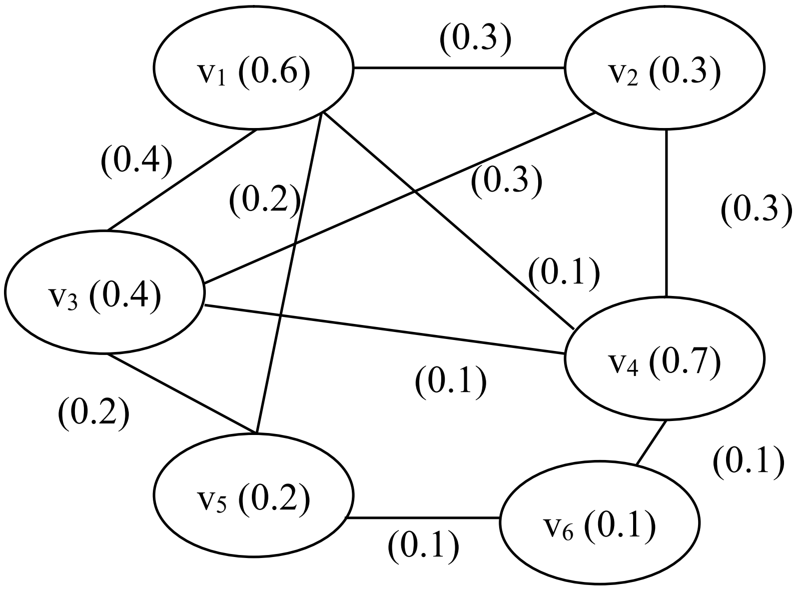


Figure 4.1 Fuzzy graph

We call this fuzzy graph as min valued fuzzy graph or in short min fuzzy graph.

Researchers can also define the notion of max fuzzy graphs and product fuzzy graphs.

Here in the definition of min fuzzy graph if a researcher or any expert replaces min by max function then fuzzy graph will be defined as the max fuzzy graph. If in the definition of min fuzzy graph if we replace the min by product we will define the resultant fuzzy graph as the product fuzzy graph. We will provide the examples only by using the min fuzzy graph given in example 4.1.

Example 4.2. The max fuzzy graph using the vertex sets {v_1 (0.6), v_2(0.3), v_3(0.4), v_4(0.7), v_5(0.2) and v_6(0.1)}. Let H = max H(s, μ) given by the following figure.

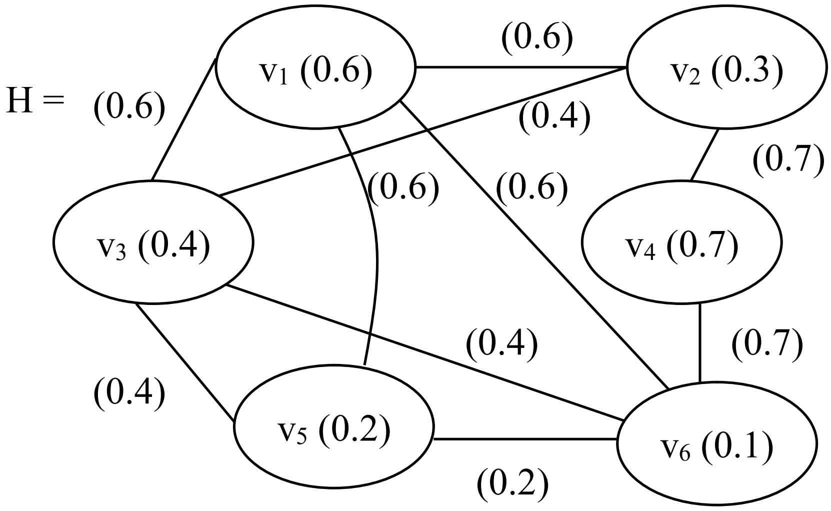


Figure 4.2

We see the edge labels of H and G are distinct.

Next we provide an example of product fuzzy graphs using the same vertex set given in example 4.1.

Example 4.3. Given V = {v_1(0.6), v_2(0.3), v_3(0.4), v_4(0.7), v_5(0.2), v_6(0.1)} as the set of vertices. The product fuzzy graph is as follows.

K =

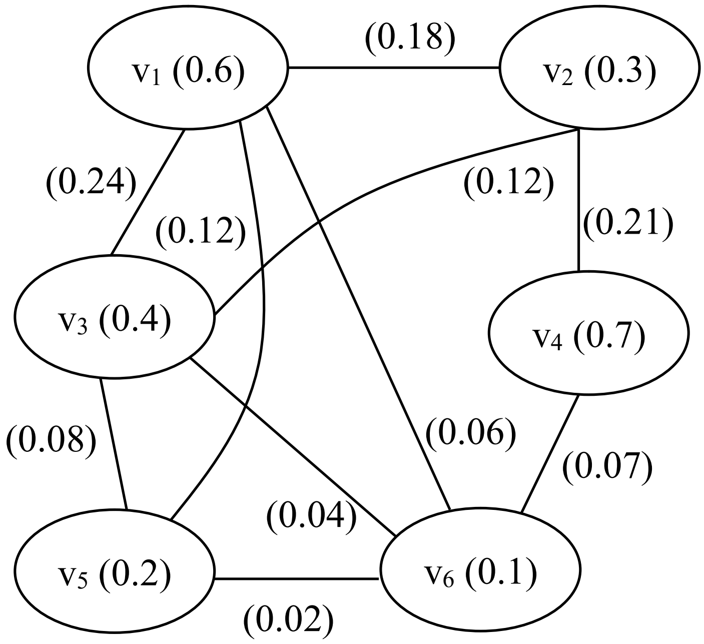


Figure 4.3

We see the edge weights of K is different from that of the edge weights of G and H.

Next we define yet another type of fuzzy graph called mean or average fuzzy graph where the min operator is replaced by mean of the vertex set values.

We will for the same set of vertex set provide the mean fuzzy graph M the following example.

Example 4.4. Let B be the mean fuzzy graph given by the same set of vertex sets V = {v_1 (0.6), v_2(0.3), v_3(0.4), v_4(0.7), v_5(0.2), v_6(0.1)} as that the one vertex sets used in example 4.

The figure of B is given by the following figure.

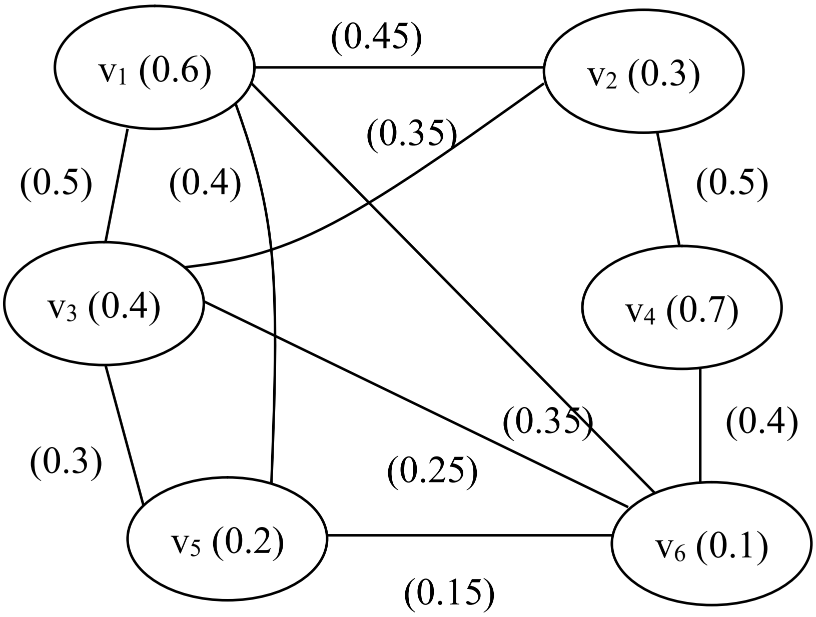
B = 

Figure 4.4

We see this mean fuzzy graph is different from all the three fuzzy graphs G, H and K.

Now we provide an example in which the fuzzy graphs of all the four types are provided and comparison of them is done.

Example 4.5. Let C = {C_1, C_2, C_3, C_4, C_5, C_6, C_7} be the six concepts which shows the marks of a student of 10th standard given in the form of membership.

C_1= v_1(0.9) membership of marks in mathematics

C_2 = v_2(0.8) membership of marks in physics

C_3 = v_3(0.5) membership of marks in chemistry

C_4 = v_4(0.4) membership of marks in languages

C_5 = v_5(0.6) membership of marks in English

C_6 = v_6(0.65) membership of marks in computer science

$C_7 = v_7(0.7)$ membership of marks in logic

Using an expert the product fuzzy graphs G was drawn.

G =

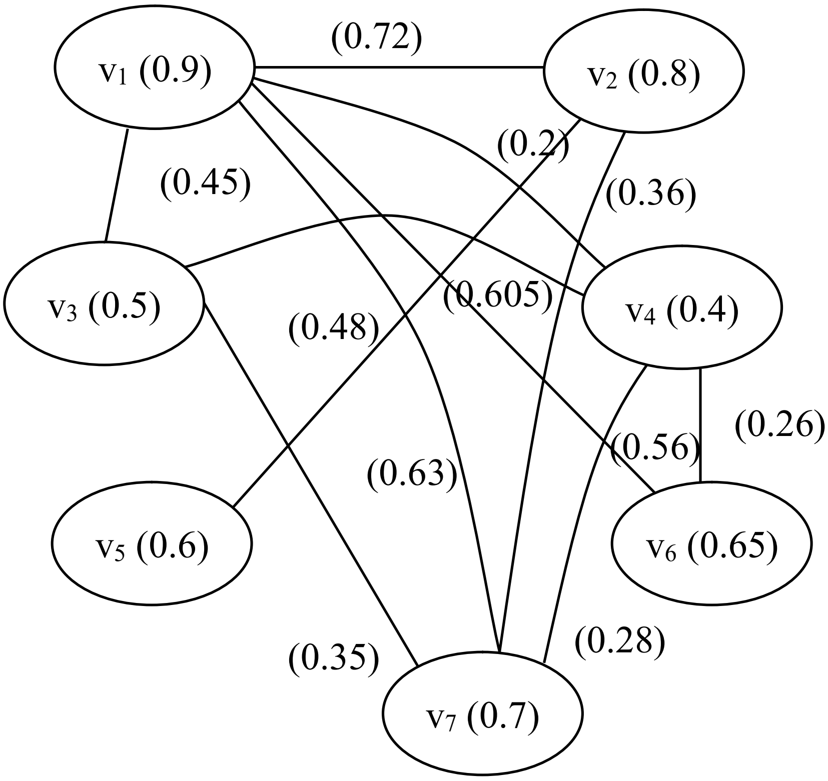


Figure 4.5

Suppose one ventures to get a complete product fuzzy graph for the same set of 7 vertices related with C_1, C_2, … C_7 we have the following figure.

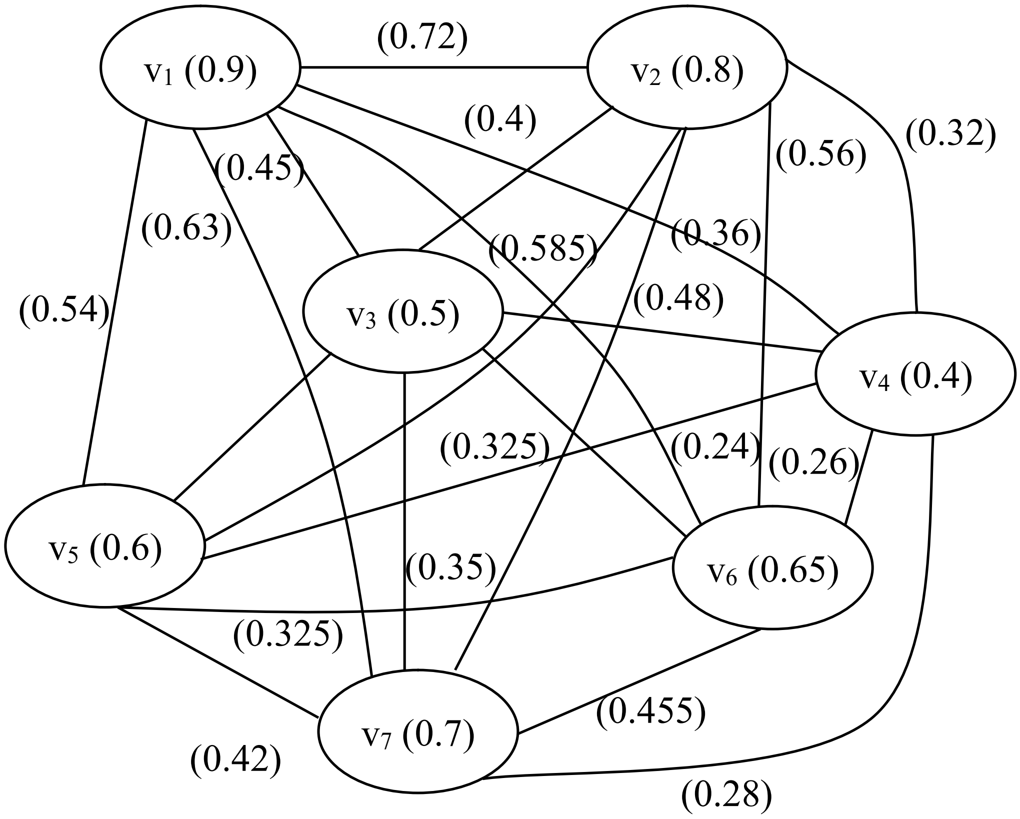


Figure 4.6

It is pertinent to record at this juncture that edge weights are so assigned to measure the weight or strength or weakness of the relation between the two nodes or vertices.

So in this case we see if a persons scores good marks in mathematics $v_1(0.9)$ and poor marks in languages $v_4(0.4)$. The edge weight adjoining v_1 to v_4 is 0.36 under product fuzzy graph model. Suppose we use min fuzzy graph model the related edge weight is 0.4 if on the other hand mean or average fuzzy graph model is used the edge weighted is 0.65.

However we cannot use max fuzzy graph model for the value would be 0.9 which is misleading for one cannot weight so which means if one is good is mathematics then his/her language expertise in languages need not in general be good in languages also for in this case we see his language marks are in fact poor just 0.4.

Hence it is mandatory that the expert uses his discretion in a rationalistic way and avoids using max fuzzy graph model.

Using min fuzzy graph model or product fuzzy graph model or mean fuzzy graph model does not that much affect the result.

Further we wish to state that the best model using fuzzy graphs is the product fuzzy graph model and other models are not that best suited.

Secondly if one really access this product the next better would be min fuzzy graph model and final and not even a better model is mean fuzzy graph model and max fuzzy graph model cannot be used for this problem.

Another factor is we can get two product fuzzy graph models (i) uses the complete fuzzy graph as the network or a dynamical system.

Other one functions on the expert's opinion, both are different.

The strengths of the edges can be get in both cases from the adjacency matrices of the product fuzzy graph which are described for the problem of marks of the student and his over all assessment.

Further in most cases it is meaningless to use complete fuzzy graph for in general for every case the results happen to be true, however there can be something known as laws of exception cases.

Sometimes there can be cases a person good in all subjects.

Finally, the marks scored is also many a times relative or system of exams measures only wrote memory and so on and so forth.

The adjacency matrix G_1 relative to the product fuzzy graph model and that of the adjacency matrix G_2 of the product complete fuzzy graph are given in the following:

$$G_1 = \begin{array}{c c}
 & \begin{array}{ccccccc} v_1(0.9) & v_2(0.8) & v_3(0.5) & v_4(0.4) & v_5(0.6) & v_6(0.65) & v_7(0.7) \end{array} \\
\begin{array}{c} v_1(0.9) \\ v_2(0.8) \\ v_3(0.5) \\ v_4(0.4) \\ v_5(0.6) \\ v_6(0.65) \\ v_7(0.7) \end{array} &
\begin{bmatrix}
0 & 0.72 & 0.45 & 0.36 & 0 & 0.585 & 0.63 \\
0.72 & 0 & 0 & 0 & 0.48 & 0 & 0.56 \\
0.45 & 0 & 0 & 0.2 & 0 & 0 & 0.35 \\
0.36 & 0 & 0.2 & 0 & 0 & 0.26 & 0.28 \\
0 & 0.48 & 0 & 0 & 0 & 0 & 0 \\
0.585 & 0 & 0 & 0.26 & 0 & 0 & 0 \\
0.63 & 0.56 & 0.35 & 0.28 & 0 & 0 & 0
\end{bmatrix}
\end{array}$$

$$G_2 = \begin{array}{cc} & \begin{array}{ccccccc} v_1(0.9) & v_2(0.8) & v_3(0.5) & v_4(0.4) & v_5(0.6) & v_6(0.65) & v_7(0.7) \end{array} \\ \begin{array}{c} v_1(0.9) \\ v_2(0.8) \\ v_3(0.5) \\ v_4(0.4) \\ v_5(0.6) \\ v_6(0.65) \\ v_7(0.7) \end{array} & \begin{bmatrix} 0 & 0.72 & 0.45 & 0.36 & 0.54 & 0.585 & 0.63 \\ 0.72 & 0 & 0.4 & 0.32 & 0.48 & 0.52 & 0.56 \\ 0.45 & 0.4 & 0 & 0.2 & 0.3 & 0.325 & 0.35 \\ 0.36 & 0.32 & 0.2 & 0 & 0.24 & 0.26 & 0.28 \\ 0.54 & 0.48 & 0.3 & 0.24 & 0 & 0.39 & 0.42 \\ 0.585 & 0.52 & 0.325 & 0.26 & 0.39 & 0 & 0.455 \\ 0.63 & 0.56 & 0.35 & 0.28 & 0.42 & 0.455 & 0 \end{bmatrix} \end{array}$$

When we compare the two matrices using product gives a more feasible solution.

One can interpret that a person in general strong in mathematics, happens to be good in physics, logic and computer science. Likewise, one can interpret the results from the adjacency matrices.

Next we proceed onto describe how this fuzzy graph theory model functions in medical diagnostics.

Example 4.6. A patient comes with the following symptoms and the product fuzzy graph model is used by the doctor after assigning the following membership values to the symptoms which he says he is suffering

Fever high	s_1 (0.9)
Shivering	s_2 (0.7)
Cold and cough	s_3 (0.4)
Headache	s_4 (0.5)
Malaria	s_5 (0.7)
Tuberculosis	s_6 (0.4)
Vomiting and Nausia	s_7 (0.1)
Food poisoning	s_8 (0.2)

Now he gives the product complete fuzzy graph using these 8 nodes.

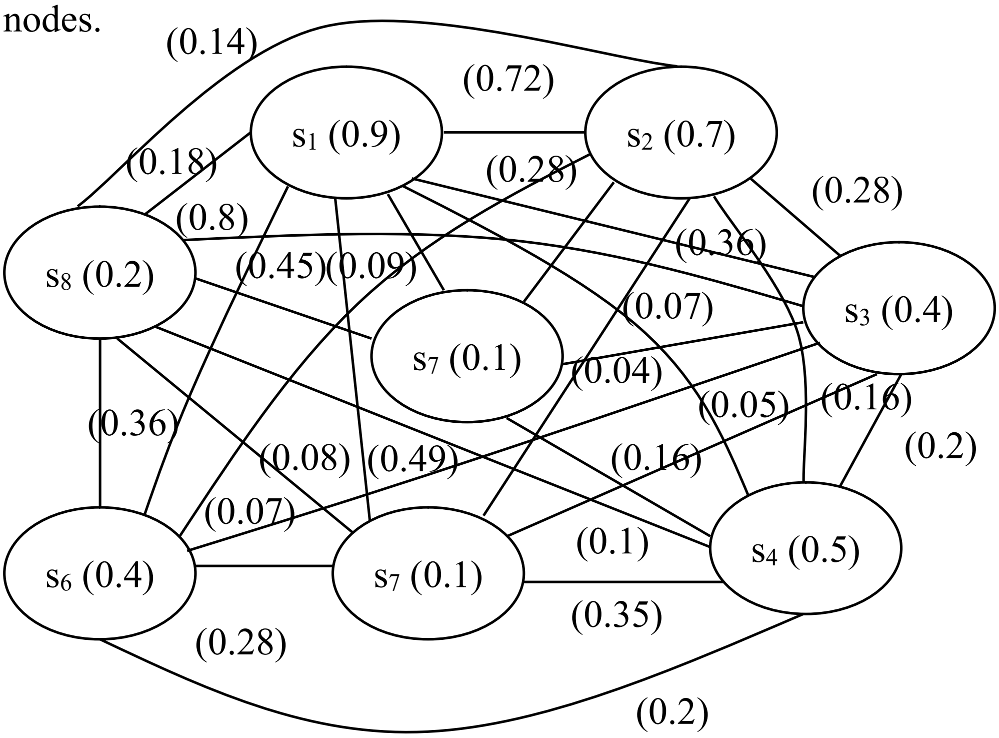


Figure 4.7

We only study the strong edges whose values are ≥ 0.5 and obtain a product fuzzy graph H which will serve as a product fuzzy graph model for the study.

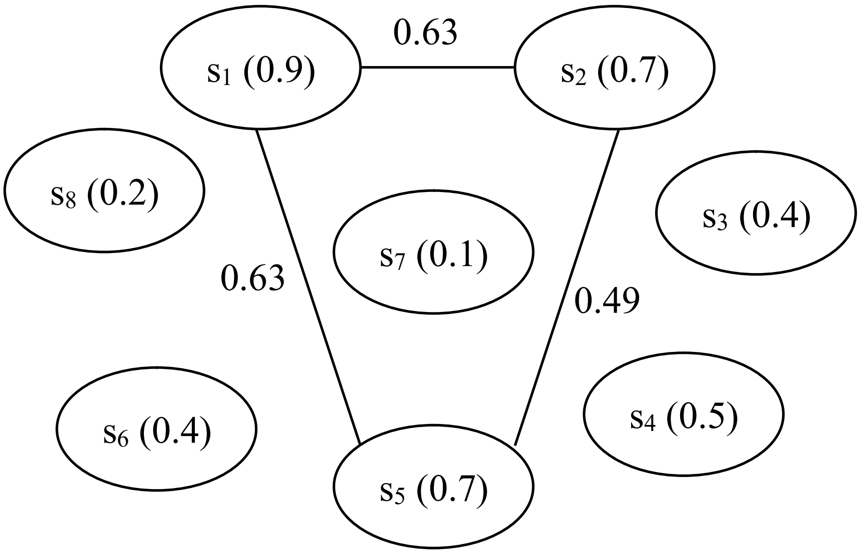
H = 

Figure 4.8

We clearly make a conclusion that the patient has malaria fever is high and he has shivering.

