Consecutive, Reversed, Mirror, and Symmetric Smarandache Sequences of Triangular Numbers*

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Abstract

We use the Maple system to check the investigations of S. S. Gupta regarding the Smarandache consecutive and the reversed Smarandache sequences of triangular numbers [Smarandache Notions Journal, Vol. 14, 2004, pp. 366–368]. Furthermore, we extend previous investigations to the mirror and symmetric Smarandache sequences of triangular numbers.

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The $n$th triangular number $t_n$, $n \in \mathbb{N}$, is defined by $t_n = \sum_{i=1}^{n} i = n(n+1)/2$. These numbers were first studied by the Pythagoreans.

The first $k$ terms of the triangular sequence $\{t_n\}_{n=1}^{\infty}$ are easily obtained in Maple:

```maple
> t:=n->n*(n+1)/2:
> first := k -> seq(t(n),n=1..k):
```

For example:

```maple
> first(20);
1, 3, 6, 10, 15, 21, 28, 36, 45, 55, 66, 78, 91, 105, 120, 136, 153, 171, 190, 210
```

In this short note we are interested in studying Smarandache sequences of triangular numbers with the help of the Maple system.

To define the Smarandache sequences, it is convenient to introduce first the concatenation operation. Given two positive integer numbers $n$ and $m$, the concatenation operation $\text{conc}$ is defined in Maple by the following function:

```maple
> conc := (n,m) -> n*10^length(m)+m:
```

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For example,

\[ \texttt{> conc(12,345); 12345} \]

Given a positive integer sequence \( \{u_n\}_{n=1}^{\infty} \), we define the corresponding Smarandache Consecutive Sequence \( \{scs_n\}_{n=1}^{\infty} \) recursively:

\[
\begin{align*}
scs_1 &= u_1, \\
scs_n &= \text{conc}(scs_{n-1}, u_n).
\end{align*}
\]

In Maple we define:

\[
\begin{align*}
\texttt{> scs_n := (u,n) -> if n=1 then u(1) else conc(scs_n(u,n-1),u(n)) fi:} \\
\texttt{> scs := (u,n) -> seq(scs_n(u,i),i=1..n):}
\end{align*}
\]

The standard Smarandache consecutive sequence, introduced by the Romanian mathematician Florentin Smarandache, is obtained when one chooses \( u_n = n, \forall n \in \mathbb{N} \). The first 10 terms are:

\[
\begin{align*}
\texttt{> scs(n->n,10);} \\
1, 12, 123, 1234, 12345, 123456, 1234567, 12345678, 123456789, 12345678910
\end{align*}
\]

Another example of a Smarandache consecutive sequence is the Smarandache consecutive sequence of triangular numbers. With our Maple definitions, the first 10 terms of such sequence are obtained with the following command:

\[
\begin{align*}
\texttt{> scs(t,10);} \\
1, 13, 136, 13610, 1361015, 136101521, 13610152128, \\
1361015212836, 136101521283645, 13610152128364555
\end{align*}
\]

Sometimes, it is preferred to display Smarandache sequences in “triangular form”.

\[
\begin{align*}
\texttt{> show := L -> map(i->print(i),L):} \\
\texttt{> show([scs(t,10)]):}
\end{align*}
\]

\[
\begin{align*}
1 \\
13 \\
136 \\
13610 \\
1361015 \\
136101521 \\
13610152128 \\
1361015212836 \\
136101521283645 \\
13610152128364555
\end{align*}
\]
The *Reversed Smarandache Sequence* \( \text{rss} \) associated with a given sequence \( \{u_n\}_{n=1}^{\infty} \), is defined recursively by

\[
\text{rss}_1 = u_1, \\
\text{rss}_n = \text{conc}(u_n, \text{rss}_{n-1}).
\]

In Maple we propose the following definitions:

\[
> \text{rss}_n := (u,n) \rightarrow \text{if } n=1 \text{ then } u(1) \text{ else } \text{conc}(u(n), \text{rss}_n(u,n-1)) \text{ fi;}
\]

\[
> \text{rss} := (u,n) \rightarrow \text{seq(rss}_n(u,i),i=1..n):\]

