# Super Mean Labeling of Some Classes of Graphs 

P.Jeyanthi<br>Department of Mathematics, Govindammal Aditanar College for Women<br>Tiruchendur-628 215, Tamil Nadu, India<br>D.Ramya<br>Department of Mathematics, Dr. Sivanthi Aditanar College of Engineering<br>Tiruchendur- 628 215, Tamil Nadu, India

E-mail: jeyajeyanthi@rediffmail.com, aymar_padma@yahoo.co.in


#### Abstract

Let $G$ be a $(p, q)$ graph and $f: V(G) \rightarrow\{1,2,3, \ldots, p+q\}$ be an injection. For each edge $e=u v$, let $f^{*}(e)=(f(u)+f(v)) / 2$ if $f(u)+f(v)$ is even and $f^{*}(e)=(f(u)+$ $f(v)+1) / 2$ if $f(u)+f(v)$ is odd. Then $f$ is called a super mean labeling if $f(V) \cup\left\{f^{*}(e): e \in\right.$ $E(G)\}=\{1,2,3, \ldots, p+q\}$. A graph that admits a super mean labeling is called a super mean graph. In this paper we prove that $S\left(P_{n} \odot K_{1}\right), S\left(P_{2} \times P_{4}\right), S\left(B_{n, n}\right),\left\langle B_{n, n}: P_{m}\right\rangle, C_{n} \odot \overline{K_{2}}, n \geq$ 3, generalized antiprism $\mathcal{A}_{n}^{m}$ and the double triangular snake $D\left(T_{n}\right)$ are super mean graphs.


Key Words: Smarandachely super $m$-mean labeling, Smarandachely super $m$-mean graph, super mean labeling, super mean graph.

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## §1. Introduction

By a graph we mean a finite, simple and undirected one. The vertex set and the edge set of a graph $G$ are denoted by $V(G)$ and $E(G)$ respectively. The disjoint union of two graphs $G_{1}$ and $G_{2}$ is the graph $G_{1} \cup G_{2}$ with $V\left(G_{1} \cup G_{2}\right)=V\left(G_{1}\right) \cup V\left(G_{2}\right)$ and $E\left(G_{1} \cup G_{2}\right)=E\left(G_{1}\right) \cup E\left(G_{2}\right)$. The disjoint union of $m$ copies of the graph $G$ is denoted by $m G$. The corona $G_{1} \odot G_{2}$ of the graphs $G_{1}$ and $G_{2}$ is obtained by taking one copy of $G_{1}$ (with $p$ vertices) and $p$ copies of $G_{2}$ and then joining the $i^{t h}$ vertex of $G_{1}$ to every vertex in the $i^{t h}$ copy of $G_{2}$. Armed crown $C_{n} \Theta P_{m}$ is a graph obtained from a cycle $C_{n}$ by identifying the pendent vertex of a path $P_{m}$ at each vertex of the cycle. Bi-armed crown is a graph obtained from a cycle $C_{n}$ by identifying the pendant vertices of two vertex disjoint paths of equal length $m-1$ at each vertex of the cycle. We denote a bi-armed crown by $C_{n} \Theta 2 P_{m}$, where $P_{m}$ is a path of length $m-1$. The double triangular snake $D\left(T_{n}\right)$ is the graph obtained from the path $v_{1}, v_{2}, v_{3}, \ldots, v_{n}$ by joining $v_{i}$ and $v_{i+1}$ with two new vertices $i_{i}$ and $w_{i}$ for $1 \leq i \leq n-1$. The bistar $B_{m, n}$ is a graph obtained from

[^0]$K_{2}$ by joining $m$ pendant edges to one end of $K_{2}$ and $n$ pendant edges to the other end of $K_{2}$. The generalized prism graph $C_{n} \times P_{m}$ has the vertex set $V=\left\{v_{i}^{j}: 1 \leq i \leq n\right.$ and $\left.1 \leq j \leq m\right\}$ and the edge set $E=\left\{v_{i}^{j} v_{i+1}^{j}, v_{n}^{j} v_{1}^{j}: 1 \leq i \leq n-1\right.$ and $\left.1 \leq j \leq m\right\} \cup\left\{v_{i}^{j} v_{i-1}^{j+1}, v_{1}^{j} v_{n}^{j+1}: 2 \leq i \leq n\right.$ and $1 \leq j \leq m-1\}$. The generalized antiprism $\mathcal{A}_{n}^{m}$ is obtained by completing the generalized prism $C_{n} \times P_{m}$ by adding the edges $v_{i}^{j} v_{i}^{j+1}$ for $1 \leq i \leq n$ and $1 \leq j \leq m-1$. Terms and notations not defined here are used in the sense of Harary [1].

