A Study of The Fundamentals of Hypersoft Set Theory

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Abstract- In this study, we discuss the fundamentals of hypersoft set such as hypersoft subset, complement, not hypersoft set and aggregation operators. After that we discuss the hypersoft set relation, sub relation, complement relation, function, matrices and operations on hypersoft matrices.

Keywords-Hypersoft set, hypersoft subset, complement, not set, aggregation operators, hypersoft set relation, sub relation, complement relation, function, hypersoft matrices.

1 Introduction

The soft-set concept was developed by [1] as a completely new math tool for solving difficulty in dealing with uncertainty. Molodtsov[1] defined a soft set that is sub-set as a parameterized family of the set of the universe where each element is considered a set approximate elements of the soft set. In the past few years, the fundamentals of soft set theory have been studied by different researchers. Maji et al.[2] presented a detailed theoretical study of soft sets, which includes subsets and super set of soft sets, equations of soft sets, operation soft sets such as unions, intersections, and more among others.

He also studied and talked the main features of these operations. Pei and Miao [3] discussed the relationship between soft sets and information system. Ali et al.[4] introduced something new operations such as restricted union, restricted intersection, restricted spacing and extension discuss the intersection of two soft sets and their basics characteristics. A gentle development by Cagman and Enginoglu [5] in matrix theory and successfully applied it to a decision making the problem. Babitha and Sunil [6] discussed the concept of soft-set relation and function. Also, Many related concepts like equivalence soft, soft relationships, soft sets distribution, ordering on soft sets. In the continuation of their work, Babita and Sunil [7] more introduced the work on soft set relation, anti-symmetric relation and transitive closure. A soft-set relationship was introduced by Yang and Guo [8]. Sezgin, A. and Atagım [9]. Ge and Yang [10], Fu Li[11] et al. modified some of Maji’s et al. [2] work and also set some new results. Sezgin and Atagım [9] also introduced a restricted symmetric difference of soft sets and examples. Singh and Onyeozili [12] obtained some results with respect to distribution and absorption properties with respect to different operations on soft sets. Singh and Onyeozili [12] also proved that the actions apply on soft sets is similar to actions on soft matrices. After that Florentin Smarandache a pioneer came forward and introduced many results about hypersoft sets. He opened many fields in this area. In this paper, we discuss the fundamentals of hypersoft set such as hypersoft subset, complement, not standard aggregation operators. After that we discuss the hypersoft set relation, sub relation, complement relation, function, matrices and operations on hypersoft matrices. The rest of this article is structured as follows: Section 2 gives some basic definitions and
results on hypersoft sets. Section 3 discusses the various works of relaxation in detail. Section 4 describes many hypersoft set properties without proof set operation. Section 5 focuses on hypersoft set relationships and functions. Final section 6 which consists of two the first sections talk about soft matrix and its basics The second section focuses on work their characteristics.

2 Preliminaries

2.1 Hypersoft set

Let \( a_1, a_2, a_3, \ldots, a_n \) be the distinct attributes whose attribute values belongs to the sets \( A_1, A_2, A_3, \ldots, A_n \) respectively, where \( A_i \cap A_j = \emptyset \) for \( i \neq j \). A pair \( (\Phi, E_1 \ast E_2 \ast E_3 \ast \cdots \ast E_n) \) is called a hypersoft set over the universal set \( U \), where \( \Phi \) is the mapping given by \( \Phi : E_1 \ast E_2 \ast E_3 \ast \cdots \ast E_n \rightarrow P(U) \)

Example 1 Let \( U = \{R_1, R_2, R_3, R_4, R_5\} \) is universal set, where \( R_1, R_2, R_3, R_4, R_5 \) represents the refrigerator. Mr. X, Mrs. X goes to market and wants to buy such a refrigerator which is feasible and having more characteristics then that their expectation level. Let \( a_1 = \text{Size}, a_2 = \text{Pressure}, a_3 = \text{Freezing point}, a_4 = \text{Price}, \) be the attributes whose attribute values belongs to the sets \( B_1, B_2, B_3, B_4 \) given as \( B_1 = \{\text{small} = e_1, \text{medium} = e_2, \text{large} = e_3\} \) \( B_2 = \{\text{Low freezing point} = e_4\} \) and hypersoft set can be written as

\[
\left(\Phi, A_1 \ast A_2 \ast A_3 \ast A_4\right) = \{\Phi(e_1, e_4, e_5, e_7), \Phi(e_1, e_4, e_6, e_7), \Phi(e_3, e_4, e_5, e_7)\} = \{(R_1, R_2, R_3), \{R_1, R_2, R_4\}, \{R_3, R_5\}, \{R_1, R_2, R_3\}\}
\]

