Two conjectures on Smarandache’s divisor products sequence

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Abstract. In this paper I make the following two conjectures on the Smarandache’s divisor products sequence where a term $P(n)$ of the sequence is defined as the product of the positive divisors of $n$: (1) there exist an infinity of $n$ composites such that the number $m = P(n) + n - 1$ is prime; (2) there exist an infinity of $n$ composites such that the number $m = P(n) - n + 1$ is prime.

The Smarandache’s divisor products sequence (see A007955 in OEIS):

\[
1, 2, 3, 8, 5, 36, 7, 64, 27, 100, 11, 1728, 13, 196, 225, 1024, 17, 5832, 19, 8000, 441, 484, 23, 331776, 125, 676, 729, 21952, 29, 810000, 31, 32768, 1089, 1156, 1225, 10077696, 37, 1444, 1521, 2560000, 41, 3111696, 43, 85184, 91125, 2116, 47, 254803968 (\ldots)
\]

Conjecture 1:

Let $P(n)$ be the Smarandache’s divisor products sequence where a term $P(n)$ of the sequence is defined as the product of the positive divisors of $n$: there exist an infinity of $n$ composites such that the number $m = P(n) + n - 1$ is prime.

Note that for $n$ primes, because $P(n) = n$, $P(n) + n - 1 = 2*n - 1$ and is already conjectured that there exist an infinity of primes of the form $2*q - 1$, where $q$ prime.

The sequence of primes $m$:

\[
\begin{align*}
& m = 3, \text{ prime, for } (n, P(n)) = (2, 2); \\
& m = 11, \text{ prime, for } (n, P(n)) = (4, 8); \\
& m = 41, \text{ prime, for } (n, P(n)) = (6, 36); \\
& m = 71, \text{ prime, for } (n, P(n)) = (8, 64); \\
& m = 109, \text{ prime, for } (n, P(n)) = (10, 100); \\
& m = 1739, \text{ prime, for } (n, P(n)) = (12, 1728); \\
& m = 239, \text{ prime, for } (n, P(n)) = (15, 225); \\
& m = 1039, \text{ prime, for } (n, P(n)) = (16, 1024); \\
& m = 5849, \text{ prime, for } (n, P(n)) = (18, 5832); \\
& m = 461, \text{ prime, for } (n, P(n)) = (21, 441); \\
& m = 149, \text{ prime, for } (n, P(n)) = (25, 125); \\
& m = 701, \text{ prime, for } (n, P(n)) = (26, 676); \\
& m = 1259, \text{ prime, for } (n, P(n)) = (35, 1225); \\
& m = 1481, \text{ prime, for } (n, P(n)) = (38, 1444); \\
& m = 2560039, \text{ prime, for } (n, P(n)) = (40, 2560000);
\end{align*}
\]
Examples of larger m:

- $m = 2161$, prime, for $(n, P(n)) = (46, 2116)$;

(...)

Note that $m$ is prime for $n = 12, 60, 96, 108, 168$. I conjecture that $m$ is prime for an infinity of $n$ of the form $12*k$.

**Conjecture 2:**

Let $P(n)$ be the Smarandache’s divisor products sequence where a term $P(n)$ of the sequence is defined as the product of the positive divisors of $n$: there exist an infinity of $n$ composites such that the number $m = P(n) - n + 1$ is prime.

Note that for $n$ primes, because $P(n) = n$, $P(n) - n + 1 = 1$.

The sequence of primes $m$:

- $m = 5$, prime, for $(n, P(n)) = (4, 8)$;
- $m = 31$, prime, for $(n, P(n)) = (6, 36)$;
- $m = 19$, prime, for $(n, P(n)) = (9, 27)$;
- $m = 211$, prime, for $(n, P(n)) = (15, 225)$;
- $m = 1009$, prime, for $(n, P(n)) = (16, 1024)$;
- $m = 421$, prime, for $(n, P(n)) = (21, 441)$;
- $m = 463$, prime, for $(n, P(n)) = (22, 484)$;
- $m = 331753$, prime, for $(n, P(n)) = (24, 331776)$;
- $m = 149$, prime, for $(n, P(n)) = (25, 125)$;
- $m = 1123$, prime, for $(n, P(n)) = (34, 1156)$;
- $m = 254803921$, prime, for $(n, P(n)) = (48, 254803968)$;

(...)

Examples of larger m:

- $m = 531440999911$, prime, for $(n, P(n)) = (90, 531441000000)$;
- $m = 389328928561$, prime, for $(n, P(n)) = (208, 389328928768)$.

Note that $m$ is prime for $n = 24, 48$. I conjecture that $m$ is prime for an infinity of $n$ of the form $12*k$. 