An ordered set of certain seven numbers that results constantly from a recurrence formula based on Smarandache function

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Abstract. Combining two of my favorite topics of study, the recurrence relations and the Smarandache function, I discovered a very interesting pattern: seems like the recurrent formula \( f(n) = S(f(n - 2)) + S(f(n - 1)) \), where \( S \) is the Smarandache function and \( f(1), f(2) \) are any given different non-null positive integers, leads every time to a set of seven values (i.e. 11, 17, 28, 24, 11, 15, 16) which is then repeating infinitely.

Conjecture:

The recurrent formula \( f(n) = S(f(n - 2)) + S(f(n - 1)) \), where \( S \) is the Smarandache function, leads every time to the set of seven consecutive values \{11, 17, 28, 24, 11, 15, 16\}, set which is then repeating infinitely, for any given different non-null positive integers \( f(1), f(2) \).

Verifying the conjecture for few pairs \([f(1), f(2)]\)

For \([f(1), f(2)] = [1, 2]\):

\[
\begin{align*}
& f(3) = S(1) + S(2) = 3; \\
& f(5) = S(3) + S(5) = 8; \\
& f(7) = S(8) + S(9) = 10; \\
& f(9) = S(10) + S(11) = 16; \\
& f(11) = S(16) + S(17) = 23; \\
& f(13) = S(23) + S(40) = 28; \\
& f(15) = S(28) + S(12) = 11; \\
& f(17) = S(11) + S(15) = 16; \\
& f(19) = S(16) + S(11) = 17; \\
& f(21) = S(17) + S(28) = 24; \\
& f(23) = S(24) + S(11) = 15; \\
\end{align*}
\]

(…)

For \([f(1), f(2)] = [7, 13]\):

\[
\begin{align*}
& f(3) = S(7) + S(13) = 20; \\
& f(5) = S(20) + S(18) = 11; \\
& f(4) = S(2) + S(3) = 5; \\
& f(6) = S(5) + S(8) = 9; \\
& f(8) = S(9) + S(10) = 11; \\
& f(10) = S(11) + S(10) = 17; \\
& f(12) = S(17) + S(23) = 40; \\
& f(14) = S(40) + S(28) = 12; \\
& f(16) = S(12) + S(11) = 15; \\
& f(18) = S(15) + S(16) = 11; \\
& f(20) = S(11) + S(17) = 28; \\
& f(22) = S(28) + S(24) = 11; \\
& f(24) = S(11) + S(15) = 16 \\
\end{align*}
\]

(…)

(...)
For \([f(1), f(2)] = [5, 11]\):

\[
\begin{align*}
\text{: } f(3) &= S(5) + S(11) = 16; & f(4) &= S(11) + S(16) = 17; \\
(\ldots) & & \\
\text{: } f(12) &= 11; & f(13) &= 17 \\
(\ldots)
\end{align*}
\]

For \([f(1), f(2)] = [531, 44]\):

\[
\begin{align*}
\text{: } f(3) &= S(531) + S(44) = 70; & f(4) &= S(44) + S(70) = 18; \\
\text{: } f(5) &= S(70) + S(18) = 13; & f(6) &= S(18) + S(13) = 19; \\
\text{: } f(7) &= S(13) + S(19) = 32; & f(8) &= S(19) + S(32) = 27; \\
\text{: } f(9) &= S(32) + S(27) = 17; & f(10) &= S(27) + S(17) = 26; \\
\text{: } f(11) &= S(17) + S(26) = 30; & f(12) &= S(26) + S(30) = 18; \\
\text{: } f(13) &= S(30) + S(18) = 11; & f(14) &= S(18) + S(19) = 17 \\
(\ldots)
\end{align*}
\]

For \([f(1), f(2)] = [341, 561]\):

\[
\begin{align*}
\text{: } f(3) &= S(341) + S(561) = 48; & f(4) &= S(561) + S(48) = 23; \\
\text{: } f(5) &= S(48) + S(23) = 29; & f(6) &= S(23) + S(29) = 52; \\
\text{: } f(7) &= S(29) + S(52) = 42; & f(8) &= S(52) + S(42) = 20; \\
\text{: } f(9) &= S(42) + S(20) = 12; & f(10) &= S(20) + S(12) = 9; \\
\text{: } f(11) &= S(12) + S(9) = 10; & f(12) &= S(9) + S(10) = 11; \\
(\ldots) & & \\
\text{: } f(22) &= 11; & f(23) &= 17 \\
(\ldots)
\end{align*}
\]

For \([f(1), f(2)] = [49, 121]\):

\[
\begin{align*}
\text{: } f(3) &= S(49) + S(121) = 35; & f(4) &= S(121) + S(35) = 29; \\
\text{: } f(5) &= S(35) + S(29) = 36; & f(6) &= S(29) + S(36) = 35; \\
\text{: } f(7) &= S(36) + S(35) = 13; & f(8) &= S(35) + S(13) = 20; \\
\text{: } f(9) &= S(13) + S(18) = 18; & f(10) &= S(20) + S(18) = 11; \\
\text{: } f(11) &= S(18) + S(11) = 17 \\
(\ldots)
\end{align*}
\]

**Open problems**

I. Is there any exception to this apparent rule?

II. Is there a finite or infinite set of exceptions?

III. Is there a superior limit for \(n\) such that eventually \(f(n) = 11\) and \(f(n + 1) = 17\)?

IV. Is the obtaining of a constant repeating set of values a characteristic of other recurrent formulas based similarly on the Smarandache function, having three or more terms?