Gen. Math. Notes, Vol. 31, No. 2, December 2015, pp.1-15
ISSN 2219-7184; Copyright ©ICSRS Publication, 2015
www.i-csrs.org
Available free online at http://www.geman.in

Smarandache Curves in Terms of Sabban Frame of Spherical Indicatrix Curves

Süleyman Şenyurt[1] and Abdussamet Çalışkan[2]

[1,2]Faculty of Arts and Sciences, Department of Mathematics
Ordu University, 52100, Ordu, Turkey
[1]E-mail: senyurtsuleyman@hotmail.com
[2]E-mail: abdussamet65@gmail.com

(Received: 4-11-14 / Accepted: 20-4-15)

Abstract

In this paper, we investigate special Smarandache curves in terms of Sabban frame of spherical indicatrix curves and we give some characterization of Smarandache curves. Besides, we illustrate examples of our results.

Keywords: *Smarandache Curves, Sabban Frame, Geodesic Curvature, Spherical Indicatrix Curves.*

1 Introduction

A regular curve in Minkowski space-time, whose position vector is composed by Frenet frame vectors on another regular curve, is called a Smarandache curve [5]. Special Smarandache curves have been studied by some authors.
Ahmad T.Ali studied some special Smarandache curves in the Euclidean space. He studied Frenet-Serret invariants of a special case, [2]. Özcan Bektaş and Salim Yüce studied some special smarandache curves according to Darboux Frame in E^3, [4]. Muhammed Çetin, Yılmaz Tuncer and Kemal Karacan investigated special smarandache curves according to Bishop frame in Euclidean 3-Space and they gave some differential geometric properties of Smarandache curves, [3]. Melih Turgut and Süha Yılmaz studied a special case of such curves and called it smarandache TB_2 curves in the space E_1^4, [5]. Nurten Bayrak, Özcan Bektaş and Salim Yüce studied some special smarandache curves in E_1^3, [6]. Kemal Taşköprü , Murat Tosun studied special Smarandache curves

according to Sabban frame on S^2, [7].

In this paper, we study special Smarandache curves such as TT_T, $T_T(T\wedge T_T)$, $TT_T(T\wedge T_T)$, NT_N, $T_N(N\wedge T_N)$, $NT_N(N\wedge T_N)$, BT_B, $T_B(B\wedge T_B)$ and $BT_B(B\wedge T_B)$ created by Sabban frame, $\{T, T_T, T\wedge T_T\}$, $\{N, T_N, N\wedge T_N\}$ and $\{B, T_B, B\wedge T_B\}$, that belongs to spherical indicatrix of a α curve are defined. Besides we have found some results.

2 Problem Formulations

The Euclidean 3-space E^3 be inner product given by

$$\langle,\rangle = x_1^2 + x_2^3 + x_3^2$$

where $(x_1, x_2, x_3) \in E^3$. Let $\alpha : I \to E^3$ be a unit speed curve denote by $\{T, N, B\}$ the moving Frenet frame . For an arbitrary curve $\alpha \in E^3$, with first and second curvature, κ and τ respectively, the Frenet formulae is given by [1]

$$\begin{cases} T' = \kappa N \\ N' = -\kappa T + \tau B \\ B' = -\tau N. \end{cases} \tag{1}$$

Accordingly, the spherical indicatrix curves of Frenet vectors are (T), (N) and (B) respectively. These equations of curves are given by [10]

$$\begin{cases} \alpha_T(s) = T(s) \\ \alpha_N(s) = N(s) \\ \alpha_B(s) = B(s) \end{cases} \tag{2}$$

For any unit speed curve $\alpha : I \to \mathbb{E}^3$, the vector W is called Darboux vector defined by

$$W = \tau(s)T(s) + \kappa(s)B(s).$$

If we consider the normalization of the Darboux $c = \frac{W}{\|W\|}$ we have

$$\cos\varphi = \frac{\kappa(s)}{\|W\|}, \sin\varphi = \frac{\tau(s)}{\|W\|}$$

and

$$c = \sin\varphi T(s) + \cos\varphi B(s)$$

where $\angle(W, B) = \varphi$.

Let $\gamma : I \to S^2$ be a unit speed spherical curve. We denote s as the arc-length parameter of γ. Let us denote by

$$\begin{cases} \gamma(s) = \gamma(s) \\ t(s) = \gamma'(s) \\ d(s) = \gamma(s) \wedge t(s). \end{cases} \tag{3}$$

