THE PRIMES IN THE SMARANDACHE POWER PRODUCT SEQUENCES OF THE FIRST KIND

Maohua Le

Abstract. In this paper we prove that if \( r \geq 1 \) and \( r \) is not a power of 2, then the Smarandache \( r \)-power product sequence of the first kind contains only one prime 2.

Key words. Smarandache power product sequence, first kind, prime.

For any positive integers \( n, r \) with \( r > 1 \), let \( P(n, r) \) be the \( n \)-th power of degree \( r \). Further, let

\[
(1) \quad V(n, r) = \prod_{k=1}^{n} P(k, r) + 1.
\]

Then the sequence \( V(r) = \{ V(n, r) \}_{n=1}^{\infty} \) is called the Smarandache \( r \)-power product sequence of the first kind.

In [2], Russo proposed the following question.

Question. How many terms in \( V(2) \) and \( V(3) \) are primes?

In fact, Le and Wu [1] showed that if \( r \) is odd, then \( V(r) \) contains only one prime 2. It implies that \( V(3) \) contains only one prime 2. In this paper we prove a general result as follows.

Theorem. If \( r \) is not a power of 2, then \( V(r) \) contains only one prime 2.

Proof. Since \( r > 1 \), if \( r \) is not a power of 2, then \( r \) has an odd prime divisor \( p \). By (1), we get

\[
V(n, r) = (n!)^r + 1 = ((n!)^{r/p} + 1)((n!)^{r(p-1)/p} - (n!)^{r(p-2)/p} + \ldots - (n!)^{r/p} + 1),
\]
Where \( r/p \) is a positive integer. Notice that if \( n>1 \), then 
\((n!)^{r/p}+1>1\) and 
\((n!)^{r(p-1)/p-1}>1\). Therefore, we see from (2) that if \( n>1 \), then \( V(n,r) \) is not a prime. Thus, the sequence \( V(r) \) contains only one prime \( V(1,r)=2 \). The theorem is proved.

References


Department of Mathematics
Zhanjiang Normal College
Zhanjiang, Guangdong
P.R. CHINA