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Rough Neutrosophic Multi-Attribute Decision-Making Based on Rough Accuracy Score Function

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Abstract. This paper presents multi-attribute decision making based on rough accuracy score function with rough neutrosophic attribute values. While the concept of neutrosophic sets is a powerful logic to handle indeterminate and inconsistent information, the theory of rough neutrosophic sets is also a powerful mathematical tool to deal with incompleteness. The rating of all alternatives is expressed with the upper and lower approximation operator and the pair of neutrosophic sets which are characterized by truth-membership degree, indeterminacy-membership degree, and falsity-membership degree. Weight of each attribute is partially known to decision maker. We introduce a multi attribute decision making method in rough neutrosophic environment based on rough accuracy score function. Information entropy method is used to obtain the unknown attribute weights. Rough accuracy score function is defined to determine rough accuracy score values. Then weighted rough accuracy score value is defined to determine the ranking order of all alternatives. Finally, a numerical example is provided to illustrate the applicability and effectiveness of the proposed approach.

Keywords: Neutrosophic set, Rough neutrosophic set, Single-valued neutrosophic set, Grey relational analysis, Information Entropy, Multi-attribute decision making.

Introduction

The concept of rough neutrosophic set is very recently proposed by Broumi et al. [1], [2]. It seems to be very interesting and applicable in realistic problems. It is a new hybrid intelligent structure. The concept of rough set was proposed by Pawlak [3] in 1982 and the concept of neutrosophic set was proposed by Smarandache [4], [5] in 1998. Wang et al. [6] introduced single valued netrosophic sets in 2010. Neutrosophic sets and rough sets are both capable of dealing with uncertainty and incomplete information. The theory of neutrosophic set has achieved success in various areas of research such as medical diagnosis [7], educational problems [8], [9], social problems [10], [11], conflict resolution [12], [13], image processing [14], [15], [16], decision making [17], [18], [19], [20], [21], [22], etc. On the other hand, rough set theory has been successfully applied in the different fields such as artificial intelligence [23], pattern recognition [24], [25], medical diagnosis [26], [27], [28], data mining [29], [30], [31], image processing [32], conflict analysis [33], decision support systems [34], [35], intelligent control [36], etc. It appears that the computational techniques based on any one of neutrosophic sets or rough sets alone will not always offer the best results but a fusion of two or more can often offer better results [2].

Rough neutrosophic set is the generalization of rough fuzzy sets [37], [38] and rough intuitionistic fuzzy sets [39]. Mondal and Pramanik [40] applied the concept of rough neutrosophic set in multi-attribte decision making based on grey relational analysis. Mondal and Pramanik [41] also studied cosine similarity measure of rough neutrosophic sets and its application in medical diagnosis. Literature review reflects that no studies have been made on multi-attribute decision making using rough neutrosophic score function.

In this paper, we develop rough neutrosophic multi-attribute decision making (MADM) based on rough accuracy score function (RASF).

Rest of the paper is organized in the following way. Section 2 presents preliminaries of neutrosophic sets and rough neutrosophic sets. Section 3 is devoted to present multi attribute decision-making method based on rough accuracy score function. Section 4 presents a numerical example of the proposed method. Finally section 5 presents concluding remarks.

2 Mathematical Preliminaries

2.1 Definitions on neutrosophic Set:

The concept of neutrosophy set [4] is derived from the new branch of philosophy, namely, neutrosophy [5]. Neutrosophy succeeds in creating different fields of studies because of its capability to deal with the origin, nature, and scope of neutralities, as well as their interactions with different ideational spectra.

Definition 2.1.1

Let *G* be a space of points (objects) with generic element in *E* denoted by *y*. Then a neutrosophic set *N1* in *G* is characterized by a truth membership function T_{N1} , an indeterminacy membership function I_{N1} and a falsity membership function F_{N1}. The functions T_{N1}, I_{N1} and F_{N1} are real standard or non-standard subsets of $\left]^{-}0, 1^{+}\right[$ that is T_{N1}: $G \to \left]^{-}0, 1^{+}\right[$; I_{N1}: $G \to \left]^{-}0, 1^{+}\right[$; F_{N1}: $G \to \left]^{-}0, 1^{+}\right[$.

The sum of $T_{N1}(y),\ I_{N1}(y),\ F_{N1}(y)$ is given by

$$^{-}0 \le \sup T_{N1}(y) + \sup I_{N1}(y) + \sup F_{N1}(y) \le 3^{+}$$

Definition 2.1.2 The complement of a neutrosophic set [5] *A* is denoted by $N1^{c}$ and is defined as follows: $T_{N1^c}(y) = \{1^{+}\} - T_{N1}(y)$; $I_{N1^c}(y) = \{1^{+}\} - I_{N1}(y)$

$F_{N1^c}(y) = \{1^{+}\} - F_{N1}(y)$

Definition 2.1.3 A neutrosophic set [5] *N1* is contained in the other neutrosophic set *N2*, $N1 \subseteq N2$ if and only if the following results hold.

$$\inf T_{N1}(y) \le \inf T_{N2}(y),\ \sup T_{N1}(y) \le \sup T_{N2}(y)$$
$$\inf I_{N1}(y) \ge \inf I_{N2}(y),\ \sup I_{N1}(y) \ge \sup I_{N2}(y)$$
$$\inf F_{N1}(y) \ge \inf F_{N2}(y),\ \sup F_{N1}(y) \ge \sup F_{N2}(y)$$

for all *y* in *G*.