This product fuzzy graphs. Next we proceed onto describe intuitionistic fuzzy graph and prove some applications of them.

An intuitionistic fuzzy graph G(V,E) with V = $\{v_1, \ldots, v_n\}$ such that $\sigma_1 : V \to [0, 1]$ and $\delta_1 : V \to [0, 1]$ denote the degree of non membership or membership of an element $v_i \in V$ (or fuzzy membership) respectively $1 \leq i \leq n$ with $0 \leq \sigma_1 (v_i) + \delta_1 (v_i) \leq 1$.

Here $E \subseteq V \times V$ where $\sigma_2: V \times V \to [0, 1]$ and $\delta_2: V \times V \to [0, 1]$ are such that

$$\sigma_2(v_i, v_j) \leq \min [\sigma_1 (v_i), \delta_1(v_j)]$$

$\delta_2 (v_i, v_j) \geq \max [\sigma_1(v_i), \delta_1(v_j)]$

$0 \leq \sigma_2 (v_i, v_j) + \delta_2 (v_i, v_j) \leq 1$

for every $(v_i, v_j) \in E$; $1 \leq i, j \leq n$.

We give one example of the intuitionistic fuzzy graph in the following.

Example 4.7. Let $V = \{v_1, v_2, v_3, v_4, v_5\}$ of the intuitionistic fuzzy graph G(V, E) with $\sigma_1 : V \to [0, 1]$ and $\delta_1: V \to [0, 1]$ denote the fuzzy memberships of the element $v_i \in V$; $1 \leq i \leq 5$ respectively.

The graph G(V, E) is as follows:

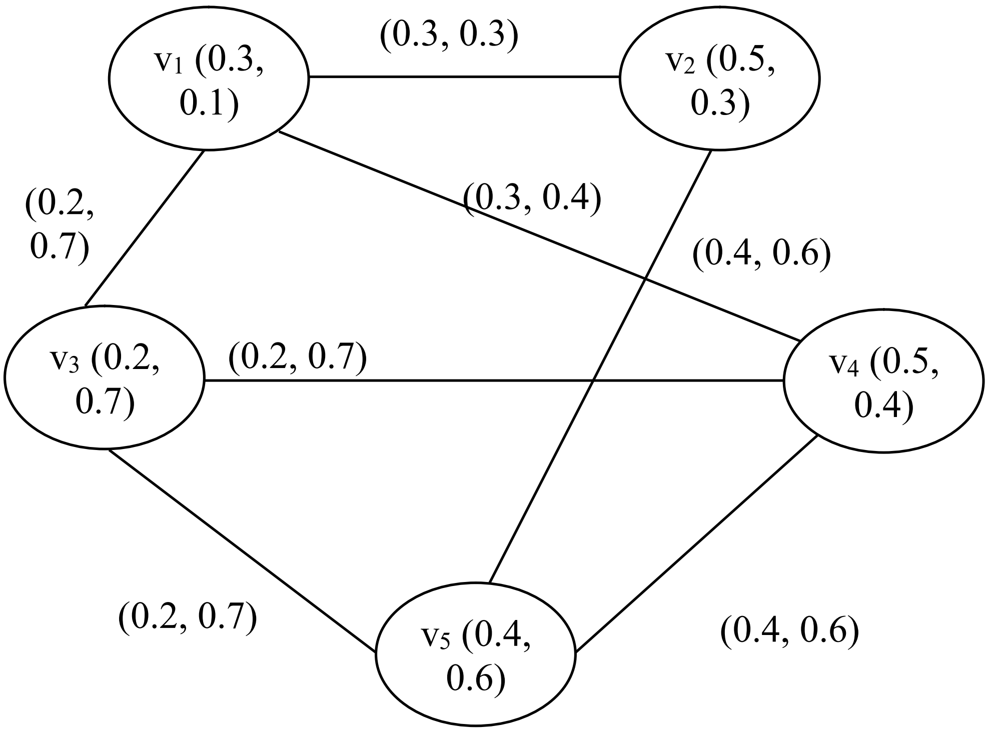


Figure 4.9

It is pertinent to record we call in this book such intuitionistic fuzzy graphs as type I intuitionistic fuzzy graphs and denote it by $G_1(V, E)$.

We now proceed onto define type II to type VI intuitionistic fuzzy graphs in the following.

Definition 4.2 : *Let $G_2(V, E)$ be a intuitionistic fuzzy graph with $V = \{v_1, v_2, \ldots, v_n\}$ where $\sigma_1: V \to [0, 1]$ and $\delta_1: V \to [0, 1]$ are fuzzy memberships which satisfies the additional condition as*

$$0 \leq \sigma_1(v_i) + \delta_1(v_i) \leq 1$$

and $\sigma_2 : V \times V \to [0, 1]$ *and*

$\delta_2: V \times V \to [0, 1]$ *such that*

$\sigma_2(v_i, v_j) \leq \min [\sigma_1(v_i), \delta_1(v_j)]$ *and*

$\delta_2(v_i, v_j) \geq \text{product} [\sigma_1(v_i), \delta_2(v_j)]$ *such that*

$0 \leq \sigma_2(v_i, v_j) + \delta_2 (v_{i, j}) \leq 1, (v_i\, v_j) \in E; 1 \leq i, j \leq n.$

We define this $G_2(V,E)$ as intuitionistic fuzzy graph of type II.

Clearly type I and type II are different only in their edge weights the structures remain the same.

We prove the graph with same vertex set given in Example 4.1 for this type II intuitionistic fuzzy graph in the following.

G_2 (V, E) =

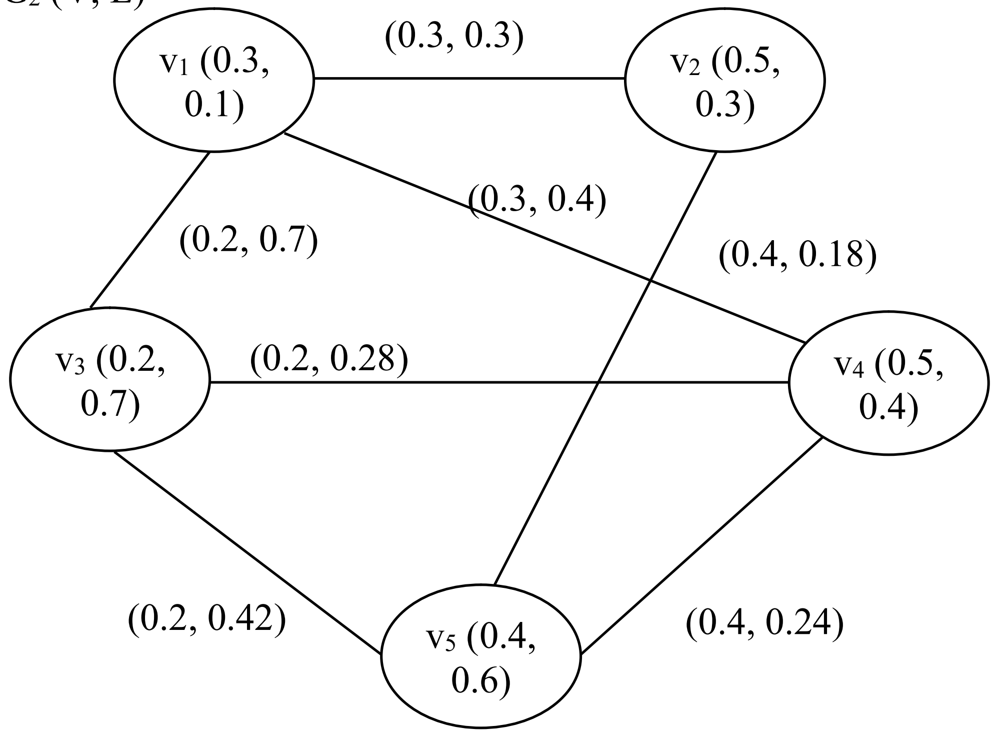


Figure 4.10

It is easily observed that G(V, E) = G_1 (V, E) and G_2(V, E) are structurally the same however only the edge weights of G_1(V, E) and G_2(V, E) are different. This will be helpful to the researchers when they do not want to boost the values using max to boost the values using max function.

Now we proceed onto define type III intuitionistic fuzzy graph in the following.

Definition 4.3 *Let $G_3(V, E)$ be the intuitionistic fuzzy graph of type III with $V = \{v_1, \ldots, v_n\}$ the vertex set $\sigma_I : V \to [0, 1]$ and $\delta_I : V \to [0, 1]$ special type of fuzzy membership functions such that*

$$0 \leq \sigma_I(v_i) + \delta_I(v_i) \leq 1; \quad 1 \leq i \leq n \text{ and}$$

σ_2: $V \times V \to [0, 1]$ *and*

δ_2: $V \times V \to [0, 1]$ *such that*

$\sigma_2 (v_i, v_j) \leq product [\sigma_1 (v_i), \delta_1(v_j)]$

and $\delta_2(v_i, v_j) \geq min (\sigma_1(v_i), \delta_1(v_j)]$

$0 \leq \sigma_2 (v_i, v_j) + \delta_2(v_i, v_j) \leq 1$ $(v_i, v_j) \in E$, $1 \leq i, j \leq n$.

We define the $G_3(V, E)$ to be the intuitionistic fuzzy graph of type III.

When a researcher wishes work with minimum occurrence of an event or attribute we can use the notion of intuitionistic fuzzy graph of type III.

We will illustrate this situation by the same example is given in 4.1.

G_3 (V, E) =

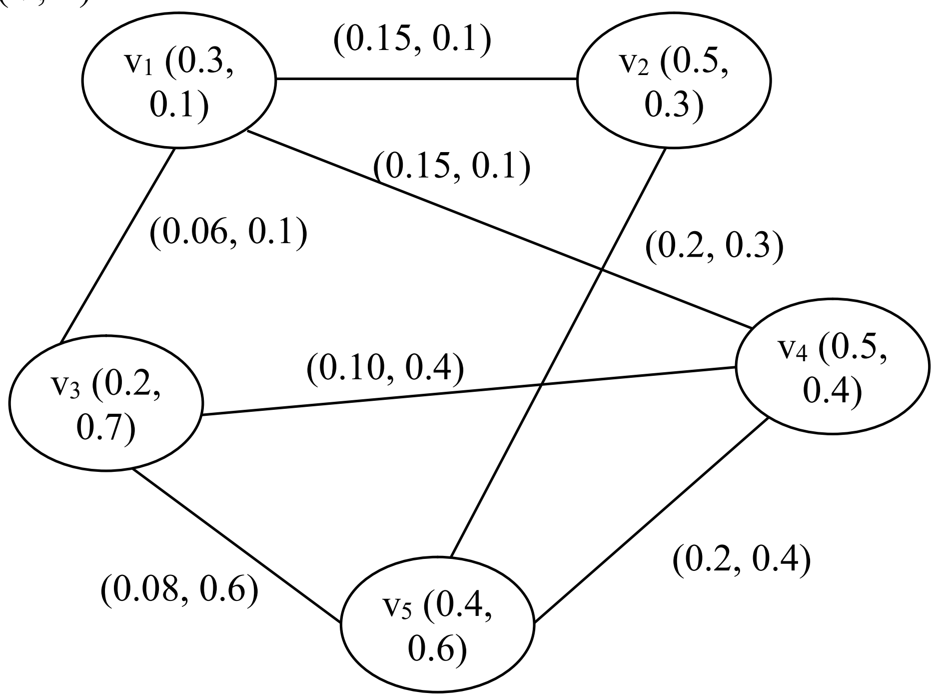


Figure 4.11

We see the intuitionistic fuzzy graph $G_3(V, E)$ is structurally the same as that of $G_2(V, E)$ and $G_1(V, E)$ but is different only in the edge weights.

Next we proceed onto define and describe type IV fuzzy intuitionistic graphs.

Definition 4.4 *Let $G_4(V, E)$ be as in definition*

$\sigma_1: V \to [0, 1]$ *and* $\delta_1: V \to V\ [0, 1]$ *such that*

$0 \leq \sigma_1(v_i) + \delta(v_i) \leq 1;\ 1 \leq i \leq n.$

Let $\sigma_2: V \times V \to [0, 1]$ *and* $\delta_2: V \times V \to [0, 1]$ *be defined as follows:*

$\sigma_2\ (v_i, v_j) \geq \max\ \{\sigma_1(v_i),\ \delta_1(v_j)\}$ *and*

$\delta_2(v_i, v_j) \leq \min\ \{\sigma_1(v_i),\ \delta_1(v_j)\}$

$0 \leq \sigma_2\ (v_i, v_j) + \delta_2(v_i, v_j) \leq 1.\ \ 1 \leq i,j \leq n.$

We define this graph as type IV fuzzy intuitionistic graph.

We will illustrate this situation by the following example for the same set of vertex set $\{v_1, v_2, \ldots, v_5\}$ given in example 4.1.

Example 4.8 Let V = $\{v_1(0.3, 0.1), v_2(0.5, 0.3), v_3(0.2, 0.7), v_4(0.5, 0.4), v_5(0.4, 0.6)\}$ be the set of vertices of the intuitionistic fuzzy graph $G_4(V, E)$. We have the following figure.

‘

$G_4(V, E) =$

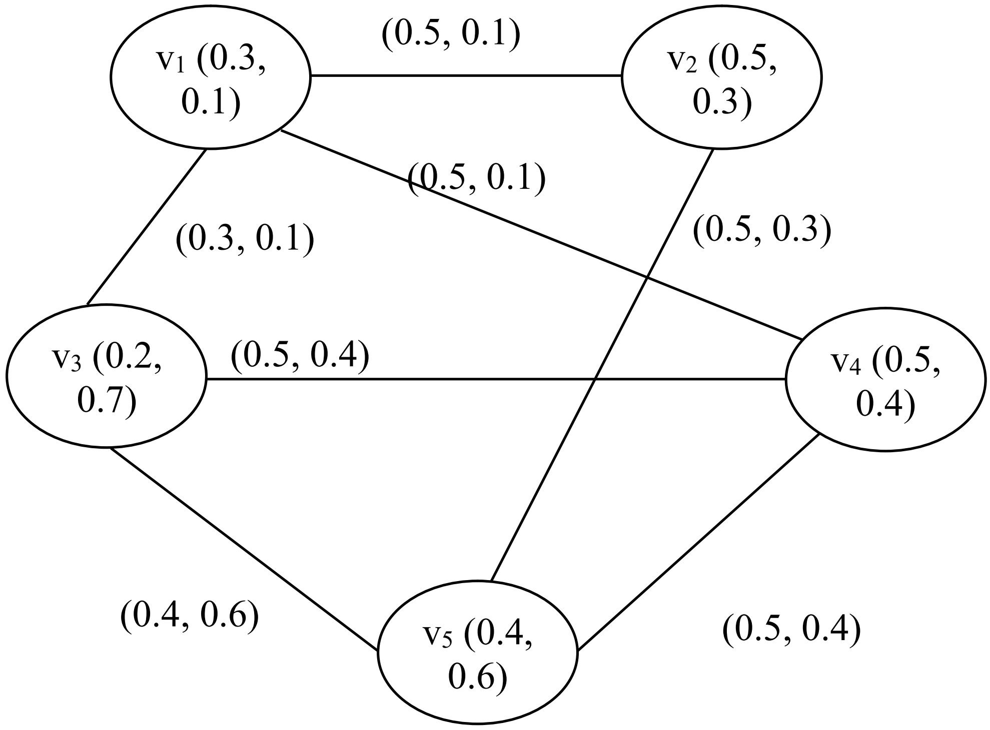


Figure 4.12

We see $G_4(V, E)$ is the intuitionistic fuzzy graph of type IV.

Next we proceed onto define type V fuzzy intuitionistic graph.

Definition 4.5 *Let $G_5(V, E)$ be as in Definition 4.2. We have σ_1: $V \to [0, 1]$, δ_1: $V \to [0, 1]$ such that $0 \leq \sigma_1(v_i) + \delta_1(v_i) \leq 1$ for all $v_i \in V$. Let σ_2: $V \times V \to [0, 1]$ and δ_2: $V \times V \to [0, 1]$ such that $\sigma_2(v_i, v_j) \geq \max \{\sigma_1(v_i)$, and $\delta_2(v_i, v_j) \leq \sigma_1 (v_i) \times \delta_1(v_j)$; $(v_i, v_j) \in V \times V$ with the additional condition $0 \leq \sigma_2(v_i, v_j) + \delta_2(v_i, v_j)$.*

We define $G(V, E) \leq 1$. $G_5(V, E)$ as the intuitionistic fuzzy graph of type V.

We provide an example of the same.

Example 4.9 Let the vertex set V = {v_1(0.3, 0.1), v_2 (0.5, 0.3), v_3(0.2, 0.7), v_4(0.5, 0.4), v_5(0.4, 0.6)}. σ_I and δ_I defined as in the above definition.

We get the following figure.

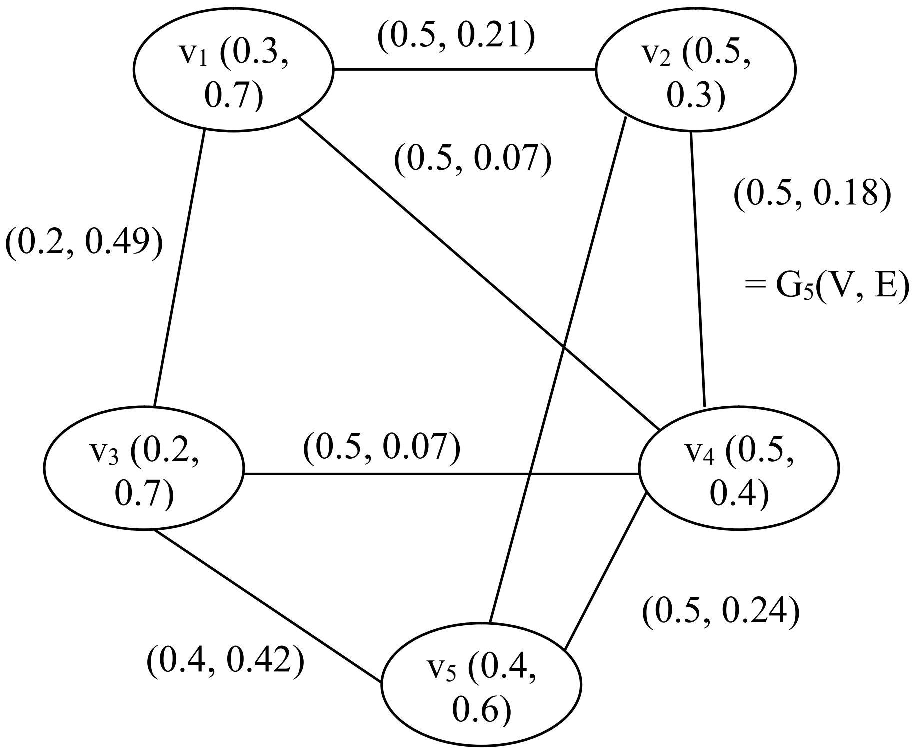


Figure 4.13

We see clearly G_5(V, E) is structurally the same as that of G_1(V, E), G_2(V, E), G_3(V, E) and G_4(V, E) but however the edge weights of G_5(V, E) is entirely different from that of the other fuzzy intuitionistic graphs.

Finally, we proceed onto define the notion of fuzzy intuitionistic graph of type VI.

Definition 4.6 *Let G_6(G, E) be the fuzzy intuitionistic graph of type VI. We as in case of the intuitionistic graph of type I or the* *classical one give in definition 4.1 take V and take $\sigma_1 : V \rightarrow [0, 1]$ and δ_1: $V \rightarrow [0, 1]$ satisfying the additional condition.*

$$0 \leq \sigma_1 (v_i) + \delta_1(v_i) \leq 1 \text{ for } 1 \leq i \leq n.$$

$$\sigma_2: V \times V \rightarrow [0, 1] \text{ and } \delta_2: V \times V \rightarrow [0, 1]$$

are defined as $\sigma_2(v_i, v_j) \geq \max \{\sigma_1(v_i), \delta_1(v_j)\}$ and $\delta_2(v_i, v_j) \leq \min \{\sigma_1(v_i), \delta_1(v_j)\}$ satisfying the additional condition $1 \leq \sigma_2 (v_i, v_j) + \delta_2(v_i, v_j) \leq 1$ for all $(v_i, v_j) \in V \times V$.

We define the resulting fuzzy intuitionistic graph as the type VI intuitionistic fuzzy graph.

We will illustrate this situation by the following example for the same vertex set given in Example 4.1.

Example 4.10 Let V = v_1(0.3, 0.7), v_2 (0.5, 0.3), v_3(0.2, 0.7), v_4(0.5, 0.1), v_5(0.4, 0.6)} be the vertex set and G_6(V, E) be the fuzzy intuitionistic graph given by the following figure.

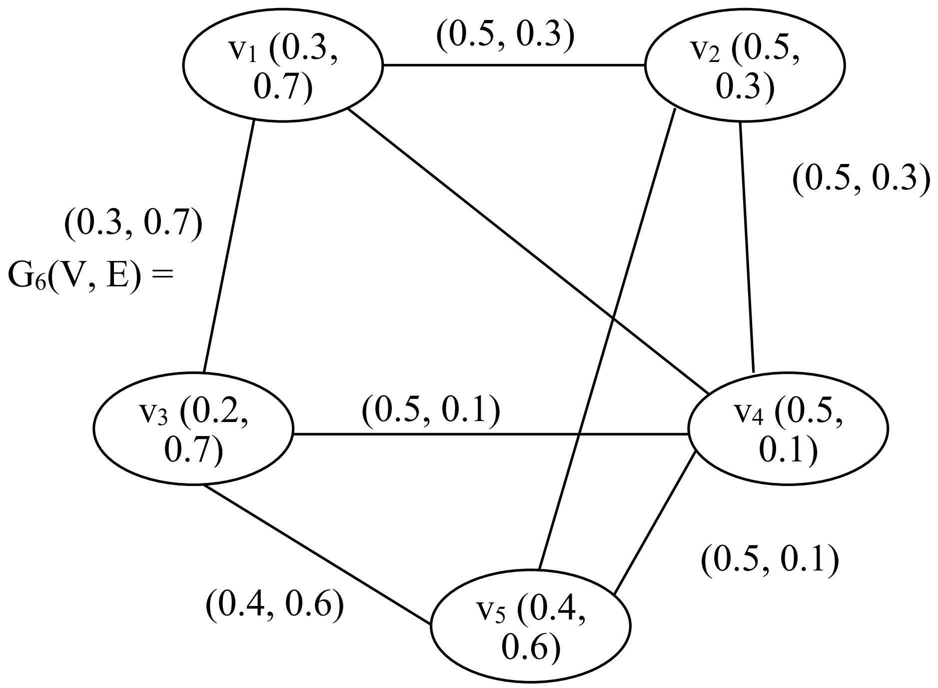


Figure 4.14

Clearly $G_6(V, E)$ is the intuitionistic fuzzy graph of type VI and structurally it is the same as that of $G_i(V, E)$; $1 \leq i \leq 5$ but however the edge weights of $G_6(V, E)$ are different from that of $G_i(V, E)$ $1 \leq i \leq 5$.

