SOME EQUIVALENT CONDITIONS OF SMARANDACHE - SOFT NEUTROSOPHIC-NEAR RING

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ABSTRACT. In this paper, we introduced Samarandache-2-algebraic structure of Soft Neutrosophic-Near ring namely Smarandache-Soft Neutrosophic-Near ring. A Samarandache-2-algebraic structure on a set \(N\) means a weak algebraic structure \(S_1\) on \(N\) such that there exist a proper subset \(M\) of \(N\), which is embedded with a stronger algebraic structure \(S_2\), stronger algebraic structure means satisfying more axioms, that is \(S_1 \prec S_2\), by proper subset one can understand a subset different from the empty set, from the unit element if any, from the Whole set. We define Smarandache - Soft Neutrosophic - Near ring and obtain the some of it characterization through sub algebraic structures of near - ring. For basic concept of near - ring we refer to G.Pilz \[^3\]\ and for soft neutrosophic algebraic structures we refer to Muhammed Shabir, MumtazAli, Munazza Naz, and Florentin Smarandache\[^4,5\].

1. Introduction

In order that, New notions are introduced in algebra to better study the congruence in number theory by Florentin smarandache\[^2\]. By \(<\)proper subset\(>\) of a set \(A\) we consider a set \(P\) included in \(A\), and different from \(A\), different from empty set, and from the unit element in \(A\) - if any they rank the algebraic structures using an order relationship:

They say that the algebraic structures \(S_1 \prec S_2\) if: both are defined on the same set; all \(S_1\) laws are also \(S_2\) laws; all axioms of an \(S_1\) law are accomplished by the corresponding \(S_2\) law; \(S_2\) law accomplish strictly more axioms that \(S_1\) laws, or \(S_2\) has more laws than \(S_1\).

For example: Semi group \(\prec\) Monoid \(\prec\) group \(\prec\) ring \(\prec\) field, or Semi group \(\prec\) to commutative semi group, ring \(\prec\) unitary ring etc. They define a general special structure to be a structure SM on a set \(A\), different from a structure SN, such that a proper subset of \(A\) is an structure, where SM \(\prec\) SN.


\(^{1}\)Key words and phrases. Soft Neutrosophic Near-ring, soft Neutrosophic Near-field, Smarandache-soft Neutrosophic near-ring.
2. Preliminaries

Definition 2.1. Let \( \langle N \cup I \rangle \) be a neutrosophic near-ring and \((F,A)\) be a soft set over \( \langle N \cup I \rangle \). Then \((F,A)\) is called soft neutrosophic near-ring if and only if \(F(a)\) is a neutrosophic sub near-ring of \(\langle N \cup I \rangle\) for all \(a \in A\).

Definition 2.2. Let \(K(I) = \langle K \cup I \rangle\) be a neutrosophic near-field and let \((F,A)\) be a soft set over \(K(I)\). Then \((F,A)\) is said to be soft neutrosophic near-field if and only if \(F(a)\) is a neutrosophic sub near-field of \(K(I)\) for all \(a \in A\).

Definition 2.3. Let \((F,A)\) be a soft neutrosophic near-ring over \(\langle N \cup I \rangle\). Then \((F,A)\) is called soft neutrosophic right near-ring if and only if \(F(a)\) is a neutrosophic sub near-ring of \(\langle N \cup I \rangle\) for all \(a \in A\).

Definition 2.4. Let \((F,A)\) be a soft neutrosophic near-ring over \(\langle N \cup I \rangle\). Then \((F,A)\) is called soft neutrosophic left near-ring if and only if \(F(a)\) is a neutrosophic sub near-ring of \(\langle N \cup I \rangle\) for all \(a \in A\).

Definition 2.5. Let \((F,A)\) be a soft neutrosophic near-ring over \(\langle N \cup I \rangle\) with more than one element. Then the non-zero elements of \((F,A)\) form a group under multiplication if and only if for every \(F(a) \neq 0\) in \((F,A)\), there exists a unique \(F(b)\) in \((F,A)\) such that \(F(a)F(b)F(c) = F(a)\).