Definition 2.1.4 Let *G* be a universal space of points (objects) with a generic element of *G* denoted by *y*.

A single valued neutrosophic set [6] *S* is characterized by a truth membership function $T_N(y)$, a falsity membership function $F_N(y)$ and indeterminacy function $I_N(y)$ with $T_N(y),\ F_N(y),\ I_N(y) \in [0, 1]$ for all *y* in *G*.

When *G* is continuous, a SNVS *S* can be written as follows:

$$S = \int_y \langle T_S(y), F_S(y), I_S(y) \rangle / y, \quad \forall y \in G$$

and when *G* is discrete, a SVNS *S* can be written as follows:

$$S = \sum \langle T_S(y), F_S(y), I_S(y) \rangle / y, \quad \forall y \in G$$

It should be observed that for a SVNS *S*,

$$0 \le \sup T_S(y) + \sup F_S(y) + \sup I_S(y) \le 3, \quad \forall y \in G$$

Definition 2.1.5 The complement of a single valued neutrosophic set [6] *S* is denoted by S^{c} and is defined as follows:

$$T_S^{\,c}(y) = F_S(y)\ ;\ I_S^{\,c}(y) = 1 - I_S(y)\ ;\ F_S^{\,c}(y) = T_S(y)$$

Definition 2.1.6 A SVNS [6] S_{N1} is contained in the other SVNS S_{N2} denoted by $S_{N1} \subseteq S_{N2}$, iff $T_{S_{N1}}(y) \le T_{S_{N2}}(y)$; $I_{S_{N1}}(y) \ge I_{S_{N2}}(y)$; $F_{S_{N1}}(y) \ge F_{S_{N2}}(y)$, $\forall y \in G$.

Definition 2.1.7 Two single valued neutrosophic sets [6] S_{N1} and S_{N2} are equal, i.e. $S_{N1} = S_{N2}$, iff $S_{N1} \subseteq S_{N2}$ and $S_{N1} \supseteq S_{N2}$

Definition 2.1.8 The union of two SVNSs [6] S_{N1} and S_{N2} is a SVNS S_{N3} , written as $S_{N3} = S_{N1} \cup S_{N2}$.

Its truth membership, indeterminacy-membership and falsity membership functions are related to S_{N1} and S_{N2} by the following equations

$$T_{S_{N3}}(y) = \max\left(T_{S_{N1}}(y), T_{S_{N2}}(y)\right);$$
$$I_{S_{N3}}(y) = \max\left(I_{S_{N1}}(y), I_{S_{N2}}(y)\right);$$
$$F_{S_{N3}}(y) = \min\left(F_{S_{N1}}(y), F_{S_{N2}}(y)\right) \text{ for all } y \text{ in } G$$

Definition 2.1.9 The intersection of two SVNSs [6] *N1* and *N2* is a SVNS *N3*, written as $N3 = N1 \cap N2$. Its truth membership, indeterminacy membership and falsity membership functions are related to *N1* an *N2* by the following equations:

$$T_{S_{N3}}(y) = \min\left(T_{S_{N1}}(y), T_{S_{N2}}(y)\right);$$
$$I_{S_{N3}}(y) = \max\left(I_{S_{N1}}(y), I_{S_{N2}}(y)\right);$$
$$F_{S_{N3}}(y) = \max\left(F_{S_{N1}}(y), F_{S_{N2}}(y)\right), \forall y \in G$$

Definition 2.1.1.10 The general SVNS can be presented in the following form as follows:

$$S = \{(y/(T_S(y), I_S(y), F_S(y))) : y \in G\}$$

Finite SVNSs can be represented as follows:

$$S = \left\{ \begin{matrix} (y_1/(T_S(y_1), I_S(y_1), F_S(y_1))), \cdots, \\ (y_m/(T_S(y_m), I_S(y_m), F_S(y_m))) \end{matrix} \right\}, \forall y \in G \quad (1)$$