## §2. Preliminary Results

Let $G$ be a graph and $f: V(G) \rightarrow\{1,2,3, \cdots,|V|+|E(G)|\}$ be an injection. For each edge $e=u v$ and an integer $m \geq 2$, the induced Smarandachely edge $m$-labeling $f_{S}^{*}$ is defined by

$$
f_{S}^{*}(e)=\left\lceil\frac{f(u)+f(v)}{m}\right\rceil .
$$

Then $f$ is called a Smarandachely super m-mean labeling if $f(V(G)) \cup\left\{f^{*}(e): e \in E(G)\right\}=$ $\{1,2,3, \cdots,|V|+|E(G)|\}$. A graph that admits a Smarandachely super mean $m$-labeling is called Smarandachely super $m$-mean graph. Particularly, if $m=2$, we know that

$$
f^{*}(e)= \begin{cases}\frac{f(u)+f(v)}{2} & \text { if } f(u)+f(v) \text { is even } \\ \frac{f(u)+f(v)+1}{2} & \text { if } f(u)+f(v) \text { is odd }\end{cases}
$$

Such a labeling $f$ is called a super mean labeling of $G$ if $f(V(G)) \cup\left(f^{*}(e): e \in E(G)\right\}=$ $\{1,2,3, \ldots, p+q\}$. A graph that admits a super mean labeling is called a super mean graph. The concept of super mean labeling was introduced in [7] and further discussed in [2-6].

We use the following results in the subsequent theorems.
Theorem 2.1([7]) The bistar $B_{m, n}$ is a super mean graph for $m=n$ or $n+1$.
Theorem 2.2([2]) The graph $\left\langle B_{n, n}: w\right\rangle$, obtained by the subdivision of the central edge of $B_{n, n}$ with a vertex $w$, is a super mean graph.

Theorem 2.3([2]) The bi-armed crown $C_{n} \Theta 2 P_{m}$ is a super mean graph for odd $n \geq 3$ and $m \geq 2$.

Theorem 2.4([7]) Let $G_{1}=\left(p_{1}, q_{1}\right)$ and $G_{2}=\left(p_{2}, q_{2}\right)$ be two super mean graphs with super mean labeling $f$ and $g$ respectively. Let $f(u)=p_{1}+q_{1}$ and $g(v)=1$. Then the graph $\left(G_{1}\right)_{f} *\left(G_{2}\right)_{g}$ obtained from $G_{1}$ and $G_{2}$ by identifying the vertices $u$ and $v$ is also a super mean graph.

## §3. Super Mean Graphs

If $G$ is a graph, then $S(G)$ is a graph obtained by subdividing each edge of $G$ by a vertex.

Theorem 3.1 The graph $S\left(P_{n} \odot K_{1}\right)$ is a super mean graph.

Proof Let $V\left(P_{n} \odot K_{1}\right)=\left\{u_{i}, v_{i}: 1 \leq i \leq n\right\}$. Let $x_{i}(1 \leq i \leq n)$ be the vertex which divides the edge $u_{i} v_{i}(1 \leq i \leq n)$ and $y_{i}(1 \leq i \leq n-1)$ be the vertex which divides the edge $u_{i} u_{i+1}(1 \leq i \leq n-1)$. Then $V\left(S\left(P_{n} \odot K_{1}\right)\right)=\left\{u_{i}, v_{i}, x_{i}, y_{j}: 1 \leq i \leq n, 1 \leq j \leq n-1\right\}$.

