Palindromic Numbers And Iterations of the Pseudo-Smarandache Function

Charles Ashbacher
Charles Ashbacher Technologies
Box 294, 119 Northwood Drive
Hiawatha, IA 52233
e-mail 71603.522@compuserve.com


Definition: For any \( n \geq 1 \), the value of the Pseudo-Smarandache function \( Z(n) \) is the smallest integer \( m \) such that \( n \) evenly divides

\[
\sum_{k=1}^{m} k .
\]

And it is well-known that the sum is equivalent to \( \frac{m(m+1)}{2} \).

Having been defined only recently, many of the properties of this function remain to be discovered. In this short paper, we will tentatively explore the connections between \( Z(n) \) and a subset of the integers known as the palindromic numbers.

Definition: A number is said to be a palindrome if it reads the same forwards and backwards. Examples of palindromes are

121, 34566543, 1111111111

There are some palindromic numbers \( n \) such that \( Z(n) \) is also palindromic. For example,

\( Z(909) = 404 \)  \( Z(2222) = 1111 \)

In this paper, we will not consider the trivial cases of the single digit numbers.

A simple computer program was used to search for values of \( n \) satisfying the above criteria. The range of the search was, \( 10 \leq n \leq 10000 \). Of the 189 palindromic values of \( n \) within that range, 37, or slightly over 19%, satisfied the criteria.

Furthermore it is sometimes possible to repeat the function again and get another palindrome.

\( Z(909) = 404, Z(404) = 303 \)

and once again, a computer program was run looking for values of \( n \) within the range.
1 \leq n \leq 10,000. Of the 37 values found in the previous test, 9 or slightly over 24%, exhibited the above properties.

Repeating the program again, looking for values of n such that n, Z(n), Z(Z(n)) and Z(Z(Z(n))) are all palindromic, we find that of the 9 found in the previous test, 2 or roughly 22%, satisfy the new criteria.

**Definition:** Let $Z^k(n) = Z(Z(Z(\ldots(n))))$ where the Z function is executed k times. For notational purposes, let $Z^0(n) = n$.

Modifying the computer program to search for solutions for a value of n so that n and all iterations $Z^i(n)$ are palindromic for i = 1, 2, 3 and 4, we find that there are no solutions in the range $1 \leq n \leq 10,000$. Given the percentages already encountered, this should not be a surprise. In fact, by expanding the search up through 100,000 one solution was found.

$Z(86868) = 17271, Z(17271) = 2222, Z(2222) = 1111, Z(1111) = 505$

Since $Z(505) = 100$, this is the largest such sequence in this region.

Computer searches for larger such sequences can be more efficiently carried out by using only palindromic numbers for n.

**Unsolved Question:** What is the largest value of m so that for some n, $Z^k(n)$ is a palindrome for all $k = 0, 1, 2, \ldots, m$?

**Unsolved Question:** Do the percentages discussed previously accurately represent the general case?

Of course, an affirmative answer to the second question would mean that there is no largest value of m in the first.

**Conjecture:** There is no largest value of m such that for some n, $Z^k(n)$ is a palindrome for all $k = 0, 1, 2, 3, \ldots, m$.

There are solid arguments in support of the truth of this conjecture. Palindromes tend to be divisible by palindromic numbers, so if we take n palindromic, many of the numbers that it divides would also be palindromic. And that palindrome is often the product of two numbers, one of which is a different palindrome. Numbers like the repunits, 11...111 and those with only a small number of different digits, like 1001 and 505 appeared quite regularly in the computer search.

**Reference**