Let

$$S_{N1} = \left\{ \begin{matrix} (y_1/(T_{S\,N1}(y_1),\ I_{S\,N1}(y_1),\ F_{S\,N1}(y_1))), \cdots, \\ (y_n/(T_{S\,N1}(y_n),\ I_{S\,N1}(y_n),\ F_{S\,N1}(y_n))) \end{matrix} \right\} \quad (2)$$

$$S_{N2} = \left\{ \begin{matrix} (x_1/(T_{S\,N2}(y_1),\ I_{S\,N2}(y_1),\ F_{S\,N2}(y_1))), \cdots, \\ (x_n/(T_{S\,N2}(y_n),\ I_{S\,N2}(y_n),\ F_{S\,N2}(y_n))) \end{matrix} \right\} \quad (3)$$

be two single-valued neutrosophic sets, then the Hamming distance [42] between two SNVS *N1*and *N2* is defined as follows:

$$d_S(S_{N1}, S_{N2}) = \sum_{i=1}^{n} \left\langle \begin{matrix} \left| T_{S_{N1}}(y) - T_{S_{N2}}(y) \right| + \\ \left| I_{S_{N1}}(y) - I_{S_{N2}}(y) \right| + \\ \left| F_{S_{N1}}(y) - F_{S_{N2}}(y) \right| \end{matrix} \right\rangle \quad (4)$$

and normalized Hamming distance [42] between two SNVSs S_{N1} and S_{N2} is defined as follows:

$$^{N}d_S\left(S_{N1},S_{N2}\right)=\frac{1}{3n}\sum_{i=1}^{n}\left\langle\begin{array}{l}\left|T_{S_{N1}}(y)-T_{S_{N2}}(y)\right|+\\\left|I_{S_{N1}}(y)-I_{S_{N2}}(y)\right|+\\\left|F_{S_{N1}}(y)-F_{S_{N2}}(y)\right|\end{array}\right\rangle \quad (5)$$

with the following properties

1. $0\le d_S\left(S_{N1},S_{N2}\right)\le 3n$ (6)
2. $0\le {}^{N}d_S\left(S_{N1},S_{N2}\right)\le 1$ (7)

2.2 Definitions on rough neutrosophic set

Definition 2.2.1

Let Z be a non-null set and R be an equivalence relation on Z. Let P be neutrosophic set in Z with the membership function T_P, indeterminacy function I_P and non-membership function F_P. The lower and the upper approximations of P in the approximation (Z, R) denoted by $\underline{N}(P)$ and $\overline{N}(P)$ are respectively defined as follows:

$$\underline{N}(P)=\left\langle\begin{array}{l}<x,T_{\underline{N}(P)}(x),I_{\underline{N}(P)}(x),\ F_{\underline{N}(P)}(x)>/\\ z\in[x]_R,x\in Z\end{array}\right\rangle, \quad (8)$$

$$\overline{N}(P)=\left\langle\begin{array}{l}<x,T_{\overline{N}(P)}(x),I_{\overline{N}(P)}(x),\ F_{\overline{N}(P)}(x)>/\\ z\in[x]_R,x\in Z\end{array}\right\rangle \quad (9)$$

Where, $T_{\underline{N}(P)}(x)=\wedge_z\in[x]_R\,T_P(z)$,

$I_{\underline{N}(P)}(x)=\wedge_z\in[x]_R\,I_P(z)$, $F_{\underline{N}(P)}(x)=\wedge_z\in[x]_R\,F_P(z)$,

$T_{\overline{N}(P)}(x)=\vee_z\in[x]_R\,T_P(z)$, $I_{\overline{N}(P)}(x)=\vee_z\in[x]_R\,T_P(z)$,

$F_{\overline{N}(P)}(x)=\vee_z\in[x]_R\,I_P(z)$

So, $0\le\sup T_{\underline{N}(P)}(x)+\sup I_{\underline{N}(P)}(x)+\sup F_{\underline{N}(P)}(x)\le 3$

$0\le\sup T_{\overline{N}(P)}(x)+\sup I_{\overline{N}(P)}(x)+\sup F_{\overline{N}(P)}(x)\le 3$

Here $\vee$ and $\wedge$ denote "max" and "min'' operators respectively, $T_P(z)$, $I_P(z)$ and $F_P(z)$ are the membership, indeterminacy and non-membership function of z with respect to P. It is easy to see that $\underline{N}(P)$ and $\overline{N}(P)$ are two neutrosophic sets in Z.

Thus NS mapping $\underline{N}$, $\overline{N}$: $N(Z)\rightarrow N(Z)$ are, respectively, referred to as the lower and upper rough NS approximation operators, and the pair $(\underline{N}(P),\overline{N}(P))$ is called the rough neutrosophic set [1], [2] in (Z, R).

From the above definition, it is seen that $\underline{N}(P)$ and $\overline{N}(P)$ have constant membership on the equivalence clases of R if $\underline{N}(P)=\overline{N}(P)$; .e. $T_{\underline{N}(P)}(x)=T_{\overline{N}(P)}(x)$,

$I_{\underline{N}(P)}(x)=I_{\overline{N}(P)}(x)$, $F_{\underline{N}(P)}(x)=F_{\overline{N}(P)}(x)$

for any x belongs to Z.