Define $f: V\left(S\left(P_{n} \odot K_{1}\right)\right) \rightarrow\{1,2,3, \ldots, p+q=8 n-3\}$ by

$$
\begin{aligned}
& f\left(v_{1}\right)=1 ; f\left(v_{2}\right)=14 ; f\left(v_{2+i}\right)=14+8 i \text { for } 1 \leq i \leq n-4 \\
& f\left(v_{n-1}\right)=8 n-11 ; f\left(v_{n}\right)=8 n-10 ; f\left(x_{1}\right)=3 \\
& f\left(x_{1+i}\right)=3+8 i \text { for } 1 \leq i \leq n-2 ; f\left(x_{n}\right)=8 n-7 \\
& f\left(u_{1}\right)=5 ; f\left(u_{2}\right)=9 ; f\left(u_{2+i}\right)=9+8 i \text { for } 1 \leq i \leq n-3 ; \\
& f\left(u_{n}\right)=8 n-5 ; f\left(y_{i}\right)=8 i-1 \text { for } 1 \leq i \leq n-2 ; f\left(y_{n-1}\right)=8 n-3 .
\end{aligned}
$$

It can be verified that $f$ is a super mean labeling of $S\left(P_{n} \odot K_{1}\right)$. Hence $S\left(P_{n} \odot K_{1}\right)$ is a super mean graph.

Example 3.2 The super mean labeling of $S\left(P_{5} \odot K_{1}\right)$ is given in Fig.1.


Fig. 1

Theorem 3.2 The graph $S\left(P_{2} \times P_{n}\right)$ is a super mean graph.
Proof Let $V\left(P_{2} \times P_{n}\right)=\left\{u_{i}, v_{i}: 1 \leq i \leq n\right\}$. Let $u_{i}^{1}, v_{i}^{1}(1 \leq i \leq n-1)$ be the vertices which divide the edges $u_{i} u_{i+1}, v_{i} v_{i+1}(1 \leq i \leq n-1)$ respectively. Let $w_{i}(1 \leq i \leq n)$ be the vertex which divides the edge $u_{i} v_{i}$. That is $V\left(S\left(P_{2} \times P_{n}\right)\right)=\left\{u_{i}, v_{i}, w_{i}: 1 \leq i \leq n\right\} \cup\left\{u_{i}^{1}, v_{i}^{1}: 1 \leq i \leq n-1\right\}$.

Define $f: V\left(S\left(P_{2} \times P_{n}\right)\right) \rightarrow\{1,2,3, \ldots, p+q=11 n-6\}$ by

$$
\begin{aligned}
& f\left(u_{1}\right)=1 ; f\left(u_{2}\right)=9 ; f\left(u_{3}\right)=27 \\
& f\left(u_{i}\right)=f\left(u_{i-1}\right)+5 \text { for } 4 \leq i \leq n \text { and } i \text { is even } \\
& f\left(u_{i}\right)=f\left(u_{i-1}\right)+17 \text { for } 4 \leq i \leq n \text { and } i \text { is odd } \\
& f\left(v_{1}\right)=7 ; f\left(v_{2}\right)=16 ; \\
& f\left(v_{i}\right)=f\left(v_{i-1}\right)+5 \text { for } 3 \leq i \leq n \text { and } i \text { is odd } \\
& f\left(v_{i}\right)=f\left(v_{i-1}\right)+17 \text { for } 3 \leq i \leq n \text { and } i \text { is even } \\
& f\left(w_{1}\right)=3 ; f\left(w_{2}\right)=12 \\
& f\left(w_{2+i}\right)=12+11 i \text { for } 1 \leq i \leq n-2 \\
& f\left(u_{1}^{1}\right)=6 ; f\left(u_{2}^{1}\right)=24
\end{aligned}
$$

$$
\begin{aligned}
& f\left(u_{i}^{1}\right)=f\left(u_{i-1}^{1}\right)+6 \text { for } 3 \leq i \leq n-1 \text { and } i \text { isodd } \\
& f\left(u_{i}^{1}\right)=f\left(u_{i-1}^{1}\right)+16 \text { for } 3 \leq i \leq n-1 \text { and } i \text { iseven } \\
& f\left(v_{1}^{1}\right)=13 ; f\left(v_{i}^{1}\right)=f\left(v_{i-1}^{1}\right)+6 \text { for } 2 \leq i \leq n-1 \text { and } i \text { iseven } \\
& f\left(v_{i}^{1}\right)=f\left(v_{i-1}^{1}\right)+16 \text { for } 2 \leq i \leq n-1 \text { and } i \text { isodd. }
\end{aligned}
$$

It is easy to check that $f$ is a super mean labeling of $S\left(P_{2} \times P_{n}\right)$. Hence $S\left(P_{2} \times P_{n}\right)$ is a super mean graph.

