Some results on Smarandache groupoids

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Abstract In this paper we prove some results towards classifying Smarandache groupoids which are in $Z^*(n)$ and not in $Z(n)$ when $n$ is even and $n$ is odd.

Keywords Groupoids, Smarandache groupoids.

§1. Introduction and preliminaries

In [3] and [4], W. B. Kandasamy defined new classes of Smarandache groupoids using $Z_n$. In this paper we prove some theorems for construction of Smarandache groupoids according as $n$ is even or odd.

Definition 1.1. A non-empty set of elements $G$ is said to form a groupoid if in $G$ is defined a binary operation called the product denoted by $*$ such that $a * b \in G$, $\forall a, b \in G$.

Definition 1.2. Let $S$ be a non-empty set. $S$ is said to be a semigroup if on $S$ is defined a binary operation $*$ such that

(i) for all $a, b \in S$ we have $a * b \in S$ (closure).

(ii) for all $a, b, c \in S$ we have $a * (b * c) = (a * b) * c$ (associative law).

$(S, *)$ is a semi-group.

Definition 1.3. A Smarandache groupoid $G$ is a groupoid which has a proper subset $S \subset G$ which is a semi-group under the operation of $G$.

Example 1.1. Let $(G, *)$ be a groupoid on the set of integer modulo 6, given by the following table.

<table>
<thead>
<tr>
<th></th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
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<tbody>
<tr>
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<tr>
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</tr>
</tbody>
</table>
Here, \{0, 5\}, \{1, 3\}, \{2, 4\} are proper subsets of \(G\) which are semigroups under \(*\).

**Definition 1.4.** Let \(Z_n = \{0, 1, 2, \cdots, n - 1\}, n \geq 3\). For \(a, b \in Z_n\) define a binary operation \(*\) on \(Z_n\) as: \(a * b = ta + ub \pmod{n}\) where \(t, u\) are 2 distinct elements in \(Z_n\) and \((t, u) = 1\). Here “+” is the usual addition of two integers and “\(ta\)” mean the product of the two integers \(t\) and \(a\).

Elements of \(Z_n\) form a groupoid with respect to the binary operation. We denote these groupoids by \(\{Z_n(t, u), *\}\) or \(Z_n(t, u)\) for fixed integer \(n\) and varying \(t, u \in Z_n\) such that \((t, u) = 1\). Thus we define a collection of groupoids \(Z(n)\) as follows

\[ Z(n) = \{Z_n(t, u), *\} \text{ for integers } t, u \in Z_n\setminus\{0\} \text{ such that } (t, u) = 1. \]

**Definition 1.5.** Let \(Z_n = \{0, 1, 2, \cdots, n - 1\}, n \geq 3\). For \(a, b \in Z_n\) define a binary operation \(*\) on \(Z_n\) as: \(a * b = ta + ub \pmod{n}\) where \(t, u\) are two distinct elements in \(Z_n\) and \(t\) and \(u\) need not always be relatively prime but \(t \neq u\). Here “+” is usual addition of two integers and “\(ta\)” means the product of two integers \(t\) and \(a\).

For fixed integer \(n\) and varying \(t, u \in Z_n\) s.t \(t \neq u\) we get a collection of groupoids \(Z^*(n)\) as: \(Z^*(n) = \{Z_n(t, u), *\} \text{ for integers } t, u \in Z_n\setminus\{0\} \text{ such that } t \neq u\).

**Remarks 1.1.**
(i) Clearly, \(Z(n) \subset Z^*(n)\).
(ii) \(Z^*(n) \setminus Z(n) = \Phi\) for \(n = p + 1\) for prime \(p = 2, 3\).
(iii) \(Z^*(n) \setminus Z(n) \neq \Phi\) for \(n \neq p + 1\) for prime \(p\).

We are interested in Smarandache Groupoids which are in \(Z^*(n)\) and not in \(Z(n)\) i.e., \(Z^*(n) \setminus Z(n)\).