P is said to be a definable neutrosophic set in the approximation (Z, R). It can be easily proved that zero neutrosophic set (0_N) and unit neutrosophic sets (1_N) are definable neutrosophic sets.

Definition 2.2.2

If $N(P)=(\underline{N}(P),\overline{N}(P))$ is a rough neutrosophic set in (Z, R) , the rough complement [1], [2] of $N(P)$ is the rough neutrosophic set denoted by $\sim N(P)=(\underline{N}(P)^c,\ \overline{N}(P)^c)$, where $\underline{N}(P)^c$, $\overline{N}(P)^c$ are the complements of neutrosophic sets of $\underline{N}(P),\overline{N}(P)$ respectively.

$$\underline{N}(P)^c=\left\langle\begin{array}{l}<x,T_{\underline{N}(P)}(x),1-I_{\underline{N}(P)}(x),\ F_{\underline{N}(P)}(x)>/\\ ,x\in Z\end{array}\right\rangle, \text{ and}$$

$$\overline{N}(P)^c=\left\langle\begin{array}{l}<x,T_{\underline{N}(P)}(x),1-I_{\overline{N}(P)}(x),\ F_{\overline{N}(P)}(x)>/\\ ,x\in Z\end{array}\right\rangle \quad (10)$$

Definition 2.2.3

If $N(P_1)$ *and* $N(P_2)$ are the two rough neutrosophic sets of the neutrosophic set P respectively in Z, then the following definitions [1], [2] hold good:

$N(P_1)=N(P_2)\Leftrightarrow\underline{N}(P_1)=\underline{N}(P_2)\ \wedge\ \overline{N}(P_1)=\overline{N}(P_2)$

$N(P_1)\subseteq N(P_2)\Leftrightarrow\underline{N}(P_1)\subseteq\underline{N}(P_2)\ \wedge\ \overline{N}(P_1)\subseteq\overline{N}(P_2)$

$N(P_1)\cup N(P_2)=<\underline{N}(P_1)\cup\underline{N}(P_2),\ \overline{N}(P_1)\cup\overline{N}(P_2)>$

$N(P_1)\cap N(P_2)=<\underline{N}(P_1)\cap\underline{N}(P_2),\ \overline{N}(P_1)\cap\overline{N}(P_2)>$

$N(P_1)+N(P_2)=<\underline{N}(P_1)+\underline{N}(P_2),\ \overline{N}(P_1)+\overline{N}(P_2)>$

$N(P_1)\ .N(P_2)=<\underline{N}(P_1)\ .\ \underline{N}(P_2),\ \overline{N}(P_1)\ .\overline{N}(P_2)>$

If N, M, L are the rough neutrosophic sets in (Z, R), then the following propositions are stated from definitions

Proposition 1 [1], [2]

1. $\sim N(\sim N)=N$
2. $N\cup M=M\cup N, M\cup N=N\cup M$
3. $(L\cup M)\cup N=L\cup(M\cup N)$,
   $(L\cap M)\cap N=L\cap(M\cap N)$
4. $(L\cup M)\cap N=(L\cup M)\cap(L\cup N)$,
   $(L\cap M)\cup N=(L\cap M)\cup(L\cap N)$

Proposition 2 [1], [2]

De Morgan's Laws are satisfied for rough neutrosophic sets

1. $\sim(N(P_1)\cup N(P_2))=(\sim N(P_1))\cap(\sim N(P_2))$
2. $\sim(N(P_1)\cap N(P_2))=(\sim N(P_1))\cup(\sim N(P_2))$

Proposition 3[1], [2]

If P_1 and P_2 are two neutrosophic sets in U such that $P_1\subseteq P_2$, *then* $N(P_1)\subseteq N(P_2)$

1. $N(P_1\cap P_2)\subseteq N(P_2)\cap N(P_2)$
2. $N(P_1\cup P_2)\supseteq N(P_2)\cup N(P_2)$

Proposition 4 [1], [2]

1. $\underline{N}(P)=\sim\overline{N}(\sim P)$
2. $\overline{N}(P)=\sim\underline{N}(\sim P)$
3. $\underline{N}(P)\subseteq\overline{N}(P)$

Definition 2.2.4

Let $N_{ij}(P) = \left(\underline{N}_{ij}(P), \overline{N}_{ij}(P)\right)$ is a rough neutrosophic set in (Z, R), where $\underline{N}_{ij}(P) = \left(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}\right)$, $\overline{N}_{ij}(P) = \left(\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij}\right)$ i = 1, 2, ..., m and j = 1, 2, ..., n. We define the rough accuracy score function (RASF) of $N_{ij}(P)$ as follows:

$$S[N_{ij}(P)] = \frac{\left(2 + \left(\frac{\underline{T}_{ij} + \overline{T}_{ij}}{2}\right) - \left(\frac{\underline{I}_{ij} + \overline{I}_{ij}}{2}\right) - \left(\frac{\underline{F}_{ij} + \overline{F}_{ij}}{2}\right)\right)}{3} \quad (11)$$

Proposition 5:

1. For any values of $N_{ij}(P)$, $0 \le S[N_{ij}(P)] \le 1$

Proof: Since both lower and upper approximations are neutrosophic sets, so the proof of the statement is obvious.