The first terms of the reversed Smarandache sequence of triangular numbers are now easily obtained:

\[
> \text{rss}(t,10);
\]

\[
1, 31, 631, 10631, 1510631, 211510631, 28211510631, 3628211510631, 453628211510631, 55453628211510631
\]

We define the *Smarandache Mirror Sequence* \( \text{sms} \) as follows:

\[
\text{sms}_1 = u_1, \\
\text{sms}_n = \text{conc}(\text{conc}(u_n, \text{sms}_{n-1}), u_n)
\]

\[
> \text{sms}_n := (u,n) \rightarrow \text{if } n=1 \text{ then}
\]

\[
> u(1)
\]

\[
> \text{else}
\]

\[
> \text{conc}(\text{conc}(u(n), \text{sms}_n(u,n-1)), u(n))
\]

\[
> \text{fi;}
\]

\[
> \text{sms} := (u,n) \rightarrow \text{seq(sms}_n(u,i),i=1..n):
\]

The first 10 terms of the Smarandache mirror sequence introduced by Smarandache are:

\[
> \text{sms}(n->n,10);
\]

\[
1, 212, 32123, 4321234, 543212345, 65432123456, 7654321234567, 8765432123456789, 9876543212345678910
\]

We are interested in the Smarandache mirror sequence of triangular numbers. The first 10 terms are:

\[
> \text{sms}(t,10);
\]

\[
1, 313, 63136, 106313610, 1510631361015, 21151063136101521, 28211510631361015212836, 362821151063136101521283645, 453628211510631361015212836455
\]

Finally, we define the Smarandache Symmetric Sequence \( \text{sss} \). For that we introduce the function “But Last Digit” \( \text{bld} \):
> bld := n -> iquo(n,10):
> bld(123);

12

If the integer number is a one-digit number, then function \( bld \) returns zero:

> bld(3);

0

This is important: with our \( \text{conc} \) function, the concatenation of zero with a positive integer \( n \) gives \( n \).

> conc(bld(1),3);

3

The Smarandache Symmetric Sequence \( (sss) \) is now easily defined, appealing to the Smarandache consecutive, and reversed Smarandache sequences:

\[
ss_{2n-1} = \text{conc}(bld(ss_{2n-1}),rs_{2n-1}),
\]
\[
ss_{2n} = \text{conc}(scs_{2n},rs_{2n}),
\]

\( n \in \mathbb{N} \). In Maple, we give the following definitions:

> sss_n := (u,n) -> if type(n,odd) then
> conc(bld(scs_n(u,(n+1)/2)),rss_n(u,(n+1)/2))
> else
> conc(scs_n(u,n/2),rss_n(u,n/2))
> fi:
> sss := (u,n) -> seq(sss_n(u,i),i=1..n):

The first terms of Smarandache’s symmetric sequence are

> sss(n->n,10);

1, 11, 121, 1221, 12321, 123321, 1234321, 12344321, 123454321

while the first 10 terms of the Smarandache symmetric sequence of triangular numbers are

> sss(t,10);

1, 11, 131, 1331, 13631, 136631, 136110631, 1361010631, 136101510631, 13610151510631

One interesting question is to find prime numbers in the above defined Smarandache sequences of triangular numbers. We will restrict our search to the first 1000 terms of each sequence. All computations were done with Maple 9 running on a 2.00Ghz Pentium 4 with 256Mb RAM. We begin by collecting four lists with the first 1000 terms of the consecutive, reversed, mirror, and symmetric Smarandache sequences of triangular numbers:

> st:=time(): Lscs1000:=[scs(t,1000)]: printf("%a seconds",round(time()-st));
20 seconds