Thus instead of having a fixed choice of ≤ min and ≥ max as the condition we can choose any one of the six conditions as per the need of the researcher or expert and the nature of the problem in hand.

We will illustrate the applications of intuitionistic fuzzy graph in case of medical diagnosis by the following example.

Example 4.11 Now after discussing with the patient about the symptoms he suffers, the doctor wishes to confirm the disease he suffers. The doctor uses the intuitionistic fuzzy graph model.

From the doctor the following vertices are provided Recall any pair say v_i (x_i, x_j) where $0 \leq x_i, y_i \leq 1$ represents the membership and non membership; here x_i denotes the membership that this person suffers from malaria and y_i denotes the non-membership that is the patient may not be suffering from malaria.

Thus v_1 intermittent fever then the doctor gives the value 0.8 for possible chances the patient suffers from malaria and 0.1 that he is not suffering from malaria. So for the intuitionistic graph we have $v_1(0.8, 0.1)$.

Likewise for chill before fever v_2 the doctor gives $v_2(0.7, 0.2)$ for one can suffer chills before fever even in case of food poisoning so says 0.2.

For the same for vomiting symptom v_3 he gives v_3 (0.3, 0.6) the diagnoses is for the malaria disease can the presence of cold and cough indicate the malaria disease. The doctor gives the membership as 0.2 and non membership as 0.5 so (0.2, 0.5). So according to him cold and cough cannot be a strong indication of malaria.

Suppose one has fever in the evening v_5 then the doctor says evening fever can indicate lung disease or TB so he gives the membership and non membership as (0.5, 0.2). Finally, can stomach upset v_6 cannot be an indication of malaria the membership value is given by (0.2, 0.6).

Now we give the intuitionistic fuzzy graph using the vertex set $v_1, v_2, \ldots, v_6$, in the following.

We can have the following complete intuitionistic fuzzy graph with (min, max) edge weights.

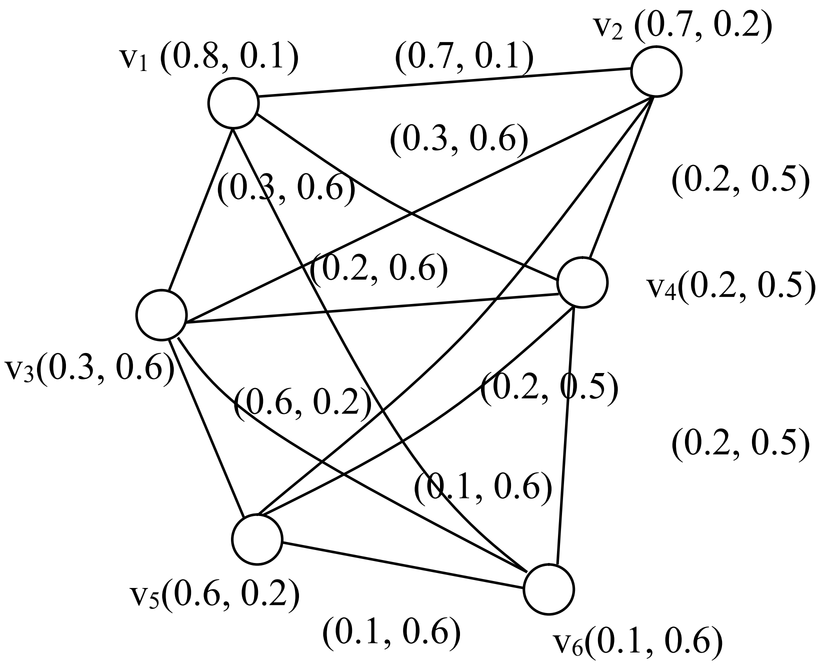


Figure 4.15

Now we collect the edge weights which has larger or bigger membership for malaria. We say the nodes are related for the malarial disease symptoms if the membership values are high. If the non membership values are high then the two nodes are unrelated for the symptoms of the malaria.

We tabulate the edges and edge with membership to malaria in the following table. Table of edge weight membership and non membership for malaria.

Table of Membership for malaria

S. No.	**Edges with weights**	**Membership with malaria**	**Non-membership for malaria**
1.	v_1v_2 (0.7, 1)	0.7	0.1
2.	v_1v_3 (0.3, 0.6)	0.3	0.6
3.	v_1v_4 (0.2, 0.5)	0.2	0.5
4.	v_1v_5 (0.3, 0.6)	0.3	0.6
5.	v_1v_6 (0.1, 0.6)	0.1	0.6
6.	v_2v_3 (0.3, 0.6)	0.3	0.6
7.	v_2v_4 (0.2, 0.5)	0.2	0.5
8.	$v_2 v_5$ (0.6, 0.2)	0.6	0.2
9.	$v_2 v_6$ (0.1, 0.6)	0.1	0.6
10.	$v_3 v_4$ (0.2, 0.6)	0.2	0.6
11.	v_3v_5 (0.3, 0.6)	0.3	0.6
12.	v_3v_6 (0.1, 0.6)	0.1	0.6
13.	v_4v_5 (0.2, 0.5)	0.2	0.5
14.	v_4v_6	0.1	0.6
15.	v_5v_6	0.1	0.6

Now we make the clusters in the following way. The edges associated with the edges that has maximum membership (say a membership greater than or equal to 0.5) will form a cluster I and those whose non membership is greater 0.4 will form another cluster II.

It is probable and possible in general c have a third cluster III which does not full in the membership side or a non membership side.

We take only the edges for if nodes are to be taken we will find the clusters to be an overlapping one.

Thus cluster I is given by these set of edges.

Cluster I = $\{v_1v_2, v_2v_5\}$

Cluster II = $\{v_1v_3, v_1v_4, v_1v_5, v_2v_3, v_1v_6, v_2v_4, v_2v_6, v_4v_6, v_3v_4, v_3v_5, v_4v_5\}$

Cluster III = $\{\phi\}$

Here in this case cluster III happens to be empty.

It is pertinent to note we have taken some symptoms suffered by a patient in general and have tried to find which of the symptoms are related with malaria using the fuzzy intuitionistic graph / network. The doctor has taken 6 symptoms $v_1, v_2, \ldots, v_6$ some of them are related and some unrelated and has formed the graph and also the membership table is formed using the edge strengths. These edge strengths are used to form the table from which clusters are evolved.

Thus from the clusters one can get those symptoms which predict the chances of malaria these will corresponds to the memberships and the non membership cluster will correspond to the symptoms which do not relate to malaria.

In fact it is pertinent to keep on record that there may be symptoms in a patient where the expert may not be in a position to say the symptom is malarial or the symptom is non malarial in such cases this fuzzy intuitionistic fuzzy graph cannot make any conclusions. So only to make our model and study more sensitive we try to model the same problem using Single Valued Neutrosophic (SVNs) graphs model.

Let $G^* = (V, E)$ denote the crisp graph and vertex set V and edge set E.

We denote the single valued neutrosophic graph by $G = (A, B)$ where the notational convention is described in the following: A is defined as the single valued neutrosophic vertex set of V and B is the symmetric single valued neutrosophic relation on A [5-6].

The functions T_A, I_A and F_A are defined in the following which denote the degree of truth membership, degree of indeterminacy membership and falsity membership of an element $v_i \in V$ respectively satisfying the condition $0 \leq T_A(v_i) + I_A(v_i) + F_A(v_i) \leq 3$ for all $v_i \in V$; $(i = 1, 2, \ldots, n)$. The functions are defined in the following will form the edge set E of the single valued neutrosophic graph $G = (A, B)$.

The 3 functions T_B, I_B and F_B from $E \subseteq V \times V \to [0, 1]$ defined by

$$\left.\begin{array}{l} T_B(\{v_i, v_j\} \leq \min [T_A(v_i), T_A(v_j)] \\ I_B(\{v_i, v_j\}) \geq \max [I_A(v_i), I_A(v_j)] \text{ and} \\ F_B(\{v_i, v_j)\} \geq \max [F_A(v_i), F_A(v_j)] \end{array}\right\} \quad \text{I}$$

where $T_B: E \subseteq V \times V \to [0, 1]$,

$I_B: E \subseteq V \times V \to [0, 1]$

and $F_B: E \subseteq V \times V \to [0, 1]$.

Denotes the degree of truth-membership, I_B the indeterminacy membership and F_B the false membership respectively.

One can as in case of fuzzy intuitionistic graphs and models explained earlier in this case also apply them and obtain all types of them.

However, it is important to mention [87] have used SVNS graphs to construct SVNS models in medical diagnostics. Thus any researcher can use other operation on the SVNS triplets and arrive at a best solution.

Now we proceed onto recall the definition plithogenic set [44]. For more about other features and properties please refer [1, 2, 44].

A Plithogenic fuzzy set is defined as a set whose each element is characterized by many attribute values. That is if (a_1, a_2, ..., a_n) are n attributes then if x_1, x_2, …, x_t then x_1's fuzzy degree of these attributes is x_1 (m_{11}, ,m_{12}, …, m_{1n}) where $m_{1i} \in [0, 1]$; $1 \leq i \leq n$.

Similarly the x_2's fuzzy degree of these attributes is x_2 $(m_{21}, m_{22}, \ldots, m_{2n})$ where $m_{2i} \in [0, 1]$; $1 \leq i \leq n$. Similar fuzzy degree values for any x_k: $1 \leq k \leq t$.

This is plithogenic fuzzy set. Suppose we assign a neutrosophic degree then for the above stated attributes.

x_1 $((n_{11}^1, n_{11}^2, n_{11}^3)$ $(n_{12}^1, n_{12}^2, n_{12}^3)$, …, $(n_{1n}^1, n_{1n}^2, n_{1n}^3)$ is the neutrosophic degree of x_1, where $n_{1i}^j \in [0, 1]$; $1 \leq i \leq n$, $1 \leq j \leq 3$. On similar lines the neutrosophic degree for x_2 is

x_2 $((n_{21}^1, n_{21}^2, n_{21}^3)$, $(n_{22}^1, n_{22}^2, n_{22}^3)$, …, $(n_{2n}^1, n_{2n}^2, n_{2n}^3))$ where $n_{2i}^j \in [0, 1]$; $1 \leq j \leq 3$, $1 \leq i \leq n$.

This is the way we define for any x_k; so technically we say a plithogenic fuzzy set is represented with n attributes, any expert as a fuzzy row matrix and that of a plithogenic neutrosophic set is a super fuzzy row matrix with each element of it is a 1×3 fuzzy matrix.

In this book we use only two types of plithogenic sets for constructing graphs and models.

We provide examples of them in the following.

Example 4.12 Suppose we want to study the symptom disease model in general using Plithogenic fuzzy sets. The symptoms are taken as the attributes and are listed in the following:

1. Vomiting - a_1
2. Fever - a_2

3. Very high fever - a_3
4. Cold - a_4
5. Cold and cough - a_5
6. Chills before fever - a_6
7. Indigestion - a_7
8. Headache - a_8
9. Dysentery - a_9
10. Stomach pain - a_{10}

The diseases under investigation is

x_1 - Malaria

x_2 - Flu

x_3 - Typhoid

x_4 - Jaundice

x_5 - Food poisoning

The experts gives the following Plithogenic fuzzy values for the disease malaria x_1 is

x_1 (0.1, 0.4, 0.6, 0.2, 0.2, 0.8, 0.1, 0.5, 0, 0.1) is the fuzzy degree of the systems associated with malaria.

For flu x_2 the fuzzy degree of the Plithogenic fuzzy set is as follows.

x_2 (0.1, 0.6, 0.7, 0.8, 0.6, 0.4, 0.2, 0.6, 0.2, 0.1)

Next we give the Plithogenic fuzzy values for the disease typhoid (x_3) is given in the following.

x_3 (0.7, 0.6, 0.7, 0.1, 0.1, 0.2, 0.5, 0.7, 0.3)

The Plithogenic fuzzy value for the disease Jaundice x_4 is as follows.

x_4 (0.7, 0.6, 0.5, 0.1, 0.1, 0.2, 0.7, 0.6, 0.1, 0.4)

Now we give the Plithogenic fuzzy set associated with the disease food poisoning.

x_5 (0.7, 0.6, 0.5, 0.1, 0.1, 0.7, 0.6, 0.8, 0.7, 0.8)

Now for the same set of attributes and experts we give the fuzzy neutrosophic plithogenic set values in the following.

x_1 {((0.2, 0.1, 0.6), (0.6, 0.3, 0.1), (0.5, 0.4, 0.2), (0.1, 0.8, 0.7), (0.1, 0.6, 0.6), (0.7, 0.3, 0.2), (0.2, 0.6, 0.5), (0.5, 0.6, 0.5), (0.1, 0.6, 0.7), (0.1, 0.5, 0.6))} is the fuzzy super matrix associated with the fuzzy neutrosophic plithogenic set for the value x_1.

The fuzzy neutrosophic plithogenic set values for flu x_2 is as follows.

x_2 {((0.1, 0.5, 0.7), (0.8, 0.2, 0.1), (0.7, 0.3, 0.2), (0.7, 0.3, 0.1), (0.7, 0.1, 0.1), (0.4, 0.7, 0.5), (0.1, 0.3, 0.6), (0.6, 0.4, 0.5), (0.1, 0.7, 0.6), (0.1, 0.6, 0.2))}.

Next the fuzzy neutrosophic plithogenic set value for typhoid x_3 is as follows.

x_3 {((0.6, 0.4, 0.1), (0.7, 0.2, 0.4), (0.6, 0.3, 0.3), (0.1, 0.6, 0.4), (0.1, 0.6, 0.4), (0.1, 0.5, 0.6), (0.6, 0.4, 0.2), (0.6, 0.2, 0.3), (0.1, 0.6, 0.6), (0.2, 0.6, 0.4))} is the super fuzzy row matrix associated with it.

Next we give the fuzzy neutrosophic plithogenic set associated with the disease jaundice (x_4) for the symptoms listed

x_4 {(0.7, 0.4, 0.1), (0.5, 0.6, 0.5), (0.4, 0.6, 0.6), (0.1, 0.3, 0.7), (0.1, 0.4, 0.3), (0.7, 0.1, 0.2), (0.6, 0.4, 0.4), (0.2, 0.6, 0.6), (0.5, 0.5, 0.5))} is the super fuzzy row matrix of the fuzzy neutrosophic plithogenic set associated with the disease jaundice.

We give the fuzzy neutrosophic set associated with food poisoning x_5 for the given symptoms

x_5 {((0.8, 0.3, 0.1), (0.5, 0.6, 0.5), (0.6, 0.6, 0.5), (0.1, 0.2, 0.7), (0.1, 0.3, 0.8), (0.6, 0.4, 0.4), (0.8, 0.2, 0.2), (0.7, 0.2, 0.1), (0.7, 0.2, 0.3), (0.7, 0.1, 0.1))}.

Next we proceed onto define intuitionistic Plithogenic fuzzy set for the symptom disease problem discussed in example.

We see the fuzzy intuitionistic plithogenic set associated with malaria x_1 for the given set of 10 symptoms is as follows.

x_1 {((0.2, 0.6), (0.6, 0.3), (0.7, 0.2), (0.2, 0.8), (0.8, 0.1), (0.4, 0.6), (0.5, 0.6), (0.1, 0.7), (0.2, 0.9))} which is a fuzzy super row matrix of order 1×10.

Similarly for the disease flu x_2 the fuzzy intuitionistic plithogenic super fuzzy 1×10 row matrix is as follows.

x_2 {((0.1, 0.8), (0.8, 0.2), (0.8, 0.1), (0.7, 0.2), (0.7,l 0.1), (0.5, 0.5), (0.2, 0.7), (0.6, 0.3), (0.1, 0.6), (0.1, 0.7))}.

Now the expert gives the fuzzy intuitionistic plithogenic set super fuzzy matrix for the disease typhoid using the 10 symptoms in the problem.

x_3 {((0.7, 0.2), (0.6, 0.3), (0.7, 0.2), (0.1, 0.8), (0.1, 0.7), (0.1, 0.5), (0.8, 0.1), (0.7, 0.1), (0.5, 0.6), (0.6, 0.6))}.

Next we give the super fuzzy 1×10 row matrix of the fuzzy intuitionistic plithogenic set associated with the disease Jaundice using the 10 symptoms listed in the problem.

x_4 {((0.7, 0.2), (0.6, 0.5), (0.4, 0.5), (0.1, 0.8), (0.1, 0.7), (0.2, 0.6), (0.8, 0.2), (0.6, 0.2), (0.3, 0.6), (0.6, 0.3))}.

Finally we give the fuzzy intuitionistic plithogenic set associated with the disease food poisoning x_5 using the 10 symptoms listed in the problem.

x_5 {((0.8, 0.1), (0.4, 0.5), (0.2, 0.7), (0.1, 0.8), (0.2, 0.8), (0.7, 0.5), (0.8, 0.2), (0.7, 0.3), (0.7, 0.2), (0.7, 0.2))}.

Now we define and illustrate for the 3 types of plithogenic sets which we have discussed in the following a medical diagnostic model.

Suppose we have for patients P_1, P_2, P_3 and P_4 who have been diagnosed for all the 5 diseases using the 10 symptoms.

The Plithogenic fuzzy sets associated with the patient P_1, P_2, P_3 and P_4 is as follows:

$P_1((0.1, 0.5, 0.7, 0.8, 0.5, 0.3, 0.2, 0.5, 0.1, 0.2)$

$P_2((0.1, 0.3, 0.7, 0.1, 0.2, 0.8, 0.1, 0.5, 0.1, 0)$

$P_3((0.7, 0.7, 0.7, 0.2, 0.2, 0.2, 0.2, 0.5, 0.7, 0.3)$ and

P_4 (0.7, 0.7, 0.6, 0.1, 0.1, 0.1, 0.6, 0.8, 0.2).

Now we use the formula for the normalized Hamming distance between two, Plithogenic fuzzy sets defined by

$$d_{FP}^{H}(A, B) = \frac{1}{n}\sum_{j=1}^{n} \left|\mu_M^{FP}(x_j) - \mu_N^{FP}(x_j)\right| \qquad \text{(I)}$$

The normalized Euclidean distance between any plithogenic set A and B is defined by

$$d_{FP}^{E}(A, B) = \left[\frac{1}{n}\sum_{j=1}^{n} (\mu_M^{FP}(z_j) - \mu_N^{FP}(z_j))^2\right]^{1/2}$$

We now proceed onto build special graphs which of two types which we call as Hamming distance-based edge graphs with row matrix labeled and Euclidean distance based edge Plithogenic graphs.

First we provide examples of them.

Example 4.13 Let V = {v_1, v_2, v_3, v_4, v_5} be the vertex set where v_i's are 1×4 matrices fuzzy or real valued.

Let us define or weight the edges with Euclidean distance value which is as follows.

G =

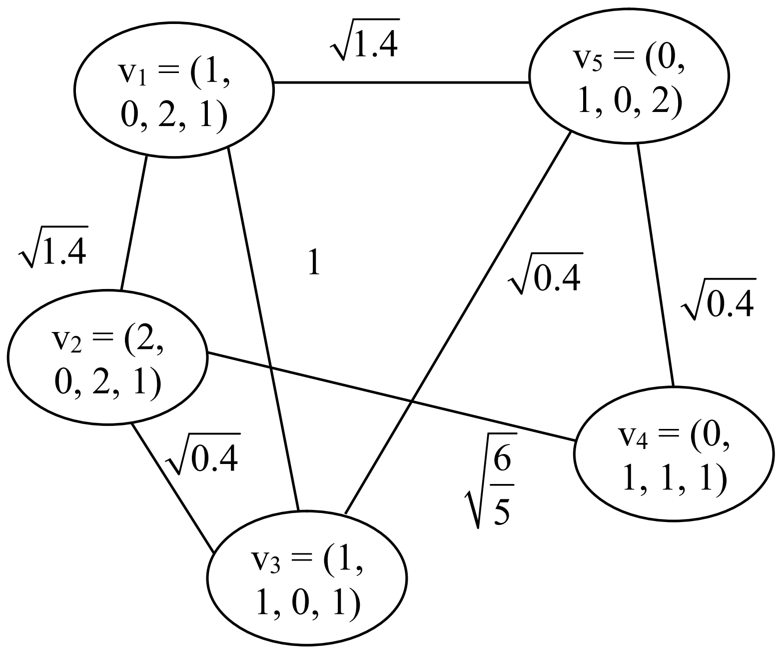


Figure 4.16

The edge weight between v_1 and v_5 is

$$= \frac{1}{5}\sqrt{1+1+2^2+1}$$

$$= \frac{\sqrt{7}}{\sqrt{5}} = \sqrt{\frac{7}{5}} = \sqrt{1.4}$$

G is a special Plithogenic graph with Euclidean distance edge weights. The edge v_iv_j if it exists (that is v_i is adjacent with v_j, $i \neq j$) then the weight of this edge v_iv_j is $d_{FP}^{E}(v_i, v_j)$ where E is the Euclidean distance satisfying the formula II.

For the same vertex set we give the Plithogenic vertex graph with Hamming distance.

The edge weight of two adjacent vertices v_i and v_j are given by formula denoted by $d^{H}_{FP}(v_i, v_j)$.

This special graph is given by the following figure H.

H =

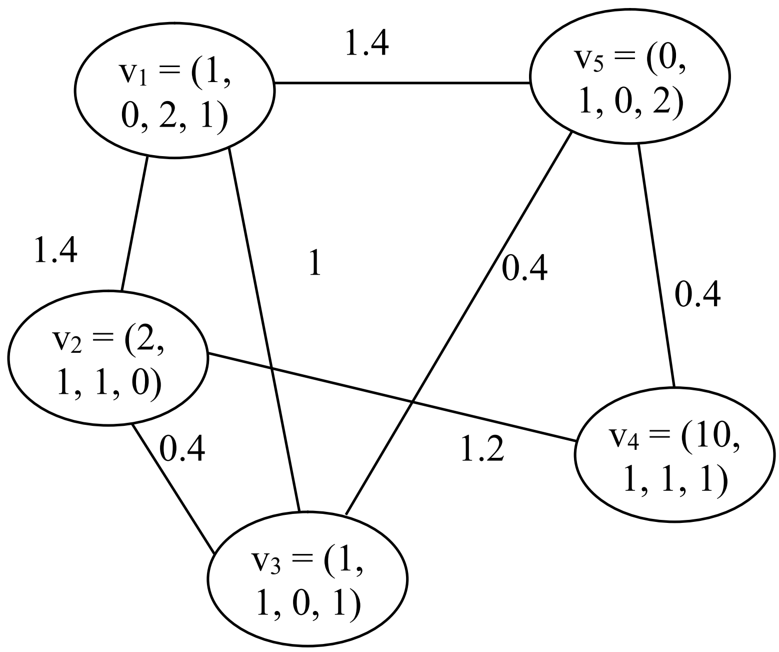


Figure 4.17

Both special graphs G and H though enjoy the same set of vertices and their adjacency is the same but the weights of the edges are different in both the cases.