Example 3.4 The super mean labeling of $S\left(P_{2} \times P_{6}\right)$ is given in Fig.2.


Fig. 2

Theorem 3.5 The graph $S\left(B_{n, n}\right)$ is a super mean graph.
Proof Let $V\left(B_{n, n}\right)=\left\{u, u_{i}, v, v_{i}: 1 \leq i \leq n\right\}$ and $E\left(B_{n, n}\right)=\left\{u u_{i}, v v_{i}, u v: 1 \leq i \leq\right.$ $n\}$. Let $w, x_{i}, y_{i},(1 \leq i \leq n)$ be the vertices which divide the edges $u v, u u_{i}, v v_{i}(1 \leq i \leq$ $n$ ) respectively. Then $V\left(S\left(B_{n, n}\right)\right)=\left\{u, u_{i}, v, v_{i}, x_{i}, y_{i}, w: 1 \leq i \leq n\right\}$ and $E\left(S\left(B_{n, n}\right)\right)=$ $\left\{u x_{i}, x_{i} u_{i}, u w, w v, v y_{i}, y_{i} v_{i}: 1 \leq i \leq n\right\}$.

Define $f: V\left(S\left(B_{n, n}\right)\right) \rightarrow\{1,2,3, \ldots, p+q=8 n+5\}$ by
$f(u)=1 ; f\left(x_{i}\right)=8 i-5$ for $1 \leq i \leq n ; f\left(u_{i}\right)=8 i-3$ for $1 \leq i \leq n ; f(w)=8 n+3$; $f(v)=8 n+5 ; f\left(y_{i}\right)=8 i-1$ for $1 \leq i \leq n ; f\left(v_{i}\right)=8 i+1$ for $1 \leq i \leq n$. It can be verified that $f$ is a super mean labeling of $S\left(B_{n, n}\right)$. Hence $S\left(B_{n, n}\right)$ is a super mean graph.

Example 3.6 The super mean labeling of $S\left(B_{n, n}\right)$ is given in Fig.3.


Fig. 3

Next we prove that the graph $\left\langle B_{n, n}: P_{m}\right\rangle$ is a super mean graph. $\left\langle B_{m, n}: P_{k}\right\rangle$ is a graph obtained by joining the central vertices of the stars $K_{1, m}$ and $K_{1, n}$ by a path $P_{k}$ of length $k-1$.

Theorem 3.7 The graph $\left\langle B_{n, n}: P_{m}\right\rangle$ is a super mean graph for all $n \geq 1$ and $m>1$.

Proof Let $V\left(\left\langle B_{n, n}: P_{m}\right\rangle\right)=\left\{u_{i}, v_{i}, u, v, w_{j}: 1 \leq i \leq n, 1 \leq j \leq m\right.$ with $\left.u=w_{1}, v=w_{m}\right\}$ and $E\left(\left\langle B_{n, n}: P_{m}\right\rangle\right)=\left\{u u_{i}, v v_{i}, w_{j} w_{j+1}: 1 \leq i \leq n, 1 \leq j \leq m-1\right\}$.

Case $1 \quad n$ is even.
Subcase $1 m$ is odd.

By Theorem 2.2, $\left\langle B_{n, n}: P_{3}\right\rangle$ is a super mean graph. For $m>3$, define $f: V\left(\left\langle B_{n, n}: P_{m}\right\rangle\right) \rightarrow$ $\{1,2,3, \ldots, p+q=4 n+2 m-1\}$ by

$$
\begin{aligned}
& f(u)=1 ; f\left(u_{i}\right)=4 i-1 \text { for } 1 \leq i \leq n \text { and for } i \neq \frac{n}{2}+1 \\
& f\left(u_{\frac{n}{2}+1}\right)=2 n+2 ; f\left(v_{i}\right)=4 i+1 \text { for } 1 \leq i \leq n ; f(v)=4 n+3 \\
& f\left(w_{2}\right)=4 n+4 ; f\left(w_{3}\right)=4 n+9 \\
& f\left(w_{3+i}\right)=4 n+9+4 i \text { for } 1 \leq i \leq \frac{m-5}{2} ; f\left(w_{\frac{m+3}{2}}\right)=4 n+2 m-4 \\
& f\left(w_{\frac{m+3}{2}+i}\right)=4 n+2 m-4-4 i \text { for } 1 \leq i \leq \frac{m-5}{2}
\end{aligned}
$$

