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Smarandache Curves of Null Quaternionic Curves in Minkowski 3-space

Tanju Kahraman

Manisa Celal Bayar University, Faculty of Arts and Sciences, Department of Mathematics, Şehit Prof. Dr. İlhan Varank Campus 45140 Manisa / Turkey.
tanju.kahraman@cbu.edu.tr

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Abstract: *In this paper, we define Smarandache curves of null quaternionic curves in the semi-Euclidean space* E_1^3 *and obtaine that curvatures of null quaternionic curves have some relations for Smarandache curves.*

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Keywords: *Null Quaternionic Curves; Smarandache curves; Serret-Frenet formulae.*

Minkowski 3-uzayında Null Kuaterniyonik Eğrilerin Smarandache Eğrileri

Özet: *Bu çalışmada,* E_1^3 *semi-Öklidyen uzaydaki null kuaterniyonik eğrilerin Smarandache eğrilerini tanımlarız ve null kuaterniyonik eğrilerin eğriliklerinin, Smarandache eğriler için bazı özelliklere sahip olduğunu elde ederiz.*

MSC: 11R52, 53A35, 53B30.

Anahtar Kelimeler: *Null Kuaterniyonik Eğriler; Smarandache Eğriler; Serret-Frenet formülleri*

1. INTRODUCTION

In the study of the fundamental theory and the characterizations of space curves, the special curves are very interesting and an important problem. The most mathematicians studied the special curves such as Mannheim curves and Bertrand curves. Recently, a new special curve which is called Smarandache curve is defined by Turgut and Yılmaz in Minkowski space-time [14]. Ali have studied Smarandache curves in the Euclidean 3-space E^3 [1]. Later, Smarandache curves are worked strikingly by many mathematicians on Euclid and Minkowski spaces [3,7-13].

Quaternions were introduced by Hamilton. His initial attempt to generalize the complex numbers by introducing a three-dimensional object failed in the sense that the algebra he constructed for these three-dimensional object did not have the desired properties. On the 16th October 1843, Hamilton discovered that the appropriate generalization is one in which the scalar(real) axis is left unchanged whereas the vector(imaginary) axis is supplemented by adding two further vectors axes.

In 1987, the Serret-Frenet formulas for a quaternionic curve in E^3 and E^4 was defined by Bharathi and Nagaraj [2] and then in 2004, Serret-Frenet formulas for quaternionic curves and quaternionic inclined curves have been defined in Semi-Euclidean space by Çöken and Tuna [4]. In 2015, Serret-Frenet formulas for null quaternionic curves was defined in semi-Euclidean spaces by Çöken and Tuna [5,6].

In this paper, we define Smarandache curves of null quaternionic curves in the semi-Euclidean space E_1^3. Moreover, we obtain that curvatures of null quaternionic curves have some relations for Smarandache curves.

2. PRELIMINARIES

In this section we give basic concepts related to the semi-real quaternions. For more detailed information, we refer ref. [5].

The set of semi-real quaternions is given by

$$Q = \{ q \mid \quad q = ae_1 + be_2 + ce_3 + d\,; \quad a,b,c,d \in IR \}$$

where $e_1, e_2, e_3 \in E_1^3$, $h(e_i, e_i) = \varepsilon(e_i), \quad 1 \le i \le 3$ and

$$e_i \times e_i = -\varepsilon(e_i)\,,$$
$$e_i \times e_j = \varepsilon(e_i)\varepsilon(e_i)e_k \in E_1^3\,.$$

The multiplication of two semi real quaternions p and q are defined by

$$p \times q = S_p S_q + S_p V_q + S_q V_p + h\left(V_p, V_q\right) + V_p \wedge V_q$$

Here, we have used the inner and cross products in semi-Euclidean space E_1^3. For a semi real quaternion $q = ae_1 + be_2 + ce_3 + d$, conjugate αq of q and inner product h_1 are defined by $\alpha q = -ae_1 - be_2 - ce_3 + d$ and

$$h(p,q) = \frac{1}{2}\left[\varepsilon(p)\varepsilon(\alpha q)\left(p \times \alpha q\right) + \varepsilon(q)\varepsilon(\alpha p)\left(q \times \alpha p\right) \right]$$

respectively.

The three dimensional semi-Euclidean space E_1^3 is identified with the space of null spatial quaternions $\left\{\gamma \in Q_{E_1^3} \middle| \quad \gamma + \alpha\gamma = 0\right\}$ in an obvious manner,

$$\gamma(s) = \sum_{i=1}^{3} \gamma_i(s) e_i, \quad 1 \le i \le 3 .$$

Let $\{l, n, u\}$ be the Frenet trihedron of the differentiable null spatial quaternionic curve in E_1^3 and e_2 be timelike vector. Then, Frenet formulae are

$$\begin{bmatrix} l' \\ n' \\ u' \end{bmatrix} = \begin{bmatrix} 0 & 0 & k \\ 0 & 0 & \tau \\ -\tau & -k & 0 \end{bmatrix} \begin{bmatrix} l \\ n \\ u \end{bmatrix}$$

where k and τ are curvatures of null spatial quaternionic curve and $h(l,l) = h(n,n) = h(l,u) = h(n,u) = 0$, $h(l,n) = h(u,u) = 1$, l and n are null vectors and u is a spacelike vector. Herein, the quaternion product is given by

$$\begin{aligned} &l \times n = -1 - u, \quad n \times l = -1 + u, \quad n \times u = -n, \quad u \times n = n \\ &u \times l = -l, \quad l \times u = l, \quad u \times u = -1 \ , \quad l \times l = n \times n = 0 \end{aligned} .$$

3. SMARANDACHE CURVES OF NULL QUATERNIONIC CURVES IN MINKOWSKI 3-SPACE

In this section, we first define the four different type of the Smarandache curves of null quaternionic curves in E_1^3. Then, by the aid of this frame, we give the characterizations for Smarandache curves of null quaternionic curves.

