

## Refined Neutrosophic Crisp Set (RNCS)

Salama and Smarandache defined in 2014 and 2015 [1] the Neutrosophic Crisp Set as follows.

### Definition of Neutrosophic Crisp Set (NCS)

Let  $X$  be a non-empty fixed space. A neutrosophic crisp set is an object  $D$  having the form

$D = \langle A, B, C \rangle$ , where  $A, B, C$  are subsets of  $X$ .

### Types of Neutrosophic Crisp Sets

The object having the form  $D = \langle A, B, C \rangle$  is called:

(a) A neutrosophic crisp set of Type 1 (NCS-Type1) if it satisfies:

$$A \cap B = B \cap C = C \cap A = \emptyset \text{ (empty set).}$$

(b) A neutrosophic crisp set of Type 2 (NCS-Type2) if it satisfies:

$$A \cap B = B \cap C = C \cap A = \emptyset \text{ and } A \cup B \cup C = X.$$

(c) A neutrosophic crisp set of Type 3 (NCS-Type3) if it satisfies:

$$A \cap B \cap C = \emptyset \text{ and } A \cup B \cup C = X.$$

Of course, more types of Neutrosophic Crisp Sets may be defined by modifying the intersections and unions of the subsets  $A, B$ , and  $C$ .

### Refined Neutrosophic Crisp Set

In 2019, Smarandache extended for the first time the Neutrosophic Crisp Set to Refined Neutrosophic Crisp Set, based on his 2013 definition of Refined Neutrosophic Set / Logic / Probability [2], i.e. the truth  $T$  was refined/split into sub-truths such as  $T_1, T_2, \dots, T_p$ , similarly indeterminacy  $I$  was refined/split into sub-indeterminacies  $I_1, I_2, \dots, I_r$ , and the falsehood  $F$  was refined/split into sub-falsehoods  $F_1, F_2, \dots, F_s$ .

#### Definition of Refined Neutrosophic Crisp Set (RNCS)

Let  $X$  be a non-empty fixed space. And let  $D$  be a Neutrosophic Crisp Set, where

$D = \langle A, B, C \rangle$ , with  $A, B, C$  as subsets of  $X$ .

We refined/split  $D$  (and denote it by  $RD = \text{Refined } D$ ) by refining/splitting  $A, B, C$  into sub-subsets as follows:

$RD = (A_1, \dots, A_p; B_1, \dots, B_r; C_1, \dots, C_s)$ , where  $p, r, s \geq 1$  are positive integers,

and

$$A = \bigcup_{i=1}^p A_i, \quad B = \bigcup_{j=1}^r B_j, \quad C = \bigcup_{k=1}^s C_k.$$

#### Types of Refined Neutrosophic Crisp Set

Similarly, we have:

- (a) A refined neutrosophic crisp set of Type 1 (RNCS-Type1) if it satisfies:  
 $A \cap B = B \cap C = C \cap A = \emptyset$  (empty set).
- (b) A refined neutrosophic crisp set of Type 2 (RNCS-Type2) if it satisfies:  
 $A \cap B = B \cap C = C \cap A = \emptyset$  and  $A \cup B \cup C = X$ .

- (c) A refined neutrosophic crisp set of Type 3 (RNCS-Type3) if it satisfies:  
 $A \cap B \cap C = \emptyset$  and  $A \cup B \cup C = X$ .

And, of course, more types of Refined Neutrosophic Crisp Sets may be defined: by modifying the intersections and unions of the subsets  $A, B, C$ , or the intersections and unions of their sub-subsets  $A_i, B_j, C_k$ , for  $i \in \{1, 2, \dots, p\}$ ,  $j \in \{1, 2, \dots, r\}$ , and  $k \in \{1, 2, \dots, s\}$ .

### References

- [1] A.A. Salama, F. Smarandache, *Neutrosophic Crisp Set Theory*, Educational Publisher, Columbus, Ohio, USA, 2015;  
<http://fs.unm.edu/NeutrosophicCrispSetTheory.pdf>
- [2] Florentin Smarandache, *n-Valued Refined Neutrosophic Logic and Its Applications in Physics*, Progress in Physics, 143-146, Vol. 4, 2013;  
<https://arxiv.org/ftp/arxiv/papers/1407/1407.1041.pdf>  
and  
<http://fs.unm.edu/n-ValuedNeutrosophicLogic-PiP.pdf>

This section, called "Refined Neutrosophic Crisp Set (RNCS)" <http://fs.unm.edu/RefinedNeutrosophicCrispSet.pdf> pp. 114-116, is extracted from the book:

Florentin Smarandache, *Nidus Idearum*, Vol. VII, third edition, 2019, Editions Pons, Brussels, Belgium;  
<http://fs.unm.edu/NidusIdearum7-ed3.pdf> .