S-DENYING A THEORY

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Abstract.

In this paper we introduce the operators of *validation* and *invalidation* of a proposition, and we extend the operator of *S-denying* a proposition, or an axiomatic system, from the geometric space to respectively any theory in any domain of knowledge, and show six examples in geometry, in mathematical analysis, and in topology.

1. Definitions.

Let $\mathcal{T}$ be a theory in any domain of knowledge, endowed with an ensemble of sentences $\mathcal{E}$, on a given space $\mathcal{M}$.

$\mathcal{E}$ can be for example an axiomatic system of this theory, or a set of primary propositions of this theory, or all valid logical formulas of this theory, etc. $\mathcal{E}$ should be closed under the logical implications, i.e. given any subset of propositions $\mathcal{P}_1$, $\mathcal{P}_2$, … in this theory, if $\mathcal{Q}$ is a logical consequence of them then $\mathcal{Q}$ must also belong to this theory.

A sentence is a logic formula whose each variable is quantified {i.e. inside the scope of a quantifier such as: $\exists$ (exist), $\forall$ (for all), modal logic quantifiers, and other various modern logics' quantifiers}.

With respect to this theory, let $\mathcal{P}$ be a proposition, or a sentence, or an axiom, or a theorem, or a lemma, or a logical formula, or a statement, etc. of $\mathcal{E}$.

It is said that $\mathcal{P}$ is *S-denied*[1] on the space $\mathcal{M}$ if $\mathcal{P}$ is valid for some elements of $\mathcal{M}$ and invalid for other elements of $\mathcal{M}$, or $\mathcal{P}$ is only invalid on $\mathcal{M}$ but in at least two different ways.

[1] The multispace operator S-denied (*Smarandachely-denied*) has been inherited from the previously published scientific literature (see for example Ref. [1] and [2]).

An ensemble of sentences $\mathcal{E}$ is considered *S-denied* if at least one of its propositions is *S-denied*.

And a theory $\mathcal{T}$ is *S-denied* if its ensemble of sentences is *S-denied*, which is equivalent to at least one of its propositions being *S-denied*.

The proposition $\mathcal{P}$ is partially or totally denied/negated on $\mathcal{M}$. The proposition $\mathcal{P}$ can be simultaneously validated in one way and invalidated in (finitely or infinitely) many different ways on the same space $\mathcal{M}$, or only invalidated in (finitely or infinitely) many different ways.

The invalidation can be done in many different ways.

For example the statement $\mathcal{A}$ = *"$x \neq 5$"* can be invalidated as *"$x=5$"* (total negation), but "$x \in \{5, 6\}$" (partial negation).

(Use a notation for *S-denying*, for invalidating in a way, for invalidating in another way a different notation; consider it as an operator: neutrosophic operator? A notation for invalidation as well.)

But the statement $\mathcal{B}$ = *"$x > 3$"* can be invalidated in many ways, such as *"$x \leq 3$"*, or *"$x = 3$"*, or *"$x < 3$"*, or *"$x = -7$"*, or *"$x = 2$"*, etc. A negation is an invalidation, but not reciprocally – since an invalidation signifies a (partial or total) degree of negation, so invalidation may not necessarily be a complete negation. The negation of $\mathcal{B}$ is $\neg\mathcal{B}$ = *"$x \leq 3$"*, while *"$x = -7$"* is a partial negation (therefore an invalidation) of $\mathcal{B}$.

Also, the statement $\mathcal{C}$ = "John's car is blue and Steve's car is red" can be invalidated in many ways, as: "John's car is yellow and Steve's car is red", or "John's car is blue and Steve's car is black", or "John's car is white and Steve's car is orange", or "John's car is not blue and Steve's car is not red", or "John's car is not blue and Steve's car is red", etc.

Therefore, we can *S-deny* a theory in finitely or infinitely many ways, giving birth to many partially or totally denied versions/deviations/alternatives theories: $\mathcal{T}1$, $\mathcal{T}2$, … . These new theories represent degrees of negations of the original theory $\mathcal{T}$.

Some of them could be useful in future development of sciences.

Why do we study such *S-denying* operator? Because our reality is heterogeneous, composed of a multitude of spaces, each space with different structures. Therefore, in one space a statement may be valid, in another space it may be invalid, and invalidation can be done in various ways.

Or a proposition may be false in one space and true in another space or we may have a degree of truth and a degree of falsehood and a degree of indeterminacy. Yet, we live in this mosaic of distinct (even opposite structured) spaces put together.

S-denying involved the creation of the multi-space in geometry and of the *S-geometries* (1969).