Now we give examples of special Plithogenic complete bipartite graph with row matrix vertices but Hamming weight weights / Euclidean weight as edges in the following.

Example 4.14 Let V = $\{v_1, v_2, v_3, v_4, v_5, u_1, u_2, u_3\}$ where u_i and v_j are 1×5 row matrices.

Let G be the special Plithogenic complete bipartite graph which Euclidean distance as edge weights given by the following figure.

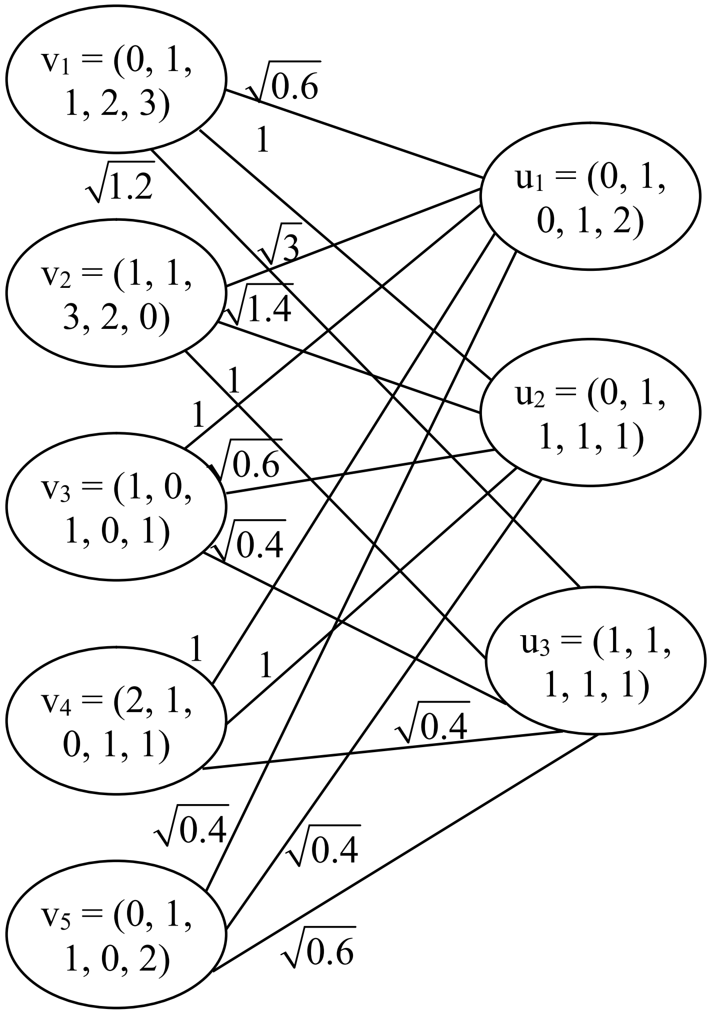


Figure 4.18

G is a bipartite complete special Plithogenic graph with edge weights as Euclidean weights as Euclidean weight between the two vertices.

Next we give the graph which is a special Plithogenic bipartite complete graph H with vertices as above but edge weights are the Hamming distance which is as follows.

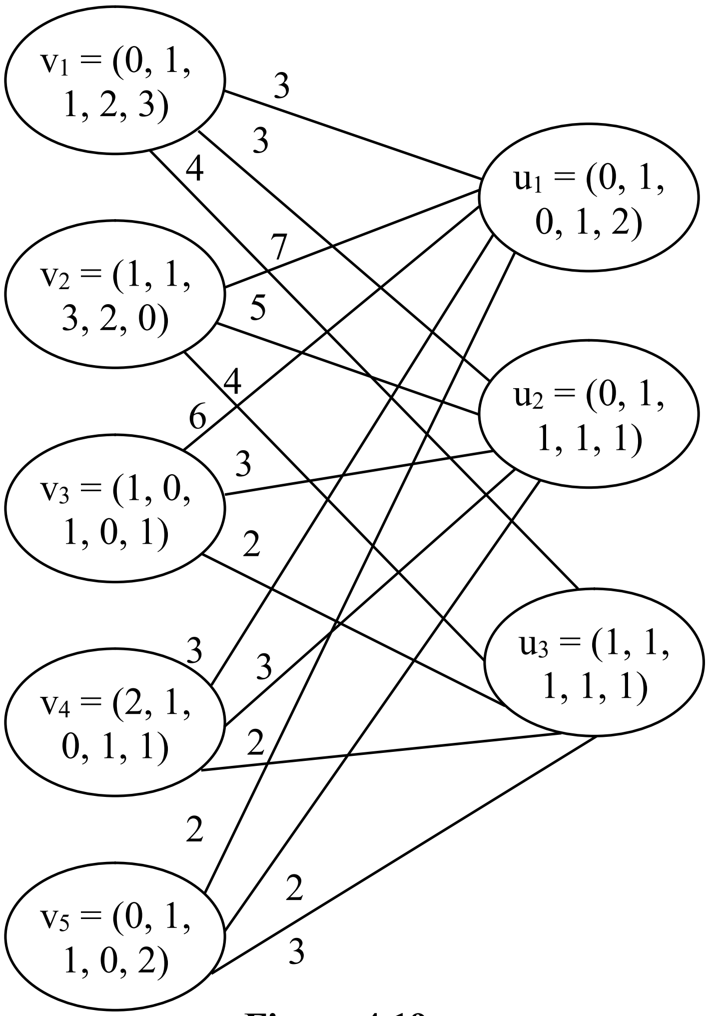


Figure 4.19

The two bipartite complete Plithogenic graphs with edge weights as Hamming distance / Euclidean distance are distinct though they share the same vertex set and edges; for the edge weights are distinctly different.

Now for the first time we apply these concepts in the case of Plithogenic fuzzy problem.

To obtain that we first define certain concepts analogous to fuzzy intuitionistic graph, fuzzy single valued neutrosophic graph the notion of Plithogenic fuzzy graphs and the applications to real world problems.

To that end we know given some n attributes about a problem we can give the membership functions related to them from the interval [0, 1].

So if U is the universal set of discourse and $a_1, \ldots, a_n$ are the n attributes and if $x_1, x_2, \ldots, x \in U$ then we have the Plithogenic fuzzy set is given by

x_1 $(b_{11}, b_{12}, \ldots, b_{1n})$,

x_2 $(b_{21}, b_{22},\ldots, b_{2n})$, and

x_m $(b_{m1}, b_{m2}, \ldots, b_{mn})$ where

$b_{ij} \in [0, 1]$; $1 \leq i \leq m$ and $1 \leq j \leq n$.

Thus we can have a graph with vertex set $v_i = x_i(b_{21}, b_{i21}, \ldots, b_{in})$; $1 \leq i \leq m$. Depending on the problem and the experts opinion the vertices in $V = \{v_1, v_2, \ldots, v_m\}$ can be adjacent or not.

We call these graphs as Plithogenic fuzzy graphs. There are 5 such types of graphs. We will be describing them by examples so that it is clear for the reader to get the plithogenic graphs of all the 5 types.

Example 4. 15 Let $V = \{v_1 = (0.5, 0.3, 0.4, 0.1, 0), v_2 = (0.1, 0, 0, 0.3, 0.4), v_3 = (0.4, 0.6, 0, 0, 0.4), v_4 = (0, 0.2, 0.3, 0.4, 0), v_5 = (0.1, 0.1, 0.3, 0.3, 0)$ and $v_6 = (0, 0, 1, 0.2, 0.5)\}$ be the vertex set of the graph G_1 given by the following figure.

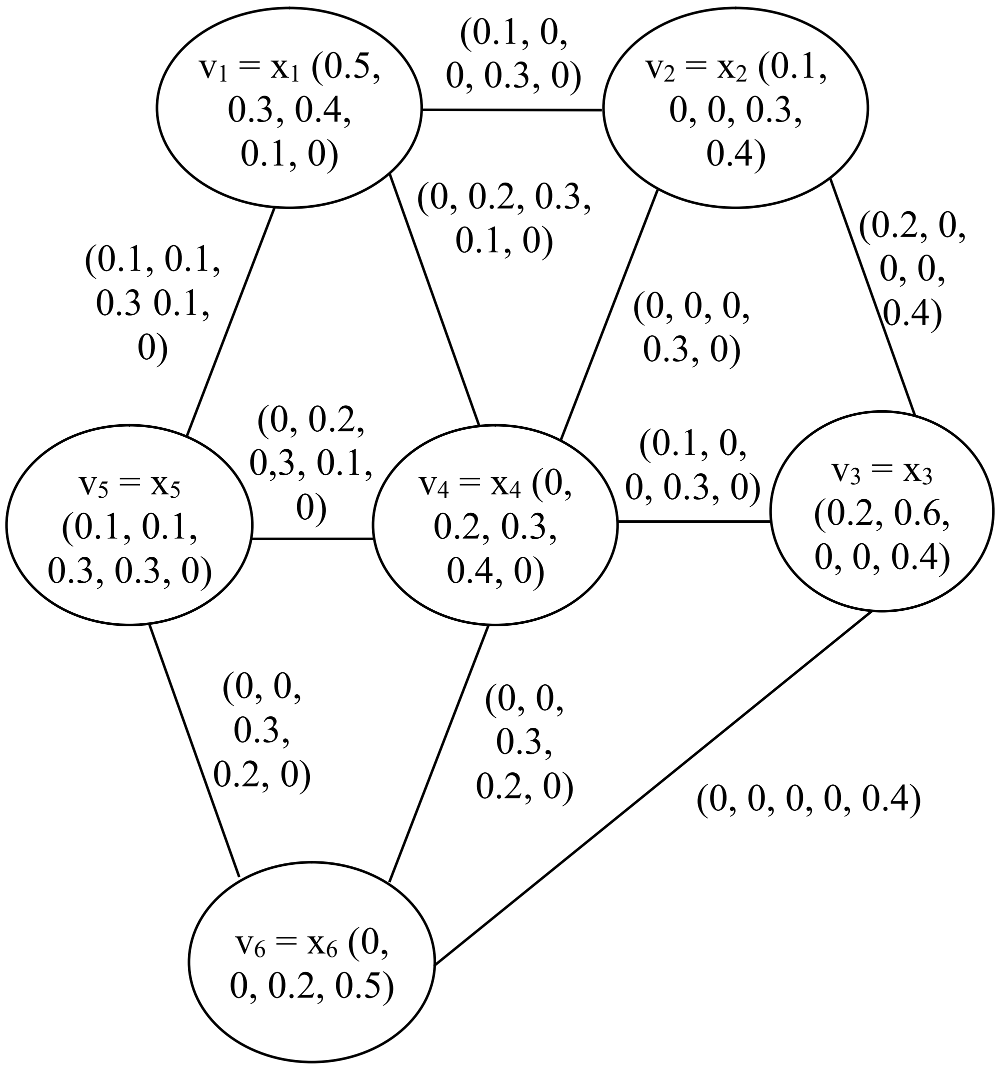


Figure 4.20

We have denoted the edge label as the min$\{v_i, v_j\}$ for relevant i and j; $1 \leq i, j \leq b$. This Plithogenic fuzzy graph G_1 is defined as the Plithogenic fuzzy graph of type I.

This graph can be used to model a plithogenic set for if we want to find from the collection of Plithogenic fuzzy sets $(x_i(b_{i1}, b_{i2}, \ldots, b_{in})$; $1 \leq i \leq m$ and $b_{ij} \in [0, 1]$, $1 \leq j \leq n$ and if one of the values say x_t $(b_{t1}, b_{t2}, \ldots, b_{tn})$ which is the target or goal solution then using this graph we can calculate the weight of each edge weight and make.

Suppose in this problem $v_4 = x_4$ (0, 0.2, 0.3, 0.4, 0) is the targeted value and the value which we choose from x_1, x_2, x_3, x_5 and x_6 by finding the weights of the edges v_1v_4, v_2v_4, v_3v_4, v_5v_4 and v_6v_4 and picking up the least weight, in this case

$v_1v_4 = | 0 | + | 0.2 | + | 0.3 | + | 0.1 | + | 0 | = 0.6$,

$v_2v_4 = | 0 | + | 0 | + | 0 | + | 0.3 | + 0 = 0.3$,

$v_4v_3 = | 0 | + | 0.2 | + | 0 | + | 0 | + | 0 | = 0.2$,

$v_4v_5 = | 0 | + | 0.1 | + | 0.2 | + | 0.3 | + | 0 | = 0.6$ and

$v_4v_6 = | 0 | + | 0 | + | 0.3 | + | 0.2 | + | 0 | = 0.5$

So v_3 that is x_3 (0.2, 0.6, 0, 0, 0.4) the first preferred value for it is close to x_4 (0, 0.2, 0.3, 0.4, 0).

$v_1 = x_1$ (0.5, 0.3, 0.4, 0.1, 0) and v_5 = (0, 0.1, 0.2, 0.5) are the values which will never be considered, as the value is very large 0.6.

The next value which is close to v_4 is $v_2 = x_2$ (0.1, 0, 0, 0.3, 0.4) is the next preferred value.

From this the following observations are mandatory. One cannot say if two values are preferred as first or second; they need not be close to each other. This example gives the answer for the same.

But under general conditions such results may not be possible, so we make it clear that this type of deviation makes the solution not only suitable but also paves way to study relatively and make different type of conclusions.

Suppose for the same problem we use type II Plithogenic fuzzy graphs but we are interested in taking the maximum value from the expected least value for the same problem. Now we assume $v_4 = x_4$ (0, 0.2, 0.3, 0.4, 0) is the least excepted value we want to find a $v_i = x_i(b_{i1}, b_{i2}, b_{i3}, b_{i4}, b_{i5})$ $1 \leq i \leq 6$ $i \neq 4$ by defining $v_iv_4 = \max \{v_i, v_4\}$; $1 \leq i \leq 6$; $i \neq 4$ and find the Hamming weight and take the largest value.

Let G_2 be the associated Plithogenic fuzzy graph of type II with same set of vertices and edges and only the edge values are max $\{v_i, v_j\}$ $i \neq j$; $1 \leq i, j \leq 6$ for relevant i and j is as follows.

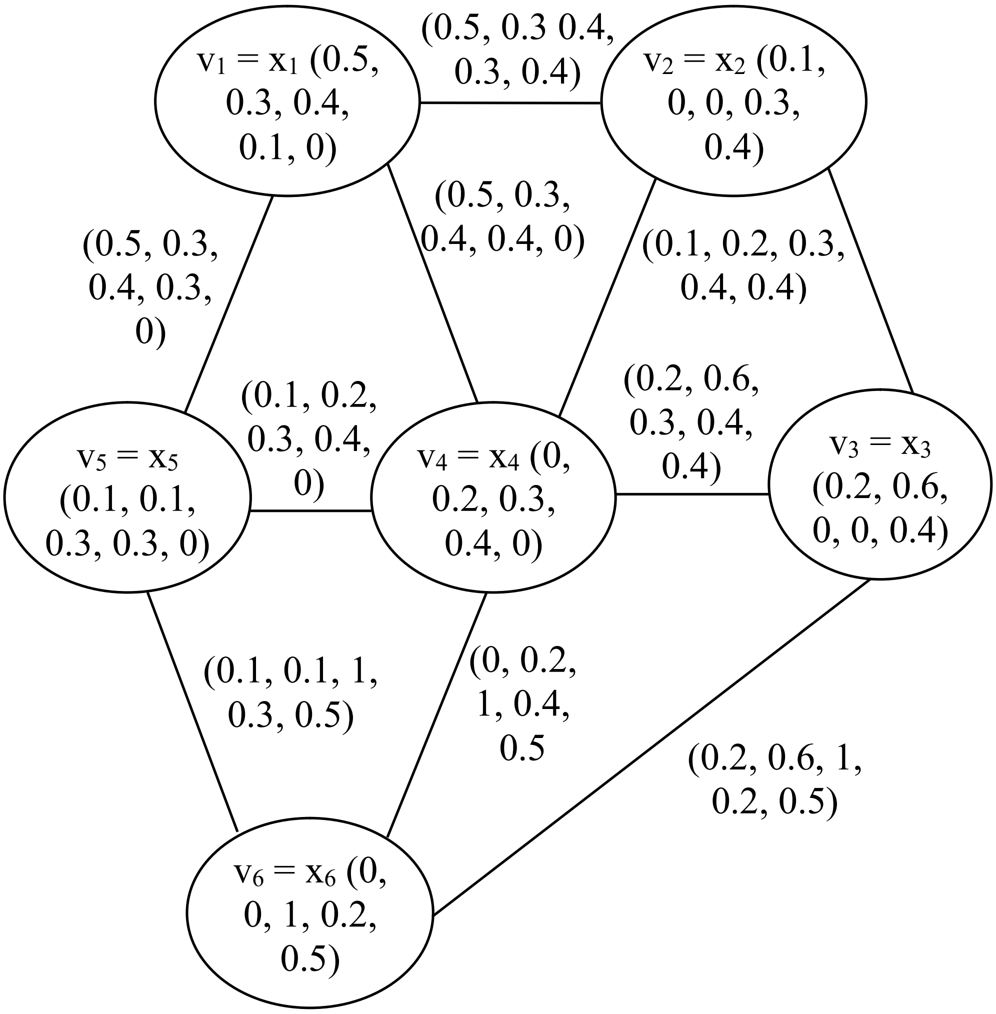


Figure 4.21

Using Hamming weight as the weights of the edges v_4v_1, v_4v_2, v_4v_3, v_4v_5 and v_4v_6 are determined as follows.

$w_H (v_4v_1) = (0.5 + 0.3 + 0.4 + 0.4) = 1.6,$

$w_H (v_4v_2) = (0.1 + 0.2 + 0.3 + 0.4 + 0.4) = 1.4;$

$w_H (v_4v_3) = (0.2 + 0.6 + 0.3 + 0.4 + 0.4) = 1.9,$

$w_H (v_5v_4) = (0.1 + 0.2 + 0.3 + 0.4 + 0 = 1$ and

$w_H(v_6v_4) = 0 + 0.2 + 1 + 0.2 + 0.5 = 2.1$

Now we are forced to choose only

$v_6 = x_6 (0, 0, 1, 0.2, 0.5).$

This choice will yield the best result.

Now we proceed onto describe the type III Plithogenic fuzzy graph for the same problem with 6 vertex sets and the same set of edges in the following.

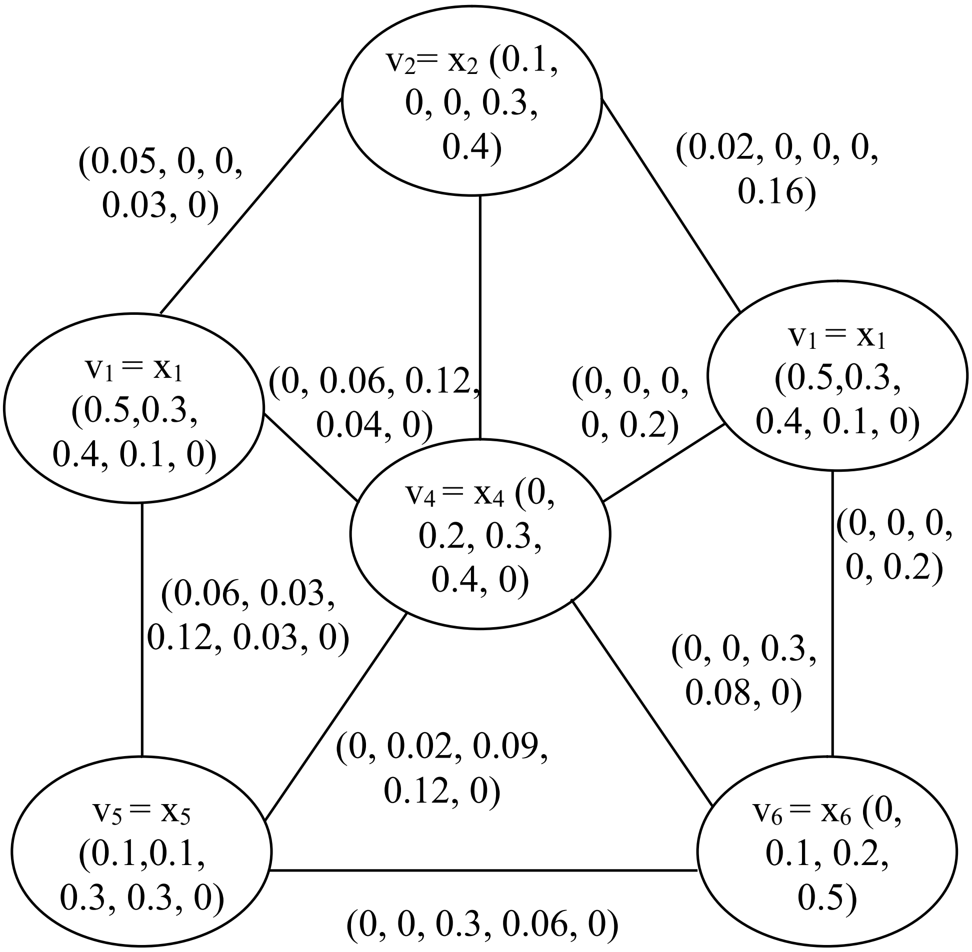


Figure 4.22

The edge weights of G_3 are got as the usual product of the fuzzy row matrices. Now we find the Hamming weights of v_4v_1, v_4v_2, v_4v_3, v_4v_5 and v_4v_6 in the following.

$w_H(v_4v_2) = 0 + 0 + 0 + 0.12 + 0 = 0.12,$

$w_H(v_4v_1) = 0 + 0.06 + 0.12 + 0.04 + 0 = 0.22,$

$w_H(v_4v_3) = 0 + 0.12 + 0 + 0 + 0 = 0.12$

$w_H(v_5v_4) = 0 + 0.02 + 0.09 + 0.12 + 0 = 0.23$ and

$w_H(v_5v_6) = 0 + 0 + 0.03 + 0.08 + 0 = 0.11.$

If the problem or expert opinion is pertaining to the greatest value, then it is the one associated with vertex v_5 which gives the greatest value as 0.23 and if one prefers the least value it is given by v_6 which is 0.11.