2. $S[N_{ij}(P)] = 0$ when $\underline{T}_{ij} = \overline{T}_{ij} = 0$, $\underline{I}_{ij} = \overline{I}_{ij} = \underline{F}_{ij} = \overline{F}_{ij} = 1$

Proof: This proof is obvious.

3. $S[N_{ij}(P)] = 1$ when $\underline{T}_{ij} = \overline{T}_{ij} = 1$, $\underline{I}_{ij} = \overline{I}_{ij} = \underline{F}_{ij} = \overline{F}_{ij} = 0$

4. For any two rough neutrosophic set $N_{ij}(P_1)$ and $N_{ij}(P_2)$, if $N_{ij}(P_1) \subseteq N_{ij}(P_2)$ then $S[N_{ij}(P_1)] \le S[N_{ij}(P_2)]$.

Proof: Since $N_{ij}(P_1) \subseteq N_{ij}(P_2)$ we have

$\overline{T}^{P_1}_{ij} \le \overline{T}^{P_2}_{ij}$, $\underline{T}^{P_1}_{ij} \le \underline{T}^{P_2}_{ij}$, $\overline{I}^{P_1}_{ij} \ge \overline{I}^{P_2}_{ij}$, and

$\underline{I}^{P_1}_{ij} \ge \underline{I}^{P_2}_{ij}$, $\overline{F}^{P_1}_{ij} \ge \overline{F}^{P_2}_{ij}$, $\underline{F}^{P_1}_{ij} \ge \underline{F}^{P_2}_{ij}$.

$\Rightarrow S[N_{ij}(P_1)] - S[N_{ij}(P_2)] \le 0$.

This proves the proposition.

5. For any two rough neutrosophic set $N_{ij}(P_1)$ and $N_{ij}(P_2)$, if $N_{ij}(P_1) = N_{ij}(P_2)$, then $S[N_{ij}(P_1)] = S[N_{ij}(P_2)]$.

Proof: Since $N_{ij}(P_1) = N_{ij}(P_2)$ we have

$\overline{T}^{P_1}_{ij} = \overline{T}^{P_2}_{ij}$, $\underline{T}^{P_1}_{ij} = \underline{T}^{P_2}_{ij}$, $\overline{I}^{P_1}_{ij} = \overline{I}^{P_2}_{ij}$, $\underline{I}^{P_1}_{ij} = \underline{I}^{P_2}_{ij}$, $\overline{F}^{P_1}_{ij} = \overline{F}^{P_2}_{ij}$,

$\underline{F}^{P_1}_{ij} = \underline{F}^{P_2}_{ij}$

$\Rightarrow S[N_{ij}(P_1)] - S[N_{ij}(P_2)] = 0$.

This completes the proof.

Definition 2.2.5: Let $N_{ij}(P_1)$ and $N_{ij}(P_2)$ be two rough neutrosophic sets. Then the ranking method is defined as follows:

If $S[N_{ij}(P_1)] > S[N_{ij}(P_2)]$ then $N_{ij}(P_1) > N_{ij}(P_2)$.

3. Multi-attribute decision making methods based on rough accuracy score function

Consider a multi-attribute decision making problem with m alternatives and n attributes. Let $A_1, A_2, ..., A_m$ and $C_1, C_2, ..., C_n$ denote the alternatives and attributes respectively.

The rating describes the performance of alternative A_i against attribute C_j. For MADM weight vector $W = \{w_1, w_2, ..., w_n\}$ is assigned to the attributes. The weight w_j (j = 1, 2, ..., n) reflects the relative importance of attributes C_j (j = 1, 2, ..., m) to the decision making process. The weights of the attributes are usually determined on subjective basis. They represent the opinion of a single decision maker or accumulate the opinions of a group of experts using a group decision technique. The values associated with the alternatives for MADM problem are presented in the table 1.

Table1: Rough neutrosophic decision matrix

$$D = \left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle_{m \times n} = \begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \langle \underline{d}_{11}, \overline{d}_{11} \rangle & \langle \underline{d}_{12}, \overline{d}_{12} \rangle & \ldots & \langle \underline{d}_{1n}, \overline{d}_{1n} \rangle \\ A_2 & \langle \underline{d}_{21}, \overline{d}_{21} \rangle & \langle \underline{d}_{22}, \overline{d}_{22} \rangle & \ldots & \langle \underline{d}_{2n}, \overline{d}_{2n} \rangle \\ . & \ldots & \ldots & \ldots & \ldots \\ . & \ldots & \ldots & \ldots & \ldots \\ A_m & \langle \underline{d}_{m1}, \overline{d}_{m1} \rangle & \langle \underline{d}_{m2}, \overline{d}_{m2} \rangle & \ldots & \langle \underline{d}_{mn}, \overline{d}_{mn} \rangle \end{array} \quad (12)$$

Here $\left\langle \underline{d}_{ij}, \overline{d}_{ij} \right\rangle$ is the rough neutrosophic number according to the i-th alternative and the j-th attribute.

