A NEW CHARACTERIZATION OF SMARANDACHE TNB CURVES OF HELICES IN THE SOL SPACE Sol³

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Abstract

In this paper, we characterize Smarandache TNB curves of helices in the Sol space Sol³. We characterize Smarandache TNB curves of helices in terms of their curvature and torsion. Finally, we find out their explicit parametric equations.

Keywords: General helix, Sol Space, Curvature, Torsion, Smarandache TNB curve.
1. INTRODUCTION

A fundamental advance in theory of curves was the advent of analytic geometry in the seventeenth century. This enabled a curve to be described using an equation rather than an elaborate geometrical construction. This not only allowed new curves to be defined and studied, but it enabled a formal distinction to be made between curves that can be defined using algebraic equations, algebraic curves. Some curves and surfaces have been also represented as motion by several authors [1-7].

The geometry of the Galilean Relativity works such as a bridge from Euclidean geometry to special Relativity. The geometry of curves in Euclidean space have been developed in the past [4]. In modern times, mathematicians have started to research curves and surfaces some different spaces [8-24].

Helices are among easy and simple styles that are located in the filamentary and molecular improvements of mechanics. A nearby physical elements of such components have a inclination to be made by way of a standard elastic potential energy dependent on bending and opinion, which is accurately what we call a pole version.

In this paper, we study Smarandache TNB curves of helices in the Sol³. We characterize Smarandache TNB curves of helices in terms of their curvature and torsion. Finally, we find out their explicit parametric equations.

2. MATERIAL AND METHODS

Sol space, one of Thurston’s eight 3-dimensional geometries, can be viewed as $\mathbb{R}^3$ provided with Riemannian metric

$$g_{Sol³} = e^{2z}dx^2 + e^{-2z}dy^2 + dz^2,$$

where $(x, y, z)$ are the standard coordinates in $\mathbb{R}^3$.

Note that the Sol metric can also be written as [25]:

$$g_{Sol³} = \sum_{i=1}^{3} \omega^i \otimes \omega^i,$$

where

$$\omega^1 = e^z dx, \quad \omega^2 = e^{-z} dy, \quad \omega^3 = dz,$$

and the orthonormal basis dual to the 1-forms is

$$e_1 = e^{-z} \frac{\partial}{\partial x}, \quad e_2 = e^z \frac{\partial}{\partial y}, \quad e_3 = \frac{\partial}{\partial z}.$$
Assume that \( \{T, N, B\} \) be the Frenet frame field along \( \gamma \). Then, the Frenet frame satisfies the following Frenet–Serret equations \([26, 27]\):

\[
\begin{align*}
\nabla_T T &= \kappa N, \\
\nabla_T N &= -\kappa T + \tau B, \\
\nabla_T B &= -\tau N,
\end{align*}
\]

where \( \kappa \) is the curvature of \( \gamma \) and \( \tau \) its torsion and

\[
\begin{align*}
g_{\text{SoS}}(T, T) &= 1, \\
g_{\text{SoS}}(N, N) &= 1, \\
g_{\text{SoS}}(B, B) &= 1, \\
g_{\text{SoS}}(T, N) &= g_{\text{SoS}}(T, B) = g_{\text{SoS}}(N, B) = 0.
\end{align*}
\]

With respect to the orthonormal basis \( \{e_1, e_2, e_3\} \), we can write

\[
\begin{align*}
T &= T_1 e_1 + T_2 e_2 + T_3 e_3, \\
N &= N_1 e_1 + N_2 e_2 + N_3 e_3, \\
B &= T \times N = B_1 e_1 + B_2 e_2 + B_3 e_3.
\end{align*}
\]

**Theorem 2.1.** \((28)\) Let \( \gamma : I \rightarrow \text{SoS}^3 \) be a unit speed non-geodesic general helix. Then, the parametric equations of \( \gamma \) are

\[
\begin{align*}
x(s) &= \sin P e^{-\cos P s - C_3} \left[ -\cos P \cos(C_1 s + C_2) + C_4 \sin(C_1 s + C_2) \right] + C_4, \\
y(s) &= \sin P e^{\cos P s + C_3} \left[ -C_1 \cos(C_1 s + C_2) + \cos P \sin(C_1 s + C_2) \right] + C_5, \\
z(s) &= \cos P s + C_3,
\end{align*}
\]

where \( C_1, C_2, C_3, C_4, C_5 \) are constants of integration.
3. RESULTS AND DISCUSSION