It can be verified that $f$ is a super mean labeling of $\left\langle B_{n, n}: P_{m}\right\rangle$.
Subcase $2 m$ is even.
By Theorem 2.1, $\left\langle B_{n, n}: P_{2}\right\rangle$ is a super mean graph. For $m>2$, define $f: V\left(\left\langle B_{n, n}: P_{m}\right\rangle\right) \rightarrow$ $\{1,2,3, \ldots, p+q=4 n+2 m-1\}$ by

$$
\begin{aligned}
& f(u)=1 ; f\left(u_{i}\right)=4 i-1 \text { for } 1 \leq i \leq n \text { and for } i \neq \frac{n}{2}+1 \\
& f\left(u_{\frac{n}{2}+1}\right)=2 n+2 ; f\left(v_{i}\right)=4 i+1 \text { for } 1 \leq i \leq n ; f(v)=4 n+3 \\
& f\left(w_{2}\right)=4 n+4 ; f\left(w_{2+i}\right)=4 n+4+2 i \text { for } 1 \leq i \leq \frac{m-4}{2} \\
& f\left(w_{\frac{m+2}{2}}\right)=4 n+m+3 \\
& f\left(w_{\frac{m+2}{2}+i}\right)=4 n+m+3+2 i \text { for } 1 \leq i \leq \frac{m-4}{2}
\end{aligned}
$$

It can be verified that $f$ is a super mean labeling of $\left\langle B_{n, n}: P_{m}\right\rangle$.
Case $2 \quad n$ is odd.
Subcase $1 \quad m$ is odd.
By Theorem 2.1, $\left\langle B_{n, n}: P_{2}\right\rangle$ is a super mean graph. For $m>2$, define $f: V\left(\left\langle B_{n, n}: P_{m}\right\rangle\right) \rightarrow$

$$
\begin{aligned}
& \{1,2,3, \ldots, p+q=4 n+2 m-1\} \text { by } \\
& \qquad f(u)=1 ; f(v)=4 n+3 ; f\left(u_{i}\right)=4 i-1 \text { for } 1 \leq i \leq n \\
& f\left(v_{i}\right)=4 i+1 \text { for } 1 \leq i \leq n \text { and for } i \neq \frac{n+1}{2} \\
& f\left(v_{\frac{n+1}{2}}\right)=2 n+2 ; f\left(w_{2}\right)=4 n+4 \\
& f\left(w_{2+i}\right)=4 n+4+2 i \text { for } 1 \leq i \leq \frac{m-4}{2} ; f\left(w_{\frac{m+2}{2}}\right)=4 n+m+3 \\
& f\left(w_{\frac{m+2}{2}+i}\right)=4 n+m+3+2 i \text { for } 1 \leq i \leq \frac{m-4}{2}
\end{aligned}
$$

It can be verified that $f$ is a super mean labeling of $\left\langle B_{n, n}: P_{m}\right\rangle$.
Subcase $2 m$ is even.
By Theorem 2.2, $\left\langle B_{n, n}: P_{3}\right\rangle$ is a super mean graph. For $m>3$, define $f: V\left(\left\langle B_{n, n}: P_{m}\right\rangle\right) \rightarrow$ $\{1,2,3, \ldots, p+q=4 n+2 m-1\}$ by

$$
\begin{aligned}
& f(u)=1 ; f(v)=4 n+3 ; f\left(u_{i}\right)=4 i-1 \text { for } 1 \leq i \leq n ; \\
& f\left(v_{i}\right)=4 i+1 \text { for } 1 \leq i \leq n \text { and for } i \neq \frac{n+1}{2} ; f\left(v_{\frac{n+1}{2}}\right)=2 n+2 ; \\
& f\left(w_{2}\right)=4 n+4 ; f\left(w_{3}\right)=4 n+9 ; \\
& f\left(w_{3+i}\right)=4 n+9+4 i \text { for } 1 \leq i \leq \frac{m-5}{2} ; f\left(w_{\frac{m+3}{2}}\right)=4 n+2 m-4 ; \\
& f\left(w_{\frac{m+3}{2}+i}\right)=4 n+2 m-4-2 i \text { for } 1 \leq i \leq \frac{m-5}{2} .
\end{aligned}
$$