We call $t(s)$ a unit tangent vector of γ. $\{\gamma, t, d\}$ frame is called the Sabban frame of γ on S^2 . Then we have the following spherical Frenet formulae of γ :

$$\begin{cases} \gamma' = t \\ t' = -\gamma + \kappa_g d \\ d' = -\kappa_g t \end{cases} \tag{4}$$

where is called the geodesic curvature of κ_g on S^2 and

$$\kappa_g = \langle t', d \rangle \ \ [8] \tag{5}$$

3 Smarandache Curves in Terms of Sabban Frame of Spherical Indicatrix Curves

In this section, we investigate Smarandache curves according to the Sabban frame of Spherical Indicatrix Curves.
Let $\alpha_T(s) = T(s)$ be a unit speed regular spherical curves on S^2. We denote s_T as the arc-lenght parameter of tangents indicatrix (T)

$$\alpha_T(s) = T(s) \tag{6}$$

Differentiating (6), we have

$$\frac{d\alpha_T}{ds_T}\frac{ds_T}{ds} = T'(s)$$

and

$$T_T\frac{ds_T}{ds} = \kappa N \tag{7}$$

From the equation (7)

$$T_T = N$$

and

$$T \wedge T_T = B$$

From the equation (3)

$$\begin{cases} T(s) = T(s) \\ T_T(s) = N(s) \\ T \wedge T_T(s) = B(s) \end{cases}$$

is called the Sabban frame of tangents indicatrix (T). From the equation (5)

$$\kappa_g = \langle T_T', T \wedge T_T \rangle \Longrightarrow \kappa_g = \frac{\tau}{\kappa}$$

Then from the equation (4) we have the following spherical Frenet formulae of (T):

$$\begin{cases} T' = T_T \\ T_T' = -T + \frac{\tau}{\kappa} T \wedge T_T \\ (T \wedge T_T)' = -\frac{\tau}{\kappa} T_T \end{cases} \tag{8}$$

Let $\alpha_N(s) = N(s)$ be a unit speed regular spherical curves on S^2.We denote s_N as the arc-lenght parameter of principal normals indicatrix (N)

$$\alpha_N(s) = N(s) \tag{9}$$

Differentiating (9), we have

$$T_N = -\cos\varphi T + \sin\varphi B$$

and

$$N \wedge T_N = \sin\varphi T + \cos\varphi B.$$

From the equation (3)

$$\begin{cases} N(s) = N(s) \\ T_N(s) = -\cos\varphi T(s) + \sin\varphi B(s) \\ N \wedge T_N(s) = \sin\varphi T(s) + \cos\varphi B(s) \end{cases}$$

is called the Sabban frame of principal normals indicatrix (N). From the equation (5)

$$\kappa_g = \frac{\varphi'}{\|W\|}$$

Then from the equation (4) we have the following spherical Frenet formulae of (N):

$$\begin{cases} N' = T_N \\ T_N' = -N + \frac{\varphi'}{\|W\|}(N \wedge T_N) \\ (N \wedge T_N)' = -\frac{\varphi'}{\|W\|} T_N \end{cases} \tag{10}$$

Let $\alpha_B(s) = B(s)$ be a unit speed regular spherical curves on S^2.We denote s_B as the arc-lenght parameter of indicatrix (B)

$$\alpha_B(s) = B(s) \tag{11}$$

Differentiating (11), we have

$$T_B = -N$$

and

$$B \wedge T_B = T$$

From the equation (3)

$$\begin{cases} B(s) = B(s) \\ T_B(s) = -N(s) \\ (B \wedge T_B)(s) = T(s) \end{cases}$$

is called the Sabban frame of binormals indicatrix (B). From the equation (5)

$$\kappa_g = \frac{\kappa}{\tau}$$

Then from the equation (4) we have the following spherical Frenet formulae of (B):

$$\begin{cases} B' = T_B \\ T_B{}' = -B + \frac{\kappa}{\tau}(B \wedge T_B) \\ (B \wedge T_B)' = -\frac{\kappa}{\tau}T_B \end{cases} \tag{12}$$

i-) TT_T-Smarandache Curves

Let S^2 be a unit sphere in E^3 and suppose that the unit speed regular curve $\alpha_T(s) = T(s)$ lying fully on S^2. In this case, TT_T - Smarandache curve can be defined by

$$\beta_1(s^*) = \frac{1}{\sqrt{2}}(T + T_T). \tag{13}$$

Now we can compute Sabban invariants of TT_T - Smarandache curves. Differentiating (13), we have

$$T_{\beta_1}\frac{ds^*}{ds} = \frac{1}{\sqrt{2}}(-T + N + \frac{\tau}{\kappa}B),$$

where

$$\frac{ds^*}{ds} = \sqrt{\frac{2 + (\frac{\tau}{\kappa})^2}{2}}. \tag{14}$$

Thus, the tangent vector of curve β_1 is to be

$$T_{\beta_1} = \frac{1}{\sqrt{2 + (\frac{\tau}{\kappa})^2}}(-T + N + \frac{\tau}{\kappa}B). \tag{15}$$