Now we proceed onto describe the type IV graphs.

This example of Plithogenic fuzzy graph of type IV takes the same set of vertices as v_1, v_2, v_3, v_4, v_5 and v_6 and the edges as in other cases. The edges are not labeled as row matrices, but they are the Hamming distance of v_iv_j for $i \neq j$; $1 \leq i, j \leq 6$ for relevant i and j's of this graph.

This Plithogenic fuzzy graph G_4 is type IV graph which is as follows.

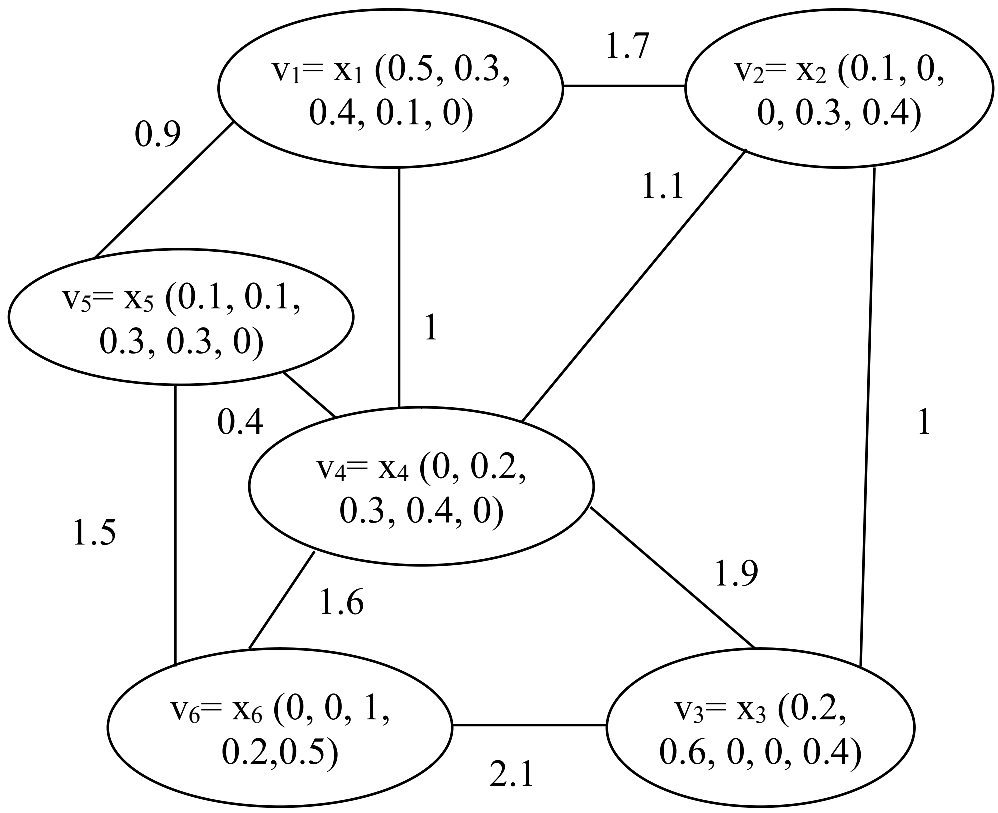


Figure 4.23

where the distances are Hamming weight. If the expert wishes to have a maximum value from v_4 he would prefer v_3 which corresponding to 1.9. If on the other hand prefers the least value from v_4 he would prefer v_5.

Now we finally describe the type V Plithogenic fuzzy graph for the same set of vertices $v_1, v_2, \ldots, v_6$ and the same set of edges but here the edge values are taken as the Euclidean distance from v_iv_j $i \neq j$; $1 \leq i, j \leq 6$ for relevant i and j.

Let G_5 be the Plithogenic fuzzy graph given by the following figure.

$G_5 =$

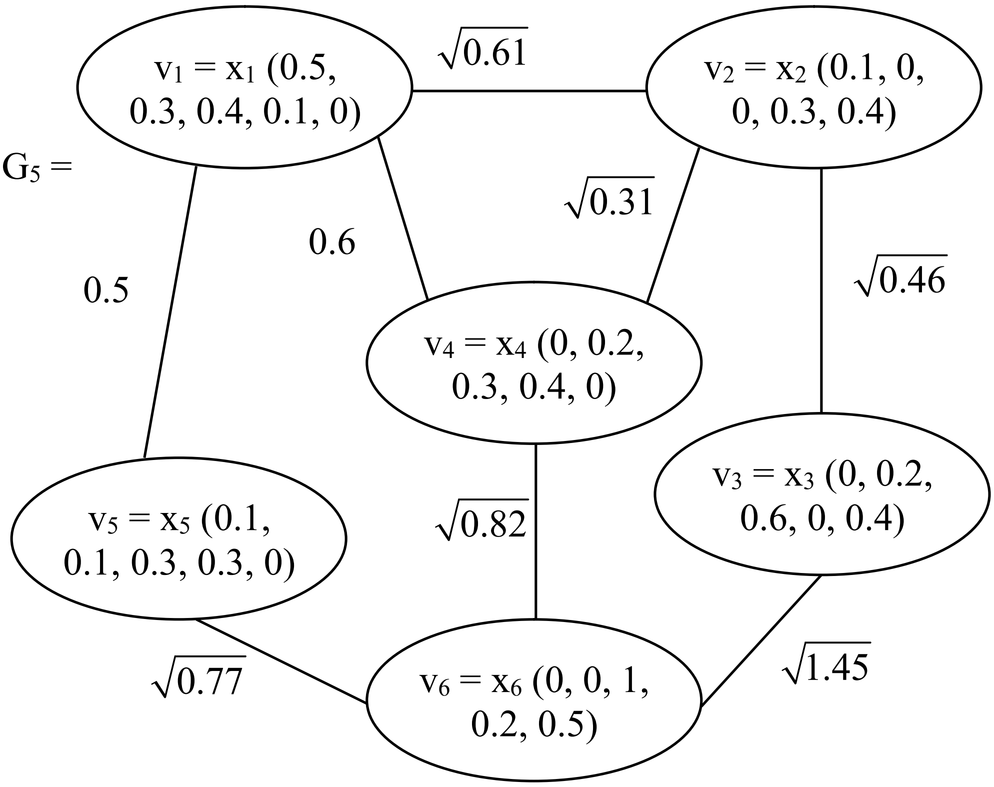


Figure 4.24

The maximum edges weight from v_4 in case of this Plithogenic fuzzy graph G_5 is $\sqrt{0.82}$ is v_4v_6 and the least value of the edges from v_4 is $\sqrt{0.03}$ and the edges v_5v_4.

All the 5 types of Plithogenic fuzzy graphs are distinct and in fact one can find max and min from the vertex v_4.

We will tabulate this information in the form of tables so that comparison is easy. One table pertains to greatest value from v_4 to other 5 nodes and the other table gives the least value from v_4 to v_1, v_2, v_3, v_5 and v_6 edges from v_4.

S. No.	**G**	**Graph G_1**	**Graph G_2**	**Graph G_3**	**Graph G_4**	**Graph G_5**
1.	Edge v_1v_4	0.6	1.6	0.12	1	0.6
2.	Edge v_2v_4	0.3	1.4	0.22	1.1	$\sqrt{0.31}$
3.	Edge v_3v_4	0.2	1.9	0.12	1.9	$\sqrt{0.61}$
4.	Edge v_5v_4	0.6	1	0.23	0.4	$\sqrt{0.03}$
5.	Edge v_6v_4	0.5	2.1	0.11	1.6	$\sqrt{0.82}$

Graph G_5	**G_1**	**G_2**	**Max G_3**	**G_4**	**G_5**
0.6	0.6 v_4v_1	-	-	-	-
$\sqrt{0.31}$		-	-	-	-
$\sqrt{0.61}$		-	-	1.9 v_3v_4	-
$\sqrt{0.03}$	0.6 v_4v_5	-	0.23 v_1v_5	-	-
$\sqrt{0.82}$		2.1 v_4v_6	-	-	$\sqrt{0.82}$ v_4v_6

G_1	Min		G_4	G_5
	G_2	G_3		
-	-	-	-	-
-	-	-	-	-
v_4v_3 0.2	-	-	-	-
-	1 v_4v_5	-	0.4 v_4v_5	$\sqrt{0.03}$ v_4v_5
-	-	v_4v_6 0.11	-	-

The flexibility of the choice of any of the 5 types of graphs makes the expert to deal better in solving the problem. Further he can use these Plithogenic fuzzy graphs to model problems which could yield a better solution using plithogenic concept.

Next, we proceed onto describe the notion of Plithogenic fuzzy complete bipartite graphs, using all the five types of operations.

First, we provide some examples then we give a real world illustration of the model in medical diagnostic.

Example 4.16 Let V = $\{v_1, v_2, v_3, v_4, u_1, u_2, u_3\}$ be a set of vertices which are 1×6 fuzzy row matrices. Let G_1 be the Plithogenic fuzzy graph of type I which is a complete bipartite graph given by the following figure which we call as the Plithogenic fuzzy complete bipartite graph. We also give the applications in the medical diagnostic in later work. The operation of finding edges is min operator.

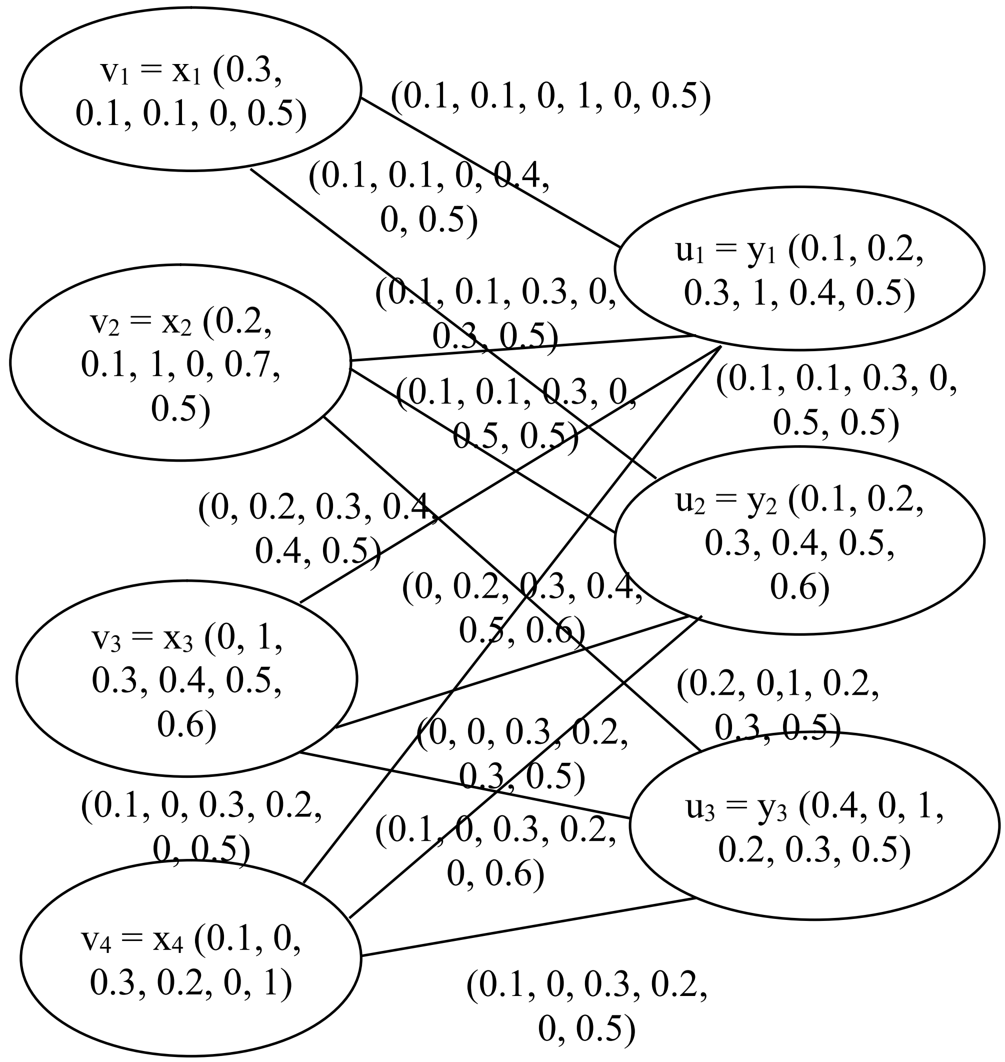


Figure 4.25

We see G_1 is a Plithogenic fuzzy bipartite complete graph with edge weights is the min$\{v_i, u_j\}$; $1 \leq i \leq 4$ and $1 \leq j \leq 3$.

Now if the expert wants to determine which of the values of u_i is best suited for a v_j; $1 \leq i \leq 3$, $1 \leq j \leq 4$. Then we find the Hamming weight of $\{u_i, v_j\}$.

For instance, to study the best suited u_i for the vertex v_1 we find the Hamming weights of v_1u_1, v_1u_2 and v_1u_3 and take the least value.

In this case $w_H(v_1u_1) = 0.1 + 0.1 + 0 + 1 + 0 + 0.5 = 1.7$

$w_H (v_1u_2) = 0.1 + 0.1 + 0 + 0.4 + 0 + 0.5 = 1.1$

$w_H(v_1u_3) = (0.3 + 0 + 0 + 0.2 + 0 + 0.5) = 1.$

So the best result for v_1 is u_3 as the Hamming weight is least in this case.

Next for v_2 we find the Hamming weights of v_2u_1, v_2u_2, v_2u_3.

$w_H(v_2u_1) = 0.1 + 0.1 + 0.3 + 0 + 0.3 + 0.5 = 1.3$

$w_H(v_2u_2) = 0.1 + 0.1 + 0.3 + 0 + 0.5 + 0.5 = 1.5$

$w_H(v_2u_3) = 0.2 + 0 + 1 + 0.2 + 0.3 + 0.5 = 2.2$

For v_2 the best choice with least Hamming weight is u_1. Consider for the Hamming weights v_3u_1, v_3u_2 and v_3u_3 as follows.

$w_H (v_3u_1) = 0 + 0.2 + 0.3 + 0.4 + 0.4 + 0.5 = 1.8$

$w_H(v_3u_2) = 0 + 0.2 + 0.3 + 0.4 + 0.5 + 0.6 = 2$

$w_H(v_3u_3) = 0 + 0 + 0.3 + 0.2 + 0.3 + 0.5 = 1.3$

The least Hamming weight being 1.3. That is the closest value for v_3 is u_3.

Now we consider the edges v_4u_1, v_4u_2 and v_4u_3 and find their Hamming weights

$w_H(v_4u_1) = 0.1 + 0 + 0.3 + 0.2 + 0 + 0.5 = 1.0$

$w_H(v_4u_2) = 0.1 + 0 + 0.3 + 0.2 + 0 + 0.6 = 1.2$

$w_H(v_4u_3) = 0.1 + 0 + 0.3 + 0.2 + 0 + 0.5 = 1.1$

We see u_1 are the values which are close to v_4.

It is pertinent to keep on record that these are just examples and not any real-world problems.

Now we consider type II Plithogenic fuzzy complete bipartite graph G_2 by labeling the edges with max operator that is max (v_iu_j); $1 \leq i \leq 4$ and $1 \leq j \leq 3$. The figure of G_2 is as follows.

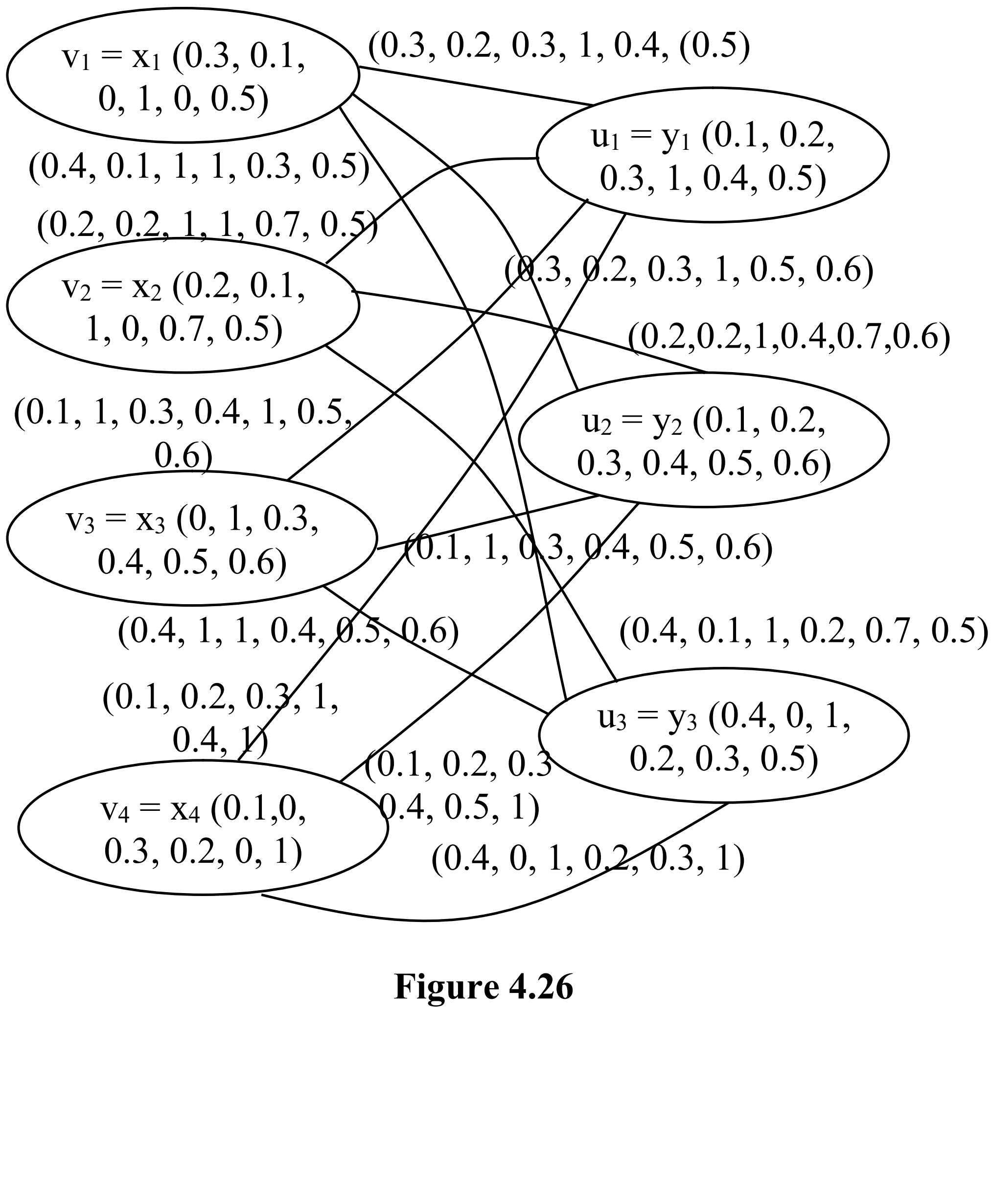


Figure 4.26

We now give the Hamming weights of v_1u_1, v_1u_2 and v_1u_3;

$w_H(v_1u_1) = 0.3 + 0.2 + 0.3 + 1 + 0.4 + 0.5 = 2.7$

$w_H(v_1u_2) = 0.3 + 0.2 + 0.3 + 0.5 + 0.6 = 2.9$

$w_H(v_1u_3) = 0.4 + 0.1 + 1 + 1 + 0.3 + 0.5 = 3.3.$

We see the greatest value is u_3 for v_1 so we prefer it.

Now we see in this model by using max or min function we see for the vertex v_1 or attribute set v_1, u_3 is the most preferred.

Now we find the Hamming weights of v_2u_1, v_2u_2 and v_2u_3 in the following.

$w_H(v_2,u_1) = 0.2 + 0.2 + 1 + 1 + 0.7 + 0.5 = 3.6$

$w_H(v_2,u_2) = 0.2 + 0.2 + 1 + 0.4 + 0.7 + 0.6 = 3.1$

$w_H(v_2, u_3) = 0.4 + 0.1 + 1 + 0.2 + 0.7 + 0.5 = 2.9.$

So the maximum weight is u_1 so u_1 is preferred for this attributes v_2.

We see it is the same as min operator.

Next we find the Hamming weights v_3u_1, v_3u_2 and v_3v_3 in the following

$w_H(v_3,u_1) = 0.1 + 1 + 0.3 + 0.4 + 1 + 0.5 + 0.6 = 3.9$

$w_H(v_3,u_2) = 0.1 + 1 + 0.3 + 0.4 + 0.5 + 0.6 = 2.6$

$w_H(v_3,u_3) = 0.4 + 1 + 1 + 0.4 + 0.5 + 0.6 = 3.9$.

The largest Hamming weight corresponds to $w_H(v_3, u_3)$ so u_3 is the preferred one.

Next we find the Hamming weights associated with the attribute v_4, v_4u_1, v_4u_2 and v_4u_3.

$w_H(v_4, u_1) = 0.1 + 0.2 + 0.3 + 1 + 0.4 + 1 = 3$

$w_H(v_4, u_2) = 0.1 + 0.2 + 0.3 + 0.4 + 0.5 + 1 = 2.5$

$w_H(v_4,u_3) = 0.4 + 1 + 0.3 + 0.2 + 1 = 2.9$

Here we get the two attributes viz u_1 for then value is 3.

Now we keep on record if we use min value as the edge values we take the min of the Hamming weight as a solution whereas if we use max value as the edge values then we take the max of Hamming weight as a solution.

The solution in both cases will be the same. So one can always choose to use only one of them.

Next proceed onto give the Plithogenic fuzzy graph G_3 where we use the natural product of the two adjacent vertices as the row label for the same Plithogenic fuzzy complete bigraph.