2.2 Hypersoft subset

Assume that \( (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \) and \( (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \) be the two hypersoft sets over the same universal sets \( U \). (a) \( (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \) is the hypersoft subset of \( (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \) denoted by \( (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \subseteq (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \) if (i) \( A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \subseteq B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n \) and (ii) \( \forall \epsilon \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \phi(\epsilon) \) and \( \Psi(\epsilon) \) are identical approximations. (b) \( (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \) is hypersoft equal set to \( (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \) and it is denoted by \( (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) = (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \) if \( (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \subseteq (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \) and \( (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \subseteq (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \).

Example 2 Let \( U = \{R_1, R_2, R_3, R_4, R_5\} \) is universal set, where \( R_1, R_2, R_3, R_4, R_5 \) represents the refrigerator. Let \( a_1 = \text{Size}, a_2 = \text{Pressure}, a_3 = \text{Freezing point}, a_4 = \text{Price}, \) be the attributes whose attribute values belongs to the sets \( B_1, B_2, B_3, B_4 \) given as \( B_1 = \{\text{small} = e_1, \text{medium} = e_2, \text{large} = e_3\} \) \( B_2 = \{\text{Low freezing point} = e_4\} \) \( B_3 = \{\text{High expectation pressure} = e_5\} \) \( B_4 = \{\text{Low condensing pressures} \neq e_6\} \) \( B_5 = \{\text{Low price} \neq e_7\} \)

2.3 Fuzzy hypersoft set

Let \( F(U) \) be the set of all fuzzy subsets in the universal set \( U \), let \( E_1 \ast E_2 \ast E_3 \ast \cdots \ast E_n \) be a set of parameters. A pair \( (\Phi, E_1 \ast E_2 \ast E_3 \ast \cdots \ast E_n) \) is called a fuzzy hypersoft set over \( U \), where \( F \) is the mapping given by \( \Phi : E_1 \ast E_2 \ast E_3 \ast \cdots \ast E_n \rightarrow F(U) \) in general, \( \Phi(\epsilon) = \{(x, \Phi(\epsilon)/x \in U) \} \) where \( \epsilon \in E_1 \ast E_2 \ast E_3 \ast \cdots \ast E_n \). It is very convenient to see that every fuzzy hypersoft set can be seen as an (fuzzy) information system and it can be represented in an data table belonging to the unit interval \([0, 1]\).

Example 3 Let \( U = \{R_1, R_2, R_3, R_4, R_5\} \) is universal set, where \( R_1, R_2, R_3, R_4, R_5 \) represents refrigerator. Let \( a_1 = \text{Size}, a_2 = \text{Pressure}, a_3 = \text{Freezing point}, \)
Remark 4
One can easily notice that from the definition of hypersoft fuzzy set
\[
\Phi(e) = \{0.5/u_2, 0.9/u_4\},
\]
we have
\[
F(e_1, e_2, e_3, e_4, e_5, e_6, e_7) = \{(e_1, e_4, e_5, e_7), (e_3, e_4, e_6, e_7), 0.5/u_1, 0.9/u_3\}.
\]

2.5 Complement of hypersoft set
If \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) be the hypersoft set and complement is denoted by \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)^c\) and it is defined in such a way that \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)^c = (\Phi', \neg A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\), where \(\Phi : A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \rightarrow P(U)\) be a mapping as follows
\[
\Phi(c)(\alpha) = U - \Phi(\neg \alpha), \forall \alpha \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n
\]