Differentiating (15), we get

$$T'_{\beta_1}\frac{ds^*}{ds} = \frac{1}{\left(2+(\frac{\tau}{\kappa})^2\right)^{\frac{3}{2}}}(\lambda_1 T + \lambda_2 N + \lambda_3 B) \tag{16}$$

where

$$\lambda_1 = \frac{\tau}{\kappa}\Big(\frac{\tau}{\kappa}\Big)' - \Big(\frac{\tau}{\kappa}\Big)^2 - 2$$

$$\lambda_2 = -\frac{\tau}{\kappa}\Big(\frac{\tau}{\kappa}\Big)' - \Big(\frac{\tau}{\kappa}\Big)^4 - 3\Big(\frac{\tau}{\kappa}\Big)^2 - 2$$

$$\lambda_3 = 2\Big(\frac{\tau}{\kappa}\Big) + \Big(\frac{\tau}{\kappa}\Big)^3 + 2\frac{\tau}{\kappa}.$$

Substituting the equation (15) into equation (16), we reach

$$T'_{\beta_1} = \frac{\sqrt{2}}{(2+(\frac{\tau}{\kappa})^2)^2}(\lambda_1 T + \lambda_2 N + \lambda_3 B). \tag{17}$$

Considering the equations (13) and (15), it easily seen that

$$(T\wedge T_T)_{\beta_1} = \frac{1}{\sqrt{4+2(\frac{\tau}{\kappa})^2}}\Big(\frac{\tau}{\kappa}T - \frac{\tau}{\kappa}N + 2B\Big). \tag{18}$$

From the equation (17) and (18), the geodesic curvature of $\beta_1(s^*)$ is

$$\kappa_g{}^{\beta_1} = \frac{1}{(2+(\frac{\tau}{\kappa})^2)^{\frac{5}{2}}}(\lambda_1\tfrac{\tau}{\kappa} - \lambda_2\tfrac{\tau}{\kappa} + 2\lambda_3).$$

ii-) $T_T(T\wedge T_T)$-Smarandache Curves

Similarly, $T_T(T\wedge T_T)$ - Smarandache curve can be defined by

$$\beta_2(s^*) = \frac{1}{\sqrt{2}}(T_T + T\wedge T_T). \tag{19}$$

In that case, the tangent vector of curve β_2 is as follows

$$T_{\beta_2} = \frac{1}{\sqrt{1+2(\frac{\tau}{\kappa})^2}}\Big(-T - \frac{\tau}{\kappa}N + \frac{\tau}{\kappa}B\Big). \tag{20}$$

Differentiating (20), it is obtained that

$$T'_{\beta_2} = \frac{\sqrt{2}}{(1+2(\frac{\tau}{\kappa})^2)^2}(\lambda_1 T + \lambda_2 N + \lambda_3 B) \tag{21}$$

where

$$\lambda_1 = \frac{\tau}{\kappa} + 2\Big(\frac{\tau}{\kappa}\Big)^3 + 2\Big(\frac{\tau}{\kappa}\Big)\Big(\frac{\tau}{\kappa}\Big)'$$

$$\lambda_2 = -2\Big(\frac{\tau}{\kappa}\Big)^4 - 3\Big(\frac{\tau}{\kappa}\Big)^2 - \frac{\tau}{\kappa} - 1$$

$$\lambda_3 = \Big(\frac{\tau}{\kappa}\Big)' - 2\Big(\frac{\tau}{\kappa}\Big)^4 - \Big(\frac{\tau}{\kappa}\Big)^2.$$

Using the equations (19) and (20), we easily find

$$(T \wedge T_T)_{\beta_2} = \frac{1}{\sqrt{2 + 4(\frac{\tau}{\kappa})^2}}\big(2\frac{\tau}{\kappa}T - N + B\big). \tag{22}$$

So, the geodesic curvature of $\beta_2(s^*)$ is as follows

$${\kappa_g}^{\beta_2} = \frac{1}{(1 + 2(\frac{\tau}{\kappa})^2)^{\frac{5}{2}}}\big(2\lambda_1\frac{\tau}{\kappa} - \lambda_2 + \lambda_3\big).$$

iii-) $TT_T(T \wedge T_T)$-Smarandache Curves

$TT_TT \wedge T_T$ - Smarandache curve can be defined by

$$\beta_3(s^*) = \frac{1}{\sqrt{3}}(T + T_T + T \wedge T_T). \tag{23}$$

Differentiating (23), we have the tangent vector of curve β_3 is

$$T_{\beta_3} = \frac{1}{\sqrt{2(1 - \frac{\tau}{\kappa} + (\frac{\tau}{\kappa})^2)}}\big(-T + (1 - \frac{\tau}{\kappa})N + \frac{\tau}{\kappa}B\big). \tag{24}$$