\section{Smarandache groupoids when \(n\) is even}

**Theorem 2.1.** Let \(Z_n(t, lt) \in Z^*(n) \setminus Z(n)\). If \(n\) is even, \(n > 4\) and for each \(t = 2, 3, \cdots, \frac{n}{2} - 1\) and \(l = 2, 3, 4, \cdots\) such that \(lt < n\), then \(Z_n(t, lt)\) is Smarandache groupoid.

**Proof.** Let \(x = \frac{n}{2}\).

Case 1. \(t\) is even.

\(x * x = xt + ltx = (l + 1)tx \equiv 0 \pmod{n}\).
\(x * 0 = xt \equiv 0 \pmod{n}\).
\(0 * x = ltx \equiv 0 \pmod{n}\).
\(0 * 0 = 0 \pmod{n}\).

\(\therefore\ \{0, x\} \text{ is semigroup in } Z_n(t, lt)\).
\(\therefore\ Z_n(t, lt) \text{ is Smarandache groupoid when } t \text{ is even}.

Case 2. \(t\) is odd.

(a) If \(l\) is even.

\(x * x = xt + ltx = (l + 1)tx \equiv x \pmod{n}\).
\(\{x\} \text{ is semigroup in } Z_n(t, lt)\).
\(\therefore\ Z_n(t, lt) \text{ is Smarandache groupoid when } t \text{ is odd and } l \text{ is even.}
If \( l \) is odd then \((l + 1)\) is even.
\[
x \ast x = xt + lt \equiv (l + 1)x \equiv 0 \mod n.
\]
\[
x \ast 0 = xt \equiv x \mod n.
\]
\[
0 \ast x = lt \equiv x \mod n.
\]
\[
0 \ast 0 \equiv 0 \mod n.
\]
\[
\Rightarrow \{0, x\} \text{ is semigroup in } Z_n(t, lt).
\]
\[
\therefore Z_n(t, lt) \text{ is Smarandache groupoid when } t \text{ is odd and } l \text{ is odd.}
\]

**Theorem 2.2.** Let \( Z_n(t, u) \in Z^*(n) \setminus Z(n) \), \( n \) is even \( n > 4 \) where \( (t, u) = r \) and \( r \neq t, u \) then \( Z_n(t, u) \) is Smarandache groupoid.

**Proof.** Let \( x = \frac{n}{2} \).

**Case 1.** Let \( r \) be even i.e \( t \) and \( u \) are even.
\[
x \ast x = tx + ux = (t + u)x \equiv 0 \mod n.
\]
\[
x \ast 0 = tx \equiv 0 \mod n.
\]
\[
x \ast 0 = tx \equiv 0 \mod n.
\]
\[
0 \ast 0 \equiv 0 \mod n.
\]
\[
\{0, x\} \text{ is semigroup in } Z_n(t, lt).
\]
\[
\therefore Z_n(t, lt) \text{ is Smarandache groupoid when } t \text{ is even and } u \text{ is even.}
\]

**Case 2.** Let \( r \) be odd.

(a) when \( t \) is odd and \( u \) is odd,
\[
\Rightarrow t + u \text{ is even.}
\]
\[
x \ast x = tx + ux = (t + u)x \equiv 0 \mod n.
\]
\[
x \ast 0 = tx \equiv x \mod n.
\]
\[
0 \ast x \equiv ux \equiv x \mod n.
\]
\[
0 \ast 0 \equiv 0 \mod n.
\]
\[
\{0, x\} \text{ is a semigroup in } Z_n(t, u).
\]
\[
\therefore Z_n(t, u) \text{ is Smarandache groupoid when } t \text{ is odd and } u \text{ is odd.}
\]