Example 6 Let \(U = \{R_1, R_2, R_3, R_4, R_5\}\) is universal set and \(A_1 \ast A_2 \ast A_3 \ast A_4\) be the set of parameters. Now we defined the hypersoft set on it
\[
(\Phi, A_1 \ast A_2 \ast A_3 \ast A_4) = \{(e_1, e_4, e_5, e_7), e_6, (e_3, e_4, e_5, e_7), (e_1, e_4, e_5, e_7), (e_3, e_4, e_5, e_7), (e_1, e_4, e_5, e_7), (e_3, e_4, e_5, e_7)\}
\]

2.4 Not Hypersoft
Let \(A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n\) be a set of parameters and not set is denoted by \(\neg A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n\)

Example 5 If
\[
(A_1 \ast A_2 \ast A_3 \ast A_4) = \{(e_1, e_4, e_5, e_7), (e_2, e_4, e_5, e_7), (e_3, e_4, e_5, e_7)\}
\]
and not set can be written as
\[
\neg (A_1 \ast A_2 \ast A_3 \ast A_4) = \{(e_1, e_4, e_5, e_7), (e_2, e_4, e_5, e_7), (e_3, e_4, e_5, e_7)\}
\]

2.6 Relative Complement of hypersoft
If \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) be the hypersoft set and relative complement is denoted by \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)^c\) and it is defined in such a way that \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)^c = (\Phi', \neg A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\), where \(\Phi : A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \rightarrow P(U)\) be a mapping as follows
\[
\Phi(c)(\alpha) = U - \Phi(\neg \alpha), \forall \alpha \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n
\]

Example 7 Let \(U = \{R_1, R_2, R_3, R_4, R_5\}\) is universal set and \(A_1 \ast A_2 \ast A_3 \ast A_4\) be the set of parameters. Now we defined the hypersoft set on it
\[
(\Phi, A_1 \ast A_2 \ast A_3 \ast A_4) = \{(e_1, e_4, e_5, e_7), (e_2, e_4, e_5, e_7), (e_3, e_4, e_5, e_7)\}
\]
then relative complement of it is

\[
(\Phi, A_1 \ast A_2 \ast A_3 \ast A_4)^c = (\Phi, A_1 \ast A_2 \ast A_3 \ast A_4)'
\]

Proposition 8 Assume that If \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) be the hypersoft set over the universe \(U\). Then

1. \(((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)c)^c = (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\)

2. \(((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)c)^c = (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\)

3. \(U_{A_1 \ast A_2 \ast A_3 \cdots \ast A_n} = \Phi_{A_1 \ast A_2 \ast A_3 \cdots \ast A_n}\)

4. \(\Phi_{A_1 \ast A_2 \ast A_3 \cdots \ast A_n} = U_{A_1 \ast A_2 \ast A_3 \cdots \ast A_n}\)

2.7 Aggregation operator of hypersoft sets

Union of hypersoft sets

Assume that \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) and \((\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) be the two hypersoft sets over the same universal sets \(U\), then union between them is denoted by \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \cup (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) is hypersoft set \((F, C)\), where \(C = (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \cup (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) and \(\forall e \in C \: F(e) = \Phi(e) \cup \Psi(e)\)

Example 10 Let \(U = \{R_1, R_2, R_3, R_4, R_5\}\) is universal set and \(A_1 \ast A_2 \ast A_3 \ast A_4\) and \(B_1 \ast B_2 \ast B_3 \ast B_4\) be the set of parameters. Now we defined the hypersoft sets on it,

\[
(\Phi, A_1 \ast A_2 \ast A_3 \ast A_4) = \{(e_1, e_4, e_5, e_7), (R_1; R_2), (e_3, e_4, e_6, e_7), (R_1, R_3)\}
\]

and

\[
(\Psi, B_1 \ast B_2 \ast B_3 \ast B_4) = \{(e_1, e_4, e_6, e_7), (R_4, R_5)\}
\]

Intersection of hypersoft sets

Assume that \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) and \((\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) be the two hypersoft sets over the same universal sets \(U\), then intersection between them is denoted by \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \cap (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) is hypersoft set \((F, C)\), where \(C = (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \cap (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) and \(\forall e \in C \: F(e) = \Phi(e) \cap \Psi(e)\)