Differentiating (24), it is obtained that

$$T'_{\beta_3} = \frac{\sqrt{3}}{4(1 - \frac{\tau}{\kappa} + (\frac{\tau}{\kappa})^2)^2}(\lambda_1 T + \lambda_2 N + \lambda_3 B). \tag{25}$$

where

$$\lambda_1 = \Big(\frac{\tau}{\kappa}\Big)'\Big(2\frac{\tau}{\kappa} - 1\Big) + 2\Big(\frac{\tau}{\kappa}\Big)^3 - 4\Big(\frac{\tau}{\kappa}\Big)^2 + 4\frac{\tau}{\kappa} - 2$$

$$\lambda_2 = -\Big(\frac{\tau}{\kappa}\Big)'\Big(\frac{\tau}{\kappa} + 1\Big) - 2\Big(\frac{\tau}{\kappa}\Big)^4 + 2\Big(\frac{\tau}{\kappa}\Big)^3 - 4\Big(\frac{\tau}{\kappa}\Big)^2 + 2\Big(\frac{\tau}{\kappa}\Big) - 2$$

$$\lambda_3 = \Big(\frac{\tau}{\kappa}\Big)'\Big(2 - \frac{\tau}{\kappa}\Big) - 2\Big(\frac{\tau}{\kappa}\Big)^4 + 4\Big(\frac{\tau}{\kappa}\Big)^3 - 4\Big(\frac{\tau}{\kappa}\Big)^2 + 2\Big(\frac{\tau}{\kappa}\Big).$$

Using the equations (23) and (24), we have

$$(T\wedge T_T)_{\beta_3}=\frac{(2\frac{\tau}{\kappa}-1)T+(-1-\frac{\tau}{\kappa})N+(2-\frac{\tau}{\kappa})B}{\sqrt{6}\sqrt{1-\frac{\tau}{\kappa}+(\frac{\tau}{\kappa})^2}}\cdot \tag{26}$$

So, the geodesic curvature of $\beta_3(s^*)$ is

$${\kappa_g}^{\beta_3}=\frac{\lambda_1(2\frac{\tau}{\kappa}-1)+\lambda_2(-1-\frac{\tau}{\kappa})+\lambda_3(2-\frac{\tau}{\kappa})}{4\sqrt{2}(1-\frac{\tau}{\kappa}+(\frac{\tau}{\kappa})^2)^{\frac{5}{2}}}.$$

iv-) NT_N-Smarandache Curves

NT_N - Smarandache curve can be defined by

$$\varsigma_1(s^*)=\frac{1}{\sqrt{2}}(N+T_N). \tag{27}$$

Differentiating (27), we have the tangent vector of curve ς_3 is

$$T_{\varsigma_1}=\frac{(-\cos\varphi+\frac{\varphi'}{\|W\|}\sin\varphi)T-N+(\sin\varphi+\frac{\varphi'}{\|W\|}\cos\varphi)B}{\sqrt{2+(\frac{\varphi'}{\|W\|})^2}}\cdot \tag{28}$$

Differentiating (28), we get

$$T'_{\varsigma_1}=\frac{1}{(2+(\frac{\varphi'}{\|W\|})^2)^2}((\lambda_3\sin\varphi-\lambda_2\cos\varphi)T+\lambda_1 N+(\lambda_2\sin\varphi+\lambda_3\cos\varphi)B). \tag{29}$$

where

$$\lambda_1=(\frac{\varphi'}{\|W\|})(\frac{\varphi'}{\|W\|})'-(\frac{\varphi'}{\|W\|})^2-2$$

$$\lambda_2=-(\frac{\varphi'}{\|W\|})(\frac{\varphi'}{\|W\|})'-(\frac{\varphi'}{\|W\|})^4-3(\frac{\varphi'}{\|W\|})^2-2$$

$$\lambda_3=2(\frac{\varphi'}{\|W\|})'+(\frac{\varphi'}{\|W\|})^3+2(\frac{\varphi'}{\|W\|}).$$

Considering the equations (27) and (28), it easily seen that

$$(N\wedge T_N)_{\varsigma_1}=\frac{(2\sin\varphi+\frac{\varphi'}{\|W\|}\cos\varphi)T+\frac{\varphi'}{\|W\|}N+(2\cos\varphi-\frac{\varphi'}{\|W\|}\sin\varphi)B}{\sqrt{4+2(\frac{\varphi'}{\|W\|})^2}}\cdot \tag{30}$$

The geodesic curvature of $\varsigma_1(s^*)$ is

$$\kappa_g{}^{\varsigma_1} = \frac{\left(\lambda_1 \frac{\varphi'}{\|W\|} - \lambda_2 \frac{\varphi'}{\|W\|} + 2\lambda_3\right)}{\left(2 + (\frac{\varphi'}{\|W\|})^2\right)^{\frac{5}{2}}}.$$

v-) $T_N(N \wedge T_N)$-Smarandache Curves

$T_N(N \wedge T_N)$ - Smarandache curve can be defined by

$$\varsigma_2(s^*) = \frac{1}{\sqrt{2}}(T_N + N \wedge T_N). \tag{31}$$

Differentiating (31), the tangent vector of curve ς_2 is

$$T_{\varsigma_2} = \frac{(\frac{\varphi'}{\|W\|}(\sin\varphi + \cos\varphi))T - N + (\frac{\varphi'}{\|W\|}(\cos\varphi - \sin\varphi)B)}{\sqrt{1 + 2(\frac{\varphi'}{\|W\|})^2}}. \tag{32}$$