(b) when \( t \) is odd and \( u \) is even,
\[
\Rightarrow t + u \text{ is odd.}
\]
\[
x \ast x = tx + ux = (t + u)x \equiv x \mod n.
\]
\[
\{x\} \text{ is a semigroup in } Z_n(t, u).
\]
\[
\therefore Z_n(t, u) \text{ is Smarandache groupoid when } t \text{ is odd and } u \text{ is even.}
\]

(c) when \( t \) is even and \( u \) is odd,
\[
\Rightarrow t + u \text{ is odd.}
\]
\[
x \ast x = tx + ux = (t + u)x \equiv x \mod n.
\]
\[
\{x\} \text{ is a semigroup in } Z_n(t, u).
\]
\[
\therefore Z_n(t, u) \text{ is Smarandache groupoid when } t \text{ is even and } u \text{ is odd.}
\]

By the above two theorems we can determine Smarandache groupoids in \( Z^*(n) \setminus Z(n) \) when \( n \) is even and \( n > 4 \).

We find Smarandache groupoids in \( Z^*(n) \setminus Z(n) \) for \( n = 22 \) by Theorem 2.1.
<table>
<thead>
<tr>
<th>t</th>
<th>l</th>
<th>(lt &lt; 22)</th>
<th>(Z_n(t, lt))</th>
<th>Proper subset which is semigroup</th>
<th>Smarandache groupoid in (\mathbb{Z}^*(n)\setminus\mathbb{Z}(n))</th>
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</thead>
<tbody>
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<tr>
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<td>8</td>
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<td>{0, 11}</td>
<td>(Z_{22}(4, 8))</td>
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<tr>
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<td>(Z_{22}(4, 12))</td>
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<tr>
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<td>4</td>
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<td>(Z_{22}(9, 18))</td>
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<tr>
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<td>20</td>
<td>(Z_{22}(10, 20))</td>
<td>{0, 11}</td>
<td>(Z_{22}(10, 20))</td>
<td></td>
</tr>
</tbody>
</table>