It was spelt *multi-space*, or *multispace*, of *S-multispace*, or *mu-space*, and similarly for its: *multi-structure*, or *multistructure*, or *S-multistructure*, or *mu-structure.*

2. **Notations.**

Let $<A>$ be a statement (or proposition, axiom, theorem, etc.).

a) For the classical Boolean logic *negation* we use the same notation. The negation of $<A>$ is noted by $\neg A$ and $\neg A = <nonA>$.

An *invalidation* of $<A>$ is noted by $i(A)$, while a *validation* of $<A>$ is noted by $v(A)$:

$$i(A) \subseteq 2^{<nonA>} \setminus \{\emptyset\} \text{ and } v(A) \subseteq 2^{<A>} \setminus \{\emptyset\}$$

where 2^X means the power-set of X, or all subsets of X.

All possible invalidations of $<A>$ form a set of invalidations, notated by $I(A)$. Similarly for all possible validations of $<A>$ that form a set of validations, and noted by $V(A)$.

b) *S-denying* of $<A>$ is noted by $S_{\neg}(A)$. S-denying of $<A>$ means some validations of $<A>$ together with some invalidations of $<A>$ in the same space, or only invalidations of <A> in the same space but in many ways.

Therefore, $S_{\neg}(A) \subseteq V(A) \cup I(A)$ or $S_{\neg}(A) \subseteq I(A)^k$, for $k \geq 2$.

3. **Examples.** Let's see some models of *S-denying*, three in a geometrical space, and other three in mathematical analysis (calculus) and topology.

3.1. The first *S-denying* model was constructed in 1969. This section is a compilation of ideas from paper [1].

An axiom is said *Smarandachely denied* if the axiom behaves in at least two different ways within the same space (i.e., validated and invalided, or only invalidated but in multiple distinct ways).

A *Smarandache Geometry* [SG] is a geometry which has at least one Smarandachely denied axiom.

Let's note any point, line, plane, space, triangle, etc. in such geometry by s-point, s-line, s-plane, s-space, s-triangle respectively in order to distinguish them from other geometries.

Why these hybrid geometries? Because in reality there does not exist isolated homogeneous spaces, but a mixture of them, interconnected, and each having a different structure.

These geometries are becoming very important now since they combine many spaces into one, because our world is not formed by perfect homogeneous spaces as in pure mathematics, but by non-homogeneous spaces. Also, SG introduce the degree of negation in geometry for the first time [for example an axiom is denied 40% and accepted 60% of the space] that's why they can become revolutionary in science and it thanks to the idea of partial denying/accepting of axioms/propositions in a space (making multi-spaces, i.e. a space formed by combination of many different other spaces), as in fuzzy logic the degree of truth (40% false and 60% true).

They are starting to have applications in physics and engineering because of dealing with non-homogeneous spaces.

<u>The first model of S-denying and of SG was the following:</u>

The axiom that through a point exterior to a given line there is only one parallel passing through it [Euclid's Fifth Postulate], was S-denied by having in the same space: no parallel, one parallel only, and many parallels.

In the Euclidean geometry, also called parabolic geometry, the fifth Euclidean postulate that there is only one parallel to a given line passing through an exterior point, is kept or validated.

In the Lobachevsky-Bolyai-Gauss geometry, called hyperbolic geometry, this fifth Euclidean postulate is invalidated in the following way: there are infinitely many lines parallels to a given line passing through an exterior point.

While in the Riemannian geometry, called elliptic geometry, the fifth Euclidean postulate is also invalidated as follows: there is no parallel to a given line passing through an exterior point.

Thus, as a particular case, Euclidean, Lobachevsky-Bolyai-Gauss, and Riemannian geometries may be united altogether, in the same space, by some SG's. These last geometries can be partially Euclidean and partially Non-Euclidean simultaneously.

3.2. **Geometric Model (particular case of SG).**

Suppose we have a rectangle *ABCD*.

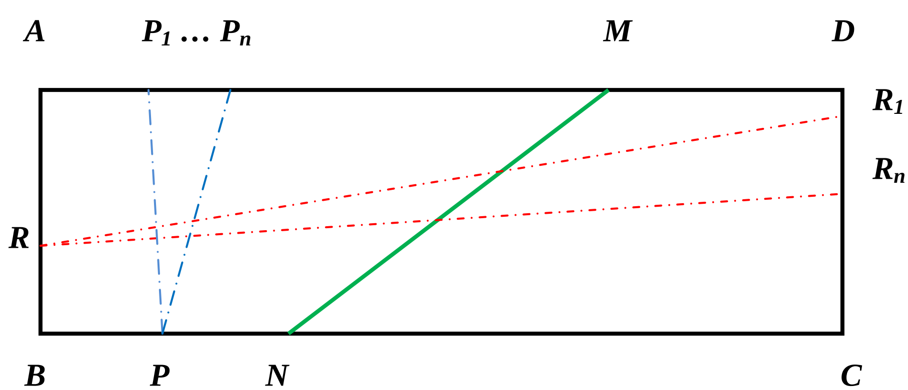

Fig. 1.