Definition 2.6. Let \((F,A)\) be a soft neutrosophic zero symmetric near-ring over \(\langle N \cup I \rangle\), which contains a distributive element \(F(a_1) \neq 0\). Then \((F,A)\) is a near-field if and only if for each \(F(a) \neq 0\) in \((F,A)\), \((F,A)\) is a distributive element \(F(a) \neq 0\) in \((F,A)\) such that \(F(a)F(b)F(c) = F(a)\).

Definition 2.7. Let \((F,A)\) be a finite soft neutrosophic zero symmetric near-ring that contains a distributive element \(F(w) \neq 0\) and for each \(F(x) \neq 0\) in \((F,A)\) there exist \(F(y)\) in \((F,A)\) such that \(F(y)F(x) \neq 0\) then \((F,A)\) is a soft neutrosophic near-field if and only if \((F,A)\) has no proper left ideal.

Definition 2.8. Let \((F,A)\) be a soft neutrosophic near-ring over \(\langle N \cup I \rangle\), Then \((F,A)\) is called soft neutrosophic zero symmetric near-ring over \(\langle N \cup I \rangle\). If \(F(n)0 = 0\) for all \(F(n)\) in \((F,A)\).

Definition 2.9. Let \((F,A)\) be a soft neutrosophic near-ring over \(\langle N \cup I \rangle\). An element \(F(e)\) in a soft neutrosophic near-ring \((F,A)\) over \(\langle N \cup I \rangle\) is called idempotent. If \(F(e^2) = F(e)\).

Definition 2.10. Let \((F,A)\) be a soft neutrosophic near-ring over \(\langle N \cup I \rangle\). An element \(F(b)\) in \((F,A)\) is called distributive if \(F(b)(F(a_1) + F(a_2)) = F(b)F(a_1) + F(b)F(a_2)\) for all \(F(a_1), F(a_2)\) in \((F,A)\).

Definition 2.11. Let \((F,A)\) be a soft neutrosophic near-ring over \(\langle N \cup I \rangle\). A soft neutrosophic subgroup \((H,A)\) of \((F,A)\) is called \((F,A)\) - subgroup if \((F,A)(H,A) \subset (H,A)\).
Definition 2.12. Let \((F, A)\) be a soft neutrosophic near-ring over \((N \cup I)\) is called regular if for each \(F(a)\) in \((F, A)\). There exist \(F(x)\) in \((F, A)\) such that \(F(a)F(x)F(a) = F(a)\).

Now we have introduced our basic concept, called SMARANDACHE - SOFT NEUTROSOPHIC - NEAR RING.

Definition 2.13. A soft neutrosophic - near ring is said to be smarandache - soft neutrosophic - near ring, if a proper subset of it is a soft neutrosophic - near field with respect to the same induced operations.

3. Equivalent Conditions

Theorem 3.1. Let \((F, A)\) soft neutrosophic near-ring over \((N \cup I)\). Then \((F, A)\) smarandache-soft neutrosophic - near ring if and only if there exist a proper non-empty subset \((H, A)\) of \((F, A)\) satisfies the following conditions:

(i) For all \(H(a), H(b)\) in \((H, A)\) such that \(H(a) - H(b)\) in \((H, A)\).

(ii) For all \(H(a), H(b)\) in \((H, A)\) such that \(H(a)[H(b)^{-1}]\) in \((H, A)\)

(iii) For all \(H(a), H(b), H(c)\) in \((H, A)\)
such that \(H(a)(H(b) + H(c)) = H(a).H(b) + H(a).H(c)\).

Proof. PART-I : (i) we assume that \((H, A)\) be a non-empty proper subset of \((F, A)\) such that \(H(a), H(b)\) in \((H, A)\) \(\Rightarrow H(a) - H(b)\) in \((H, A)\).

To prove that \((H, A)\) is a abelian group under \('+'\). Since \((H, A) \neq \phi\), there exist an element \(H(a)\) in \((H, A)\). Hence \(H(a) - H(a)\) in \((H, A)\).

Thus \(H(e)\) in \((H, A)\). Also since \(H(e), H(a)\) in \((H, A)\), \(H(e) - H(a)\) in \((H, A)\).

Hence \(-H(a)\) in \((H, A)\).