Next, we find Smarandache groupoids in \(\mathbb{Z}^*(n)\setminus\mathbb{Z}(n)\) for \(n = 22\) by Theorem 2.2.
\[
\begin{array}{|c|c|c|c|c|}
\hline
 t & u & (t, u) = r & \mathbb{Z}_n(t, u) & \text{Proper subset which is semigroup} & \text{Smarandache groupoid} \\
\hline
\hline
4 & 6 & (4,6) & \mathbb{Z}_2(4,6) & \{0,11\} & \mathbb{Z}_2(4,6) \\
10 & (4,10) & \mathbb{Z}_2(4,10) & \{0,11\} & \mathbb{Z}_2(4,10) \\
14 & (4,14) & \mathbb{Z}_2(4,14) & \{0,11\} & \mathbb{Z}_2(4,14) \\
18 & (4,18) & \mathbb{Z}_2(4,18) & \{0,11\} & \mathbb{Z}_2(4,18) \\
\hline
6 & 8 & (6,8) & \mathbb{Z}_2(6,8) & \{0,11\} & \mathbb{Z}_2(6,8) \\
9 & (6,9) & \mathbb{Z}_2(6,9) & \{11\} & \mathbb{Z}_2(6,9) \\
10 & (6,10) & \mathbb{Z}_2(6,10) & \{0,11\} & \mathbb{Z}_2(6,10) \\
14 & (6,14) & \mathbb{Z}_2(6,14) & \{0,11\} & \mathbb{Z}_2(6,14) \\
16 & (6,16) & \mathbb{Z}_2(6,16) & \{0,11\} & \mathbb{Z}_2(6,16) \\
20 & (6,20) & \mathbb{Z}_2(6,20) & \{0,11\} & \mathbb{Z}_2(6,20) \\
21 & (6,21) & \mathbb{Z}_2(6,21) & \{11\} & \mathbb{Z}_2(6,21) \\
\hline
8 & 10 & (8,10) & \mathbb{Z}_2(8,10) & \{0,11\} & \mathbb{Z}_2(8,10) \\
12 & (8,12) & \mathbb{Z}_2(8,12) & \{0,11\} & \mathbb{Z}_2(8,12) \\
14 & (8,14) & \mathbb{Z}_2(8,14) & \{0,11\} & \mathbb{Z}_2(8,14) \\
18 & (8,18) & \mathbb{Z}_2(8,18) & \{0,11\} & \mathbb{Z}_2(8,18) \\
20 & (8,20) & \mathbb{Z}_2(8,20) & \{0,11\} & \mathbb{Z}_2(8,20) \\
\hline
9 & 21 & (9,21) & \mathbb{Z}_2(9,21) & \{0,11\} & \mathbb{Z}_2(9,21) \\
10 & (10,12) & \mathbb{Z}_2(10,12) & \{0,11\} & \mathbb{Z}_2(10,12) \\
14 & (10,14) & \mathbb{Z}_2(10,14) & \{0,11\} & \mathbb{Z}_2(10,14) \\
16 & (10,16) & \mathbb{Z}_2(10,16) & \{0,11\} & \mathbb{Z}_2(10,16) \\
18 & (10,18) & \mathbb{Z}_2(10,18) & \{0,11\} & \mathbb{Z}_2(10,19) \\
\hline
12 & 14 & (12,14) & \mathbb{Z}_2(12,14) & \{0,11\} & \mathbb{Z}_2(12,14) \\
15 & (12,15) & \mathbb{Z}_2(12,15) & \{11\} & \mathbb{Z}_2(12,15) \\
16 & (12,16) & \mathbb{Z}_2(12,16) & \{0,11\} & \mathbb{Z}_2(12,16) \\
18 & (12,18) & \mathbb{Z}_2(12,18) & \{0,11\} & \mathbb{Z}_2(12,18) \\
20 & (12,20) & \mathbb{Z}_2(12,20) & \{0,11\} & \mathbb{Z}_2(12,20) \\
21 & (12,21) & \mathbb{Z}_2(12,21) & \{11\} & \mathbb{Z}_2(12,21) \\
\hline
14 & 16 & (14,16) & \mathbb{Z}_2(14,16) & \{0,11\} & \mathbb{Z}_2(14,16) \\
18 & (14,18) & \mathbb{Z}_2(14,18) & \{0,11\} & \mathbb{Z}_2(14,18) \\
20 & (14,20) & \mathbb{Z}_2(14,20) & \{0,11\} & \mathbb{Z}_2(14,20) \\
21 & (14,21) & \mathbb{Z}_2(14,21) & \{11\} & \mathbb{Z}_2(14,21) \\
\hline
15 & 20 & (15,20) & \mathbb{Z}_2(15,20) & \{11\} & \mathbb{Z}_2(15,20) \\
18 & (16,18) & \mathbb{Z}_2(16,18) & \{0,11\} & \mathbb{Z}_2(16,18) \\
20 & (16,20) & \mathbb{Z}_2(16,20) & \{0,11\} & \mathbb{Z}_2(16,20) \\
\hline
16 & 20 & (18,20) & \mathbb{Z}_2(18,20) & \{0,11\} & \mathbb{Z}_2(18,20) \\
\hline
\end{array}
\]
§3. Smarandache groupoids when \( n \) is odd

**Theorem 3.1.** Let \( Z_n(t, u) \in Z^*(n) \setminus Z(n) \). If \( n \) is odd, \( n > 4 \) and for each \( t = 2, \ldots, \frac{n-1}{2} \), and \( u = n - (t - 1) \) such that \( (t, u) = r \) then \( Z_n(t, u) \) is Smarandache groupoid.

**Proof.** Let \( x \in \{0, \ldots, n-1\} \).

\[
 x \ast x = xt + xu = (n + 1)x \equiv x \mod n.
\]

\( \therefore \{x\} \) is semigroup in \( Z_n \).

\( \therefore Z_n(t, u) \) is Smarandache groupoid.

