AN INEQUALITY CONCERNING THE SMARANDACHE FUNCTION

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Abstract. For any positive integer n, let S(n) denote the Smarandache function of n. In this paper we prove that
S(mn) ≤ S(m)+S(n).

Let N be the set of all positive integers. For any positive integer n, let S(n) denote the Smarandache function of n. By [2], we have

\( S(n) = \min\{k \mid k \in \mathbb{N}, n \mid k!\}. \)

Recently, Jozsef[1] proved that

\( S(mn) \leq mS(n), \ m, n \in \mathbb{N}. \)

In this paper we give a considerable improvement for the upper bound (2). We prove the following result.

Theorem. For any positive integers m, n, we have

\( S(mn) \leq S(m)+S(n). \)

Proof. Let \( a = S(m) \) and \( b = S(n) \). Then we have

\( n \mid b!, \)

by (1). Let \( x \) be a positive integer with \( x \geq a \), and let

\[
\binom{x}{a} = \frac{x(x-1)\ldots(x-a+1)}{a!}
\]

be a binomial coefficient. It is a well known fact that \( \binom{x}{a} \) is a positive integer. So we have

\( a! \mid x(x-1)\ldots(x-a+1), \)

by (4). Further, since \( m \mid a! \), we get from (5) that

\( m \mid x(x-1)\ldots(x-a+1), \)

for any positive integer \( x \) with \( x \geq a \). Put \( x = a + b \). We see from (3) and (6) that

\( mn \mid b!(b+1)\ldots(b+a) = (a+b)!. \)
Thus we get from (7) that $S(mn) \leq a-b=S(m)-S(n)$. The theorem is proved.

References

2. P. Smarandache "A function in the number theory", "Smarandache Function J". 1(1990), No.1, 3-17.