In real life situation, the decision makers may have personal biases and some indiviguals may give unduly low or unduly high preferences with respect to their preferences. In this case it is necessary to assign very low weights to these baised options. The steps of RASF method under rough neutrosophic environment are described as follows:

Step 1: Construction of the decision matrix with rough neutrosophic form

For multi-attribute decision making problem, the rating of alternative A_i (i = 1, 2,…m) with respect to attribute C_j (j = 1, 2,…n) is assumed as rough neutrosophic set. It can be represented with the following forms:

$$A_i = \left[\begin{array}{l} C_1 \Big/ \left\langle \underline{N}_1(\underline{T}_{i1}\, \underline{I}_{i1}\, \underline{F}_{i1}), \overline{N}_1(\overline{T}_{i1}\, \overline{I}_{i1}\, \overline{F}_{i1}) \right\rangle, \\ C_2 \Big/ \left\langle \underline{N}_2(\underline{T}_{i2}\, \underline{I}_{i2}\, \underline{F}_{i2}), \overline{N}_2(\overline{T}_{i2}\, \overline{I}_{i2}\, \overline{F}_{i2}) \right\rangle, \cdots, \\ C_n \Big/ \left\langle \underline{N}_n(\underline{T}_{in}\, \underline{I}_{in}\, \underline{F}_{in}), \overline{N}_n(\overline{T}_{in}\, \overline{I}_{in}\, \overline{F}_{in}) \right\rangle : C_j \in C \end{array} \right]$$

$$= \left[C_j \Big/ \left\langle \underline{N}_j(\underline{T}_{ij}\, \underline{I}_{ij}\, \underline{F}_{ij}), \overline{N}_j(\overline{T}_{ij}\, \overline{I}_{ij}\, \overline{F}_{ij}) \right\rangle : C_j \in C \right] \text{for} \quad (13)$$

$j = 1, 2, \cdots, n$

Here $\overline{N}$ and $\underline{N}$ are neutrosophic sets, and $\left\langle \overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij} \right\rangle$ *and* $\left\langle \underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij} \right\rangle$ are the degrees of truth membership, degree of indeterminacy and degree of falsity membership of the alternative A_i satisfying the attribute C_j, respectively where

$0 \le \underline{T}_{ij}, \overline{T}_{ij} \le 1,\quad 0 \le \underline{I}_{ij}, \overline{I}_{ij} \le 1,\quad 0 \le \underline{F}_{ij}, \overline{F}_{ij} \le 1,$

$0 \le \underline{T}_{ij} + \underline{I}_{ij} + \underline{F}_{ij} \le 3,\ 0 \le \overline{T}_{ij} + \overline{I}_{ij} + \overline{F}_{ij} \le 3$

The rough neutrosophic decision matrix can be presented in the following form (See the table 2):

Table 2: Rough neutrosophic decision matrix

$$d = \left\langle \underline{N}_{ij}(F), \overline{N}_{ij}(F) \right\rangle_{m\times n} =$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \langle \underline{N}_{11}, \overline{N}_{11} \rangle & \langle \underline{N}_{12}, \overline{N}_{12} \rangle & \cdots & \langle \underline{N}_{1n}, \overline{N}_{1n} \rangle \\ A_2 & \langle \underline{N}_{21}, \overline{N}_{21} \rangle & \langle \underline{N}_{22}, \overline{N}_{22} \rangle & \cdots & \langle \underline{N}_{2n}, \overline{N}_{2n} \rangle \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & \langle \underline{N}_{n1}, \overline{N}_{n1} \rangle & \langle \underline{N}_{n2}, \overline{N}_{n2} \rangle & \cdots & \langle \underline{N}_{mn}, \overline{N}_{mn} \rangle \end{array} \quad (14)$$

Here $\underline{N}_{ij}$ *and* $\overline{N}_{ij}$ are lower and upper approximations of the neutrosophic set *P*.

Step 2: Determination of the rough accuracy score matrix

Let us consider a rough neutrosophic set in the form: $N_{ij}(P) = \left\langle \left(\underline{T}_{ij}, \underline{I}_{ij}, \underline{F}_{ij}\right), \left(\overline{T}_{ij}, \overline{I}_{ij}, \overline{F}_{ij}\right) \right\rangle$

The rough accuracy score matrix is formed by using equation (11) and it is presented in the table 3.