Now, let \(H(a), H(b)\) in \((H, A)\).

Then \(H(a), -H(b)\) in \((H, A)\).

Hence \(H(a) - (-H(b)) = H(a)H(b)\) in \((H, A)\).

Thus \((H, A)\) is closed under \('+'\).

Therefore \((H, A)\) is an abelian soft neutrosophic group.

(ii) we assume that \((H, A)\) be a non-empty proper subset of \((H, A)\) such that \(H(a), H(b)\) in \((H, A)\) \(\Rightarrow H(a)H(b)^{-1}\) in \((H, A)\).

To prove that \((H, A)\) is a abelian group under \(\cdot\).

Since \((H, A) \neq \phi\), there exist an element \(H(a)\) in \((H, A)\).

Hence \(H(a)H(a)^{-1}\) in \((H, A)\).

Thus \(H(e)\) in \((H, A)\).

Also since \(H(e), H(a)\) in \((H, A)\), \(H(e)H(a)^{-1}\) in \((H, A)\).

Hence \(H(a)^{-1}\) in \((H, A)\).

Now, let \(H(a), H(b)\) in \((H, A)\).

Then \(H(a), H(b)^{-1}\) in \((H, A)\).

Hence \(H(a)(H(b)^{-1}) = H(a)H(b)\) in \((H, A)\). Thus \((H, A)\) is closed under \(\cdot\).

Therefore \((H, A)\) is an abelian soft neutrosophic group.

Evidently by \((i)\) and \((ii)\), \((iii)\) is satisfied.
Therefore \((H, A), \cdot, +\) is a soft neutrosophic field.
Since every soft neutrosophic field is a soft neutrosophic near-field.
Therefore \((H, A)\) is soft neutrosophic near-field.
Then by definition, \((F, A)\) is smarandache - soft neutrosophic - near ring.

**PART-II :** Conversely, we assume that \((F, A)\) is smarandache- soft neutrosophic - near ring.
By definition, there exist a proper subset \((H, A)\) is soft neutrosophic near-field.
Therefore in \((H, A)\), the conditions are trivially hold. \(\square\)

**Theorem 3.2.** Let \((F, A)\) be a soft neutrosophic near-ring over \((N \cup I)\). Where \(N\) is a near-ring. Then \((F, A)\) is a smarandache- soft neutrosophic near - ring if and only if for every \(H(a) \neq 0\) in \((H, A)\), there exist a unique \(H(b)\) in \((H, A)\) such that \(H(a)H(b)H(a) = H(a)\), where \((H, A)\) is a soft neutrosophic near-ring which is a proper subset of \((F, A)\).

**Proof.** Part-I: we assume that for every \(H(a) \neq 0\) in \((H, A)\). There exist a unique \(H(b)\) in \((H, A)\) such that \(H(a)H(b)H(a) = H(a)\).
Now to claim that \((H, A)\) is a Soft neutrosophic near-field. Let \(H(a) \neq 0\) and \(H(b) \neq 0\) in \((H, A)\) then \(H(a)H(b) \neq 0\).
For, if not there exist \(H(x)\) in \((H, A)\), such that \(H(b)H(x)H(b) = H(b)\)
Now \(H(b)H(x - a)H(b) = H(b)H(x)H(b) = H(b)\)
By uniqueness \(H(x - H(a) = H(x)\). Hence \(H(a) = 0\), contradiction.
Hence \((H, A)\) is without zero divisors. \((H, A)\) is zerosymmetric,

Since for any \(H(n)\) in \((H, A)\), \((H(n)0)(H(n)0) = H(n)0\) and \((H(n)0)(H(n)0) = H(n)0\). Hence by uniqueness \(H(n)0 = 0\).
Given \(H(a) \neq 0\) in \((H, A)\), there exist a unique \(H(b)\) in \((H, A)\) such that \(H(a)H(b)H(a) = H(a)\).
So \(H(a)(H(b)H(a))H(b)H(a) = (H(a)H(b)H(a))H(b)H(a)\)
\(= (H(a)H(b)H(a) = H(a)\). By uniqueness \(H(b)H(a)H(b) = H(b)\).
Note \(H(a)H(b), H(b)H(a)\) are non-zero idempotents.
Let \(H(e), H(f)\) be any two non-zero idempotents. Then \(H(e)H(f) \neq 0\) and there exist \(H(x)\) in \((H, A)\) such that \((H(e)H(f))H(x)(H(e)H(f)) = H(e)H(f)\) and \(H(x)(H(e)H(f))H(x) = H(x)\).
Let \(H(y) = H(x)H(e)\), then \((H(e)H(f))H(y)(H(e)H(f)) = H(e)H(f)\).
Hence by uniqueness \(H(x) = H(y) = H(x)H(e)\).