In this model we define as:
Point = any point inside or on the sides of this rectangle;
Line = a segment of line that connects two points of opposite sides of the rectangle;
Parallel lines = lines that do not have any common point (do not intersect);
Concurrent lines = lines that have a common point.

Let's take the line *MN*, where *M* lies on side *AD* and *N* on side *BC* as in the above *Fig. 1*. Let *P* be a point on side *BC,* and *R* a point on side *AB*.

Through *P* there are passing infinitely many parallels (PP_1, ..., PP_n, …) to the line *MN*, but through *R* there is no parallel to the line *MN* (the lines RR_1, ..., RR_n cut line *MN*). Therefore, the Fifth Postulate of Euclid (that though a point exterior to a line, in a given plane, there is only one parallel to that line) in *S-denied* on the space of the rectangle *ABCD* since it is invalidated in two distinct ways.

3.3. Another Geometric Model (another particular case of SG).

We change a little the Geometric Model 1 such that:

The rectangle *ABCD* is such that side *AB* is smaller than side *BC*. And we define as *line* the arc of circle inside (and on the borders) of *ABCD*, centered in the rectangle's vertices *A, B, C,* or *D.*

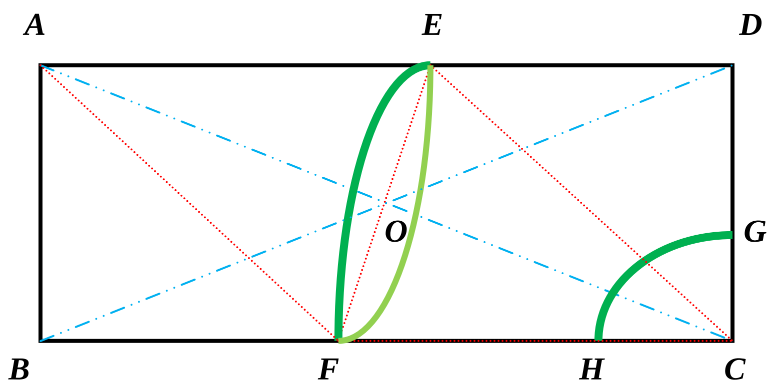

Fig. 2.

The axiom that: through two distinct points there exist only one line that passes through is S-denied (in three different ways):

a) Through the points *A* and *B* there is <u>no passing line</u> in this model, since there is no arc of circle centered in *A, B, C,* or *D* that passes through both points. See *Fig. 2*.
b) We construct the perpendicular $EF \perp AC$ that passes through the point of intersection of the diagonals *AC* and *BD*. Through the points *E* and *F* there are <u>two distinct lines</u> the dark green (left side) arc of circle centered in *C* since $CE \equiv FC$, and the light green (right side) arc of circle centered in *A* since $AE \equiv AF$. And because the right triangles $\triangle COE$, $\triangle COF$, $\triangle AOE$, and $\triangle AOF$ are all four congruent, we get $CE \equiv FC \equiv AE \equiv AF$.

c) Through the points G and H {such that $CG \equiv CH$ (their lengths are equal)} there is only one passing line (the dark green arc of circle *GH,* centered in *C*) since $AG \neq AH$ (their lengths are different), and similarly $BG \neq BH$ and $DG \neq DH$.

3.4. **Example for the Axiom of Separation.**

The Axiom of Separation of Hausdorff is the following:

$\forall\ x, y \in M,\ \ \exists N(x), N(y):\ N(x) \cap N(y) = \emptyset,$

where $N(x)$ is a neighborhood of x, and respectively $N(y)$ is a neighborhood of y.

We can *S-deny* this axiom on a space M in the following way:

a) $\exists\ x_1, y_1 \in M:\ \exists N_1(x_1), N_1(y_1):\ \ N_1(x_1) \cap N_1(y_1) = \emptyset,$
where $N_1(x_1)$ is a neighborhood of x_1, and respectively $N_1(y_1)$ is a neighborhood of y_1; [validated].

b) $\exists\ x_2, y_2 \in M:\ \forall N_2(x_2), N_2(y_2):\ \ N_2(x_2) \cap N_2(y_2) \neq \emptyset;$
where $N_2(x_2)$ is a neighborhood of x_2, and respectively $N_2(y_2)$ is a neighborhood of y_2; [invalidated].