It can be verified that $f$ is a super mean labeling of $\left\langle B_{n, n}: P_{m}\right\rangle$. Hence $\left\langle B_{n, n}: P_{m}\right\rangle$ is a super mean graph for all $n \geq 1$ and $m>1$.

Example 3.8 The super mean labeling of $\left\langle B_{4,4}: P_{5}\right\rangle$ is given in Fig.4.


Fig. 4

Theorem 3.9 The corona graph $C_{n} \odot \overline{K_{2}}$ is a super mean graph for all $n \geq 3$.
Proof Let $V\left(C_{n}\right)=\left\{u_{1}, u_{2}, \ldots, u_{n}\right\}$ and $V\left(C_{n} \odot \overline{K_{2}}\right)=\left\{u_{i}, v_{i}, w_{i}: 1 \leq i \leq n\right\}$. Then $E\left(C_{n} \odot \overline{K_{2}}\right)=\left\{u_{i} u_{i+1}, u_{n} u_{1}, u_{j} v_{j}, u_{j} w_{j}: 1 \leq i \leq n-1\right.$ and $\left.1 \leq j \leq n\right\}$.

Case $1 \quad n$ is odd.

The proof follows from Theorem 2.3 by taking $m=2$.
Case $2 n$ is even.
Take $n=2 k$ for some $k$. Define $f: V\left(C_{n} \odot \overline{K_{2}}\right) \rightarrow\{1,2,3, \ldots, p+q=6 n\}$ by

$$
\begin{aligned}
& f\left(u_{i}\right)=6 i-3 \text { for } 1 \leq i \leq k-1 ; f\left(u_{k}\right)=6 k-2 \\
& f\left(u_{k+i}\right)=6 k-2+6 i \text { for } 1 \leq i \leq k-2 ; f\left(u_{2 k-1}\right)=12 k-2 \\
& f\left(u_{2 k}\right)=12 k-9 ; f\left(v_{i}\right)=6 i-5 \text { for } 1 \leq i \leq k-1 ; f\left(v_{k}\right)=6 k-6 \\
& f\left(v_{k+1}\right)=6 k+2 ; f\left(v_{k+1+i}\right)=6 k+2+6 i \text { for } 1 \leq i \leq k-3 ; f\left(v_{2 k-1}\right)=12 k ; \\
& f\left(v_{2 k}\right)=12 k-6 ; f\left(w_{i}\right)=6 i-1 \text { for } 1 \leq i \leq k-1 ; f\left(w_{k}\right)=6 k ; \\
& f\left(w_{k+i}\right)=6 k+6 i \text { for } 1 \leq i \leq k-2 ; f\left(w_{2 k-1}\right)=12 k-4 ; \\
& f\left(w_{2 k}\right)=12 k-11 .
\end{aligned}
$$

It can be verified that $f(V) \cup\left(f^{*}(e): e \in E\right\}=\{1,2,3, \ldots, 6 n\}$. Hence $C_{n} \odot \overline{K_{2}}$ is a super mean graph.

Example 3.10 The super mean labeling of $C_{8} \odot \overline{K_{2}}$ is given in Fig.5.


Fig. 5

Theorem 3.11 The double triangular snake $D\left(T_{n}\right)$ is a super mean graph.
Proof We prove this result by induction on $n$. A super mean labeling of $G_{1}=D\left(T_{2}\right)$ is given in Fig.6.