Example 9 Let \(U = \{R_1, R_2, R_3, R_4, R_5\}\) is universal set and \(A_1 \ast A_2 \ast A_3 \ast A_4\) and \(B_1 \ast B_2 \ast B_3 \ast B_4\) be the set of parameters. Now we defined the hypersoft sets on it,

\[
(\Phi, A_1 \ast A_2 \ast A_3 \ast A_4) = \{(e_1, e_4, e_5, e_7), (R_1; R_2), (e_3, e_4, e_6, e_7), (R_1, R_3)\}
\]

and

\[
(\Psi, B_1 \ast B_2 \ast B_3 \ast B_4) = \{(e_1, e_4, e_6, e_7), (R_4, R_5)\}
\]
Then intersection between them is given as follows
\[(\Phi, A_1 \ast A_2 \ast A_3 \ast A_4) \cap (\Psi, B_1 \ast B_2 \ast B_3 \ast B_4) = \{(e_3, e_4, e_5, e_7), (R_3)\}\]

2.8 Hypersoft Set Relation and Function

Cartesian Product of hypersoft sets

Assume that \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) and \((\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) be the two hypersoft sets over the same universal sets \(U\). Then the Cartesian product is denoted by \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) = \{(F, (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n))\}, \text{where } F : (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \cap (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \rightarrow P(U \ast U) \text{ and } F(x, y) = \Phi(x) \ast \Psi(y), \forall (x, y) \in (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)
\]

Hypersoft Set Relation

\[(\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \text{ be the two hypersoft sets over the same universal sets } U. \text{ Then the relation from } (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \text{ to } (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \text{ is called a hypersoft set relation (}R, C) \text{ or it is simple way } R \text{ is a hypersoft subset and it is denoted by } (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n), \text{ where } C \subseteq (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \text{ and } \forall (x, y) \in C \text{, } R(x, y) = H(x, y), \text{ where } x = (a_1, a_2, a_3, \cdots, a_n) \text{ and } y = (b_1, b_2, b_3, \cdots, b_n) \text{ and } (H, (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)) = (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n). \text{ A hypersoft set relation on } (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \text{ is a hypersoft subset of } (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n). \text{ In similar way, the parameterized form of relation } R \text{ on the hypersoft set } (F, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \text{ is defined as follows. If } (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) = (\Phi(a), (b), \text{ then } \Phi(a) \ast \Phi(b) \in R \text{ Domain, Range and Inverse of Hypersoft Set \}}

Let \(R\) be the hypersoft set relation from \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) to \((\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) such that \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n)\) \(= (F, (A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \ast (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n))\) then \(a\) The domain of \(R(\text{dom } R)\) is the hypersoft set \((D, C_1 \ast C_2 \ast C_3 \ast \cdots \ast C_n) \subseteq (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n)\) \[\text{where } C_1 \ast C_2 \ast C_3 \ast \cdots \ast C_n = F(x, y) \in R \text{ for some } x \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \text{ and } y \in (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \text{ and } D(a) = \Phi(a) \forall a \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \text{ (b) The range of } R(\text{ran } R) \text{ is the hypersoft set } (D, X_1 \ast X_2 \ast \cdots \ast X_n) \subseteq B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n \text{ where } X_1 \ast X_2 \ast X_3 \ast \cdots \ast X_n = H(x, y) \in R \text{ for some } x \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \text{ and } y \in (B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \text{ and } E(b_1) = (G(a)) \forall b_1 \in X_1 \ast X_2 \ast \cdots \ast X_n \text{ (c) The inverse of } R \text{ denoted by } R^{-1} \text{ is a hypersoft set relation from } (\Psi, B_1 \ast B_2 \ast B_3 \ast \cdots \ast B_n) \text{ to } (\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n) \text{ defined by } R^{-1} = \{((\Psi(y), (x)) : (x) \ast (\Psi(y))\}