Therefore we have two categories of points in *M*: some points that verify The Axiom of Separation of Hausdorff and other points that do not verify it. So M becomes a partially separable and partially inseparable space, or we can see that *M* has some degrees of separation.

3.5. Example for the Norm.

If we remove one or more axioms (or properties) from the definition of a notion <*A*> we get a pseudo-notion <*pseudoA*>.

For example, if we remove the third axiom (inequality of the triangle) from the definition of the <*norm*> we get a <*pseudonorm*>.

The axioms of a **norm** on a real or complex vectorial space V over a field F, $x \mapsto ||\ .\ ||$, are the following:

a) $||x|| = 0 \Leftrightarrow x = 0.$
b) $\forall x \in V,\ \forall \alpha \in F,\ ||\ \alpha \cdot x\ || = |\ \alpha\ | \cdot ||x\ ||.$
c) $\forall x, y \in V,\ ||x+y|| \leq ||x|| \cdot ||y||$ (inequality of the triangle).

For example, a **pseudo-norm** on a real or complex vectorial space V over a field F, $x \mapsto_p ||\ .\ ||$, may verify only the first two above axioms of the norm.

A pseudo-norm is a particular case of an *S-denied norm* since we may have vectorial spaces over some given scalar fields where there are some vectors and scalars that satisfy the third axiom [validation], but others that do not satisfy [invalidation]; or for all vectors and scalars we may have either $||x+y|| = 5\cdot||x||\cdot||y||$ or $||x+y|| = 6\cdot||x||\cdot||y||$, so invalidation (since we get $||x+y|| > ||x||\cdot||y||$) in two different ways.

Let's consider the complex vectorial space $\mathscr{C} = \{a+b\cdot i$, where $a, b \in R$, $i = \sqrt{-1}\}$ over the field of real numbers R.

If $z = a+b\cdot i \in \mathscr{C}$ then its pseudo-norm is $\| z \| = \sqrt{a^2+b^2}$. This verifies the first two axioms of the norm, but do not satisfy the third axiom of the norm since:

For x = 0 + $b\cdot i$ and $y = a + 0\cdot i$ we get:

$||x+y|| = ||a+b\cdot i|| = \sqrt{a^2+b^2} \leq ||x||\cdot||y|| = ||0+b\cdot i||\cdot||a+0\cdot i|| = |a\cdot b|$, or $a^2 + b^2 \leq a^2b^2$;

But this is true for example when $a = b \geq \sqrt{2}$ (validation), and false if one of a or b is zero and the other is strictly positive (invalidation).

Pseudo-norms are already in use in today's scientific research, because for some applications the norms are considered too restrictive.

Similarly one can define a pseudo-manifold (relaxing some properties of the manifold), etc.

3.6. **Example in Topology**.

A topology $\mathscr{O}$ on a given set $\mathcal{E}$ is the ensemble of all parts of $\mathcal{E}$ verifying the following properties:

a) $\mathcal{E}$ and the empty set Ø belong to $\mathscr{O}$.
b) Intersection of any two elements of $\mathscr{O}$ belongs to $\mathscr{O}$ too.
c) Union of any family of elements of $\mathscr{O}$ belongs to $\mathscr{O}$ too.

Let's go backwards. Suppose we have a topology $\mathscr{O}_1$ on a given set $\mathcal{E}_1$, and the second or third (or both) previous axioms have been *S-denied*, resulting an <u>*S-denied topology*</u> $\mathcal{S}_\neg(\mathscr{O}_1)$ on the given set $\mathcal{E}_1$.

In general, we can go back and "recover" (reconstruct) the original topology $\mathscr{O}_1$ from $\mathcal{S}_\neg(\mathscr{O}_1)$ by recurrence: if two elements belong to $\mathcal{S}_\neg(\mathscr{O}_1)$ then we set these elements and their intersection to belong to $\mathscr{O}_1$, and if a family of elements belong to $\mathcal{S}_\neg(\mathscr{O}_1)$ then we set these family elements and their union to belong to $\mathcal{O}_1$; and so on: we continue this recurrent process until it does not bring any new element to $\mathcal{O}_1$.

Conclusion.

Decidability changes in an *S-denied theory*, i.e. a defined sentence in an S-denied theory can be partially deducible and partially undeducible (we talk about degrees of deducibility of a sentence in an *S-denied theory*).

Since in classical deducible research, a theory $\mathcal{T}$ of language $\mathcal{L}$ is said complete if any sentence of $\mathcal{L}$ is decidable in $\mathcal{T}$, we can say that an *S-denied theory* is partially complete (or has some degrees of completeness and degrees of incompleteness).

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