ON THE SMARANDACHE FUNCTION AND THE FIXED-POINT THEORY OF NUMBERS

by

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This brief note points out several basic connections between the Smarandache function, fixed-point theory [1] and prime-number theory. First recall that fixed-point theory in function spaces provides elegant, if not short, proofs of the existence of solutions to many kinds of differential equations, integral equations, optimization problems and game-theoretic problems. Further, fixed-point theory in the ring of rational integers and fixed-lattice-point theory provide many results on the existence of solutions in diophantine theory. Here are four fundamental examples of fixed-point theory in number theory. (1) There is the well-known basic result that for \( p > 4 \), \( p \) is prime iff \( S(p) = p \). (2) Recall that the present author defined [2] the number-theoretic function \( \Psi(n) \) as the product of the primes alone in the mosaic of \( n \), where the mosaic of \( n \) is obtained from \( n \) by recursively applying the unique factorization theorem/fundamental theorem of arithmetic to itself! Now the asymptotic density of fixed points of \( \Psi(n) \) is \( 7/\pi^2 \), just as the asymptotic density of square-free numbers is \( 6/\pi^2 \). Indeed, (3) the theory of perfect numbers is also connected to fixed-point theory, since if one puts \( \delta(n) = \delta(n) - n \), where \( \delta(n) \) is the sum of the divisors on \( n \), then \( n \) is perfect iff \( \delta(n) = n \). Finally, (4) the present author defined [2] the number-theoretic function \( \Psi^*(n) \) as the sum of the primes alone in the mosaic of \( n \). Here we have a striking similarity to the Smarandache function itself (see example (1) above), since \( \Psi^*(n) = n \) iff \( n = 4 \) or \( n = p \) for some prime \( p \); i.e., if \( n > 4 \), \( n \) is prime iff \( \Psi^*(n) = n \). Thus, the distribution function for the fixed points of \( S(n) \) or of \( \Psi^*(n) \) is essentially the distribution function for the primes, \( \Pi(n) \).

Problems

(1) Put \( S^2(n) = S(S(n)) \) and define \( S^m(n) \) recursively, where \( S(n) \) is the Smarandache function. (Note: This approach aligns Smarandache function theory more closely with recursive function theory/computer theory.) For each \( n \), determine the least \( m \) for which \( S^m(n) \) is prime.
(2) Prove that \( S(n) = S(n+3) \) for only finitely many \( n \).
(3) Prove that \( S(n) = S(n+2) \) for only finitely many \( n \).
(4) Prove that \( S(n) = S(n+1) \) for no \( n \).

References


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