2.8.1 Sub relation

Let \(R_1, R_2\) be two hypersoft relations on a hypersoft set \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n), R_1 \subseteq R\), \(\forall x, y \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \Phi(x) \ast \Phi(y) \in R_1 \Rightarrow (\Phi(x) \ast \Phi(y) \in R_2\)

Complement of Relation

Let \(R_1\) be a hypersoft relations on a hypersoft set \((\Phi, A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n), \text{ then complement of } R_1 \text{ is } R'_1 = \{(\Phi(x) \ast \Phi(y) : (x) \ast (\Phi(y)) \notin R_1\}

Example 11: Let \(U\) denotes the washing machine \(U = \{w_1, w_2, w_3, w_4\}\) , let \(a_1 = \text{size }, a_2 = \text{color}, a_3 = \text{country made}, \text{ be the attributes whose attribute values belongs to } A_1 = \{\text{Small} = c_1, \text{Medium} = c_2, \text{Large} = c_3\}, A_2 = \{\text{White} = e_4, \text{Yellow} = e_5\}, A_3 = \{\text{Pakistan} = e_6, \text{Japan} = e_7\} \text{ respectively. Then the hypersoft set is given by } (\Phi, A_1 \ast A_2 \ast A_3) = (\Phi(e_1, c_1, e_6) = \{(w_1, w_2), \Phi(e_2, c_4, e_7) = \{(w_1, w_4), \Phi(e_3, e_5, e_6) = \{(w_2, w_3)\}, \text{ let } b_1 = \text{Services, b_2 = vacuum system, be the attributes whose attribute values belongs to B_1 = \{Self services system = e_1', \text{ Water recycling = e_2'}, \text{ and B_2 = \{Excellent vacuum system = e_3', \text{ Self service= e_4'}\}} \text{ respectively. Then the hypersoft set is given by } (\Phi, B_1 \ast B_2) = (\Phi(e_1', e_3') = \{(w_1, w_2, w_3), \Phi(e_4', e_5') = \{(w_2, w_3, w_4), \Phi(e_6', e_7') = \{(w_2, w_4)\}, \text{ if we want to define a relation R from } (\Phi, A_1 \ast A_2 \ast A_3) \text{ to } (\Psi, B_1 \ast B_2) \text{ in such a way } \Phi(x) \ast \Phi(y) \text{ then } a) R = \{\Phi(e_1, e_4, e_6) \ast \Phi(e_1', e_3'), \Phi(e_3, e_5, e_6) \ast \Phi(e_2', e_7')\} \text{ (b) Dom R = (D, C_1 \ast C_2 \ast C_3) = \{(e_1, c_1, e_6), (e_3, c_5, e_6) \subseteq A_1 \ast A_2 \ast A_3\}, \text{ and } D(a) = \Phi(a) \forall a \in A_1 \ast A_2 \ast A_3 \text{ (c) Ran R = } (E, X_1 \ast X_2) \ast \{(e_1', e_3'), (e_2', e_7') \subseteq B_1 \ast B_2\)}

5
2.9 Hypersoft function

Let \((\Phi, A_1 * A_2 * A_3 * \cdots * A_n)\) and \((\Psi, B_1 * B_2 * B_3 * \cdots * B_n)\) be the two hypersoft sets over the same universal sets \(U\). Then the hypersoft relation from \((\Phi, A_1 * A_2 * A_3 * \cdots * A_n)\) to \((\Psi, B_1 * B_2 * B_3 * \cdots * B_n)\) defined as \(h : (\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \rightarrow (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)\) is called a hypersoft set function. If every element of domain has unique element in range of \(h\). If it is closed \(\Phi(x)h\Psi(y)\) i.e \(\Phi(x) * \Psi(y) \in h\) for \(x \in A_1 * A_2 * A_3 * \cdots * A_n\) and \(y \in B_1 * B_2 * B_3 * \cdots * B_n\) then we can represent it in the form \(h(\Phi(x)) = \Psi(y)\)

Example 12 From the previous example the hypersoft function over the universe \(U\) is given by

\[(i) \quad h = \{\Phi(e_1), \Psi(e_1'), \Phi(e_2), \Psi(e_2'), \Phi(e_3), \Psi(e_3')\}\]

\[(ii) \quad h = \{\Phi(e_2), \Psi(e_2'), \Phi(e_3), \Psi(e_3')\}\]

but \(\{\Phi(e_1), \Psi(e_1'), \Phi(e_1), \Psi(e_2), \Phi(e_2), \Psi(e_2')\}\) is not function