Differentiating (32), it is obtained that

$$T'_{\varsigma_2} = \frac{\sqrt{2}}{\left(1 + 2(\frac{\varphi'}{\|W\|})^2\right)^2}\left[(\lambda_3 \sin\varphi - \lambda_2 \cos\varphi)T + \lambda_1 N + (\lambda_2 \sin\varphi + \lambda_3 \cos\varphi)B\right] \tag{33}$$

where

$$\lambda_1 = \left(\frac{\varphi'}{\|W\|}\right) + 2\left(\frac{\varphi'}{\|W\|}\right)^3 + 2\left(\frac{\varphi'}{\|W\|}\right)\left(\frac{\varphi'}{\|W\|}\right)'$$

$$\lambda_2 = -2\left(\frac{\varphi'}{\|W\|}\right)^4 - 3\left(\frac{\varphi'}{\|W\|}\right)^4 - \left(\frac{\varphi'}{\|W\|}\right) - 1$$

$$\lambda_3 = \left(\frac{\varphi'}{\|W\|}\right)' - 2\left(\frac{\varphi'}{\|W\|}\right)^4 - \left(\frac{\varphi'}{\|W\|}\right)^2.$$

Using the equations (31) and (32), we easily find

$$(N \wedge T_N)_{\varsigma_2} = \frac{(\sin\varphi + \cos\varphi)T + 2\frac{\varphi'}{\|W\|}N + (\cos\varphi - \sin\varphi)B}{\sqrt{2 + 4(\frac{\varphi'}{\|W\|})^2}}. \tag{34}$$

So, the geodesic curvature of $\varsigma_2(s^*)$ is as follows

$$\kappa_g{}^{\varsigma_2} = \frac{\left(\frac{\varphi'}{\|W\|}\lambda_1 - \lambda_2 + \lambda_3\right)}{\left(1 + 2(\frac{\varphi'}{\|W\|})^2\right)^{\frac{5}{2}}}.$$

vi-) $NT_N(N \wedge T_N)$-Smarandache Curves

$NT_N N \wedge T_N$ - Smarandache curve can be defined by

$$\varsigma_3(s^*) = \frac{1}{\sqrt{3}}(N + T_N + N \wedge T_N). \tag{35}$$

Differentiating (35), the tangent vector of curve ς_2 is

$$T_{\varsigma_3} = \frac{\big(-\cos\varphi + \frac{\varphi'}{\|W\|}(\cos\varphi + \sin\varphi)\big)T - N + \big(\sin\varphi + \frac{\varphi'}{\|W\|}(\cos\varphi - \sin\varphi)\big)B}{\sqrt{2\Big(1 - \frac{\varphi'}{\|W\|} + \big(\frac{\varphi'}{\|W\|}\big)^2\Big)}}. \tag{36}$$

Differentiating (36), it is obtained that

$$T'_{\varsigma_3} = \frac{\sqrt{3}}{4\Big(1 - \frac{\varphi'}{\|W\|} + (\frac{\varphi'}{\|W\|})^2\Big)^2}\big[(-\lambda_2\cos\varphi + \lambda_3\sin\varphi)T + \lambda_1 N + (\lambda_3\cos\varphi + \lambda_2\sin\varphi)B\big]. \tag{37}$$

where

$$\begin{aligned}
\lambda_1 &= (\frac{\varphi'}{\|W\|})'(2\frac{\varphi'}{\|W\|} - 1) + 2(\frac{\varphi'}{\|W\|})^3 - 4(\frac{\varphi'}{\|W\|})^2 + 4(\frac{\varphi'}{\|W\|}) - 2 \\
\lambda_2 &= -(\frac{\varphi'}{\|W\|})'(\frac{\varphi'}{\|W\|} + 1) - 2(\frac{\varphi'}{\|W\|})^4 + 2(\frac{\varphi'}{\|W\|})^3 - 4(\frac{\varphi'}{\|W\|})^2 + 2(\frac{\varphi'}{\|W\|}) - 2 \\
\lambda_3 &= (\frac{\varphi'}{\|W\|})'(2 - \frac{\varphi'}{\|W\|}) - 2(\frac{\varphi'}{\|W\|})^4 + 4(\frac{\varphi'}{\|W\|})^3 - 4(\frac{\varphi'}{\|W\|})^2 + 2(\frac{\varphi'}{\|W\|}).
\end{aligned}$$