Table3: The rough accuracy score matrix

$$RASF_{m\times n} =$$

$$\begin{array}{c|cccc} & C_1 & C_2 & \cdots & C_n \\ \hline A_1 & \langle S[N_{11}(P)] \rangle & \langle S[N_{12}(P)] \rangle & \cdots & \langle S[N_{1n}(P)] \rangle \\ A_2 & \langle S[N_{21}(P)] \rangle & \langle S[N_{22}(P)] \rangle & \cdots & \langle S[N_{2n}(P)] \rangle \\ . & \cdots & \cdots & \cdots & \cdots \\ . & \cdots & \cdots & \cdots & \cdots \\ A_m & \langle S[N_{m1}(P)] \rangle & \langle S[N_{m2}(P)] \rangle & \cdots & \langle S[N_{mn}(P)] \rangle \end{array} \quad (15)$$

Step 3: Determination of the weights of attribute

During decision-making process, decision makers may enconter unknown attribute weights. In many cases, the importance of the decision makers are not equal. So, it is necessary to determine attribute weight for making a proper decision.

In this paper, we have adopted the entropy method proposed by Majumder and Samanta [42], in rough neutrosophic environment for determining attribute weight as follows.

Let us consider $\left[T_{N(P)}\right]_{ij}(x_i) = \left(\dfrac{\underline{T}_{ij} + \overline{T}_{ij}}{2}\right)$,

$\left[I_{N(P)}\right]_{ij}(x_i) = \left(\dfrac{\underline{I}_{ij} + \overline{I}_{ij}}{2}\right)$, $\left[F_{N(P)}\right]_{ij}(x_i) = \left(\dfrac{\underline{F}_{ij} + \overline{F}_{ij}}{2}\right)$

Now,

$S_N = \left\langle T_{N(P)}(x_i), I_{N(P)}(x_i), F_{N(P)}(x_i) \right\rangle,$

$$E_i(S_N) = 1 - \frac{1}{n} \sum_{i=1}^{m} \left(T_{N(P)}(x_i) + F_{N(p)}(x_i)\right) \left| I_{N(P)}(x_i) - I^c{}_{N(P)}(x_i) \right| \quad (16)$$

which has the following properties:

1. $E_i(S_N) = 0 \Rightarrow S_N$ *is a crisp set and* $I_{S_N}(x_i) = 0\ \forall x \in E.$
2. $E_i(S_N) = 1 \Rightarrow \left\langle T_{N(P)}(x_i), I_{N(P)}(x_i), F_{N(P)}(x_i) \right\rangle = \langle 0.5, 0.5, 0.5 \rangle\ \forall x \in E.$
3. $E_i(S_{N1}) \ge E_i(S_{N2}) \Rightarrow$ $(T_{N1(P)}(x_i) + F_{N1(P)}(x_i) \le (T_{N2(P)}(x_i) + F_{N2(P)}(x_i)$ *and* $\left| I_{N1(P)}(x_i) - I^c{}_{N1(P)}(x_i) \right| \le \left| I_{N2(P)}(x_i) - I^c{}_{N2(P)}(x_i) \right|$
4. $E_i(S_N) = E_i(S_{N^c})\ \forall x \in E.$

In order to obtain the entropy value E_j of the *j*-th attribute C_j (*j* = 1, 2,…, n), equation (16) can be written as:

$$E_j = 1 - \frac{1}{n} \sum_{i=1}^{m} \left(\left[T_{NP}\right]_{ij}(x_i) + \left[F_{NP}\right]_{ij}(x_i)\right). \left| \left[I_{NP}\right]_{ij}(x_i) - \left[I^c{}_{NP}\right]_{ij}(x_i) \right| \quad (17)$$

For *i* = 1, 2, …, n; *j* = 1, 2, …, m

It is observed that $E_j \in [0,1]$. Due to Hwang and Yoon [43], and Wang and Zhang [44], the entropy weight of the *j*-th attibute C_j is presented as follows:

$$w_j = \frac{1 - E_j}{\sum_{j=1}^{n} \left(1 - E_j\right)} \quad (18)$$

We have weight vector W = $(w_1, w_2, \ldots, w_n)^T$ of attributes C_j (*j* = 1, 2, …, n) with $w_j \ge 0$ and $\sum_{i=1}^{n} w_j = 1$

Step 4: Determination of the over all weighted rough accuracy score values of the alternatives

To rank alternatives, we can sum all values in each row of the rough accuracy score matrix corresponding to the attribute weights by the over all weighted rough accuracy score value (*WRASV*) of each alternative A_i (i = 1, 2, ..., n). It is defined as follows:

$$WRASV(A_i) = \sum_{i=1}^{n} w_j \left\langle S[N_{ij}(P)] \right\rangle \quad (19)$$

Step 5: Ranking the alternatives

According to the over all weighted rough accuracy score values *WRASV*(A_i) (i = 1, 2, ..., n), we can rank alternatives A_i (i = 1, 2, ..., n). The highest value of *WRASV*(A_i) (i = 1, 2, ..., n) reflects the best alternative.