Similarly if \(H(y) = H(f)H(x)\).
We have \(H(x)H(e) = H(e)H(x)\)
So \(H(x^2) = (H(x)H(e))((H(f)H(x)) = H(x)((H(e)H(f))H(x) = H(x)\).
Hence \(H(x^2) = H(x)\) and \(H(x)((H(e)H(f))H(x) = H(x)\).
By uniqueness \(H(x) = H(e)H(f), which is an idempotent.
So \((H(e)H(f))H(e)(H(e)H(f)) = H(e)H(f)\ and (H(e)H(f))H(f)(H(e)H(f)) = H(e)H(f)\).
Then by definition, there exist a proper subset \((H, A)\). We have \((H, A)\) contains only one non-zero idempotent say \(H(e)\).

We have shown that \((H, A)^*\) is closed. For any \(H(a)\) in \((H, A)^*\), there exist \(H(b)\) in \((H, A)\) such that \(H(a)H(b)H(a) = H(a)\).

Since \(H(b)H(a)\) is a non-zero idempotent \(H(a)H(e) = H(a)\).

Hence \((H, A)^*\) is right identity for \((H, A)^*\).

Hence \((H, A)^*\) is the right inverse of \(H(a)\).

Hence \((H, A)^* \cdot \cdot \) is a group.

So \((H, A)\) is a soft neutrosophic near-field.

By definition, \((F, A)\) is a smarandache-soft neutrosophic-near ring.

PART-II: we assume that \((F, A)\) is a smarandache-soft neutrosophic-near ring. Then by definition, there exist a proper subset \((H, A)\) of \((F, A)\) is a soft neutrosophic near-field. Now to prove that \((H, A)\) is a smarandache-soft neutrosophic-near ring.

We have \((H, A)\) be a soft neutrosophic near-ring over \((N \cup I)\) with more than one element. Then the non-zero elements of \((H, A)\) form a group under multiplication if and only if for every \(H(a) \neq 0\) in \((H, A)\), there exist a unique \(H(b)\) in \((H, A)\) such that \(H(a)H(b)H(a) = H(a)\).

Theorem 3.3. Let \((F, A)\) be a soft neutrosophic near-ring over \((N \cup I)\). Then \((F, A)\) is a smarandache-soft neutrosophic-near ring if and only if for each \(H(a) \neq 0\) in \((H, A)\), \((H, A)H(a) = (H, A)\) and \((H, A)0 \neq (H, A)\), where \((H, A)\) is a soft neutrosophic near-ring, which is a proper subset of \((F, A)\), in which idempotents commute.

Proof. PART-I: we assume that for each \(H(a) \neq 0\) in \((H, A)\), \((H, A)H(a) = (H, A)\) and \((H, A)0 \neq (H, A)\).

To prove that \((H, A)\) is a soft neutrosophic near-field.

Let \(H(a) \neq 0\) and \(H(b) \neq 0\) be in \((H, A)\).

Then \(H(a)H(b) \neq 0\). If not then \((H, A)H(a) = (H, A)\), so \((H, A)H(a)H(b) = (H, A)H(b)\).

Hence \((H, A)0 = (H, A)H(b) = (H, A)\), a contradiction.

Hence \((H, A)\) is a without zero divisors. Given \(H(a) \neq 0\) in \((H, A)\), there exist \(H(y)\) in \((H, A)\) such that \(H(y)H(a) = H(a)\).

So \((H(a) - H(a)H(y))H(a) = 0\). Hence \(H(a) = H(a)H(y) = H(y)H(a)\).