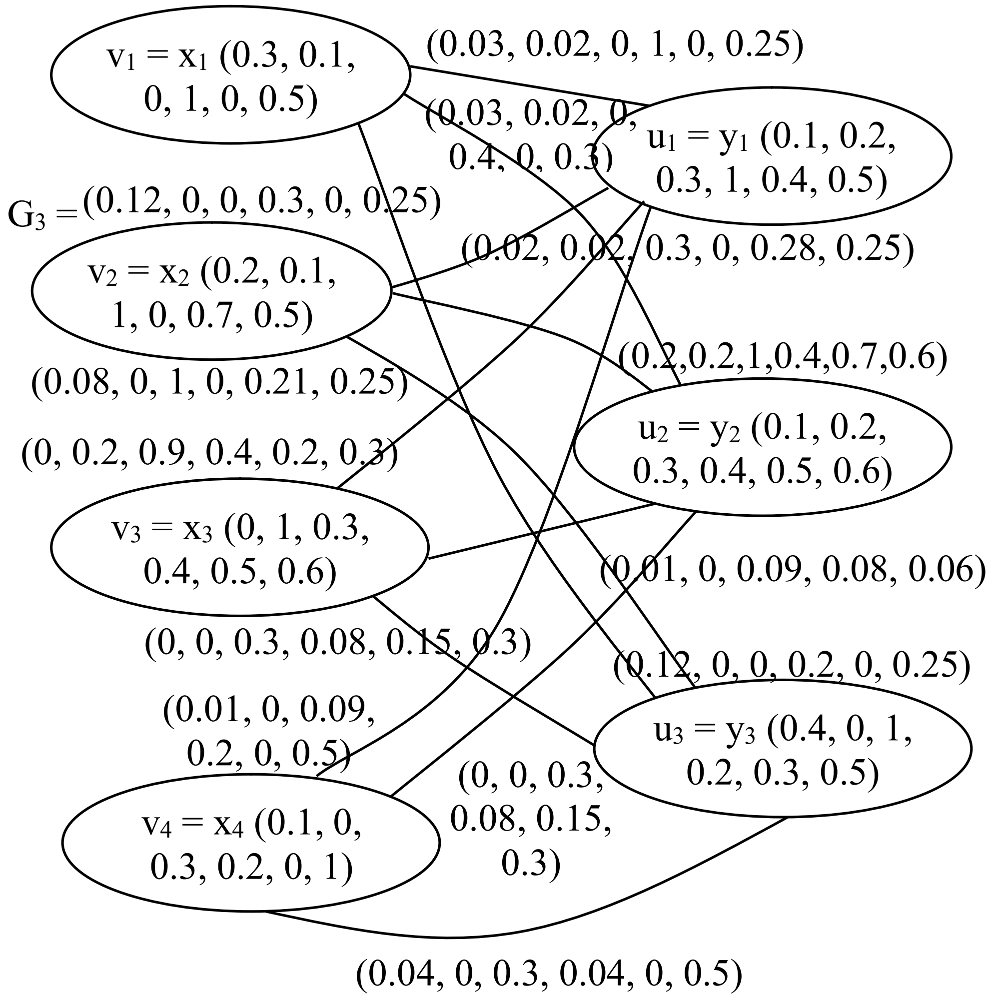


Figure 4.27

Now we find the Hamming weights w_H of the edges v_1u_1, v_1u_2 and v_1u_3.

$w_H(v_1u_1) = 0.03 + 0.02 + 0 + 1 + 0 + 0.25 = 1.3$

$w_H(v_1u_2) = 0.03 + 0.02 + 0 + 0.4 + 0 + 0.3 = 0.75$

$w_H(v_1u_3) = 0.12 + 0 + 0 + 0.3 + 0 + 0.25 = 0.4.$

We see u_3 is value of choice for v_1.

Now we find the Hamming weights w_H of the edges v_2u_1, v_2u_2 and v_2u_3.

$w_H(v_2u_1) = 0.02 + 0.02 + 0.3 + 0 + 0.28 + 0.25 = 0.87$

$w_H(v_2u_2) = 0.02 + 0.02 + 0.3 + 0 + 0.35 + 0.3 = 0.99$

$w_H(v_2u_3) = 0.08 + 0 + 1 + 0 + 0.21 + 0.25 = 1.54.$

u_1 happens to be closest value for v_2. Consider the Hamming weights of v_3u_1, v_3u_2 and v_3u_3;

$w_H(v_3u_1) = 0 + 0.2 + 0.09 + 0.4 + 0.2 + 0.3 = 1.19$

$w_H(v_3u_2) = 0 + 0.2 + 0.09 + 0.16 + 0.25 + 0.36 = 1.06$

$w_H(v_3u_3) = 0 + 0 + 0.3 + 0.08 + 0.15 + 0.3 = 0.83.$

We see u_3 is the nearest to v_3.

We find the Hamming weights of v_4u_1, v_4u_2 and v_4u_3 in the following.

$w_H(v_4u_1) = 0\ .01 + 0 + 0.09 + 0.02 + 0 + 0.5 = 0.7$

$w_H(v_4u_2) = 0.01 + 0 + 0.09 + 0.08 + 0 + 0.6 = 0.78$

$w_H(v_4u_3) = 0.04 + 0 + 0.3 + 0.04 + 0 + 0.5 = 0.88.$

The value which is with least weight is v_4u_1 so u_1 is the one close to v_4. We see the answer for all the 3 Plithogenic fuzzy graphs G_1, G_2 and G_3 yield the same solution so one can choose any one graph for the solution is identical. Now for the same set of vertices we for the Plithogenic fuzzy complete bipartite graph give the edge values as the Hamming distance from v_iu_j; $1 \leq i \leq 4$ and $1 \leq j \leq 3$ in the following figure using the formula I.

Further we denote this Plithogenic fuzzy bipartite complete graph by G_4 with the above said edge labels.

G_4 =

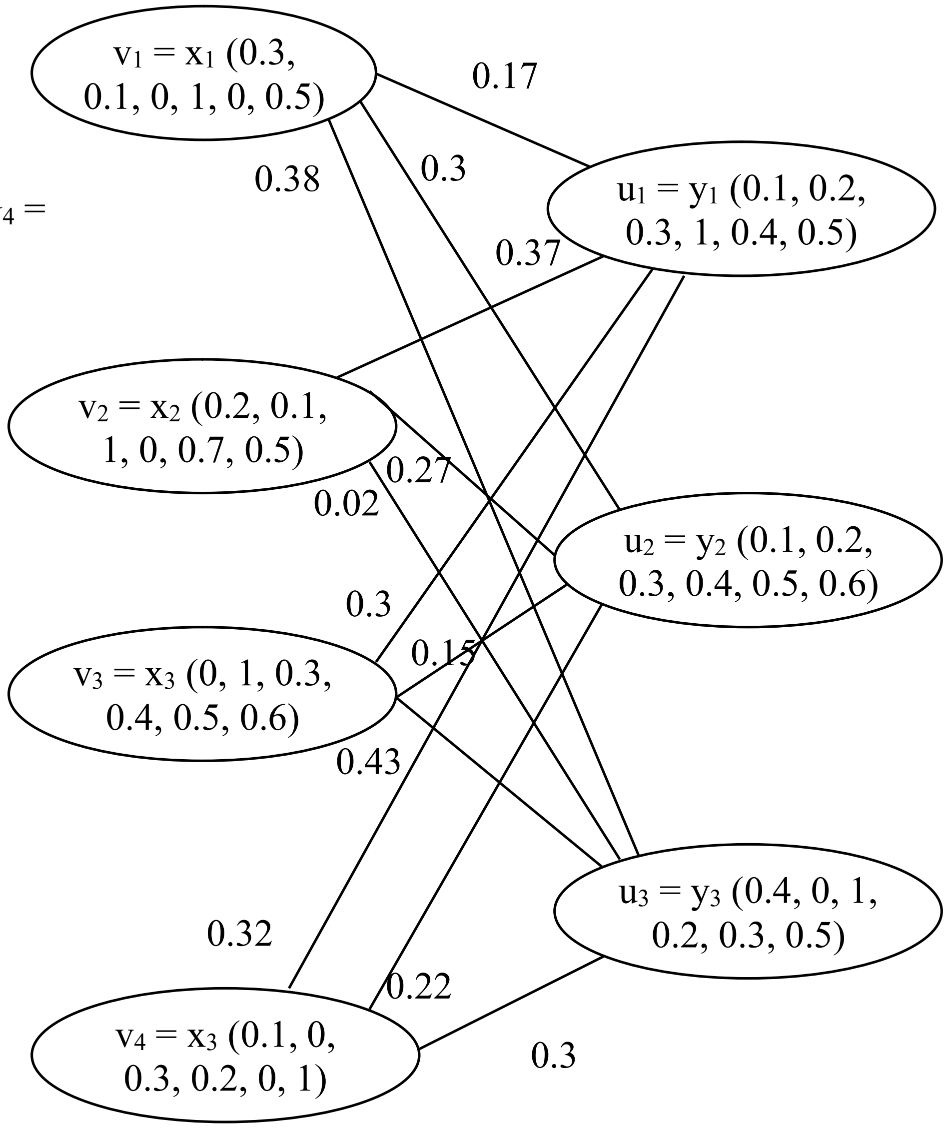


Figure 4.28

Taking for every vertex attribute $v_i = x_i$ (….), $1 \leq i \leq 4$ the greatest weight is taken and this result gives the analogously the same values as in G_1, G_2 and G_3.

Finally, we introduce the notion of Euclidean distance weighted Plithogenic fuzzy complete bigraph with the same set of vertex sets and edges in example.

We use formula II for the calculation of the Euclidean distance. Let G_5 be the Plithogenic fuzzy bipartite complete graph given by the following figure.

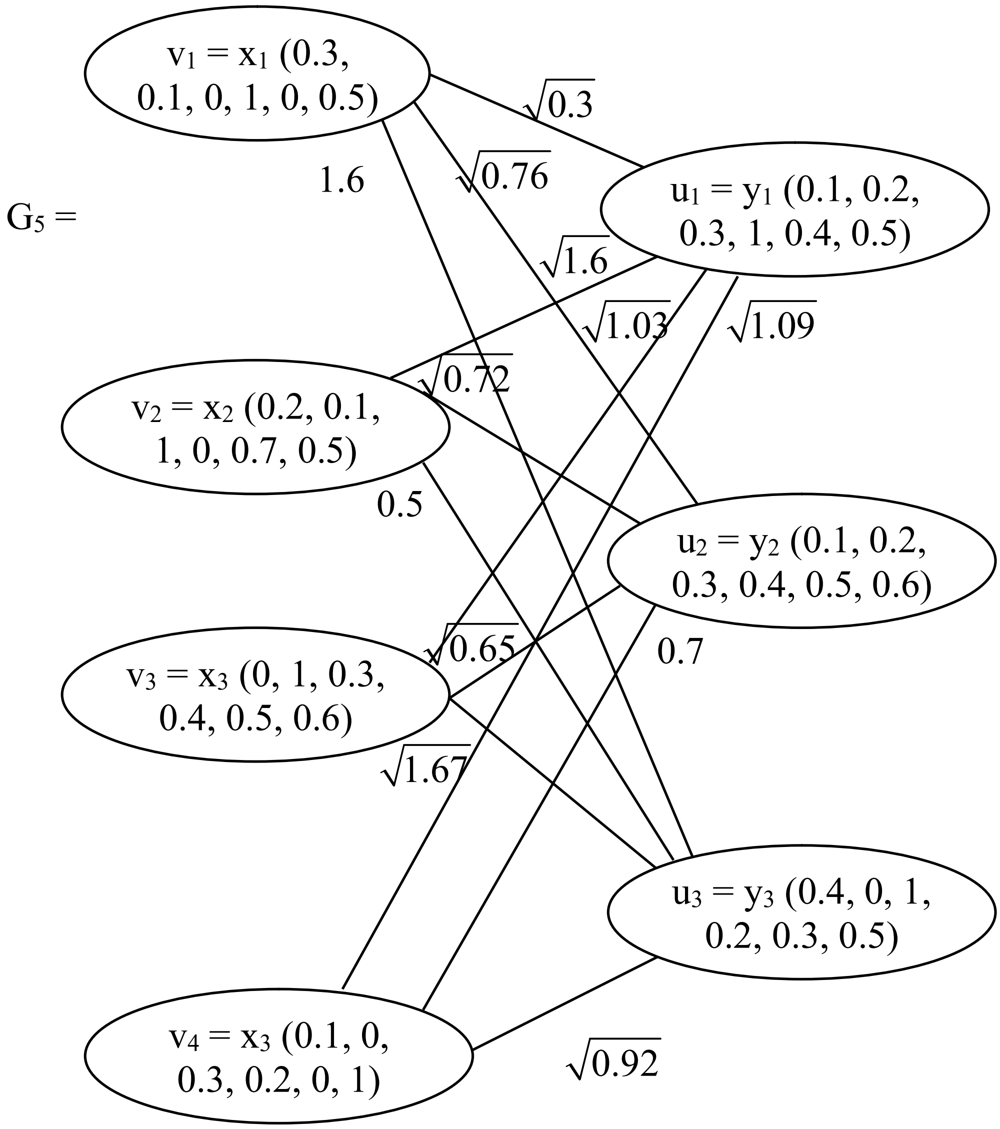


Figure 4.29

We see the height weight for the attribute v_1 is u_3 and so on that of v_4 is 0.7.

Here it is important to note any of the Plithogenic fuzzy graphs by using min or max or product or Hamming distance or Euclidean distance the result happens to be the same in general.

Now we proceed onto give one example of this Plithogenic fuzzy graph when applied to medical diagnostics.

Example 4.17 Let us consider the problem of medical diagnostic of four patients for 5 types of diseases.

The 5 diseases for which they are diagnosed for are

1. Malaria - D_1
2. Typhoid - D_2
3. Flu - D_3
4. Food poisoning - D_4
5. Jaundice - D_5

The symptoms associated in the general are given as attributes

1. Fever - a_1
2. High fever - a_2
3. Cold - a_3
4. Cold and cough - a_4
5. Chills before and after fever - a_5
6. Stomach pain - a_6
7. Vomiting - a_7

8. Headache - a_8
9. Dysentery - a_9

Now problem of malaria as given by an expert doctor is given as

$D_1 = d_1$ (0.7, 0.5, 0.1, 0.2, 0.8, 0.2, 0.3, 0.7, 0.1).

$D_2 = d_2$ (0.6, 0.7, 0.1, 0.1, 0.2, 0.6, 0.7, 0.8, 0.3) is the attributes associated by the expert / doctor.

The expert for the disease flu D_3 gives the following values.

$D_3 = d_3$(0.8, 0.8, 0.7, 0.7, 0.5, 0.1, 0.1, 0.6, 0.2).

The attribute vector given by the expert doctor for the problems of food poisoning D_4 is as follows:

$D_4 = d_4$ (0.5, 0.6, 0.1, 0.1, 0.7, 0.6, 0.8, 0.8, 0.8).

Finally, the attribute vector of symptoms given b y the expert doctor for jaundice D_5 is as follows.

$D_5 = d_5$ (0.6, 0.5, 0.1, 0.1, 0.1, 0.6, 0.7, 0.7, 0.3).

Now the doctor maps the symptoms of the four patients P_1, P_2, P_3 and P_4 in the following:

$P_1 = x_1$ (0.6, 0.6, 0.1, 0.2, 0.7, 0.5, 0.7, 0.8, 0.9)

$P_2 = x_2$ (0.8, 0.7, 0.7, 0.8, 0.5, 0.1, 0.1, 0.5, 0.2)

$P_3 = x_3$ (0.8, 0.5, 0.1, 0.2, 0.8, 0.1, 0.2, 0.7, 0.1)

and $P_4 = x_3$ (0.7, 0.6, 0.2, 0.2, 0.7, 0.2, 0.3, 0.7, 0.2).

Now for this model we use the Plithogenic fuzzy bipartite complete graph using Hamming distance between the vertices.

Then we will find the Hamming weight of these edge row matrices and the least weight will correspond to the disease which they suffer from. It may be recalled these type of Plithogenic fuzzy graphs are widely discussed in chapters II and III of this book.

The graph G of this is given in the following.

G =

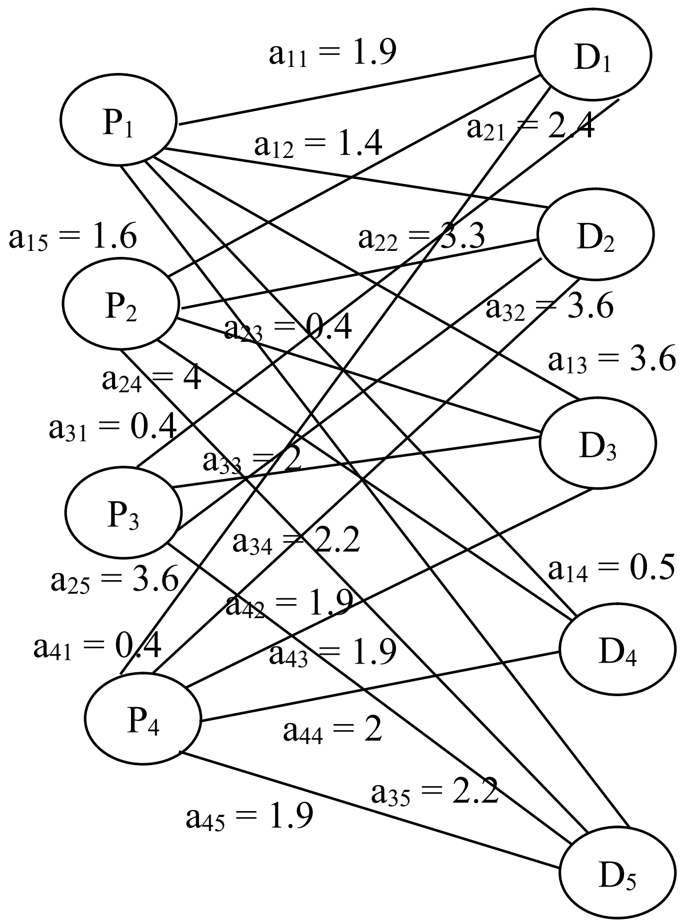


Figure 4.30

The Hamming distance $d_H(P_1D_1) = (0.1, 0.1, 0, 0, 0.1, 0.3, 0.4, 0.1, 0.8) = a_{11}$

$w_H(a_{11}) = 0.1 + 0.1 + 0 + 0 + 0.1 + 0.3 + 0.4 + 0.1 + 0.8 = 1.9$ and so on

We see the first patient suffers from food poisoning as the least weight between P_1D_4 is $0.5 = a_{14}$.

For the patient P_2 the least Hamming weight corresponds to $a_{23} = 0.4$ that is D_3 so the second patient suffers from flu.

Consider the 3rd patient P_3 the least weight is given by the edge a_{31} so the P_3 suffers from Malaria.

Consider the fourth patient the least weight is associated with the edge P_5D_1 that is the Hamming weight is a_{41} is 0.4.

Thus we can use Plithogenic fuzzy graphs to model the medical diagnostic. This will not as in case of SVNS or fuzzy intuitionistic graphs gives membership or non-membership in fact considers the overall or consolidated situation of the scenario.

Thus without any doubt one can certainly claim that when this problem uses the method of Plithogenic fuzzy graphs which we can also analogously call as Plithogenic fuzzy model will yield a better and a sensitive result by taking into account all the systems associated with the disease.

Next we proceed onto describe fuzzy intuitionistic plithogenic set from [1-3, 44]. However, we for the first time give the plithogenic fuzzy intuitionistic graph.

Definition 4.7 *Suppose we have some n attributes in the universal set and some m experts work with it using the fuzzy intuitionistic neutrosophic values. Then the plithogenic fuzzy* *intuitionistic neutrosophic graph G will have the vertex set labeled by row super matrices of the form.*

$v_1 = x_1((m_{11}\ n_{11}), (m_{12}\ n_{12}), \ldots, (m_{1n}, n_{1n})$

$v_2 = x_2((m_{21}, n_{21}), (m_{22}, n_{22}), \ldots, (m_{2n}, n_{2n}))$ *and so on.*

$v_m = x_m((m_{m,1}, n_{m1}), (m_{m2}, n_{m2}), \ldots, (m_{mn}, n_{mn}))$

where $m_{ij}, n_{ij} \in [0, 1]; 1 \leq i \leq m$ *and* $1 \leq j \leq n$ *with* m_{ij} *the degree of membership and* n_{ij} *the non-membership degree respectively.*

Clearly each vertex set v_i *is a super row matrix.*

Now the edge weights can be any value using min or max or product or Hamming distance or Euclidean distance or a combination of any of them. The edge weight for these plithogenic fuzzy intuitionistic neutrosophic graphs are in the hands of the expert or the researcher which ultimately depends on the problem.

G = {V = $\{v_1 v_2, \ldots, v_m\}$, min or max or product or Hamming distance or Euclidean distance or any combination of min or max or product is taken as the edge labels of G.

We will illustrate this situation by an example.

Example 4.18 Let V = $\{v_1, v_2, v_3, v_4, v_5\}$ be the set of vertices given by

$v_1 = x_1 \{((0.1, 0.6), (0.7, 0.2), (0.5, 0.4), (0.9, 0.2))\}$

$v_2 = x_2\{((0.6, 0.4), (0.2, 0.8), (0.6, 0.2), (0.6, 0.3))\}$

$v_3 = x_3\{((0.1, 0.7), (0.6, 0.3), (0.1, 0.7), (0.1, 0.7))\}$

$v_4 = x_4$ $\{((0.7, 0.2), (0.1, 0.7), (0.2, 0.6), (0.3, 0.8))\}$ and

$v_5 = x_5\{((0.8, 0.1), (0.2, 0.6), (0.8, 0.1), (0.2, 0.7)\}$.

The edges are labeled $\{\min (m_{ij}, m_{kt}), \max \{(n_{ij}, n_{kt})\}$ for suitable values of i, j, k, t.

We give the plithogenic fuzzy intuitionistic graph in the following.

$G_1 =$

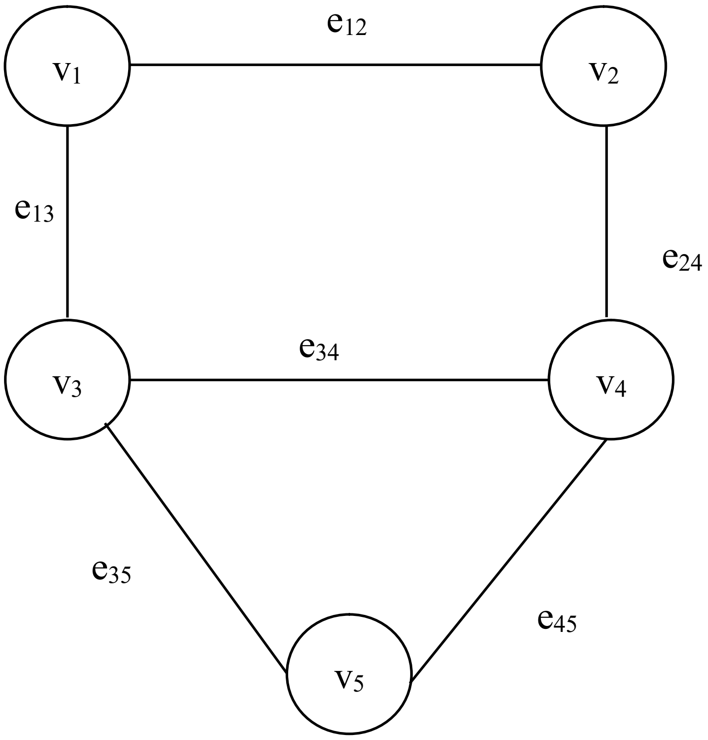


Figure 4.31

We give the values of the edges

$e_{12} = \{((0.1, 0.6), (0.2, 0.8), (0.5, 0.4), (0.6, 0.3))\}$

$e_{13} = \{((0.1, 0.7), (0.6, 0.3), (0.1, 0.7), (0.1, 0.7))\}$

$e_{15} = \{((0.1, 0.6), (0.2, 0.6), (0.5, 0.4), (0.2, 0.7))\}$,

$e_{24} = \{\{(0.6, 0.4), (0.1, 0.8), (0.2, 0.6), (0.6, (0.3))\}$,

$e_{34} = \{((0.1, 0.7), (0.1, 0.7), (0.1, 0.7), (0.1, 0.8))\}$,

$e_{35} = \{((0.1, 0.7), (0.2, 0.6), (0.1, 0.7), (0.1, 0.8))\}$ and

$e_{45} = \{((0.7, 0.2), (0.1, 0.7), (0.2, 0.6), (0.2, 0.7))\}$

We can on similar lines use {max min} operator or {product max} operator or so on. These concepts have been elaborately discussed in the earlier part of this chapter. We can also obtain the edge label as Hamming distance or Euclidean distance.