One-One, Onto, Bijection

A function from \((\Phi, A_1 * A_2 * A_3 * \cdots * A_n)\) to \((\Psi, B_1 * B_2 * B_3 * \cdots * B_n)\) is said to be (i) Injective(One to one), \(\Phi(x) \neq \Phi(y) \implies h(\Phi(x)) \neq h(\Phi(y))\) (ii) Surjective(On to), If range \(h = (\Psi, B_1 * B_2 * B_3 * \cdots * B_n)\) (iii) Bijective, If \(h\) is both one-one and onto.

Example 13 From the previous example (i) is on to but (ii) is not onto

2.10 Identity hypersoft function

The identity hypersoft function \(I\) on a hypersoft set \((\Phi, A_1 * A_2 * A_3 * \cdots * A_n)\) is defined as \(I : (\Phi, A_1 * A_2 * A_3 * \cdots * A_n) \rightarrow (\Phi, A_1 * A_2 * A_3 * \cdots * A_n)\) such that \(I(\Phi(a)) = \Phi(a), \forall a \in (\Phi, A_1 * A_2 * A_3 * \cdots * A_n)\) where \(a \in A_1 * A_2 * A_3 * \cdots * A_n\)

2.11 Hypersoft Matrices

Let \(U\) be universe of discourse, let \(a_1, a_2, a_3, \ldots, a_n\) be the attributes whose corresponding attribute values belongs the set \(E_1, E_2, E_3, \ldots, E_n\) respectively. Let \(A_1 * A_2 * A_3 * \cdots * A_n \subseteq E_1 * E_2 * E_3 * \cdots * E_n\) and \((\Phi, A_1 * A_2 * A_3 * \cdots * A_n, E_1 * E_2 * E_3 * \cdots * E_n)\) be the hypersoft set over the universal set \(U\). Then a relation \(R_{A_1 * A_2 * A_3 * \cdots * A_n}\) of \(U * (E_1 * E_2 * E_3 * \cdots * E_n)\) is defined as below \(R_{A_1 * A_2 * A_3 * \cdots * A_n} = \{(u, e) : e \in A_1 * A_2 * A_3 * \cdots * A_n, u \in f_{A_1 * A_2 * A_3 * \cdots * A_n}(e)\}\). The characteristic function of \(R_{A_1 * A_2 * A_3 * \cdots * A_n}\) is defined as, \(\zeta_{A_1 * A_2 * A_3 * \cdots * A_n} : U * A_1 * A_2 * A_3 * \cdots * A_n \rightarrow [0, 1]\)

\[H(x) = \begin{cases} 
1 & \text{if } (u, e) \in R_{A_1 * A_2 * A_3 * \cdots * A_n} \\
0 & \text{if } (u, e) \notin R_{A_1 * A_2 * A_3 * \cdots * A_n}
\end{cases} \tag{1}
\]

Then a hypersoft set \((\Phi, A_1 * A_2 * A_3 * \cdots * A_n, E_1 * E_2 * E_3 * \cdots * E_n)\) can be represented unique in the form of matrix and its is denoted by \([x_{ij}]_{m \times n}\)

\[x_{ij} = \zeta_{A_1 * A_2 * A_3 * \cdots * A_n}(u_i, e_j), e_j \in A_1 * A_2 * A_3 * \cdots * A_n\]

Example 14 From the previous example, if we take \(A_1' * A_2' * A_3' \subseteq A_1 * A_2 * A_3\) then the hypersoft set is given as follows \((f_{A_1' * A_2' * A_3'}, A_1 * A_2 * A_3) \quad (\Phi, A_1 * A_2 * A_3) = \{\Phi(e_1, e_4, e_6) = \{w_1, w_2\}, \Phi(e_1, e_4, e_7) = \{w_1, w_4\}, \Phi(e_3, e_4, e_6) = \{w_2\}, \Phi(e_3, e_4, e_7) = \{w_3\}\}

