SMARANDACHE-R-MODULES AND ALGORITHMS

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ABSTRACT

In this paper we introduced Smarandache-2-algebraic structure of R-modules namely Smarandache-R-modules. A Smarandache-2-algebraic structure on a set N means a weak algebraic structure A₀ on N such that there exists a proper subset M of N, which is embedded with a stronger algebraic structure A₁, stronger algebraic structure means satisfying more axioms, by proper subset one understands a subset different from the empty set, form the unit element if any, from the whole set. We define Smarandache-R-modules and obtain some of its algorithms through CS-Algebras, on BF-Algebras, and on BRK-Algebras. We refer to Raul Padilla[10].

Keywords: R-modules, Smarandache-R-modules, CS-Algebras, BF-Algebras, and BRK-Algebras

INTRODUCTION

In order that new notions are introduced in algebra to better study the congruence in number theory by Florentin Smarandache [1]. By <proper subset> of a set A we consider a set P included in A, and different from A, different from the empty set, and from the unit element in A-if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures S₁ << S₂ if: both are defined on the same set; all S₁ laws are also S₂ laws; all axioms of an S₁ law are accomplished by the corresponding S₂ law; S₂ law accomplish strictly more axioms that S₁ laws, or S₂ has more laws than S₁.

For example: Semi group << Monoid << group << ring << field, or Semi group << commutative semi group, ring << unitary, ring etc. They define a General special structure to be a structure SM on a set A, different form a structure SN, such that a proper subset of A is on structure, where SM << SN <<.
PRELIMINARIES:

DEFINITION: 1.1

A left R-modules A is a system with two binary operations, addition and multiplication, such that

(i) the elements of A form a group \((A,+)\) under addition,
(ii) the elements of A form a multiplicative semi-group,
(iii) \(x(y + z) = xy + xz\) for all \(x, y, z \in A\)

In particular, if A contains a multiplicative semi-group \(S\) whose elements generate \((A,+)\) and satisfy

(iv) \((x+y)s = xs + ys\), for all \(x, y \in A\) and \(s \in S\), then we say that A is a distributively generated R-modules.

DEFINITION: 1.2

A \(R\)-modules \((B,+,.\)) is said to be Smarandache R-modules whose proper subset A is a \(S\)-algebra with respect to same induced operation of B.

DEFINITION: 1.3 (Alternative definition for \(S\)-R-modules)

If there exists a non-empty set A which is a R-modules such that it superset B of A is a \(S\)-algebra with respect to the same induced operation, then B is called Smarandache R-modules. It can also written as \(S\)-R-modules.

ALGORITHMS

BE algebras: Ahn.S.S and So.K.S has introduced BE algebras and satisfies the following conditions for all \(x, y\) and \(z\) in A

1) \(x * x = 1\)
2) \(x * 1 = 1\)
3) \(1 * x = x\)
4) \(x * (y * z) = y * (x * z)\).

According to Raul Padilla (4, Thm. 1.60d) R is a module, now by definition, R is a Smarandache-R-modules

ALGORITHM:I

Step 1: Consider a R-module R
Step 2: Let x, y and z in A
Step 3: Choose \(x * y \leq yN * xN\)
Step 4: Choose \(x \leq y\) implies \(yN \leq xN\)
Step 5: Verify \(x * (y * z) = (x * y) * (x * z)\)
Step 6: If step 5 is true then R is a Smarandache-R-module

ALGORITHM:II

Step 1: Consider a R-module R
Step 2: Let x, y and z in A
Step 3: Choose \((y * x) * y \leq x * y\).
Step 4: Choose \(x * (x * y) = x * y\).
Step 5: Verify \(x * (y * z) = (x * y) * (x * z)\).
Step 6: If step 5 is true then by definition, we write R is a Smarandache-R-module

BRK ALGEBRA

BRK algebras: Imai and Iseki has introduced BRK algebras and satisfies the following conditions

1) \((x * y) * (x * z) \leq (z * y)\)
2) \(x * (x * y) \leq y\)
3) \(x \leq x\)
4) \(x \leq y\) and \(y \leq x\) imply \(x = y\)
5) \(x \leq 0\) implies \(x = 0\), where \(x \leq y\) is defined by \(x * y = 0\), for all \(x, y, z \in X\).

According to Raul Padilla (4, Thm. 1.60d) R is a module, now by definition, R is a Smarandache-R-modules.
ALGORITHMS III

Step 1: Consider a R-module R
Step 2: Let x, y in A
Step 3: Let x * x = 0
Step 4: Choose x * y = 0
Step 5: Choose y * x = 0
Step 6: Verify that 0 * x = 0 * y.
Step 7: If step 6 is true then we write R is a Smarandache-R-module.

ALGORITHM: IV

Step 1: Consider a R-module R
Step 2: Let a, b and c in A
Step 3: Choose a * b
Step 4: Choose a * c
Step 5: Verify a * b = a * c then 0 * b = 0 * c
Step 6: If step 5 is true then R is a Smarandache-R-module

BF-ALGEBRA

According to Andrzej Walendziak has introduced on BF algebras for the following conditions

a) 0 * (x * y) = y * x.

b) 0 * (0 * x) = x

c) if 0 * x = 0 * y, then x = y

d) if x * y = 0, then y * x = 0

for any x, y ∈ A;

According to Raul Padilla (4, Thm. 1.60d) R is a module, now by definition, R is a Smarandache-R-modules

Given R be a Smarandache-R-module, if there exists a proper subset A of R in which satisfies the following statements

(a) A is a BF-algebra;

(b) x = [x * (0 * y)] * y for all x, y ∈ A;

(c) x = y * [(0 * x) * (0 * y)] for all x, y ∈ A.

According to Raul Padilla (4, Thm. 1.60d) R is a module, now by definition, R is a Smarandache-R-modules

ALGORITHM: V

Step 1: Consider a R-module R
Step 2: Let x, y in A
Step 3: Choose x * y = 0
Step 4: Choose y * x = 0
Step 5: Check x = 0 * (0 * x) = x * 0
Step 6: Verify that x = y
Step 7: If step 6 is true then we write R is a Smarandache-R-module
ALGORITHM: VI

Step 1: Consider a R-module R
Step 2: Let x, y in A
Step 3: Choose x * y = 0
Step 4: Choose y * x = 0
Step 5: Check x = y * (0 * x) * (0 * y)
Step 6: Verify that x = y
Step 7: If step 6 is true then we write R is a Smarandache-R-module

Given R be a smarandache-R-module, if there exists a proper subset A of R in which BG-algebra satisfies the following statements

(a) A is a BG-algebra;
(b) For x, y ∈ A, x * y = 0 implies x = y;
(c) The right cancellation law holds in A. i.e., If x * y = z * y, then x = z for any x, y, z ∈ A;
(d) The left cancellation law holds in A. i.e., if y * x = y * z, then x = z for any x, y, z ∈ A.

According to Raul Padilla (4, Thm. 1.60d) R is a module, now by definition, R is a Smarandache-R – Modules

ALGORITHM: VII

Step 1: Consider a R-module R
Step 2: Let x, y in A
Step 3: Choose x * y = 0
Step 4: Choose y * x = 0
Step 5: Check x = (x * y) * (0 * y) = 0 * (0 * y)
Step 6: Verify that x = y
Step 7: If step 6 is true then we write R is a Smarandache-R-module

References:

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