AN IMPROVEMENT ON THE SMARANDACHE DIVISIBILITY THEOREM

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Abstract. Let \( a, n \) be positive integers. In this paper we prove that
\[
\frac{(a^n - a)(n - 1)!}{n / 2}!
\]

For any positive integer \( a \) and \( n \), Smarandache [3] proved that

\[
(1) \quad n \mid (a^n - a)(n - 1)!. \]

The above division relation is the Smarandache divisibility theorem (see [1, Notions 126]). In this paper we give an improvement on (1) as follows:

Theorem. For any positive integers \( a \) and \( n \), we have

\[
(2) \quad n \mid (a^n - a)[n / 2]!,
\]

where \([n / 2]\) is the largest integer which does not exceed \( n / 2 \).

Proof. The division relation (2) holds for \( n \leq 9 \), we may assume that \( n > 9 \). By Fermat's theorem (see [2, Theorem 71]), if \( n \) is a prime, then we have

\[
(3) \quad n \mid (a^n - a),
\]
for any \( a \). We see from (3) that (2) is true.

If \( n \) is a composite number, then we have \( n = pd \), where \( p, d \) are integers satisfying \( p \geq q \geq 2 \). Further, if \( p \neq q \), then we have \( n \mid p! \). It implies that \( n \mid (n/q)! \). Since \( q \geq 2 \), we get

\[
(4) \quad n \mid \frac{n}{2}! \\
\]

If \( p = q \), then \( n = p^2 \) and

\[
(5) \quad n \mid (2p)! \\
\]

Since \( n > 9 \), we have \( n \geq 4^2 \), \( p \geq 4 \) and \( 2p \leq n/2 \). Hence, we see from (5) that (4) is also true in this case. The combination of (3) and (4), the theorem is proved.

References