The relation of \((\Phi, A_1 * A_2 * A_3)\) is given by \(R_{A_1' * A_2' * A_3'} = \{(e_1, e_4, e_6), (e_1, e_4, e_7), (w_1, e_4, e_7), (w_4, e_3, e_4, e_6),(w_3, e_3, e_4, e_6)\}\)

We can write hypersoft matrix as follows \([x_{ij}] = \begin{pmatrix} 
1 & 1 & 1 \\
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 0 
\end{pmatrix}\)

2.12 Special hypersoft Matrices

Let the set of all hypersoft matrices over the universal set of discourse \(U\) be denoted \(HSM(U)\) \(m \times n\) or just \(HSM(U)\). Suppose \([x_{ij}] \in HSM(U)\). Then \([x_{ij}]\) is called (a) A zero hypersoft matrix denoted
by \([0]\), if \([x_{ij}] = 0 \forall i \text{ and } j\). (b) An A universal hypersoft matrix denoted by \([x_{ij}]\), if \([x_{ij}] = 1, \forall j \in I_{A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n}\) is \(\{J : e_j \in A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n\}\). It is noted that it is occur only for the parameter appearing in the set \(A_1 \ast A_2 \ast A_3 \ast \cdots \ast A_n \subseteq E_1 \ast E_2 \ast E_3 \ast \cdots \ast E_n\), and it is denoted by \([I]\), if \([x_{ij}] = 1, \forall i \text{ and } j\).

**Example 15** From the previous example, let \([x_{ij}]\), \([y_{ij}]\), \([z_{ij}]\) \(\in HSM(U)_{4 \times 4}\) (i) \(\zeta_{A_1 \ast A_2 \ast A_3}(e_1, e_4, e_6) = \phi\) \(\zeta_{A_1 \ast A_3 \ast A_3}(e_1, e_4, e_7) = \phi\) \(\zeta_{A_1 \ast A_2 \ast A_3}(e_3, e_4, e_6) = \phi\), then \([x_{ij}]\) is the zero hypersoft set matrix given by

\[
[0] = \begin{pmatrix}
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0
\end{pmatrix}
\]

If \(C_1 \ast C_2 \ast C_3 \subseteq A_1 \ast A_2 \ast A_3\) as \(C_1 \ast C_2 \ast C_3 = \{(e_1, e_4, e_6), (e_1, e_4, e_7)\}\), \(\zeta_{C_1 \ast C_2 \ast C_3}(e_1, e_4, e_6) = \zeta_{C_1 \ast C_2 \ast C_3}(e_1, e_4, e_7) = U\)

\[
\begin{pmatrix}
1 & 1 & 1 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0 \\
1 & 1 & 0 & 0
\end{pmatrix}
\]

(iii) If \(C = A_1 \ast A_2 \ast A_3\), \(\zeta_{C_1 \ast C_2 \ast C_3}(e_1) = U\) for each \(i\) where \(e_i \in A_1 \ast A_2 \ast A_3\), then \([z_{ij}] = [I]\) is the universal hypersoft matrix given by

\[
I = \begin{pmatrix}
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1 \\
1 & 1 & 1 & 1
\end{pmatrix}
\]

2.13 Hypersoft Sub Matrices

Let \(P = [x_{ij}], Q = [y_{ij}]\) \(\in HSM(U)\). Then (i) We call \(P\) is hypersoft matrix of \(Q\), denoted by \(P \subseteq Q\) if \(x_{ij} \leq y_{ij}\) for each \(i\) and \(j\). In this case, we also say that \(P\) is dominated by \(Q\). On the other hand one can say that \(Q\) dominates \(P\). Now we define \(P\) and \(Q\) are comparable and it is denoted by \(P/\phi\) if \(P \subseteq Q\) or \(Q \subseteq P\) (ii) \(P\) is proper hypersoft sub matrix of \(Q\) if \([x_{ij}] \subset [y_{ij}]\) and for at least one term \(x_{ij} < y_{ij}\) for all \(i\) and \(j\). In this case, we call \(P\) is properly dominated by \(Q\). (iii) \(P\) is strictly proper hypersoft sub matrix of \(Q\) and it is denoted by \(P \lessdot Q\), if \(P \subset Q\) and for each \(i\) and \(j\). In this case, we call \(P\) is strictly dominated by \(Q\). (iv) If \(P\) and \(Q\) are said to be hypersoft equal matrices if \(x_{ij} = y_{ij}\) and it is denoted by \(P \cong Q\) for each \(i\) and \(j\) and it is equivalent to \(P \subset Q\) and \(Q \subset P\), then \(P \cong Q\) It is prompted to observe that \(\preceq\) is a partial ordering (reflexive, Anti symmetric, and transitive) on the class of hypersoft matrices.