There exist \(H(x)\) in \((H, A)\) such that \(H(x)H(a) = H(y)\).

Now \(H(a)H(x)H(a) = H(a)H(y) = H(a)\). Hence \((H, A)\) is regular.

Let \(H(b) = H(x)H(a)H(x)\).

Then \(H(a)H(b)H(a)\)

\[= H(a)(H(x)H(a)H(x))H(a)\]

\[= (H(a)H(x)H(a))H(x)H(a)\]

\[= H(a)H(x)H(a)\]

\[= H(a)\]. and

\[H(b)H(a)H(b)\]
We have, 
If possible let us assume 
H = H = H = H = H.

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Hence (H, A) is a near-field. So (H, A) is a soft neutrosophic near-field. By definition, (F, A) is a smarandache - soft neutrosophic - near ring.

PART-II: We assume that (H, A) is a smarandache - soft neutrosophic - near ring. Then by, definition, there exist a proper subset (H, A) of (F, A) is a soft neutrosophic near-field. Now to prove that, for each H(a) ≠ 0 in (H, A).

(H, A)H(a) = (H, A) and (H, A)0 ≠ (H, A).

As given H(a) ≠ 0 in (H, A) and for any H(n) in (H, A).

H(n) = H(n)1
= H(n)H(a−1)H(a)
= (H(n)H(a−1))H(a).

Hence H(n) in (H, A)H(a).

So (H, A) ⊂ (H, A)H(a).

Hence (H, A) = (H, A)H(a) and clearly (H, A)0 ≠ (H, A). □

Definition 3.1. A soft neutrosophic subgroup (H, A) of (F, A) over ⟨N ∪ I⟩ is called a (F, A) - subgroup if (F, A)(H, A) ⊆ (H, A) and an invariant (F, A) - subgroup if in addition (H, A)(F, A) ⊆ (H, A).

Theorem 3.4. Let (F, A) be a soft neutrosophic near-ring over ⟨N ∪ I⟩. Where N is a near-ring. Then (F, A) is a Smarandache - soft neutrosophic near - ring. if and only if (H, A) has no proper (H, A) - subgroup, where (H, A) is a soft neutrosophic near-ring which is a proper subset of (F, A) in which idempotents commute and suppose that for each H(x) ≠ 0 in (H, A), there exist H(y) in (H, A) possibly depending on H(x). Such that H(y)H(x) ≠ 0.
Proof. PART-I: we assume that \((H, A)\) has no proper \((H, A)\) - subgroup.
To prove that \((H, A)\) is a near-field.
Given for each \(H(x) \neq 0\) in \((H, A)\), there exist \(H(y)\) in \((H, A)\) possibly depending on \(H(x)\) such that \(H(y)H(x) \neq 0\).
For each \(H(x) \neq 0\) in \((H, A)\), \((H, A)H(x)\) is a \((H, A)\) - subgroup of \((H, A)\).
Since there exist \(H(y)\) in \((H, A)\). Such that \(H(y)H(x) \neq 0\), \((H, A)H(x) \neq 0\).
Hence \((H, A)H(x) = (H, A)\) and clearly \((H, A)0 \neq (H, A)\).
Hence by theorem 3, \((H, A)\) is a near-field.
By definition, \((H, A)\) is a soft neutrosophic near-field.
Therefore \((F, A)\) is a smarandach - soft neutrosophic - near ring.

PART-II: We assume that \((F, A)\) is a smarandach - soft neutrosophic - near ring.
Then by definition, there exist a proper subset \((H, A)\) of \((F, A)\) is a soft neutrosophic near-field.
Now to prove that \((H, A)\) has no proper \((H, A)\) - subgroup.
Since let \((G, A) \neq 0\) be a \((F, A)\) - subgroup and let \(G(a) \neq 0\) in \((G, A)\).
Then \(G(a^{-1})G(a) = 1\) in \((G, A)\).
So for any \(G(n)\) in \((G, A)\), \(G(n) = G(n)1\) in \((G, A)\). So \((H, A) = (G, A)\). \(\square\)

4. Conclusion
In this paper four equivalent conditions are obtained for a soft neutrosophic near-ring to be a Smarandache- soft neutrosophic - near ring.

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