Using the equations (35) and (36), we have

$$\begin{aligned}
(N \wedge T_N)_{\varsigma_3} = {} & \frac{1}{\sqrt{6}\sqrt{1 - \frac{\varphi'}{\|W\|} + (\frac{\varphi'}{\|W\|})^2}}\Big((2\sin\varphi + \cos\varphi \\
& + \frac{\varphi'}{\|W\|}(\cos\varphi - \sin\varphi))T + (-1 + 2\frac{\varphi'}{\|W\|})N \\
& + (2\cos\varphi - \sin\varphi - \frac{\varphi'}{\|W\|}(\cos\varphi - \sin\varphi))B\Big)
\end{aligned} \tag{38}$$

The geodesic curvature of $\varsigma_3(s^*)$ is

$$\kappa_g^{S3} = \frac{1}{4\sqrt{2}(1-\frac{\varphi'}{\|W\|}+(\frac{\varphi'}{\|W\|})^2)^{\frac{5}{2}}}[\lambda_1\big(2\frac{\varphi'}{\|W\|}-1\big)+\lambda_2(-1-\frac{\varphi'}{\|W\|}) +\lambda_3\big(2-\frac{\varphi'}{\|W\|}\big)].$$

vii-) BT_B-Smarandache Curves

BT_B - Smarandache curve can be defined by

$$\eta_1(s^*) = \frac{1}{\sqrt{2}}(B+T_B). \tag{39}$$

Differentiating (39), the tangent vector of curve η_1 is to be

$$T_{\eta_1} = \frac{1}{\sqrt{2+\left(\frac{\kappa}{\tau}\right)^2}}\big(\frac{\kappa}{\tau}T-N-B\big). \tag{40}$$

Differentiating (40), we get

$$T'_{\eta_1} = \frac{\sqrt{2}}{\left(2+\left(\frac{\kappa}{\tau}\right)^2\right)^2}(\lambda_3 T-\lambda_2 N+\lambda_1 B). \tag{41}$$

where

$$\lambda_1 = \big(\frac{\kappa}{\tau}\big)'\big(\frac{\kappa}{\tau}\big)-\big(\frac{\kappa}{\tau}\big)^2-2$$

$$\lambda_2 = -2-3\big(\frac{\kappa}{\tau}\big)^2-\big(\frac{\kappa}{\tau}\big)^4-\big(\frac{\kappa}{\tau}\big)\big(\frac{\kappa}{\tau}\big)'$$

$$\lambda_3 = 2\big(\frac{\kappa}{\tau}\big)'+\big(\frac{\kappa}{\tau}\big)^3+2\big(\frac{\kappa}{\tau}\big).$$

Considering the equations (39) and (40), it easily seen that

$$(B\wedge T_B)_{\eta_1} = \frac{1}{\sqrt{4+2\left(\frac{\kappa}{\tau}\right)^2}}\big(2T+\frac{\kappa}{\tau}N+\frac{\kappa}{\tau}B\big). \tag{42}$$

So, the geodesic curvature of $\eta_1(s^*)$ is

$$\kappa_g{}^{\eta_1} = \frac{1}{\left(2+\left(\frac{\kappa}{\tau}\right)^2\right)^{\frac{5}{2}}}\big(\frac{\kappa}{\tau}\lambda_1-\frac{\kappa}{\tau}\lambda_2+2\lambda_3\big).$$

viii-) $T_B(B \wedge T_B)$-Smarandache Curves

$T_B(B \wedge T_B)$ - Smarandache curve can be defined by

$$\eta_2(s^*) = \frac{1}{\sqrt{2}}(T_B + B \wedge T_B). \tag{43}$$

Differentiating (43), the tangent vector of curve η_2 is as follows

$$T_{\eta_2} = \frac{1}{\sqrt{1+2(\frac{\kappa}{\tau})^2}}\big(\frac{\kappa}{\tau}T + \frac{\kappa}{\tau}N - B\big). \tag{44}$$

Differentiating (44), it is obtained that

$$T'_{\eta_2} = \frac{\sqrt{2}}{\big(1+2(\frac{\kappa}{\tau})^2\big)^2}(\lambda_3 T - \lambda_2 N + \lambda_1 B) \tag{45}$$

where

$$\lambda_1 = \big(\frac{\kappa}{\tau}\big) + 2\big(\frac{\kappa}{\tau}\big)^3 + 2\big(\frac{\kappa}{\tau}\big)\big(\frac{\kappa}{\tau}\big)'$$

$$\lambda_2 = -2\big(\frac{\kappa}{\tau}\big)^4 - 3\big(\frac{\kappa}{\tau}\big)^2 - \big(\frac{\kappa}{\tau}\big) - 1$$

$$\lambda_3 = \big(\frac{\kappa}{\tau}\big)' - 2\big(\frac{\kappa}{\tau}\big)^4 - \big(\frac{\kappa}{\tau}\big)^2.$$