By the above theorem we can determine the Smarandache groupoids in \( Z^*(n) \setminus Z(n) \) when \( n \) is odd and \( n > 4 \).

Also we note that all \( \{x\} \) where \( x \in \{0, \ldots, n-1\} \) are proper subsets which are semigroups in \( Z_n(t, u) \).

Let us consider the examples when \( n \) is odd. We will find the Smarandache groupoids in \( Z^*(n) \setminus Z(n) \) by Theorem 3.1.

<table>
<thead>
<tr>
<th>( n )</th>
<th>( t )</th>
<th>( u = n - (t - 1) )</th>
<th>( (t, u) = r )</th>
<th>( Z_n(t, u) ) Smarandache groupoid (S.G.) in ( Z^*(n) \setminus Z(n) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>2</td>
<td>4</td>
<td>(2, 4) = 2</td>
<td>( Z_5(2, 4) ) is S.G. in ( Z^*(5) \setminus Z(5) )</td>
</tr>
<tr>
<td>7</td>
<td>2</td>
<td>6</td>
<td>(2, 6) = 3</td>
<td>( Z_7(2, 6) ) is S.G. in ( Z^*(7) \setminus Z(7) )</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td>8</td>
<td>(2, 8) = 2</td>
<td>( Z_9(2, 8) ) is S.G. in ( Z^*(9) \setminus Z(9) )</td>
</tr>
<tr>
<td>4</td>
<td>6</td>
<td>(4, 6) = 2</td>
<td>( Z_9(4, 6) ) is S.G. in ( Z^*(9) \setminus Z(9) )</td>
<td></td>
</tr>
<tr>
<td>11</td>
<td>2</td>
<td>10</td>
<td>(2, 10) = 2</td>
<td>( Z_{11}(2, 10) ) is S.G. in ( Z^*(11) \setminus Z(11) )</td>
</tr>
<tr>
<td>3</td>
<td>9</td>
<td>(3, 9) = 3</td>
<td>( Z_{11}(3, 9) ) is S.G. in ( Z^*(11) \setminus Z(11) )</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>8</td>
<td>(4, 8) = 4</td>
<td>( Z_{11}(4, 8) ) is S.G. in ( Z^*(11) \setminus Z(11) )</td>
<td></td>
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<td>13</td>
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<td>12</td>
<td>(2, 12) = 2</td>
<td>( Z_{13}(2, 12) ) is S.G. in ( Z^*(13) \setminus Z(13) )</td>
</tr>
<tr>
<td>4</td>
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<td>( Z_{13}(4, 10) ) is S.G. in ( Z^*(13) \setminus Z(13) )</td>
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<tr>
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<tr>
<td>17</td>
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<td>16</td>
<td>(2, 16) = 2</td>
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<tr>
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<td>( Z_{17}(8, 10) ) is S.G. in ( Z^*(17) \setminus Z(17) )</td>
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</table>
\[ n \quad t \quad u = n - (t - 1) \quad (t, u) = r \quad Z_n(t, u) \text{ Smarandache groupoid} \]
\[
\text{S.G.) in } Z^*(n) \setminus Z(n)
\]

<table>
<thead>
<tr>
<th>n</th>
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<th>u = n - (t - 1)</th>
<th>(t, u) = r</th>
<th>Smarandache groupoid in Z^*(n) \setminus Z(n)</th>
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<td>19</td>
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<td>4</td>
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<tr>
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<tr>
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<td>12</td>
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<td>(Z_{21}(10, 12)) is S.G. in (Z^*(21) \setminus Z(21))</td>
</tr>
</tbody>
</table>

Open Problems:

1. Let \(n\) be a composite number. Are all groupoids in \(Z^*(n) \setminus Z(n)\) Smarandache groupoids?

2. Which class will have more number of Smarandache groupoids in \(Z^*(n) \setminus Z(n)\)?
   
   (a) When \(n + 1\) is prime.
   
   (b) When \(n\) is prime.

References