> st:=time(): Lrss1000:=[rss(t,1000)]: printf("%a seconds",round(time()-st));
75 seconds
We note that $scs_{1000}$ and $rss_{1000}$ are positive integer numbers with 5354 digits;

$$\text{length}(Lscs1000[1000]), \text{length}(Lrss1000[1000]) = 5354, 5354$$

while $sms_{1000}$ and $sss_{1000}$ have, respectively, 10707 and 4708 digits.

$$\text{length}(Lsms1000[1000]), \text{length}(Lsss1000[1000]) = 10707, 4708$$

There exist two primes (13 and 136101521) among the first 1000 terms of the Smarandache consecutive sequence of triangular numbers;

$$\text{select}(\text{isprime}, Lscs1000);$$

$$\text{printf}("%a minutes", \text{round}((\text{time()}-\text{st})/60));$$

$$[13, 136101521]$$

9 minutes

six primes among the first 1000 terms of the reversed Smarandache sequence of triangular numbers;

$$\text{select}(\text{isprime}, Lrss1000);$$

$$\text{printf}("%a minutes", \text{round}((\text{time()}-\text{st})/60));$$

$$[31, 631, 10631, 55453628211510631, 786655453628211510631, 10591786655453628211510631]$$

31 minutes

only one prime (313) among the first 600 terms of the Smarandache mirror sequence of triangular numbers;\(^1\)

$$\text{length}(Lsms1000[600]); \# \text{sms}_600 \text{ is a number with } 5907 \text{ digits}$$

$$5907$$

$$\text{select}(\text{isprime}, Lsms1000[1..600]);$$

$$\text{printf}("%a minutes", \text{round}((\text{time()}-\text{st})/60));$$

$$[313]$$

\(^1\)Our computer runs low in memory when one tries the first 1000 terms of the Smarandache mirror sequence of triangular numbers. For this reason, we have considered here only the first 600 terms of the sequence.
and five primes among the first 1000 terms of the Smarandache symmetric sequence of triangular numbers (the fifth prime is an integer with 336 digits).

```maple
> st := time():
> select(isprime,Lsss1000);
> printf("%a minutes",round((time()-st)/60));

[11, 131, 136110631, 1361015212836455566789110512012010591786655453628211510631, 1361015212836455566789110512013615317119021023125327630032535137840643546549652856159 563066703741780820861903946990103510811128117612251275132613781431148515401596165316 5315961540148514311378132612751225117611281081103599094690386182078074170366663059556 152849646543540637835132530027625323121019017115313612010591786655453628211510631]

19 minutes
> length(%[5]);
336
```

How many primes are there in the above defined Smarandache sequences of triangular numbers? This seems to be an open question.

Another interesting question is to find triangular numbers in the Smarandache sequences of triangular numbers. We begin by defining in Maple the boolean function `istriangular`.

```maple
> istriangular := n -> evalb(nops(select(i->evalb(whattype(i)=integer), [solve(t(k)=n)])) > 0):
```

There exist two triangular numbers (1 and 136) among the first 1000 terms of the Smarandache consecutive sequence of triangular numbers;

```maple
> st := time():
> select(istriangular,Lscs1000);
> printf("%a seconds",round(time()-st));

[1,136]
```

6 seconds

while the other Smarandache sequences of triangular numbers only show, among the first 1000 terms, the trivial triangular number 1:

```maple
> st := time():
> select(istriangular,Lrss1000);
> printf("%a seconds",round(time()-st));

[1]
```

6 seconds
> st := time():
> select(istriangular,Lsms1000);
> printf("%a seconds",round(time()-st));

[1]

10 seconds

> st := time():
> select(istriangular,Lsss1000);
> printf("%a seconds",round(time()-st));

[1]

6 seconds

Does exist more triangular numbers in the Smarandache sequences of triangular numbers? This is, to the best of our knowledge, an open question needing further investigations. Since checking if a number is triangular is much faster than to check if a number is prime, we invite the readers to continue our search of triangular numbers for besides the 1000th term of the Smarandache sequences of triangular numbers. We look forward to readers discoveries.

References