Fig. 6

Therefore the result is true for $n=2$. Let $f$ be the super mean labeling of $G_{1}$ as in the above figure. Now $D\left(T_{3}\right)=\left(G_{1}\right)_{f} *\left(G_{1}\right)_{f}$, by Theorem 2.4, $D\left(T_{3}\right)$ is a super mean graph. Therefore the result is true for $n=3$. Assume that $D\left(T_{n-1}\right)$ is a super mean graph with the super mean labeling $g$. Now by Theorem 2.4, $\left(D\left(T_{n-1}\right)\right)_{g} *\left(G_{1}\right)_{f}=D\left(T_{n}\right)$ is a super mean graph. Therefore the result is true for $n$. Hence by induction principle the result is true for all $n$. Thus $D\left(T_{n}\right)$ is a super mean graph.

Example 3.12 The super mean labeling of $D\left(T_{6}\right)$ is given in Fig.7.


Fig. 7

Theorem 3.13 The generalized antiprism $\mathcal{A}_{n}^{m}$ is a super mean graph for all $m \geq 2, n \geq 3$ except for $n=4$.

Proof Let $V\left(\mathcal{A}_{n}^{m}\right)=\left\{v_{i}^{j}: 1 \leq i \leq n, 1 \leq j \leq m\right\}$ and $E\left(\mathcal{A}_{n}^{m}\right)=\left\{v_{i}^{j} v_{i+1}^{j}, v_{n}^{j} v_{1}^{j}: 1 \leq i \leq\right.$ $n-1,1 \leq j \leq m\} \cup\left\{v_{i}^{j} v_{i-1}^{j+1}, v_{1}^{j} v_{n}^{j+1}: 2 \leq i \leq n, 1 \leq j \leq m-1\right\} \cup\left\{v_{i}^{j} v_{i}^{j+1}: 1 \leq i \leq n\right.$ and $1 \leq j \leq m-1\}$.

Case $1 \quad n$ is odd.
Define $f: V\left(\mathcal{A}_{n}^{m}\right) \rightarrow\{1,2,3, \ldots, p+q=4 m n-2 n\}$ by

$$
\begin{aligned}
& f\left(v_{i}^{j}\right)=4(j-1) n+2 i-1 \text { for } 1 \leq i \leq \frac{n+1}{2} \text { and } 1 \leq j \leq m \\
& f\left(v_{\frac{n+3}{2}}^{j}\right)=4(j-1) n+n+3 \text { for } 1 \leq j \leq m \\
& f\left(v_{\frac{n+3}{2}+i}^{j}\right)=4(j-1) n+n+3+2 i \text { for } 1 \leq i \leq \frac{n-3}{2} \text { and } 1 \leq j \leq m
\end{aligned}
$$

Then $f$ is a super mean labeling of $\mathcal{A}_{n}^{m}$. Hence $\mathcal{A}_{n}^{m}$ is a super mean graph.
Case $2 n$ is even and $n \neq 4$.
Define $f: V\left(\mathcal{A}_{n}^{m}\right) \rightarrow\{1,2,3, \ldots, p+q=4 m n-2 n\}$ by

$$
\begin{aligned}
& f\left(v_{1}^{j}\right)=4(j-1) n+1 \text { for } 1 \leq j \leq m ; f\left(v_{2}^{j}\right)=4(j-1) n+3 \text { for } 1 \leq j \leq m \\
& f\left(v_{3}^{j}\right)=4(j-1) n+7 \text { for } 1 \leq j \leq m ; f\left(v_{4}^{j}\right)=4(j-1) n+12 \text { for } 1 \leq j \leq m \\
& f\left(v_{4+i}^{j}\right)=4(j-1) n+12+4 i \text { for } 1 \leq j \leq m \text { and } 1 \leq i \leq \frac{n-6}{2} \\
& f\left(v_{\frac{n+2}{2}+i}^{j}\right)=4(j-1) n+2 n+1-4 i \text { for } 1 \leq j \leq m \text { and } 1 \leq i \leq \frac{n-6}{2} \\
& f\left(v_{n-1}^{j}\right)=4(j-1) n+9 \text { for } 1 \leq j \leq m ; f\left(v_{n}^{j}\right)=4(j-1) n+6 \text { for } 1 \leq j \leq m
\end{aligned}
$$

Then $f$ is a super mean labeling of $\mathcal{A}_{n}^{m}$. Hence $\mathcal{A}_{n}^{m}$ is a super mean graph.
Example 3.14 The super mean labeling of $\mathcal{A}_{6}^{3}$ is given in Fig.8.


Fig. 8

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