4 Numerical example

In this section, rough neutrosophic MADM is considered to demonstrate the applicability and the effectiveness of the proposed approach. Let us consider a decision-making problem stated as follows. A person wants to purchase a SIM card for mobile connection. Now it is necessary to select suitable SIM card for his/her mobile connection. After initial screening there is a panel with three possible alternatives (SIM cards) for mobile connection. The alternatives (SIM cards) are presented as follows:

A_1: Airtel,

A_2: Vodafone and

A_3: BSNL.

The person must take a decision based on the following four attributes of SIM cards:

(1) C_1 is service quality of the corresponding company;

(2) C_2 is the cost and innitial talktime;

(3) C_3 is the call rate per second; and

(4) C_4 is the internet and other facilities.

Step 1: **Construction of the decision matrix with rough neutrosophic form**

We construct the following rough neutrosophic decision matrix (see the table 4)based on the experts' assessment.

Table 4. Decision matrix with rough neutrosophic number

$$d_S=\left\langle \underline{N}(P), \overline{N}(P) \right\rangle_{3\times 4} =$$

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline A_1 & \left\langle (.7,.3,.3), (.8,.2,.2) \right\rangle & \left\langle (.6,.4,.4), (.8,.3,.2) \right\rangle & \left\langle (.6,.3,.3), (.8,.2,.2) \right\rangle & \left\langle (.7,.4,.4), (.8,.2,.2) \right\rangle \\ A_2 & \left\langle (.7,.3,.3), (.8,.1,.2) \right\rangle & \left\langle (.6,.3,.3), (.8,.1,.2) \right\rangle & \left\langle (.7,.2,.2), (.8,.4,.1) \right\rangle & \left\langle (.7,.3,.3), (.8,.3,.3) \right\rangle \\ A_3 & \left\langle (.7,.2,.2), (.8,.1,.1) \right\rangle & \left\langle (.7,.3,.3), (.8,.1,.1) \right\rangle & \left\langle (.7,.2,.2), (.9,.2,.2) \right\rangle & \left\langle (.8,.3,.2), (.9,.1,.1) \right\rangle \end{array} \quad (23)$$

The selection process using proposed approach is done based on the following steps:

Step 2: Calculation of the rough accuracy score matrix

Using the rough accuracy score function of $N_{ij}(P)$ from equation (11), the rough accuracy score matrix is presented in the table 5.

Step 3: Determination of the weights of attribute

Rough entropy value E_j of the j-th (j = 1, 2, 3) attributes can be determined from the decision matrix d_S (23) and equation (17) as: E_1= 0.4233, E_2 = 0.5200, E_3 = 0.5150, E_4 = 0.5200.

Table 5. Rough accuracy score matrix

$$\begin{array}{c|cccc} & C_1 & C_2 & C_3 & C_4 \\ \hline A_1 & \langle 0.7500 \rangle & \langle 0.6833 \rangle & \langle 0.7333 \rangle & \langle 0.7167 \rangle \\ A_2 & \langle 0.7667 \rangle & \langle 0.7500 \rangle & \langle 0.7667 \rangle & \langle 0.7333 \rangle \\ A_3 & \langle 0.8167 \rangle & \langle 0.7833 \rangle & \langle 0.8000 \rangle & \langle 0.8333 \rangle \end{array} \quad (24)$$

Then the corresponding rough entropy weights w_1, w_2, w_3, w_4 of all attributes according to equation (18) are obtained as follows: w_1 = 0.2853, w_2 = 0.2374, w_3 = 0.2399, w_4 = 0.2374 such that $\sum_{j=1}^{n} w_j = 1$.

Step 4: Determination of the over all weighted rough accuracy score values of the alternatives

Using equation (19), the over all weighted rough accuracy score value (*WRASV*) of each alternative A_i (i = 1, 2, 3) is presented as follows:

$WRASV(A_1)$ = 0.72225, $WRASV(A_2)$ = 0.754806,

$WRASV(A_3)$ = 0.808705.

Step 5: Ranking the alternatives.

According to the over all weighted rough accuracy score values $WRASV(A_i)$ (i = 1, 2, 3), we can rank alternatives A_i (i = 1, 2, 3) as follows:

$WRASV(A_3) > WRASV(A_2) > WRASV(A_1)$

Therefore A_3 (BSNL) is the best SIM card.

Conclusion

In this paper, we have defined rough accuracy score function and studied some of it's properties. Entropy based weighted rough accuracy score value is proposed. We have introduced rough neutrosophic multi-attribute decision-making problem with incompletely known or completely unknown attribute weight information based on rough accuracy score function. Finally, an illustrative example is provided to show the effectiveness of the proposed approach.

However, we hope that the concept presented here will open new avenue of research in current rough neutrosophic decision-making arena. In future the proposed approach can be used for other practical MADM problems in hybrid environment.

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