Definition 3.1. Let $\gamma : I \rightarrow \text{Sol}^3$ be a unit speed helix in the Sol Space $\text{Sol}^3$ and $\{T, N, B\}$ be its moving Frenet frame. Smarandache TNB curves are defined by

$$\gamma_{TNB} = \frac{1}{\sqrt{2\kappa^2 - 2\kappa\tau + \tau^2}} (T + N + B).$$

Theorem 3.2. Let $\gamma : I \rightarrow \text{Sol}^3$ be a unit speed non-geodesic helix in the Sol Space $\text{Sol}^3$. Then, the equation of Smarandache TNB curve of a unit speed non-geodesic helix is given by

$$\gamma_{TNB} = W[\sin P \cos [C, s + C_2] + \frac{1}{\kappa'} [\frac{1}{C_1} \sin P \sin [C, s + C_2] + \cos P \sin P \cos [C, s + C_2]]]$$

$$+ [\frac{1}{\kappa} \sin P \sin [C, s + C_2] \sin^2 P \sin^2 [C, s + C_2] - \sin^2 P \cos^2 [C, s + C_2]]$$

$$- \frac{1}{\kappa} \cos P [\frac{1}{C_1} \sin P \cos [C, s + C_2] - \cos P \sin P \sin [C, s + C_2]]]e_1$$

$$+ W[\sin P \sin [C, s + C_2] + \frac{1}{\kappa'} [\frac{1}{C_1} \sin P \cos [C, s + C_2] - \cos P \sin P \sin [C, s + C_2]]]$$

$$- [\frac{1}{\kappa} \sin P \cos [C, s + C_2] \sin^2 P \sin^2 [C, s + C_2] - \sin^2 P \cos^2 [C, s + C_2]]$$

$$- \frac{1}{\kappa} \cos P [\frac{1}{C_1} \sin P \sin [C, s + C_2] + \cos P \sin P \cos [C, s + C_2]]]e_2$$

$$+ W[\cos P + \frac{1}{\kappa'} [\sin^2 P \sin^2 [C, s + C_2] - \sin^2 P \cos^2 [C, s + C_2]]$$

$$+ \frac{1}{\kappa} \sin P \cos [C, s + C_2] \frac{1}{C_1} \sin P \cos [C, s + C_2] - \cos P \sin P \sin [C, s + C_2]]$$

$$- \frac{1}{\kappa} \sin P \sin [C, s + C_2] \frac{1}{C_1} \sin P \sin [C, s + C_2] + \cos P \sin P \cos [C, s + C_2]]]e_3,$$

where $C_1, C_2$ are constants of integration and

$$W = \frac{1}{\sqrt{2\kappa^2 - 2\kappa\tau + \tau^2}}.$$
Corollary 3.3. Let \( \gamma : I \rightarrow \text{Sol}^3 \) be a unit speed non-geodesic helix in the Sol Space \( \text{Sol}^3 \). Then, the parametric equations of Smarandache TNB curves of a unit speed non-geodesic helix are given by

\[
x_{\text{TNB}}(s) = \exp[-W[\cos P + \frac{1}{\kappa}([\sin^2 P \sin^2 [C_1 s + C_2] - \sin^2 P \cos^2 [C_1 s + C_2]]) + \frac{1}{\kappa} \sin P \cos[C_1 s + C_2] - \frac{1}{C_1} \sin P \sin[C_1 s + C_2] - \frac{1}{\kappa} \sin P \sin[C_1 s + C_2] + \cos P \sin P \cos[C_1 s + C_2]])] \\
+y_{\text{TNB}}(s) = \exp[W[\cos P + \frac{1}{\kappa}([\sin^2 P \sin^2 [C_1 s + C_2] - \sin^2 P \cos^2 [C_1 s + C_2]]) + \frac{1}{\kappa} \sin P \cos[C_1 s + C_2] - \frac{1}{C_1} \sin P \sin[C_1 s + C_2] - \frac{1}{\kappa} \sin P \sin[C_1 s + C_2] + \cos P \sin P \cos[C_1 s + C_2]])] \\
z_{\text{TNB}}(s) = W[\cos P + \frac{1}{\kappa}([\sin^2 P \sin^2 [C_1 s + C_2] - \sin^2 P \cos^2 [C_1 s + C_2]]) + \frac{1}{\kappa} \sin P \cos[C_1 s + C_2] - \frac{1}{C_1} \sin P \sin[C_1 s + C_2] - \frac{1}{\kappa} \sin P \sin[C_1 s + C_2] + \cos P \sin P \cos[C_1 s + C_2]])],
\]

where \( C_1, C_2 \) are constants of integration and

\[
W = \frac{1}{\sqrt{2\kappa^2 - 2\kappa \tau + 2\tau^2}}.
\]
REFERENCES


