ON PRIMES IN THE SMARANDACHE PIERCED CHAIN

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Abstract. Let \( C = \{c_n\}_{n=1}^{\infty} \) be the Smarandache pierced chain. In this paper we prove that if \( n > 2 \), then \( c_n/101 \) is not a prime.

For any positive integer \( n \), let

\[
(1) \quad c_n = 101 \times 10001001\ldots0001.
\]

Then the sequence \( C = \{c_n\}_{n=1}^{\infty} \) is called the Smarandache pierced chain (see [2, Notion 19]). In [3], Smarandache asked the following question:

Question. How many \( c_n/101 \) are primes?

In this paper we give a complete answer as follows:

Theorem. If \( n > 2 \), then \( c_n/101 \) is not a prime.

Proof. Let \( \zeta_n = e^{2\pi i / n} \) be a primitive root of unity with the degree \( n \), and let

\[
f_n(x) = \prod_{1 \leq k < n, \gcd(k,n) = 1} (x - \zeta_n^k).
\]

Then \( f_n(x) \) is a polynomial with integer coefficients. Further, it is a well known fact that if \( x > 2 \), then \( f_n(x) > 1 \) (see [1]). This implies that if \( x \) is an integer with \( x > 2 \), then \( f_n(x) \) is an integer with \( f_n(x) > 1 \). On the other hand, we have

\[
(2) \quad x^n - 1 = \prod_{d | n} f_d(x).
\]

We see from (1) that if \( n > 1 \), then
By the above definition, we find from (2) and (3) that

\[ \frac{c_n}{101} = \frac{1 + 10 + 10 + \ldots + 10}{10^4 - 1} \]

Since \( n > 2 \), we get \( 2n > 4 \) and \( 4n > 4 \). It implies that both \( 2n \) and \( 4n \) are divisors of \( 4n \) but not of \( 4 \). Therefore, we get from (4) that

\[ \frac{c_n}{101} = \frac{f_{2n}(10) f_{4n}(10)t}{10} \]

where \( t \) is not a positive integer. Notice that \( f_{2n}(10) > 1 \) and \( f_{4n}(10) > 1 \). We see from (5) that \( \frac{c_n}{101} \) is not a prime. The theorem is proved.

References