We will give the same graph G_6 for the same set of vertices but the edges being labeled with Hamming distance by the following figure.

$G_6 =$

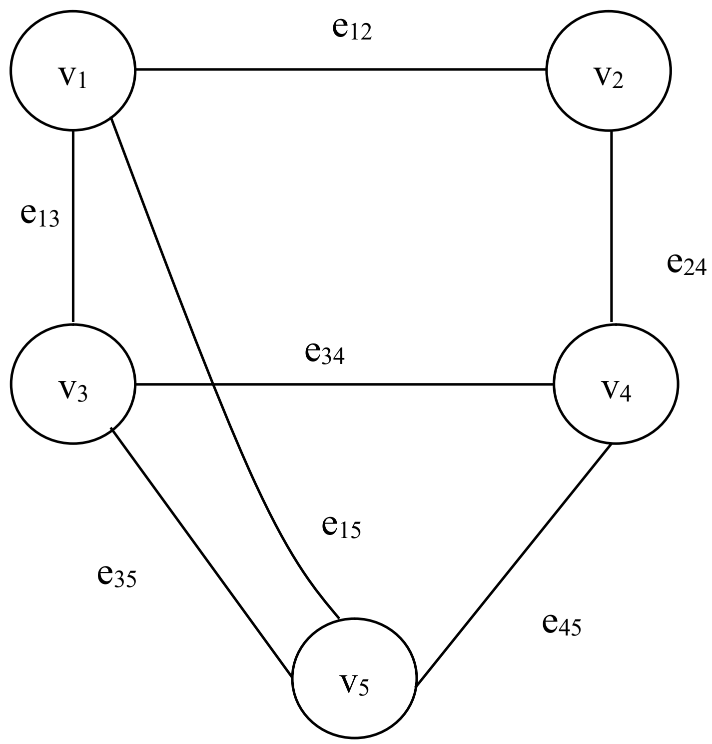


Figure 4.32

The values of e_{ij} for G_6 is calculated in the following $1 \leq i, j \leq 5$.

$e_{12} = \{((0.5, 0.2), (0.5, 0.6), (0.1, 0.2), (0.3, 0.1))\}$

$e_{13} = \{((0, 0.1), (0.1, 0.1), (0.4, 0.3), (0.8, 0.5))\}$

$e_{15} = \{((0.7, 0.5), (0.5, 0.4), (0.3, 0.3), (0.7, 0.5))\}$

$e_{24} = \{((0.1, 0.2), (0.1, 0.1), (0.4, 0.4), (0.3, 0.5))\}$

$e_{25} = \{((0.2, 0.3), (0, 0.2), (0.2, 0.1), (0.4, 0.4))\}$

$e_{34} = \{((0.6, 0.6), (0.4, 0.3), (0.7, 0.6), (0.1, 0.3))\}$

$e_{35} = \{((0.7, 0.6), (0.4, 0.3), (0.7, 0.6), (0.1, 0.3))\}$ and

$e_{45} = \{((0.1, 0.1), (0.1, 0.1), (0.6, 0.5), (0.1, 0.1))\}$

We will illustrate this situation by an example.

Example 4.19 Let us study children studying in 5^{th} standard in a private school.

We will study them for five qualities / problems, very poor performance, P_1, average performance P_2, very good or good performance P_3 and P_4 and slow learner P_5 A slow learner cannot be categorized as very poor performance or average performance or a very good or good performer.

The expert gives the following Plithogenic fuzzy intuitionistic sets associated with these five attributes. $P_1, P_2, \ldots, P_5$ denotes the Plithogenic fuzzy intuitionistic values of P_i; $1 \leq i \leq 5$.

He takes to work with the following attributes

C_1 – Regular to class

C_2 – Does homework regularly

C_3 – Gets good marks

C_4 – Interested in the classroom interacts with teacher

C_5 – Cannot cope up or is indifferent in this class

C_6 – Maintain the notebooks and books well and comes to class regularly.

In fact as per the expert wishes one can add several such attributes for the study.

As this is only an illustrate example and not based on real world data we have restricted to the minimum number of vital attributes associated with the problem of performance of school in the class room.

We take the following values for performance of students

P_1 = p{((0.2, 0.6), (0.1, 0.7), (0.1, 0.6), (0.1, 0.8), (0.1, 0.8), (0.2, 0.7))}

P_2 = p_2 {((0.6, 0.4), (0.5, 0.4), (0.6, 0.4), (0.6, 0.3), (0.5, 0.5), (0.5, 0.4))}

P_3 = p_3{((0.8, 0.1), (0.9, 0), (0.9, 0.1), (0.8, 0.1), (0.1, 0.8), (0.8, 0.1))}

P_4 = p_4 = {((0.7, 0.3), (0.6, 0.4), (0.7, 0.2), (0.7, 0.2), (0.4, 0.4) (0.6, 0.2))} and

$P_5 = p_5\{((0.5, 0.5), (0.4, 0.5), (0.3, 0.6), (0.3, 0.6), (0.7, 0.2), (0.5\ 0.2))\}$

Now we have the following feedback for the six students S_1, S_2, S_3, S_4, S_5 and S_6 which is as follows.

$S_1 = s_1\{((0.5, 0.4), (0.5, 0.4), (0.3, 0.7), (0.3, 0.5), (0.7, 0.3), (0.5, 0.3))\}$

$S_2 = s_2\{((0.4, 0.5), (0.4, 0.5), (0.4, 0.6), (0.4, 0.6), (0.7, 0.1), (0.5, 0.2))\}$

$S_3 = s_3\{((0.9, 0.1), (0.9, 0.1), (0.9, 0), (0.9, 0.1), (0.1, 0.8), (0.9, 0.1))\}$

$S_4 = s_4\{((0.9, 0), (0.8, 0.1), (0.9, 0.1), (0.9, 0), (0, 0.9), (0.9, 0))\}$

$S_5 = s_5\{((0.1, 0.6), (0.1, 0.6), (0.1, 0.7), (0.1, 0.8), (0.1, 0.7), (0.2, 0.7))\}$

$S_6 = s_6\{(0.1, 0.7), (0.1, 0.6), (0.1, 0.6), (0.1, 0.8), (0.1, 0.8), (0.3, 0.6))\}$

Using these vertex set $\{P_1, \ldots, P_5, S_1, S_2, \ldots, S_6\}$ we give the Plithogenic fuzzy intuitionistic complete bipartite graph G in the following figure.

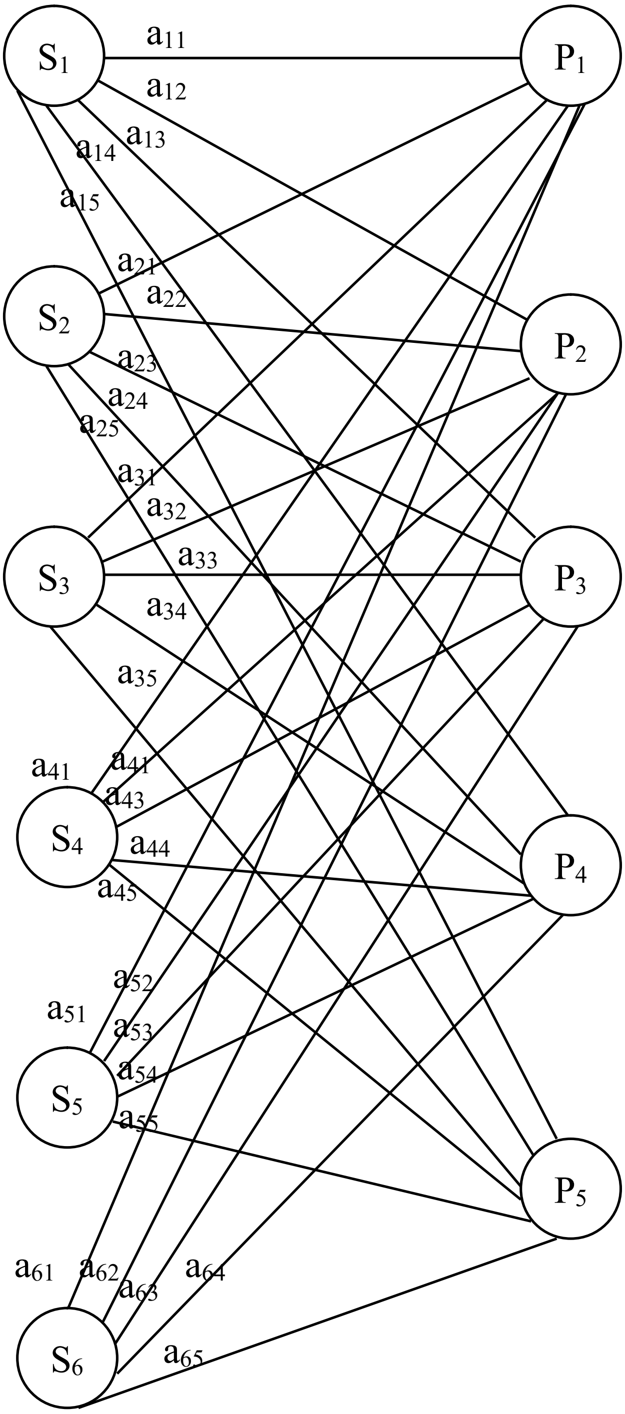


Figure 4.33

We give the Hamming distance d_{ij} and using this d_{ij} we find the Hamming weight w_H (d_{ij}) a_{ij} : $1 \leq i \leq 6$ and $1 \leq j \leq 5$ in the following.

$d_{11} = \{(0.3, 0.2), (0.4, 0.3), (0.2, 0.1), (0.2, 0.3), (0.6, 0.5), (0.2, 0.4))\}$

$w_H(d_{11}) = (0.3 + 0.4 + 0.2 + 0.2 + 0.6 + 0.2 + 0.3, 0.1 + 0.3 + 0.5 + 0.4) = (1.9, 1.8) = a_{11}$

$d_{12} = \{((0.1, 0), (0, 0), (0.3, 0.3), (0.3, 0.2), (0.2, 0.2), (0, 0.1))\}$

$w_H(d_{12}) = (0.9, 0.8) = a_{12}$

$d_{13} = \{((0.3, 0.3), (0.4, 0.4), (0.6, 0.6), (0.5, 0.4), (0.6, 0.5), (0.3, 0.2))\}$

$w_H (d_{13}) = (2.7, 2.4) = a_{13}$

$d_{14} = \{((0.2, 0.1), (0.1, 0), (0.4, 0.5), (0.4, 0.3), (0.3, 0.1), (0.1, 0.1))\}$

$w_H(d_{14}) = (1.5, 1.1) = a_{14}$

$d_{15} = ((0, 0.1), (0.1, 0.1), (0, 0.1), (0, 0.1), (0, 0.1), (0, 0.1))$

$w_H(d_{15}) = (0.1, 0.6) = a_{15}$

We see the first student S_1 is very close to the fuzzy intuitionistic set P_5 so student S_1 is a slow learner.

Next we find the Hamming distances the Hamming distances d_{2j}, and $a_{21}, a_{22}, \ldots, a_{25}$ the Hamming weights ($1 \leq j \leq 5$) in the following.

$d_{21} = ((0.2, 0.1), (0.3, 0.2), (0.3, 0), (0.3, 0.2), (0.6, 0.7), (0.2, 0.5))$

$w_H(d_{21}) = (1.9, 1.7) = a_{21}$

d_{22} = ((0.2, 0.1), (0.1, 0.1), (0.2, 0.2), (0.2, 0.3), (0.2, 0.4), (0.3, 0.2))

$w_H(d_{22})$ = (1.2, 1.2) = a_{22}

d_{23} = ((0.1, 0), (0, 0.1), (0, 0.1), (0.1, 0), (0, 0), (0.1, 0))

w_H (d_{23}) = (0.3, 0.2) = a_{23}

d_{24} = ((0.3, 0.2), (0.2, 0.1), (0.3, 0.4), (0.3, 0.4), (0.3, 0.3), (0.1, 0))

$w_H(d_{24})$ = (1.5, 1.4) = a_{24}

d_{25} = ((0.1, 0), (0, 0), (0, 0.2), (0.1, 0), (0, 0.1), (0, 0))

w_H (d_{25}) = (0.2, 0.3) = a_{25}.

We see in the case of student S_2 also he is a slow learner so the minimum weight of the Plithogenic fuzzy intuitionistic set with that of slow learner P_5 is the least (0.2, 0.3) = a_{25}.

Hence S_2 is a slow learners.

Next we find the Hamming distance of S_3 from P_1, P_2, P_3, P_4 and P_5 in the following using which we find the Hamming weight of them.

d_{31} = ((0.7, 0.5), (0.8, 0.6), (0.8, 0.6), (0.8, 0.7), (0, 0), (0.6, 0.6))

w_H (d_{31}) = (3.7, 3) = a_{31}

d_{32} = ((0.3, 0.3), (0.4, 0.3), (0.3, 0.4), (0.3, 0.2), (0.4, 0.3), (0.4, 0.3))

$w_H(d_{32}) = (2.1, 1.8) = a_{32}$

$d_{33} = ((0.1, 0), (0, 0.1), (0, 0.1), (0.1, 0), (0, 0), (0.1, 0))$

$w_H(d_{33}) = (0.3, 0.2) = a_{33}$

$d_{34} = ((0.2, 0.2), (0, 0.1), (0, 0.1), (0.1, 0), (0, 0), (0.8, 0))$

$w_H(d_{34}) = (1.1, 0.4) = a_{34}$

$d_{35} = ((0.4, 0.4), (0.5, 0.4), (0.6, 0.6), (0.6, 0.5), (0.6, 0.6), (0.4, 0.1))$

$w_H(d_{35}) = (3.1, 2.6) = a_{35}$.

From the values of a_{3j}; $1 \leq j \leq 4$ we see the student S_3 is very good performer.

Next we find the values of $d_{41}, d_{42}, \ldots, d_{45}$ and the corresponding Hamming weights $a_{41}, a_{42}, \ldots, a_{45}$ of them respectively.

$d_{41} = \{((0.7, 0.6), (0.7, 0.6), (0.8, 0.5), (0.8, 0.8), (0.1, 0.1), (0.6, 0.7))\}$

$w_H(d_{41}) = (3.7, 3.3) = a_{41}$

$d_{42} = \{((0.3, 0.4, (0.3, 0.3), (0.3, 0.3), (0.3, 0.3), (0.5, 0.4), (0.5, 0.4))\}$

$w_H(d_{41}) = (2.2, 2.1) = a_{42}$

$d_{43} = \{((0.1, 0.1), (0.1, (0.1), (0, 0), (0.1, 0.1), (0.1, 0.1), (0.1, 0.1))\}$

$w_H(d_{42}) = (0.5, 0.5) = a_{43}$

d_{44} = {((0.2, 0.3), (0.2, 0.3), (0.2, 0.1), (0.2, 0.2), (0.4, 0.5), (0.3, 0.2))}

$w_H(d_{44})$ = (1.5, 1.6) = a_{44}

d_{45} = {((0.4, 0.5), (0.4, 0.4), (0.6, 0.5), (0.6, 0.6), (0.7, 0.7), (0.4, 0.2))}

$w_H(d_{45})$ = (3.1, 2.9) = a_{45}.

The student S_4 is somewhat close to the good student.

However, the value is (0.5, 0.5) but is less than all the other four values.

Now we work for the student S_5 by finding the values d_{51}, d_{52}, d_{55}, d_{54} and d_{55} using then we will find the Hamming weight of them a_{51}, a_{52}, a_{53}, a_{54} and a_{55} respectively.

d_{51} = {((0.1, 0), (0, 0.1), (0, 0.1), (0, 0), (0, 0.1), (0, 0))}

$w_H(d_{51})$ = (0.1, 0.3) = a_{51}

d_{52} = {((0.5, 0.2), (0.4, 0.2), (0.5, 0.3), (0.5, 0.5), (0.4, 0.2), (0.3, 0.3))}

$w_H(d_{52})$ = (2.6, 1.7) = a_{52}

d_{53} = {((0.7, 0.5), (0.8, 0.6), (0.6, 0.5), (0.7, 0.7), (0, 0.1), (0.6, 0.5))}

$w_H(d_{53})$ = (3.4, 2.9) = a_{53}

d_{54} = {((0.6, 0.3), (0.5, 0.2), (0.6, 0.5), (0.6, 0.6), (0.3, 0.3), (0.4, 0.5))}

$w_H(d_{54}) = (3, 2.4) = a_{54}$

$d_{55} = \{((0.4, 0.1), (0.3, 0.1), (0.2, 0.1), (0.2, 0.2), (0.6, 0.5), (0.3, 0.5))\}$

$w_H(d_{55}) = (2, 1.5) = a_{55}$

S_5 is very close to the performer P_1 which means the student is a very poorly performed child.

Now for the student S_6 we find the Hamming distance from $P_1, P_2, \ldots, P_5$ viz $d_{61}, d_{62}, \ldots, d_{65}$ and find the weights $a_{61}, a_{62}, \ldots, a_{65}$ respectively.

$d_{61} = \{((0.1, 0.1), (0, 0.1), (0, 0), (0, 0), (0, 0), (0, 0.1))\}$

$w_H(d_{61}) = (0.1, 0.3) = a_{61}$

$d_{62} = \{((0.5, 0.3), (0.4, 0.2), (0.5, 0.2), (0.5, 0.5), (0.4, 0.3), (0.2, 0.4))\}$

$w_H(d_{62}) = (2.5, 1.9) = a_{62}$

$d_{63} = \{((0.7, 0.6), (0.8, 0.6), (0.8, 0.5), (0.7, 0.7), (0, 0), (0.5, 0.5))\}$

$w_H(d_{63}) = (3.5, 2.9) = a_{63}$

$d_{64} = \{((0.6, 0.3), (0.5, 0.2), (0.6, 0.4), (0.6, 0.6), (0.3, 0.4), (0.3, 0.4))\}$

$w_H(d_{64}) = (2.9, 2.3) = a_{64}$

$d_{65} = \{((0.4, 0.2), (0.3, 0.1), (0.2, 0), (0.2, 0.2), (0.6, 0.6), (0.2, 0.4))\}$

$w_H(d_{65}) = (1.9, 1.5) = a_{65}$.

We see the least Hamming weight is given by d_{61}. Hence this student S_6 is again a student who is very poor in studies.

Thus, this fuzzy intuitionistic the model can be used in medical diagnostic, employee, employer problem, student performance and so on.

Next, we proceed onto describe the Plithogenic single valued neutrosophic graph and the plithogenic model associated with it.

We know any Plithogenic single valued neutrosophic set is of the form

$x_i((a_{11}\ a_{12}\ a_{13}), (a_{21}\ a_{22}\ a_{23}), \ldots, (a_{m1}\ a_{m2}\ a_{m3}))$ where a_{ji} are the truth membership values a_{j2} are the indeterminate membership values and a_{j3} are the false membership values $1 \leq j \leq m$. We see $1 \leq i \leq t$ are the t values.

The are special super row matrices.

We give examples of Plithogenic single valued neutrosophic graph.

Example 4.20 Let $V = \{V_1, V_2, V_3, V_4, V_5\}$ be the vertex set which are Plithogenic Single Valued Neutrosophic Sets (SVNS). The edges can be got as min or max or product or a combination of all.

The values of $v_1, v_2, \ldots, v_5$ are given,

$v_1 = x_1$ {((0.7, 0.5, 0.3), (0.2, 0.6, 0.7), (0.9, 0.1, 0.2), (0.4, 0.8, 0.8))},

$v_2 = x_2$ {((0.6, 0.4, 0.4), (0.8, 0.2, 0.4), (0.8, 0.2, 0.3), (0.1, 0.6,0.7))}

$v_3 = x_3$ {((0.5, 0.7, 0.2), (0.8, 0.3, 0.5), (0.7, 0.4, 0.2), (0.2, 0.7, 0.9))}

$v_4 = x_4${((0.2, 0.5, 0.8), (0.8, 0.7, 0.1), (0.1, 0.8, 0.4), (0.6, 0.6, 0.2))} and

$v_5 = x_5${((0.3, 0.7, 0.2), (0.8, 0.1, 0.6), (0.4, 0.5, 0.8), (0.7, 0.8, 0.3))}

G =

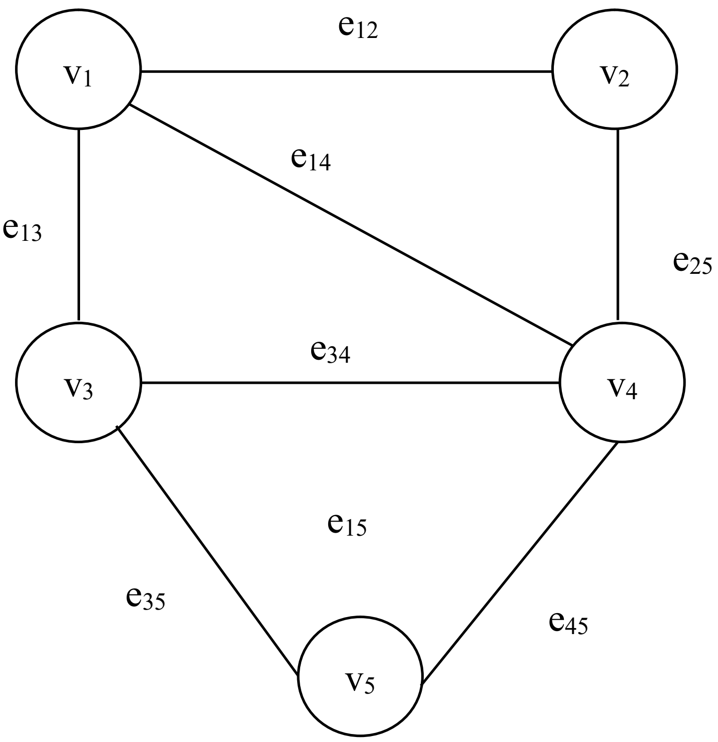


Figure 4.34

Let G be Plithogenic single valued neutrosophic graph with edges e_{ij} if v_i and v_j are adjacent vertices.