Using the equations (43) and (44), we easily find

$$(B \wedge T_B)_{\eta_2} = \frac{1}{\sqrt{2+4(\frac{\kappa}{\tau})^2}}\big(T + N + 2\frac{\kappa}{\tau}B\big). \tag{46}$$

So, the geodesic curvature of $\eta_2(s^*)$ is as follows

$$\kappa_g{}^{\eta_2} = \frac{1}{(1+2(\frac{\kappa}{\tau})^2)^{\frac{5}{2}}}(2\frac{\kappa}{\tau}\lambda_1 - \lambda_2 + \lambda_3).$$

ix-) $BT_B(B \wedge T_B)$-Smarandache Curves

$BT_B B \wedge T_B$ - Smarandache curve can be defined by

$$\eta_3(s^*) = \frac{1}{\sqrt{3}}(B + T_B + B \wedge T_B). \tag{47}$$

Differentiating (47), the tangent vector of curve η_3 is

$$T_{\eta_3} = \frac{1}{\sqrt{2\Big(1 - \frac{\kappa}{\tau} + \big(\frac{\kappa}{\tau}\big)^2\Big)}}\Big(\frac{\kappa}{\tau}T + (-1 + \frac{\kappa}{\tau})N - B\Big) \tag{48}$$

Differentiating (48), it is obtained that

$$T'_{\eta_3} = \frac{\sqrt{3}}{4\Big(1 - \frac{\kappa}{\tau} + (\frac{\kappa}{\tau})^2\Big)^2}(\lambda_3 T - \lambda_2 N + \lambda_1 B). \tag{49}$$

where

$$\lambda_1 = \big(\frac{\kappa}{\tau}\big)'\big(2\frac{\kappa}{\tau} - 1\big) + 2\big(\frac{\kappa}{\tau}\big)^3 - 4\big(\frac{\kappa}{\tau}\big)^2 + 4\frac{\kappa}{\tau} - 2$$

$$\lambda_2 = -\big(\frac{\kappa}{\tau}\big)'\big(\frac{\kappa}{\tau} + 1\big) - 2\big(\frac{\kappa}{\tau}\big)^4 + 2\big(\frac{\kappa}{\tau}\big)^3 - 4\big(\frac{\kappa}{\tau}\big)^2 + 2\big(\frac{\kappa}{\tau}\big) - 2$$

$$\lambda_3 = \big(\frac{\kappa}{\tau}\big)'\big(2 - \frac{\kappa}{\tau}\big) - 2\big(\frac{\kappa}{\tau}\big)^4 + 4\big(\frac{\kappa}{\tau}\big)^3 - 4\big(\frac{\kappa}{\tau}\big)^2 + 2\big(\frac{\kappa}{\tau}\big).$$

Using the equations (47) and (48), we have

$$(B \wedge T_B)_{\eta_3} = \frac{(2 - \frac{\kappa}{\tau})T + (1 + \frac{\kappa}{\tau})N + (-1 + 2\frac{\kappa}{\tau})B}{\sqrt{6}\sqrt{1 - \frac{\kappa}{\tau} + (\frac{\kappa}{\tau})^2}}. \tag{50}$$

The geodesic curvature of $\eta_3(s^*)$ is

$$\kappa_g^{\eta_3} = \frac{\lambda_1(2\frac{\kappa}{\tau} - 1) + \lambda_2(-1 - \frac{\kappa}{\tau}) + \lambda_3(2 - \frac{\kappa}{\tau})}{4\sqrt{2}(1 - \frac{\kappa}{\tau} + (\frac{\kappa}{\tau})^2)^{\frac{5}{2}}}.$$

Example

Let us consider the unit speed spherical curve:

$$\alpha(s) = \{\frac{9}{208}\sin 16s - \frac{1}{117}\sin 36s, -\frac{9}{208}\cos 16s + \frac{1}{117}\cos 36s, \frac{6}{65}\sin 10s\}.$$

In terms of definitions, we obtain Spherical indicatrix curves (T), (N), (B), (see Figure 1) and Smarandache curves according to Sabban frame on S^2, TT_T, $T_T(T \wedge T_T)$, $TT_T(T \wedge T_T)$, NT_N, $T_N(N \wedge T_N)$, $NT_N(N \wedge T_N)$, BT_B, $T_B(B \wedge T_B)$, $BT_B(B \wedge T_B)$, (see Figure 2, 3, 4).