3.1. Smarandache ln-Curves of Null Quaternionic Curves

Definition 3.1.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve and $\{l, n, u\}$ be its moving Frenet frame. The curve γ_1 defined by

$$\gamma_1(s) = \frac{1}{2}\big(l(s) + n(s)\big) \tag{1}$$

is called the Smarandache ln-curve of γ and γ_1 fully lies on Lorentzian sphere S_1^2.

Now, we can give the relationships between γ and its Smarandache ln-curve γ_1 as follows.

Theorem 3.1.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve. If curve γ_1 is Smarandache ln-curve of γ. Then, the relation between curvatures of γ holds that

$$k = \tau . \tag{2}$$

Proof. Let curve γ_1 be Smarandache ln-curve of γ. Then, (1) is hold. Differentiating the equation (1) with respect to s, we get

$$l_1 \frac{ds_1}{ds} = \frac{1}{2}(k-\tau)u\,. \tag{3}$$

Taking the inner product of (3) by itself, desired equality is obtained.

3.2. Smarandache lu-Curves of Null Quaternionic Curves

Definition 3.2.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve and $\{l, n, u\}$ be its moving Frenet frame. The curve γ_2 defined by

$$\gamma_2(s) = l(s) + u(s) \tag{4}$$

is called the Smarandache lu-curve of γ and γ_2 fully lies on Lorentzian sphere S_1^2.
Now, we can give the relationships between γ and its Smarandache lu-curve γ_2 as follows.

Theorem 3.2.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve. If curve γ_2 is Smarandache lu-curve of γ. Then, the relation between curvatures of γ holds that

$$k = 2\tau\,. \tag{5}$$

Proof. Let curve γ_2 be Smarandache lu-curve of γ. Then, differentiating the equation (4) with respect to s, we get

$$l_2 \frac{ds_2}{ds} = -\tau l + k\,n + k\,u\,. \tag{6}$$

Taking the inner product of (6) by itself, we obtain the desired equality.

3.3. Smarandache nu-Curves of Null Quaternionic Curves

Definition 3.3.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve and $\{l, n, u\}$ be its moving Frenet frame. The curve γ_3 defined by

$$\gamma_3(s) = n(s) + u(s) \tag{7}$$

is called the Smarandache nu-curve of γ and γ_3 fully lies on Lorentzian sphere S_1^2.
Now, we can give the relationships between γ and its Smarandache nu-curve γ_3 as follows.

Theorem 3.3.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve. If curve γ_3 is Smarandache nu-curve of γ. Then, the relation between curvatures of γ holds that

$$\tau = 2k\,. \tag{8}$$

Proof. Let curve γ_3 be Smarandache nu-curve of γ. Then, differentiating the equation (7) with respect to s, we get

$$l_2 \frac{ds_2}{ds} = -\tau l + k\,n + k\,u. \tag{9}$$

Taking the inner product of (9) by itself, we have equation (8).

3.4. Smarandache lnu-Curves of Null Quaternionic Curves

Definition 3.4.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve and $\{l, n, u\}$ be its moving Frenet frame. The curve γ_4 defined by

$$\gamma_4(s) = \frac{1}{3}\left(l(s) + n(s) + u(s)\right) \tag{10}$$

is called the Smarandache lnu-curve of γ and γ_4 fully lies on Lorentzian sphere S_1^2.
Now, we can give the relationships between γ and its Smarandache lnu-curve γ_4 as follows.
Theorem 3.4.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve. If curve γ_4 is Smarandache lnu-curve of γ. Then, the relation between curvatures of γ holds that

$$(k - \tau)^2 = 2\tau k. \tag{11}$$

Proof. Let curve γ_4 be Smarandache lnu-curve of γ. Then, differentiating the equation (10) with respect to s, we get

$$l_4 \frac{ds_4}{ds} = \frac{1}{3}\left(-\tau l + k\,n + (k - \tau)u\right). \tag{12}$$

Taking the quaternionic inner product of (12) by itself, we have equation (11).

Corollary 3.4.1. Let $\gamma = \gamma(s)$ be a unit speed null quaternionic curve. If curve γ_4 is Smarandache lnu-curve of γ, then, curvatures of null quaternionic curve γ are not equal.

Proof. From (11), it is clear.

4. CONCLUSION

To get the Smarandache curves of null quaternionic curves, Curvatures of null quaternionic curves must be non-zero. Otherwise, can not talk about Smarandache curves. Also, there is some relations between curvatures of null quaternionic curves for Smarandache curves.

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