2.14 Operations on Hypersoft Matrices

Now we are going to discuss the concept of union, intersection, complement, difference and product of hypersoft matrices and their fundamental features. Let \(P = [x_{ij}], Q = [y_{ij}]\) \(\in HSM(U)\) is called the (i) Union of \(P\) and \(Q\) denoted by \(P \cup Q\) if \(z_{ij} = \max\{x_{ij}, y_{ij}\}\) for all \(i\) and \(j\). (ii) And intersection of two hypersoft set is denoted by \(P \cap Q\), where \(z_{ij} = \min\{x_{ij}, y_{ij}\}\) for all \(i\) and \(j\) and it is called disjoint if \(P \cap Q = \phi\) (iii) Complement of \(P\) is denoted by \(P^c\), if \(z_{ij} = 1 - x_{ij}\) for all \(i\) and \(j\). (iv) Difference \(P\) \& \(Q\) is denoted by \(P - Q\) which is also called relative complement of \(Q\) in \(P\).
(ii) \( P \cap Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} \)

(iii) \( P^c = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \)

(iv) \( Q^c = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \)

(v) \( P - Q = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} \)

(vi) \( Q - P = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \)

Characteristics of hypersoft Matrix Operations
Let \( P = [x_{ij}], Q = [y_{ij}] \) and \( R = [z_{ij}] \in HSM(U) \)
(i) \( P \cup \phi = P \) (Idempotent law) (ii) \( P \cup Q = P \) (Identity law)

2.14.1 Product of hypersoft set
Let \( P = [x_{ij}], Q = [y_{ij}] \) and \( R = [z_{ik}] \in HSM(U) \)
(i) Product is the function that maps element from \( HSM(U)^{m \times n} \times HSM(U)^{m \times n} \)

### Remark 17
If the two matrices have same order then the product hold.

2.15 Features of Hypersoft Matrices

Let \( P = [x_{ij}], Q = [y_{ij}] \in HSM(U)^{m \times n} \). Then the following features hold
(i) \( P^\cap Q^c = P^c \wedge Q^c \) \( (P \wedge Q)^c = P^c \vee Q^c \) (De Morgan law)

(ii) \( P \cap Q = P \cup Q, (P \cup Q)^c = P^c \cap Q^c \) (De Morgan law) This can be proved easily by using definition.

2.15.1 Example
Suppose \( P = [x_{ij}], Q = [y_{ij}] \in HSM(U)^{3 \times 3} \), given

\[
\begin{pmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
1 & 0 & 0
\end{pmatrix}
\]

and \( Q = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix} \)

\( P \wedge Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \)

\( P^c = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 1 \end{pmatrix} \)

\( P^c \wedge Q^c = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \)

\( P^c \vee Q^c = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix} \)

which shows that De Morgan laws are valid for product of hypersoft matrix.
3 Conclusions
In this paper, we have discussed the fundamentals of hypersoft set such as hypersoft subset, complement, not set and aggregation operators. After that we have discussed the hypersoft set relation, sub relation, complement relation, function, matrices and operations on hypersoft matrices. In this study, we also observed that some properties of classical sets do not hold for hypersoft set operations. This results will be very fruitful for future experts to enhance the work for fuzzy hypersoft set, intuitionistic fuzzy hypersoft set, neutrosophic fuzzy hypersoft set, Plithogenic hypersoft set and hypersoft multi sets among others.

References