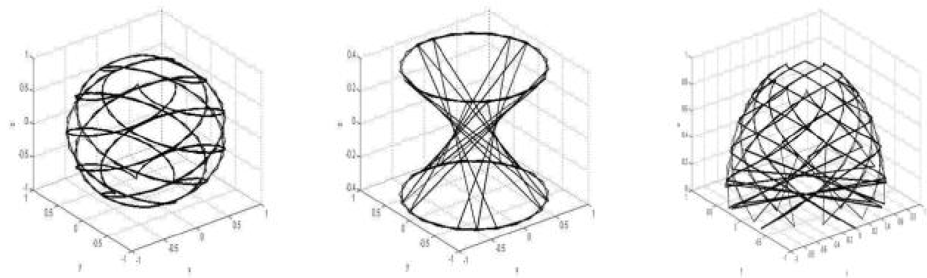

Figure 1: (T) (N) (B)

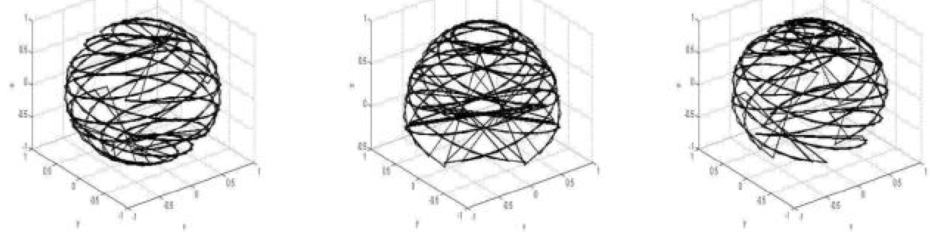

Figure 2: TT_T $T_T(T \wedge T_T)$ $TT_T(T \wedge T_T)$

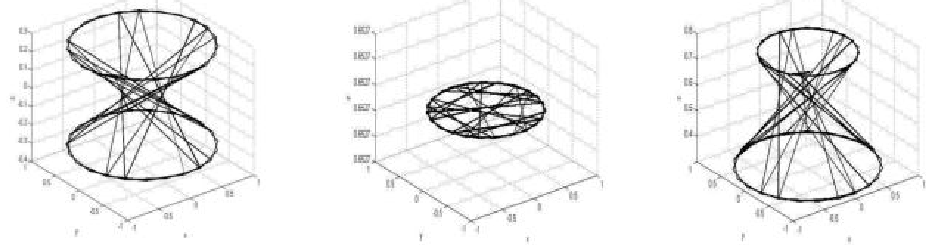

Figure 3: NT_N $T_N(N \wedge T_N)$ $NT_N(N \wedge T_N)$

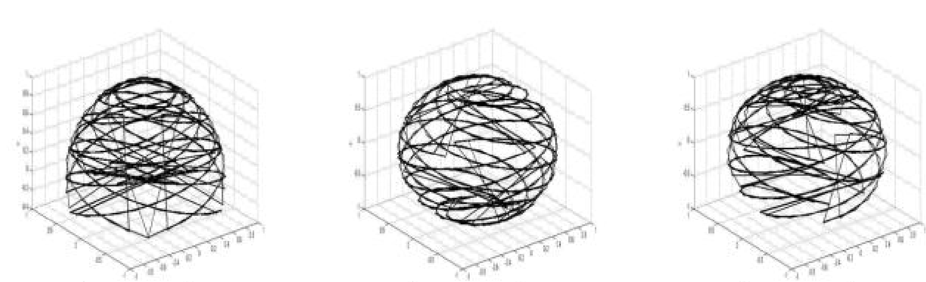

Figure 4: BT_B $T_B(B \wedge T_B)$ $BT_B(B \wedge T_B)$

References

[1] M.P. Do Carmo, *Differential Geometry of Curves and Surfaces*, Prentice Hall, Englewood Cliffs, (1976).

[2] A.T. Ali, Special Smarandache curves in the Euclidean space, *International Journal of Mathematical Combinatorics*, 2(2010), 30-36.

[3] M. Çetin, Y. Tunçer and M.K. Karacn, Smarandache curves according to Bishop frame in Euclidean 3- space, *Gen. Math. Notes*, 20(2014), 50-66.

[4] Ö. Bektaş and S. Yüce, Special Smarandache curves according to Darboux frame in Euclidean 3- space, *Romanian Journal of Mathematics and Computer Science*, 3(2013), 48-59.

[5] M. Turgut and S. Yılmaz, Smarandache curves in Minkowski space-time, *International J. Math. Combin.*, 3(2008), 51-55.

[6] N. Bayrak, Ö. Bektaş and S. Yüce, Special Smarandache curves in E_1^3, *International Conference on Applied Analysis and Algebra*, 20-24(2012), 209.

[7] K. Taşköprü and M. Tosun, Smarandache curves on S^2, *Boletim da Sociedade Paranaense de Matemtica*, 32(2014), 51-59.

[8] J. Koenderink, *Solid Shape*, MIT Press, Cambridge, MA, (1990).

[9] B. O'Neill, *Elemantary Differential Geometry*, Academic Press Inc., New York, (1966).

[10] H.H. Hacısalihoğlu, *Differantial Geometry*, Academic Press Inc., Ankara, (1994).