The values of e_{ij} are again super fuzzy row matrices given in the following using the min and max operator; {min true values, min of indeterminate values, max false values}.

e_{12} = {((0.6, 0.4, 0.4), (0.2, 0.2, 0.7), (0.8, 0.1, 0.3), (0.1, 0.6, 0.8))},

e_{13} = {((0.5, 0.5, 0.3), (0.2, 0.3, 0.7), (0.7, 0.1, 0.2), (0.2, 0.7, 0.9))}

e_{14} = {((0.2, 0.5, 0.8), (0.2, 0.6, 0.7), (0.1, 0.1, 0.4), (0.4, 0.6, 0.8))}

e_{25} = {((0.3, 0.4, 0.4), (0.8, 0.1, 0.6), (0.4, 0.2, 0.8), (0.1, 0.6, 0.7))}

e_{34} = {((0.2, 0.5, 0.8), (0.8, 0.3, 0.5), (0.1, 0.4, 0.4), (0.2, 0.6, 0.9))}

e_{35} = {(0.3, 0.7, 0.2), (0.8, 0.1, 0.6), (0.4, 0.4, 0.8), (0.2, 0.7, 0.9))} and

e_{45} = {((0.2, 0.5, 0.8), (0.8, 0.1, 0.6), (0.1, 0.5, 0.8), (0.6, 0.6, 0.3))}.

We can label the edges with min or max or min and max or product or product and min or max and product or by Euclidean distance or Hamming distance.

We will now give an example of Plithogenic single valued neutrosophic graph whose edge values correspond to

Hamming distance for any two adjacent vertices. This graph can give the distance between any two attributes which have to compared. If the Hamming weight of this distance is very small then we can conclude that the values which we calculate is approximately the same as the expected or the targeted values. So, this labeling will be a boon in modeling real world problems using Plithogenic SVNS.

Example 4.21 Let us assume we want to study the Example 4.19, in which we used plithogenic fuzzy intuitionistic graph about primary school now we use Plithogenic Single Value Neutrosophic graph, so in this case we replace the pair (membership, non-membership) by value of true membership, value of indeterminate membership, value of false membership that is by using SVNS.

We for the same set of students S_i and the performance in general $P_1, P_2, \ldots, P_5$; $1 \leq i \leq 6$ we give the plithogenic SVNS vertices for $\{S_i \text{ and } P_j \mid 1 \leq i \leq 6 \text{ and } 1 \leq j \leq 6\}$ in the following

$S_1 = \{((0.6, 0.2, 0.3), (0.6, 0.1, 0.2), (0.2, 0.4, 0.6), (0.2, 0.6, 0.4), (0.7, 0.2, 0.1), (0.6, 0.3, 0.1))\}$,

$S_2 = \{((0.6, 0.3, 0.2), (0.7, 0.2, 0.1), (0.3, 0.3, 0.5), (0.2, 0.5, 0.3), (0.6, 0.3, 0.2), (0.5, 0.2, 0.2)\}$,

$S_3 = \{((0.8, 0, 0.1), (0.9, 0.1, 0), (0.9, 0.1, 0.1), (0.9, 0.1, 0.1), (0.1, 0.2, 0.9), (0.8, 0.1, 0.2))\}$,

$S_4 = \{((0.6, 0.2, 0.2), (0.6, 0.3, 0.2), (0.7, 0.2, 0.1), (0.7, 0.3, 0.1), (0.3, 0.5, 0.2), (0.6, 0.3, 0.1))\}$

S_5 = {((0.2, 0.2, 0.6), (0.1, 0.2, 0.6), (0.2, 0.2, 0.7), (0.1, 0.2, 0.8), (0.1, 0.3, 0.7), (0.2, 0.3, 0.6))} and

S_6 = {((0.1, 0.3, 0.6), (0.1, 0.2, 0.7), (0.1, 0.1, 0.8), (0.1, 0.2, 0.8), (0.1, 0.2, 0.8), (0.2, 0.3, 0.7))}

Now we proceed onto give the plithogenic SVNS set for $P_1, P_2, \ldots, P_5$ in the following.

P_1 = {((0.1, 0.2, 0.6), (0.1, 0.1, 0.7), (0.1, 0.1, 0.7), (0.1, 0.2, 0.8), (0.2, 0.2, 0.8), (0.2, 0.3, 0.6))}

P_2 = {((0.8, 0.1, 0.1), (0.9, 0.2, 0.1), (0.9, 0.2, 0.2), (0.8, 0.1, 0.1), (0.1, 0.3, 0.8), (0.7, 0.2, 0.3))}

P_3 = {((0.8, 0.1, 0.1), (0.9, 0.1, 0.1), (0.9, 0.1, 0.1), (0.9, 0.2, 0.1), (0.1, 0.1, 0.9), (0.8, 0.1, 0.2))}

P_4 = {((0.7, 0.2, 0.2), (0.6, 0.3, 0.2), (0.7, 0.2, 0.2), (0.7, 0.3, 0.2), (0.3, 0.4, 0.2), (0.6, 0.2, 0.1))}

P_5 = {((0.6, 0.2, 0.2), (0.6, 0.2, 0.2), (0.2, 0.5, 0.6), (0.1, 0.6, 0.4), (0.7, 0.2, 0.1), (0.6, 0.3, 0.2))}

Now we give the plithogenic SVNS complete bipartite graph using the vertex sets as

$\{\{S_1, S_2, \ldots, S_6\}, \{P_1, P_2, \ldots, P_5\}\}$ where S_i are taken as the vertex set of the domain space and P_j are taken as the vertex set of the range space of the complete bipartite graph G; $1 \leq i \leq 6$ and $1 \leq j \leq 5$.

G =

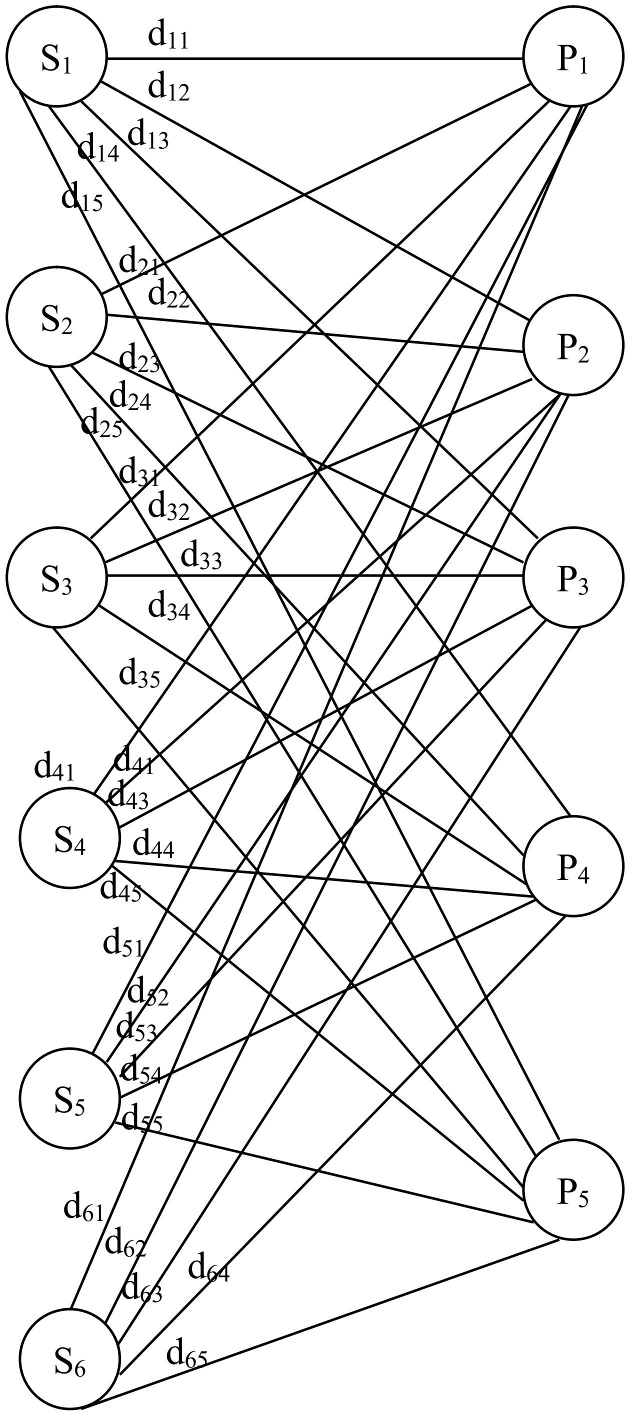


Figure 4.35

The plithogenic SVNS set values of S_i and P_j; $1 \leq i \leq 6$ and $1 \leq j \leq 5$ are given earlier.

Now we find the Hamming distance d_{ij}; $1 \leq j \leq 5$ between S_1P_j.

$d_H(S_1P_1) = d_{11} = \{((0.5, 0, 0.3), (0.5, 0, 0.5) (0.1, 0.3, 0.1), (0.1, 0.4, 0.4), (0.5, 0, 0.7), (0.4, 0, 0.5))\}$ $w_H(d_{11}) = (3.1, 0.7, 1.8), = a_{11}$

$d_H(S_1P_2) = \{((0.3, 0.1, 0.1), (0.3, 0.1, 0.1), (0.7, 0.2, 0.4), (0.6, 0.5, 0.3), (0.6, 0.1, 0.7), (0.1, 0.1, 0.1))\} = d_{12}$

$w_H(d_{12}) = (2.5, 1.1, 1.7) = a_{12}$

$d_h(S_1P_3) = \{((0.2, 0.1, 0.1), (0.3, 0, 0.1), (0.7, 0.3, 0.5), (0.7, 0.4\ 0.3), (0.6, 0.1, 0.8), (0.2\ 0.2, 0.1) = d_{13}$

$w_H(d_{13}) = (2.7, 1.1, 1.9) = a_{13}$

$d_H(S_1P_4) = \{((0.1, 0,0.1), (0, 0.2, 0), (0.5, 0.2, 0.4), (0.1, 0.3, 0.2), (0.4, 0.2,0.1), (0, 0.1, 0))\} = d_{14}$

$w_H(d_{14}) = (1.1, 1, 0.8) = a_{14}$

$d_H(S_1P_5) = \{((0, 0, 0.1), (0, 0, 0), (0, 0.1, 0), (0, 0, 0;1), (0, 0.1, 0), (0.1, 0, 0))\} = d_{15}$

$w_H(d_{15}) = (0.1, 0.2, 0.2)$.

Thus we see the least Hamming weight is associated with the edge S_1P_5 that is S_1 is very close to P_5 which in turn implies the student S_1 enjoys the attribute P_5 (or is very close to the plithogenic SVNS set P_5).

Hence S_1 is a slow learner.

Thus the student is a slow learner.

Next we calculate the edges d_{21}, d_{22}, …, d_{25} and their respective Hamming weights $w_H(d_{21})$, $w_H(d_{22})$, $w_H(d_{23})$, $w_H(d_{24})$ and $w_H(d_{25})$ respectively.

$d_H(S_2P_1) = \{((0.7, 0.2, 0.5), (0.8, 0, 0.7), (0.8, 0, 0.6), (0.8, 0.1, 0.7), (0.1, 0, 0.1), (0.6, 0.2, 0.4))\} = d_{21}$

$w_H(d_{21}) = (3.8, 0.5, 3.0) = a_{21}$

$d_H(S_2P_2) = \{((0.2, 0.2, 0.1), (0.2, 0, 0), (0.6, 0.1, 0), (0.6, 0.4, 0.2), (0.5, 0, 0.6), (0.2, 0, 0))\}\ d_{22}$

$w_H(d_{22}) = (2.3, 0.9, 0.9) = a_{22}$

$d_H(S_2P_3) = \{((0.6, 0.2, 0.1), (0.2, 0.1, 0), (0.6, 0.2, 0.4), (0.7, 0.3, 0.2), (0.5, 0.2, 0.7), (0.3, 0.1, 0))\} = d_{23}$

$w_H(d_{23}) = (2.9, 1, 1.4) = a_{23}$

$d_H(S_2P_4) = \{((0.1, 0.1, 0), (0.4, 0.2, 0.1), (0.4, 0.1, 0.3), (0.5, 0.2, 0.1), (0.3, 0.1, 0), (0.1, 0, 0.1))\} = d_{24}$

$w_H(d_{24}) = (1.8, 0.7, 0.6) = a_{24}$

$d_H(S_2P_5) = \{((0, 0.1, 0), (0.1, 0, 0.1), (0.1, 0.2, 0.1), (0.1, 0.1, 0.1), (0.1, 0.1, 0.1), (0.1, 0.1, 0))\} = d_{25}$

$w_H(d_{25}) = (0.5, 0.6, 0.4) = a_{25}$

We see a_{25} is somewhat the least value of the Hamming weight of the edges between d_{21}, d_{22}, d_{23}, d_{24} and d_{25}. Thus the second student S_2 is again a slow learner.

Next we find the Hamming distances S_3P_i; $1 \leq i \leq 5$ in the following.

$d_H(S_3P_1)$ = {((0.7, 0.2, 0.5), (0.8, 0, 0.7), (0.8, 0, 0.6), (0.8, 0.1, 0.7), (0.1, 0, 0.1), (0.6, 0.2, 0.4))} = d_{31}

$w_H(d_{31})$ = (3.8, 0.5, 3) = a_{31}

$d_H(S_3P_2)$ = {((0, 0.1, 0), (0, 0.1, 0.1), (0, 0.1, 0.1), (0.1, 0, 0), (0, 0.1, 0.1), (0.6, 0.2, 0.4))} = d_{32}

$w_H(d_{32})$ = (0.7, 0.6, 0.7) = a_{32}

$d_H(S_3P_3)$ = {((0, 0.1, 0), (0, 0, 0.1), (0, 0, 0), (0, 0.1, 0), (0, 0.1, 0), (0, 0, 0))} = d_{33}

$w_H(d_{33})$ = (0, 0.3, 0.1) = a_{33}

$d_H(S_3P_4)$ = {((0.1, 0.2, 0.1), (0.3, 0.2, 0.2), (0.2, 0.1, 0.1), (0.2, 0.2, 0.1), (0.2, 0.2, 0.7), (0.2, 0.1, 0.1))} = d_{34}

$w_H(d_{34})$ = (1.2, 1, 1.3) = a_{34}

$d_H(S_3P_5)$ = {((0.2, 0.2, 0.1), (0.3, 0.1, 0.2) (0.7, 0.4, 0.5), (0.8, 0.5, 0.3), (0.6, 0, 0.8), (0.2, 0.2, 0))} = d_{35}

$w_H(d_{35})$ = (2.8, 1.4, 1.9) = a_{35}.

We see the least value is given by a_{33}, which clearly show that the student S_3 is very close to the performance of P_3. P_3 is the very good student so S_3 is a very good student.

Next we try to find the edge weights of d_{41}, d_{42}, d_{43}, d_{44} and d_{45} in the following :

$d(S_4P_1)$ = {((0.5, 0, 0.4), (0.5, 0.2, 0.5), (0.6, 0.1, 0.6), (0.6, 0.1, 0.7), (0.1, 0.3, 0.6), (0.4, 0, 0.5))} = d_{41}

$w_H(d_{41}) = (2.7, 0.7, 3.3) = a_{41}$

$d(S_4P_2) = \{((0.2, 0.1, 0.1), (0.3, 0.1, 0.1), (0.2, 0, 0.1), (0.1, 0.2, 0), (0.2, 0.2, 0.6), (0.1, 0.1, 0.1))\} = d_{42}$

$w_H(d_{42}) = (1.1, 0.7, 1) = a_{42}$

$d(S_4P_3) = \{((0.2, 0.1, 0.1), (0.3, 0.2, 0.1), (0.2, 0.1, 0), (0.2, 0.2, 0), (0.2, 0.4, 0.7), (0.2, 0.2, 0.1))\} = d_{43}$

$w_H(d_{43}) = (1.3, 1.2, 1) = a_{43}$

$d(S_4P_4) = \{((0.1, 0, 0), (0, 0, 0), (0, 0, 0.1), (0, 0, 0.1), (0, 0.1, 0), (0, 0.1, 0))\} = d_{44}$

$w_H(d_{44}) = (0.1, 0.2, 0.2) = a_{44}$

$d(S_4P_5) = \{((0, 0, 0), (0, 0.1, 0), (0.5, 0.3, 0.5), (0.6, 0.3, 0.3), (0.4, 0.3, 0.2), (0, 0, 0.1))\} = d_{45}$

$w_H(d_{45}) = (1.5, 1, 1.1) = a_{45}$.

The student S_4 is very close to P_4 as the value a_{44} is (0.1, 0.2, 0.2). So the student S_4 is a good student.

Now we proceed onto work for the edges and Hamming weights of the edges d_{5j}; $1 \leq j \leq 5$, in the following.

$d_H(S_5P_1) = \{((0.1, 0, 0), (0, 0.1, 0.1), (0.2, 0, 0.1), (0.1, 0.2, 0), (0.2, 0.2, 0.6), (0.1, 0.1, 0.1))\} = d_{51}$

$w_H(d_{51}) = (0.7, 0.6, 0.9) = a_{51}$

$d_H(S_5P_2) = \{((0.6, 0.1, 0.5), (0.8, 0, 0.5), (0.7, 0, 0.5), (0.7, 0.1, 0.7), (0, 0, 0.1), (0.5, 0.1, 0.4))\} = d_{52}$

$w_H(d_{52}) = (3.3, 0.3, 2.7) = a_{52}$

$d_H(S_5P_3) = \{((0.6, 0.1, 0.5), (0.8, 0.1, 0.5), (0.7, 0.1, 0.6), (0.8, 0, 0.7), (0, 0.2, 0.2), (0.6, 0.2, 0.4))\} = d_{53}$

$w_H(d_{53}) = (3.5, 0.7, 2.9) = a_{53}$

$d_H(S_5P_4) = \{((0.5, 0, 0.4), (0.5, 0.1, 0.4), (0.5, 0, 0.5), (0.6, 0.1, 0.6), (0.2, 0.1, 0.5), (0.4, 0.1, 0.5))\} = d_{54}$

$w_H(d_{54}) = (2.7, 0.4, 2.9) = a_{54}$

$d_H(S_5P_5) = \{((0.4, 0, 0.4), (0.5, 0, 0.4), (0.1, 0.1, 0), (0, 0, 0), (0.6, 0.1, 0.6), (0.4, 0, 0.4))\} = d_{55}$

$w_H(d_{55}) = (2, 0.2, 1.8) = a_{55}$

Clearly the least weight is given by a_{51} which implies the student S_5 is a slow learner.

Next, we the Hamming distances d_{61}, d_{62}, d_{63}, d_{64} and d_{65} and their respective Hamming weights for the student S_6.

$d_H(S_6P_1) = \{((0, 0.1, 0), (0, 0.1, 0), (0, 0, 0.1), (0, 0, 0), (0.1, 0, 0), (0, 0, 0.1))\} = d_{61}$

$w_H(d_{61}) = (0.1, 0.2, 0.2) = a_{61}$

$d_H (S_6P_2) = \{((0.7, 0.2, 0.5), (0.8, 0, 0.6), (0.8, 0.1, 0.6), (0.7, 0.1, 0.7), (0.1, 0, 0), (0, 0, 0.1))\} = d_{62}$

$w_H(d_{62}) = (3.1, 0.4, 2.5) = a_{62}$

$d_H(S_6P_3) = \{((0.7, 0.2, 0.5), (0.8, 0, 0.6), (0.8, 0.1, 0.6), (0, 0, 0), (0.1, 0, 0), (0, 0, 0.1))\} = d_{63}$

$w_H(d_{63}) = (2, 3, 1.8) = a_{63}$

$d_H(S_6P_4) = \{((0.6, 0.1, 0.4), (0.5, 0.1, 0.5), (0.6, 0.1, 0.6), (0.6, 0.1, 0.6), (0.2, 0.3, 0.6), (0.4, 0.1, 0.5))\} = d_{64}$

$w_H(d_{64}) = (2.9, 0.8, 3.2) = a_{64}$

$d_H(S_6P_5) = \{((0.5, 0.1, 0.4), (0.5, 0, 0.5), (0.1, 0.4, 0.6), (0, 0.4, 0.4), (0, 0.4, 0.4), (0.6, 0, 0.7), (0.4, 0, 0.5))\}$

$w_H(d_{65}) = (2.1, 1.3, 3.5) = a_{65}$

We see the least weight in this case is a_{61} that is the student S_6 performance is close to P_1 which implies the student S_6 is a slow learner. This is the way we construct plithogenic models. So, the models exploit only the structure of the graph and the operations we perform on the edges in all the 3 types of plithogenic sets.

Now we proceed on to pose some problems.

Problems

1. Find all special features associated with Plithogenic fuzzy sets.
2. Give an example of a Plithogenic fuzzy graph which is complete.
3. Construct using the appropriate Plithogenic fuzzy graph in the study of a real-world problem.
4. Show that the Plithogenic fuzzy graph mentioned in problem 3 can be used to model the problem.
5. Describe the special features associated with Plithogenic fuzzy models.

6. What are Plithogenic fuzzy intuitionistic sets? How are they different from the usual Plithogenic fuzzy sets?
7. Give an example of a Plithogenic bipartite fuzzy intuitionistic graph.
8. Prove by using Plithogenic fuzzy intuitionistic models in the place of Plithogenic fuzzy models one can obtain a better solution!

   a) Justify your claim by giving a real-world problem.

   b) Compare the results got by using Plithogenic fuzzy model with the Plithogenic fuzzy intuitionistic model.
9. Enumerate all the special features associated with Plithogenic single valued neutrosophic sets.
10. Give an example of a complete bipartite Plithogenic SVNS graph.
11. Distinguish between the plithogenic SVNS models and plithogenic fuzzy intuitionistic models.
12. Distinguish the Plithogenic fuzzy models from the plithogenic SVNS models.
13. For a real-world problem use

    i) Plithogenic fuzzy model.

    ii) Plithogenic Fuzzy intuitionistic model.

    iii) Plithogenic SVNS model and compare the results.

    Which model in your opinion is best suited for the problem you have investigated.

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